Illumination Models and Surface Rendering Methods

Illumination and Surface Rendering

- Realistic displays of a scene are obtained by perspective projections and applying natural lighting effects to the visible surfaces of object.
- An **illumination model** is also called **lighting model** and some times called as a **shading model** which is used to calculate the intensity of light that we should see at a given point on the surface of a object.
- ***Surface rendering*** is a procedure for applying a lighting model to obtain pixel intensities for all the projected surface positions in a scene.

Illumination Models

Given the parameters:

- the optical properties of surfaces (opaque/transparent, shiny/dull, surface-texture);
- the relative positions of the surfaces in a scene;
- the color and positions of the light sources;
- the position and orientation of the viewing plane.

Illumination models calculate the intensity projected from a particular surface point in a specified viewing direction.

Light Source

- Sometimes light sources are referred as **light emitting object** and **light reflectors**. Generally light source is used to mean an object that is emitting radiant energy e.g. Sun, bulb

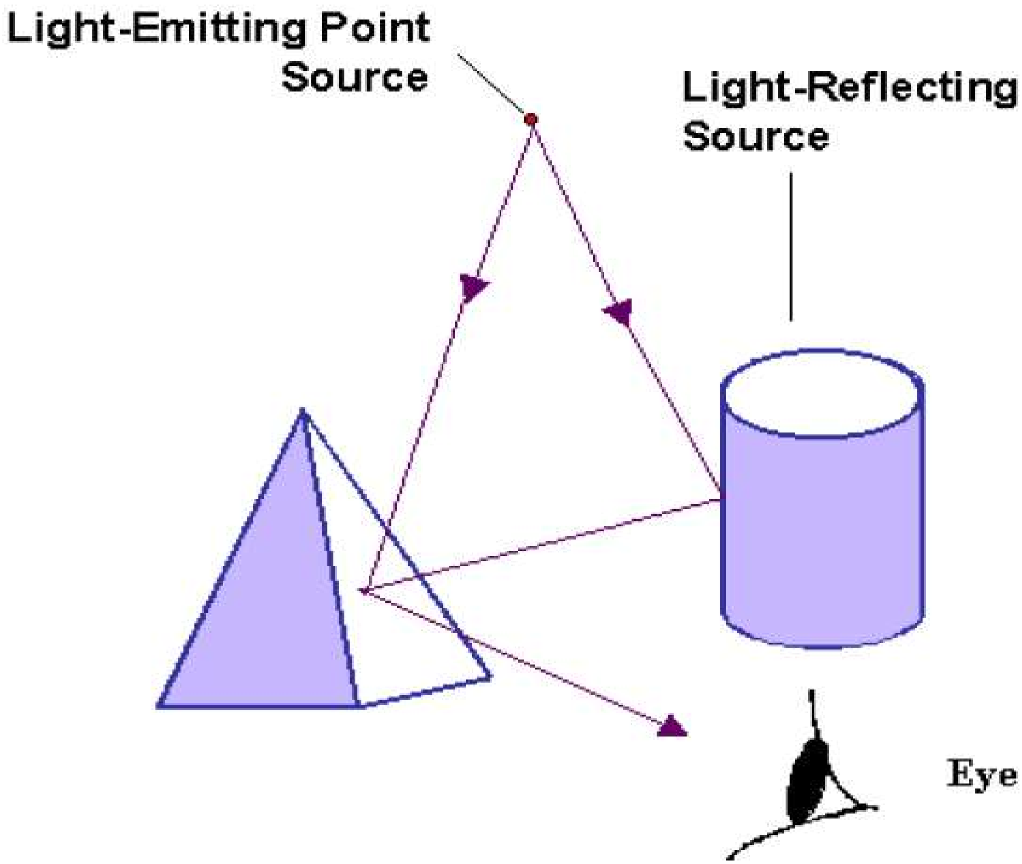


Fig. 1 Light viewed from an opaque surface is in general a combination of reflected light from a light source and reflections of light reflections from other surfaces.

Point Source

- Point source is the simplest model for light emitter.
- The rays emitted from a point light radially diverge from the source.
- Point sources are abstraction of real-world sources of light such as light bulbs, candles, or the sun.
- The light originates at a particular place; it comes from a particular direction over a particular distance.
- This light source is a reasonable approximation for sources whose dimensions are small compared to the surfaces in the scene.
- Surfaces facing towards and positioned near the light source will receive more light than those facing away from or far removed from the source following radially diverging paths as shown in fig. 2.

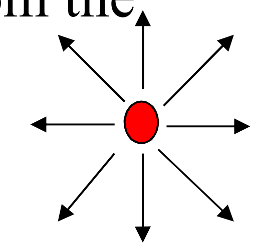


Fig. 2
Diverging ray paths from a point light source.

Distributed Source

- A nearby source, such as the long fluorescent light.
- All of the rays from a directional/distributed light source have the same direction, and no point of origin.

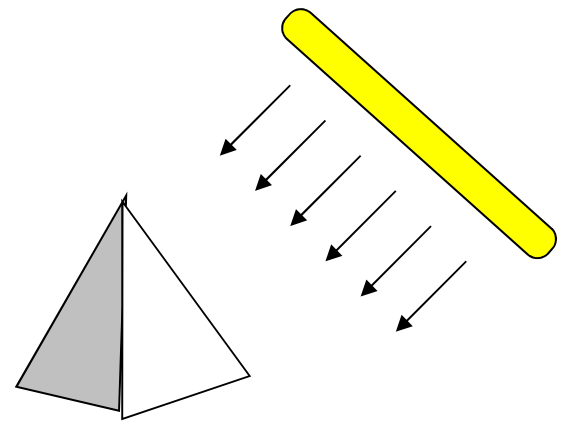


Fig. 3
An object illuminated with a distributed light source.

- It is as if the light source was infinitely far away from the surface that it is illuminating.
- Sunlight is an example of an infinite light source

Materials

- When light is incident on an opaque surface, part of it is reflected and part is absorbed.
- Shiny materials reflect more of the incident light, and dull surface absorb more of the incident light.
- For an illuminated transparent surface, some of the incident light will be reflected and some will be transmitted through the material.

Diffuse Reflection

- Rough or Grainy surfaces scatter the reflected light in all directions. This scattered light is called ***diffuse reflection***.
- The surface appears equally bright from all viewing directions.
- What we call the color of an object is the color of the diffuse reflection of the incident light

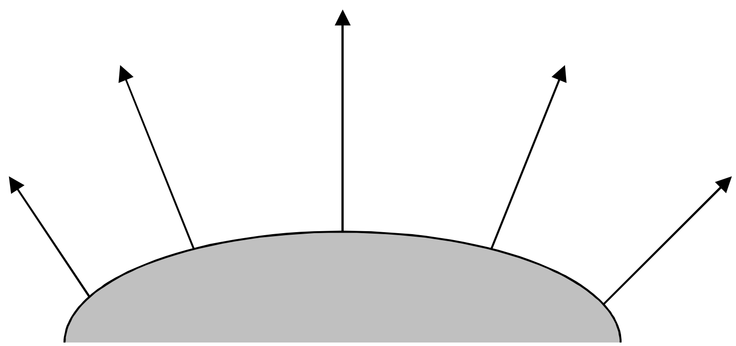


Fig. 4
Diffuse reflection from a surface.

Specular Reflection

- Light sources create highlights, bright spots, called ***specular reflection***. More pronounced on shiny surfaces than on dull.

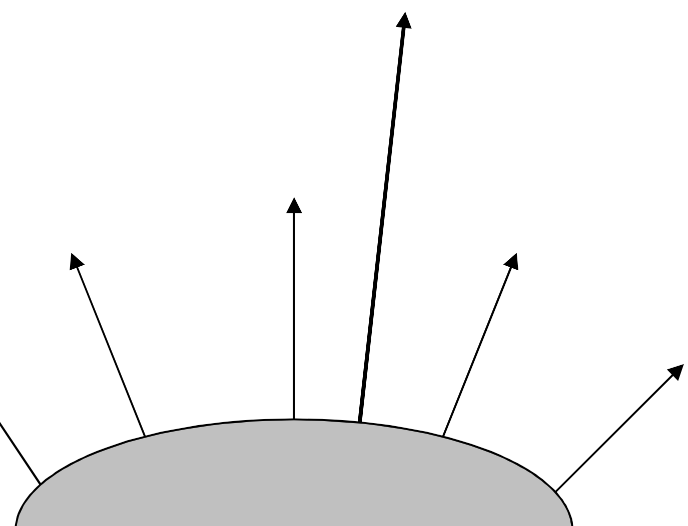


Fig. 5
Specular reflection superimposed on diffuse reflection vectors.

Basic Illumination Models

Lighting calculations are based on:

- Optical properties of surfaces, such as glossy, matte, opaque, and transparent. This controls the amount of reflection and absorption of incident light.
- The background lighting conditions.
- The light-source specifications. All light sources are considered to be point sources, specified with a coordinate position and intensity value (color).

Ambient Light (Background light)

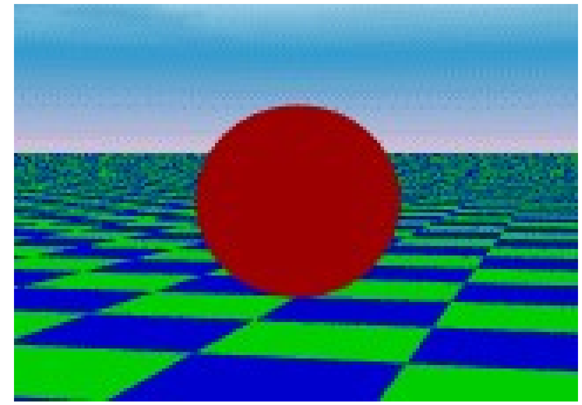


Fig. 6
Ambient light shading.

- A surface that is not directly exposed to a light source will still be visible if nearby objects are illuminated due to light reflecting from nearby objects.
- Ambient light has no spatial or directional characteristics.
- The amount of ambient light incident on each object is a constant for all surfaces and over all directions.
- The amount of ambient light that is reflected by an object is constant for all surfaces and over all direction, but the intensity of reflected light for each surface depends only on the optical properties of the surface.

- The level of ambient light in a scene is a parameter I_a , and each surface illuminated with this constant value.
- Illumination equation for ambient light is

$$I = k_a I_a$$

where

I is the resulting intensity

I_a is the incident ambient light intensity

k_a is the object's basic intensity, ***ambient-reflection coefficient or ambient reflectivity***.

Diffuse Reflection

- Diffuse reflections are constant over each surface in a scene, independent of the viewing direction.
- The amount of the incident light that is diffusely reflected can be set for each surface with parameter k_d, the ***diffuse-reflection coefficient***, or ***diffuse reflectivity***.

  $0 \leq k_d \leq 1$;

  k_d near 1 – highly reflective surface;

  k_d near 0 – surface that absorbs most of the incident light;

  k_d is a function of surface color;

Even though there is equal light scattering in all direction from a surface, the brightness of the surface does depend on the orientation of the surface relative to the light source:

(a) (b)

Fig. 8 A surface perpendicular to the direction of the incident light (a) is more illuminated than an equal-sized surface at an oblique angle (b) to the incoming light direction.

- As the angle between the surface normal and the incoming light direction increases, less of the incident light falls on the surface.
- We denote the ***angle of incidence*** between the incoming light direction and the surface normal as θ. Thus, the amount of illumination depends on $\cos\theta$. If the incoming light from the source is perpendicular to the surface at a particular point, that point is fully illuminated.

If I_l is the intensity of the point Light source, then the diffuse reflection equation for a point on the surface can be written as

$$I_{l,diff} = k_d I_l \cos\theta$$

or

$$I_{l,diff} = k_d I_l (\mathrm{N} \cdot \mathrm{L})$$

where

N is the unit normal vector to a surface and **L** is the unit direction vector to the point light source from a position on the surface

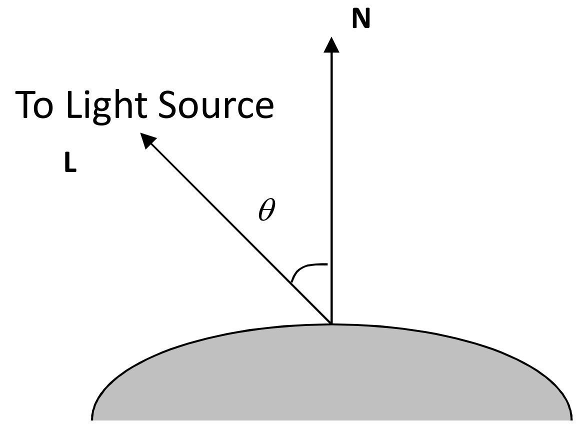


Fig. 9

Angle of incidence θ between the unit light-source direction vector **L** and the unit surface normal **N**.

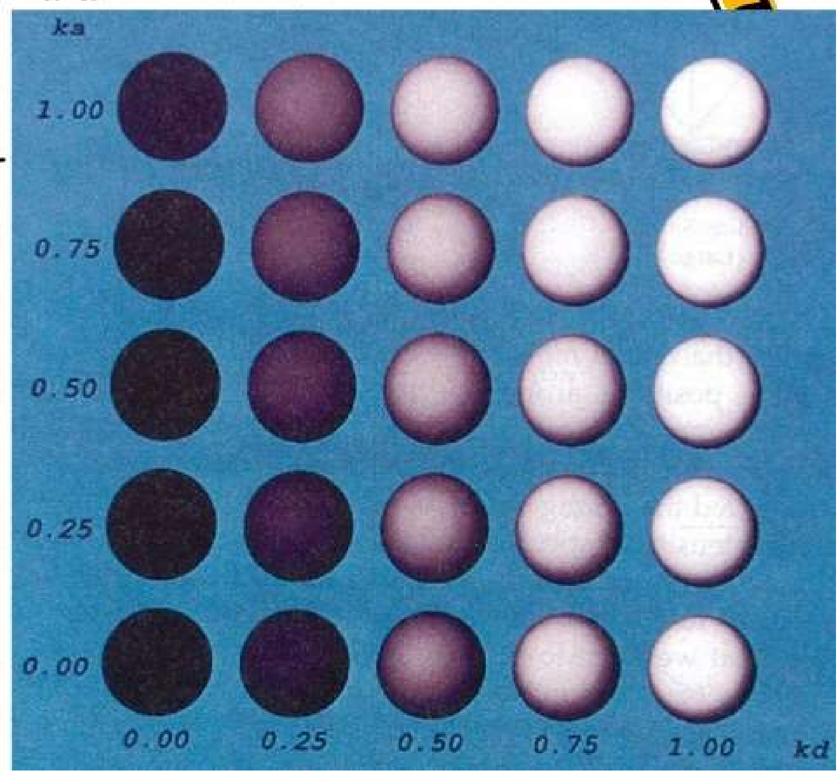
We can combine the ambient and point-source intensity calculations to obtain an expression for the total diffuse reflection.

$$I_{diff} = k_a I_a + k_d I_l (\mathbf{N} \cdot \mathbf{L})$$

where both k_a and k_d depend on surface material properties and are assigned values in the range from 0 to 1.

$$I_{diff} = \begin{cases} k_a I_a + k_d I_l (\mathbf{N} \cdot \mathbf{L}), & \text{if } \mathbf{N} \cdot \mathbf{L} > 0 \\ k_a I_a, & \text{if } \mathbf{N} \cdot \mathbf{L} \le 0 \end{cases}$$

- Combine the ambient & point-source intensity calculations the total diffuse reflection
  - Ambient-reflection coefficient k_a



Specular Reflection and the Phong Model

- It is the phenomenon in which we see an illuminated shiny surface, we observe a highlight spot at certain viewing directions.
  - Polished metal surface, person's forehead, apple etc. exhibit specular reflection.
- It is the result of total or near total internal reflection of the incident light in a concentrated region around the specular reflection angle.

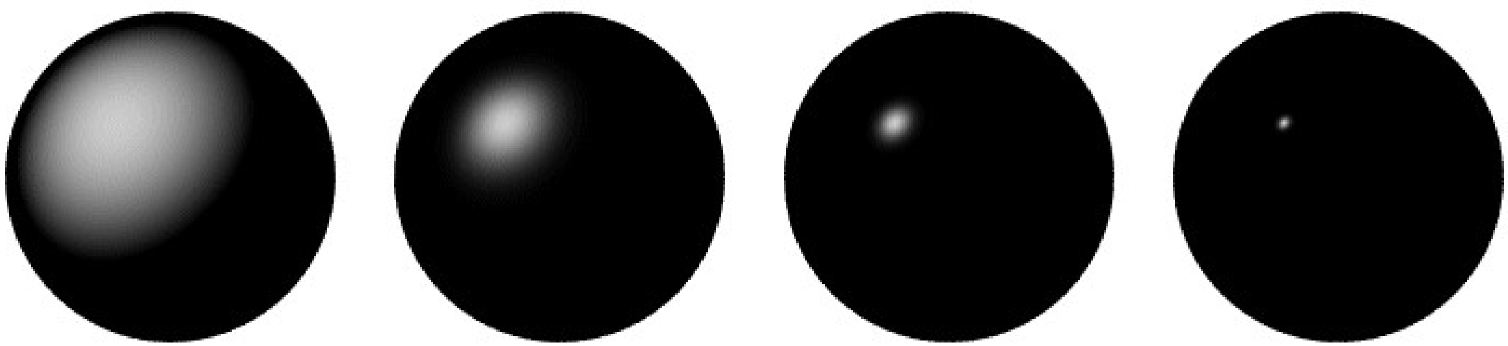
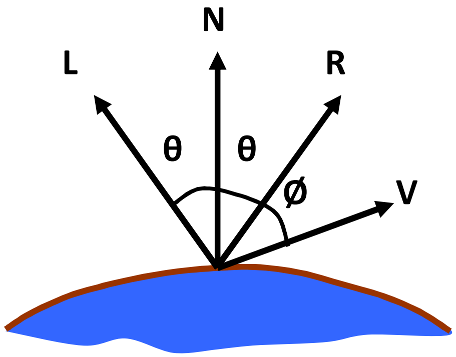


Figure shows the specular reflection direction at a point on the illuminated surface. In this figure,



- **R** = the unit vector in the direction of specular reflection;
- **N** = the unit normal surface vector
- **L** = unit vector directed toward the point light source;
- **V** = unit vector pointing to the viewer from the surface position;
- Angle *Φ* is the viewing angle relative to the specular-reflection direction **R**. It equals the angle of incident, θ.
- Ideal reflector exhibit specular reflection in the direction of R only (i.e Ø=0) but for non-ideal case specular reflection is seen over finite range of viewing positions.

Phong Model

- ***Phong specular-reflection model*** is an empirical model for calculating the specular-reflection range
- The Phong model Sets the intensity of specular reflection proportional to $\cos^{ns} \Phi$;
  - Angle Φ assigned values in the range 0^o to 90^o, so that $\cos\Phi$ varies from 0 to 1;
  - ***Specular-reflection parameter*** n_s is determined by the type of surface,
  - ***Specular-reflection coefficient*** k_s equal to some value in the range 0 to 1 for each surface.

- The intensity of specular reflection depends on
  - the materials properties of the surface
  - the angle of incidence θ,
  - other factors such as the polarization and color of the incident light.
- According to *Fresnel's Laws of Reflection*,
  Phong specular-reflection model is given as:

$$I_{spec} = W(\Theta) I_l \cos^{ns} \Phi$$

Where $W(\Theta)$ is a monochromatic specular-reflection coefficient and $\Theta=0^o$ to $\Theta=90^o$ *or* $W(\Theta)=0$ *to* $W(\Theta)=1$

At $\Theta=90^o$, $W(\Theta)=1$ → all incident light is reflected

- Very shiny surface is modeled with a large value for n_s (say, 100 or more);
- Small values are used for duller surfaces.
- For perfect reflector (perfect mirror), n_s is infinite;

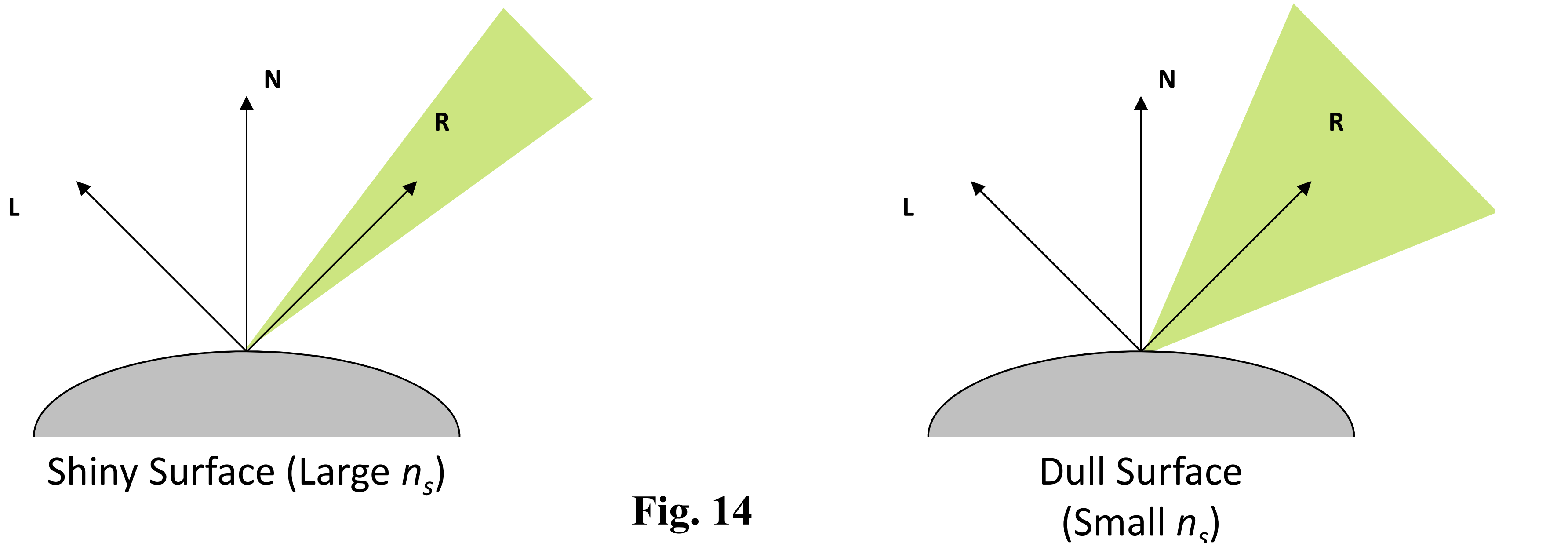
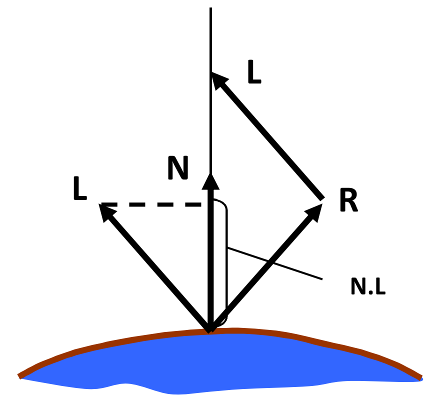


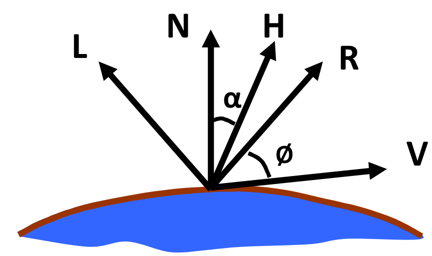
Fig. 14
Modeling specular reflection with parameter n_s.

- Simplified form: assume $w(\theta) = k_s = constant$



$$I_{spec} = k_s I_l (V.R)^{n_s}$$

- Further simplified by replacing **V**.**R** with **N**.**H** where **H** is halfway vector between **L** and **V** (i.e **H** is unit bisector vector of angle between **L** and **V**)



$$H = \frac{L+V}{|L+V|}$$

- Thus $I_{spec} = k_s I_l (N.H)^{n_s}$
- If we add ambient light and diffuse reflection component then total intensity is given as:

$$I = I_{diff} + I_{spec}$$
$$= k_a I_a + k_d I_l (N.L) + k_s I_l (N.H)^{n_s}$$

When v is coplanar with L and R
α = Ø/2 otherwise α > Ø/2

- For multiple light sources (n light sources)

$$I = k_a I_a + \sum_{i=1}^{n} I_{li} \left[k_d (N.L_i) + k_s (N.H_i)^{n_s} \right]$$

Phong shading polygons with specular reflection.

Intensity Attenuation

- As radiant energy from a point light source travels through space, its amplitude is attenuated by the factor $1/d^2$, where d is the distance that the light has traveled.
- That means, a surface close to the light source (small d) receives a higher incident intensity from the source than a distant surface (large d).
- So for realistic lighting effect, this intensity attenuation should be taken into account.

- Using merely $1/d^2$ as attenuation factor for our simple single point light source model, too much intensity variation is produced when *d* is small and a little variation when *d* is large.
- Graphical packages have compensated the problem by using inverse linear quadratic function of d for intensity attenuation as:

$$f(d) = \frac{1}{a_0 + a_1 d + a_2 d^2}$$

- The value of the constant term a_0 can be adjusted to prevent $f(d)$ from becoming too large when *d* is very small.

Attenuation function

- With a given set of attenuation coefficients, we can limit the magnitude of the attenuation function to 1 with the calculation

$$f(d) = \min\left(1, \frac{1}{a_0 + a_1 d + a_2 d^2}\right)$$

- Using this function, we can then write our basic illumination model as

$$I = k_a I_a + \sum_{i=1}^{n} f(d_i) I_{li} \left[k_d (N \bullet L_i) + k_s (N \bullet H_i)^{n_s} \right]$$

where d_i is the distance light has traveled from light source i.

Transparency

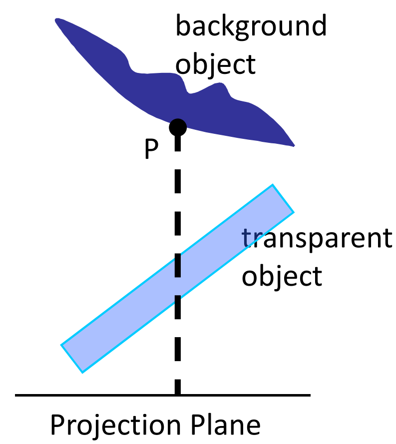
- A transparent surface, in general, produces both **reflected** and **transmitted light**.
- The relative contribution of the transmitted light depends upon the degree of transparency of the surface and the position of light source or illuminated object behind or in-front of the transparent light source.
- When a transparent surface is to be modeled, the intensity equations must be modified to include contributions from light passing through the surface.

- When light is incident upon a transparent surface, part of it is reflected and part is **refracted** as shown in figure.
- The Snell's law is used to calculate the refracted ray direction:

$$\sin\theta_r = \frac{\eta_i}{\eta_r}\sin\theta_i$$

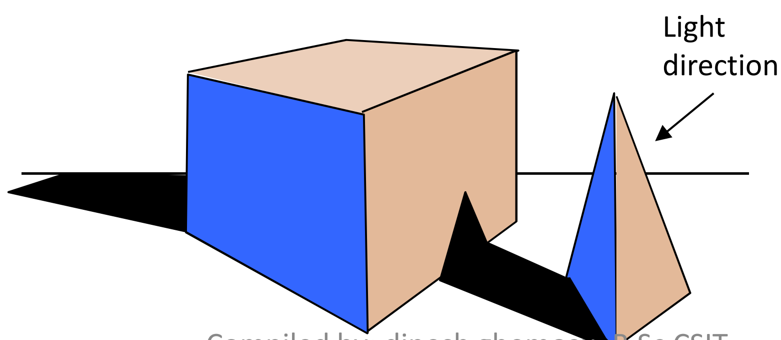
- θ_i = angle of incidence
- θ_r = angle of refraction
- η_i = refractive index of incident material
- η_r = refractive index of refracting material

- Total surface intensity due to the transmitted intensity I_{trans} through a surface from a background object with the reflected intensity I_{refl} from the transparent surface using a transparency coefficient k_t is given by:
- $I = (1-k_t)I_{refl} + k_t I_{trans}$
- The term $(1-k_t)$ is the **opacity factor**.
  - $k_t=1$ for highly transparent object
  - $k_t=0$ for nearly opaque object



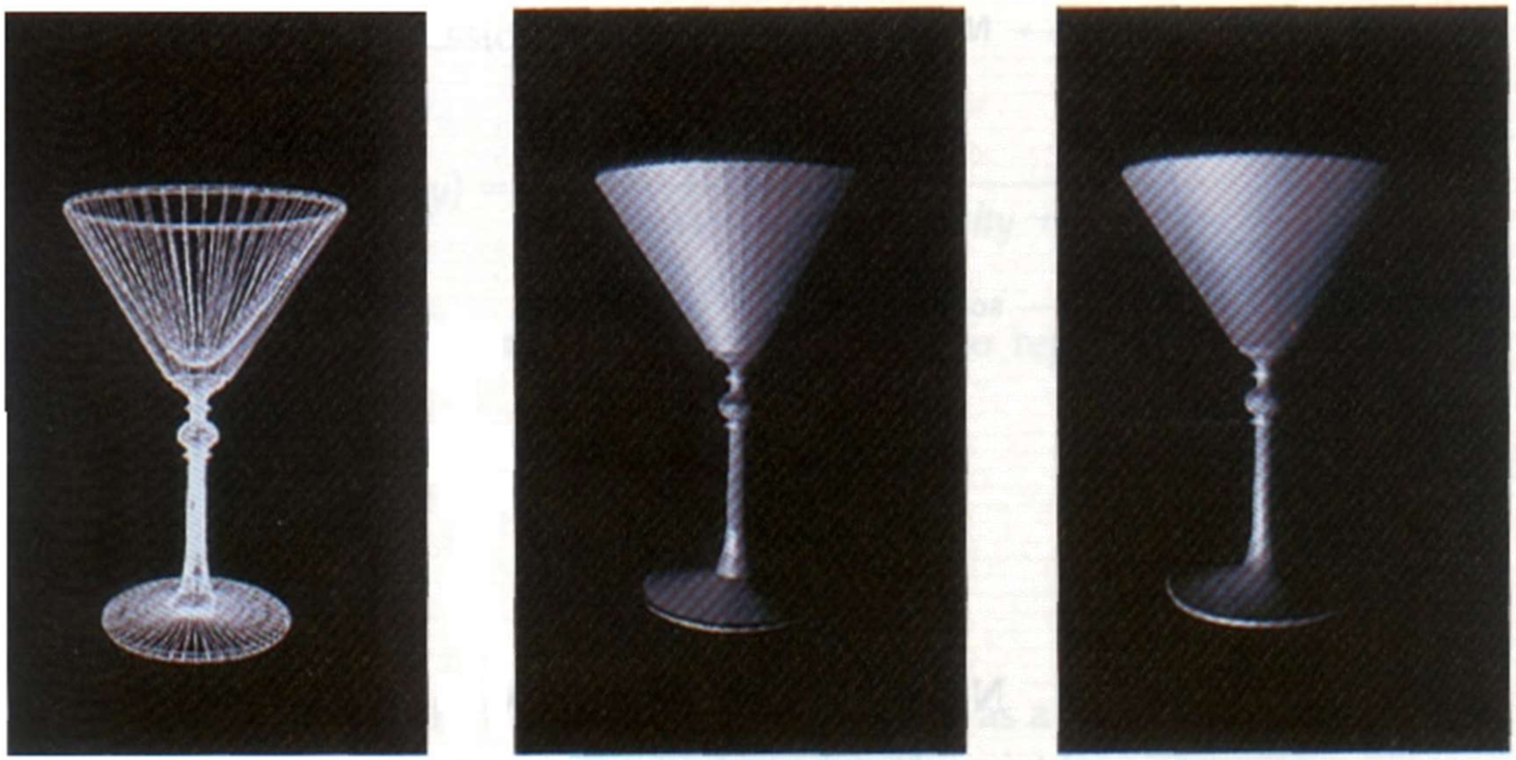
Shadow

- Hidden-surface methods can be used to locate areas where light sources produce shadows.
  - Apply a hidden-surface method with a light source at a view position.
  - Shadow patterns generated by a hidden-surface method are valid for any selected viewing position, as long as the light-source positions are not changed.
- In polygon-based system, we can add surface-detail polygons that correspond to shadow areas of surface polygons.
- We can display shadow areas with ambient light intensity only, or we can combine the ambient light with specified surface texture.



Polygon Rendering Methods

- The application of an illumination model to the rendering of standard graphic objects;
- Three methods
  - Constant-Intensity Shading / Flat Shading;
  - Intensity-Interpolation Shading / Gourad Shading;
  - Normal-vector Interpolation Shading / Phong Shading
- Each polygon can be rendered with a single intensity, or the intensity can be obtained at each point of the surface using an interpolation scheme

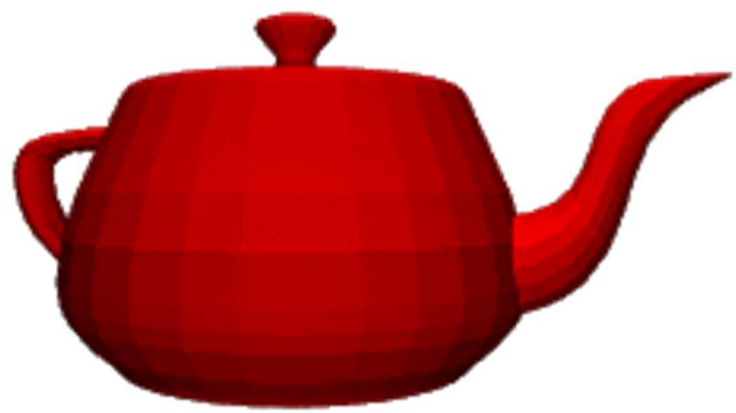


(a) (b) (c)

A polygon mesh approximation of an object (a) is rendered with flat shading (b) and with Gouraud shading (c).

Constant-Intensity Shading
Flat Shading

- Fast & simple.
- A single intensity is calculated for each polygon.
- All points over the surface of the polygon are displayed with the same intensity value.
- Useful for quickly displaying the general appearance of a curved surface.



Flat Shading

- Flat shading provides an accurate rendering for an object if all of the following assumptions are valid:
  - The object is a polyhedron and is not an approximation of an object with a curved surface;
  - All light sources illuminating the object are far from the surface so that **N·L** and the attenuation function are constant over the surface;
  - The viewing position is also far from the surface so that **V·R** is constant over the surface;

Gouraud Shading

- ***Intensity-interpolation*** scheme, referred to as ***Gouraud shading***, renders a polygon surface by linearly interpolating intensity values across the surface.
- Intensity values for each polygon are matched with the values of adjacent polygons along the common edges, thus eliminating the intensity discontinuities that can occur in flat shading.

Each polygon surface is rendered with Gouraud shading by performing the following calculations:

- **Step 1**: Determine the average unit normal vector at each polygon vertex;
- **Step 2**: Apply an illumination model to each vertex to calculate the vertex intensity;
- **Step 3**: Linearly interpolate the vertex intensities over the surface of the polygon;

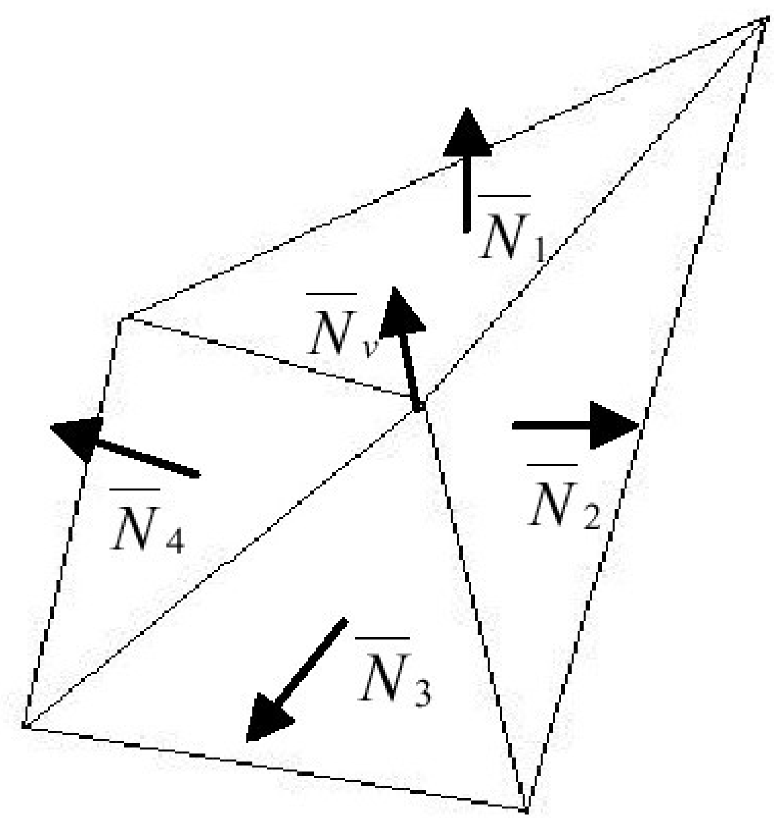
Gouraud Shading step-1

$$\overline{N}_v = \frac{\left(\overline{N}_1 + \overline{N}_2 + \overline{N}_3 + \overline{N}_4\right)}{\left\|\overline{N}_1 + \overline{N}_2 + \overline{N}_3 + \overline{N}_4\right\|}$$

More generally:

$$\vec{N}_v = \frac{\sum_{i=1}^{n} \vec{N}_i}{\left\|\sum_{i=1}^{n} \vec{N}_i\right\|}$$

$n = 3$ or 4 usually



Step 1:

The normal vector N_v is calculated as the average of the surface normals for each polygon sharing that vertex

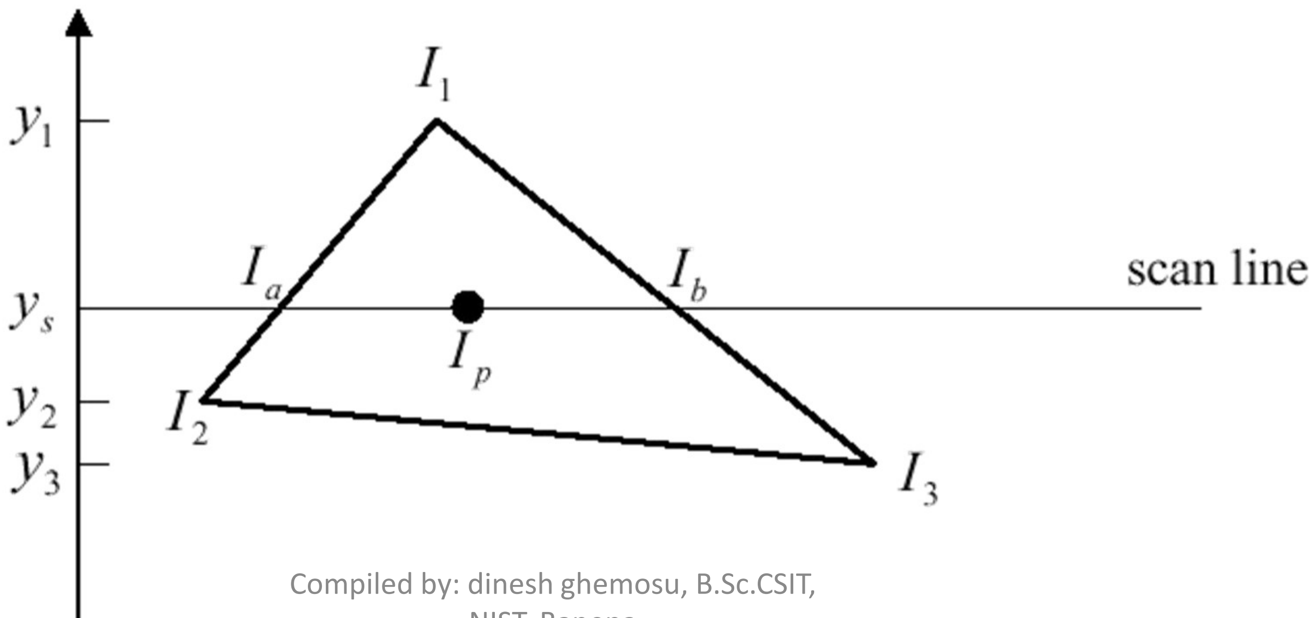
Step 2

- Apply an illumination model to each vertex to calculate the vertex intensity using following equation:

$$I = k_a I_a + \sum_{i=1}^{n} f(d_i) I_{li} \left[k_d (N \bullet L_i) + k_s (N \bullet H_i)^{n_s} \right]$$

Gouraud Shading Step-3

- For each scan line, the intensity at the intersection of the scan line with a polygon edge is linearly interpolated from the intensities at the edge endpoints.

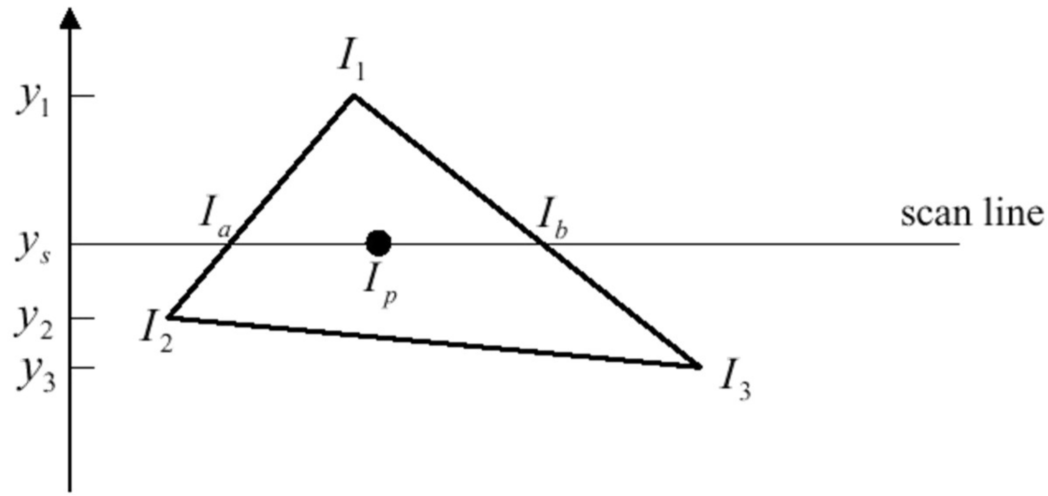


- A fast method for obtaining this intensity is to interpolate between intensities of endpoints by using only the vertical displacement of the scan line:

$$I_a = I_1 \frac{y_s - y_2}{y_1 - y_2} + I_2 \frac{y_1 - y_s}{y_1 - y_2}$$

and

$$I_b = I_1 \frac{y_s - y_3}{y_1 - y_3} + I_3 \frac{y_1 - y_s}{y_1 - y_3}$$

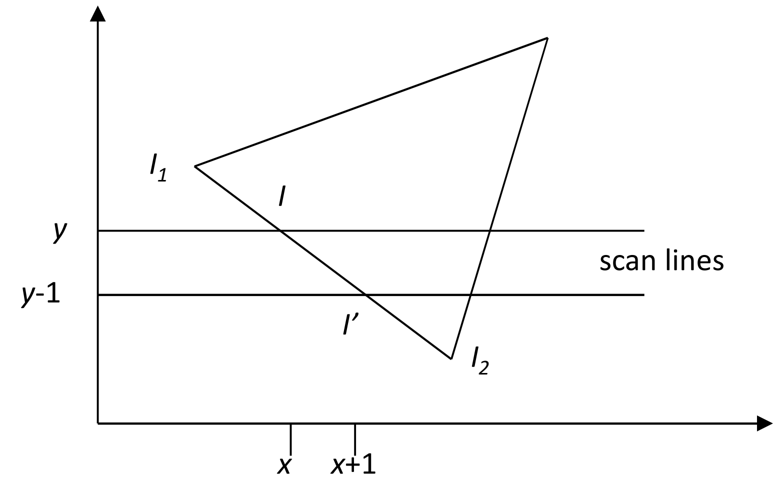


Once these bounding intensities are established for a scan line, an interior point (such as *p*) is interpolated from the bounding intensities at points *a* and *b* as

$$I_p = I_a \frac{x_b - x_p}{x_b - x_a} + I_b \frac{x_p - x_a}{x_b - x_a}$$

Incremental calculations:
If the intensity at edge position (x,y) is interpolated as

$$I = I_1 \frac{y - y_2}{y_1 - y_2} + I_2 \frac{y_1 - y}{y_1 - y_2}$$



then we can obtain the intensity along this edge for the next scan line, y-1, as

$$I' = I + \frac{I_2 - I_1}{y_1 - y_2}$$

Similar calculations are used to obtain intensities at horizontal pixel positions along each scan line.

Disadvantages

- Highlights on the surface are sometimes displayed with anomalous shapes.
- Can cause bright or dark intensity streaks to appear on the surface (***Mach-band effect***).

Dividing the surface into a greater number of polygon faces can reduce these effects.

Phong Shading

- A more accurate method for rendering a polygon surface.
- Interpolates normal vectors, and then applies the illumination model to each surface point.
- Method developed by Phong Bui Tuong.
- Called ***Phong shading,*** or ***normal-vector interpolation shading***.
- More realistic highlights.
- Greatly reduces the Mach-band effect.

Steps:

- **Step 1**: Determine the average unit normal vector at each polygon vertex.
- **Step 2**: Linearly interpolate the vertex normals over the surface of the polygon.
- **Step 3:** Apply illumination model along each scan line to calculate projected pixel intensities for the surface points.

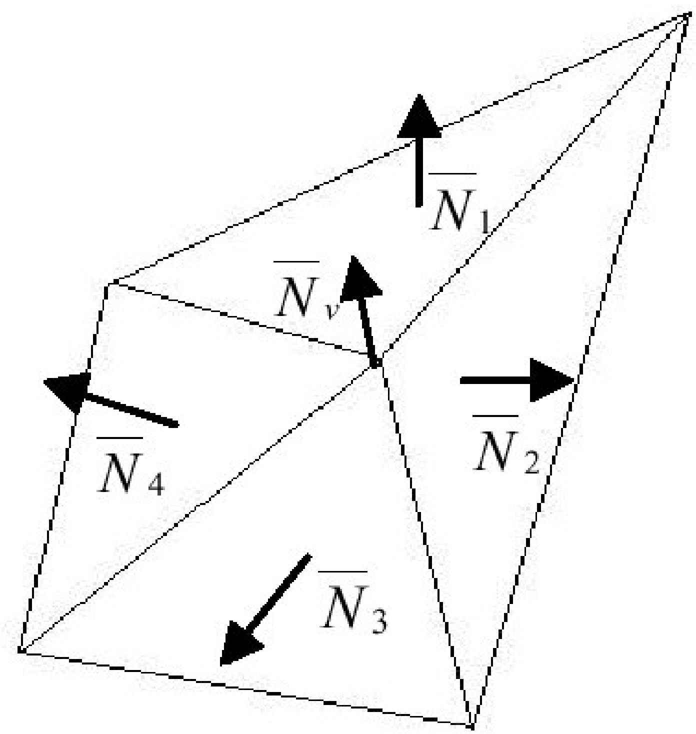
Step 1:

$$\overline{N}_v = \frac{\left(\overline{N}_1 + \overline{N}_2 + \overline{N}_3 + \overline{N}_4\right)}{\left\|\overline{N}_1 + \overline{N}_2 + \overline{N}_3 + \overline{N}_4\right\|}$$

More generally:

$$\vec{N}_v = \frac{\sum_{i=1}^{n} \vec{N}_i}{\left\|\sum_{i=1}^{n} \vec{N}_i\right\|}$$

$n = 3$ or 4 usually

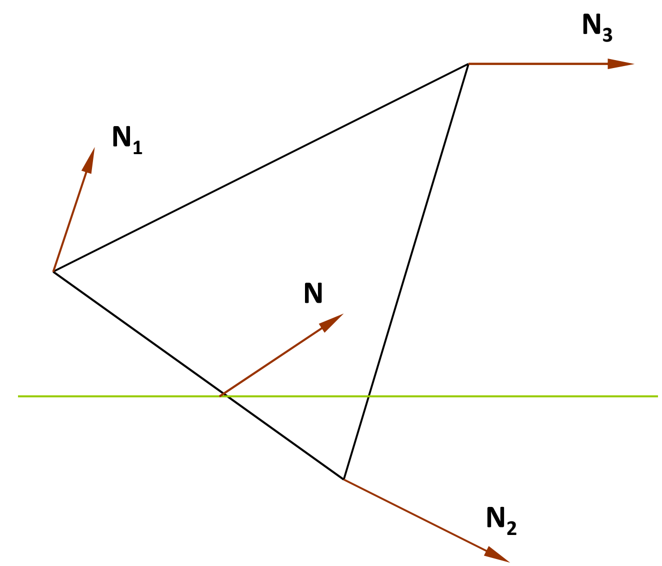


Step 1:

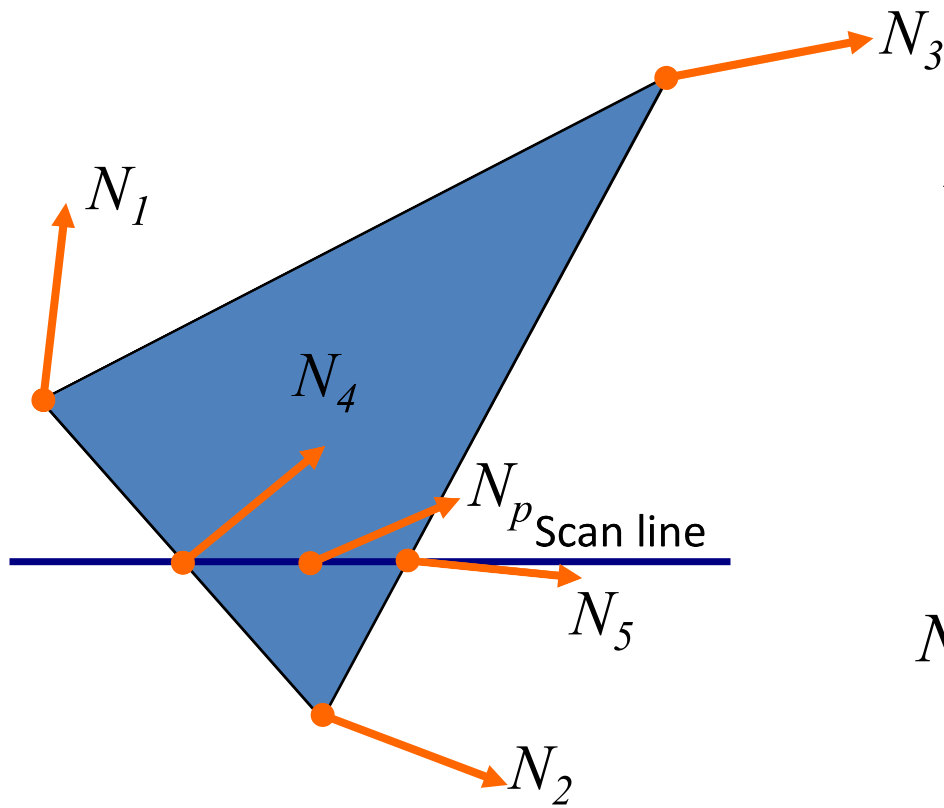
The normal vector N_v is calculated as the average of the surface normals for each polygon sharing that vertex

Step 2:

The normal vector **N** for the scan line intersection point along the edge between vertices 1 and 2 can be obtained by vertically interpolating between edge endpoint normal:



Incremental methods are used to evaluate normals between scan lines and along each individual scan line.



$$N_4 = \frac{y_4 - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y_4}{y_1 - y_2} N_2$$

$$N_5 = \frac{y_5 - y_2}{y_3 - y_2} N_3 + \frac{y_3 - y_5}{y_3 - y_2} N_2$$

$$N_p = \frac{x_5 - x_p}{x_5 - x_4} N_4 + \frac{x_p - x_4}{x_5 - x_4} N_5$$

Step 3:

- Apply illumination model along each scan line to calculate projected pixel intensities for the surface points.

$$I = k_a I_a + \sum_{i=1}^{n} f(d_i) I_{li} \left[k_d (N \bullet L_i) + k_s (N \bullet H_i)^{n_s} \right]$$

- Produce more accurate results.
- Trade-off: Phong shading requires a lot of calculations.
- Bishop & Weimer developed **Fast Phong Shading** approximation using Taylor series expansion.
- **Fast Phong Shading** (See yourself)

Fast Phong Shading

- Fast Phong shading approximates the intensity calculations using a Taylor series expansion and Triangular surface patches.
- Since Phong shading interpolates normal vectors from vertex normal, we can express the surface normal N at any point (x, y) over a triangle as:

$$\mathbf{N=Ax + By+ C}$$

- Where A, B and C are determined from three vertex equations.
- $N_k=Ax_k + By_K + C$, k=1,2,3 for (x_k , y_k) vetex

Omitting the reflecting and attenuation parameters

- $I_{diff\,(x,\,y)} = \frac{L.N}{|L||N|} = \frac{L.(Ax+By+C)}{|L|.|Ax+By \quad |} = \frac{(L.A)x+(L.B)y+(L.C)}{|L|.|Ax+By \quad |}$ …………… (i)

Re writing this

- $I_{diff\,(x,\,y)} = \frac{ax+by}{\sqrt{(dx^2+exy+fy^2+gx+hy+i)}}$

where a,b,c,d…. Are used to represent the various dot product as:

$a=\frac{L.N}{|L|}$………… and so on

Finally, denominator of equation (i) can be express as Taylor series expansion and relations terms up to second degree in x, y. This yield:

$I_{diff\,(x,y)} = T_5x^2 + T_4xy + T_3y^2 + T_1y + T_0$

where each T_k is a function of parameters a, b, c, d….and so forth.

This method still takes twice as long as Gouraud Shading. Normal Phong Shading takes six to seven times that of Gouraud Shading.