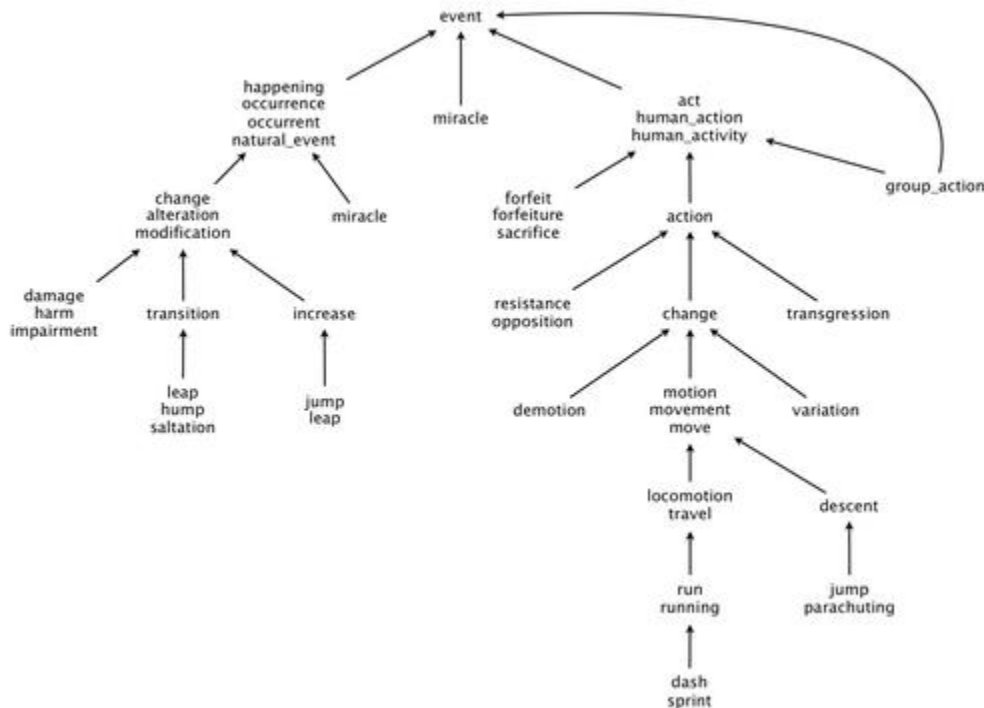


## History: WordNet

Suppose you had a graph between sets of nouns in which an arc  $A \rightarrow B$  indicates that the set of nouns  $A$  is a more specific case of the set of nouns  $B$ . Such a graph would allow us to reason about similarities and differences. The graph below is an example of what this might look like.



**Measuring the semantic relatedness of two nouns.** Semantic relatedness refers to the degree to which two concepts are related. Measuring semantic relatedness is a challenging problem. For example, you consider *George W. Bush* and *John F. Kennedy* (two U.S. presidents) to be more closely related than *George W. Bush* and *chimpanzee* (two primates). It might not be clear whether *George W. Bush* and *Eric Arthur Blair* are more related than two arbitrary people. However, both *George W. Bush* and *Eric Arthur Blair* (aka George Orwell) are famous communicators and, therefore, closely related.

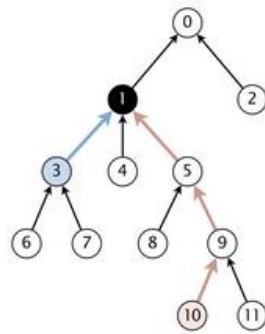
[WordNet](#) is a semantic lexicon for the English language that computational linguists and cognitive scientists use extensively. For example, WordNet was a key component in IBM's Jeopardy-playing [Watson](#) computer system.

## Your Assignment:

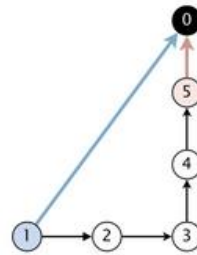
Given a directed acyclic graph, find the shortest path between any two nodes, the length of the path, and the shortest common ancestor.

**Shortest common ancestor.** An *ancestral path* between two vertices ( $v$  and  $w$ ) in a rooted DAG is a directed path from  $v$  to a common ancestor  $x$ , together with a directed path from  $w$  to the same ancestor  $x$ . A *shortest ancestral path* is an ancestral path of minimum total length. We refer to the common ancestor in a shortest ancestral path as a *shortest*

*common ancestor*. Note that a shortest common ancestor always exists because the root is an ancestor of every vertex. Note also that an ancestral path is a path, but not a directed path.

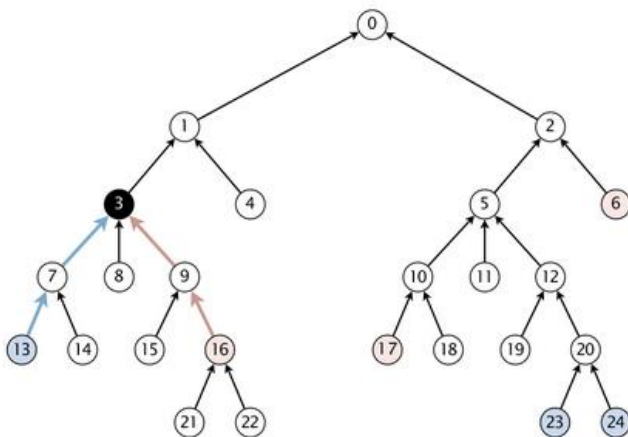


$v = 3, w = 10$   
 shortest ancestral path: 3-1-5-9-10  
 associated length: 4  
 shortest common ancestor: 1



$v = 1, w = 5$   
 ancestral path: 1-2-3-4-5  
 shortest ancestral path: 1-0-5  
 associated length: 2  
 shortest common ancestor: 0

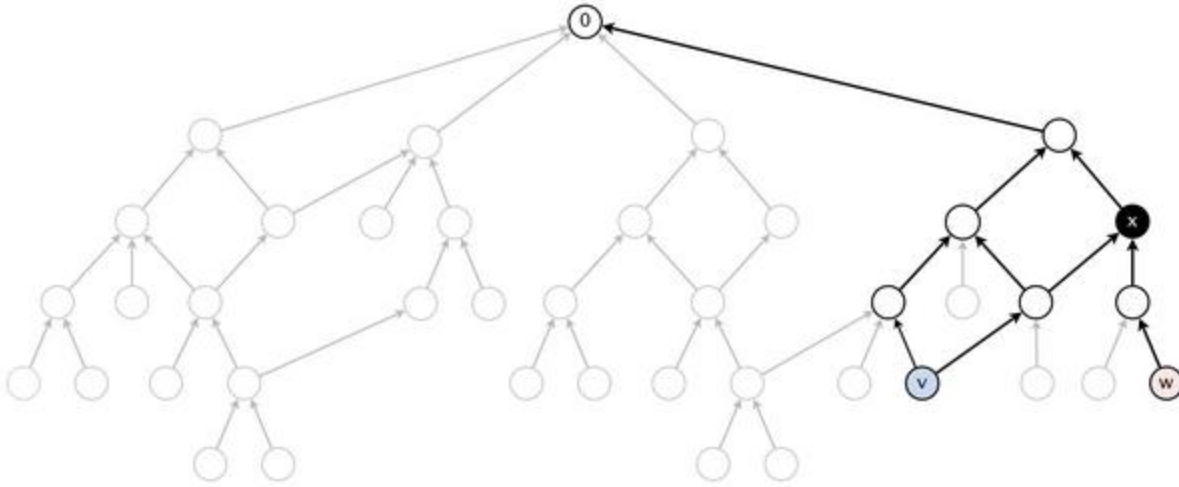
We generalize the notion of shortest common ancestor to *subsets* of vertices. A shortest ancestral path of two subsets of vertices  $A$  and  $B$  is a shortest ancestral path over all pairs of vertices  $v$  and  $w$ , with  $v$  in  $A$  and  $w$  in  $B$ .



$A = \{13, 23, 24\}, B = \{6, 16, 17\}$   
 ancestral path: 13-7-3-1-0-2-6  
 ancestral path: 23-20-12-5-10-17  
 ancestral path: 23-20-12-5-2-6

shortest ancestral path: 13-7-3-9-16  
 associated length: 4  
 shortest common ancestor: 3

**Additional performance requirements.** For full credit, the methods `length()` and `ancestor()` should take time proportional to the number of vertices and edges reachable from the argument vertices (or better). For example, to compute the shortest common ancestor of  $v$  and  $w$  in the digraph below, your algorithm should examine only the highlighted vertices and edges.



### Hint:

Every ancestor of a node  $v$  (in the above example) needs to know (1) the length of the shortest path to it from  $v$  (2) the predecessor responsible for that shortest path. If you do a breadth first traversal from  $v$  and recorded the best length, you wouldn't have to keep changing the length of the best path. Storing the predecessor help you it recreating the best path.

**BONUS feature: Outcast detection.** Given a list of nouns  $x_1, x_2, \dots, x_n$ , which noun is the least related to the others? To identify *an outcast*, compute the sum of the lengths between each noun and every other one:

$$d_i = \text{length}(x_i, x_1) + \text{length}(x_i, x_2) + \dots + \text{length}(x_i, x_n)$$

and return a noun  $x_i$  for which  $d_i$  is maximum. Note that because  $\text{length}(x_i, x_i) = 0$ , it will not contribute to the sum.

### Input:

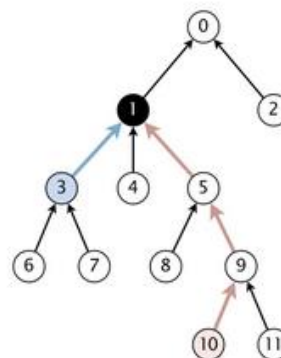
The input will be the number of nodes, number of edges, and each edge ( $A \rightarrow B$  is represented by  $A \ B$ )

### Testing:

- Allow the user to select which file he/she will use for input: digraph1.txt, digraph2.txt, or digraph3.txt. The first two files are provided. digraph3.txt is one you make up.
- All sets of nodes will be entered by a list of node numbers followed by a negative number.
- Allow the user to select from the following commands:
  - (10 points) Given two nodes in the graph, find the shortest common ancestor, shortest ancestral path, and associated length
  - (10 points) Given two subsets of nodes, find the shortest common ancestor, shortest ancestral path, and associated length
  - (5 points) BONUS: Given a set of nodes find the outcast

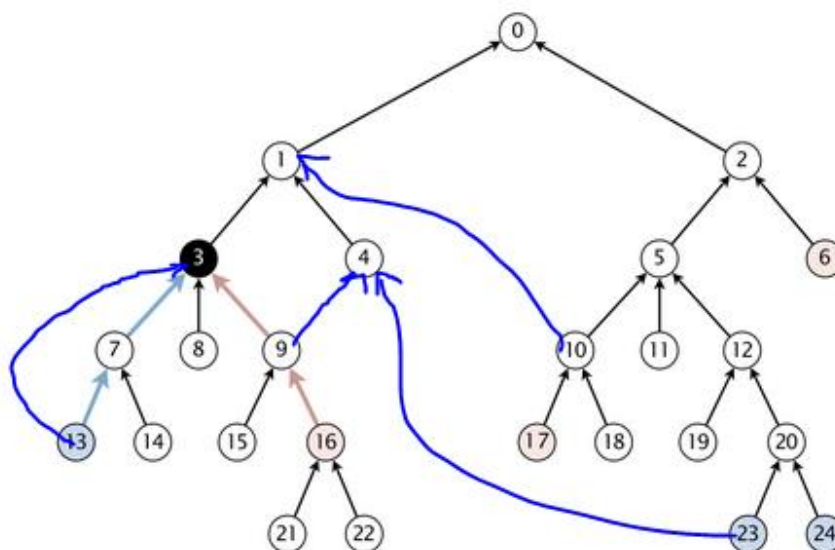
Here are the two provided input files and their associated graphs.

12  
11  
6 3  
7 3  
3 1  
4 1  
5 1  
8 5  
9 5  
10 9  
11 9  
1 0  
2 0



$v = 3, w = 10$   
shortest ancestral path: 3-1-5-9-10  
associated length: 4  
shortest common ancestor: 1

25  
28  
1 0  
2 0  
3 1  
4 1  
5 2  
6 2  
7 3  
8 3  
9 3  
10 6  
11 5  
12 6  
13 7  
14 7  
15 9  
16 9  
17 10  
18 10  
19 12  
20 12  
21 16  
22 16  
23 20  
24 20  
13 3  
9 4  
10 1  
23 4



ADDITIONAL BONUS (10 points) If you have time, try doing this for actual noun sets. See the instructions on the assignment page.