- Examples (8-bit word)

A= 0b10110011

B= 0b01101001

A&B= 0b00100001

A|B= 0b11111011

A^B= 0b11011010

~A= 0b01001100

A>>3= 0b00010110

A<<2= 0b11001100

Bitwise tricks

① Check if a number is even or odd

x & 1 ← 1 odd

0 even

Exp: x=9 bin(9) = 1001 9&1 = 1

x=6 bin(6) = 110 6&1 = 0

least significant bit

$x = 9$

x	1001
1	0001
$x \& 1$	0001

→ odd

$x = 6$

x	0110
1	0001
$x \& 1$	0000

→ even

② Check if a number is a power of 2.

$x \& (x-1)$ → 0 — power of 2
→ ≠0 — not a power of 2

≠ doesn't work if $x = 0$

```
bool powerof2 (int x) {
    return x && !(x & (x-1))
}
```

$x = 0001\,000000$

$x-1 = 0000\,111111$

$x \& (x-1) = 0000\,000000$

$x = 10$	$bin(10) =$	1010
$x-1 = 9$	$bin(9) =$	1001
$x \& (x-1)$		1000

③. Playing with the kth bit.

$(1 << k) = 2^k$ Kth bit = Kth least significant

a. Check if Kth bit is set $x \& (1 << k)$

b. Toggle the kth bit $x \wedge (1 << k)$

c. Set the kth bit $x \mid (1 << k)$

d. Unset the kth bit $x \& \sim(1 << k)$

Exp:

a. check if kth bit is set

$x = 100100111000$ (bit positions 6 5 4 3 2 1 0)

$\&$ ← kth bit from the right

0000001000 $k=3$

000 0000 0 1000

100100111000
000000000100 K=2

0

b. toggle the bit

x = 100100

100100 ^ 000100 = 100000

100100 ^ 001000 = 101100

c. set the bit

x = 100100

100100 OR 001000 = 101100

100100 AND 111011 = 100000

d. 8 = 001000

~8 = 110111

4. Multiply or Divide a number by 2^k

Multiplication: (x << k) Left shift

Division : (x >> k) Right shift

Exp:

- division: 10/2 = 5

9/2 = 4.5 = 4

bin(10) = 1010 (if we lightshift by 1)

it will become $2^2 + 2^0$ 101 => bin(5)

bin(9) = 1001

100 => 4

$x/2 \rightarrow x >> 1$

$x/4 \Rightarrow x >> 2$

$x/8 \rightarrow x >> 3$

$x / 2^k \rightarrow x >> k$

$x * 2^k \rightarrow x << k$

$x = 3 \qquad 3 \times 4 \Rightarrow 3 << 2$

$x = 3 \qquad bin(3) = 11$

binary

$(11) << 2 =$ binary $1100 \Rightarrow 12$

5. find out $x \% 2^k$

$x \& ((1 << k) - 1)$

Why does this work??

$2^k = 100000\ldots$ (k bits)

$2^{k-1} = 011111\ldots$ (k bits)

$10 \% 4 = 2$

$10 \% 2^2 = 2$

$10 \& (2^2 - 1) = 10 \& 3$

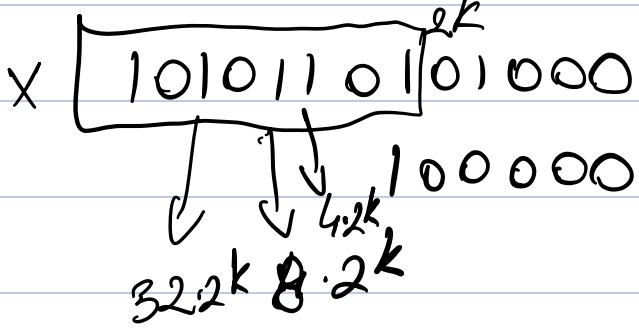
$bin(10) = 1010$
$bin(3) = 0011$
$\& = 0010 \Rightarrow 2$

How it works:

$2^k = 1000000\ldots$ (k bits)
$2^k - 1 = 0111111111\ldots$ (k bits)
$x \& (\downarrow)$
$x = 00011001110111 \&$

$$\frac{000000001111}{000001011}$$

$x \% 2^k$



$O(1)$

$x \;,\; 2^k$

$x \,\&\, (2^k - 1)$

6. Swapping 2 numbers x and y without a temporary variable.

$x = x \wedge y \quad \{ x = x \wedge y, y = y \}$

$y = x \wedge y \quad \{ x = x \wedge y, y = x \wedge y \wedge y = x \}$

$x = x \wedge y \quad \{ x = x \wedge y \wedge x = y, y = x \}$

Exp:

$x = 2,\ y = 5,\ z$

$z = x$

$x = y$

$y = z$

Exp using XOR :

$x = x \wedge y$

$x = 2 \wedge 5$, $x \Rightarrow 7$, $y \Rightarrow 5$

0010

0101

0111 ^ $= 7$

$y = x \wedge y$

$x = 7$, $y = 7 \wedge 5 \Rightarrow 2$

0111

0101

0010 ^ $= 2$

$x = x \wedge y$

$x = 7 \wedge 2$, $y = 2$

$\Rightarrow 5$

0111

0010

$0101 = 5$

7. Property : no. of set bits in $A = X$
no. of set bits in $B = y$
no. of set bits in $(A \wedge B) = z$

then

z is even if $x+y$ is even
z is odd if $x+y$ is odd

Exp:

$A = 5$ $\quad$ $B = 3$

$\downarrow$ $\quad$ $\downarrow$

101 XOR 011

$A \wedge B = 110 \Rightarrow 2$

no. of set bits 2

no. of set bits 2

Total = 4 set bits

even

4 => even

Exp:

A=6 , B=8

110 (2) 1000 (1) } 1110 odd

Exp:

A = (1100 1100 01) = x

B = (0100 1100 10) = y

A ^ B = x + y − 2

(x + y − 2k) set bits

8. if (X == A)
   X = B

↑ X = A ^ B ^ X

else if (X == B)
X = A

or just use this

Useful in a lot of if problems to code faster.

X 10
5

X = 10 ^ 5 ^ X

if X = 5 then we only have 10, if X = 10 then we only got 5.

if (X == 10)
X = 5
else if (X == 5)
X = 10

9. 2 important identities useful in some if problems.

a. $A + B = (A \wedge B) + 2(A \& B)$

b. $A + B = (A \mid B) + (A \& B)$

a.

$A = 1001 = 9 = 8 + 1$

$B = 0011 = 3 = 2 + 1$

$A + B = 8 + 2 + 2 \cdot (1)$

$A \wedge B = (1001) \wedge (0011) = 1010 =$
$= 8 + 2$

$A \& B = (1001) \& (0011) = 0001 = 1$

$A + B = (A \wedge B) + 2 \cdot (A \& B)$

b. $A = 9 \rightarrow 8 + 1 \rightarrow 1001$
$B = 3 \rightarrow 2 + 1 \rightarrow 0011$

$A | B = (1001) | (0011) = (1011)$
$= 8 + 2 + 1$

$A \& B = (1001) \& (0011) = (0001)$

(+1)

$$\Rightarrow A+B = (A \mid B) + (A \& B)$$

10. finding number of set bits in a number X

C++ O(1)

__builtin_popcount(x) X is an int
__builtin_popcountll(x) X is a long long