Optimisation Methods: Assignment 1

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The purpose of this first assignment is to refresh some notions of linear algebra and their implementation in Python.

It is due for **Wednesday 26th February at 2 pm**. Only the second part (Python implementation) will be graded but you can also submit your answers for the first part and the latter will also be corrected (though not graded). For the first part, you need to submit a Python notebook on iCorsi and follow these instructions:

- Put the answers for each part of the question into separate cells.
- Before each cell, put a markdown header that says which part of the question comes in the following cell.
- Good coding style is part of the grade: add clear comments throughout the code when it is necessary.
- Before you submit your notebook, make sure it runs.

Part 0: reading

Read Chapter 1 of the book "Numerical Optimisation" (Nocedal & Wright).

Part 1: exercise

1. What is the definition of a symmetric matrix?
2. What are the definitions of eigenvalues and eigenvectors?
3. What is a singular matrix?
4. What is the definition of a positive definite matrix? definition of a positive semi-definite matrix?
5. Let $M \in \mathbb{R}^{n\times n}$ a nonsingular square matrix and let $A = M^T M$ (here the supscript ^{T} means that we take the transpose). Prove that A is positive definite. *Hint: recall that* $x^T x = \|x\|^2$ *for* $x \in \mathbb{R}^n$.

Part 2: programming problem

In this problem you will need to use the `numpy` library. You can refer to the documentation for finding the relevant functions.

1. Create a vector b with the elements

$$b = \begin{bmatrix} -3 \\ -8 \\ 7 \end{bmatrix}$$

   and print it.

2. Create a matrix A with the elements

$$A = \begin{bmatrix} -10 & 3 & 11 \\ 3 & -5 & -4 \\ 11 & -4 & -7 \end{bmatrix}$$

   and print it.

3. Is the matrix A symmetric? Write your answer in a cell in the notebook and then check it with Python.

4. Compute x as the solution of the linear system $Ax = b$ and print it.

5. To check if the solution is correct, compute r as the product Ax and print it.

6. Print the difference of r and b as $\|r - b\|$. (This should be zero. Is it? Briefly discuss this in the cell for this question.)

7. Print the determinant of A.

8. Repeat questions 2-7 for the matrix $A1$ with the elements

$$A = \begin{bmatrix} -1 & 3000 & 1 \\ 2000 & -1.5 & -2.5 \\ -1999 & -2998.5 & 1.5 \end{bmatrix}$$

   In a cell, describe and explain what you observe.