

# ToC Spring 2025, HW2

Due: Monday, 7 April 2025, 23:55

## Decidability

Please complete the following exercises:

1. **(50pt)** Let  $L$  be a decidable language. Prove that its complement  $L'$  is decidable.
2. **(50pt)** Let  $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ , that is, the set of all the encodings of a DFA such that the language recognized by the automaton is the set of all words (over the alphabet  $\Sigma$ ). Show that  $ALL_{DFA}$  is decidable.

**Note:** To show that a language is decidable, you need to give a Turing machine that is a decider for it. In your solutions, check that you have covered the following points:

- You have given pseudocode/explanation for how your Turing machine works.
- You have proved that your TM is a decider, i.e., it will always halt.
- You have proved that the given string belongs to the prescribed language **iff** it is accepted by your TM (note that this is equivalence, so you need to prove both directions).