Week - 2

1. (a) $Y \in \{0, 1\}$ be the binary response
Parameters $\beta_0\ \beta_1\ \beta_2$
Predictors x_1, x_2

The model would be

$$P(Y=1 \mid x_1, x_2) = \frac{1}{1 + \exp(-1(\beta_0 + \beta_1 x_1 + \beta_2 x_2))}$$

(b) Likelihood fxn

$$L(\beta_0, \beta_1, \beta_2) = \prod p_i^{y_i} (1 - p_i)^{1 - y_i}$$

log-likelihood fxn

$$l(\beta_0, \beta_1, \beta_2) = \sum [y_i \log(p_i) + (1 - y_i) \log(1 - p_i)]$$

Question 2 Hours = 40 GPA = 3.5

linear predictor = $-6 \times 0.05 (40) + 1 (3.5) = -0.5$

$$P = \frac{1}{1 + e^{0.5}} = 0.377$$

(b) Hours required for 0.5 probability

$$0.5 = \frac{1}{1 + e^{x}} \qquad x = 0 \quad 1 + e^{x} = 2$$
$$e^{x} = 1 \quad x = 0$$

$0 = -6 + 0.05\ x_1 + 3.5$. $x_1 = 50$ hours

Q 3

dividend = 10 (μ_1)

No dividend = 0 (μ_2)

$\sigma^2 = 36$

$\pi_1 = 0.8$

$\pi_0 = 0.2$

log - odds

$$\log\left(\frac{P(Y=1|x)}{P(Y=0|x)}\right) = \log\left(\frac{\pi_1}{\pi_0}\right) + \left(\frac{\mu_1 - \mu_0}{\sigma^2}\right)x - \left(\frac{\mu_1^2 - \mu_0^2}{2\sigma^2}\right)$$

$$= \log(4) + \frac{10}{36}x - \frac{100}{72}$$

for $x = 4$ $= 1.108$

$$p = \frac{1}{1 + e^{-1.108}} \approx 0.75$$

Assignment - 3

Q 1