Two Dimensional Algorithm

Output Primitives

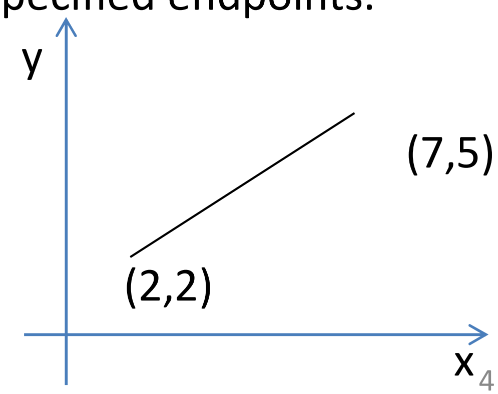
- The basic building blocks for pictures are referred to as output primitives.
- Output primitives are the geometric structures that used to describe the shapes and colors of the objects.
- They include character strings, and geometric entities such as points, straight lines, curved lines, polygons, circles etc.
- Points and straight line segments are the most basic components of a picture.
- In raster scan systems, a picture is completely specified by the set of intensities for the pixel positions in the display. The process that converts picture definition into a set of pixel intensity values is called scan conversion. This is done by display processor.

Lines and Points

- With raster-scan system, a point can be plotted, by simply turning on the electron beam at that point.
  - putpixel(20, 20, RED)
- And a random-scan system stores the point plotting instructions in the display list file.
  - LDXA 100 Load data value 100 into the X register.
  - LDYAP 450 Draw point at(100, 450)
- In raster scan systems line drawing is accomplished by calculating the intermediate positions along the line path between two specified endpoints.
- In random scan systems, line drawing is accomplished by retrieving line drawing commands from the display list.
- Scan lines are numbered consecutively from 0, starting at the bottom of the screen; and pixel columns are numbered from 0, from left to right across each scan line.

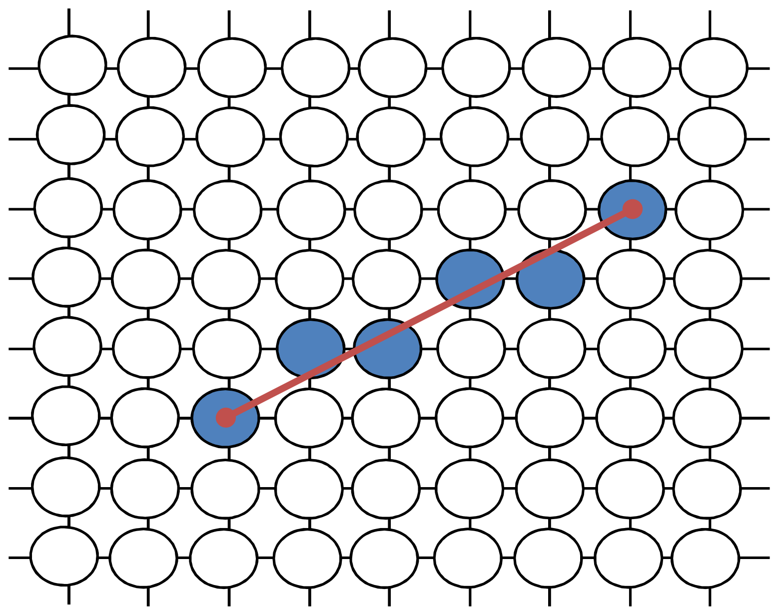
Points and Lines

- Points
  - Plotted by converting co-ordinate position to appropriate operations for the output device (e.g. : in CRT monitor, the electron beam is turned on to illuminate the screen phosphor at the selected location.)
- Line
  - A line segment in a scene is defined by the coordinate positions of the line end-points
  - Plotted by calculating intermediate positions along the line path between two specified endpoint positions.
  - Screen locations are referenced with integer values, so plotted positions may only approximate actual line positions between two specified endpoints.
  - E.g: position (10.48,20.51) → (10,21).



The Problem…..

- But what happens when we try to draw this on a pixel based display?



- How do we choose which pixels to turn on?

Consideration

- Considerations to keep in mind:
  - The line has to look good
    - Avoid *jaggies (i.e. the rounding of coordinates values to integers causes lines to be displayed with a stairstep appearance as represented in the figure)*
  - It has to be lightening fast!
    - How many lines need to be drawn in a typical scene?

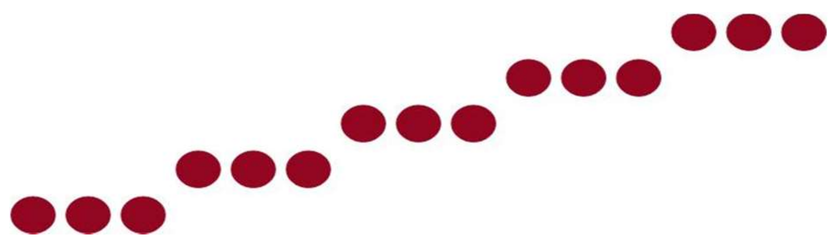


Figure: Stairstep effect (jaggies) produced when a line is generated as a series of pixel position.

Line Drawing Algorithms

- Direct use of Line Equation
- Digital Differential Analyzer Algorithm(DDA)
- Bresenham's Line Drawing Algorithm(BSA)

Direct Use of Line Equation

- The slope-intercept equation of a straight line is:

  $y = mx + b$

  where, m = slope of line and, b = y-intercept.

- For any two given points (x_1, y_1) and (x_2, y_2)
- slope (m) = $\dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

$b = y_1 - m.x_1$ i.e. from above equation

- At any point (x_k, y_k)

  $y_k = mx_k + b$ ………… 1
- At (x_{k+1}, y_{k+1}),

  $y_{k+1} = mx_{k+1} + b$ ……….. 2
- Subtracting 1 from 2 we get,

  $y_{k+1} - y_k = m(x_{k+1} - x_k)$
- Here $(y_{k+1} - y_k)$ incremental in y as corresponding increment in x.

  therefore, $\Delta y = m.\Delta x$

  or $\Delta x = \dfrac{\Delta y}{m}$
- For incremental algorithm in line drawing ,
  - Increment *x* by 1
  - Computer corresponding *y* and display pixel at position $(x_i, round(y_i))$

For |m| <1

- Set Δx proportional to horizontal deflection voltage. Then

$$\Delta y = m.\Delta x$$

For |m|>1

- Set Δy set proportional to vertical deflection voltage. Then

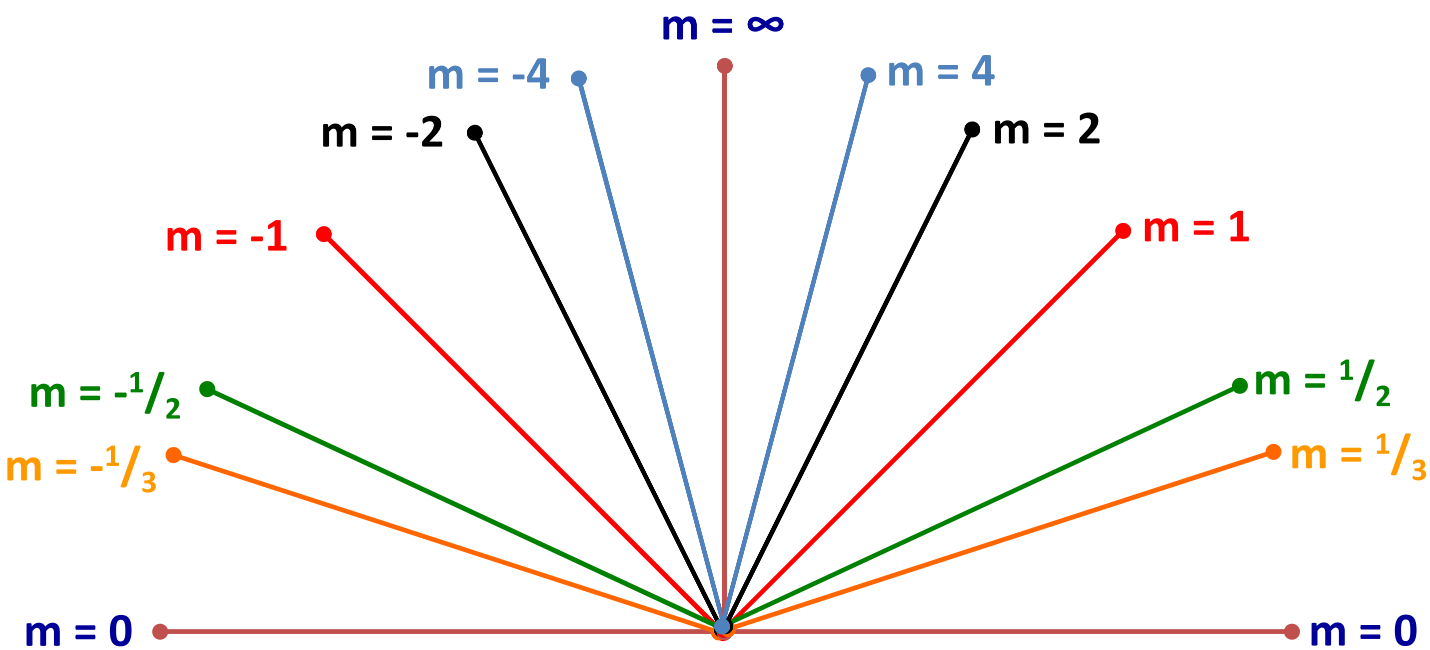
$$\Delta x = \frac{\Delta y}{m}$$

For |m|=1

- Δx = Δy ➔ horizontal and vertical deflection voltages are equal

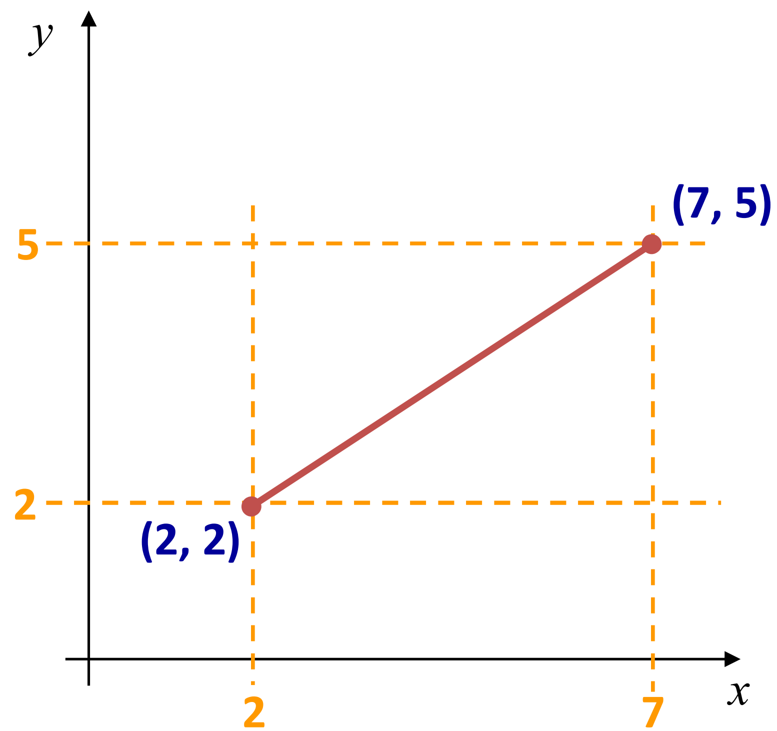
Lines and Slopes

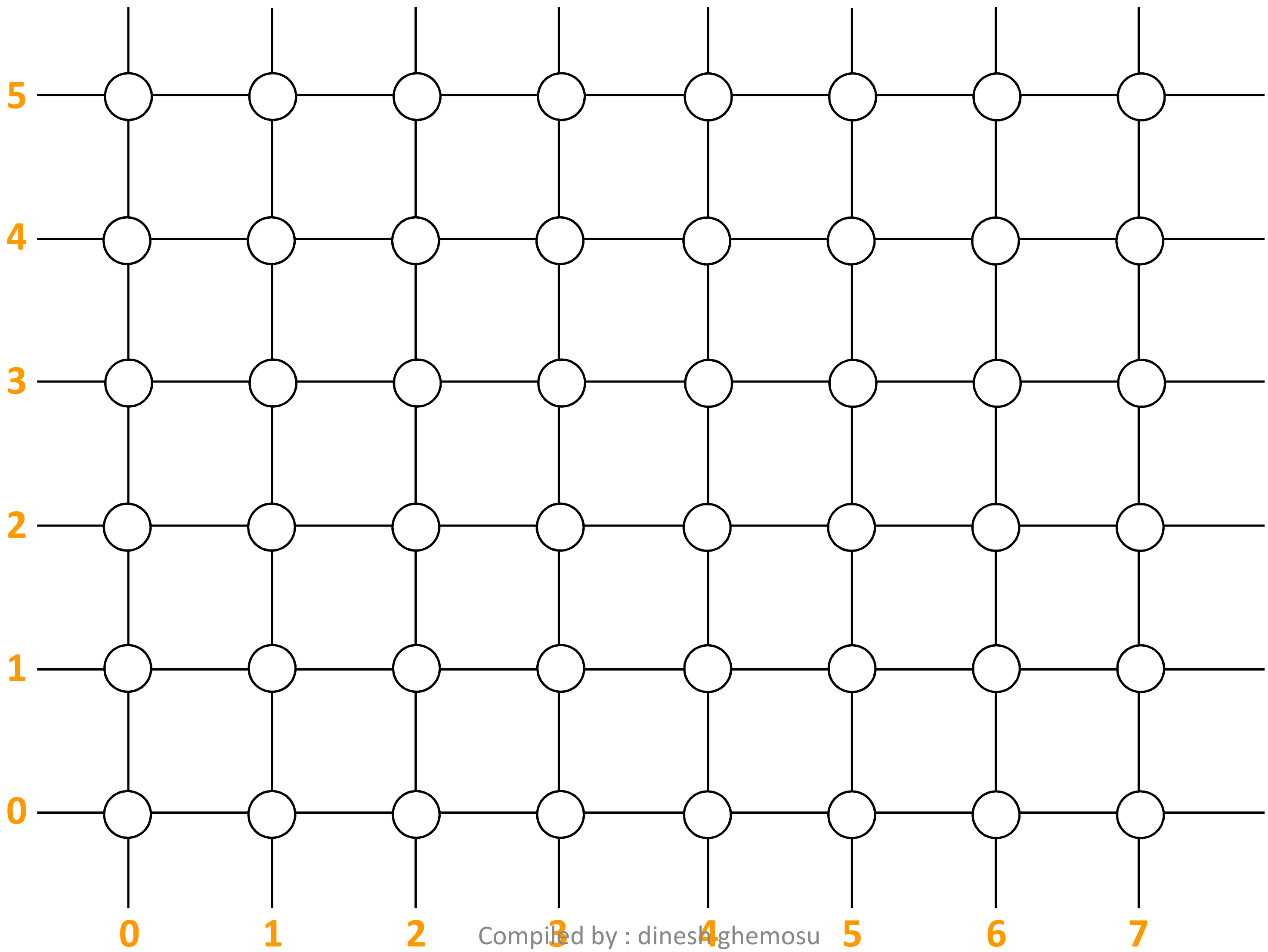
- The slope of a line (m) is defined by its start and end coordinates
- The diagram below shows some examples of lines and their slopes

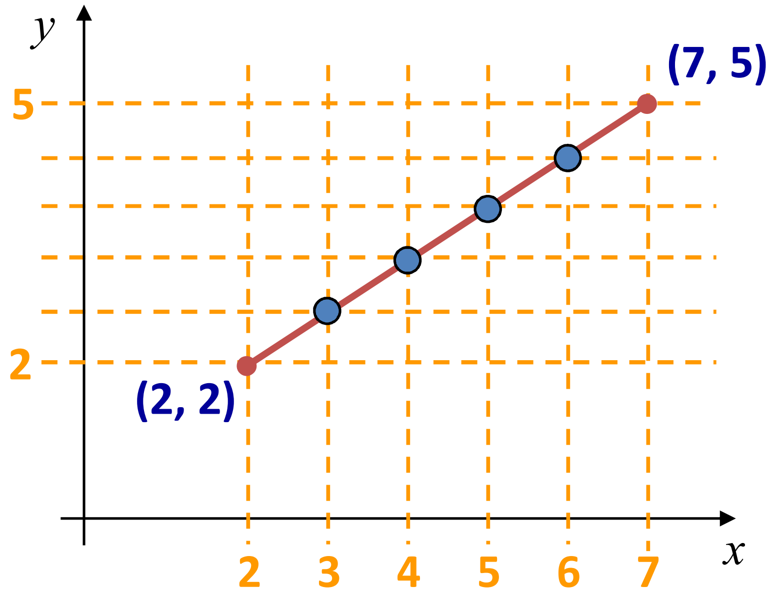


A very simple solution

- We could simply work out the corresponding y coordinate for each unit x coordinate
- Let's consider the following example:




5
4
3
2
1
0
0
1
2
3
4
5
6
7



- First work out m and b:

$$m = \frac{5-2}{7-2} = \frac{3}{5}$$

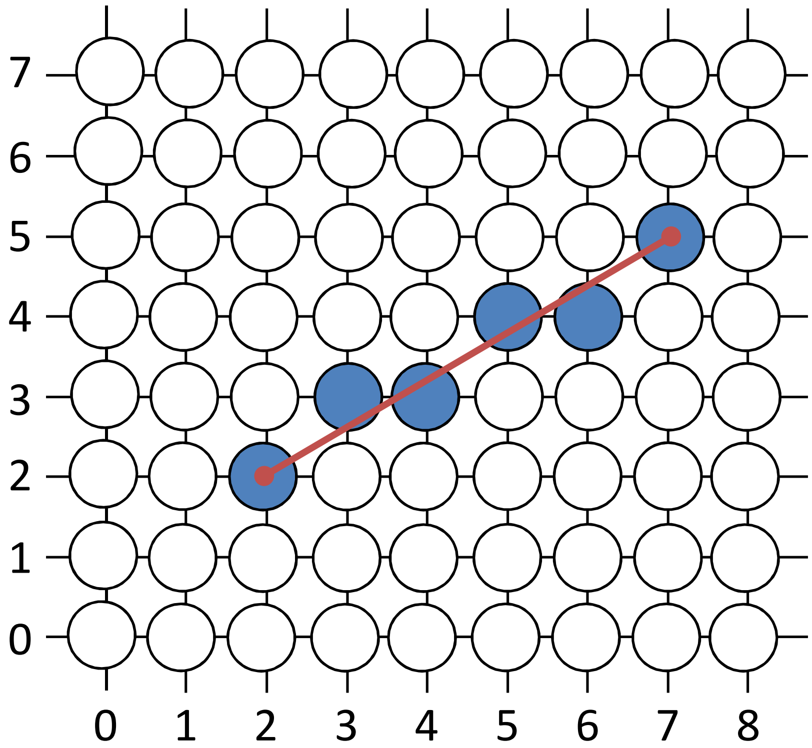
$$b = 2 - \frac{3}{5} * 2 = \frac{4}{5}$$

Now for each x value work out the y value:

$$y(3) = \frac{3}{5} \cdot 3 + \frac{4}{5} = 2\frac{3}{5} \qquad y(4) = \frac{3}{5} \cdot 4 + \frac{4}{5} = 3\frac{1}{5}$$

$$y(5) = \frac{3}{5} \cdot 5 + \frac{4}{5} = 3\frac{4}{5}$$

- Now just round off the results and turn on these pixels to draw our line.



$$y(3) = 2\frac{3}{5} \approx 3$$

$$y(4) = 3\frac{1}{5} \approx 3$$

$$y(5) = 3\frac{4}{5} \approx 4$$

$$y(6) = 4\frac{2}{5} \approx 4$$

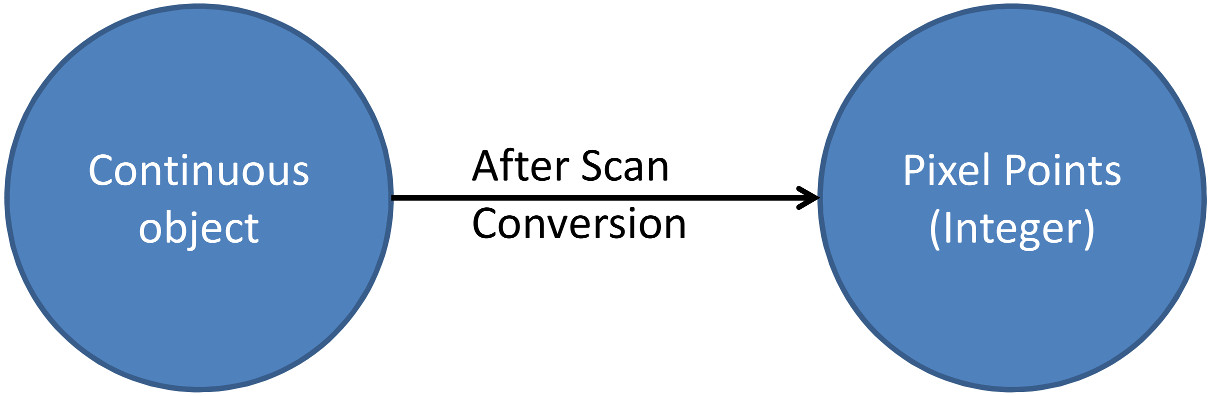
Some Terms

Vector Generation

- On our raster system, we can generate images by turning the pixels ON or OFF. The process of turning ON of the pixel for a line segment is called as Vector Generation.

Scan Conversion

- The process of conversion of the rasterized picture stored in a frame buffer to the rigid display pattern of video is called as scan conversion.



DDA Algorithm

- Digital Differential Analyzer (DDA) is a scan conversion line drawing algorithm based on calculating either Δx or Δy from the equation

$$\Delta y = m. \Delta x$$

- We sample the line at unit intervals in one coordinate and determine the corresponding integer values nearest the line path in another co-ordinate.

Consider a line with positive slope and proceed from left to right

- If m <= 1, we sample x-coordinate. So Δx =1 and compute each successive y values as:
  - $y_{k+1} = y_k + m$ where $m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} = \frac{\Delta y}{\Delta x}$ $\quad \Delta x = x_{k+1} - x_k = 1$
  - Here k takes from starting point and increase by 1 until final end point.
  - m can be any real value between 0 and 1.
- If m>1, we sample Δy =1 and calculate corresponding x value as:

$$x_{k+1} = x_k + \frac{1}{m} \qquad \Delta y = y_{k+1} - y_k = 1$$

Consider a line with positive slope that proceed from right to left.

- If |m | <=1, we sample Δx = -1 and calculate

$$y_{k+1} = y_k - m$$

- If |m|>1, we sample Δy = -1 and calculate

$$x_{k+1} = x_k - \frac{1}{m}$$

DDA Algorithm

Step 1: Input the line endpoints and store the left endpoint in (x_1, y_1) and right endpoint in (x_2, y_2).

Step 2: Calculate the values of dx and dy, $dx = x_2 - x_1$, $dy = y_2 - y_1$.

Step 3: if(abs(dx) > abs(dy))

- *steplength* = abs(dx)

else

- *steplength* = abs(dy)

Step 4: Calculate the values of x-increment and y-increment.

- xIncrement = dx/ *steplength*
- yIncrment = dy/ *steplength*

Step 5:Set $x = x_1$ and $y = y_1$

Step 6: Plot(x, y).

Step 7: for k=1 to *steplength* do

- x = x + xIncrement
- y = y + yIncrement
- Perform round off, and plot each calculated (x, y) i.e. Plot(round(x), round(y)).

Step 8: End

Advantages of DDA Algorithm

- Faster than direct use of the line equation since it calculates the line without any floating point multiplication
- Simplest algorithm since does not require special skills for its implementation

DDA Algorithm

Step 1: Input the line endpoints and store the left endpoint in (x_1, y_1) and right endpoint in (x_2, y_2).

Step 2: Calculate the values of dx and dy, $dx = x_2 – x_1$, $dy = y_2 – y_1$.

Step 3: if(abs(dx) > abs(dy))

steplength = abs(dx)

else

steplength = abs(dy)

Step 4: Calculate the values of x-increment and y-increment.

xIncrement = dx**/** *steplength*

yIncrment = dy**/** *steplength*

Step 5:Set $x = x_1$ and $y = y_1$

Step 6: Plot(x, y).

Step 7: for k=1 to *steplength* do

x = x + xIncrement

y = y + yIncrement

Perform round off, and plot each calculated (x, y) i.e. Plot(round(x), round(y)).

Step 8: End

Disadvantages of DDA Algorithm

- It is orientation dependent, due to this, the point accuracy is poor.
- A floating point addition is still needed in determining each successive point which is time consuming.
- Involves, continuous round offs which can cause the calculated pixel positions to drift away from the actual line path.

Digitize the line with endpoints (1, 5) and (7, 2) using DDA algorithm.

Here, dx = 7-1=6, and dy = 2-5 = -3,

So, steplength = 6   (since abs(dx)>abs(dy)).

Therefore, xIncrement = dx/steplength = 6/6 = 1, and

yIncrement = dy/steplength = -3/6 = -1/2 = -0.5

Based on these values the intermediate pixel calculation is shown in the table below.

k	x_{k+1}	y_{k+1}	(x_{k+1}, y_{k+1})	Plot in screen (x_{k+1}, y_{k+1})
1	2	4.5	(2, 4.5)	(2, 5)
2	3	4	(3, 4)	(3, 4)
3	4	3.5	(4, 3.5)	(4,4)
4	5	3	(5,3)	(5,3)
5	6	2.5	(6,2.5)	(6,3)
6	7	2	(7,2)	(7,2)

Questions

- **Consider a line from (0, 0) to (6, 7). Using simple DDA algorithm, rasterize this line.**
- ***Digitize the line with endpoints (1, -6) and (4, 4) using DDA algorithm.***
- ***Digitize the line with endpoints (1, 6), (6, 10) using DDA algorithm.***
- ***Trace DDA algorithm for line with endpoints (1, 2), (5, 6).***
- ***Trace DDA algorithm for endpoints (1, 7), (6, 3).***