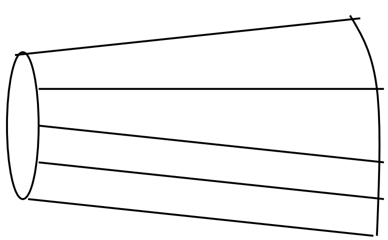
Three Dimensional Representation

Polygon Surface

- It is the most common representation for 3D graphics object.
- In this representations, a 3D object is represented by a set of surfaces that enclose the object interior.
- Many graphics system use this method. Set of polygons are stored for object description. This simplifies and speeds up the surface rendering and display of object since all surfaces can be described with linear equations.
- Used in design and solid modeling application



A 3D object represented by polygons

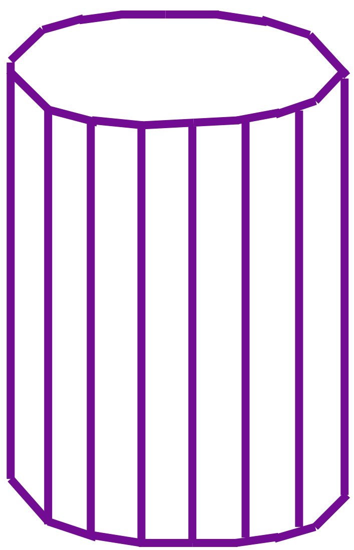
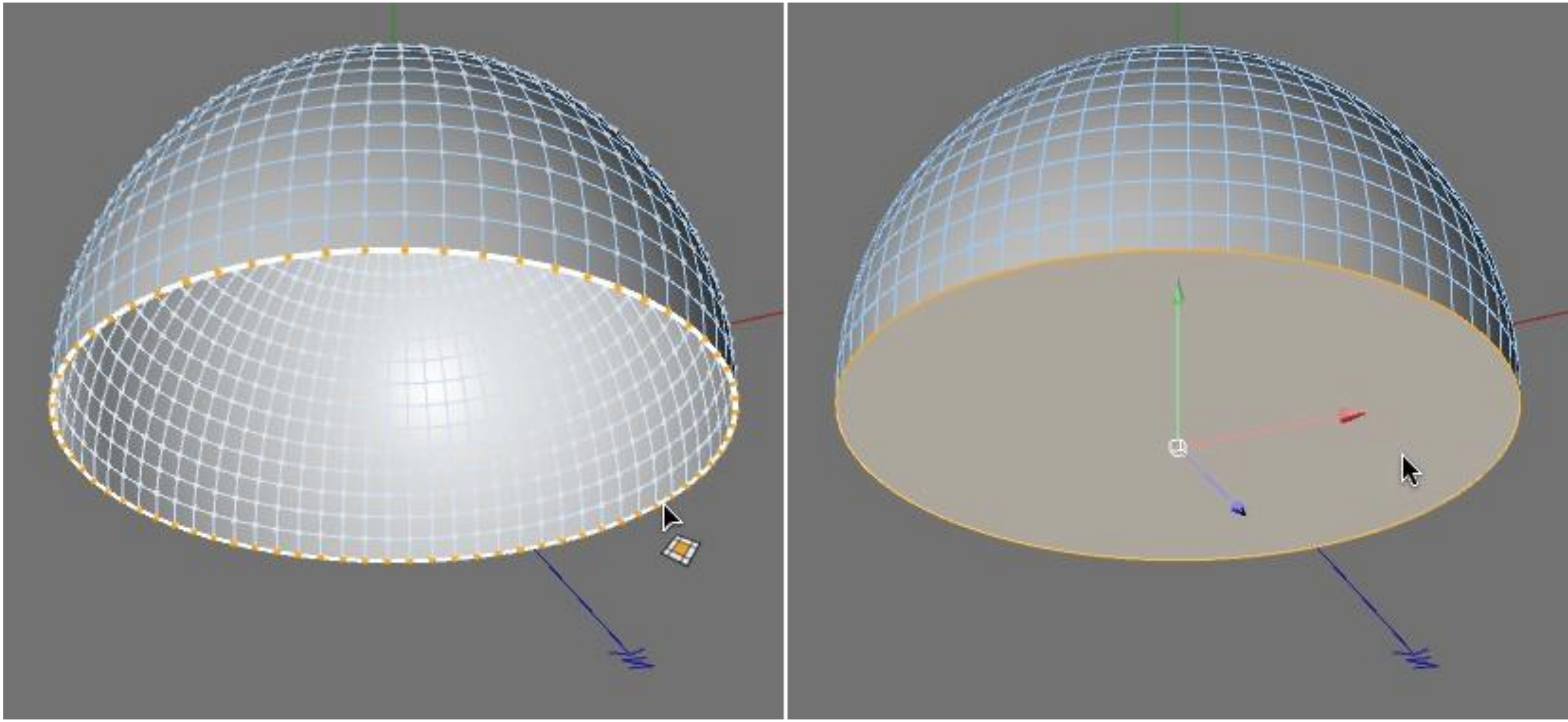
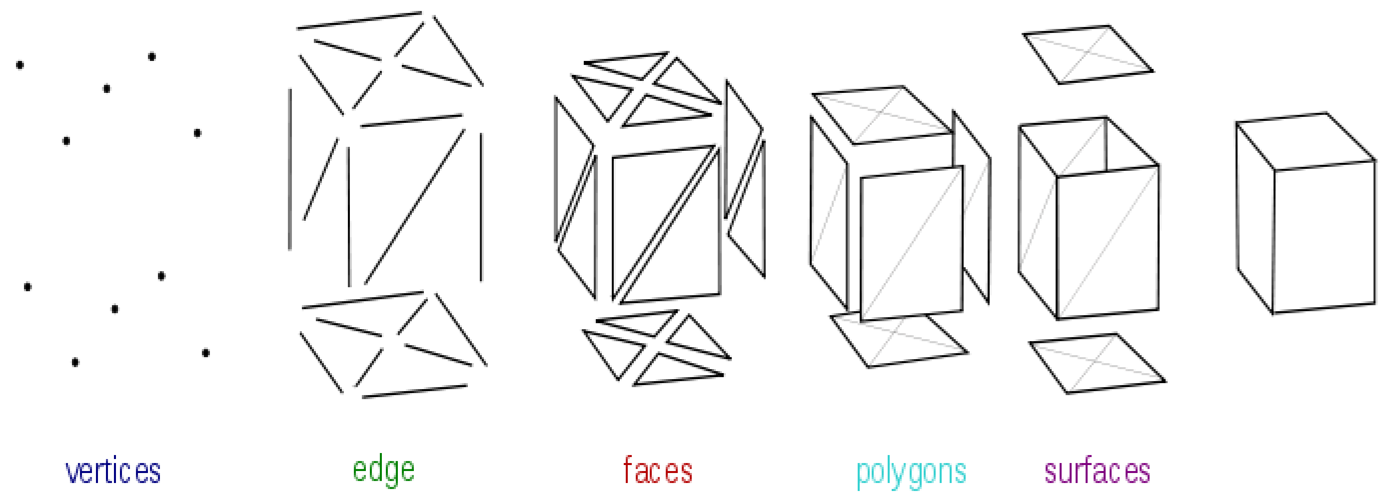


Figure: Polygon Surface Approximation of Cylinder



3-D Object Representations

- Graphical scenes can contain many different kinds of objects like trees, flowers, rocks, waters...etc.
- There is no one method that we can use to describe objects that will include all features of those different materials.
- To produce realistic display of scenes, we need to use representations that accurately model object characteristics.
- Representation schemes for solid objects are often divided into two broad categories:
  - **Boundary representations (B-reps)**: describes a 3D object as a set of polygonal surfaces, separate the object interior from environment.
-
  - **Space-partitioning representation**: used to describe interior properties, by partitioning the spatial region, containing an object into a set of small, non overlapping, contiguous solids. e.g. 3D object as Octree representation.


vertices
edge
faces
polygons
surfaces

Polygon Tables

- A polygon surface is specified with a **set of vertex co-ordinates** and associated **attribute parameters**.
- Organized into two groups:

Geometric Data Table

  – Contains vertex coordinates and parameters to identify the spatial orientation of polygon surface

Attributes Tables:

  – Includes parameters specifying the degree of transparency of the object and its surface reflectivity and texture characteristics.

- A convenient organization for storing geometric data is to create 3 lists:

1. A vertex table
2. An edge table
3. A polygon surface table.

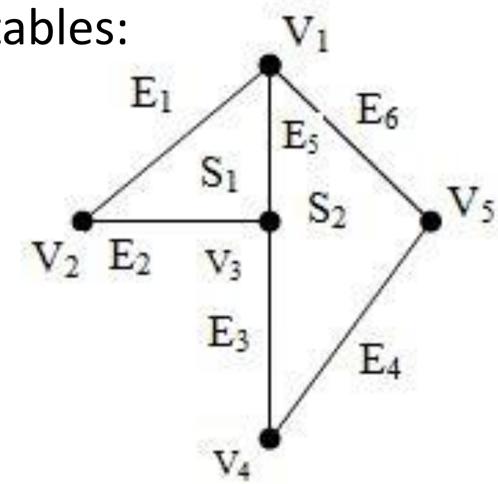
- ➢ Vertex table stores co-ordinates of each vertex in the object.
- ➢ The edge table stores the Edge information of each edge of polygon facets.
- ➢ The polygon surface table stores the surface information for each surface i.e. each surface is represented by edge lists of polygon.

- Consider the surface contains polygonal facets as shown in figure (only two polygon are taken here)
- S_1 and S_2 are two polygon surface that represent the boundary of some 3D object.
- For storing geometric data, we can use following three tables:

VERTEX TABLE
V_1: x_1,y_1,z_1
V_2: x_2,y_2,z_2
V_3: x_3,y_3,z_3
V_4: x_4,y_4,z_4
V_5: x_5,y_5,z_5

EDGE TABLE
E_1: V_1,V_2
E_2: V_2,V_3
E_3: V_3,V_4
E_4: V_4,V_5
E_5: V_1,V_3
E_6: V_5,V_1

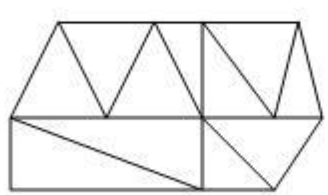
POLYGON SURFACE TABLE
S_1: E_1,E_2,E_3
S_2: E_3,E_4,E_5,E_6



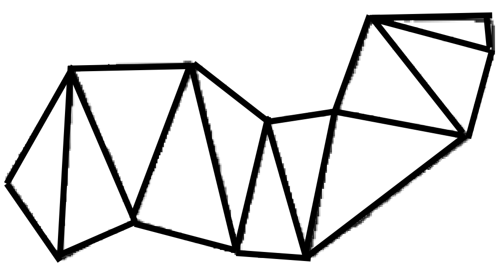
- The object can be displayed efficiently by using data from tables and processing them for surface rendering and visible surface determination.

Polygon Meshes

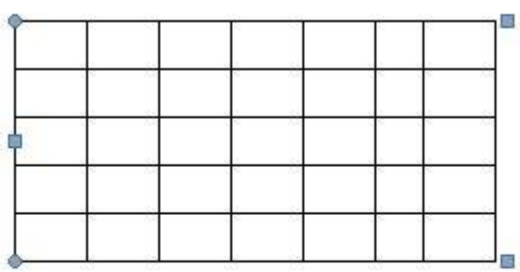
- A polygon mesh is collection of edges, vertices and polygons connected such that each edge is shared by at most two polygons. An edge connects two vertices and a polygon is a closed sequence of edges. An edge can be shared by two polygons and a vertex is shared by at least two edges.
- When object surface is to be tiled, it is more convenient to specify the surface facets with a mesh function. One type of polygon mesh is triangle strip. This function produce n-2 connected triangles., for n coordinate vertices.



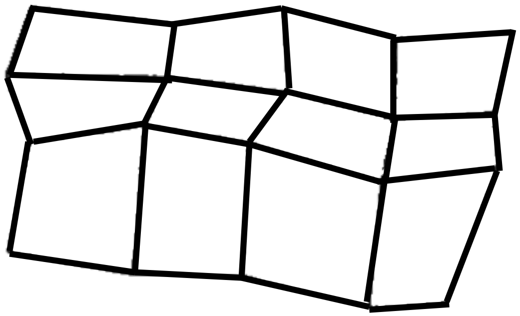
Triangular Mesh



- Another similar function is the quadrilateral mesh, which generates a mesh of (n-1) by (m-1) quadrilaterals, given the co-ordinates for an n *m array of vertices.



6 by 8 vertices array , 35 element quadrilateral mesh



Plane Equation

- Plane equation method is another method for representation the polygon surface for 3D object.
- The information about the spatial orientation of object is described by its individual surface, which is obtained by the vertex co-ordinates and the equation of each surface.
- The equation for a plane surface can be expressed in the form,

$$Ax + By + Cz + D = 0$$

where (x,y,z) is any point on the plane, and A,B,C,D are constants describing the spatial properties of the plane. The values of A,B,C,D can be obtained by solving a set of three plane equations using co-ordinate values of 3 non collinear points on the plane.

- Let (x_1,y_1,z_1), (x_2,y_3,z_2) and (x_3,y_3,z_3) are three such points on the plane, then-

$$Ax_1 + By_1 + Cz_1 + D = 0$$
$$Ax_2 + By_2 + Cz_2 + D = 0$$
$$Ax_3 + By_3 + Cz_3 + D = 0$$

- The solution of these equations can be obtained in determinant from using Cramer's rule as:-

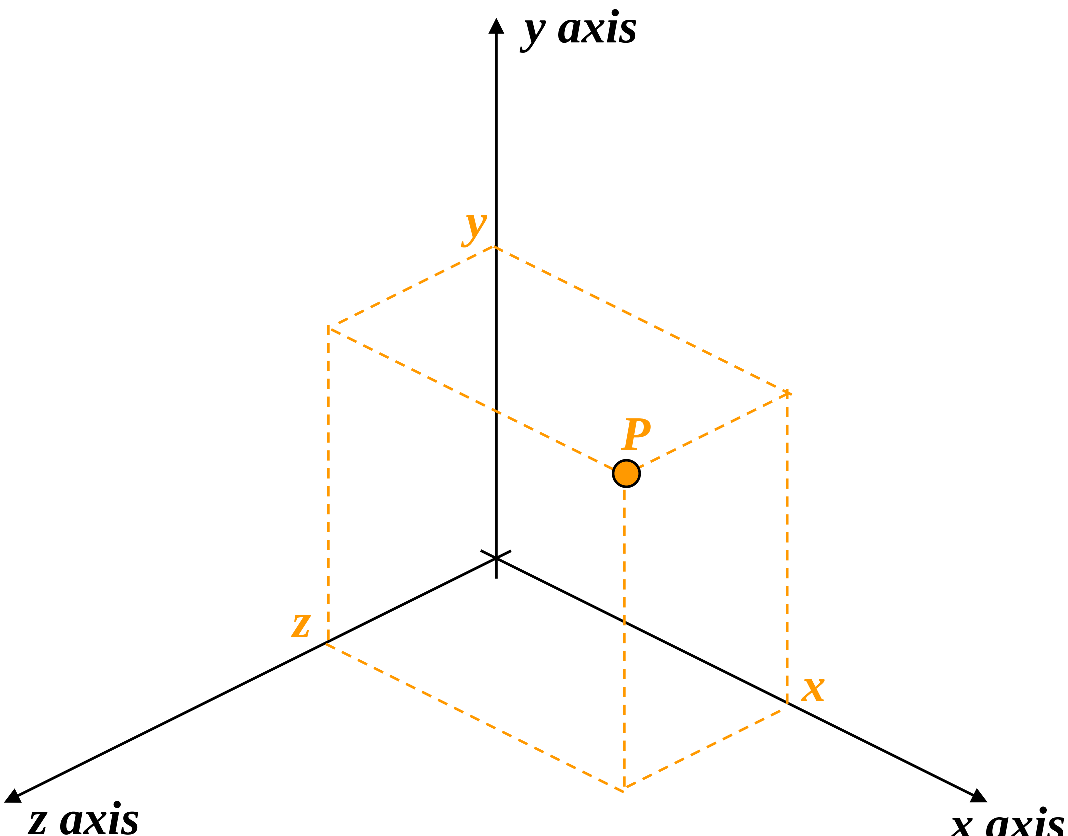
$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \qquad B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix}$$

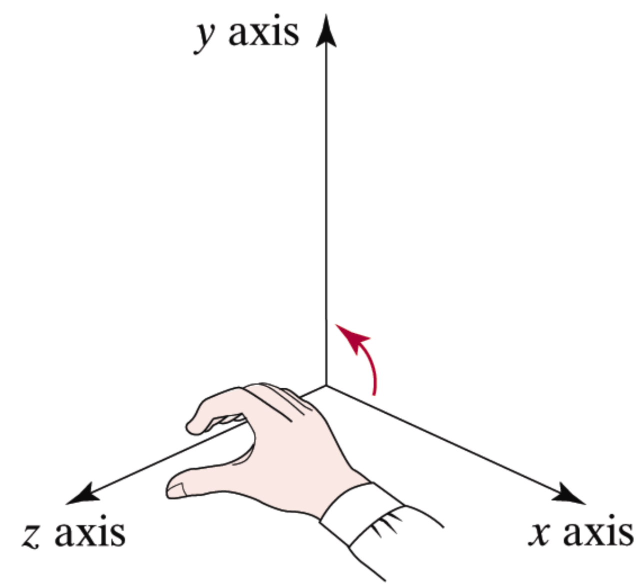
$$C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

For any points (x,y,z)

- If $Ax + By + Cz + D \neq 0$, then (x,y,z) is not on the plane.
- If $Ax + By + Cz + D < 0$, then (x,y,z) is inside the surface.
- If $Ax + By + Cz + D > 0$, then (x,y,z) is lies out side the surface.

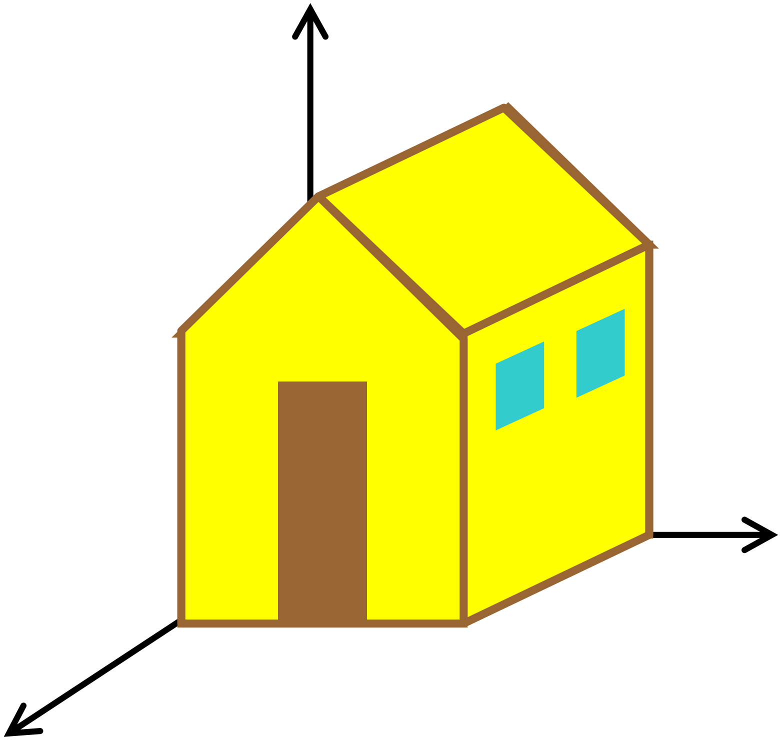
3D coordinates space



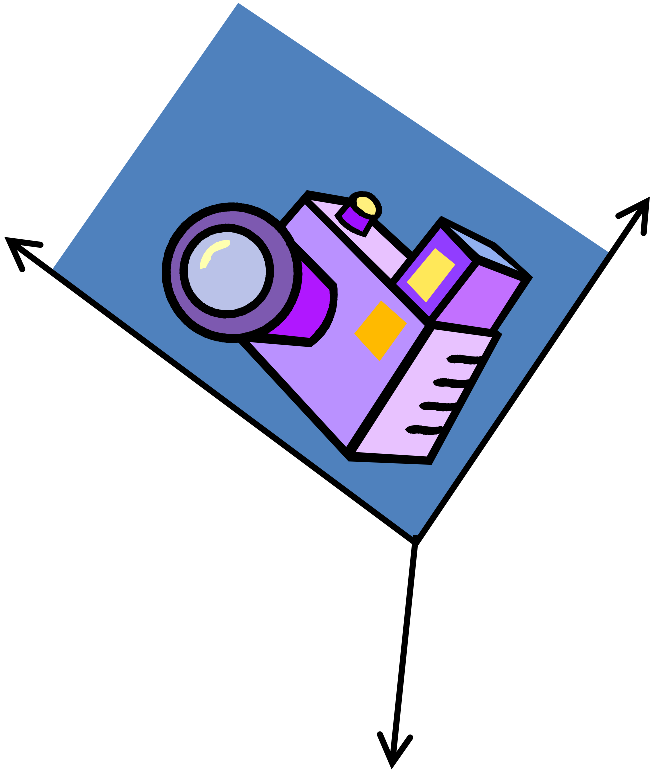


Right-Hand
Reference System

World Coordinate System

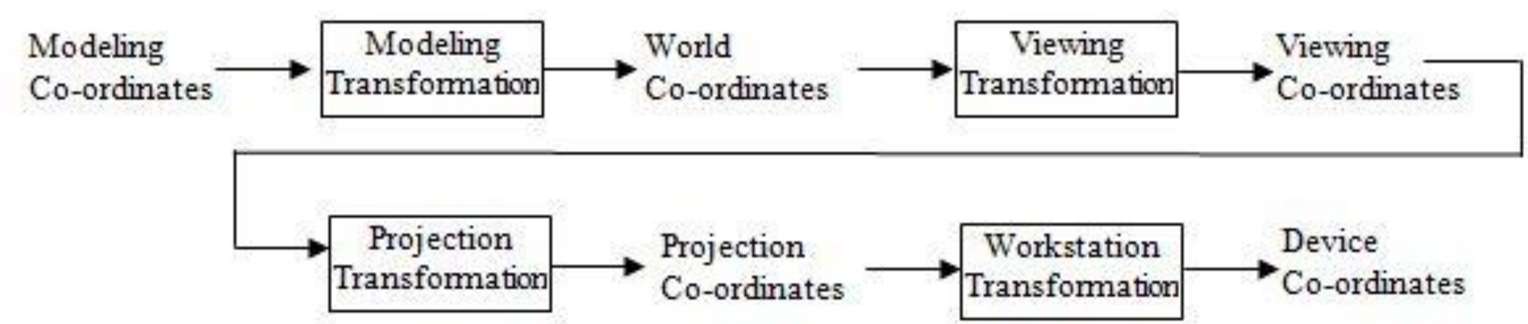


Viewing Coordinate System



3D Viewing pipeline

- The steps for computer generation of a view of 3D scene are analogous to the process of taking photograph by a camera. For a snapshot, we need to position the camera at a particular point in space and then need to decide camera orientation. Finally when we snap the shutter, the seen is cropped to the size of window of the camera and the light from the visible surfaces is projected into the camera film.



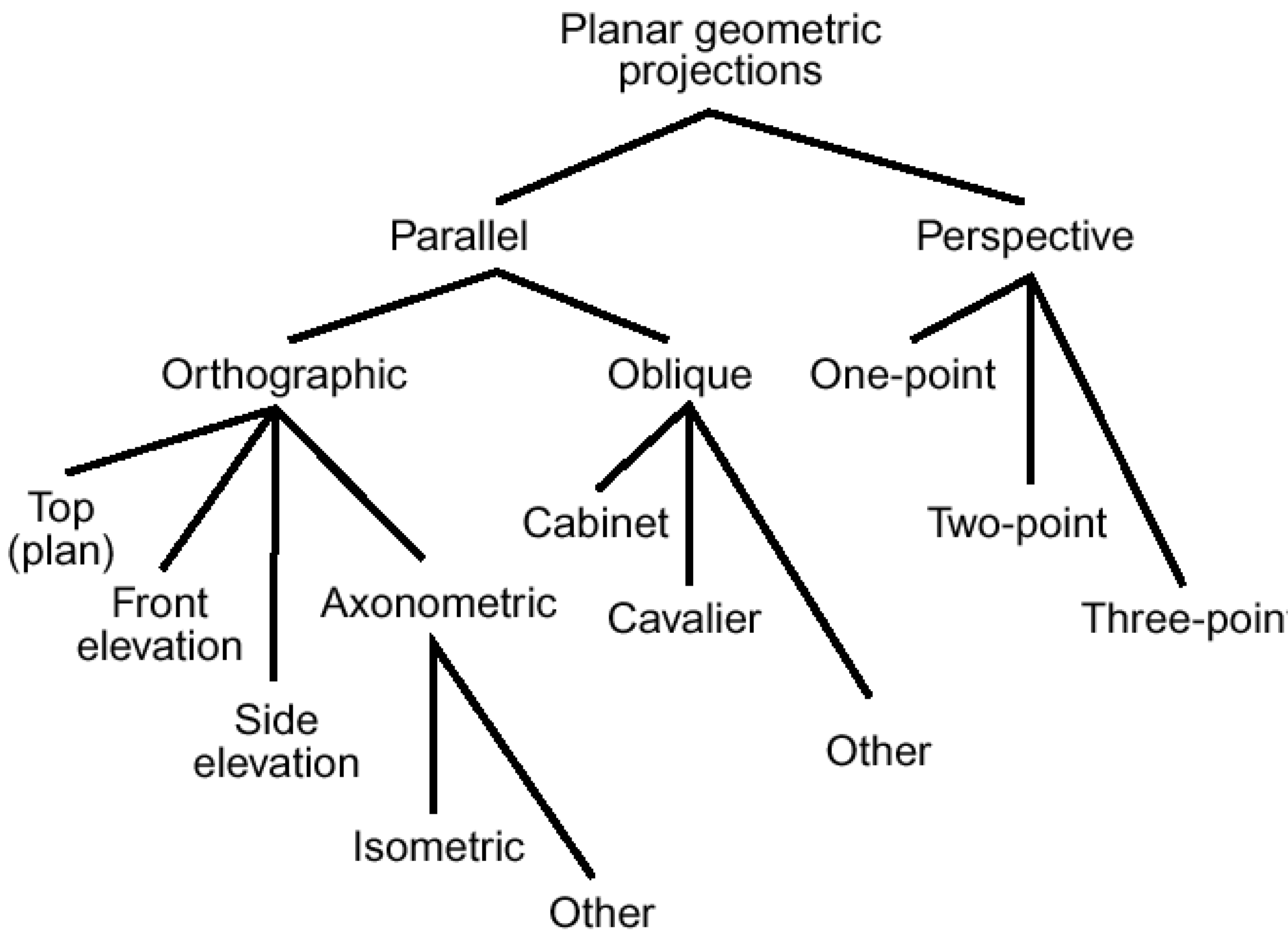
Projections Concept

- Process of representing 3D object or scene on a 2D medium.
- Transforms points in coordinate system of dimension ‘n’ into points in coordinate system of dimension less that ‘n’.
- Projection of 3D object is defined by straight projection rays (projectors) emanating from center of projection, passing through each point of object and intersecting a projection plane to form projection.

Terms

- **Centre of projection**:
  - the point from where projection is taken. It can either be a light source or eye position.
- **Projection plane:**
  - the plane on which projection of the object is formed.
- **Projectors:**
  - lines emerging from center of projection and hitting the projection plane after passing through a point in the object to be projected.

Projection Types



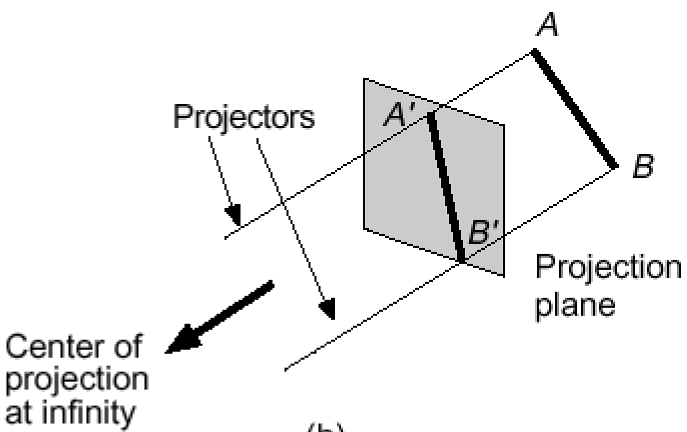
Projection Types

- Once world co-ordinate description of the objects in a scene are converted to viewing co-ordinates, we can project the three dimensional objects onto the two dimensional view plane. There are two basic projection methods:

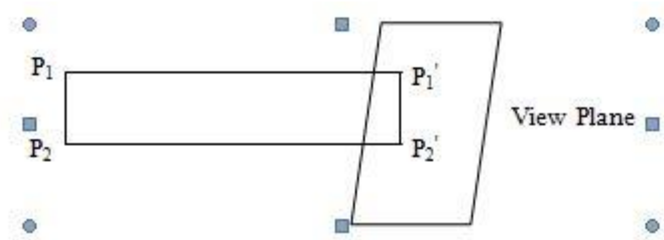
1. Parallel Projection
2. Perspective Projection

Parallel Projection

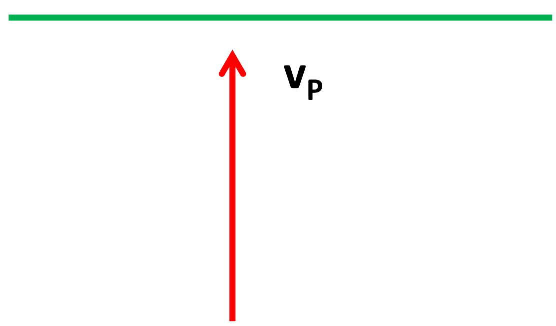
- In parallel projection, co-ordinates positions are transformed to the view plane along parallel lines.
- Center of projection is infinity
- A parallel projection preserve relative proportions of objects and this is the method used in drafting in drafting to produce scale drawing of three-dimensional objects. Accurate view of various sides of 3D object is obtained with parallel projection. But it does not given a realistic appearance of a 3D-object.



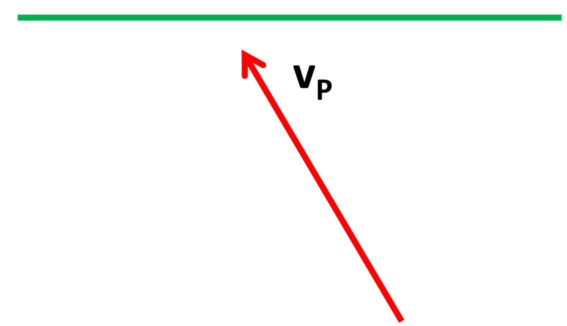
(b)



- We can specify parallel projection with the projection vector that specifies the direction of projection line. When the projection lines are perpendicular to view plane, the projection is **orthographic parallel projections**.
- It projection line are not parallel to view plane then it is **oblique parallel projection.**



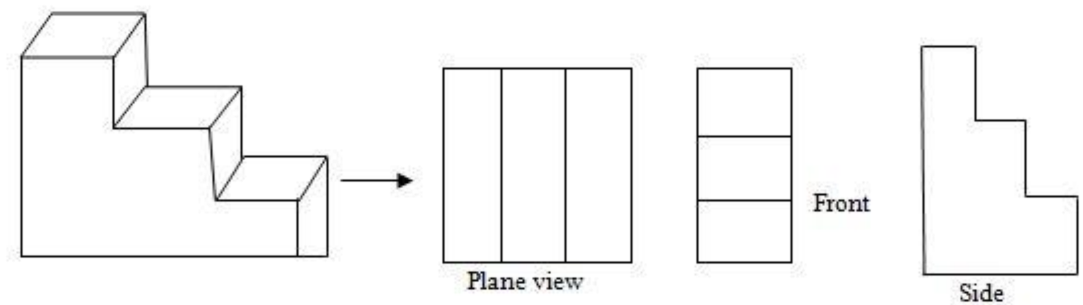
Orthographic Projection
(a)



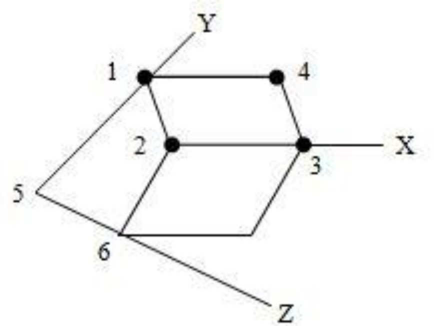
Oblique Projection
(b)

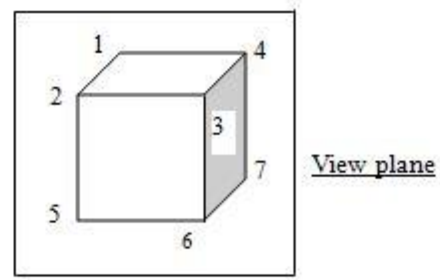
Orientation of projection vector $\mathbf{V_p}$ to produce an orthographic projection (a) and oblique projection (b)

- Orthographic projections are most often used to produce the front, side, and top views of an object. Front, side and rear orthographic are called **elevations** and the top orthographic view of object is known as **plane view**. Engineering and Architectural drawings commonly employ these orthographic projections.



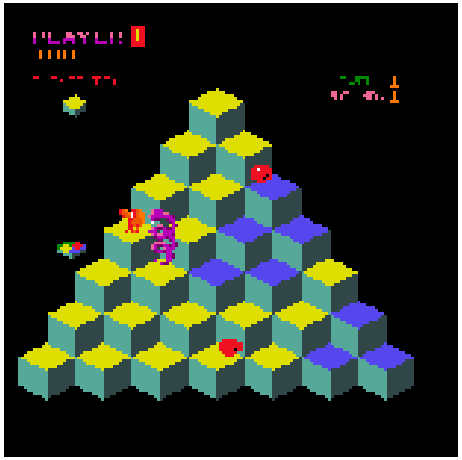
- We also from orthographic projections that display more than one face of an object. Such views are called **axonometric orthographic projections**. the most commonly used axonometric projection is the **isometric projection.**



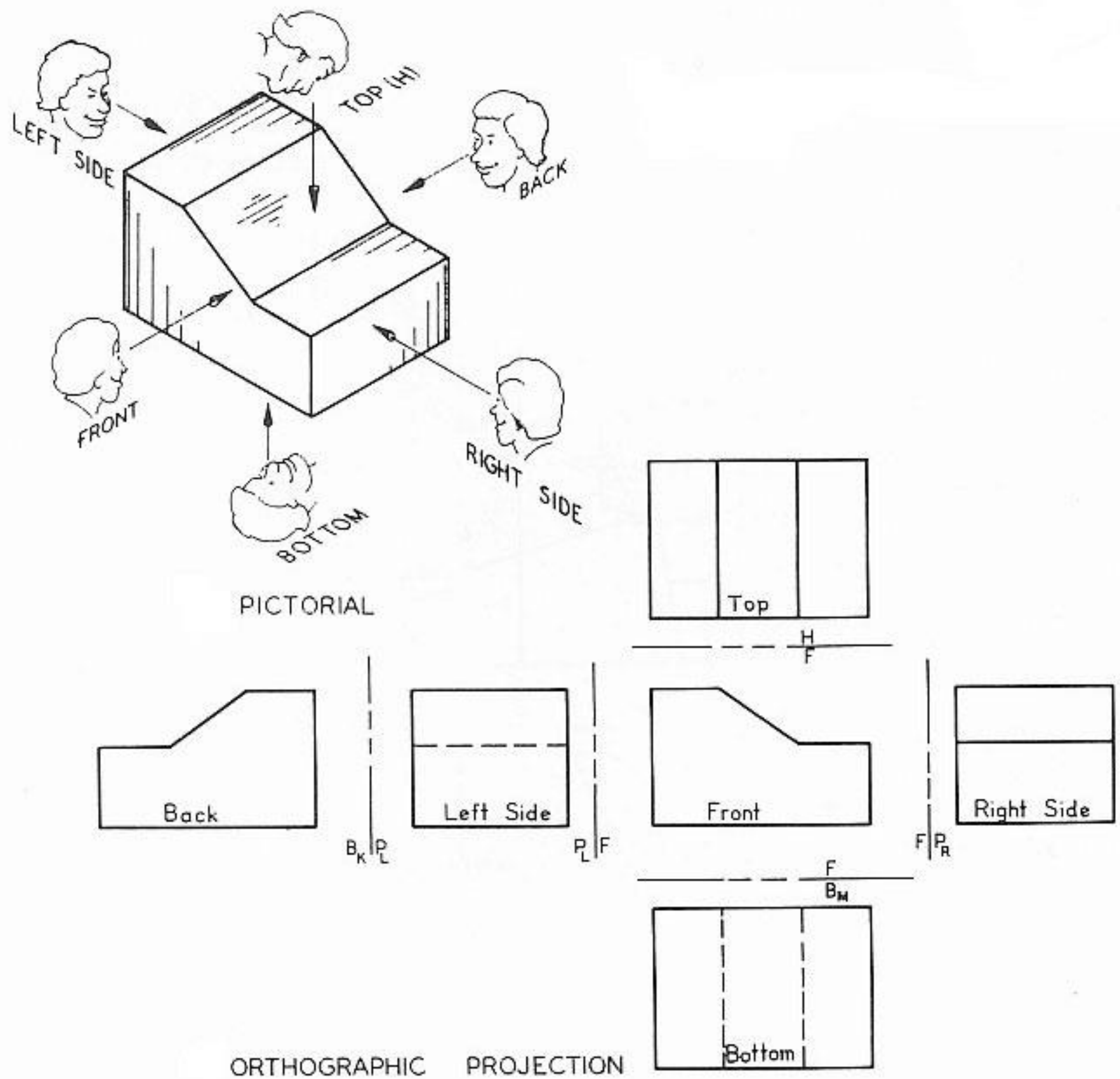


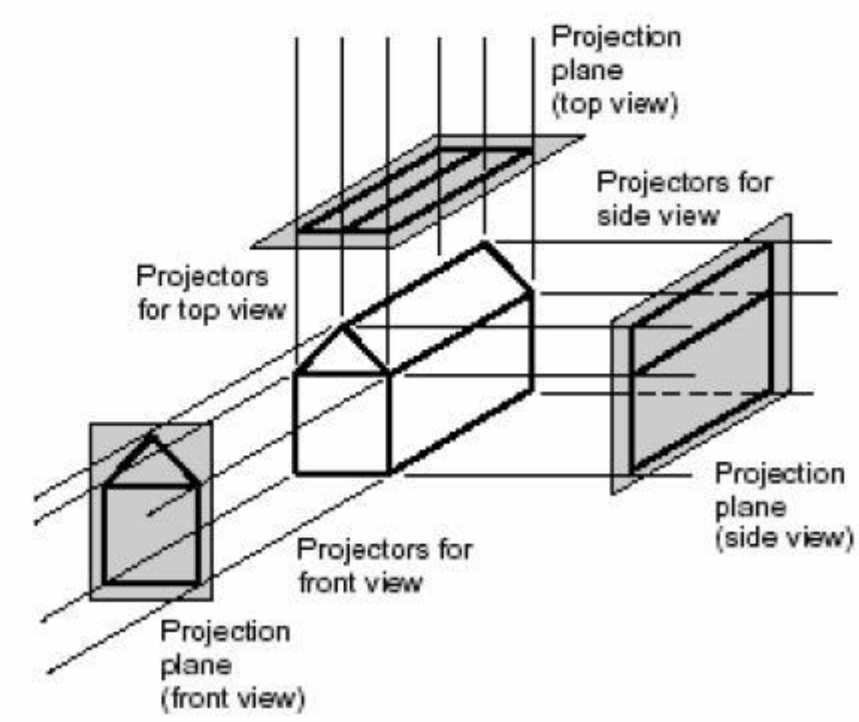
- The transformation equations for orthographic projection is $x_p = x, y_p = y$ where original z coordinate value is preserved for the depth information

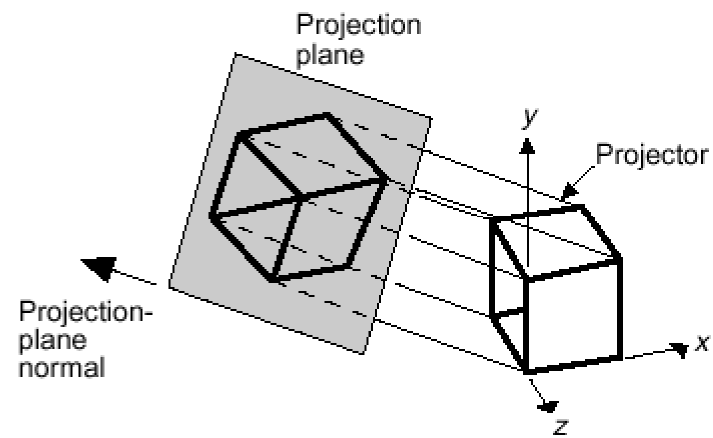
- Isometric projections have been used in computer games from the very early days of the industry up to today.

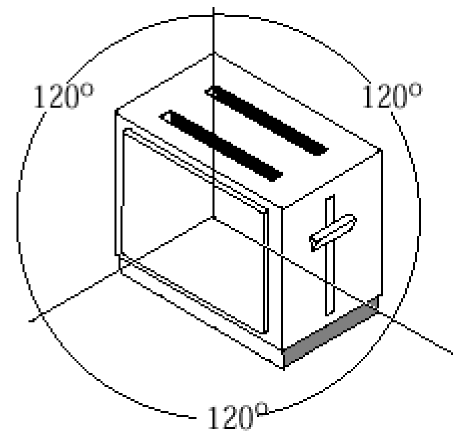


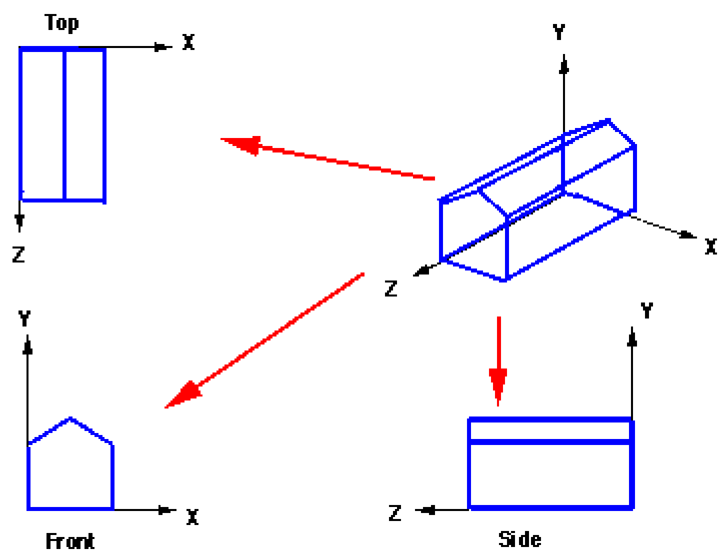
Q*Bert **Sim City** **Virtual Magic Kingdom**


LEFT SIDE
TOP (H)
BACK
FRONT
RIGHT SIDE
BOTTOM
PICTORIAL
Top
H
F
Back
Left Side
Front
Right Side
B_K P_L
P_L F
F P_R
F
B_M
Bottom
ORTHOGRAPHIC PROJECTION


Projection
plane
(top view)
Projectors for
side view
Projectors
for top view
Projection
plane
(side view)
Projectors for
front view
Projection
plane
(front view)


Projection
plane
y
Projector
Projection-
plane
normal
x
z

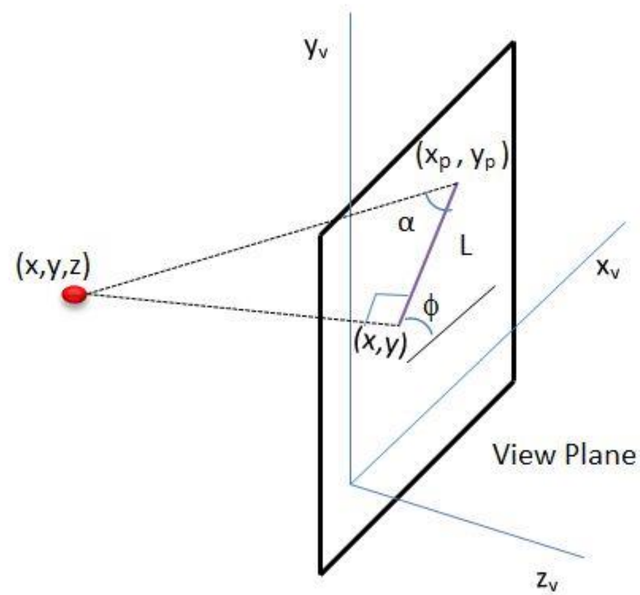

120°
120°
120°

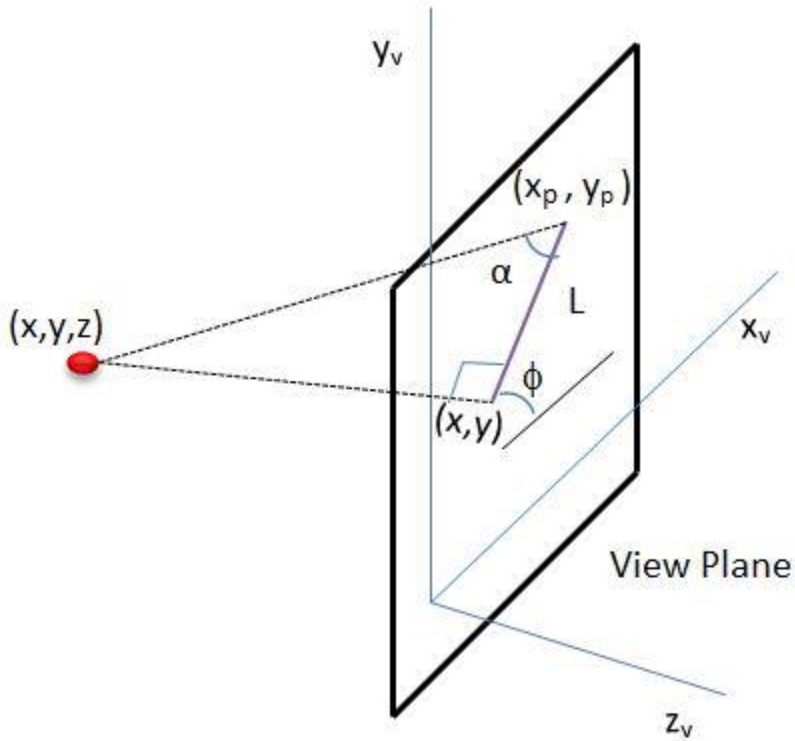

Top
X
Z
Y
X
Z
Y
Y
X
Z
Front
Side

Oblique Parallel Projection

- Obtained by projecting points along parallel lines that are not perpendicular to the projection plane. i.e. the angle between the projectors and the plane of projection is not equal to 90^0 .
- Often specified by two angles, α and ϕ.
- Point (x, y, z) is projected to position (x_p, y_p).
- Orthographic projection coordinates on the plane are (x, y).

- The oblique projection line from (x, y, z) to (x_p, y_p) makes an angle α with the line on the projection plane that joins (x_p, y_p) and (x, y).
- The line of length L is at an angle ϕ with the horizontal direction in the projection plane.





y_v
(x_p , y_p)
α
L
(x,y,z)
x_v
ϕ
(x,y)
View Plane
z_v

Expressing projection coordinates in terms of x, y, L and ϕ as

$x_p = x + L\cos\phi$

$y_p = y + L\sin\phi$

Also,

$\tan\alpha = z/L \rightarrow L = z / \tan\alpha = zL_1$ where, L_1 = inverse of $\tan\alpha$

So oblique projection are

$x_p = x + z(L_1 \cos\phi)$

$y_p = y + z(L_1 \sin\phi)$

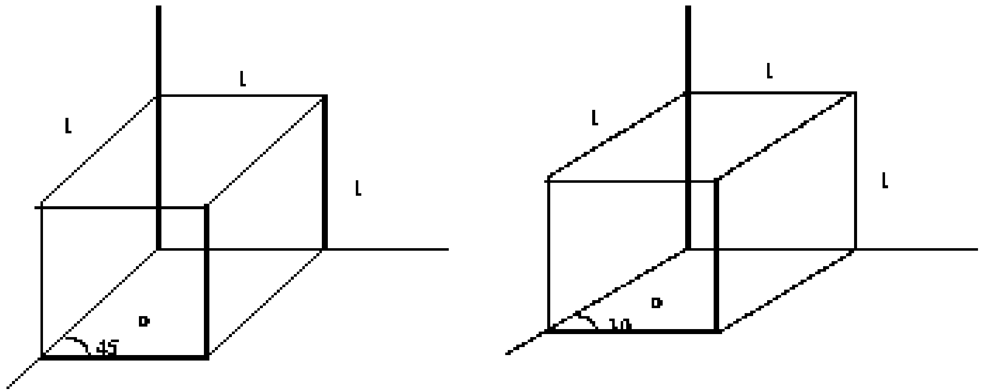
The transformation matrix for producing any parallel projection onto the $x_v y_v$ plane can be written as:

$$M_{parallel} = \begin{bmatrix} 1 & 0 & L_1\cos\phi & 0 \\ 0 & 1 & L_1\sin\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

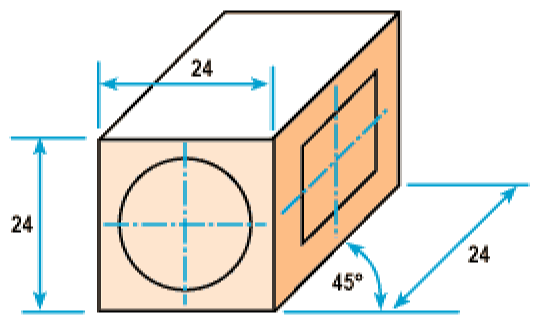
Cases:

- When $L_1 = 0$ ($\phi = 90^0$) → **orthographic projection** on z=0 plane.
- When $\phi = 45^0$ ($\tan \phi = 1$) → **cavalier oblique projection**
  - Angle between the oblique projector and plane of projection is 45^0 .
  - All the lines perpendicular to the projection plane are projected with no change in length.
- When $\phi = 63.4^0$ ($\tan \phi = 2$) → **cabinet oblique projection**
  - Lines perpendicular to the viewing surface are projected with one-half their length → more realistic than cavalier projection

Cavalier Projection

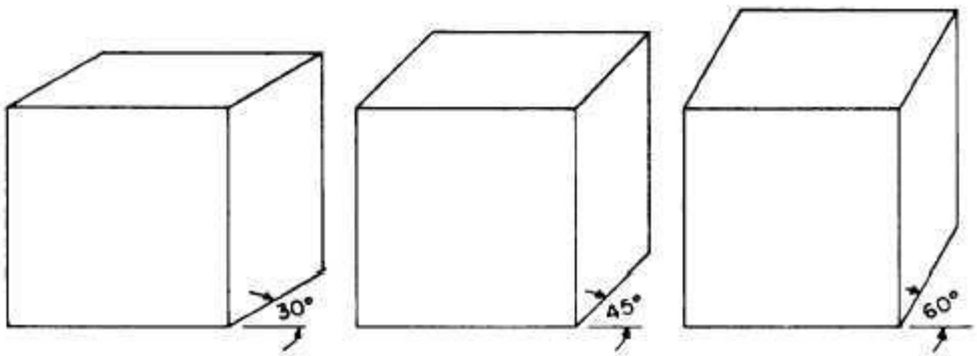


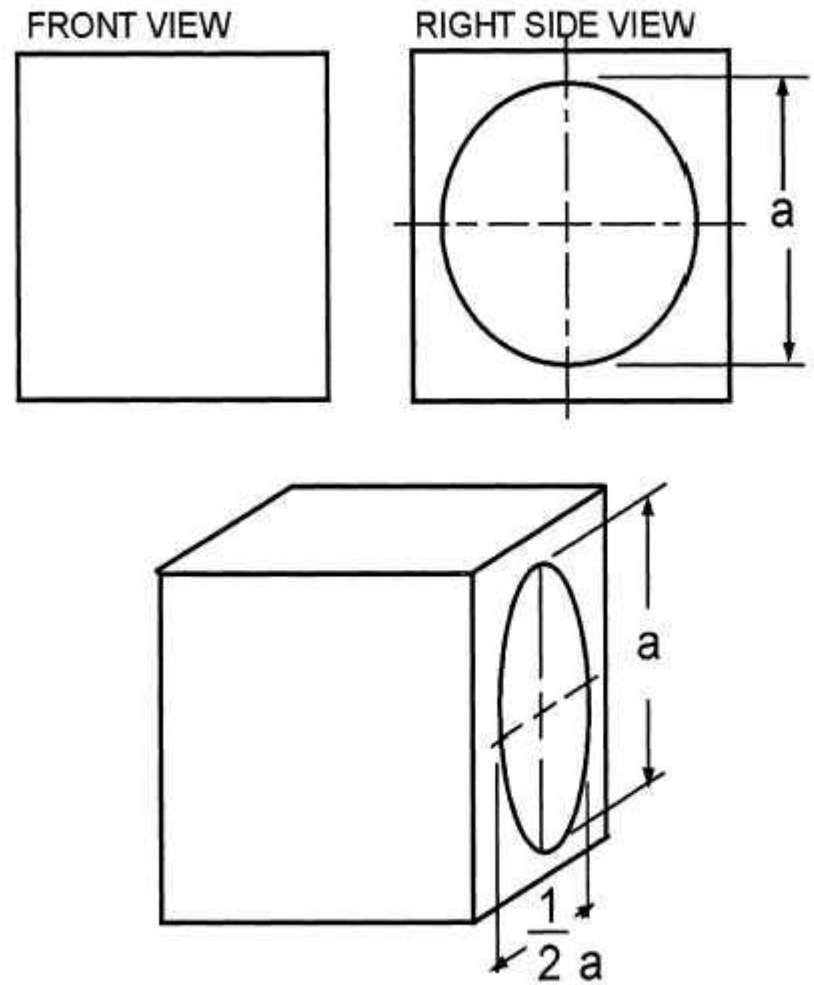
cavalier projection
of unit cube



Cavalier projection

Cabinet projection

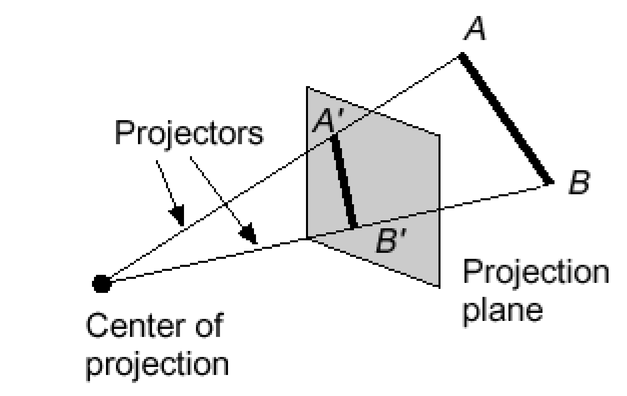




DMV2Ch06f08

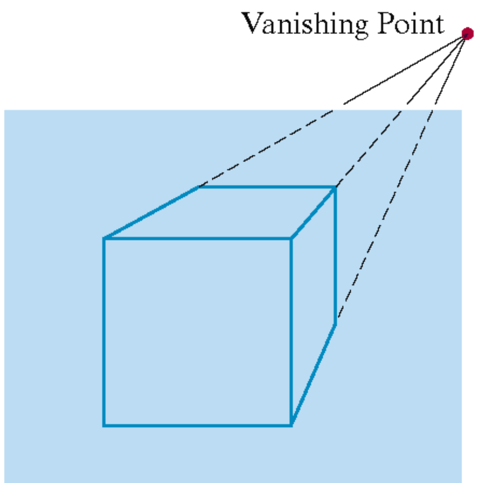
Perspective Projection

- In prospective projection, objects positions are transformed to the view plane along lines that converge to a point called **projection reference point** (center of projection). The projected view of an object is determined by calculating the intersection of the projection lines with the view plane.
- It produces realistic views but does not preserve relative proportions. Projections of distance objects from view plane are smaller than the projections of objects of the same size that are closer to the projection place.

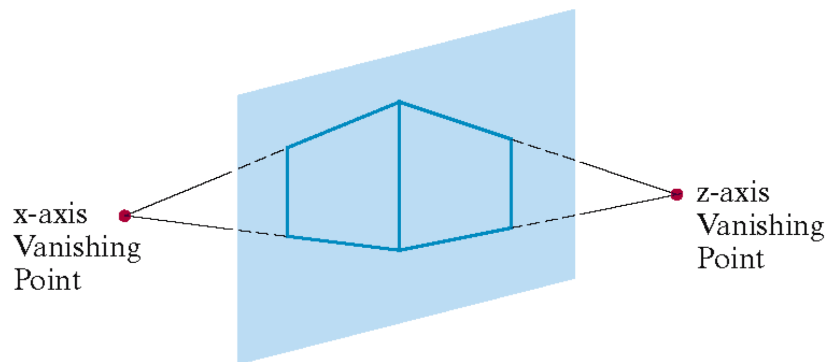


(a)

- There are a number of different kinds of perspective views.
- The most common are one-point and two point perspectives.



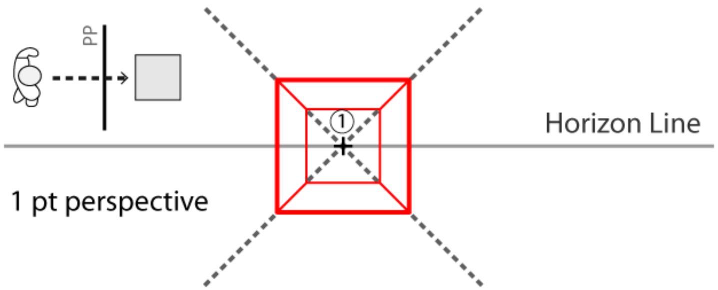
(b)
One-Point
Perspective
Projection



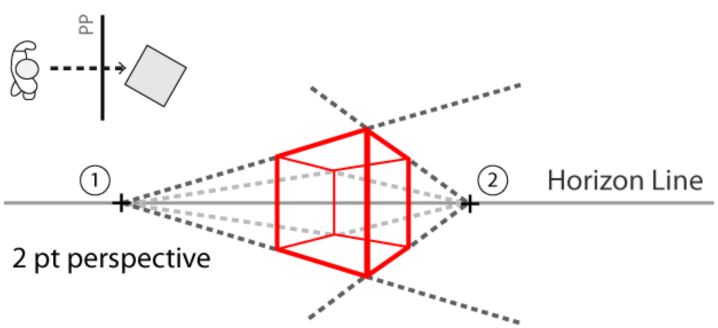
(c)
Two-Point
Perspective
Projection

Type of Perspective Projection

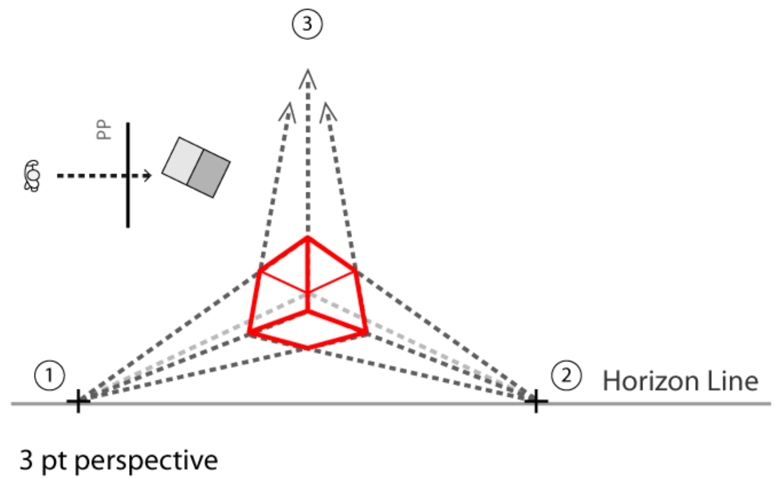
- One point perspective projection
  - Only one principal axis intersect the plane of projection
- Two point perspective projection
  - The plane of projection intersects two principal axis
- Three point perspective projection
  - The plane of projection intersects three principal axis



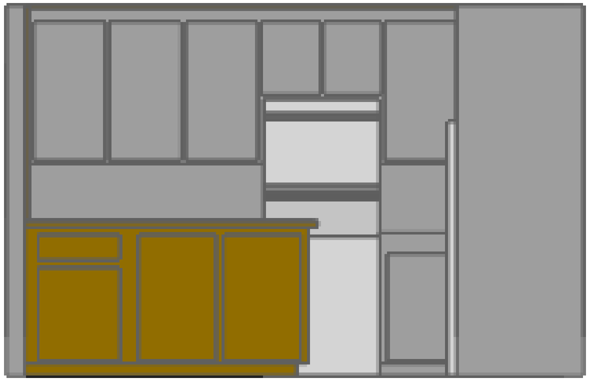
PP
①
Horizon Line
1 pt perspective



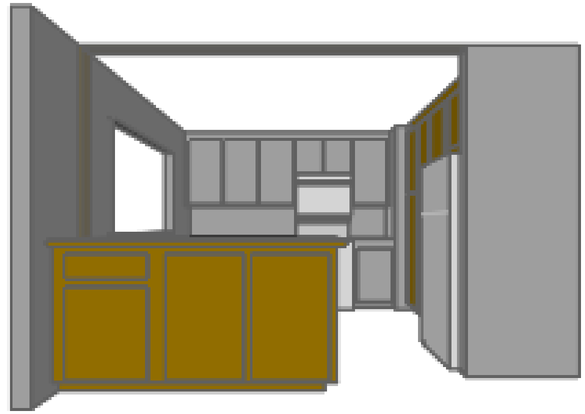
PP
①
②
Horizon Line
2 pt perspective



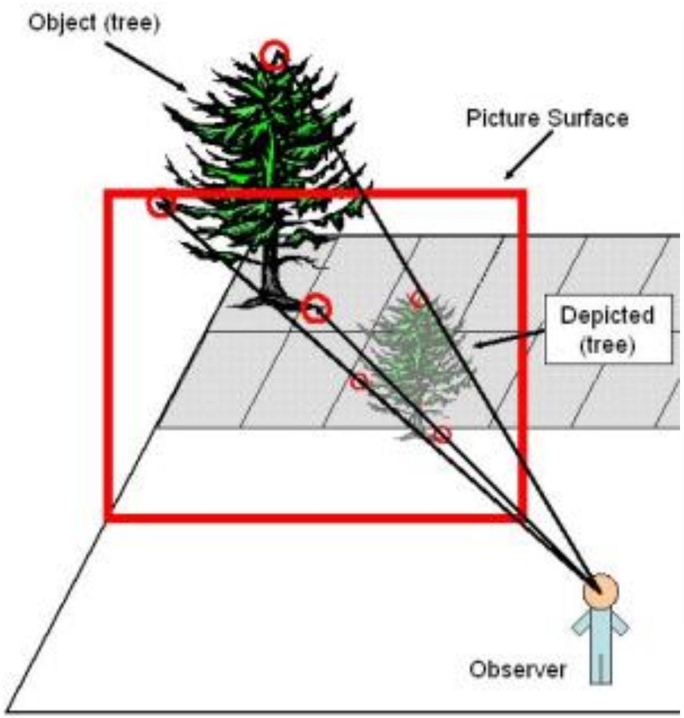
③
PP
①
②
Horizon Line
3 pt perspective

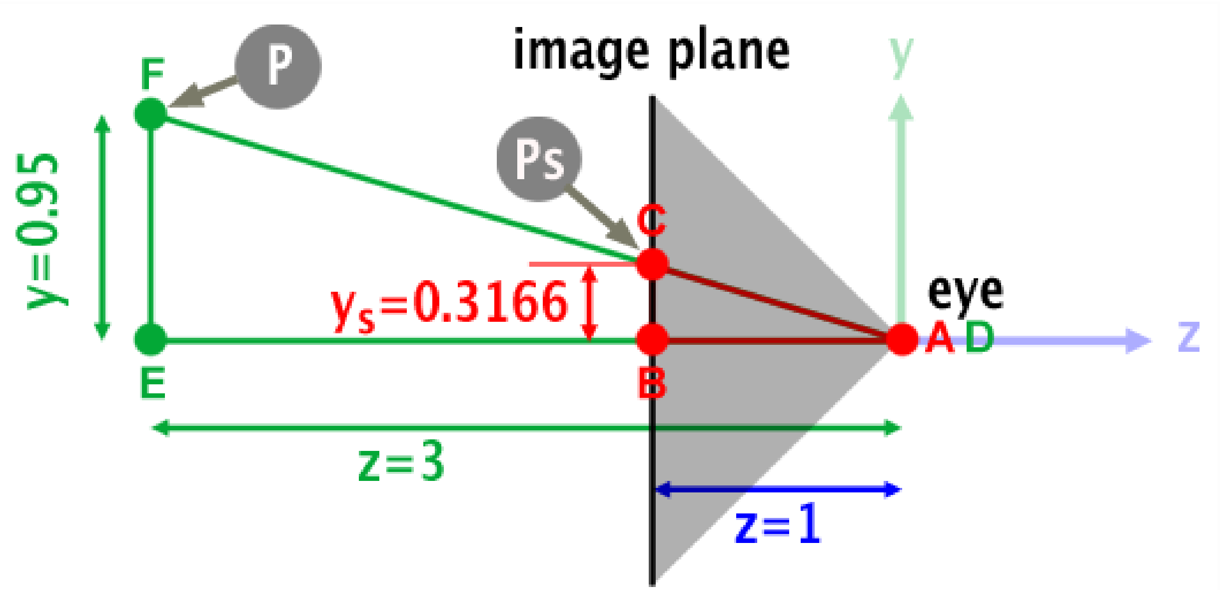


Parallel projection



Perspective projection





Formula Derivation

- To obtain a prospective projection of a three-dimensional object, we transform points along projection lines that meet at a point called projection reference point.
- Suppose we set the projection reference point at position z_{prp} along z_v the axis, and we place the view plane at z_{vp} as shown is figure below:

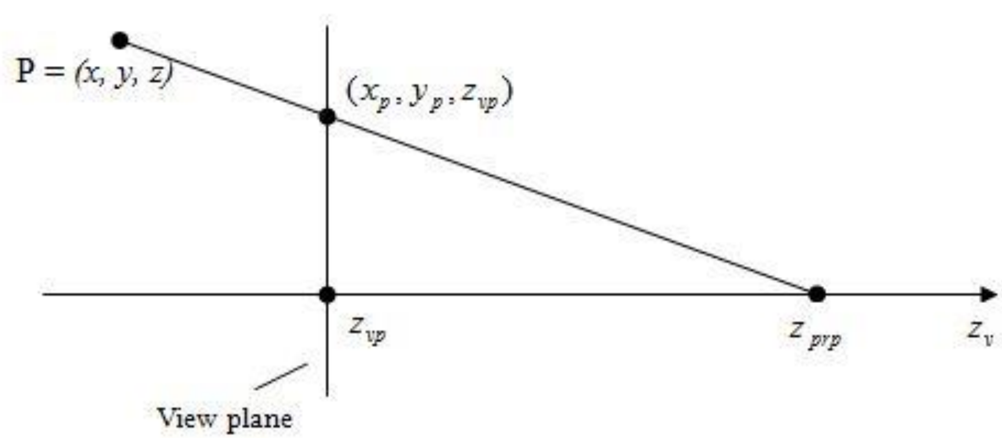
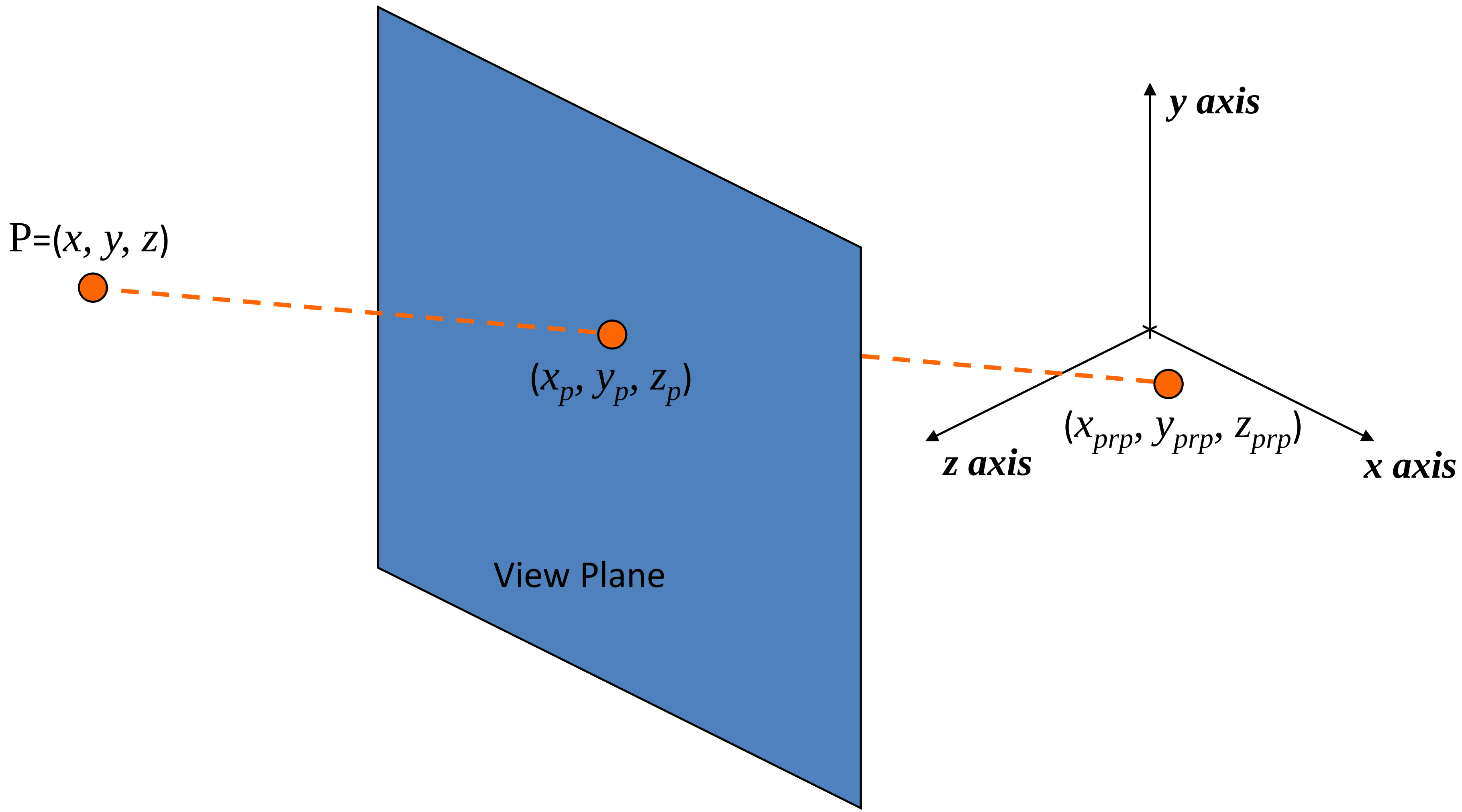


Figure: Perspective projection of a point P with coordinates (x, y, z) to position (x_p, y_p, z_{vp}) on the view plane


P=(x, y, z)
(x_p, y_p, z_p)
View Plane
y axis
z axis
x axis
(x_{prp}, y_{prp}, z_{prp})

Any point along the projector (x', y', z') can be given as:

$x' = x - xu$

$y' = y - yu$

$z' = z - (z - z_{prp})u$

$$0 \le u \le 1$$

When $u=0$, we are at the position P = (x,y,z) and when $u=1$, we have projection reference coordinates ($0,0,z_{prp}$).

On the view plane $z' = z_{vp}$, we can solve the z' equation for parameter u :

$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

- Armed with this we can restate the equations for x' and y' for general perspective:

$$x_p = x - x\left(\frac{z_{vp} - z}{z_{prp} - z}\right) = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{dp}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = y\left(\frac{dp}{z_{prp} - z}\right)$$

- Where $d_p = z_{prp} - z_{vp}$, the distance of the view plane from the projection reference point.

- Using 3-D homogenous coordinate representations, the perspective-projection transformation matrix is given as:

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & z_{vp}/dp & \dfrac{-z_{vp}}{z_{prp}/dp} \\ 0 & 0 & 1/dp & z_{prp}/dp \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- In this representation homogeneous factor $h = \dfrac{z_{prp} - z}{d_p}$
- Projection coordinates are:

$$xp = {xh}/{h}, yp = {yh}/{h}$$