

Program

Week	Lecture, Lecture hall Chip	
1	14 Feb	Today, Fuzzing
2	21 Feb	No lecture
3	28 Feb	Test Case Generation
4	7 Mar	Concolic Execution
5	14 Mar	Mutation Analysis
6	21 Mar	State Machine Learning
7	28 Mar	Model-Based Testing
8	4 Apr	Binary Analysis
9	11 Apr	What's next?

Lectures on Tuesday, 13:45 till 15:45

Office hours Sicco Verwer and Andy Zaidman (online)

Skype: live:9e3207a8ea5fdf16, azaidman

Thursdays 10:00 till 12:00

TODAY: (CONCRETE & SYMBOLIC) CONCOLIC EXECUTION

From Wikipedia

```
1. void f(int x, int y) {  
2.     int z = 2*y;  
3.     if (x == 100000) {  
4.         if (x < z) {  
5.             assert(0); /* error */  
6.         }  
7.     }  
8. }
```

“Simple random testing, trying random values of x and y , would require an impractically large number of tests to reproduce the failure.”

Q. Why?

From Wikipedia

```
1. void f(int x, int y) {  
2.     int z = 2*y;  
3.     if (z > 100000) {  
4.         System.out.println("z = " + z);  
5.     }  
6. }  
7.  
8. }
```

AFL finds it in milliseconds

So does EvoSuite....

“Simple random testing, trying random values of x and y , would require an impractically large number of tests to reproduce the failure.”

Q. Why?

From Wikipedia

```
1. void f(int x, int y) {  
2.     int z = 2*y;  
3.     if (z > 100000) {  
4.         // ...  
5.     }  
6. }  
7.  
8. }
```

AFL finds it in milliseconds

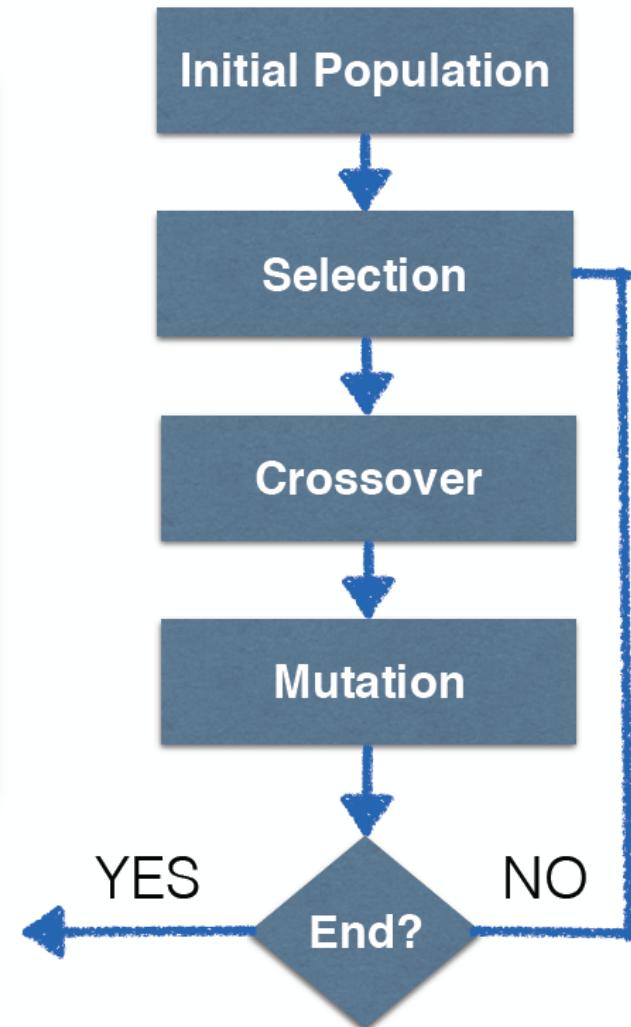
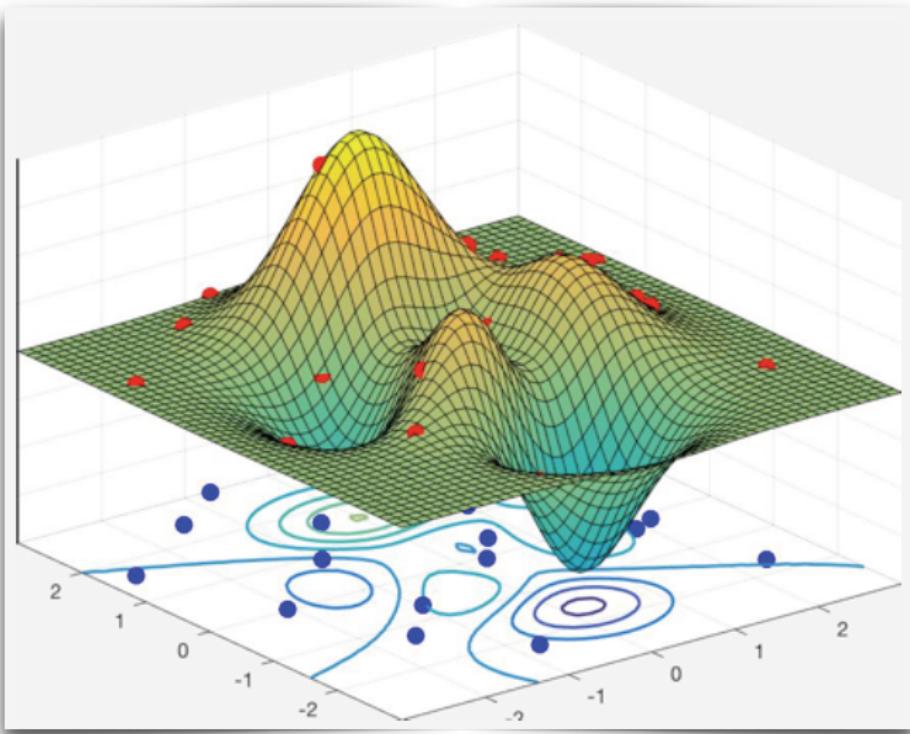
So does

Why?

“Simple random testing, trying random values of x and y , would require an impractically large number of tests to reproduce the failure.”

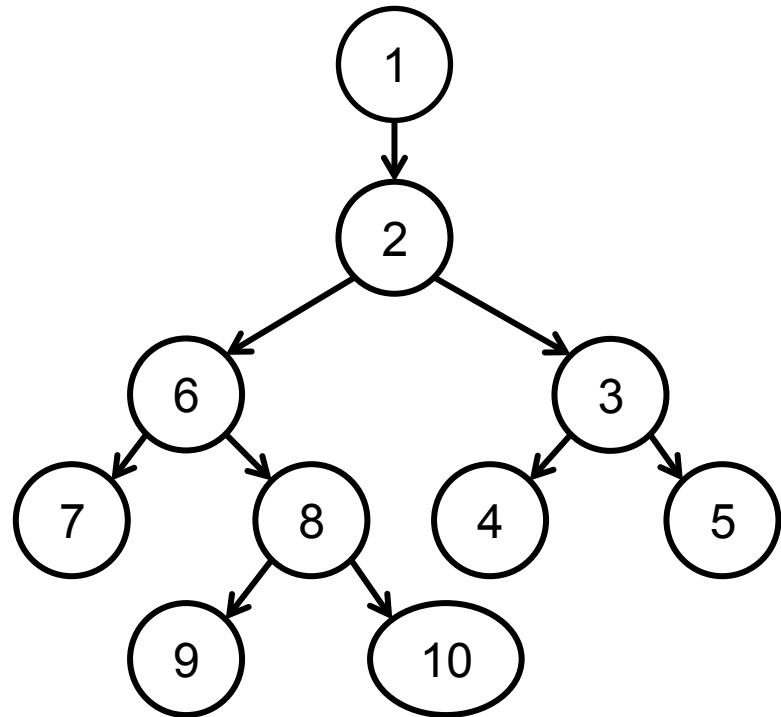
Q. Why?

Genetic Algorithms



Genetic Algorithms and Code

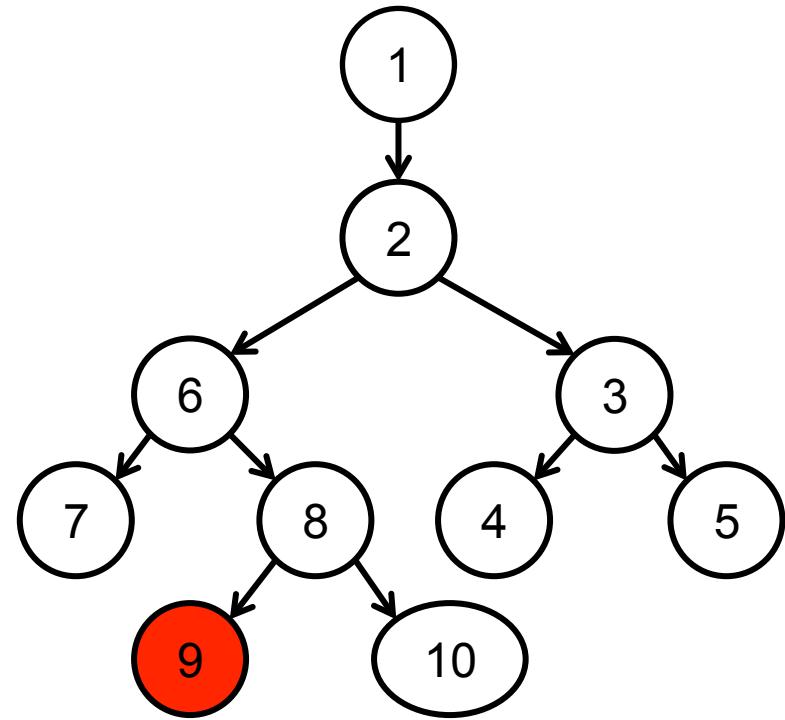
```
class Triangle {  
    void computeTriangleType() {  
1.        if (isTriangle()) {  
2.            if (side1 == side2) {  
3.                if (side2 == side3)  
4.                    type = "EQUILATERAL";  
5.                else  
6.                    type = "ISOSCELES";  
7.                } else {  
8.                    if (side1 == side3) {  
9.                        type = "ISOSCELES";  
10.                   } else {  
11.                       if (side2 == side3)  
12.                           type = "ISOSCELES";  
13.                       else  
14.                           checkRightAngle();  
15.                   }  
16.               }  
17.           }  
18.       }  
19.   }
```



Dependency Graph

Fitness: does it reach the target?

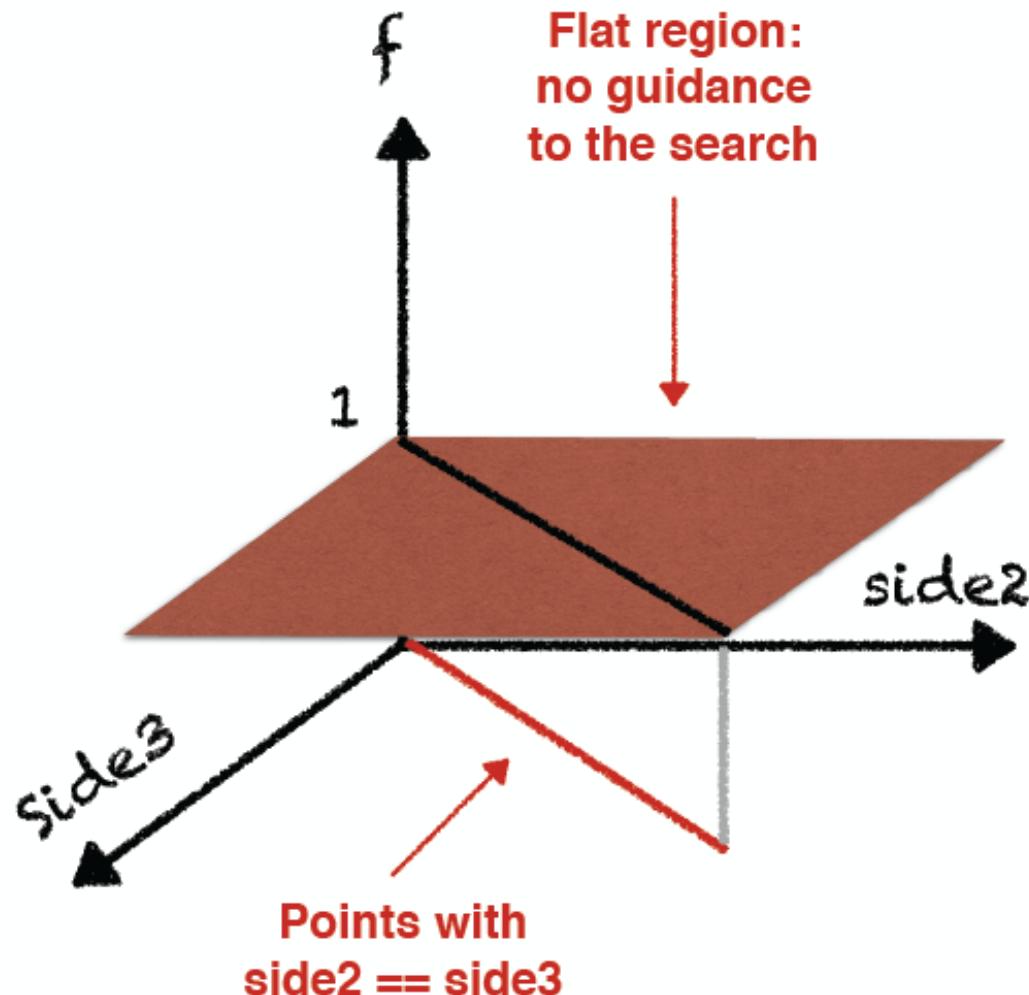
```
class Triangle {  
    void computeTriangleType() {  
1.        if (isTriangle()) {  
2.            if (side1 == side2) {  
3.                if (side2 == side3)  
4.                    type = "EQUILATERAL";  
5.                else  
6.                    type = "ISOSCELES";  
7.                } else {  
8.                    if (side1 == side3) {  
9.                        type = "ISOSCELES";  
10.                   } else  
11.                      type = "SCALENE";  
12.                }  
13.            }  
14.        }  
15.    }  
16.}
```



A simple Fitness Function could be:

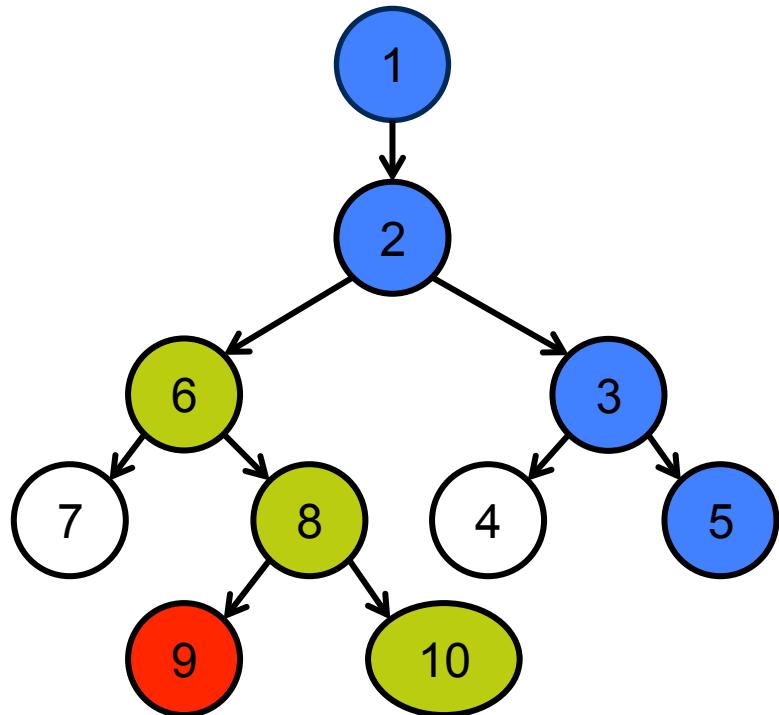
$$f(\text{side2}; \text{side3}) = \begin{cases} 0 & \text{if } \text{side2} == \text{side3} \\ 1 & \text{if } \text{side2} != \text{side3} \end{cases}$$

Result: a very flat optimization surface



Measuring distance to target

```
class Triangle {  
    void computeTriangleType() {  
1.        if (isTriangle()) {  
2.            if (side1 == side2) {  
3.                if (side2 == side3)  
4.                    type = "EQUILATERAL";  
5.                else  
6.                    type = "ISOSCELES";  
7.                } else {  
8.                    if (side1 == side3) {  
9.                        type = "ISOSCELES";  
10.                   } else  
11.                      if (side2 == side3)  
12.                          type = "ISOSCELES";  
13.                      else  
14.                          checkRightAngle();  
15.                }  
16.            }  
17.        }  
18.    }
```



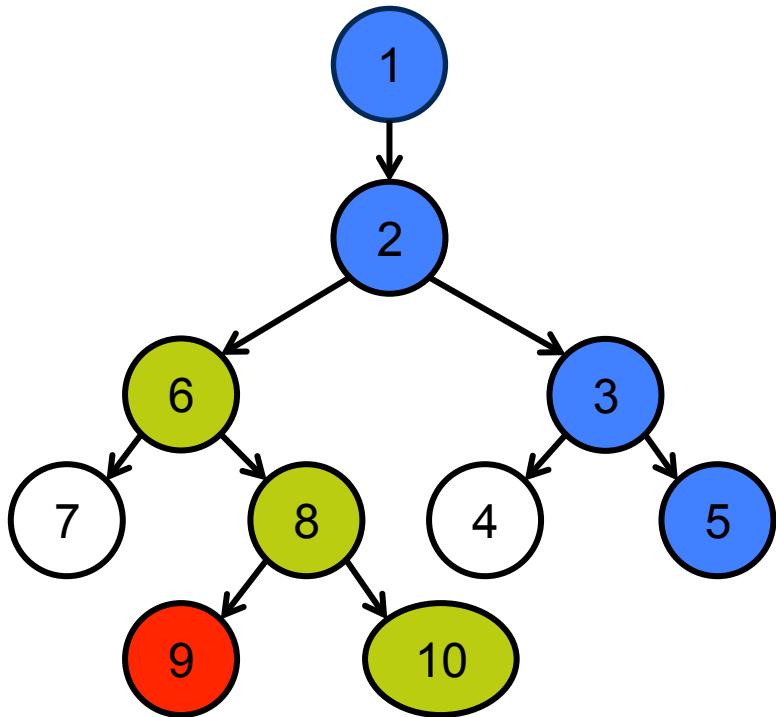
Some traces:

$x_1 = (2, 2, 3)$ Path(x_1) = $\langle 1, 2, 3, 5 \rangle$
 $x_2 = (2, 3, 5)$ Path(x_2) = $\langle 1, 2, 6, 8, 10 \rangle$

Fitness: Approach Level

The number of nodes between the target node and the execution trace:

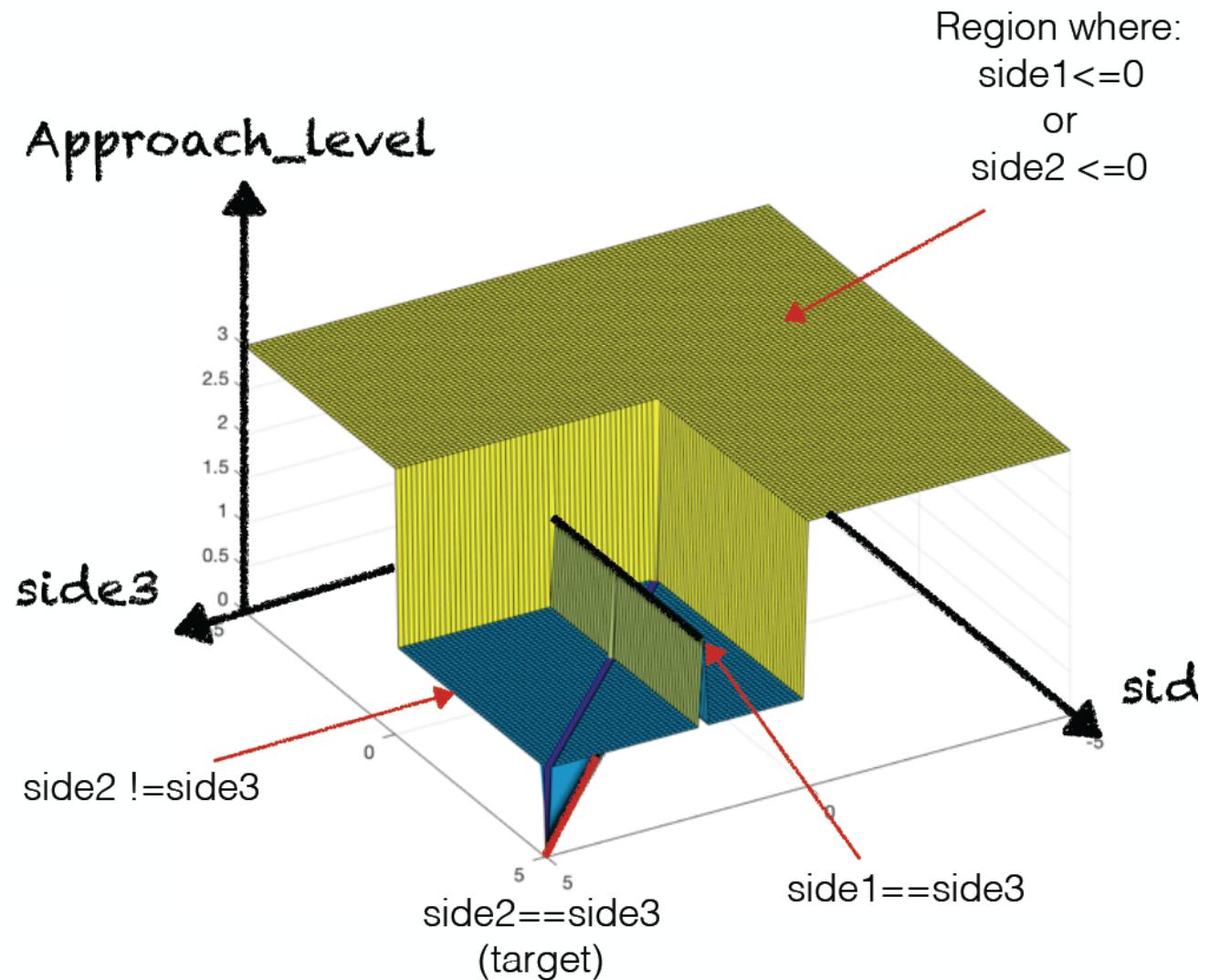
$$\begin{aligned}f(x_1) &= 2 \\f(x_2) &= 0\end{aligned}$$



Some traces:

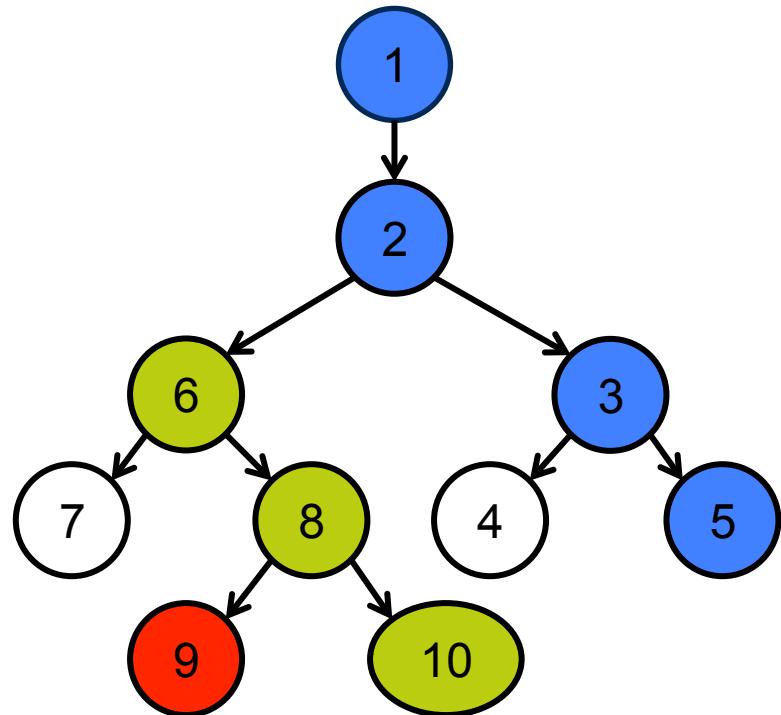
$$\begin{aligned}x_1 &= (2, 2, 3) \text{ Path}(x_1) = <1, 2, 3, 5> \\x_2 &= (2, 3, 5) \text{ Path}(x_2) = <1, 2, 6, 8, 10>\end{aligned}$$

Result: still too flat....



Fitness: which one is closer x2 or x3?

```
class Triangle {  
    void computeTriangleType() {  
1.        if (isTriangle()) {  
2.            if (side1 == side2) {  
3.                if (side2 == side3)  
4.                    type = "EQUILATERAL";  
5.                else  
6.                    type = "ISOSCELES";  
7.                } else {  
8.                    if (side1 == side3) {  
9.                        type = "ISOSCELES";  
10.                   } else  
11.                      if (side2 == side3)  
12.                          type = "ISOSCELES";  
13.                      else  
14.                          checkRightAngle();  
15.                }  
16.            }  
17.        }  
18.    }
```



Some traces:

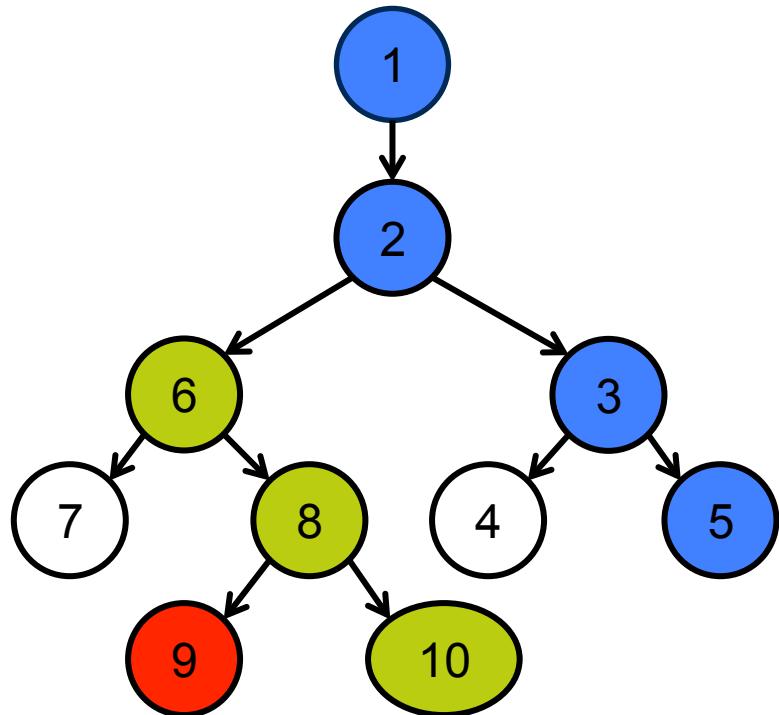
$x_1 = (2, 2, 3)$ Path(x_1) = $<1, 2, 3, 5>$

$x_2 = (2, 3, 5)$ Path(x_2) = $<1, 2, 6, 8, 10>$

$x_3 = (2, 3, 10)$ Path(x_3) = $<1, 2, 6, 8, 10>$

Fitness: which one is closer x2 or x3?

```
class Triangle {  
    void computeTriangleType() {  
1.        if (isTriangle()) {  
2.            if (side1 == side2) {  
3.                if (side2 == side3)  
4.                    type = "EQUILATERAL";  
  
5.                line 8:  
6.                x2: if 3 == 5  
7.                x3: if 3 == 10  
  
8.            if (side2 == side3)  
9.            target      type = "ISOSCELES";  
10.           else  
               checkRightAngle();  
    }  
}  
}
```

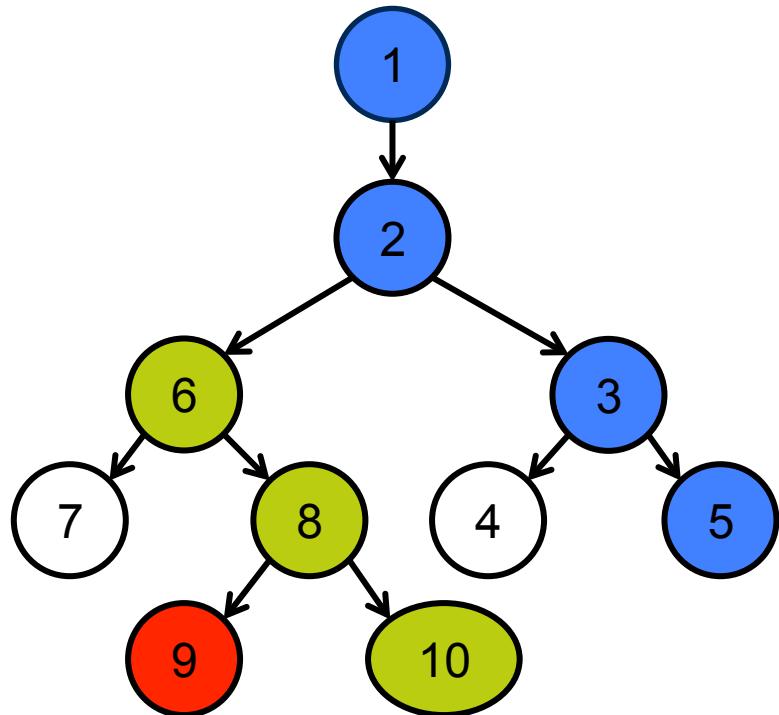


Some traces:

$x_1 = (2, 2, 3)$ Path(x_1) = $\langle 1, 2, 3, 5 \rangle$
 $x_2 = (2, 3, 5)$ Path(x_2) = $\langle 1, 2, 6, 8, 10 \rangle$
 $x_3 = (2, 3, 10)$ Path(x_3) = $\langle 1, 2, 6, 8, 10 \rangle$

Fitness: which one is closer x2 or x3?

```
class Triangle {  
    void computeTriangleType() {  
1.        if (isTriangle()) {  
2.            if (side1 == side2) {  
3.                if (side2 == side3)  
4.                    type = "EQUILATERAL";  
  
5.                branch distance:  
6.                f(x2) = abs(3-5) = 2  
7.                f(x3) = abs(3-10) = 7  
  
8.            if (side2 == side3)  
9.                target = "ISOSCELES";  
10.           else  
11.               checkRightAngle();  
12.           }  
13.       }  
14.   }  
15. }
```



Some traces:

$x_1 = (2, 2, 3)$ Path(x_1) = $\langle 1, 2, 3, 5 \rangle$
 $x_2 = (2, 3, 5)$ Path(x_2) = $\langle 1, 2, 6, 8, 10 \rangle$
 $x_3 = (2, 3, 10)$ Path(x_3) = $\langle 1, 2, 6, 8, 10 \rangle$

Normalized branch distance

- For numerical and Boolean variables

Condition $c = \text{atomic predicate}$	Distance $BD(c) = d / (d + 1)$
a	$d = \{0 \text{ if } a == \text{true}; K \text{ otherwise}\}$
$\neg a$	$d = \{K \text{ if } a == \text{true}; 0 \text{ otherwise}\}$
$a == b$	$d = \{0 \text{ if } a == b; \text{abs}(a - b) + K \text{ otherwise}\}$
$a != b$	$d = \{0 \text{ if } a != b; K \text{ otherwise}\}$
$a < b$	$d = \{0 \text{ if } a < b; a - b + K \text{ otherwise}\}$
$a \leq b$	$d = \{0 \text{ if } a \leq b; a - b + K \text{ otherwise}\}$
$a > b$	$d = \{0 \text{ if } a > b; b - a + K \text{ otherwise}\}$
$a \geq b$	$d = \{0 \text{ if } a \geq b; b - a + K \text{ otherwise}\}$

Normalized branch distance

- For Strings

Condition $c = \text{atomic predicate}$	Distance $BD(c) = d / (d + 1)$
$a == b$	$d = \{0 \text{ if } a == b; \text{edit_dist}(a, b) + K \text{ otherwise}\}$
$a != b$	$d = \{0 \text{ if } a != b; K \text{ otherwise}\}$
$a < b$	$d = \{0 \text{ if } a < b; a[j] - b[j] + K \text{ otherwise}\}$
$a <= b$	$d = \{0 \text{ if } a <= b; a[j] - b[j] + K \text{ otherwise}\}$
$a > b$	$d = \{0 \text{ if } a > b; b[j] - a[j] + K \text{ otherwise}\}$
$a >= b$	$d = \{0 \text{ if } a >= b; b[j] - a[j] + K \text{ otherwise}\}$

- j is the position of the first different character such that $a[j] \neq b[j]$, while $a[i] == b[i]$ for $i < j$ ($a[j]-b[j]$) is set to zero if $a == b$)

Normalized branch distance

- For Composite Predicates

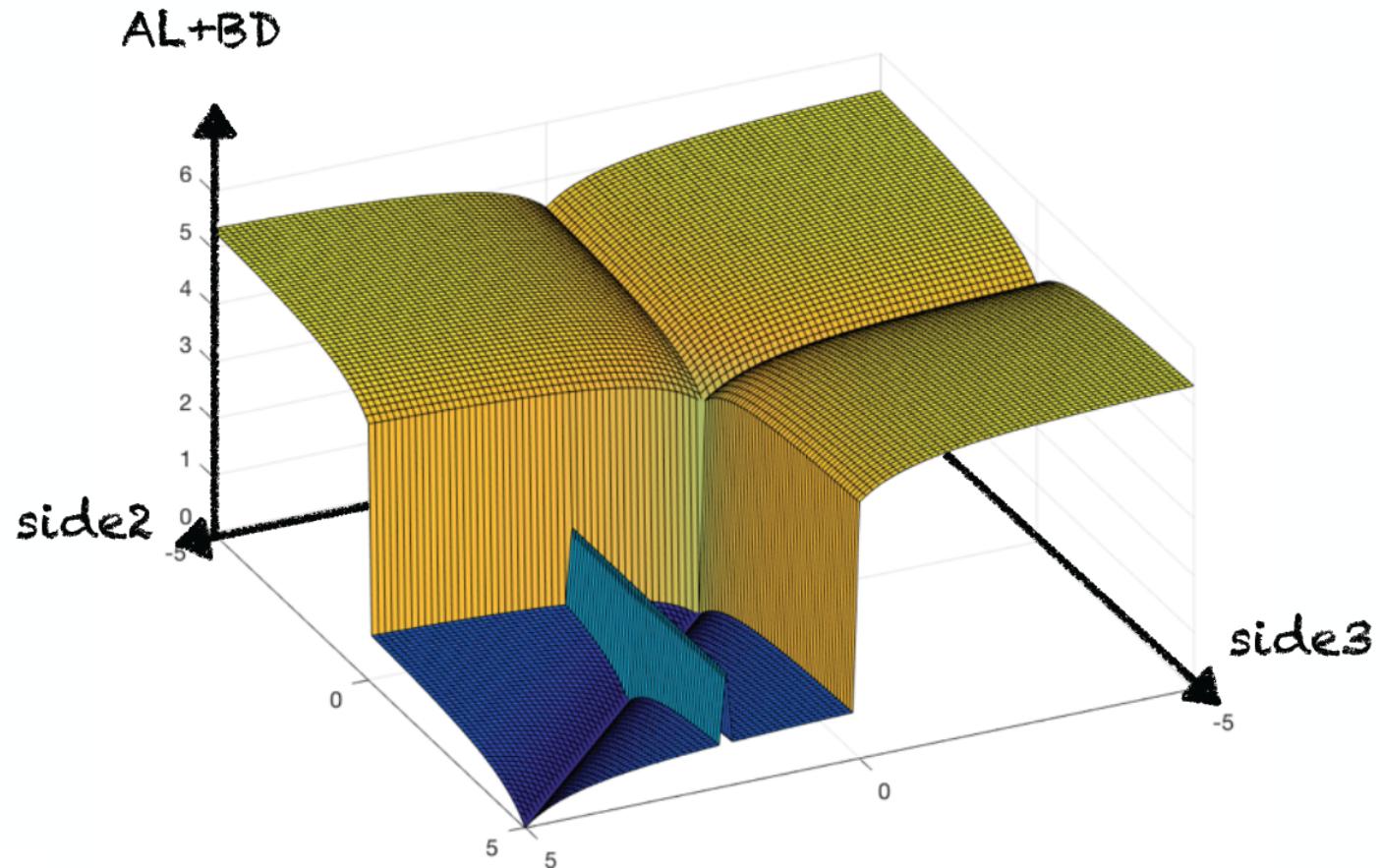
Condition $c = \text{composite predicate}$	Distance $BD(c) = d / (d + 1)$
$\neg p$	Negation is propagated inside p
$p \wedge q$	$d = d(p) + d(q)$
$p \vee q$	$d = \min(d(p), d(q))$
$p \oplus q = p \wedge \neg q \vee \neg p \wedge q$	$d = \min(d(p)+d(\neg q), d(\neg p)+d(q))$

- The final fitness is a combination of Approach Level and Branch Distance:

$$f(x) = \text{approach_level}(x, t) + \text{branch_distance}(x, t)$$

where t is the target

Result: a nice smooth optimization surface!



EvoSuite and AFL

- EvoSuite computes distances using such a fitness function
 - *Future generations are guaranteed to get closer to the target*
- What AFL does exactly is not very clear, but see:
 - <http://lcamtuf.blogspot.nl/2014/08/binary-fuzzing-strategies-what-works.html>
 - Walking bit/byte flips, simple arithmetic, ..
 - *But, from http://lcamtuf.coredump.cx/afl/technical_details.txt:*
 - *“AFL generally does not try to reason about the relationship between specific mutations and program states; the fuzzing steps are nominally blind, and are guided only by the evolutionary design of the input queue”*

EvoSuite and AFL

- EvoSuite computes distances using such a fitness function
 - *Future generations are guaranteed to get closer to the target*
- What AFL does exactly is not very clear, but see:
 - <http://lcamtuf.coredump.cx/afl/>
 - <https://github.com/lcamtuf/testng-afl>

Challenge:

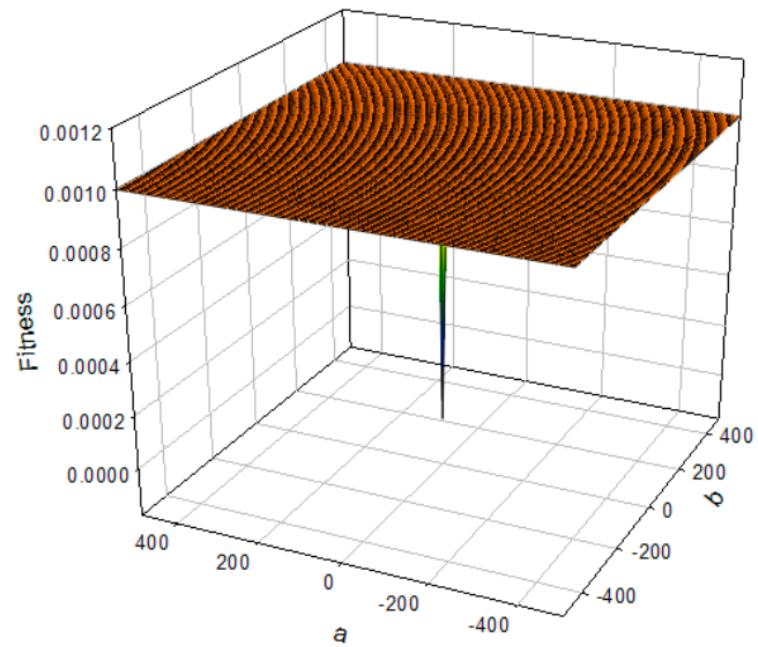
Find small code samples (+- 20 lines) that takes amazingly long to Fuzz/Test!

Code for AFL/EvoSuite is on Git

One famous example: *the flag problem*

```
void testme(int a, int b, ....)
{
    ...
    flag = (a == 0 && b == 0);

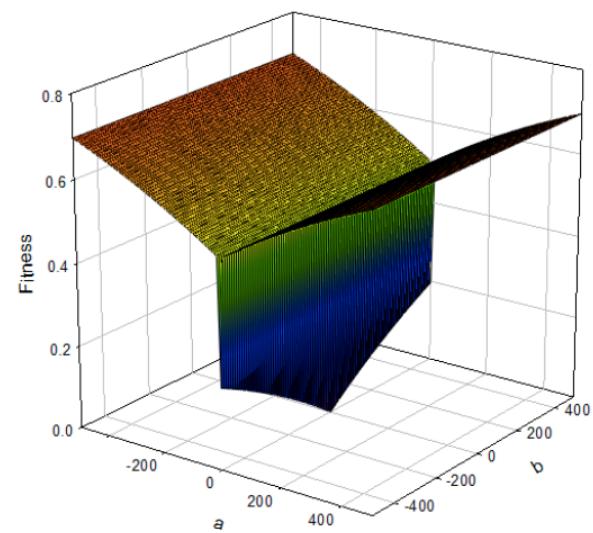
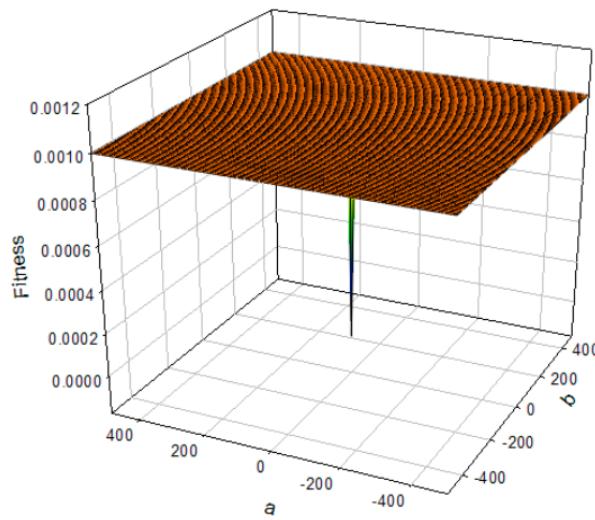
    if (flag) {
        ...
    }
}
```



With a fix, but does this always work?

```
void testme(int a, int b, ....)
{
    ...
    flag = (a == 0 && b == 0);
    if (flag) { [REDACTED LINE] }
```

```
void testme(int a, int b, ....)
{
    ...
    flag = (a == 0 && b == 0);
    if (a == 0 && b == 0) { [REDACTED LINE] }
```



TODAY: (CONCRETE & SYMBOLIC) CONCOLIC EXECUTION

Satisfiability & Validity

$$p \vee q \Rightarrow q \vee p$$

$$p \vee q \Rightarrow q$$

$$p \wedge \neg q \wedge (\neg p \vee q)$$

ϕ	A	B	$\neg A$	$A \vee B$	$A \wedge \neg A$	$A \Rightarrow B$	$A \Rightarrow (B \vee A)$
$\mathcal{M}_1(\phi)$	\perp	\perp	\top	\perp	\perp	\top	\top
$\mathcal{M}_2(\phi)$	\perp	\top	\top	\top	\perp	\top	\top
$\mathcal{M}_3(\phi)$	\top	\perp	\perp	\top	\perp	\perp	\top
$\mathcal{M}_4(\phi)$	\top	\top	\perp	\top	\perp	\top	\top

Satisfiability & Validity

$$p \vee q \Rightarrow q \vee p \quad \text{VALID}$$

$$p \vee q \Rightarrow q \quad \text{SATISFIABLE}$$

$$p \wedge \neg q \wedge (\neg p \vee q) \quad \text{UNSATISFIABLE}$$

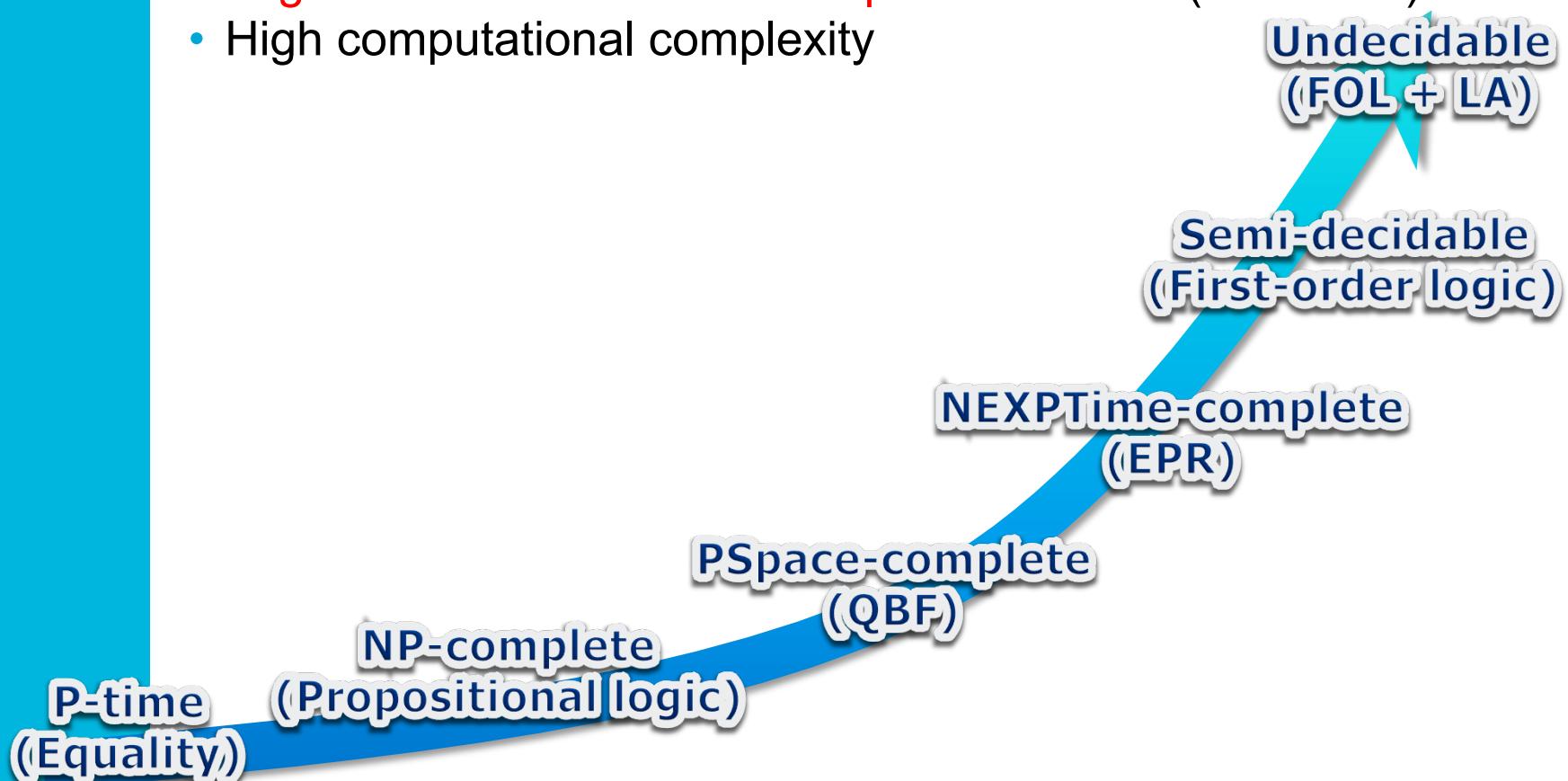
ϕ	A	B	$\neg A$	$A \vee B$	$A \wedge \neg A$	$A \Rightarrow B$	$A \Rightarrow (B \vee A)$
$\mathcal{M}_1(\phi)$	\perp	\perp	\top	\perp	\perp	\top	\top
$\mathcal{M}_2(\phi)$	\perp	\top	\top	\top	\perp	\top	\top
$\mathcal{M}_3(\phi)$	\top	\perp	\perp	\top	\perp	\perp	\top
$\mathcal{M}_4(\phi)$	\top	\top	\perp	\top	\perp	\top	\top

SAT solvers

- Decades of experience in solving NP-hard problems by translating them to Satisfiability
 - *problem has solution iff formula is satisfiable*
- Yearly competitions pushing the state-of-the-art
 - highly optimized
 - competitive for many problems
 - very general: *every NP-hard problem can be translated to SAT*
 - *(Interested in MSc project? Let me know!)*
- Recently used to solve long-standing open problems:
 - <http://www.nature.com/news/two-hundred-terabyte-maths-proof-is-largest-ever-1.19990>
- Simply very fast at solving formulas in propositional logic

Symbolic Reasoning

- Logic is “The Calculus of Computer Science” (Z. Manna)
- High computational complexity



A CNF formula

$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$

SAT solver inner workings --- DPLL

- ▶ Standard backtrack search
- ▶ DPLL(F) :
 - ▶ Apply unit propagation
 - ▶ If conflict identified, return UNSAT
 - ▶ Apply the pure literal rule
 - ▶ If F is satisfied (empty), return SAT
 - ▶ Select decision variable x
 - ▶ If $\text{DPLL}(F \wedge x) = \text{SAT}$ return SAT
 - ▶ return $\text{DPLL}(F \wedge \neg x)$

Unit Propagation

(Davis–Putnam–Logemann–Loveland)

DPLL = Unit resolution + Split rule.

$$\frac{\Gamma}{\Gamma, p \mid \Gamma, \neg p} \text{split} \quad p \text{ and } \neg p \text{ are not in } \Gamma.$$
$$\frac{C \vee \bar{l}, l}{C, l} \text{unit}$$

Used in the most efficient SAT solvers.

Pure Literals

A literal is **pure** if only occurs positively or negatively.

Example :

$$\varphi = (\neg x_1 \vee x_2) \wedge (x_3 \vee \neg x_2) \wedge (x_4 \vee \neg x_5) \wedge (x_5 \vee \neg x_4)$$

$\neg x_1$ and x_3 are pure literals

Pure literal rule :

Clauses containing pure literals can be removed from the formula (i.e. just satisfy those pure literals)

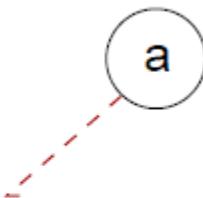
$$\varphi_{\neg x_1, x_3} = (x_4 \vee \neg x_5) \wedge (x_5 \vee \neg x_4)$$

Preserve satisfiability, not logical equivalency !

DPLL (example)

Guess

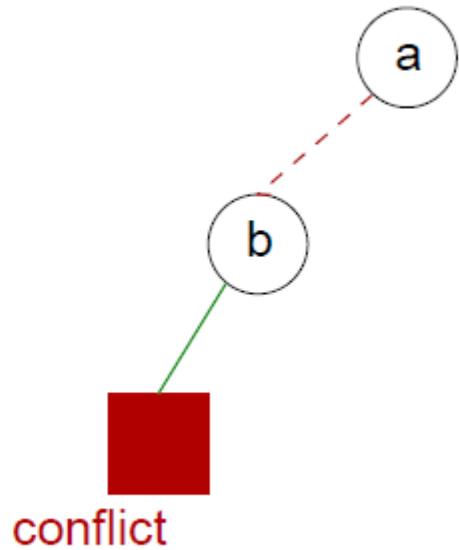
$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



DPLL (example)

Deduce

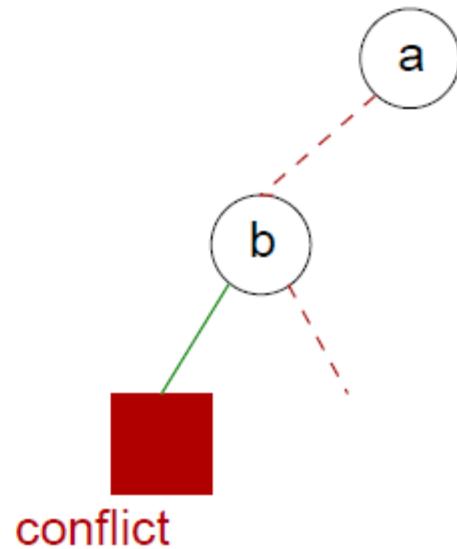
$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



DPLL (example)

Backtrack

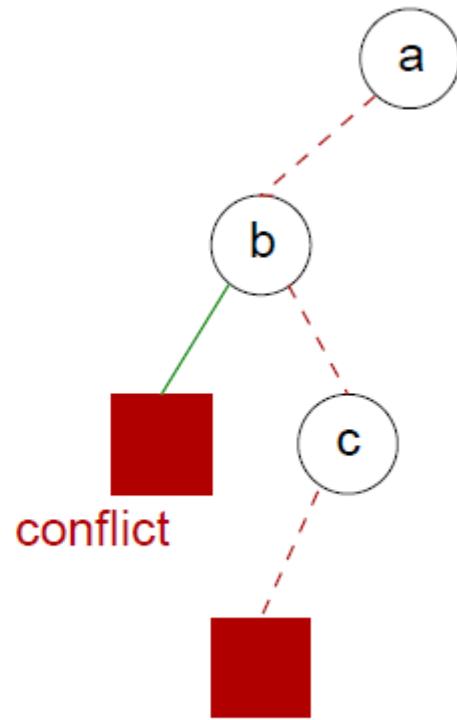
$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



DPLL (example)

Deduce

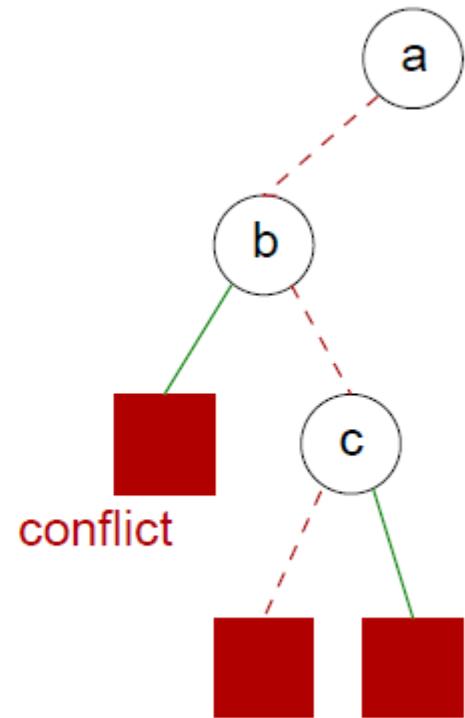
$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



DPLL (example)

Deduce

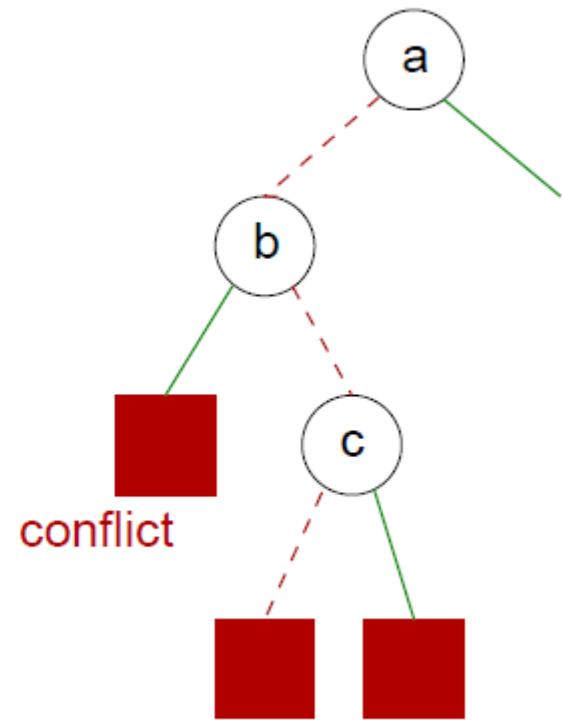
$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



DPLL (example)

Backtrack

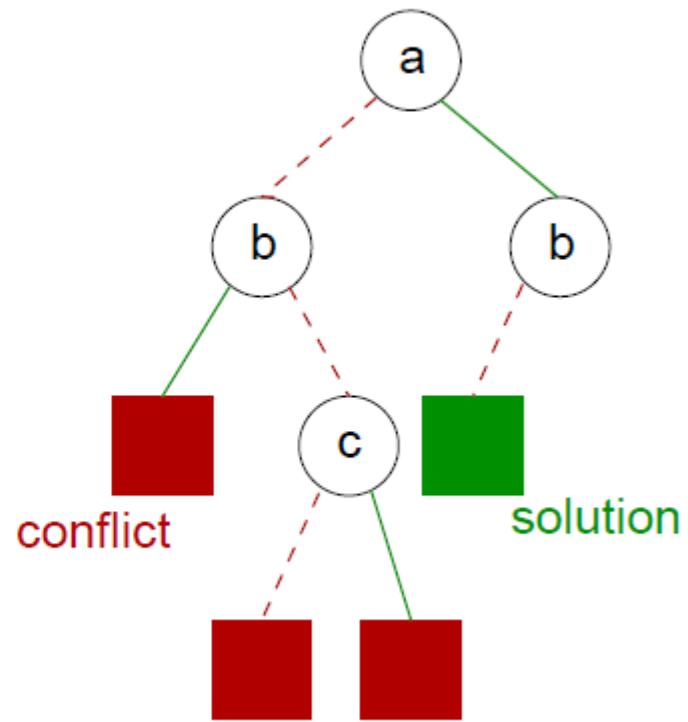
$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



DPLL (example)

Guess

$$\begin{aligned}\varphi = & (a \vee \neg b \vee d) \wedge (a \vee \neg b \vee e) \wedge \\& (\neg b \vee \neg d \vee \neg e) \wedge \\& (a \vee b \vee c \vee d) \wedge (a \vee b \vee c \vee \neg d) \wedge \\& (a \vee b \vee \neg c \vee e) \wedge (a \vee b \vee \neg c \vee \neg e)\end{aligned}$$



Modern DPLL

- Do **guess-deduce-backtrack** very efficiently
 - all represented using bits and binary operators
- In addition
 - Efficient indexing (two-watch literal)
 - Non-chronological backtracking (backjumping)
 - Lemma learning
- Google if interested...building a SAT solver from scratch is unlikely competitive with the state-of-the-art:
 - They improve every year...
 - See SAT Live!: <http://www.satlive.org/solvers/>

SAT solvers 2

- ..
- Simply very fast at solving **formulas in propositional logic**
 - In contrast to GAs and local search, SAT solvers are complete!
- But not so good at modeling **numeric variables**:
 - integers, floats, (non-)linear arithmetic, ...
- Possible using Boolean representation:
 - $0 = 000, 1 = 001, 2 = 010, 3 = 011, \dots$
- But representing **arithmetic** in logic is slow...
- So use a **seperate solver** for such operations!
 - or represent the problem as an integer program...
 - or use other approaches such as genetic algorithms...

Satisfiability Modulo Theories (SMT)

**Is formula F satisfiable
modulo theory T ?**

SMT solvers have
specialized algorithms for T

SAT + Theory solvers

Basic Idea

$$x \geq 0, y = x + 1, (y > 2 \vee y < 1)$$



Abstract (aka “naming” atoms)

$$p_1, p_2, (p_3 \vee p_4)$$

$$\begin{aligned} p_1 &\equiv (x \geq 0), \\ p_2 &\equiv (y = x + 1), \\ p_3 &\equiv (y > 2), \\ p_4 &\equiv (y < 1) \end{aligned}$$

SAT + Theory solvers

Basic Idea

$$x \geq 0, y = x + 1, (y > 2 \vee y < 1)$$



Abstract (aka “naming” atoms)

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$$\begin{aligned} p_1 &\equiv (x \geq 0), p_2 \equiv (y = x + 1), \\ p_3 &\equiv (y > 2), p_4 \equiv (y < 1) \end{aligned}$$

SAT
Solver

SAT + Theory solvers

Basic Idea

$$x \geq 0, y = x + 1, (y > 2 \vee y < 1)$$



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$$\begin{aligned} p_1 &\equiv (x \geq 0), p_2 \equiv (y = x + 1), \\ p_3 &\equiv (y > 2), p_4 \equiv (y < 1) \end{aligned}$$

SAT
Solver



$$p_1, p_2, \neg p_3, p_4$$

SAT + Theory solvers

Basic Idea

$$x \geq 0, y = x + 1, (y > 2 \vee y < 1)$$



Abstract (aka “naming” atoms)

$$p_1, p_2, (p_3 \vee p_4)$$



SAT
Solver



$$p_1, p_2, \neg p_3, p_4$$



$$\begin{aligned} p_1 &\equiv (x \geq 0), p_2 \equiv (y = x + 1), \\ p_3 &\equiv (y > 2), p_4 \equiv (y < 1) \end{aligned}$$

$$\begin{aligned} x &\geq 0, y = x + 1, \\ \neg(y > 2), y &< 1 \end{aligned}$$

SAT + Theory solvers

Basic Idea

$$x \geq 0, y = x + 1, (y > 2 \vee y < 1)$$



Abstract (aka “naming” atoms)

$$p_1, p_2, (p_3 \vee p_4)$$



SAT
Solver

Assignment

$$p_1, p_2, \neg p_3, p_4$$



$$\begin{aligned} p_1 &\equiv (x \geq 0), p_2 \equiv (y = x + 1), \\ p_3 &\equiv (y > 2), p_4 \equiv (y < 1) \end{aligned}$$

$$\begin{aligned} x &\geq 0, y = x + 1, \\ &\neg(y > 2), y < 1 \end{aligned}$$



Unsatisfiable

$$x \geq 0, y = x + 1, y < 1$$

Theory
Solver

SAT + Theory solvers

Basic Idea

$$x \geq 0, y = x + 1, (y > 2 \vee y < 1)$$



Abstract (aka “naming” atoms)

$$p_1, p_2, (p_3 \vee p_4)$$



$$\begin{aligned} p_1 &\equiv (x \geq 0), p_2 \equiv (y = x + 1), \\ p_3 &\equiv (y > 2), p_4 \equiv (y < 1) \end{aligned}$$



SAT
Solver



Assignment

$$p_1, p_2, \neg p_3, p_4$$



$$\begin{aligned} x &\geq 0, y = x + 1, \\ &\neg(y > 2), y < 1 \end{aligned}$$



Theory
Solver

New Lemma

$$\neg p_1 \vee \neg p_2 \vee \neg p_4$$

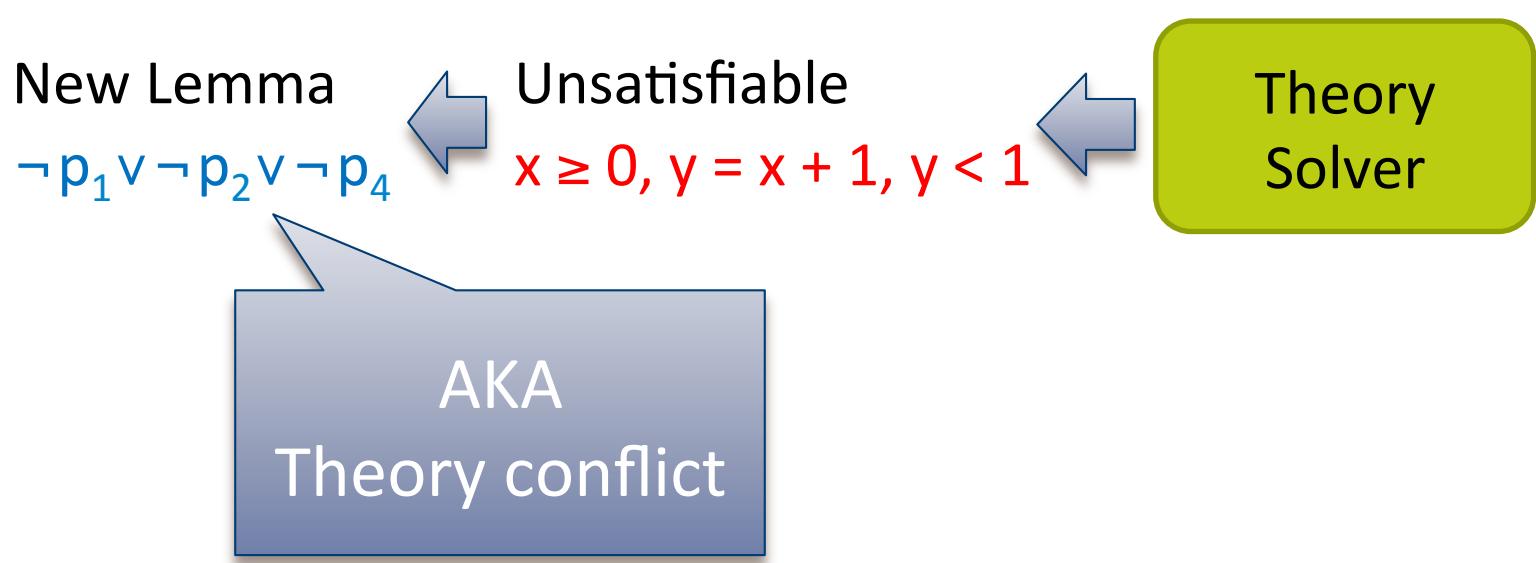


Unsatisfiable

$$x \geq 0, y = x + 1, y < 1$$



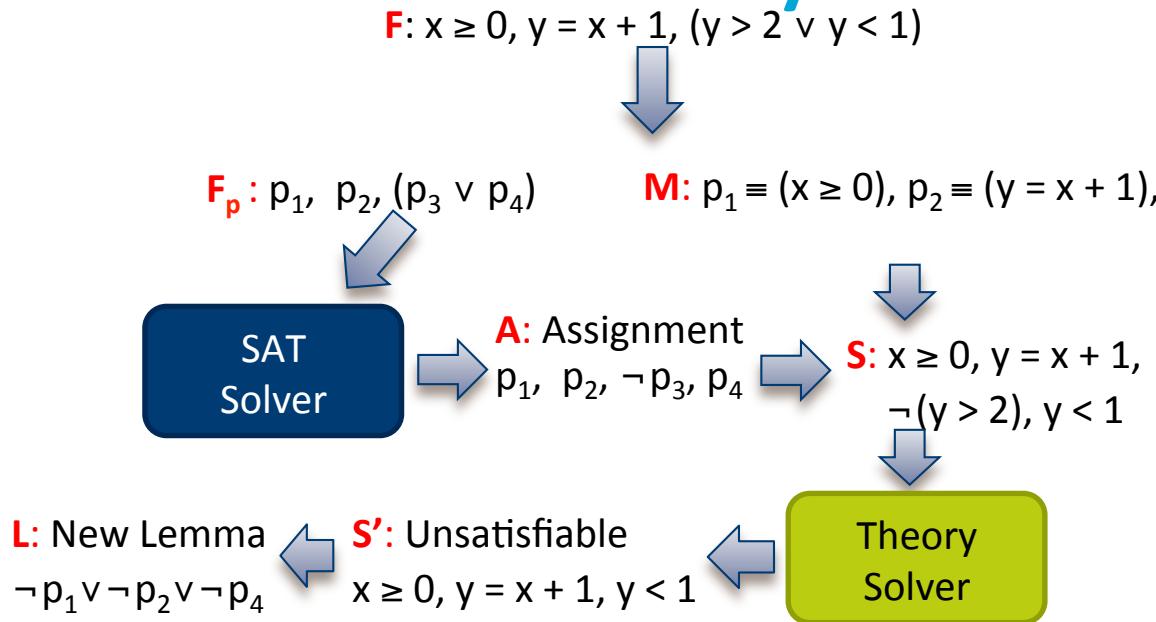
SAT + Theory solvers



SAT + Theory solvers: Main loop

```
procedure SmtSolver(F)
    ( $F_p$ , M) := Abstract(F)
    loop
        (R, A) := SAT_solver( $F_p$ )
        if R = UNSAT then return UNSAT
        S := Concretize(A, M)
        (R, S') := Theory_solver(S)
        if R = SAT then return SAT
        L := New_Lemma(S', M)
        Add L to  $F_p$ 
```

SAT + Theory solvers



```
procedure SMT_Solver(F)
  ( $F_p, M$ ) := Abstract( $F$ )
  loop
    (R, A) := SAT_solver( $F_p$ )
    if R = UNSAT then return UNSAT
    S = Concretize(A, M)
    (R, S') := Theory_solver(S)
    if R = SAT then return SAT
    L := New_Lemma(S, M)
    Add L to  $F_p$ 
```

How can this be fast?

- SAT solvers are extremely efficient
- The obtained theory S is often easy to solve (not NP-hard)
- and

**State-of-the-art SMT solvers implement
many improvements.**

SAT + Theory solvers

Incrementality

Send the literals to the Theory solver as they are assigned by the SAT solver

$$p_1 \equiv (x \geq 0), p_2 \equiv (y = x + 1),$$

$$p_3 \equiv (y > 2), p_4 \equiv (y < 1), p_5 \equiv (x < 2),$$

$$p_1, p_2, p_4 \quad | \quad p_1, p_2, (p_3 \vee p_4), (p_5 \vee \neg p_4)$$

Partial assignment is already
Theory inconsistent.

SAT + Theory solvers

Efficient Lemma Generation (computing a small S')

Avoid lemmas containing redundant literals.

$$p_1 \equiv (x \geq 0), p_2 \equiv (y = x + 1),$$

$$p_3 \equiv (y > 2), p_4 \equiv (y < 1), p_5 \equiv (x < 2),$$

$$p_1, p_2, p_3, p_4 \quad | \quad p_1, p_2, (p_3 \vee p_4), (p_5 \vee \neg p_4)$$

$$\neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \neg p_4$$

Imprecise Lemma

SAT + Theory solvers

Theory Propagation

$p_1 \equiv (x \geq 0)$, $p_2 \equiv (y = x + 1)$,

$p_3 \equiv (y > 2)$, $p_4 \equiv (y < 1)$, $p_5 \equiv (x < 2)$,

$p_1, p_2 \mid p_1, p_2, (p_3 \vee p_4), (p_5 \vee \neg p_4)$



p_1, p_2 imply $\neg p_4$ by theory propagation

$p_1, p_2, \neg p_4 \mid p_1, p_2, (p_3 \vee p_4), (p_5 \vee \neg p_4)$

From code to test

```
unsigned GCD(x, y) {  
    requires(y > 0);  
    while (true) {  
        unsigned m = x % y;  
        if (m == 0) return y;  
        x = y;  
        y = m;  
    }  
}
```

Give a trace where the loop is executed twice.

From code to test

```
unsigned GCD(x, y) {  
    requires(y > 0);  
    while (true) {  
        unsigned m = x % y;  
        if (m == 0) return y;  
        x = y;  
        y = m;  
    }  
}
```

For the Break:

Find such an assignment

You may use KLEE, see Git

Give a trace where the loop is
executed twice.

From code to test

```
unsigned GCD(x, y) {  
    requires(y > 0);  
    while (true) {  
        unsigned m = x % y;  
        if (m == 0) return y;  
        x = y;  
        y = m;  
    }  
}
```

SSA

Static Single Assignment

$(y_0 > 0)$ and
 $(m_0 = x_0 \% y_0)$ and
not $(m_0 = 0)$ and
 $(x_1 = y_0)$ and
 $(y_1 = m_0)$ and
 $(m_1 = x_1 \% y_1)$ and
 $(m_1 = 0)$

Solver

Variable Assignment
 $x_0 = 2$
 $y_0 = 4$
 $m_0 = 2$
 $x_1 = 4$
 $y_1 = 2$
 $m_1 = 0$

Give a trace where the loop is executed twice.

From code to test

```
unsigned GCD(x, y) {  
    requires(y > 0);  
    while (true) {  
        unsigned m = x % y;  
        if (m == 0) return y;  
        x = y;  
        y = m;  
    }  
}
```

SSA

Static Single Assignment

$(y_0 > 0)$ and
 $(m_0 = x_0 \% y_0)$ and
not $(m_0 = 0)$ and
 $(x_1 = y_0)$ and
 $(y_1 = m_0)$ and
 $(m_1 = x_1 \% y_1)$ and
 $(m_1 = 0)$

Solver

Variable Assignment
 $x_0 = 2$
 $y_0 = 4$
 $m_0 = 2$
 $x_1 = 4$
 $y_1 = 2$
 $m_1 = 0$

AKA Path Constraint

Symbolic execution

- Maps code and a target branch/line entirely into sets of constraints that can be solved using, e.g., an SMT solver
- Used to be a **static analysis** tool:
 - ***code is only interpreted, not executed!***
- Nowadays it is combined with actually running the code via **dynamic analysis** (concrete execution)
 - ***run, analyze, and iterate***
- Balance concrete and symbolic parts
- Many tools developed, and used to find real bugs:
 - PEX, KLEE, Angr, SAGE, ...
 - see https://en.wikipedia.org/wiki/Symbolic_execution

The theory: Difference Logic: $a - b \leq 5$

Very useful in practice!

Most arithmetical constraints in software verification/analysis are in this fragment.

$$\begin{array}{c} x := x + 1 \\ \downarrow \\ x_1 = x_0 + 1 \\ \downarrow \\ x_1 - x_0 \leq 1, x_0 - x_1 \leq -1 \end{array}$$

Difference Logic

Satisfiable?

$$z - t_{1,1} \leq 0$$

$$z - t_{2,1} \leq 0$$

$$z - t_{3,1} \leq 0$$

$$t_{3,2} - z \leq 5$$

$$t_{3,1} - t_{3,2} \leq -2$$

$$t_{2,1} - t_{3,1} \leq -3$$

$$t_{1,1} - t_{2,1} \leq -2$$

Difference Logic

NO!

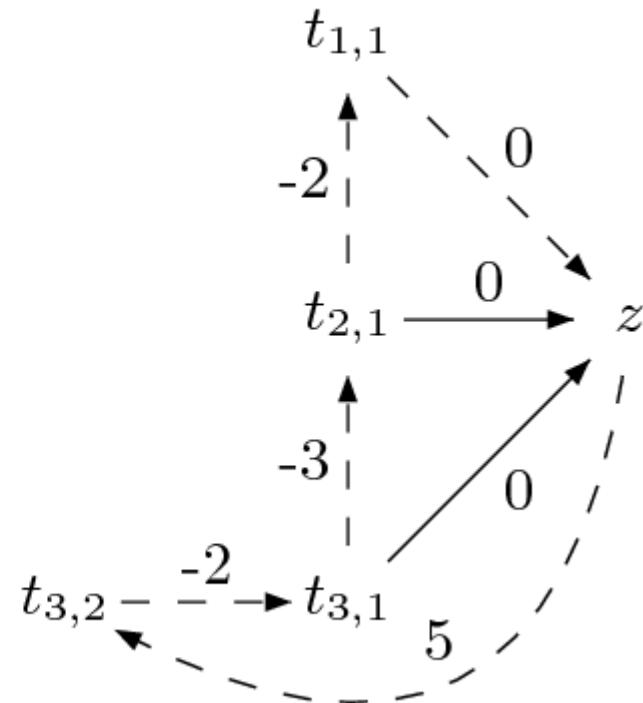
$$\begin{array}{lcl} z - t_{1,1} \leq 0 \\ z - t_{2,1} \leq 0 \\ z - t_{3,1} \leq 0 \\ t_{3,2} - z \leq 5 \\ t_{3,1} - t_{3,2} \leq -2 \\ t_{2,1} - t_{3,1} \leq -3 \\ t_{1,1} - t_{2,1} \leq -2 \end{array}$$

Difference Logic complexity

Satisfiable if and only if there are no negative cycles!

Algorithms based on Bellman-Ford ($O(mn)$).

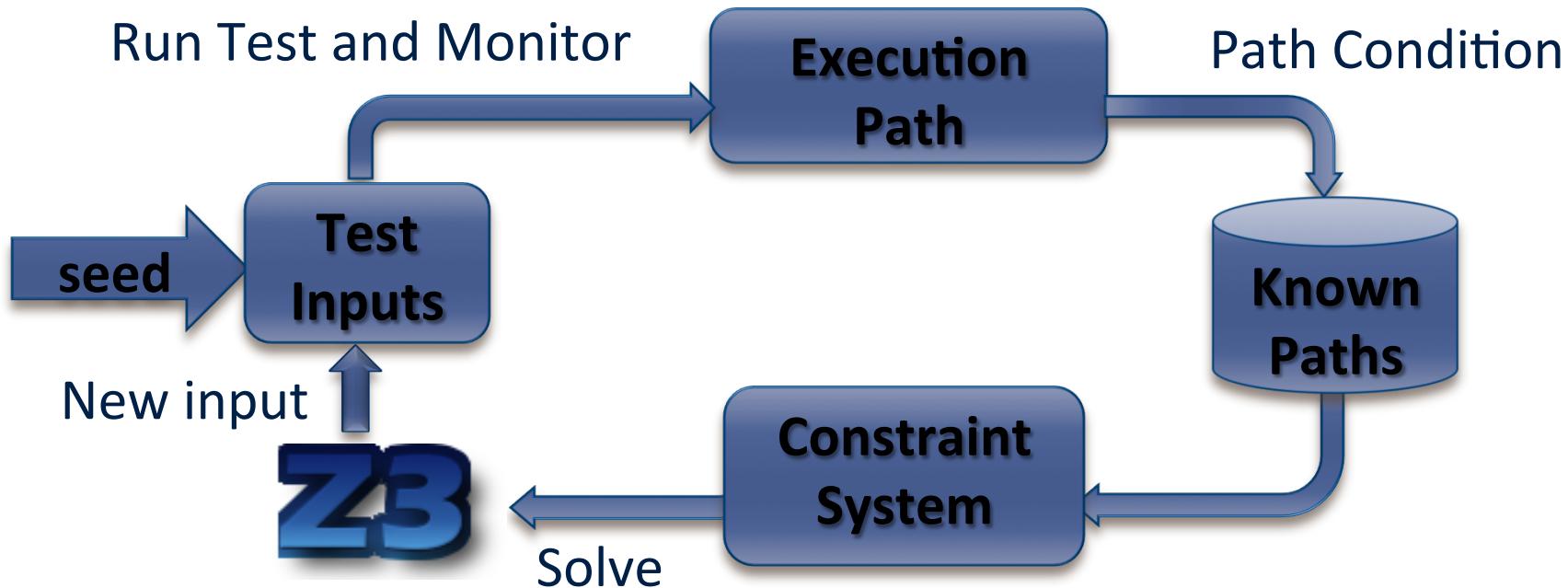
$$\begin{array}{lcl} z - t_{1,1} \leq 0 \\ z - t_{2,1} \leq 0 \\ z - t_{3,1} \leq 0 \\ t_{3,2} - z \leq 5 \\ t_{3,1} - t_{3,2} \leq -2 \\ t_{2,1} - t_{3,1} \leq -3 \\ t_{1,1} - t_{2,1} \leq -2 \end{array}$$



Z3 – a frequently used solver

- <https://github.com/Z3Prover/z3>
- <https://github.com/Z3Prover/z3/wiki>
- One of the most powerful SMT solvers, developed by Microsoft, check out <http://rise4fun.com/>
- Can quickly solve problems such as Sudoku, Scheduling, ...
- *and efficiently analyze code!*

Example: Directed Automated Random Testing (DART) in Microsoft PEX



PEX concolic testing of ArrayList

The screenshot shows two versions of the MSDN .NET Framework Developer Center website. The top version is a larger view of the page, and the bottom version is a smaller, more detailed view of the same content.

.NET Framework Developer Center

ArrayList.Add Method

Adds an object to the end of the [ArrayList](#).

Namespace: [System.Collections](#)
Assembly: mscorelib (in mscorelib.dll)

Remarks

[ArrayList](#) accepts a null reference (**Nothing** in Visual Basic) as a valid value and allows duplicate elements.

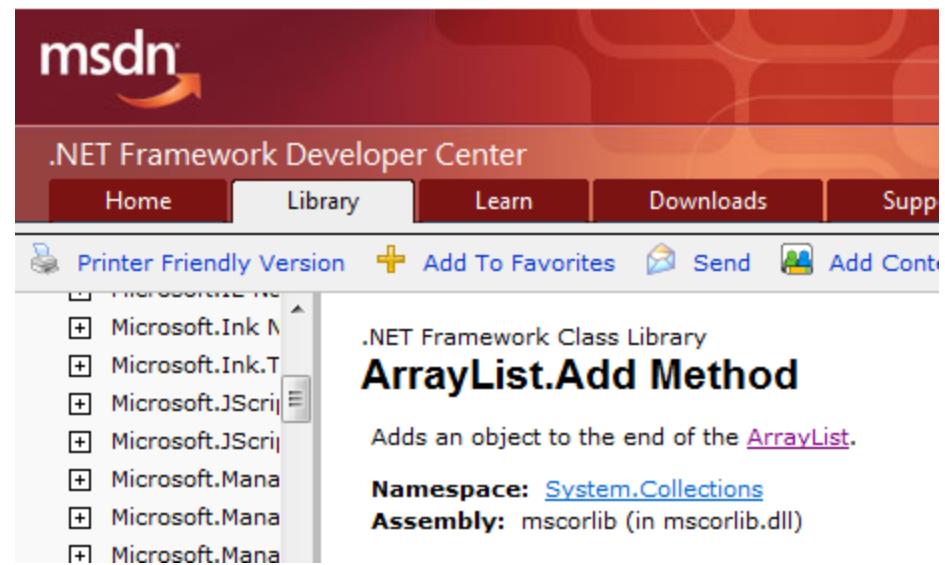
If [Count](#) already equals [Capacity](#), the capacity of the [ArrayList](#) is increased by automatically reallocating the internal array, and the existing elements are copied to the new array before the new element is added.

If [Count](#) is less than [Capacity](#), this method is an O(1) operation. If the capacity needs to be increased to accommodate the new element, this method becomes an O(n) operation, where n is [Count](#).

ArrayList: AddItem Test

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
}
```



The screenshot shows a web browser displaying the MSDN .NET Framework Developer Center. The URL in the address bar is [https://msdn.microsoft.com/en-us/library/39f1w43e\(v=vs.110\).aspx](https://msdn.microsoft.com/en-us/library/39f1w43e(v=vs.110).aspx). The page title is ".NET Framework Class Library ArrayList.Add Method". The content area describes the `Add` method, stating it "Adds an object to the end of the [ArrayList](#)". It specifies the **Namespace:** [System.Collections](#) and the **Assembly:** `mscorlib` (in `mscorlib.dll`). On the left, there is a navigation tree for the `ArrayList` class, listing various members like `Capacity`, `Count`, `Insert`, etc.

ArrayList: Starting Pex...

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

Inputs

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
}
```

ArrayList: Run 1, (0,null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

Inputs

(0, null)

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
}
```

ArrayList: Run 1, (0,null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

Inputs	Observed Constraints
(0, null)	!(c < 0)

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
TODent
```

c < 0 → false

ArrayList: Run 1, (0,null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

Inputs	Observed Constraints
(0,null)	!(c<0) && 0==c

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length) 0 == c → true  
        ResizeArray();  
  
        items[this.count++] = item; }  
...  
TODent
```

if (count == items.Length) 0 == c → true

ArrayList: Run 1, (0,null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

Inputs	Observed Constraints
(0,null)	$!(c < 0) \ \&\& \ 0 == c$

item == item → true

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...
```

This is a *tautology*,
i.e. a constraint that is always true,
regardless of the chosen values.

We can ignore such constraints.

ArrayList: Run 1, (0,null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
}
```

Inputs	Observed Constraints
(0,null)	!(c<0) && 0==c

Q: How to trigger another branch?

ArrayList: Picking next branch

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
TUDent
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) && 0!=c		



Negate the observed constraints!

ArrayList: Solve using SMT solver

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
TUDent
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) &&	(1,null)	



ArrayList: Run 2, (1, null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length) 0 == c → false  
        ResizeArray();  
  
        items[this.count++] = item; }  
    ...
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) &&	(1,null)	!(c<0) && 0!=c

ArrayList: Pick new branch

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
TUDent
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) &&	(1,null)	!(c<0) && 0!=c
c<0		

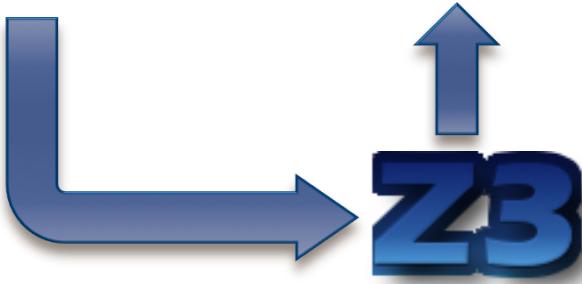


ArrayList: Run 3, (-1, null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
...  
TUDent
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	$!(c < 0) \ \&\& \ 0 == c$
$!(c < 0) \ \&\&$	(1,null)	$!(c < 0) \ \&\& \ 0 != c$
$c < 0$	(-1,null)	



ArrayList: Run 3, (-1, null)

```
class ArrayListTest {  
    [PexMethod]  
    void AddItem(int c, object item) {  
        var list = new ArrayList(c);  
        list.Add(item);  
        Assert(list[0] == item); }  
}
```

Constraints to solve	Inputs	Observed Constraints
	(0,null)	!(c<0) && 0==c
!(c<0) &&	(1,null)	!(c<0) && 0!=c
c<0	(-1,null)	c<0

```
class ArrayList {  
    object[] items;  
    int count;  
  
    ArrayList(int capacity) {  
        if (capacity < 0) throw ...;  
        items = new object[capacity];  
    }  
  
    void Add(object item) {  
        if (count == items.Length)  
            ResizeArray();  
  
        items[this.count++] = item; }  
    ...
```

c < 0 → true

Overview

- Never simply random testing of code
 - This can indeed take forever...
- Mutation-based Fuzzing
 - manipulate and combine existing valid inputs
- Genetic Algorithms:
 - use some form of branch distance/approach level
- Concolic Execution:
 - represent code in logic and analyze using SMT solvers
- Both:
 - generate tests, execute tests, collect traces, analyze branches, generate new tests, etc.

KLEE

- An LLVM based concolic execution engine
- <http://klee.github.io/docs/>
- Very easy to use, simply declare parameters as symbolic, and run KLEE!

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- Very easy to use, simply declare parameters as symbolic, and run KLEE!

Now:

Let's get KLEE running on RERS

About the second assignment

- **Be Inspired!**
- We provide you with a set of papers/blogposts/videos, either:
 1. Pick one of these and replicate the performed study
 2. Find your own paper/blogpost/video to replicate, needs teacher's approval
- You are asked to make a video (max 10 mins) demonstrating the code, tool, and study results
- In your study, it is obligatory to also use one of the methods from the second part of the course:
 1. Mutation analysis
 2. Model-based testing
 3. State machine learning
 4. Binary reverse engineering