Continuous time Fourier series

$$x(t)=\sum_{k=-\infty}^{\infty} c_k^x e^{jk\omega_0 t} \longrightarrow c_k^x = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt,\quad \omega_0 = 2\pi/T$$

Table 4.1 Properties of exponential Fourier series

Periodic signal	Fourier coefficients
$x(t)=x(t+T),\ \ \omega_0=2\pi/T$	$\{c_k^x\}$
$y(t)=y(t+T),\ \ \omega_0=2\pi/T$	$\{c_k^y\}$
$ax(t)+by(t)$	$\{ac_k^x+bc_k^y\}$
$x(t-t_0),\quad t_0\in\mathbb{R}$	$\{e^{-jk\omega_0 t_0}c_k^x\}\quad \omega_0=2\pi/T$
$x(t)e^{jk_0\omega_0 t},\quad k_0\in\mathbb{Z},\ \ \omega_0=2\pi/T$	$\{c_{k-k_0}^x\}$
$x^*(t)$	$\{(c_{-k}^x)^*\}$
$x(-t)$	$\{c_{-k}^x\}$
$x(at),\quad a\in\mathbb{R}_+^*,\quad \text{period } T/a$	$\{c_k^x\}$
$x(t)\circledast y(t)=\int_T x(\tau)y(t-\tau)d\tau$	$\{Tc_k^x c_k^y\}$
$\frac{1}{T}\int_T x^*(\tau)y(t+\tau)d\tau$	$\{(c_k^x)^* c_k^y\}$
$x(t)y(t)$	$\{c_k^x * c_k^y\}=\left\{\sum_{n=-\infty}^{\infty} c_{k-n}^x c_n^y\right\}$
$\frac{dx(t)}{dt}$	$\{jk\omega_0 c_k^x\}$
$\int_{-\infty}^{t} x(\tau)d\tau$	$\left\{\frac{c_k^x}{jk\omega_0}\right\}\qquad c_0^x=0$
$x(t)\in\mathbb{R}$	$c_k^x=(c_{-k}^x)^*$ $\lvert c_k^x\rvert=\lvert c_{-k}^x\rvert;\qquad \operatorname{Arg} c_k^x=-\operatorname{Arg} c_{-k}^x$ $\operatorname{Re}\{c_k^x\}=\operatorname{Re}\{c_{-k}^x\};\quad \operatorname{Im}\{c_k^x\}=-\operatorname{Im}\{c_{-k}^x\}$
$x_e(t),\quad x(t)\in\mathbb{R}$	$\{\operatorname{Re} c_k^x\}$
$x_o(t),\quad x(t)\in\mathbb{R}$	$\{j\operatorname{Im} c_k^x\}$
$\frac{1}{T}\int_T \lvert x(t)\rvert^2 dt=\sum_{k=-\infty}^{\infty}\lvert c_k\rvert^2$	

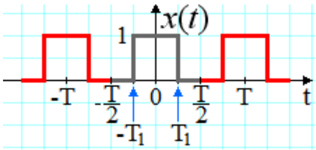
Continuous-time Fourier transform

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}d\omega \longrightarrow X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$$

Table 5.3 Properties of continuous-time Fourier transform

Aperiodic signal	Fourier transform
$x(t)$	$X(\omega)$
$y(t)$	$Y(\omega)$
$ax(t)+by(t)$	$aX(\omega)+bY(\omega)$
$x(t-t_0),\quad t_0\in\mathbb{R}$	$e^{-j\omega t_0}X(\omega)$
$e^{j\omega_0 t}x(t),\quad \omega_0\in\mathbb{R}$	$X(\omega-\omega_0)$
$x^*(t)$	$X^*(-\omega)$
$x(-t)=\breve{x}(t)$	$X(-\omega)$
$x(at),\quad a\in\mathbb{R}$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
$x(t)*y(t)$	$X(\omega)\cdot Y(\omega)$
$\breve{x}^*(t)*y(t)$	$X^*(\omega)\cdot Y(\omega)$
$x(t)y(t)$	$\frac{1}{2\pi}X(\omega)*Y(\omega)$
$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
$x(t)\in\mathbb{R}$	$X(\omega)=X^*(-\omega)$ $\lvert X(\omega)\rvert=\lvert X(-\omega)\rvert;\quad \mathrm{Arg}\,X(\omega)=-\mathrm{Arg}\,X(-\omega)$ $\mathrm{Re}\{X(\omega)\}=\mathrm{Re}\{X(-\omega)\};\quad \mathrm{Im}\{X(\omega)\}=-\mathrm{Im}\{X(-\omega)\}$
$x_e(t),\quad x(t)\in\mathbb{R}$	$\mathrm{Re}\{X(\omega)\}$
$x_o(t),\quad x(t)\in\mathbb{R}$	$j\,\mathrm{Im}\{X(\omega)\}$
$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega$	
$x(t)\longleftrightarrow X(\omega)$ $X(t)\longleftrightarrow 2\pi x(-\omega)$	

Table 5.4 Tables of Fourier transforms for different signals

Signal	Fourier transform	Exponential Fourier series (for periodic signals only)
$\sum_k c_k e^{jk\omega_0 t}$	$\sum_k 2\pi c_k \delta(\omega - k\omega_0)$	$\{c_k\}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$c_1 = 1;\ \ c_k = 0, k \neq 1$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$c_1 = c_{-1} = \frac{1}{2};$ $c_k = 0, k \notin \{-1,1\}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$c_1 = -c_{-1} = \frac{1}{2j};$ $c_k = 0, k \notin \{-1,1\}$
$x(t) = 1(\text{const})$	$2\pi\delta(\omega)$	$c_0 = 1;\ c_k = 0, k \neq 0$
	$\sum_{k=-\infty}^{\infty} \frac{2\sin k\omega_0 T_1}{k}\delta(\omega - k\omega_0)$	$c_k = \frac{\sin k\omega_0 T_1}{k\pi}, k \neq 0;$ $c_0 = \frac{2T_1}{T}$
$\delta_T(t) = \sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\omega_0\delta_{\omega_0}(t) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - k\frac{2\pi}{T}\right)$	$c_k = \frac{1}{T}$
$p_\tau(t) = \begin{cases} 1, & \lvert t\rvert < \tau \\ 0, & \lvert t\rvert > \tau \end{cases}$	$\frac{2\sin\omega\tau}{\omega}$	——
$\frac{\sin\omega_0 t}{\pi t}$	$p_{\omega_0}(\omega) = \begin{cases} 1, & \lvert\omega\rvert < \omega_0 \\ 0, & \lvert\omega\rvert > \omega_0 \end{cases}$	——
$\delta(t)$	$1(\text{const})$	——
$\delta(t-t_0),\quad t_0 \in \mathbb{R}$	$e^{-j\omega t_0}$	——
$\sigma(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	——
$e^{-at}\sigma(t), \text{Re}\{a\} > 0$	$(a+j\omega)^{-1}$	——
$te^{-at}\sigma(t), \text{Re}\{a\} > 0$	$(a+j\omega)^{-2}$	——
$\frac{t^{n-1}e^{-at}}{(n-1)!}\sigma(t), \text{Re}\{a\} > 0$	$(a+j\omega)^{-n}$	——

Discrete-time Fourier series

$$x[n]=\sum_{k=0}^{N-1}c_k e^{jk\Omega_0 n}, n=\overline{0,N-1}\longrightarrow c_k=\frac{1}{N}\sum_{n=0}^{N-1}x[n]e^{-jk\Omega_0 n}, k=\overline{0,N-1},\quad \Omega_0=\frac{2\pi}{N}$$

Table 6.1 Properties of discrete-time Fourier series

Discrete-time periodic signal	Exponential Fourier series
$x[n], x[n+N]=x[n]=x\left[(n)_N\right]$	$\left\{c_k^x\right\}; c_{k+N}^x=c_k^x=c_{(k)_N}^x$
$y[n], y[n+N]=y[n]=y\left[(n)_N\right]$	$\left\{c_k^y\right\}; c_{k+N}^y=c_k^y=c_{(k)_N}^y$
$ax[n]+by[n]$	$\left\{ac_k^x+bc_k^y\right\}$
$x\left[n-n_0\right], n_0\in\mathbb{Z}$	$\{e^{-jk\frac{2\pi}{N}n_0}c_k^x\}$
$x^*[n]$	$\{\left(c_{-k}^x\right)^*\}$
$x[-n]$	$\left\{c_{-k}^x\right\}$
$x_{(m)}[n]=\begin{cases}x[n/m]; & \text{if } \; n\vdots m\\ 0 \; ; & \text{otherwise}\end{cases}$, period mN	$\left\{\frac{1}{m}c_k^x\right\}$, period mN
$e^{jk_0\frac{2\pi}{N}n}x[n]$	$\left\{c_{k-k_0}^x\right\}$
$x[n]\circledast y[n]=\sum_{k\in\langle N\rangle}x[k]y[n-k]$	$\left\{Nc_k^x c_k^y\right\}$
$\frac{1}{N_0}\sum_{k=0}^{N_0}x^*[n]y[n+k]$	$\left\{\left(c_k^x\right)^*\cdot c_k^y\right\}$
$x[n]y[n]$	$\left\{c_k^x\circledast c_k^y\right\}=\{\sum_{m\in\langle N\rangle}c_{k-m}^x c_m^y\}$
$x[n]-x[n-1]$	$\left\{\left(1-e^{-j2\pi k/N}\right)c_k^x\right\}$
$\sum_{k=-\infty}^{n}x[k],\; c_0^x=0$	$\{c_k^x/(1-e^{-j2\pi k/N})\},\; c_0^x=0$
$x[n]\in\mathbb{R}$	$c_k=c_{-k}^*=c_{(-k)_N}^*$ $\lvert c_k\rvert=\lvert c_{-k}\rvert;\quad \operatorname{Arg}c_k=-\operatorname{Arg}c_{-k}$ $\operatorname{Re}\left\{c_k\right\}=\operatorname{Re}\left\{c_{-k}\right\};\quad \operatorname{Im}\left\{c_k\right\}=-\operatorname{Im}\left\{c_{-k}\right\}$
$x_e[n],\quad x[n]\in\mathbb{R}$	$\left\{\operatorname{Re}\left\{c_k^x\right\}\right\}$
$x_o[n],\quad x[n]\in\mathbb{R}$	$\left\{j\operatorname{Im}\left\{c_k^x\right\}\right\}$
$\frac{1}{N}\sum_{n\in\langle N\rangle}\lvert x[n]\rvert^2=\sum_{k\in\langle N\rangle}\lvert c_k\rvert^2$	

Discrete-time Fourier transform

$$x[n]=\frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{j\Omega n}d\Omega \longrightarrow X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

Table 6.2 Properties of the discrete-time Fourier transform

Discrete-time aperiodic signal	Exponential Fourier series
$x[n]$	$X(\Omega);\quad X(\Omega+2\pi)=X(\Omega)$
$y[n]$	$Y(\Omega);\quad Y(\Omega+2\pi)=Y(\Omega)$
$ax[n]+by[n]$	$aX(\Omega)+bY(\Omega)$
$x[n-n_0]$	$e^{-j\Omega n_0}X(\Omega)$
$e^{j\Omega_0 n}x[n]$	$X(\Omega-\Omega_0)$
$x^*[n]$	$X^*(-\Omega)$
$x[-n]$	$X(-\Omega)$
$x_{(k)}[n]=\begin{cases} x[n/k]; & \text{if } \; n \vdots k \\ 0 & ; \text{ otherwise}\end{cases}$	$X(k\Omega)$
$x[n]*y[n]$	$X(\Omega)Y(\Omega)$
$\breve{x}^*[n]*y[n]$	$X^*(\Omega)Y(\Omega)$
$x[n]y[n]$	$\frac{1}{2\pi}X(\Omega)\circledast Y(\Omega)=\frac{1}{2\pi}\int_{2\pi}X(u)Y(\Omega-u)du$
$x[n]-x[n-1]$	$\left(1-e^{-j\Omega}\right)X(\Omega)$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{X(\Omega)}{1-e^{-j\Omega}}+\pi X(0)\sum_{k=-\infty}^{\infty}\delta(\Omega-k2\pi)$
$nx[n]$	$j\frac{dX(\Omega)}{d\Omega}$
$x[n]\in\mathbb{R}$	$X(\Omega)=X^*(-\Omega);$ $\lvert X(\Omega)\rvert=\lvert X(-\Omega)\rvert;\qquad \operatorname{Arg}X(\Omega)=-\operatorname{Arg}X(-\Omega)$ $\operatorname{Re}\{X(\Omega)\}=\operatorname{Re}\{X(-\Omega)\};\ \operatorname{Im}\{X(\Omega)\}=-\operatorname{Im}\{X(-\Omega)\}$
$x_e[n],\quad x[n]\in\mathbb{R}$	$\operatorname{Re}\{X(\Omega)\}$
$x_o[n],\quad x[n]\in\mathbb{R}$	$j\operatorname{Im}\{X(\Omega)\}$
$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2=\frac{1}{2\pi}\int_{2\pi}\lvert X(\Omega)\rvert^2 d\Omega$	

Table 6.3 Tables of discrete-time Fourier transform for different signals

Signal	Fourier transform	Exponential Fourier series (periodic signals)
$\sum_{k\in\langle N\rangle} c_k e^{jk\frac{2\pi}{N}n}$	$\sum_{k\in\langle N\rangle} 2\pi c_k \delta(\Omega - k2\pi/N);\quad c_k = c_{(k)_N}$	$\{c_k\}$
$e^{j\Omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega-\Omega_0-k2\pi)$	For $\frac{\Omega_0}{2\pi}=\frac{m}{N}\in\mathbb{Q}:\{c_k\};c_k=\begin{cases}1, & (k)_N=m\\ 0, & \text{otherwise}\end{cases}$ For $\frac{\Omega_0}{2\pi}\notin\mathbb{Q}:$ the signal isn't periodic
$\cos\Omega_0 n$	$\sum_{k=-\infty}^{\infty} \pi\left[\delta(\Omega-\Omega_0-k2\pi)+\delta(\Omega+\Omega_0-k2\pi)\right]$	For $\frac{\Omega_0}{2\pi}=\frac{m}{N}\in\mathbb{Q}:\{c_k\},c_k=c_{-k}=\begin{cases}1/2, & (k)_N=m\\ 0, & \text{otherwise}\end{cases}$ For $\frac{\Omega_0}{2\pi}\notin\mathbb{Q}:$ the signal isn't periodic
$\sin\Omega_0 n$	$\sum_{k=-\infty}^{\infty} \frac{\pi}{j}\left[\delta(\Omega-\Omega_0-k2\pi)-\delta(\Omega+\Omega_0-k2\pi)\right]$	For $\frac{\Omega_0}{2\pi}=\frac{m}{N}\in\mathbb{Q}:c_k=c_{-k}=\begin{cases}\frac{1}{2j}, & k=m\pm pN\\ -\frac{1}{2j}, & k=-m\pm pN\\ 0, & \text{otherwise}\end{cases}$ For $\frac{\Omega_0}{2\pi}\notin\mathbb{Q}:$ the signal isn't periodic
$x[n]=1$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega-k2\pi)$	$c_k=\begin{cases}1, & (k)_N=m\\ 0, & \text{otherwise}\end{cases}$
$x[n]=\begin{cases}1, & \lvert n\rvert\le N_1\\ 0, & N_1\le\lvert n\rvert\le\frac{N}{2}\end{cases}$ $x[n+N]=x[n]$	$\sum_{k=-\infty}^{\infty} 2\pi\frac{\sin\left(\frac{2N_1+1}{2}\cdot\frac{2k\pi}{N}\right)}{N\sin\frac{2k\pi}{2N}}\cdot\delta\left(\Omega-k\frac{2\pi}{N}\right)$	$c_k=\frac{\sin\left(\frac{2N_1+1}{2}\cdot\frac{2k\pi}{N}\right)}{N\sin\frac{2k\pi}{2N}},(k)_N\neq 0;$ $c_k=\frac{2N_1+1}{2},\ (k)_N=0$

Signal	Fourier transform	Exponential Fourier series (periodic signals)
$\delta_N[n]=\sum_{k=-\infty}^{\infty}\delta[n-kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\Omega-k\frac{2\pi}{N}\right)$	$c_k=\frac{1}{N}$
$a^n\sigma[n],\quad \lvert a\rvert<1$	$\frac{1}{1-ae^{-j\Omega}}$	-
$x[n]=\begin{cases}1, & \lvert n\rvert\le N_1\\ 0, & \lvert n\rvert>N_1\end{cases}$	$\frac{\sin\left[\left(2N_1+1\right)/2\right]\Omega}{\sin\left(\Omega/2\right)}$	-
$\frac{\sin\Omega_0 n}{\pi n};\ 0<\Omega_0<\pi$	$X\left(\Omega\right)=\begin{cases}1, & \lvert\Omega\rvert\le\Omega_0\\ 0, & \Omega_0<\lvert\Omega\rvert\le\pi\end{cases},$ $X\left(\Omega+2\pi\right)=X\left(\Omega\right)$	-
$\delta[n]$	$1(\text{constant})$	-
$\sigma[n]$	$\frac{1}{1-e^{-j\Omega}}+\pi\sum_{k=-\infty}^{\infty}\delta\left(\Omega-2k\pi\right)$	-
$\delta[n-n_0]$	$e^{-j\Omega n_0}$	-
$\left(n+1\right)a^n\sigma[n];$ $\lvert a\rvert<1$	$\frac{1}{\left(1-ae^{-j\Omega}\right)^2}$	-
$\frac{\left(n+m-1\right)!}{n!\left(m-1\right)!}a^n\sigma[n];$ $\lvert a\rvert<1$	$\frac{1}{\left(1-ae^{-j\Omega}\right)^m}$	-

Laplace transform

$$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds, \mathrm{Re}\{s\}\in ROC \longrightarrow X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}dt;\ \ (s=\sigma+j\omega)$$ - bilateral Laplace transform

$$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds, \mathrm{Re}\{s\}>\sigma_0 \longrightarrow X_u(s)=\int_{0^+}^{\infty} x(t)e^{-st}dt;\ \ (s=\sigma+j\omega)$$ - unilateral Laplace transform

Table 7.1 Properties of Laplace transform

Signal	**Bilateral transform**		**Unilateral transform**
	Transform	**Region of convergence**	
$x(t)$	$X(s)$	ROC^x	$X_u(s)$
$y(t)$	$Y(s)$	ROC^y	$Y_u(s)$
$ax(t)+by(t)$	$aX(s)+bY(s)$	$ROC^x \cap ROC^y$ at least	$aX_u(s)+bY_u(s)$
$x(t-t_0),\ t_0\in\mathbb{R}$	$e^{-st_0}X(s)$	ROC^x	—
$x(t-t_0),\ t_0>0$	$e^{-st_0}X(s)$	ROC^x	$e^{-st_0}X_u(s)$
$e^{s_0t}x(t)$	$X(s-s_0)$	ROC^x shifted	$X_u(s-s_0)$
$x(at),\ \ a\in\mathbb{R}^*$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	ROC^x scaled	—
$x(at),\ \ a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$	ROC^x scaled	$\frac{1}{a}X_u\left(\frac{s}{a}\right)$
$x(t)*y(t)$	$X(s)Y(s)$	$ROC^x \cap ROC^y$ at least	$X_u(s)Y_u(s)$
$\frac{d}{dt}x(t)$	$sX(s)$	ROC^x at least	$sX_u(s)-x(0^+)$

Signal	Bilateral transform		Unilateral transform
	Transform	Region of convergence	
$-tx(t)$	$\frac{d}{ds}X(s)$	ROC^x	$\frac{d}{ds}X_u(s)$
$(-t)^n x(t)$	$\frac{d^n}{ds^n}X(s)$	ROC^x	$\frac{d^n}{ds^n}X_u(s)$
$x(t)y(t)$	$\frac{1}{2\pi j}\oint_\Gamma X(u)Y(s-u)du$	$\Gamma \subset ROC^x \cap ROC^y$ at least	$\frac{1}{2\pi j}\oint_\Gamma X(u)Y(s-u)du$
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	$ROC^x \cap \{\mathrm{Re}\{s\}>0\}$ at least	—
$\int_{0}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	$ROC^x \cap \{\mathrm{Re}\{s\}>0\}$ at least	$\frac{X_u(s)+x^{(-1)}(0)}{s}$ $x^{(-1)}(0)$ – initial value of integral
$x(t)=x(t)\sigma(t)$	$x(0^+)=\lim_{s\to\infty} sX(s)$		$x(0^+)=\lim_{s\to\infty} sX_u(s)$
$x(t)=x(t)\sigma(t)$	$x(\infty)=\lim_{s\to 0} sX(s)$		$x(\infty)=\lim_{s\to 0} sX_u(s)$

Table 7.2 Pairs signal-Laplace transform

Signal	Transform	Region of convergence
$\delta(t)$	$1(\text{constant})$	$\forall s$
$\sigma(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}>0$
$^{*}-\sigma(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}<0$
$\frac{t^{n-1}}{(n-1)!}\sigma(t)$	$\frac{1}{s^n}$	$\mathrm{Re}\{s\}>0$
$^{*}-\frac{t^{n-1}}{(n-1)!}\sigma(-t)$	$\frac{1}{s^n}$	$\mathrm{Re}\{s\}<0$
$e^{-at}\sigma(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}>-a$
$^{*}-e^{-at}\sigma(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}<-a$
$\frac{t^{n-1}}{(n-1)!}e^{-at}\sigma(t)$	$\frac{1}{(s+a)^n}$	$\mathrm{Re}\{s\}>-a$
$^{*}-\frac{t^{n-1}}{(n-1)!}e^{-at}\sigma(-t)$	$\frac{1}{(s+a)^n}$	$\mathrm{Re}\{s\}<-a$
$\delta(t-t_0);\ t_0>0$	e^{-st_0}	$\forall s$
$(\cos\omega_0 t)\sigma(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
$(\sin\omega_0 t)\sigma(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
$(e^{-at}\cos\omega_0 t)\sigma(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$
$(e^{-at}\sin\omega_0 t)\sigma(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$
$J_0(at)\sigma(t)$	$\frac{1}{\sqrt{s^2+a^2}}$	$\mathrm{Re}\{s\}>-\lvert a\rvert$

For causal signals, $X(s)=X_u(s)$, and for non-causal signals only $X(s)$ exists.

Z transform

$$x[n]=\frac{1}{2\pi j}\oint_{\Gamma} X(z)z^{n-1}dz,\ \ \Gamma\subset ROC \longrightarrow X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$$ - bilateral Z transform,

$$z=r\cdot e^{j\Omega},\ \ r\geq 0$$

$$x[n]\longrightarrow X_u(z)=\sum_{n=0}^{\infty} x[n]z^{-n}$$ - unilateral Z transform

Table 10.1 Properties of the Z transform

The signals marked by $*$ correspond to the unilateral transform. Otherwise the properties of the two transforms are identical.

Signal	Z transform	Region of convergence
$x[n]$	$X(z)$	$R_-^x<\lvert z\rvert<R_+^x$
$y[n]$	$Y(z)$	$R_-^y<\lvert z\rvert<R_+^y$
$ax[n]+by[n]$	$aX(z)+bY(z)$	$\max\{R_-^x,R_-^y\}<\lvert z\rvert$ $\lvert z\rvert<\min\{R_+^x,R_+^y\}$
$x[n-n_0],\ n_0\in\mathbb{Z}$	$z^{-n_0}X(z)$	$R_-^x<\lvert z\rvert<R_+^x$
$*\ x[n-n_0],\ n_0\in\mathbb{N}$	$z^{-n_0}\left(X(z)+\sum_{n=-n_0}^{-1}x[n]z^{-n}\right)$	$R_-^x<\lvert z\rvert$
$*\ x[n+n_0],\ n_0\in\mathbb{N}$	$z^{n_0}\left(X(z)-\sum_{n=0}^{n_0-1}x[n]z^{-n}\right)$	$R_-^x<\lvert z\rvert$
$e^{j\Omega_0 n}x[n]$	$X\left(e^{-j\Omega_0}z\right)$	$R_-^x<\lvert z\rvert<R_+^x$
$x[-n]$	$X(1/z)$ only for the bilateral transform	$\frac{1}{R_+^x}<\lvert z\rvert<\frac{1}{R_-^x}$
$x[n]-x[n-1]$	$\left(1-z^{-1}\right)X(z)$	$R_-^x<\lvert z\rvert<R_+^x$
$*\ x[n]-x[n-1]$	$\left(1-z^{-1}\right)X(z)-x[-1]$	$R_-^x<\lvert z\rvert$
$\sum_{k=-\infty}^{n}x[k]$	$\frac{X(z)}{1-z^{-1}}$	$R_-^x<\lvert z\rvert<R_+^x$
$*\ \sum_{k=-\infty}^{n}x[k]$	$\frac{X(z)+\sum_{k=-\infty}^{-1}x[k]}{1-z^{-1}}$	$R_-^x<\lvert z\rvert$
$nx[n]$	$-z\frac{d}{dz}X(z)$	$R_-^x<\lvert z\rvert<R_+^x$
$x[n];\ x[n]\equiv 0\ n<0$	$\lim_{z\to\infty}X(z)=x[0]$	$R_-^x<\lvert z\rvert$
$x[n];\ x[n]\equiv 0\ n<0$	$\lim_{z\to 1}(z-1)X(z)=x[\infty]$	$R_-^x<\lvert z\rvert$
$x^*[n]$	$X^*\left(z^*\right)$	$R_-^x<\lvert z\rvert<R_+^x$

Signal	Z transform	Region of convergence
$x[n] * y[n]$	$X(z)Y(z)$	$\max\{R_-^x, R_-^y\} < \lvert z\rvert$ $\lvert z\rvert < \min\{R_+^x, R_+^y\}$
$x[n]y[n]$	$\frac{1}{2\pi j}\oint_\Gamma X(u)Y\left(\frac{z}{u}\right)\frac{du}{u}$	$R_-^x R_-^y < \lvert z\rvert < R_+^x R_+^y$
$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2 = \frac{1}{2\pi j}\oint_\Gamma X(u)X^*(1/u^*)\frac{du}{u}$		

Table 10.2 Pairs signal-Z transform

With the exception of the signals marked in the table with $*$ that do not have unilateral transform, the two transforms (unilateral and bilateral) are identical.

Signal	Z transform	Region of convergence
$\delta[n]$	1 (constant)	$\forall z \in \mathbb{C}$
$\sigma[n]$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$\lvert z\rvert > 1$
$^*-\sigma[-n-1]$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$\lvert z\rvert < 1$
$\delta[n-n_0]$	z^{-n_0}	$\forall z \in \mathbb{C}\setminus\{0\}$ if $n_0 > 0$ or $\forall z \in \mathbb{C}\setminus\{\infty\}$ if $n_0 < 0$
$a^n\sigma[n]$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$^*\ -a^n\sigma[-n-1]$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$\lvert z\rvert < \lvert a\rvert$
$na^n\sigma[n]$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^2} = \frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
$^*\ -na^n\sigma[-n-1]$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^2} = \frac{az}{(z-a)^2}$	$\lvert z\rvert < \lvert a\rvert$
$\sigma[n]\cos\Omega_0 n$	$\frac{1-z^{-1}\cos\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}} = \frac{z(z-\cos\Omega_0)}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert > 1$
$\sigma[n]\sin\Omega_0 n$	$\frac{z^{-1}\sin\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}} = \frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert > 1$
$\sigma[n]a^n\cos\Omega_0 n$	$\frac{1-z^{-1}a\cos\Omega_0}{1-2z^{-1}a\cos\Omega_0+a^2z^{-2}} = \frac{z(z-a\cos\Omega_0)}{z^2-2za\cos\Omega_0+a^2}$	$\lvert z\rvert > \lvert a\rvert$
$\sigma[n]a^n\sin\Omega_0 n$	$\frac{z^{-1}a\sin\Omega_0}{1-2z^{-1}a\cos\Omega_0+a^2z^{-2}} = \frac{za\sin\Omega_0}{z^2-2za\cos\Omega_0+a^2}$	$\lvert z\rvert > \lvert a\rvert$