

Super Brief FRAP Note

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Abstract

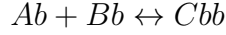
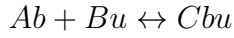
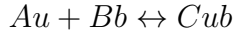
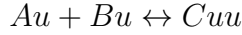
This is an extension of my previous FRAP code in which I better encode the reaction process, and I further allow the product particles to diffuse (if someone doesn't like that, they can just put the k_{off} value for that particle to be 0). This is going to fucking suck, so I am (for the moment) only writing this note to help me to verify the validity of my equations.

MODEL

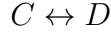
I will allow all the particles to possible diffuse in order to induce symmetry in my program. I want the only exceptional particles to be due to the binding process. The reaction I am modeling is



In order to take into account the FRAP process, I need to define what happens when you bind bleached particles with unbleached particles. Therefore, I define the more detailed reaction scheme as follows:



The ultimate goal is to model the following reaction scheme under a FRAP experiment:



In order to model the first reaction, I need the following equations:

- Au

- Initial Volume: $Au(x, 0) = \widetilde{Au}(x)$

- Initial Surface: $\sigma_{Au}(0) = \widetilde{\sigma_{Au}}$

- Diffusion: $\frac{\partial Au}{\partial t} = D_{Au} \frac{\partial^2 Au}{\partial x^2}$

- Left Flux: $\left. \frac{\partial Au}{\partial x} \right|_{x=0} = 0$

- Right Flux: $-D_{Au} \left. \frac{\partial Au}{\partial x} \right|_{x=L} = k_{on}^{Au} Au(L, t) - k_{off}^{Au} \sigma_{Au}(t)$

- Reaction:

$$\frac{d\sigma_{Au}}{dt} = k_{on}^{Au} Au(L, t) - (k_{off}^{Au} + k_{bind}^{uu} \sigma_{Bu} + k_{bind}^{ub} \sigma_{Bb}) \sigma_{Au} + k_{dis}^{uu} \sigma_{cuu} + k_{dis}^{ub} \sigma_{Cub}$$

- Bu

- Initial Volume: $Bu(x, 0) = \widetilde{Bu}(x)$
- Initial Surface: $\sigma_{Bu}(0) = \widetilde{\sigma_{Bu}}$
- Diffusion: $\frac{\partial Bu}{\partial t} = D_{Bu} \frac{\partial^2 Bu}{\partial x^2}$
- Left Flux: $\left. \frac{\partial Bu}{\partial x} \right|_{x=0} = 0$
- Right Flux: $-D_{Bu} \left. \frac{\partial Bu}{\partial x} \right|_{x=L} = k_{on}^{Bu} Bu(L, t) - k_{off}^{Bu} \sigma_{Bu}(t)$
- Reaction:

$$\frac{d\sigma_{Bu}}{dt} = k_{on}^{Bu} Bu(L, t) - (k_{off}^{Bu} + k_{bind}^{uu} \sigma_{Au} + k_{bind}^{bu} \sigma_{Ab}) \sigma_{Bu} + k_{dis}^{uu} \sigma_{Cuu} + k_{dis}^{bu} \sigma_{Cbu}$$

- Ab

- Initial Volume: $Ab(x, 0) = \widetilde{Ab}(x)$
- Initial Surface: $\sigma_{Ab}(0) = \widetilde{\sigma_{Ab}}$
- Diffusion: $\frac{\partial Ab}{\partial t} = D_{Ab} \frac{\partial^2 Ab}{\partial x^2}$
- Left Flux: $\left. \frac{\partial Ab}{\partial x} \right|_{x=0} = 0$
- Right Flux: $-D_{Ab} \left. \frac{\partial Ab}{\partial x} \right|_{x=L} = k_{on}^{Ab} Ab(L, t) - k_{off}^{Ab} \sigma_{Ab}(t)$
- Reaction:

$$\frac{d\sigma_{Ab}}{dt} = k_{on}^{Ab} Ab(L, t) - (k_{off}^{Ab} + k_{bind}^{bu} \sigma_{Bu} + k_{bind}^{bb} \sigma_{Bb}) \sigma_{Ab} + k_{dis}^{bu} \sigma_{Cbu} + k_{dis}^{bb} \sigma_{Cbb}$$

- Bb

- Initial Volume: $Bb(x, 0) = \widetilde{Bb}(x)$
- Initial Surface: $\sigma_{Bb}(0) = \widetilde{\sigma_{Bb}}$
- Diffusion: $\frac{\partial Bb}{\partial t} = D_{Bb} \frac{\partial^2 Bb}{\partial x^2}$
- Left Flux: $\left. \frac{\partial Bb}{\partial x} \right|_{x=0} = 0$
- Right Flux: $-D_{Bb} \left. \frac{\partial Bb}{\partial x} \right|_{x=L} = k_{on}^{Bb} Bb(L, t) - k_{off}^{Bb} \sigma_{Bb}(t)$
- Reaction:

$$\frac{d\sigma_{Bb}}{dt} = k_{on}^{Bb} Bb(L, t) - (k_{off}^{Bb} + k_{bind}^{ub} \sigma_{Au} + k_{bind}^{bb} \sigma_{Ab}) \sigma_{Bb} + k_{dis}^{ub} \sigma_{Cub} + k_{dis}^{bb} \sigma_{Cbb}$$

- Cuu

- Initial Volume: $Cuu(x, 0) = \widetilde{Cuu}(x)$
- Initial Surface: $\sigma_{Cuu}(0) = \widetilde{\sigma_{Cuu}}$
- Diffusion: $\frac{\partial Cuu}{\partial t} = D_{Cuu} \frac{\partial^2 Cuu}{\partial x^2}$
- Left Flux: $\frac{\partial Cuu}{\partial x} \Big|_{x=0} = 0$
- Right Flux: $-D_{Cuu} \frac{\partial Cuu}{\partial x} \Big|_{x=L} = k_{on}^{Cuu} Cuu(L, t) - k_{off}^{Cuu} \sigma_{Cuu}(t)$
- Reaction:

$$\frac{d\sigma_{Cuu}}{dt} = k_{on}^{Cuu} Cuu(L, t) - (k_{off}^{Cuu} + k_{dis}^{uu}) \sigma_{Cuu} + k_{bind}^{uu} \sigma_{Au} \sigma_{Bu}$$

- Cub

- Initial Volume: $Cub(x, 0) = \widetilde{Cub}(x)$
- Initial Surface: $\sigma_{Cub}(0) = \widetilde{\sigma_{Cub}}$
- Diffusion: $\frac{\partial Cub}{\partial t} = D_{Cub} \frac{\partial^2 Cub}{\partial x^2}$
- Left Flux: $\frac{\partial Cub}{\partial x} \Big|_{x=0} = 0$
- Right Flux: $-D_{Cub} \frac{\partial Cub}{\partial x} \Big|_{x=L} = k_{on}^{Cub} Cub(L, t) - k_{off}^{Cub} \sigma_{Cub}(t)$
- Reaction:

$$\frac{d\sigma_{Cub}}{dt} = k_{on}^{Cub} Cub(L, t) - (k_{off}^{Cub} + k_{dis}^{ub}) \sigma_{Cub} + k_{bind}^{ub} \sigma_{Au} \sigma_{Bb}$$

- Cbu

- Initial Volume: $Cbu(x, 0) = \widetilde{Cbu}(x)$
- Initial Surface: $\sigma_{Cbu}(0) = \widetilde{\sigma_{Cbu}}$
- Diffusion: $\frac{\partial Cbu}{\partial t} = D_{Cbu} \frac{\partial^2 Cbu}{\partial x^2}$
- Left Flux: $\frac{\partial Cbu}{\partial x} \Big|_{x=0} = 0$
- Right Flux: $-D_{Cbu} \frac{\partial Cbu}{\partial x} \Big|_{x=L} = k_{on}^{Cbu} Cbu(L, t) - k_{off}^{Cbu} \sigma_{Cbu}(t)$
- Reaction:

$$\frac{d\sigma_{Cbu}}{dt} = k_{on}^{Cbu} Cbu(L, t) - (k_{off}^{Cbu} + k_{dis}^{bu}) \sigma_{Cbu} + k_{bind}^{bu} \sigma_{Ab} \sigma_{Bu}$$

- Cbb

- Initial Volume: $Cbb(x, 0) = \widetilde{Cbb}(x)$
- Initial Surface: $\sigma_{Cbb}(0) = \widetilde{\sigma_{Cbb}}$
- Diffusion: $\frac{\partial Cbb}{\partial t} = D_{Cbb} \frac{\partial^2 Cbb}{\partial x^2}$
- Left Flux: $\frac{\partial Cbb}{\partial x} \Big|_{x=0} = 0$
- Right Flux: $-D_{Cbb} \frac{\partial Cbb}{\partial x} \Big|_{x=L} = k_{on}^{Cbb} Cbb(L, t) - k_{off}^{Cbb} \sigma_{Cbb}(t)$
- Reaction:

$$\frac{d\sigma_{Cbb}}{dt} = k_{on}^{Cbb} Cbb(L, t) - (k_{off}^{Cbb} + k_{dis}^{bb}) \sigma_{Cbb} + k_{bind}^{bb} \sigma_{Ab} \sigma_{Bb}$$

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