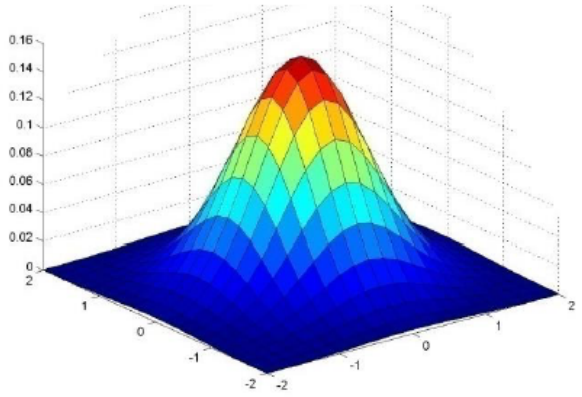
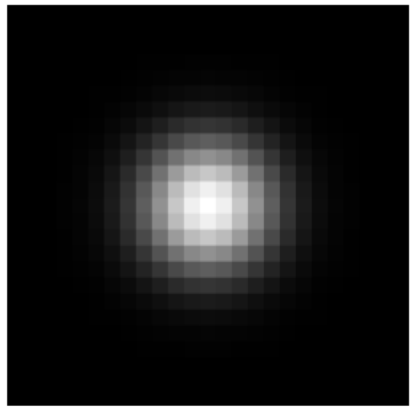
Image Enhancement and Frequency domain operations

Gaussian filters

Weight contributions of neighboring pixels by nearness





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

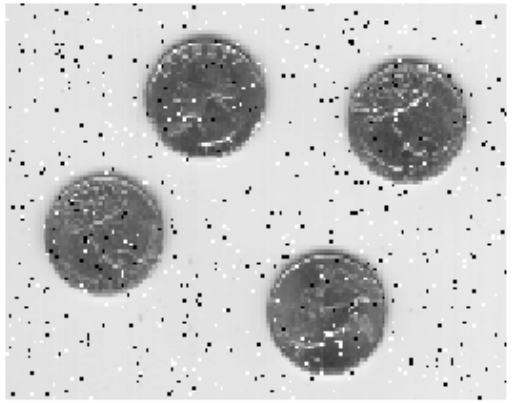
5 x 5, $\sigma = 1$

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

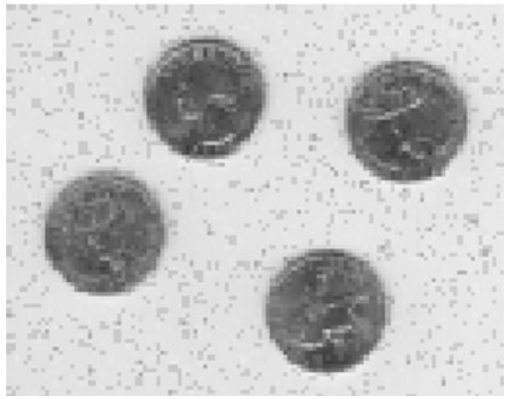
Like the box filter, the Gaussian is a smoothing filter.

Comparison with a Gaussian

(a)



(b)



(c)

(a) original image, (b) image+ ‘salt and pepper’ noise (c) After Gaussian filtering

Effect of σ on Gaussian filters

σ= 2

σ= 4

σ= 8

Gaussian filtering

Resolution
Sequence

Original Image

$\sigma_0 = 0$

Gaussian filtering

Resolution Sequence

Gaussian LPF

$\sigma_1 = 1$

Gaussian filtering

Resolution Sequence

Gaussian LPF

$\sigma_2 = 2$

Gaussian filtering

Resolution Sequence

Gaussian LPF

$\sigma_3 = 4$

Gaussian filtering

Resolution Sequence

Gaussian LPF

$\sigma_4 = 8$

Gaussian filtering

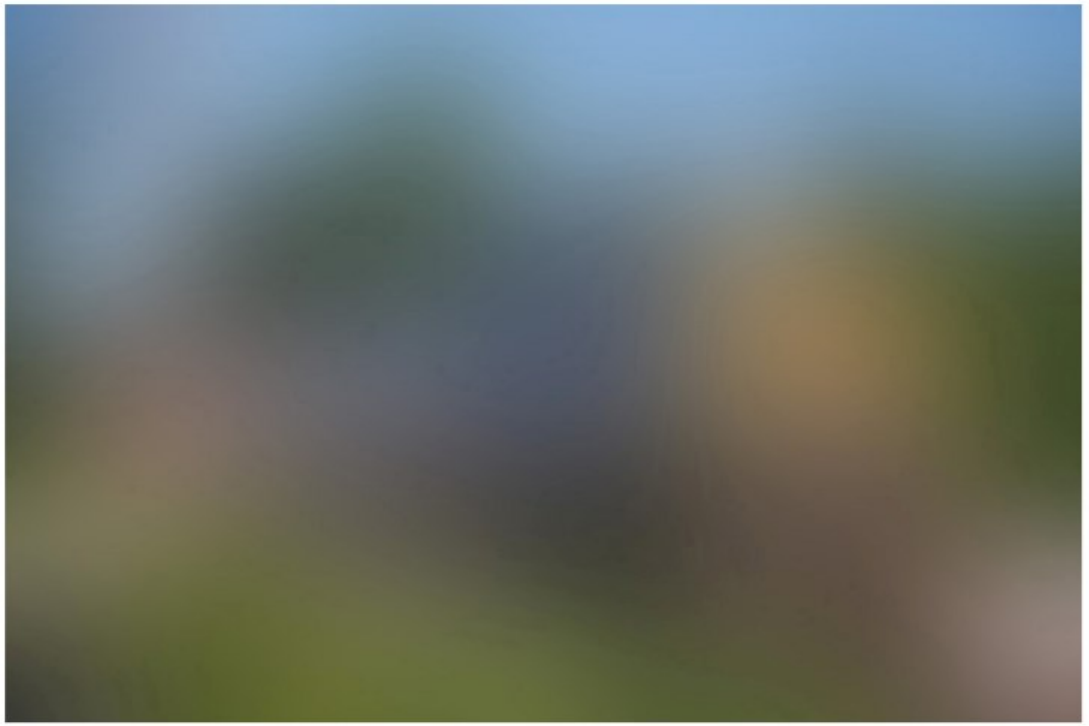
Resolution Sequence

Gaussian LPF

$\sigma_5 = 16$

Gaussian filtering

Resolution Sequence

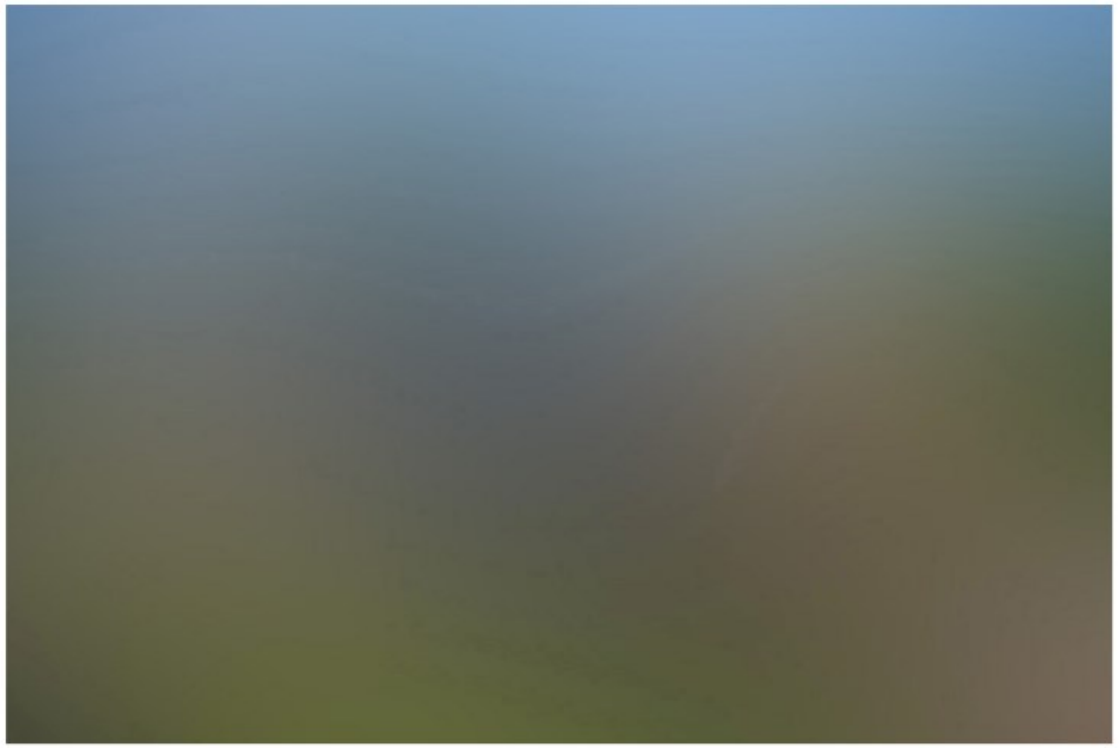


Gaussian LPF

$\sigma_6 = 32$

Gaussian filtering

Resolution Sequence



Gaussian LPF

$\sigma_7 = 64$

Gaussian filtering

Resolution
Sequence

Gaussian LPF

$\sigma_8 = 128$

Gaussian filtering

Resolution Sequence

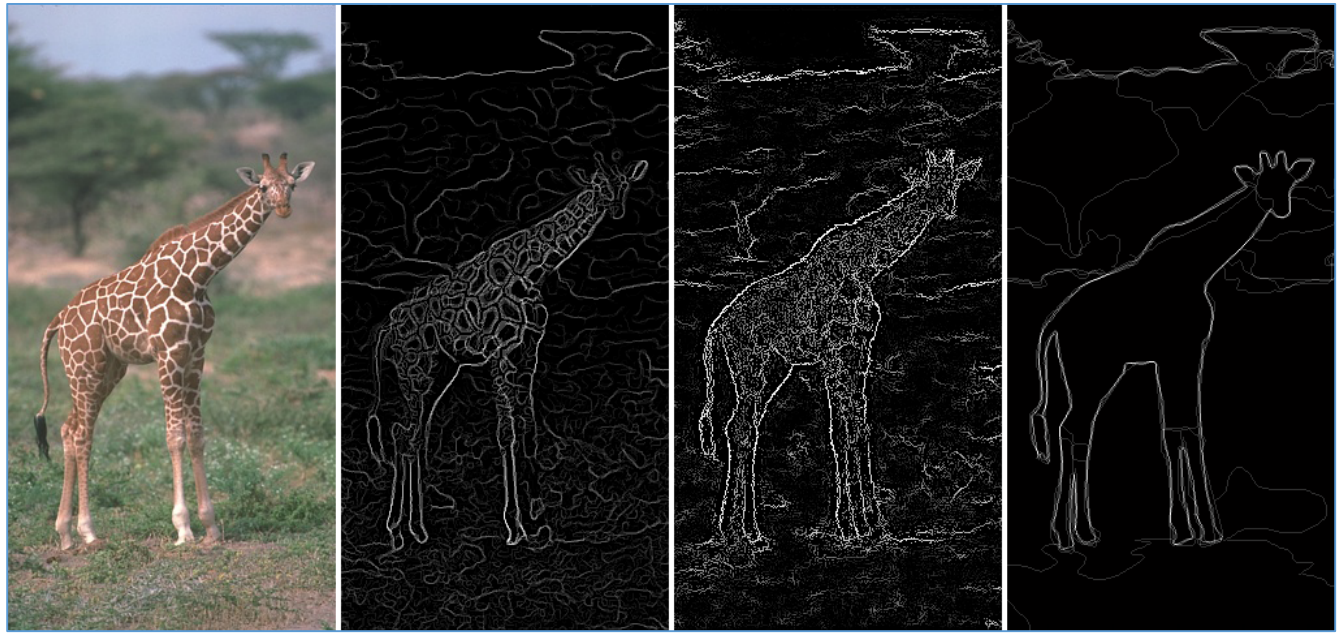
Gaussian LPF

$\sigma \rightarrow \infty$

Using a sequence of Gaussian filters the image converges to a constant plan with a value that is equal to the mean Intensity value in the image

Using filters for edge detection and image enhancement

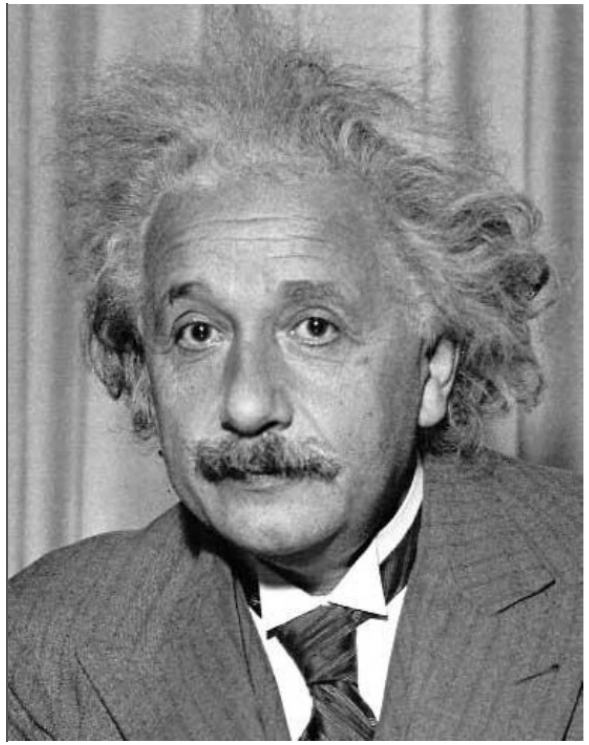
- So far we have been focusing on smoothing filters.
- Another class of filters are the edge detection filters and they allow use to detect edges and other regions where high magnitude changes of intensity occurs.



Simple edge detectors

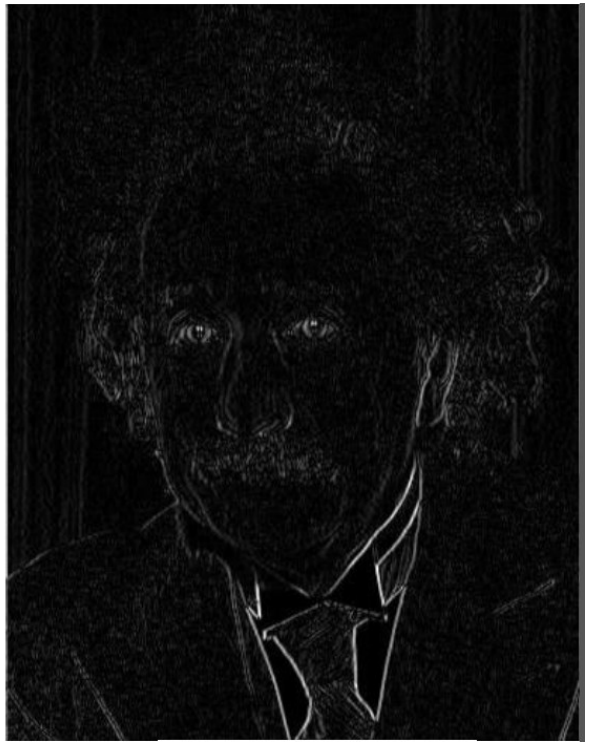
Roberts			Prewitt			Sobel		
0	−1	X derivative	1	0	-1	1	0	−1
1	0		1	0	-1	2	0	−2
			1	0	-1	1	0	−1
−1	0	Y derivative	1	1	1	1	2	1
0	1		0	0	0	0	0	0
			-1	-1	-1	−1	−2	−1

Simple edge detectors

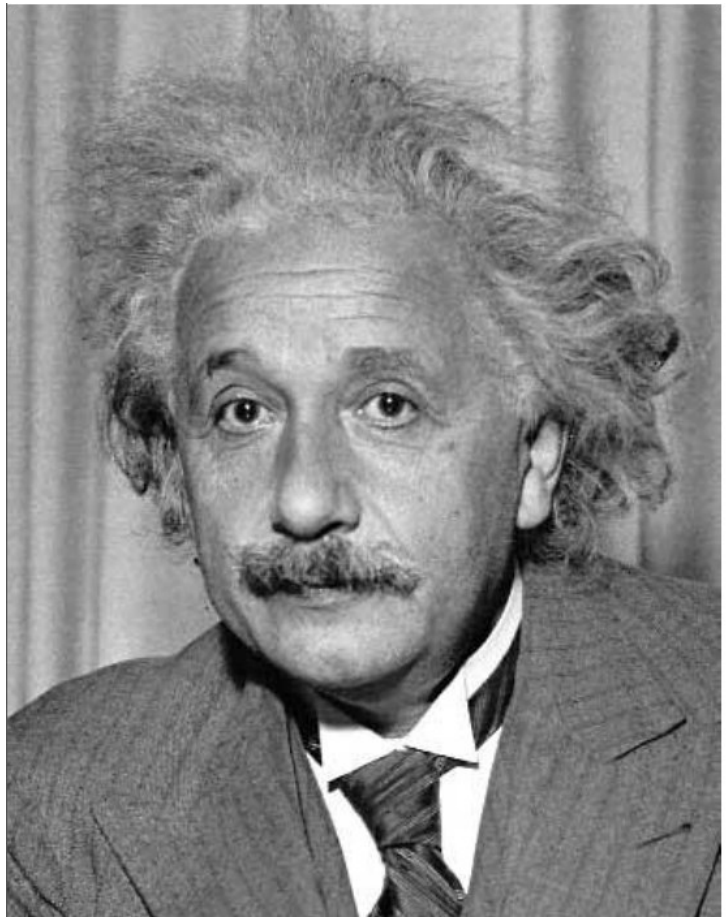


1	0	-1
2	0	-2
1	0	-1

Sobel

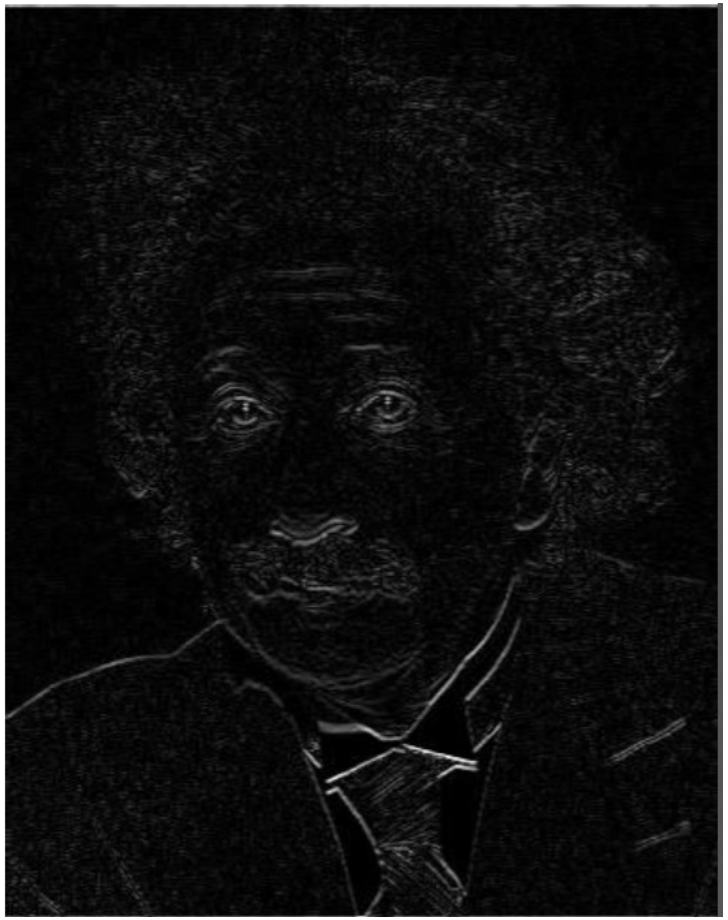


Simple edge detectors



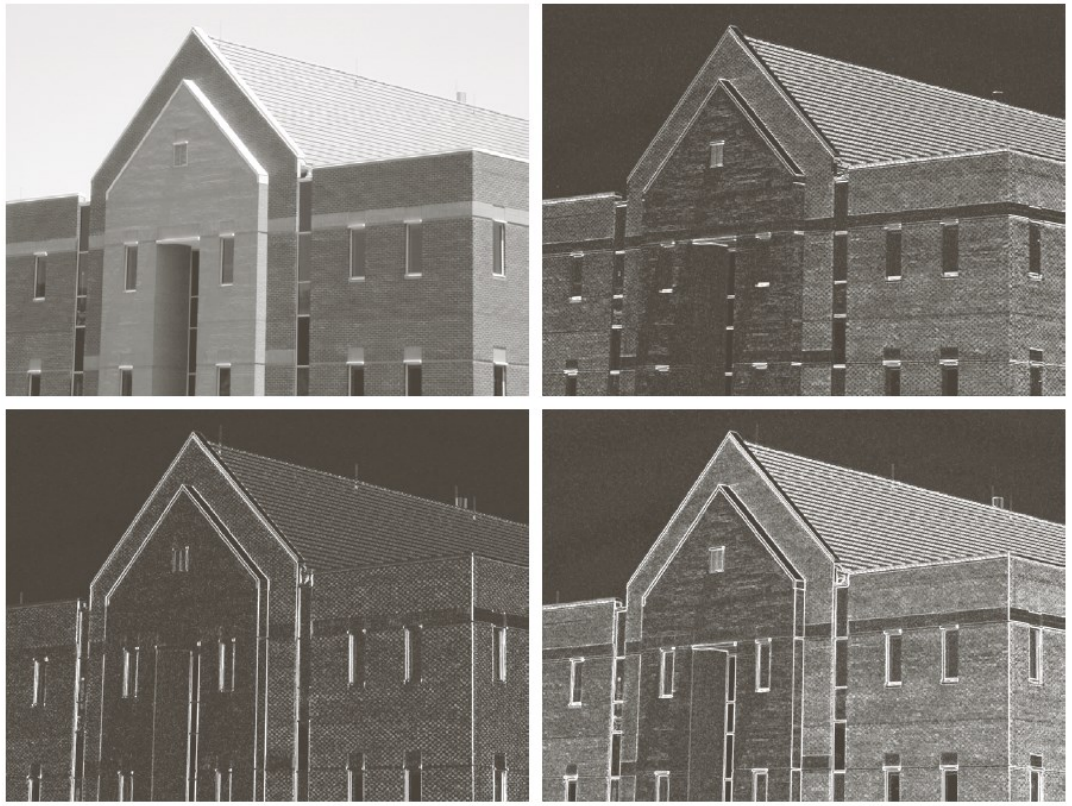
1	2	1
0	0	0
-1	-2	-1

Sobel



Smoothing prior to edge detection

- Smoothing an image can improve the quality of edge detection.



a b
c d

FIGURE 10.16
(a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) $|g_x|$, the component of the gradient in the x-direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image. (c) $|g_y|$, obtained using the mask in Fig. 10.14(g). (d) The gradient image, $|g_x| + |g_y|$.

Smoothing prior to edge detection

a b
c d

FIGURE 10.18
Same sequence as in Fig. 10.16, but with the original image smoothed using a 5×5 averaging filter prior to edge detection.

Derivative operators: First order derivatives

$$I = \begin{bmatrix} Kul & Ku & Kur \\ Kl & K & Kr \\ Kdl & Kd & Klr \end{bmatrix}$$

- Foreword difference method

$$Gx(I)_{2,2} = \left.\frac{\partial f}{\partial x}\right|_{x=2} = Kr - K \rightarrow \left.\frac{\partial f}{\partial x}\right|_{x} = f(x+1,y) - f(x,y)$$

- Backward difference method

$$Gx(I)_{2,2} = \left.\frac{\partial f}{\partial x}\right|_{x=2} = K - Kl \rightarrow \left.\frac{\partial f}{\partial x}\right|_{x} = f(x,y) - f(x-1,y)$$

First order derivatives

$$I = \begin{bmatrix} Kul & Ku & Kur \\ Kl & K & Kr \\ Kdl & Kd & Klr \end{bmatrix}$$

- Centered difference method

$$Gx(I)_{2,2} = \left.\frac{\partial f}{\partial x}\right|_{x=2} = \frac{Kr - Kl}{2} \rightarrow \left.\frac{\partial f}{\partial x}\right|_{x} = \frac{f(x+1,y) - f(x-1,y)}{2}$$

- At boundaries use forward difference or backward difference.

- Central difference is the preferred method for computing derivatives since it generates less error

Second order derivatives

- Using centered difference in second order derivate

$$\dot{f}\Big|_x = \frac{f(x+1,y) - f(x-1,y)}{2}$$

$$\ddot{f}\Big|_x = \frac{\dot{f}\Big|_{x+1} - \dot{f}\Big|_{x-1}}{2}$$

$$\ddot{f}\Big|_x = \frac{f(x+1,y)-f(x,y)}{2} - \frac{f(x,y)-f(x-1,y)}{2}$$

$$\boxed{\ddot{f}\Big|_x = \frac{f(x+1,y) - 2f(x,y) + f(x-1,y)}{2}}$$

- What is $\ddot{f}\Big|_y$?

Laplacian filter

- Laplacian operator

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Discrete Laplacian operator

$$\Delta f = f(x+1, y) + f(x-1, y) - 4f(x, y) + f(x, y+1) + f(x, y-1)$$

Laplacian filter

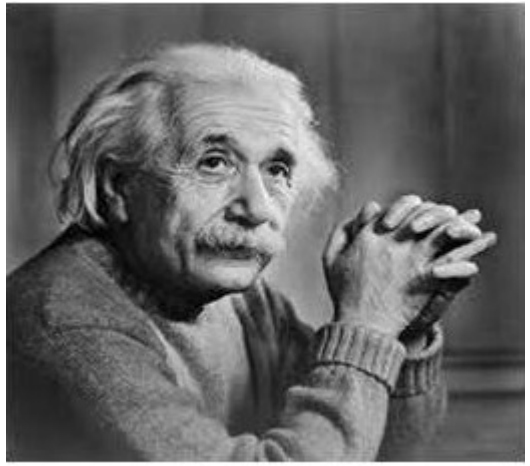
- Laplacian filter

0	1	0
1	-4	1
0	1	0

- Insensitive to diagonal changes.
- Solution

1	1	1
1	-8	1
1	1	1

Laplacian filter



The Laplacian filter highlights discontinuities.

Laplacian edge enhancement

- Take the original image and subtract the laplacian

$$I_{output}(x,y) = I_{in}(x,y) - \nabla^2 I_{in}(x,y)$$

Laplacian

Original Image

Laplacian “edges”

Sharpened Image

Laplacian of Gaussian

- Smooth the image with a Gaussian filter before applying the Laplacian (Laplacian of Gaussian, LOG)
- Recall that smoothing improves edge detection

Original Image

LoG “edges”

Sharpened Image

Image Sharpening

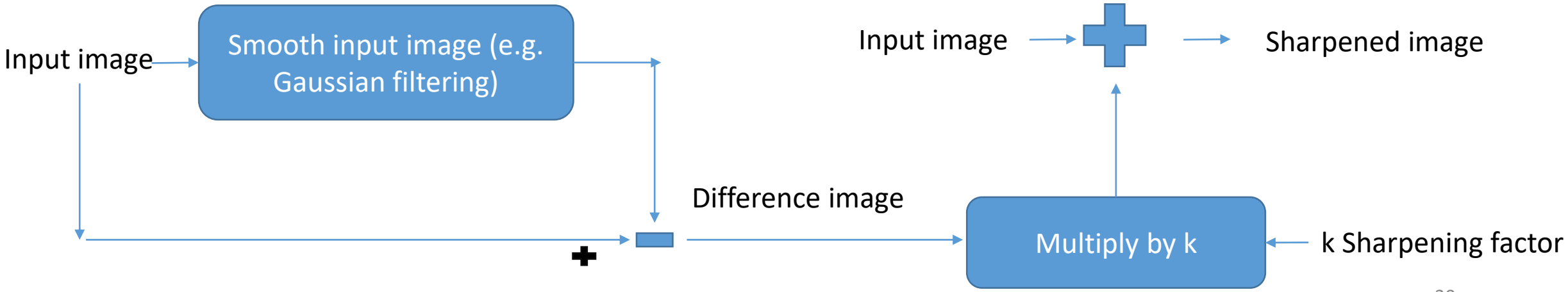
- Sharpening is achieved by enhancing high-frequency content of images (*i.e.* edges)
- In Unsharp masking, an image is sharpened by adding some of the difference between the original image and the smoothed image

$$g_{sharp} = f + k\,(\,f - h_{smoothing} * f)$$

Unsharp mask filter

1. Generate a smoothed image $I_{smoothed}$ ($h_{smoothing}$ * f)
2. Compute $I_{diff} = I_{original} - I_{smoothed}$.
3. Add the difference image to the original

   $I_{enhanced} = I_{original} + k (I_{diff})$



Unsharp mask filter

Original Image

Edge image

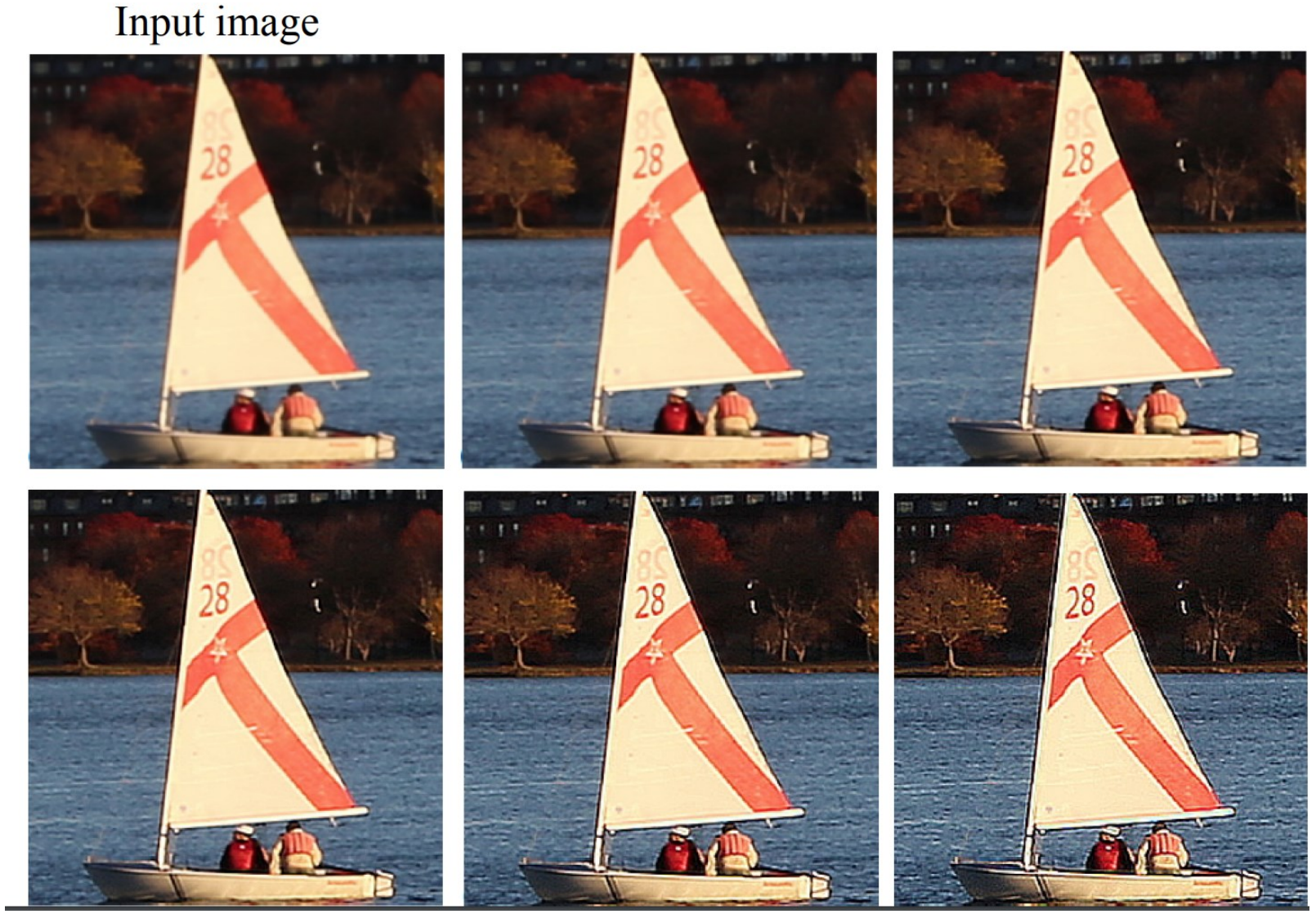
k=0.3

k=0.5

k=0.7

k=2.0

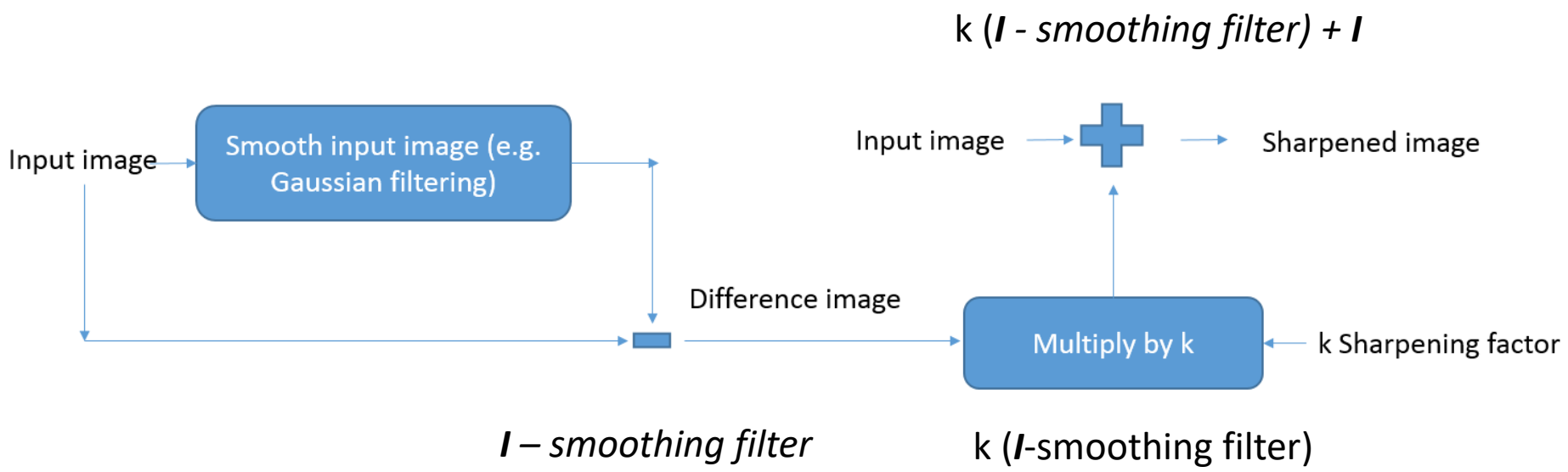
Unsharp mask filter



Unsharp mask filter: faster implementation

- The identity filter returns the original image.

$$\boldsymbol{I} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Unsharp mask filter

- Unsharp mask filtering can be implemented using a single filtering operation with the filter *k(**I** - smoothing filter)+**I***

$$\textit{Where } \mathbf{I} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Exercise

- Is this a sharpening or smoothing filter?
- Why is (2) used as the center value of the first filter.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Hint: Compare this with unsharp mask filtering

Exercise

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

Exercise

What is the effect of applying this filter to a sample image?

0	0	0	0	0
0	0	0	0	0
0	0	0	0	1
0	0	0	0	0
0	0	0	0	0

Answer: The filter replaces every intensity with the intensity two pixels to the right, shifting the image to the left by two pixels.

Filter Separability

- A separable 2D filter kernel can be written as a multiplication of two 1D kernels.
- Separable filters reduce computational cost.
- 2K operations for 1D kernels, K^2 for 2D kernels

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

- In separable filters kernel rows or columns are multiples of each other
- If the 2D filter kernel has a rank of 1 then it is separable
- The rank of a matrix is defined as the maximum number of linearly independent column vectors in the matrix OR the maximum number of linearly independent row vectors in the matrix.

Filter Separability

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

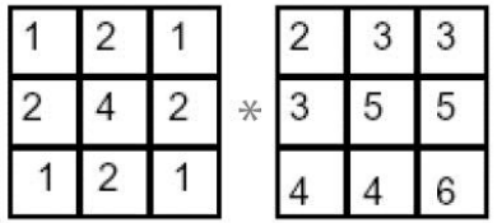
$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

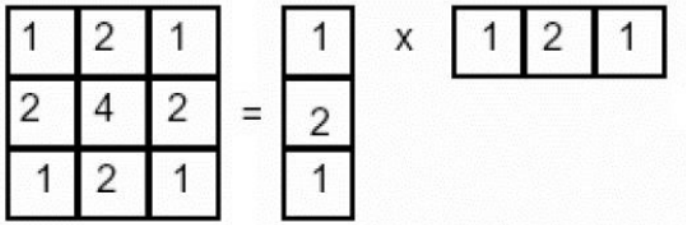
In this case, the two functions are the (identical) 1D Gaussian

Filter Separability

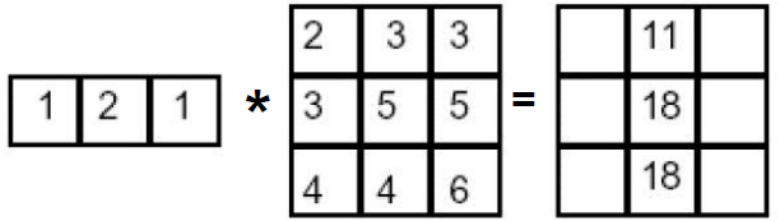
2D convolution
(center location only)



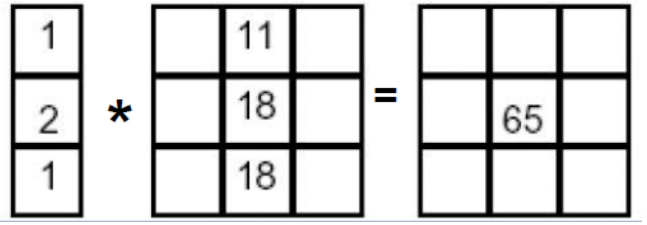
The filter factors
into a product of 1D
filters:



Perform convolution
along rows:



Followed by convolution
along the remaining column:



Filter Separability

- We would like to smooth a 3D image by a Gaussian kernel of size 10x10x10
- Smoothing using the 3D filter kernel requires 1000 operations per voxel
- A better approach is to smooth using three 1D Gaussian kernels of sizes 10x1x1, 1x10x1, and 1x1x10
- Operations required becomes 30 operations per voxel.

Significant saving !!

Filter Separability

Assume that you have an image of size M x N and 2D separable kernel of size k x k.

Computational advantage = # of operations using the 2D kernel/# of operations using the 1D kernel

$$= \frac{MNk^2}{MNk+MNK} = \frac{MNk^2}{MN2k} = \frac{k^2}{2k} = \frac{k}{2}$$

Frequency domain filtering

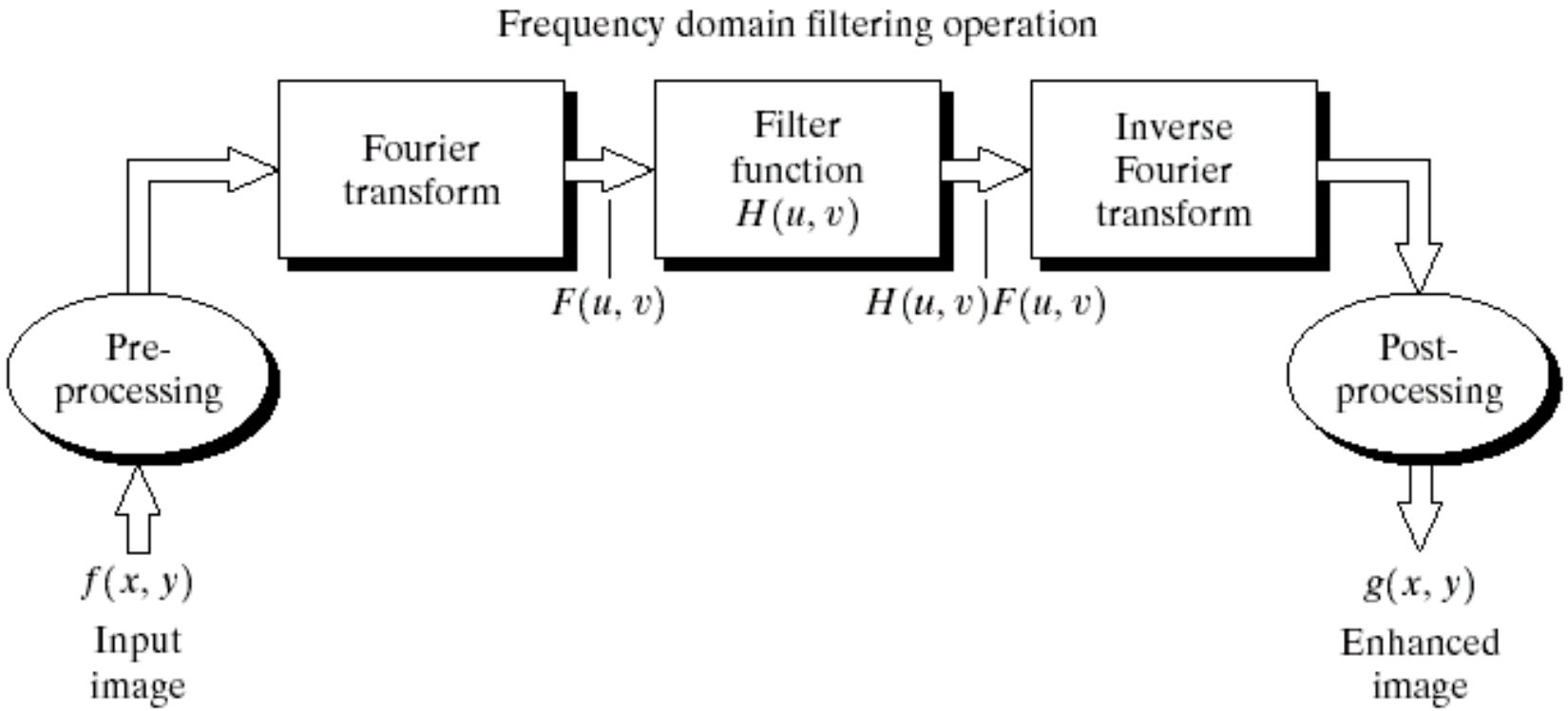


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Using DFTs to represent signals

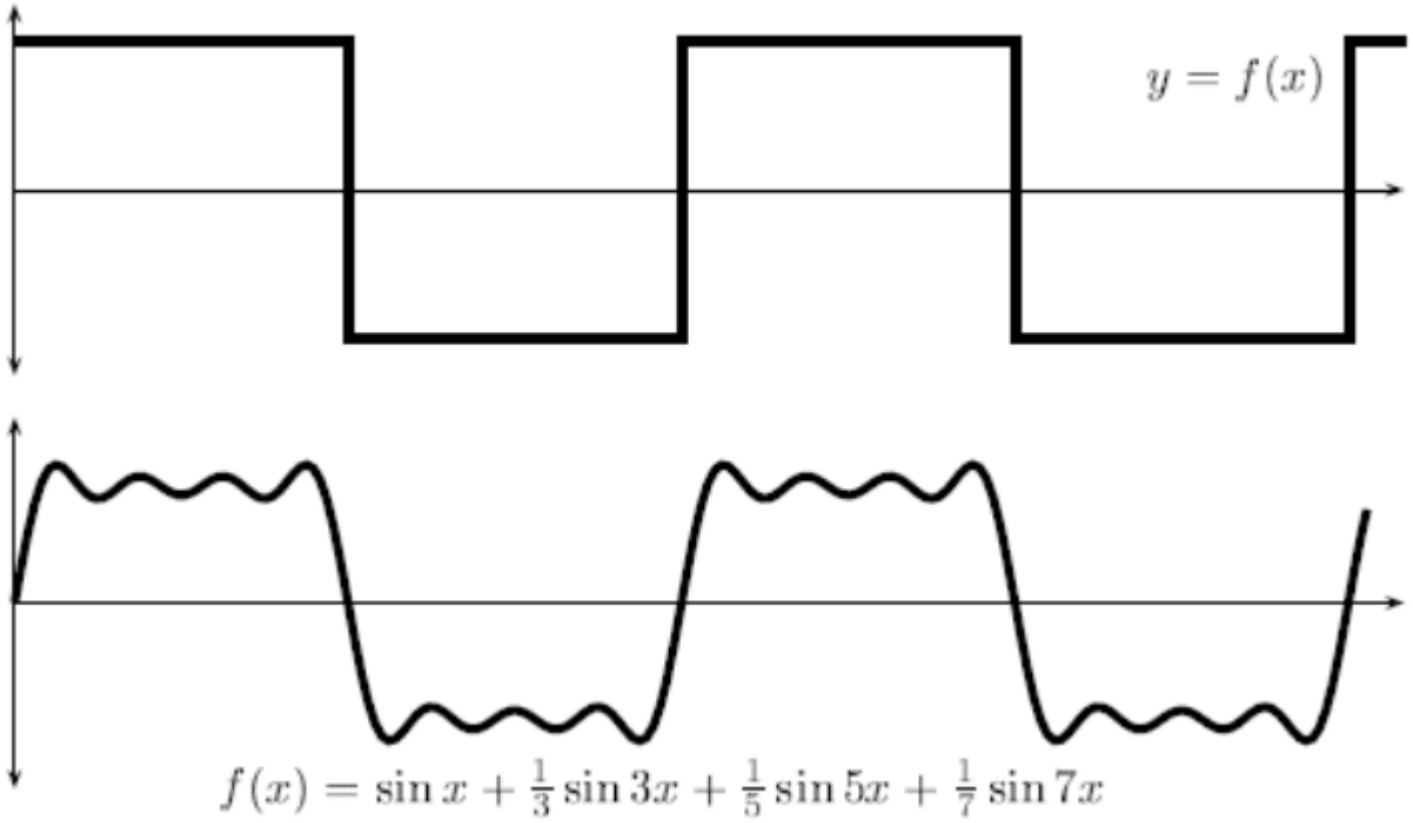
- Theorem (by Fourier, 1768-1830)

  A periodic signal x(t) with period T can be written as an infinite sum of sinusoids, all of which have frequencies that are multiples of 1/T.

- But most signals in practice are not infinite?

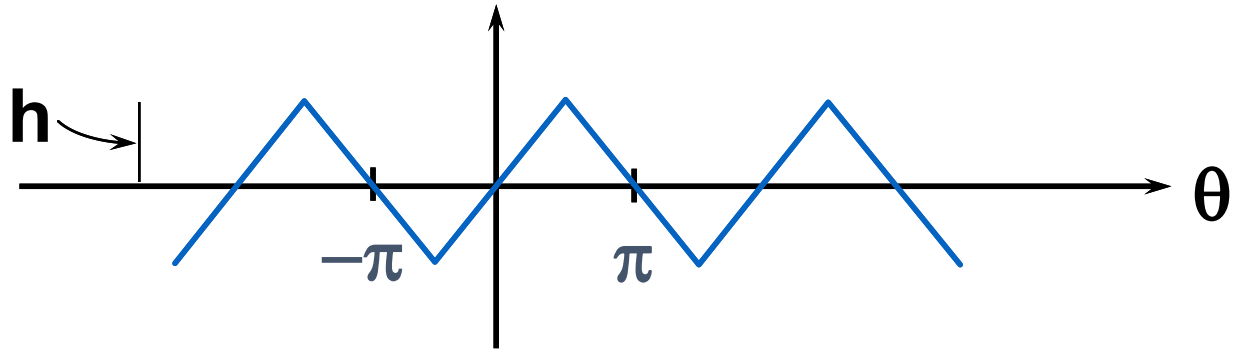
  In this case, its assumed they extend to infinity by replication

Fourier Analysis



Fourier Analysis

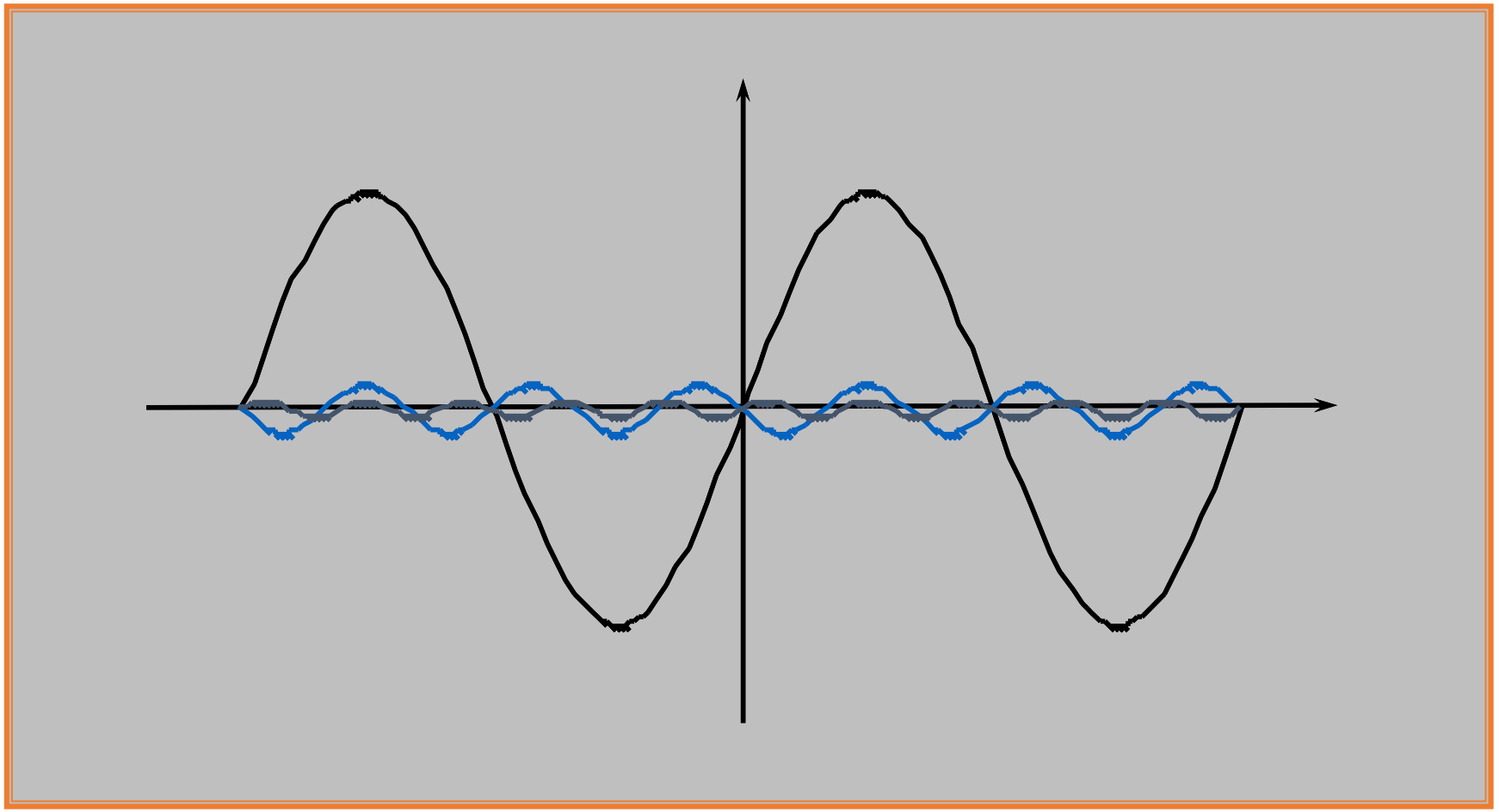
Triangular wave



$$f(\theta) = \left(8h/\pi^2\right)\left(\frac{\sin\theta}{1^2} - \frac{\sin 3\theta}{3^2} + \ldots + \frac{\sin 5\theta}{5^2} - \frac{\sin 7\theta}{7^2} \ldots\right)$$

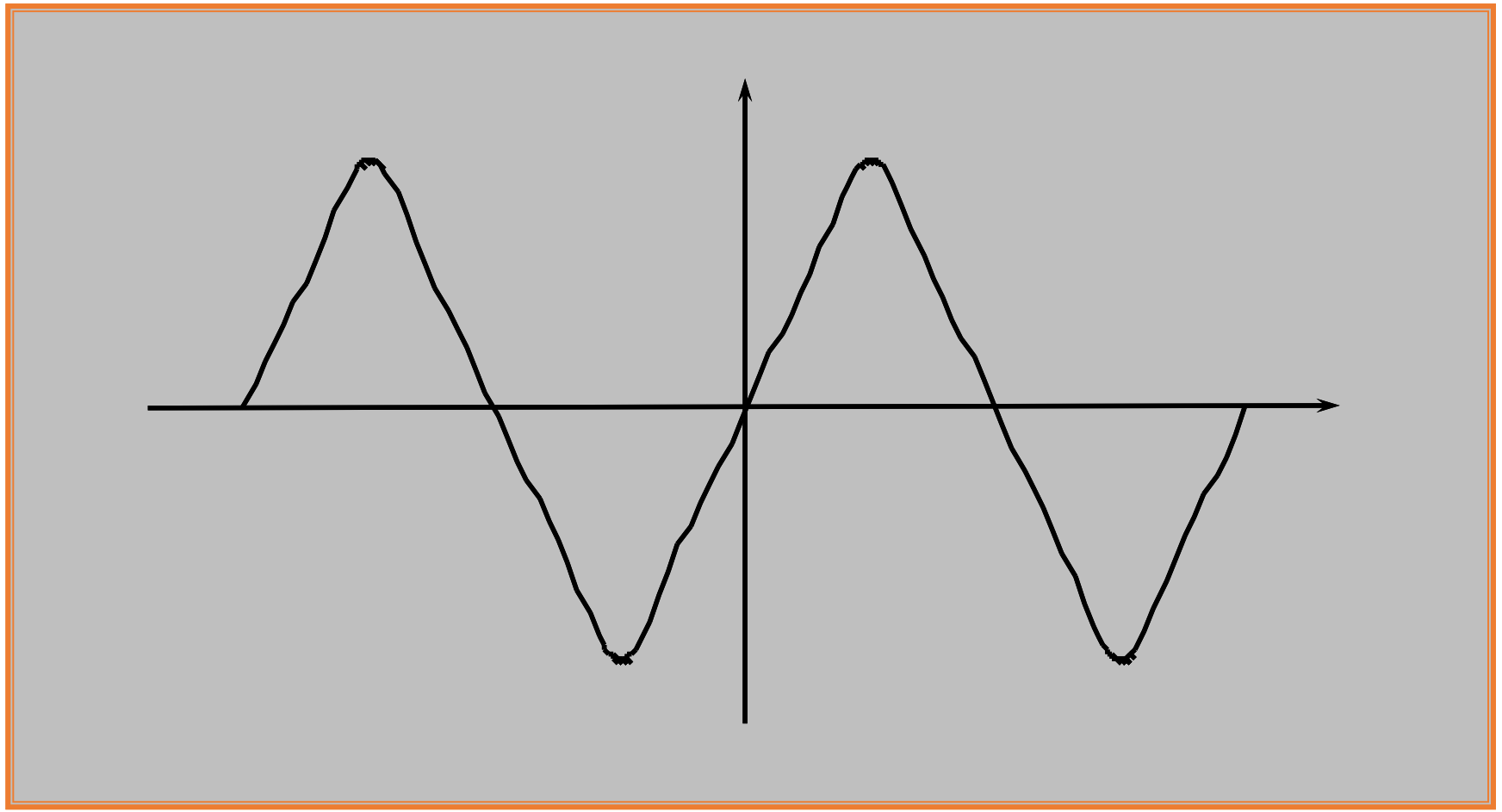
Triangular wave

- First 3 series terms:



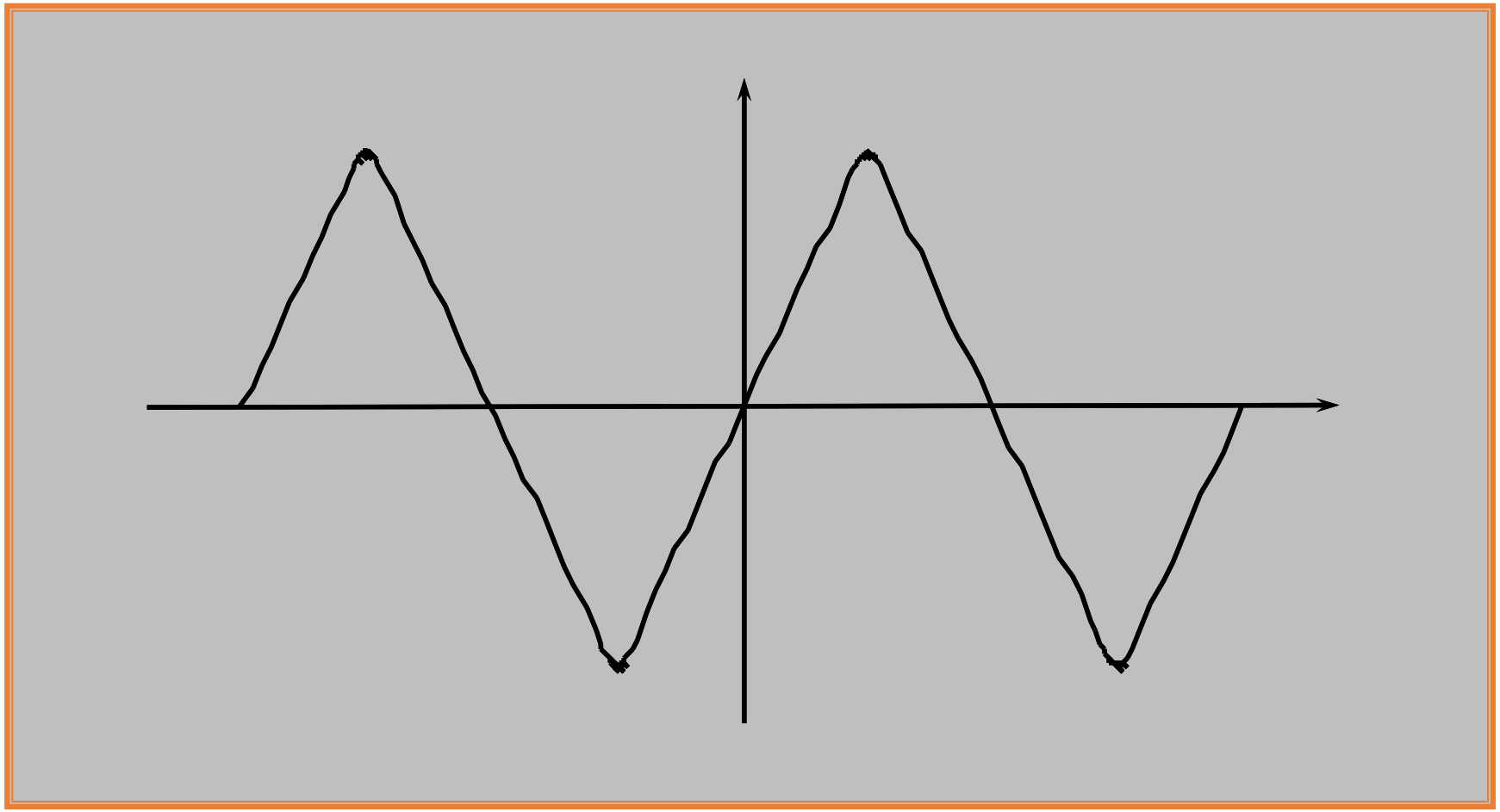
Triangular wave

- Sum of first 3 series terms:



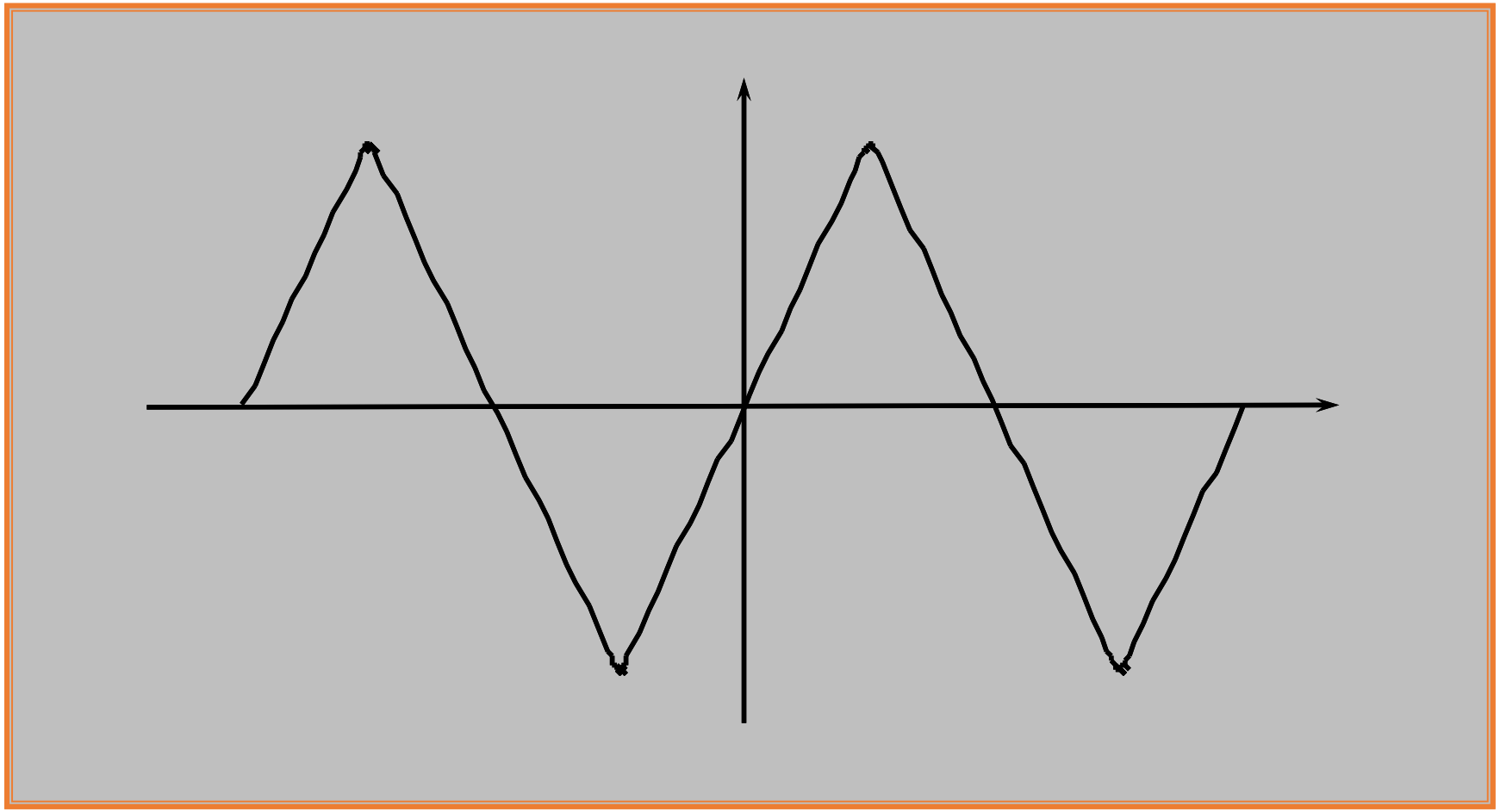
Triangular wave

- Sum of first five terms:



Triangular wave

- Sum of first 10 terms:



Fourier coefficients

- A periodic signal f(x) with period T can be constructed exactly using

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi n x}{T}\right) + \sum_{n=0}^{\infty} b_n \sin\left(\frac{2\pi n x}{T}\right)$$

- a_n and b_n represent the contribution of the nth frequency to the signal.
- a_n and b_n at lower values for *n* describe the low frequency content (i.e. broad structural details).
- a_n and b_n at high n values describe fine details in the signal.

Fourier coefficients

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi n x}{T}\right) + \sum_{n=0}^{\infty} b_n \sin\left(\frac{2\pi n x}{T}\right)$$

- For most meaningful signals and images a_n and b_n at the first "few" n values can reconstruct a signal close-enough to the original signal

- This idea "sparsity" is the basis of audio, image, and video compression formats (e.g. MP3, JPEG, MPG)

Computation of Fourier coefficients

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi n x}{T}\right) + \sum_{n=0}^{\infty} b_n \sin\left(\frac{2\pi n x}{T}\right)$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos\left(\frac{2\pi n}{T}\right)dx$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\sin\left(\frac{2\pi n}{T}\right)dx$$

This can be expressed in complex form as

$$f(x) = \sum_{n=-\infty}^{\infty} c_n exp\left(\frac{-i2\pi n x}{T}\right)$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\, exp\left(\frac{-i2\pi n}{T}\right)dx$$

Discrete Fourier transform

DFT $$F[k] = \sum_{n=0}^{N-1} f[n] e^{-i2\pi uk} = \sum_{n=0}^{N-1} f[n] e^{-i2\pi \frac{n}{N} k}$$

IDFT $$\mathrm{f}[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{i2\pi un} = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{i2\pi \frac{n}{N} k}$$

Properties of Fourier transform

Properties of the DFT

Property	Time Domain	Frequency Domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
Time-shifting	$x[n-m]$	$e^{-j2\pi km}X(k)$
Frequency-shifting (modulation)	$e^{-j2\pi k_0 n/N}x[n]$	$X(k-k_0)$
Time-convolution	$x_1[n] \otimes x_2[n]$	$X_1[k]X_2[k]$
Frequency-convolution	$x_1[n]x_2[n]$	$X_1[k] \otimes X_2[k]$
Parseval's relation	$E_x = \sum_{n=0}^{N-1} \lvert x[n] \rvert^2$	$E_x = \sum_{k=0}^{N-1} \lvert X[k] \rvert^2$

Fourier transform in two dimensions

- The two-dimensional Fourier transform and its inverse
  - Fourier transform

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$\text{for } u = 0,1,2,\ldots,M-1, v = 0,1,2,\ldots,N-1$$

  - Inverse Fourier transform:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

$$\text{for } x = 0,1,2,\ldots,M-1, y = 0,1,2,\ldots,N-1$$

- *u*, *v* : the transform or frequency variables
- *x*, *y* : the spatial or image variables

Fourier transform in two dimensions

- We define the Fourier spectrum, phase angle, and power spectrum as follows:

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{\frac{1}{2}} \quad \text{(Magnitude)}$$

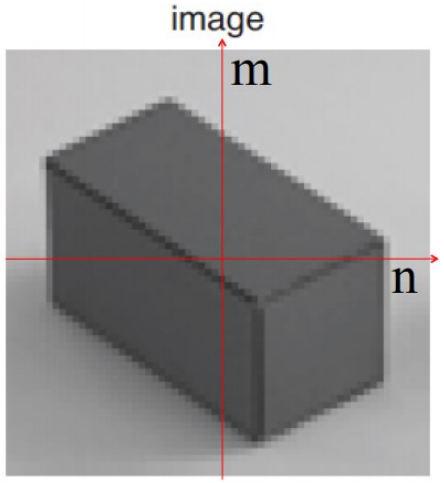
$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right] \quad \text{(phase angle)}$$

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v) \quad \text{(power spectrum)}$$

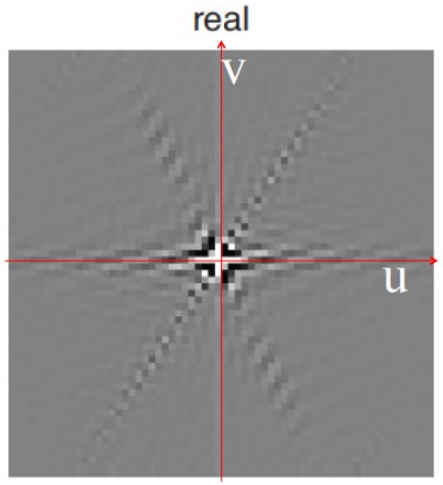
  - $R(u,v)$: the real part of $F(u,v)$
  - $I(u,v)$: the imaginary part of $F(u,v)$

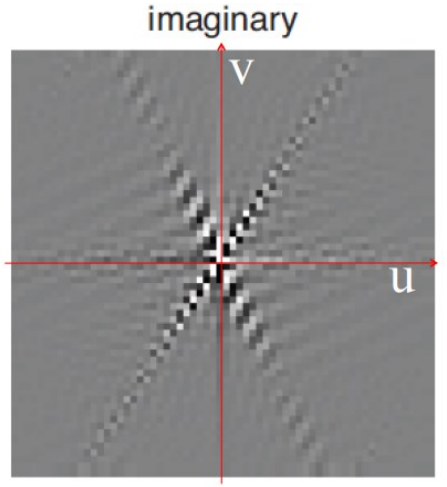
Visualizing the 2D Fourier Transform

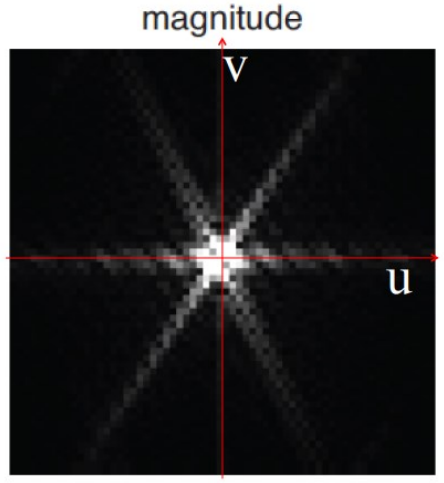
$f[n, m]$

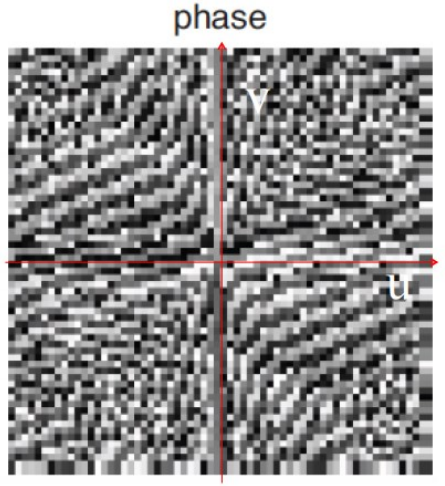


$F[u, v]$

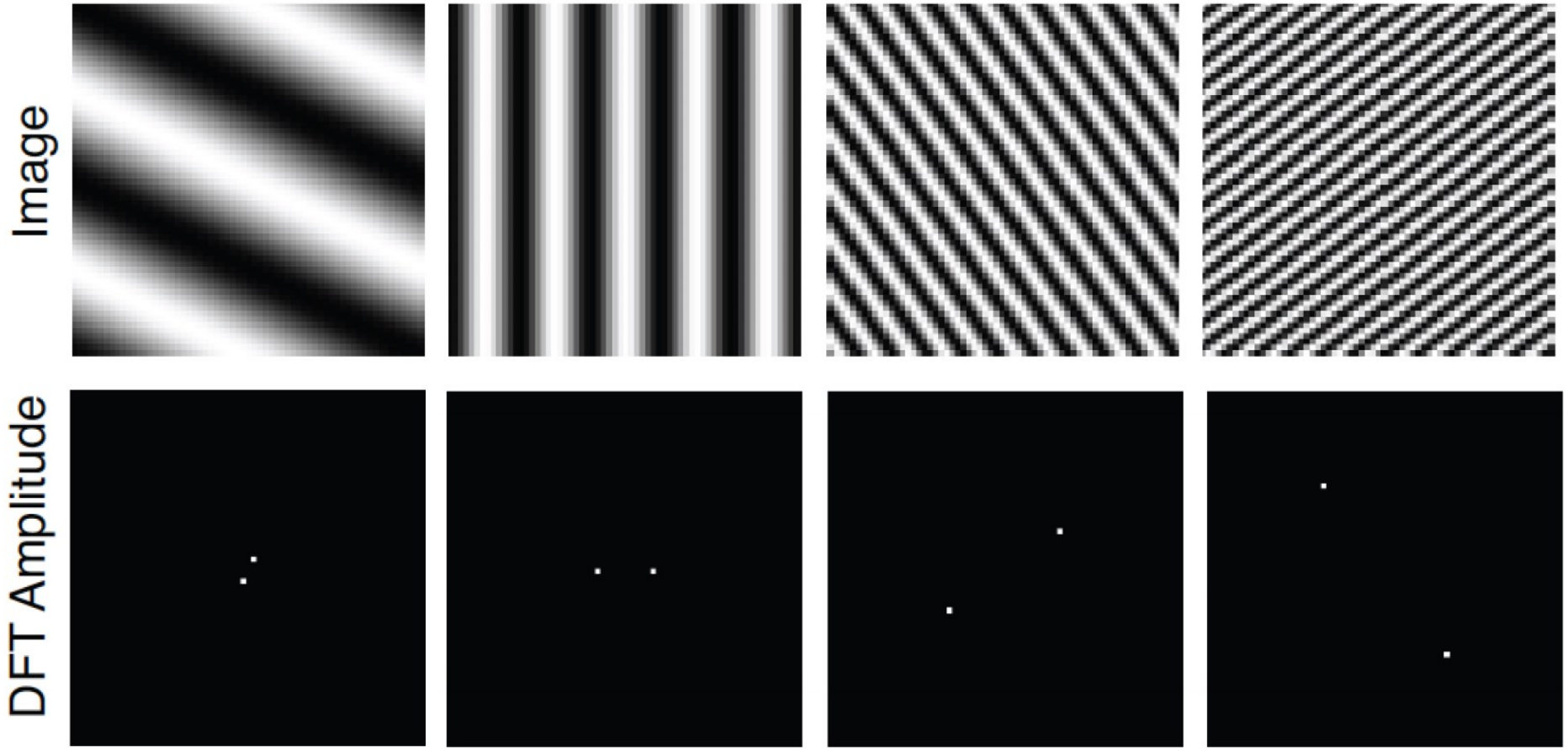




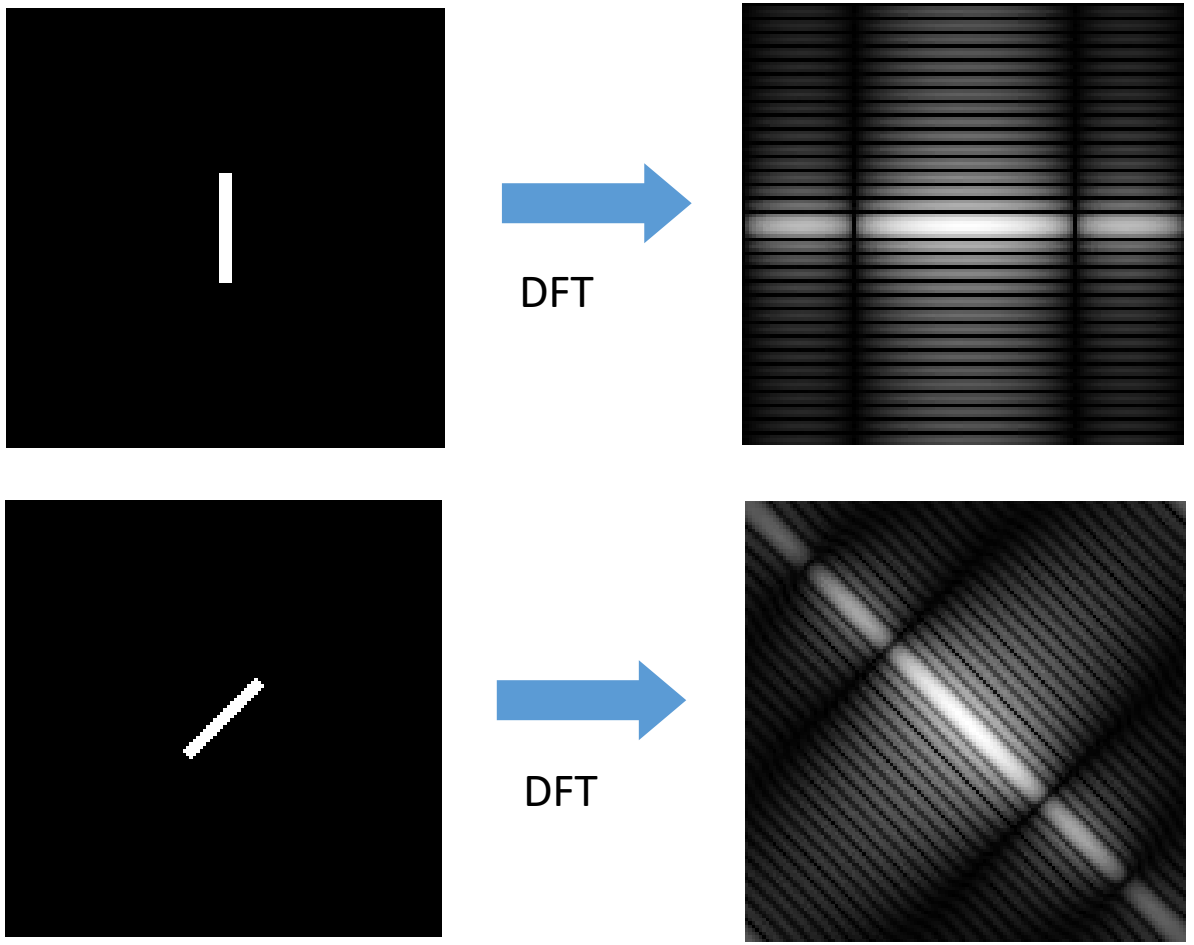




Examples of 2D DFT Pairs

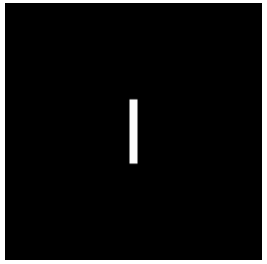


2D DFT Rotation property

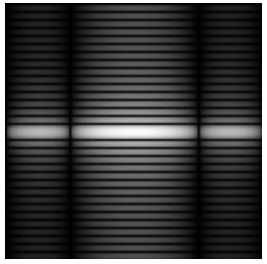


DFT properties: Linearity

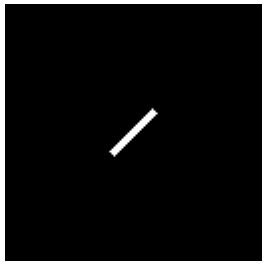
A



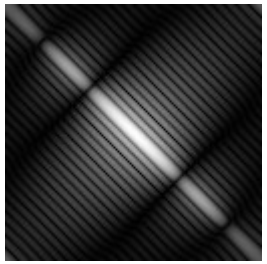
DFT



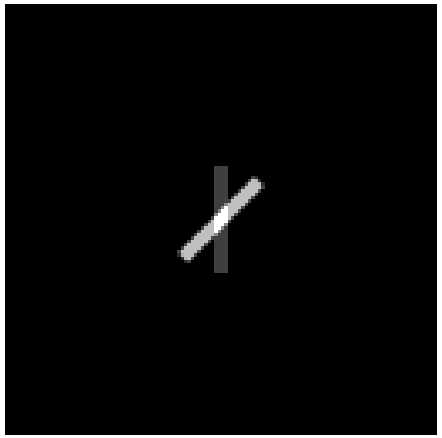
B



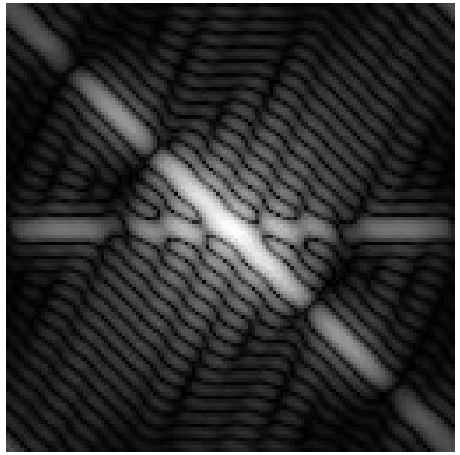
DFT



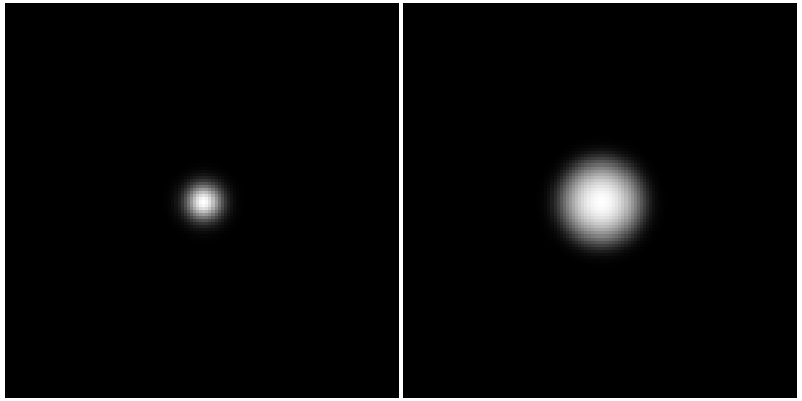
0.25 * A
+ 0.75 * B



DFT



2D DFT of sample functions



2D Gaussian function

Its DFT magnitude is another Gaussian

2D DFT in Matlab

- fft2 command, using fftshift to relocate the center frequency.

```
>> A=imread('boat.png');
>> B=fft2(A);
>> B(3,6)

ans =

  -3.5586e+05 + 1.6945e+05i

>> imagesc(log(abs(B)));
>> C=fftshift(fft2(A));
>> figure;imagesc(log(abs(C)));
```

Source code

Original image

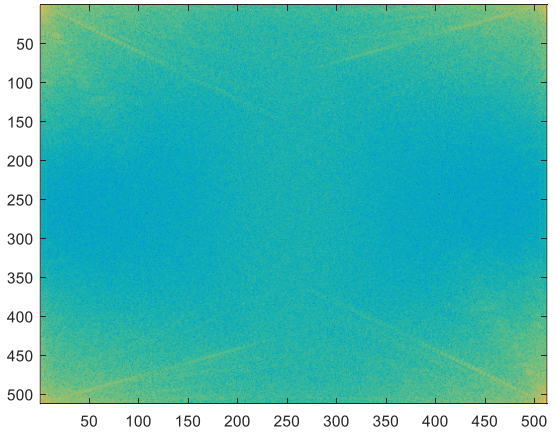


Image B log-magnitude spectrum

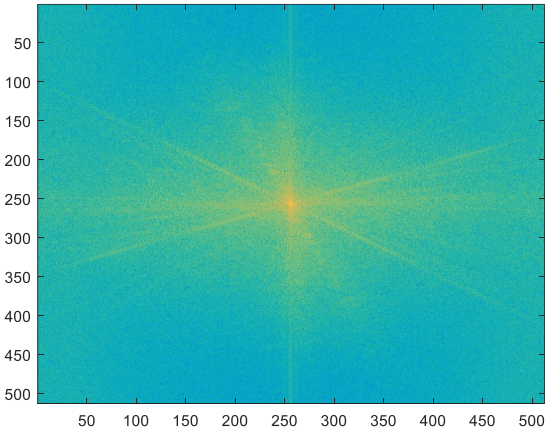
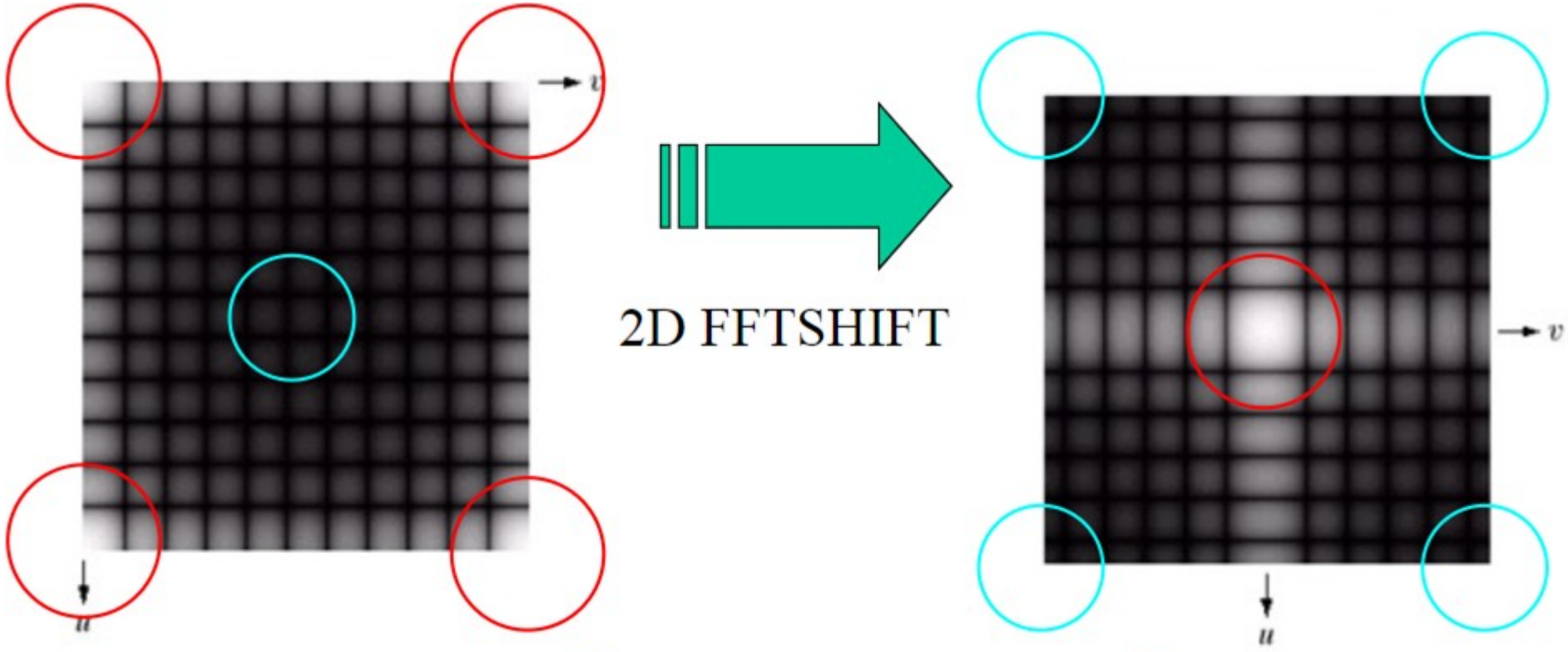


Image C log-magnitude spectrum

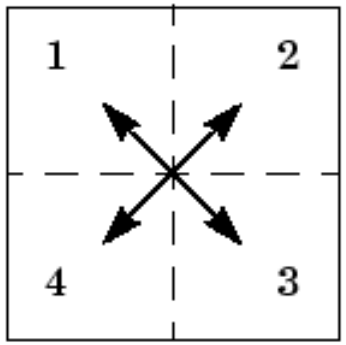
Shifting quadrants



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Shifting quadrants ‘fftshift’

- fftshift shifts quadrants to move the zero-frequency component to the center of the array



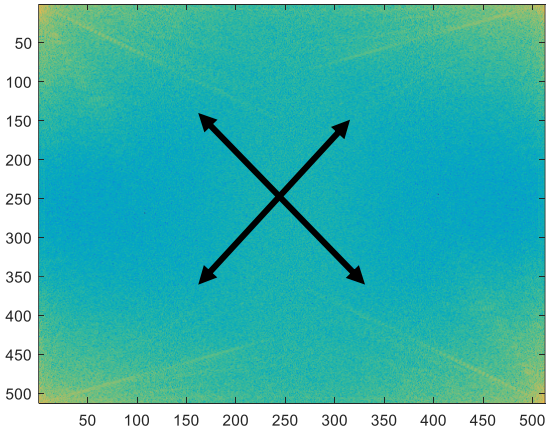


Image B

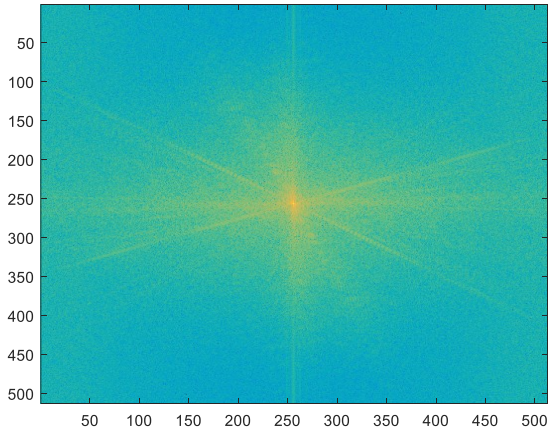


Image C

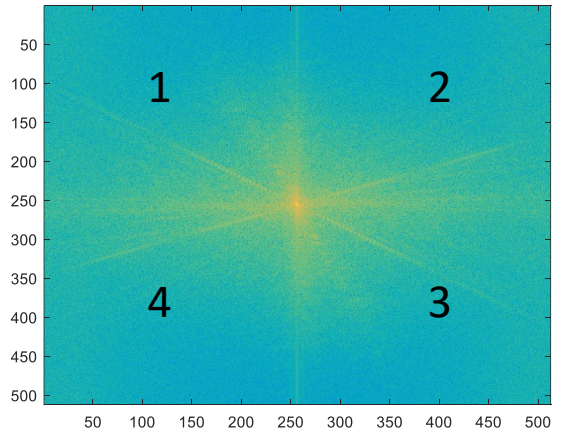
Properties of the 2D DFT

- Some properties of 2D DFT:

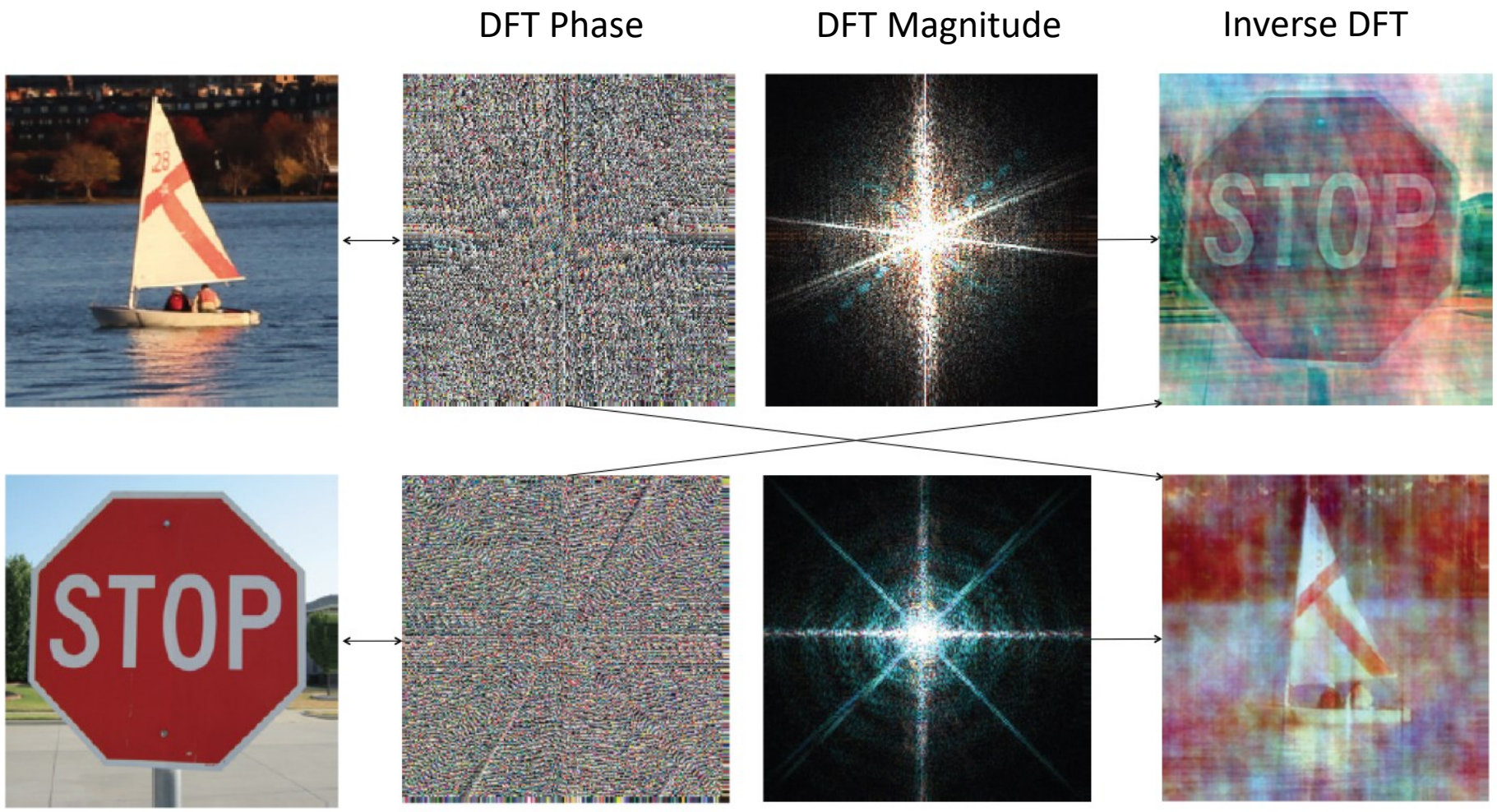
$$F(u,v) = F^*(-u,-v) \qquad \text{(conujgate symmetric)}$$
$$|F(u,v)| = |F(-u,-v)| \qquad \text{(symmetric)}$$

This is symmetry around the origin. DFT values in the first quadrant are the complex conjugate of values in the third quadrant.

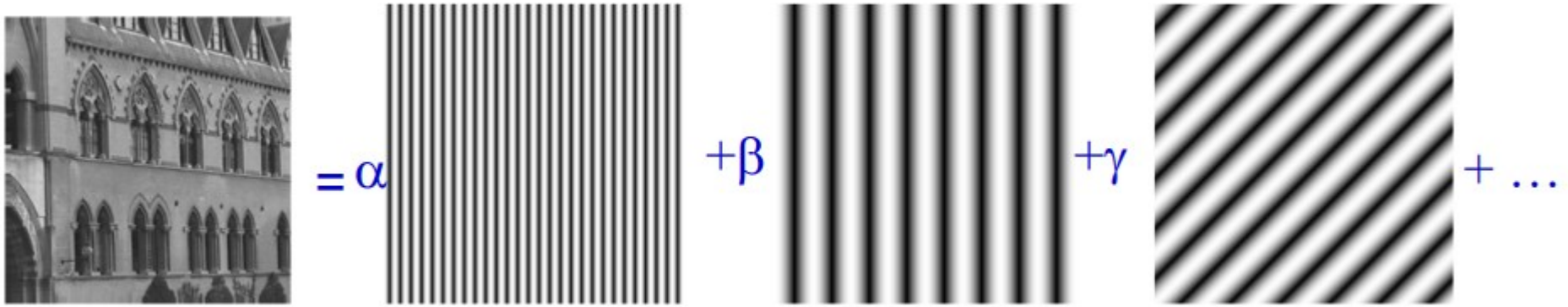
Similarly, DFT values in the second quadrant are complex conjugates of values in the fourth quadrants.



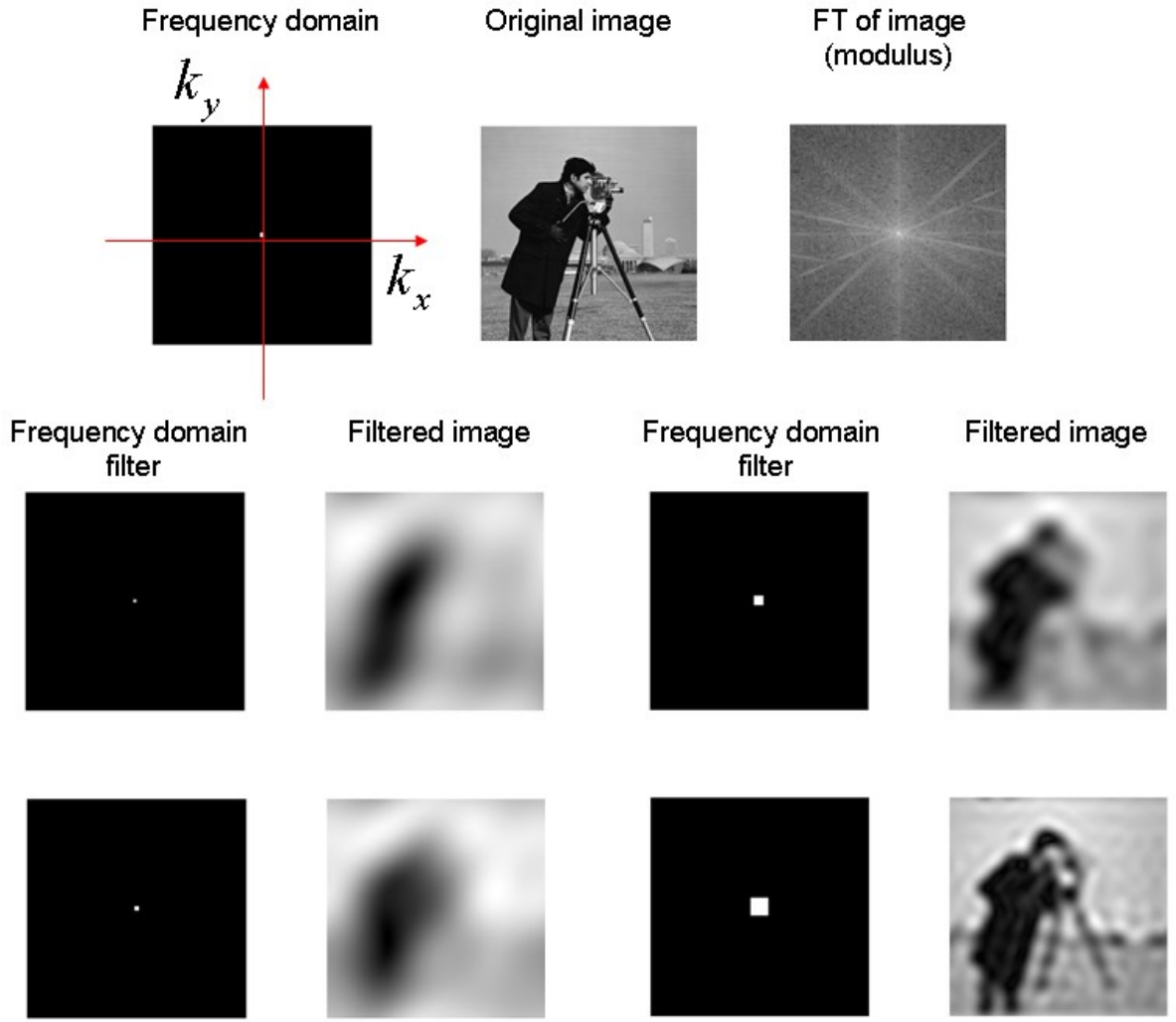
Mixing the phase and magnitude



Reconstructing an image from its Fourier coefficients

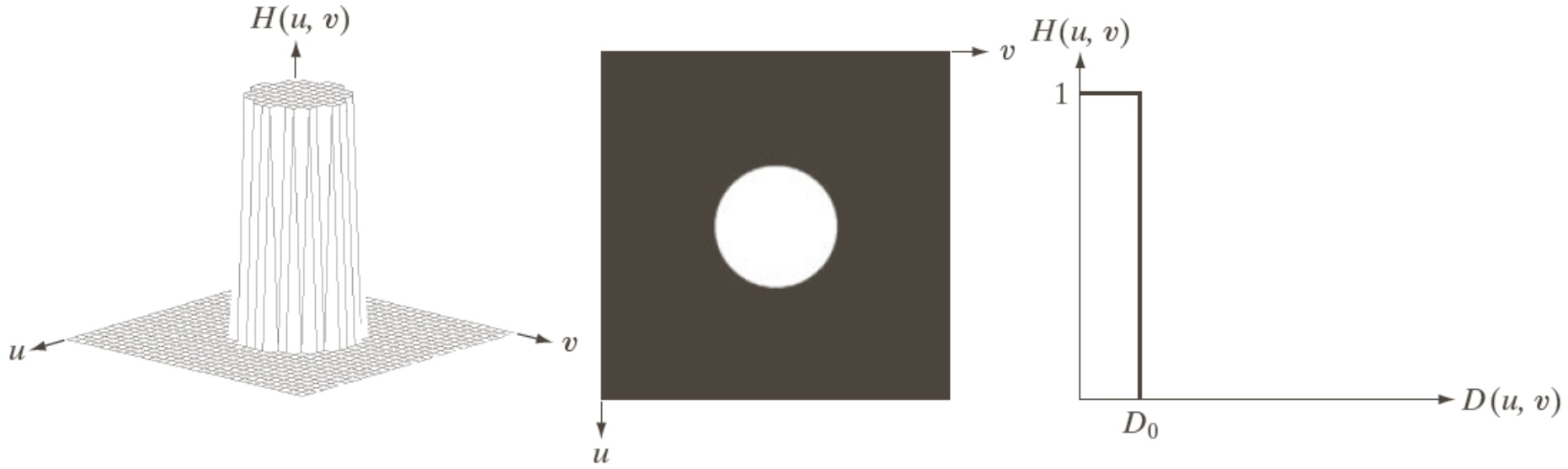


Frequency domain filtering example



Low pass filtering of a sample image

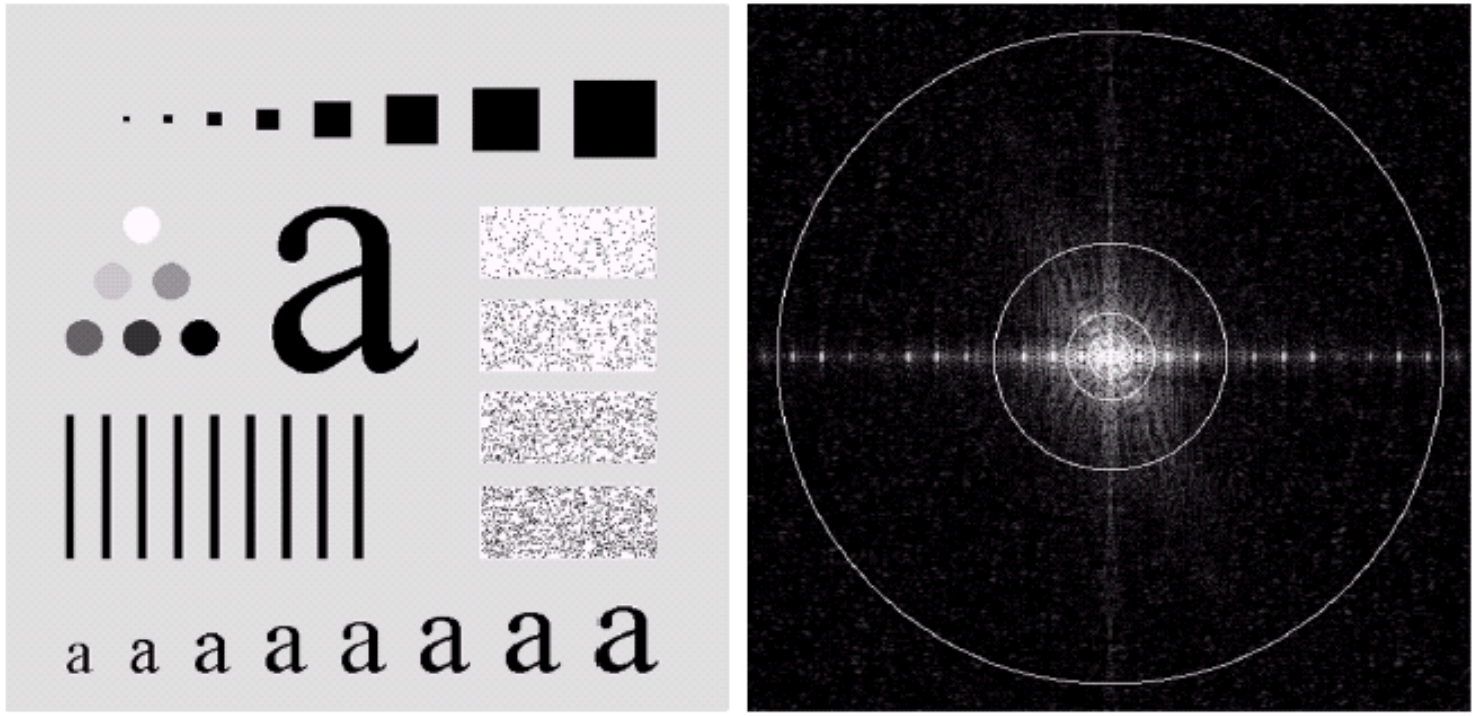
Ideal Lowpass filter



a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

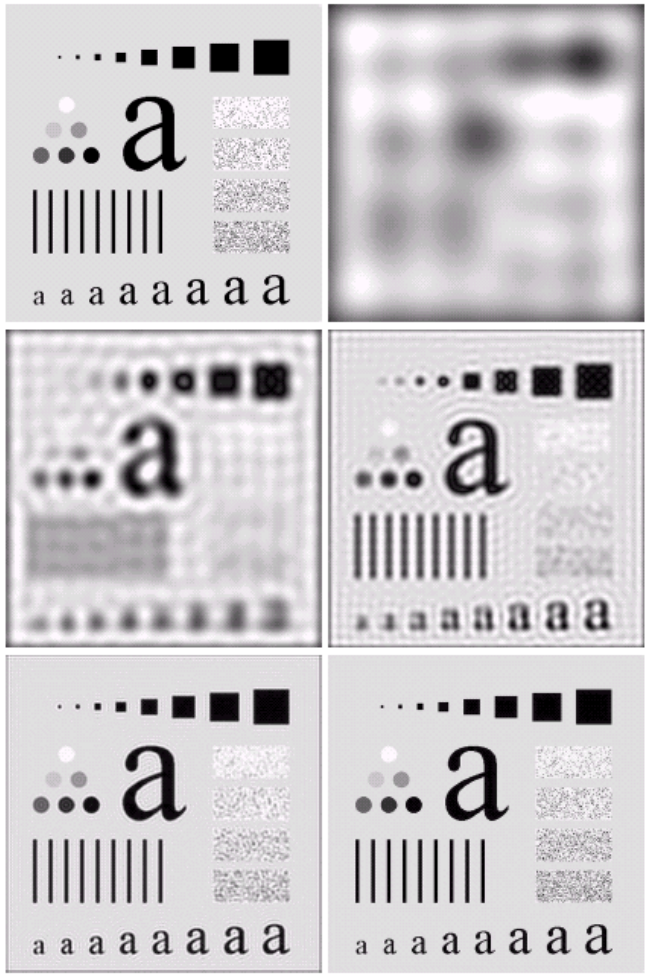
Ideal Lowpass filter



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Ideal Lowpass filter

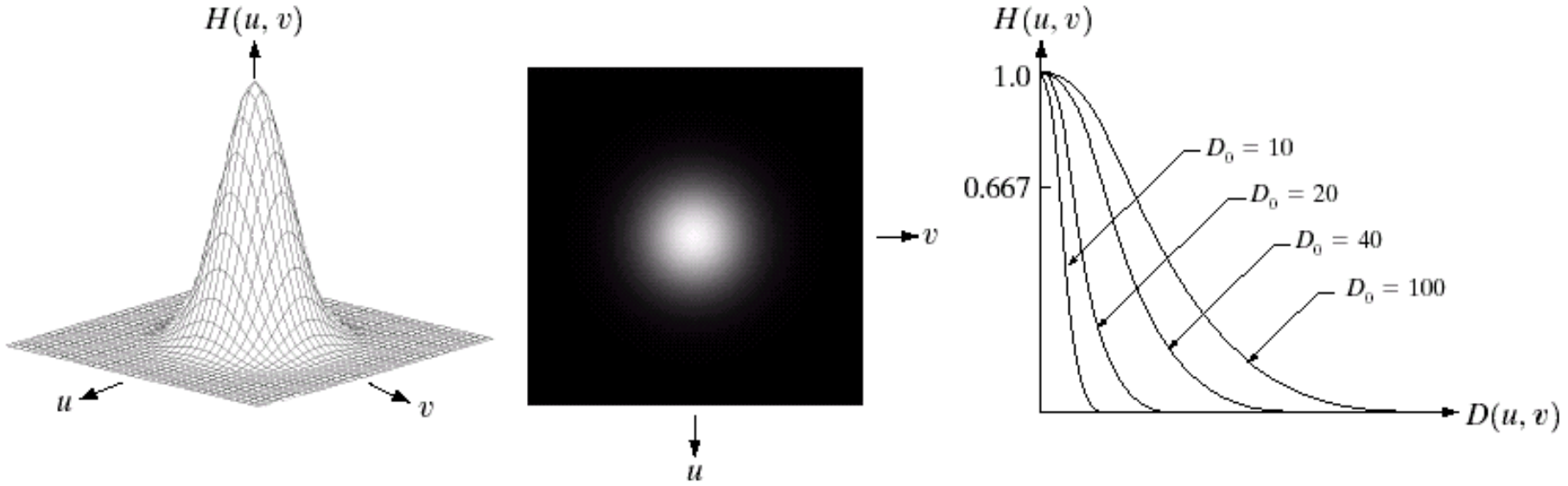


a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Gaussian Lowpass filters

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Gaussian LPF's

D_0=5,15,30,80,and 230

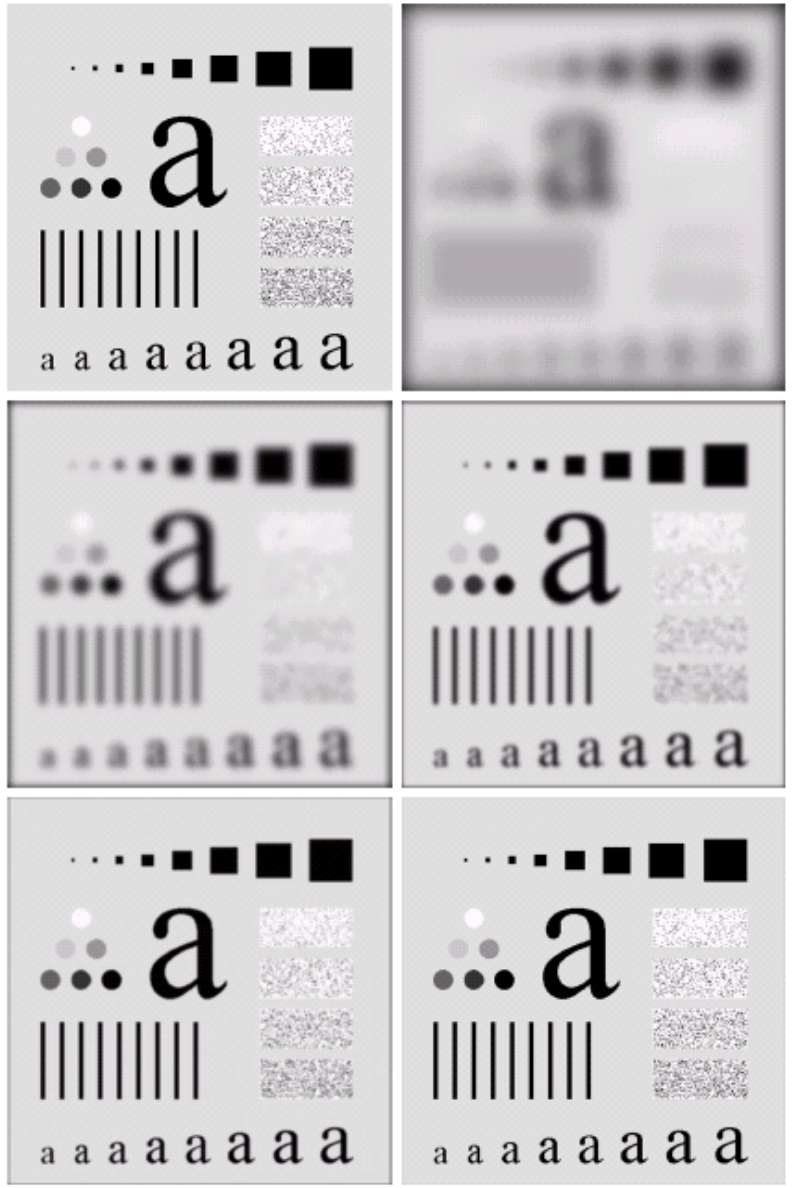


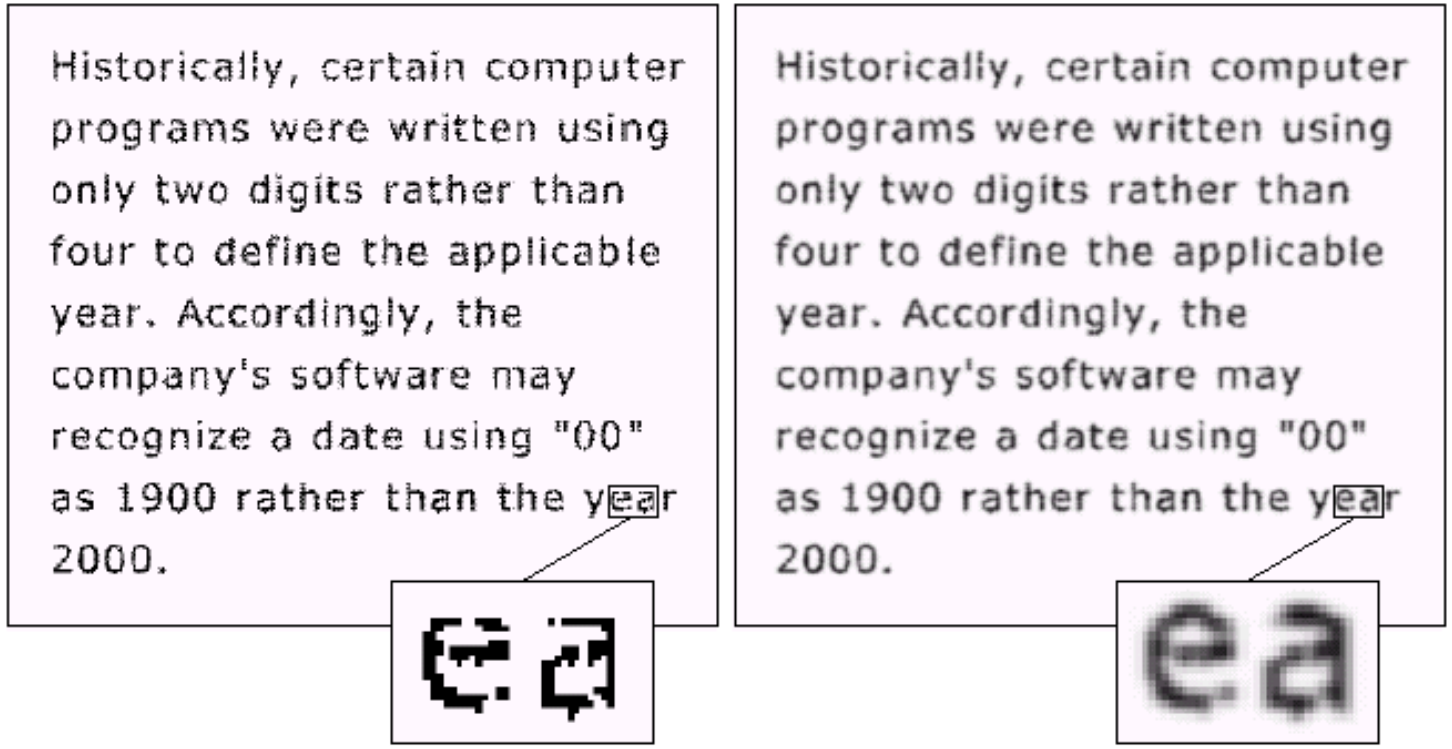
FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

Gaussian Lowpass filters

a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



Laplacian in the frequency domain

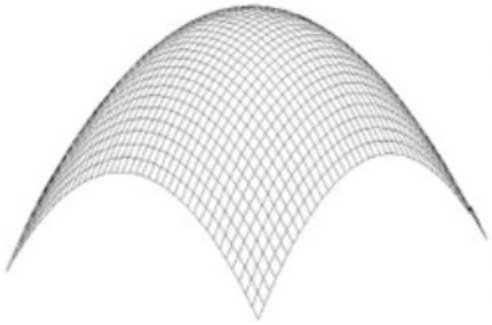
From Fourier Tr. Property: $\frac{d^n f(x)}{dx^n} \Leftrightarrow (ju)^n F(u)$

Then for Laplacian operator

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \Leftrightarrow -(u^2 + v^2)F(u,v)$$

We get

$$\nabla^2 \Leftrightarrow -(u^2 + v^2)$$



Surface plot

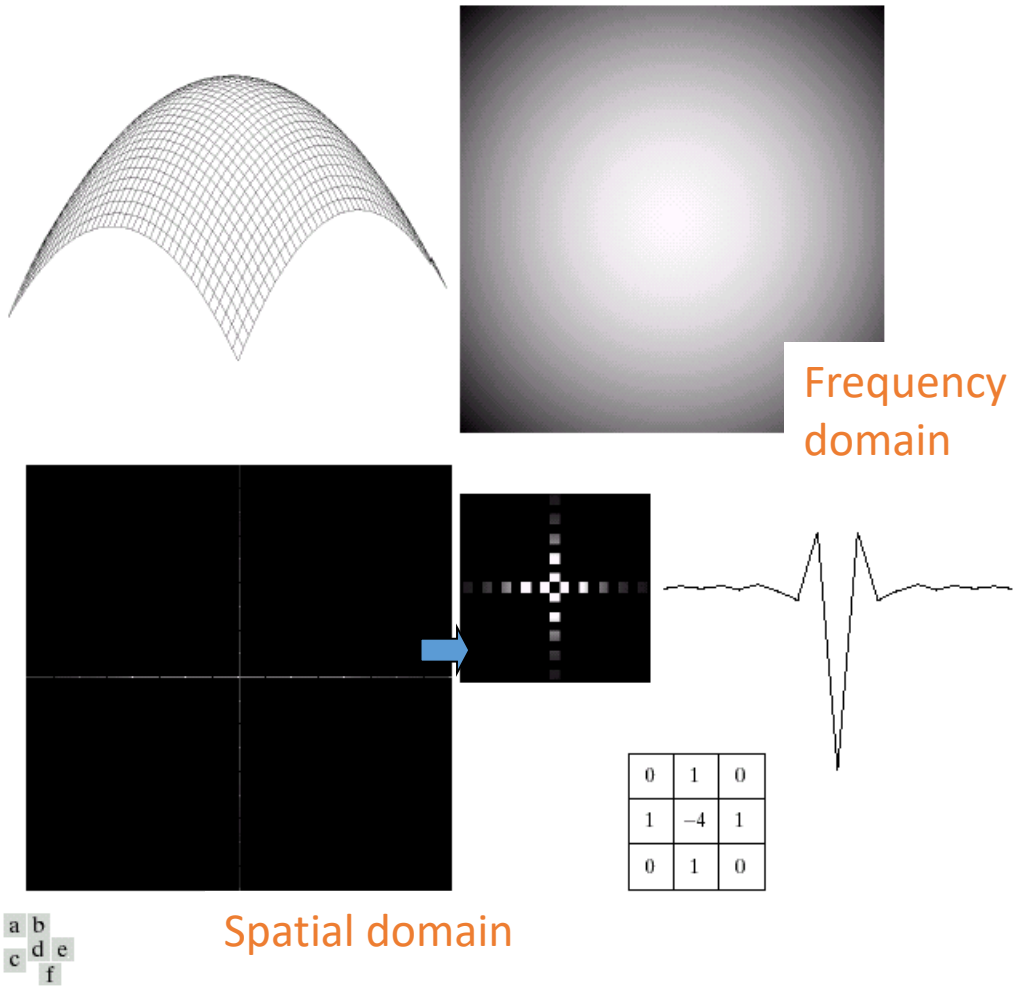
Image of $-(u^2+v^2)$

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Laplacian in the frequency domain

- The Laplacian filter

$$H(u,v) = -(u^2 + v^2)$$



a b
c d e
f

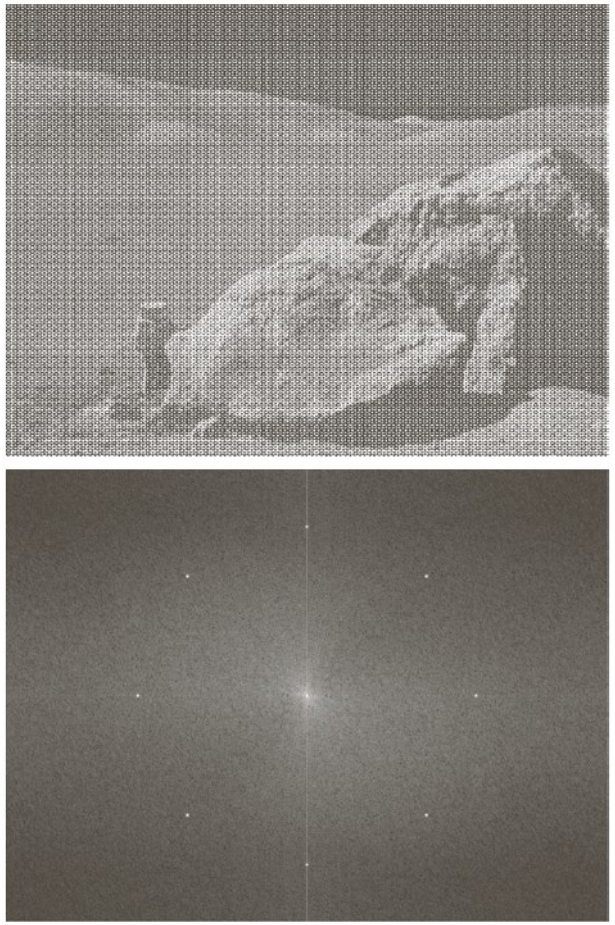
FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

Laplacian sharpening (edge enhancement)

a b

FIGURE 4.58
(a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain.

Selective filtering



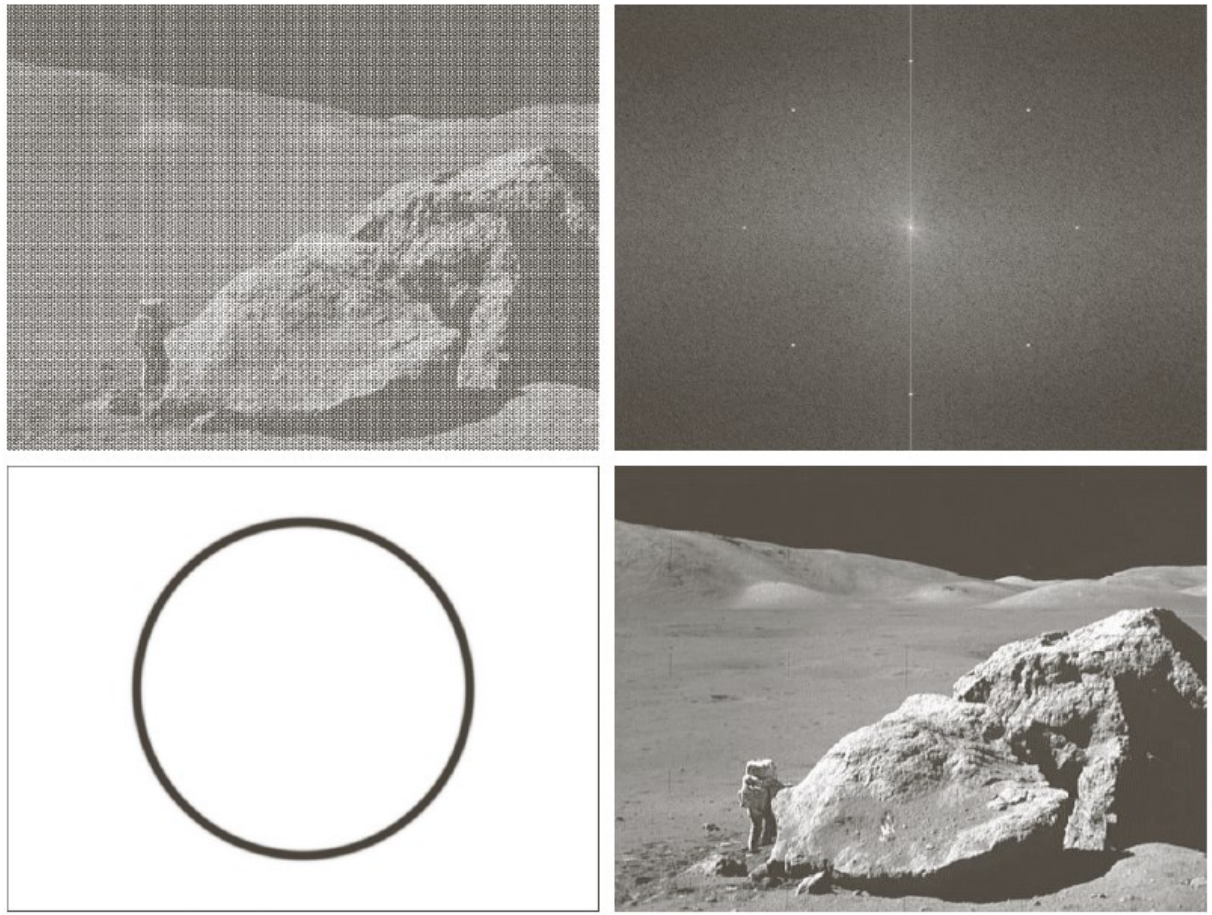
a
b

FIGURE 5.5
(a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Selective filtering

a b
c d

FIGURE 5.16
(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)



Selective filtering

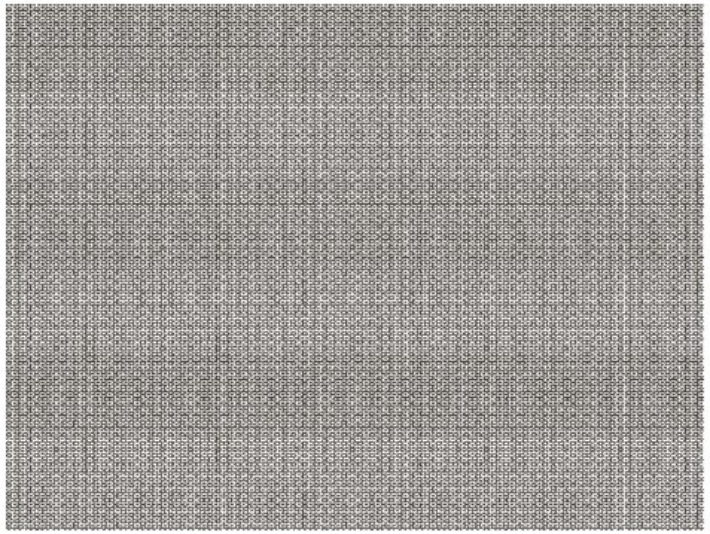
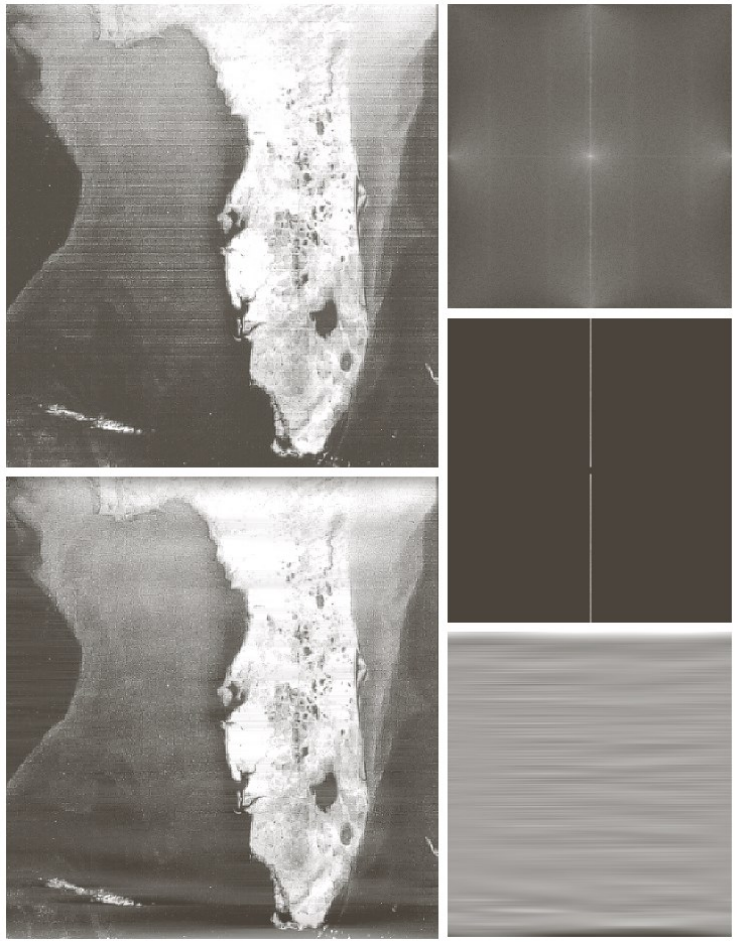


FIGURE 5.17
Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

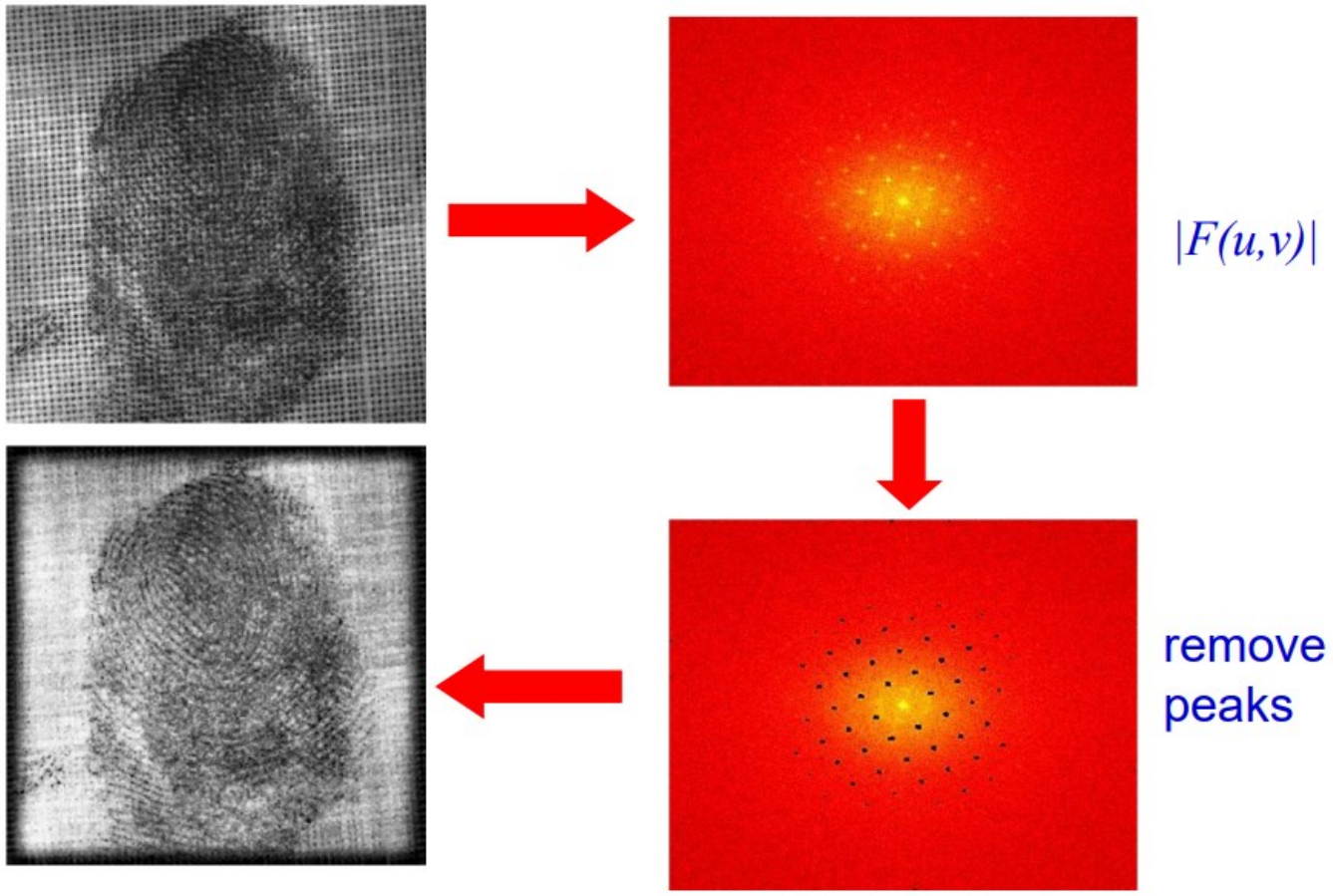
Selective filtering



a b
e c
d

FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Frequency domain operations

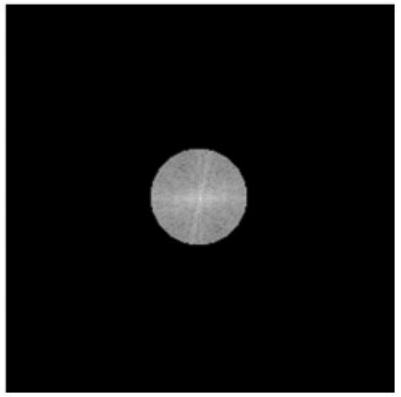


Noisy fingerprint data

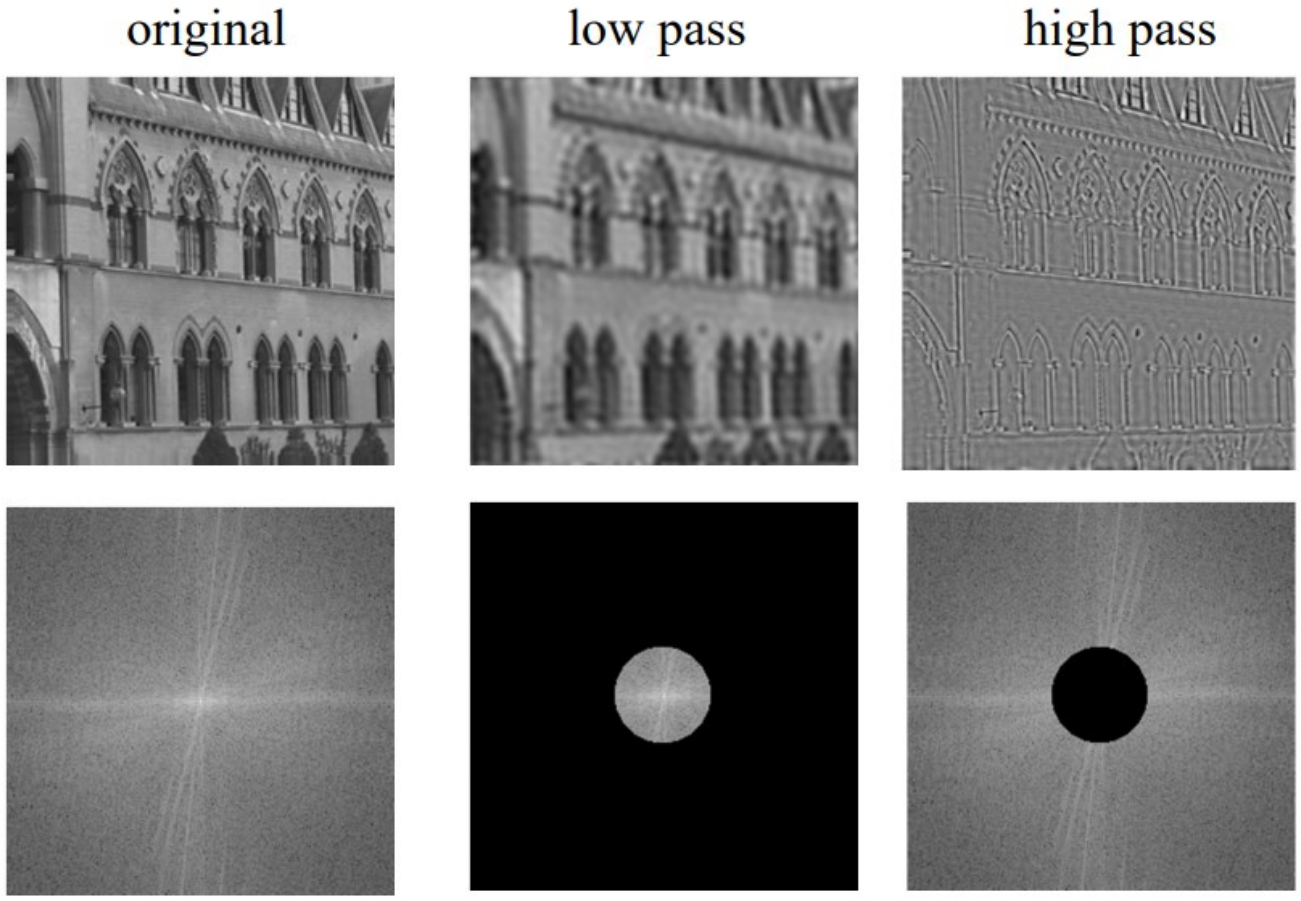
Frequency domain operations

- To get the low pass image:
  1. Define a binary mask that is equal to 1 inside a circle and zero outside
  2. Take the 2D Fourier transform of the image.
  3. Multiply the 2D Fourier transform of the image by the binary mask
  4. Take the inverse Fourier transform to get the low-passed image back

low pass

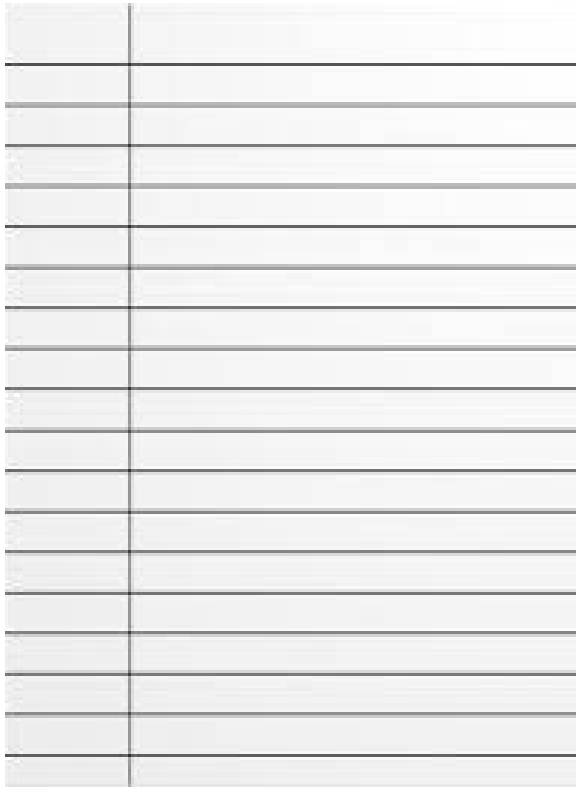


Frequency domain operations

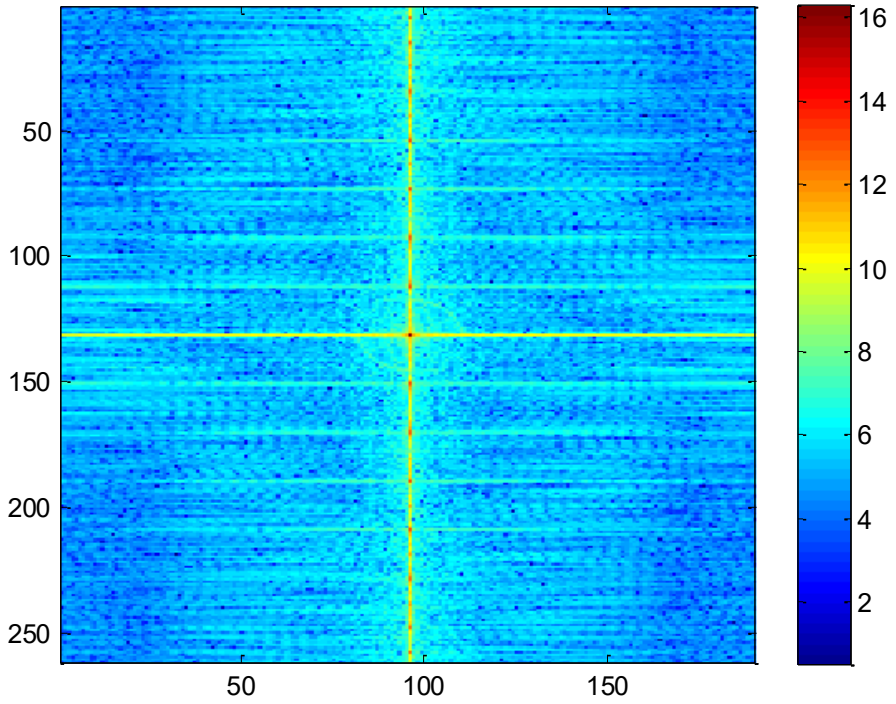


Frequency domain filtering

- Example: Removing vertical/horizontal lines



Original image



Log-magnitude of its Fourier coefficients

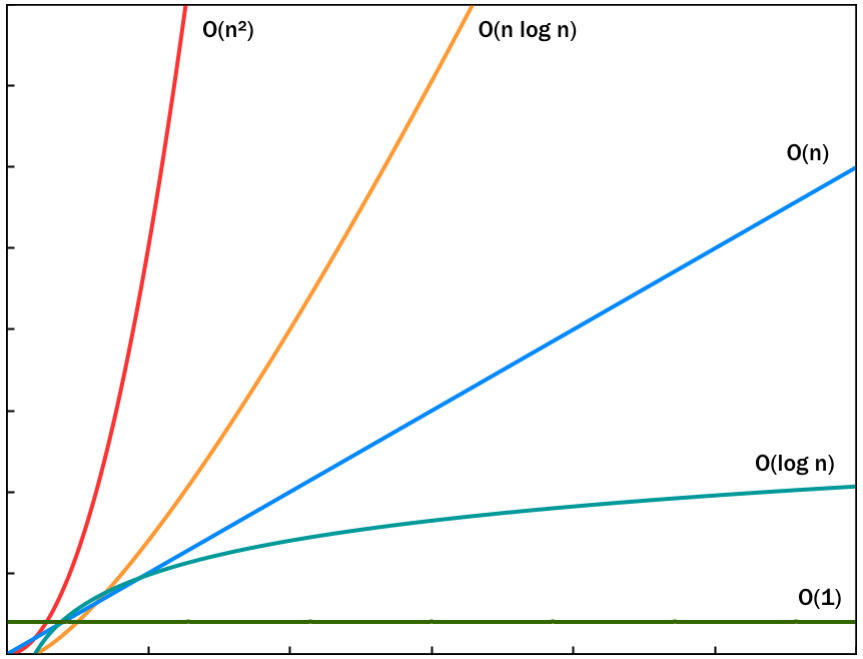
Frequency domain filtering

- Matlab example removeverticalline.m

The Fast Fourier transform

- The Fast Fourier Transform (FFT) is a method for implementing the DFT.
- FFT computes Fourier coefficients in an efficient manner.
  - ➢FFT uses O(*nlogn*) instead of O(n^2) for DFT

    O(*n*) means that computational cost scales linearly with the number of elements in the signal/image '*n*'



Filtering in the frequency domain

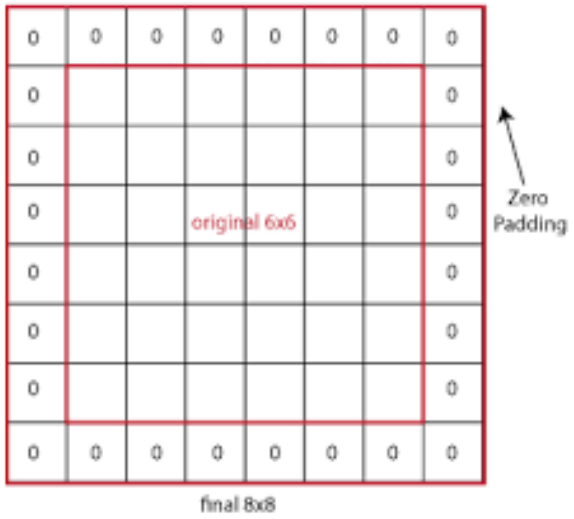
- Convolution in the time domain is equivalent to multiplication in the frequency domain

$$x_1(t) * x_2(t) \xleftrightarrow{F} X_1(f) \cdot X_2(f)$$

- In the spatial domain the filter and the image differ in size and the equation assumes that the signal is periodic. Zero-padding can be used to address both issues.

Frequency Domain Filtering

- Assume that we like to filter an image in the frequency domain using a spatial domain filtering (e.g. Gaussian, box,…etc)
- Filtering in the frequency domain is normally faster since it requires one multiplication per pixel
- Because of the mismatch in size, the filter should be zeropadded.
- Periodic extension or image zero padding will also improve results as they facilitate the periodic assumption in the Fourier convolution property.



Filtering in the frequency domain

```
img = imread('cameraman.jpg');
%img = rgb2gray(img);    %Use of the image is in color
img=double(img);
[imh, imw] = size(img);
hs = 50; % filter half-size
fil = fspecial('gaussian', hs*2+1, 10); %size of the filter is hs*2+1 (odd) and sigma=10
fftsize = 1024; % preferred be be a power of 2 (for speed) and include padding
img_fft = fft2(img, fftsize, fftsize); % 1) fft im with padding; automatically zero-pads
fil_fft = fft2(fil, fftsize, fftsize); % 2) fft fil, pad to same size as image
img_fil_fft = img_fft .* fil_fft; % 3) multiply fft images
img_fil = ifft2(img_fil_fft); % 4) inverse fft2
img_fil = img_fil(1+hs:size(img,1)+hs, 1+hs:size(img, 2)+hs); % 5) remove padding
figure(1), imagesc(log(abs(fftshift(img_fft)))), axis image, colormap jet
figure(2), imagesc(img_fil), axis image, colormap gray
```

Hybrid Images

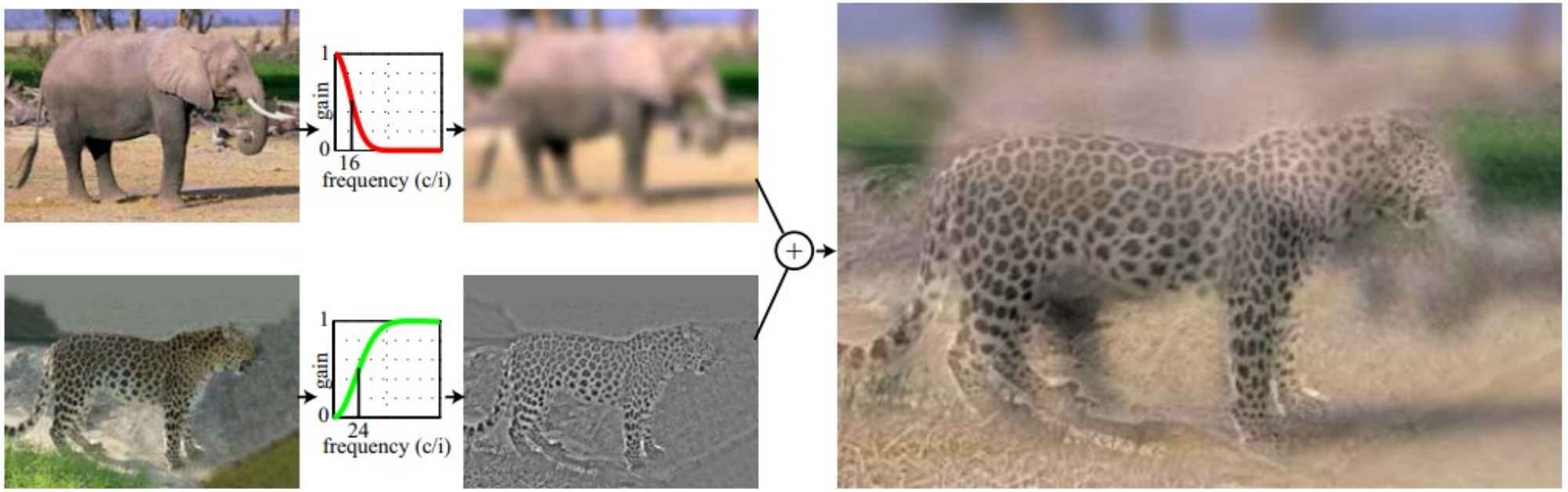


Figure 2: hybrid images are generated by superimposing two images at two different spatial scales: the low-spatial scale is obtained by filtering one image with a low-pass filter, and the high spatial scale is obtained by filtering a second image with a high-pass filter. The final hybrid image is composed by adding these two filtered images.

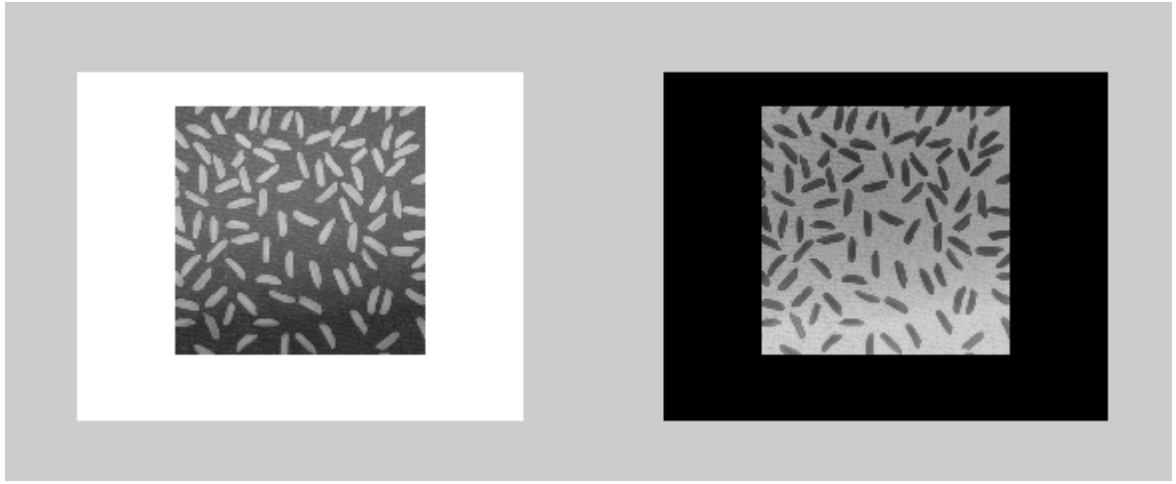
Combining two images:
Low resolution image- the one to be seen at a far distance
High resolution image- the one to be seen at a close distance

Hybrid images paper

Logical operations

- Inverting an image
  - For binary images this is accomplished by flipping ones and zeros.
  - For grayscale and color images

$$I_{inverted} = MAX - I(i,j) \; \forall i,j \quad \text{for grayscale,}$$
$$I_{inverted} = MAX - I(i,j,k) \; \forall i,j,k \quad \text{for color images.}$$



Inverting an image

Logical operations

Inputs A and B		and A & B	or A | B	xor xor(A,B)
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

- OR/XOR: Useful for detecting objects that moved from a binary video sequence or a set of images.
- AND : Detecting objects that stayed at their location in a set of binary images or video sequences.