$\cancel{G(t_1, t}$

$$G(\tau) = 2\gamma\lambda\left[\cot\left(\frac{\beta\gamma}{2}\right) - i\right]e^{-\gamma\tau} + \sum_{n=1}^{\infty} \frac{4\lambda\gamma}{\beta} \frac{\gamma_n}{\gamma_n^2 - \gamma^2} e^{-\gamma_n\tau}$$

$$\approx c_0 e^{-\gamma\tau} + \sum_{n=1}^{K} c_n e^{-\gamma_n\tau} + c_\infty \delta(\tau)$$

$$c_0 = 2\gamma\lambda\left[\cot\left(\frac{\beta\gamma}{2}\right) - i\right]$$

$$c_n = \frac{4\lambda\gamma}{\beta} \frac{\gamma_n}{\gamma_n^2 - \gamma^2}$$

$\downarrow$

$$c_\infty \delta(\tau) = \sum_{n=K+1}^{\infty} c_n e^{-\gamma_n\tau}$$

$\downarrow$

$$c_\infty \underbrace{\int_0^\infty d\tau\, \delta(\tau)}_{1/2} = \sum_{n=K+1}^{\infty} c_n \underbrace{\int_0^\infty d\tau\, e^{-\gamma_n\tau}}_{-\frac{1}{\gamma_n}\left[e^{-\infty} - e^0\right] = \frac{1}{\gamma_n}}$$

$\downarrow$

$$c_\infty = 2\sum_{n=K+1}^{\infty} \frac{c_n}{\gamma_n} = \frac{8\lambda\gamma}{\beta} \sum_{n=K+1}^{\infty} \frac{1}{\gamma_n^2 - \gamma^2}$$

$$\Delta\ \gamma_n = \frac{2\pi n}{\beta}$$

$$= \frac{2\lambda\gamma\beta}{\pi^2} \sum_{n=K+1}^{\infty} \frac{1}{n^2 - \left(\frac{\gamma\beta}{2\pi}\right)^2}$$

$$\boxed{\pi\cot(\pi z) = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{1}{z^2-n^2}}$$

$$= \frac{1}{z} - 2z\sum_{n=1}^{\infty}\frac{1}{n^2-z^2}$$

$$\pi z\cot(\pi z) - 1 = -2z^2\sum_{n=1}^{\infty}\frac{1}{n^2-z^2}$$

$\downarrow$

$$\sum_{n=1}^{\infty}\frac{1}{n^2-z^2} = \frac{1-\pi z\cot(\pi z)}{2z^2}$$

$\downarrow$

$$\sum_{n=1}^{K}\frac{1}{n^2-z^2} + \sum_{n=K+1}^{\infty}\frac{1}{n^2-z^2} = \frac{1-\pi z\cot(\pi z)}{2z^2}$$

$\downarrow$

$$\sum_{n=K+1}^{\infty}\frac{1}{n^2-z^2} = \frac{1-\pi z\cot(\pi z)}{2z^2} - \sum_{n=1}^{K}\frac{1}{n^2-z^2}$$

$\downarrow$

$$\sum_{n=K+1}^{\infty}\frac{1}{n^2-z^2} = \frac{1-\cancel{\pi}\left(\frac{\gamma\beta}{2\cancel{\pi}}\right)\cot\left[\cancel{\pi}\left(\frac{\gamma\beta}{2\cancel{\pi}}\right)\right]}{2\left(\frac{\gamma\beta}{2\pi}\right)^2} - \sum_{n=1}^{K}\frac{1}{n^2-\left(\frac{\gamma\beta}{2\pi}\right)^2}$$

$\downarrow$

$$C_\infty = \frac{2\lambda\cancel{\gamma\beta}}{\cancel{\pi^2}}\cdot\overset{2}{\cancel{4\pi^2}}\cdot\frac{1-\frac{\gamma\beta}{2}\cot\left(\frac{\gamma\beta}{2}\right)}{\cancel{2}\gamma^{\cancel{2}}\beta^{\cancel{2}}} - \frac{2\lambda\gamma\beta}{\pi^2}\sum_{n=1}^{K}\frac{1}{n^2-\left(\frac{\gamma\beta}{2\pi}\right)^2}$$

$$= \frac{\overset{4}{\cancel{2}}\lambda - 2\lambda\gamma\beta\cot\left(\frac{\gamma\beta}{2}\right)}{\gamma\beta} - \overset{8}{\cancel{2}}\lambda\gamma\beta\sum_{n=1}^{K}\frac{1}{(2\pi n)^2-(\gamma\beta)^2}$$

Thus,

$$C_\infty = \frac{4\lambda}{\gamma\beta} - 2\lambda\cot\left(\frac{\gamma\beta}{2}\right) - 8\lambda\gamma\beta\sum_{n=1}^{K}\frac{1}{(2\pi n)^2-(\gamma\beta)^2}$$

and therefore,

$$\boxed{G(\tau) \approx 2\lambda\gamma\left[\cot\left(\frac{\beta\gamma}{2}\right) - i\right] + \frac{4\lambda\gamma}{\beta}\sum_{n=1}^{K}\frac{\gamma_n}{\gamma_n^2-\gamma^2}e^{-\gamma_n\tau} + \left[\frac{4\lambda}{\gamma\beta} - 2\lambda\cot\left(\frac{\gamma\beta}{2}\right) - 8\lambda\gamma\beta\sum_{n=1}^{K}\frac{1}{(2\pi n)^2-(\gamma\beta)^2}\right]\delta(\tau)}$$

where $\gamma_n \equiv \frac{2\pi n}{\beta}$