Project-II: Design of Experiments

Mukesh Ghimire

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1 Introduction

The primary objective of this project is to get familiar with the "Design of Experiments", which is a powerful tool used for optimization. We take the problem of designing brake disc for a four-wheeled vehicle. By the end of the project, we aim to determine the optimum dimension of the brake disk such that the volume of the brake disk is minimized all while minimizing the stress on the disk, maximizing its natural frequency and minimizing the temperature. ANSYS is used for modeling and optimization. The steps taken to achieve the optimum dimension are discussed in this report.

1.1 ANSYS Schematics

Fig. 1 shows the project schematics in the ANSYS software package after setting up all the subsystems and Design of Experiments.

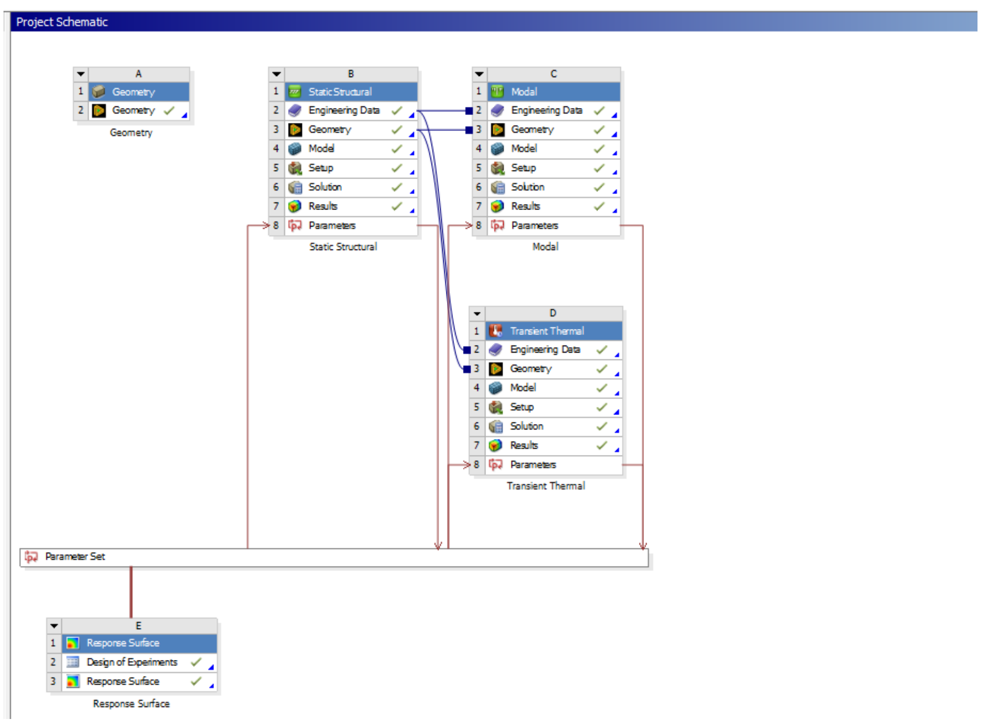

Figure 1: Project Schematics

2 Analyses

The 3-D model of the brake disk is shown in Fig. 2. It consists of a disk and two brake pads. The materials for brake pads and brake disk are structural steel and gray cast iron respectively.

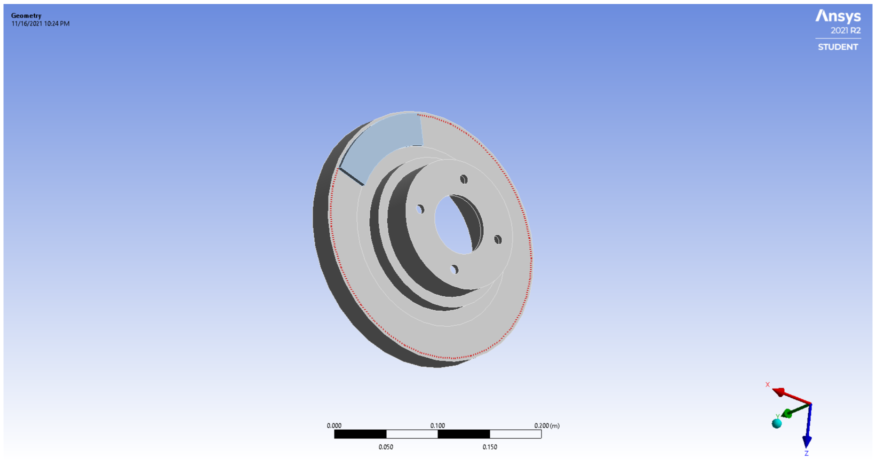

Figure 2: Disk Brake Model

Before beginning the DOE (Design of Experiments), we perform three different analysis on the brake disk – Structural, Modal, and Thermal analysis.

2.1 Structural Analysis

In structural analysis, we evaluate the maximum stress that the brake disk assembly goes through when subjected to some loading. We subject the brake disk to an angular velocity of 250 rad/s in the y-direction. We assume the pressure applied on the brakes is $1.0495 \times 10^7 Pa$. Furthermore, we fix the displacement of the brake pads in x and z direction and allow the displacement in the y-direction. The stress distribution obtained after solving for the equivalent von-Misses stress in shown in Fig. 3.

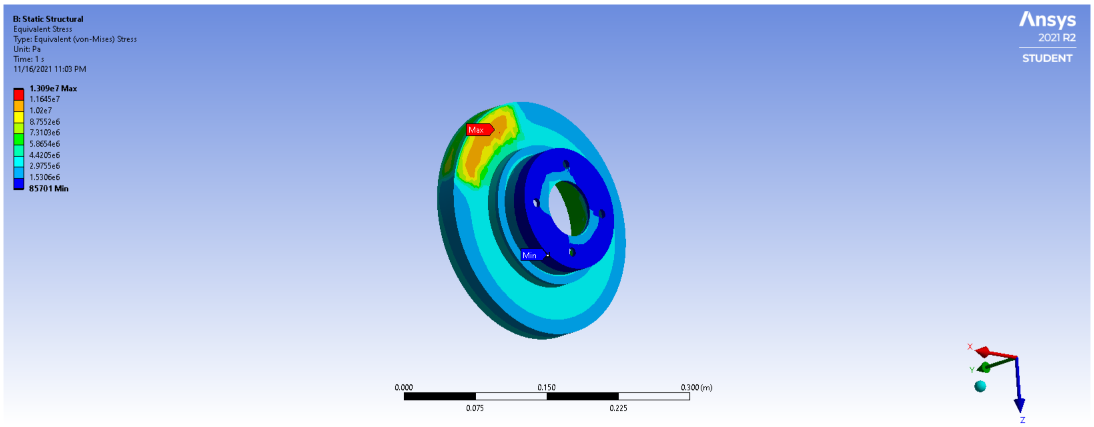

Figure 3: Stress Distribution on the Brake Disk

As expected, the stress is maximum in the contact region between the brake pads and the brake disk. The maximum stress is $1.3097 \times 10^7 Pa$.

2.2 Modal Analysis

Modal analysis is performed to determine the natural frequencies of the brake disk as the maximum deformation occurs at the natural frequency due to resonance. We want the disk's first natural frequency to be higher than the operating frequency of the disk. The result of the modal analysis is shown in Fig. 4. The natural frequency of the brake disk was found to be 1590.4 Hz and the maximum deformation experienced by the assembly was 0.80965 m.

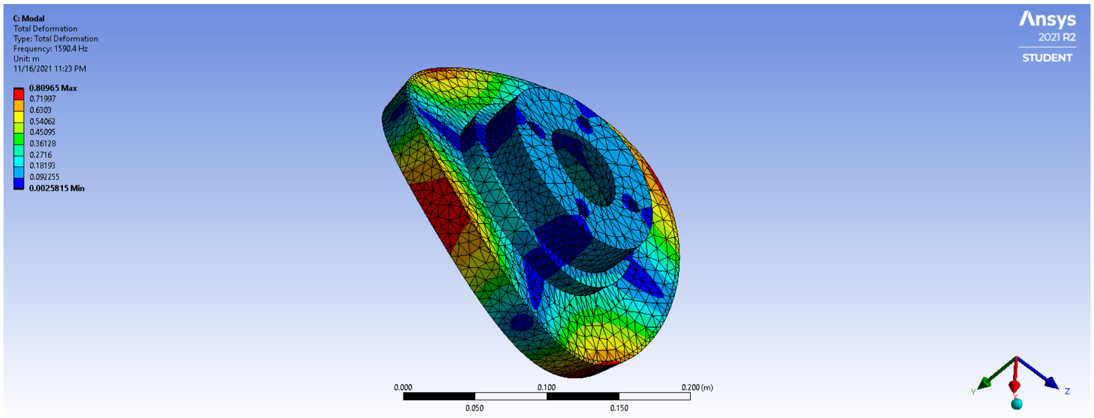

Figure 4: Modal Analysis Results

2.3 Thermal Analysis

Transient Thermal Analysis is done on the brake disk to evaluate the maximum temperature that the disk experienced as a result of braking. Heat flux of $1.5395 \times 10^6 \frac{W}{m^2}$ was applied on the two surfaces of the brake disk. The initial temperature of the disk was set to $35°C$. Convection of $5\frac{W}{m^2K}$was applied on all surfaces of the disk. The braking time was assumed to be 5 s.The result of the thermal analysis is shown in Fig. 5.

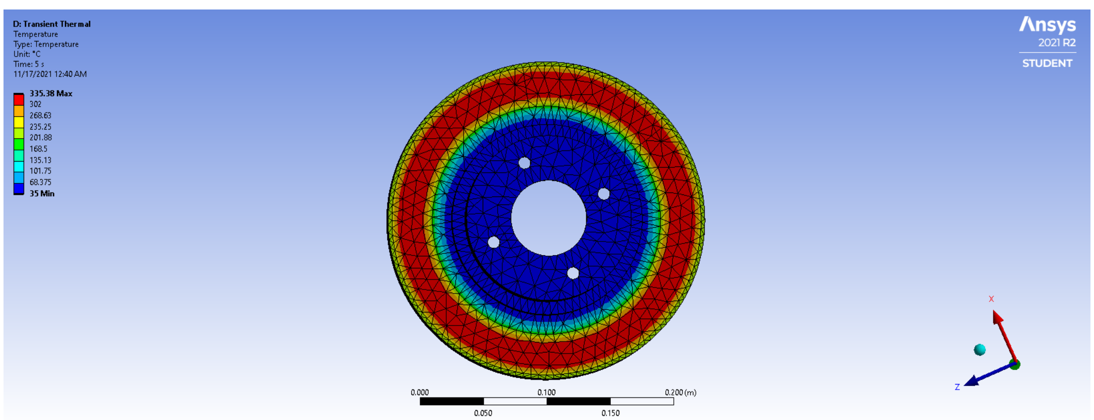

Figure 5: Thermal Analysis Result

The maximum temperature was observed at the contact surface between the brake pads and the disk, as expected. The maximum temperature was found out to be 335.38 $°C$.

3 Design of Experiments

After performing initial system analyses, design of experiments was carried out. In order to find the parameters (dimension of the brake disk), it is necessary to sample range of points from the upper and lower bounds of the design parameters. The bounds of the parameters of the brake disk are tabulated in table 1.

Table 1: Upper and Lower Bounds (mm) of the Brake Disk for DOE

Parameter	Lower Bound	Upper Bound
Thickness	15	28
Outer Diameter	112	138
Inner Diameter	66	84

In order to sample the points for the DOE, Latin Hypercube Sampling (LHS) method with user defined sample points was used to create a response surface. 10 points were used to determine the response surface. The design points are shown in Fig. 7. Kriging method was used to calculate the response surface. After calculating the maximum stress and volume of the brake disk for all the points, the dimensions corresponding to minimum stress and minimum volume of the brake disk was determined. The local sensitivity curve for the equivalent maximum stress, volume, total deformation reported frequency, and the maximum temperature of the brake disk are shown in Fig. 8, Fig. 9, Fig. 10, and Fig. 11 respectively. The goodness of the fit of the model is shown in Fig. 6.

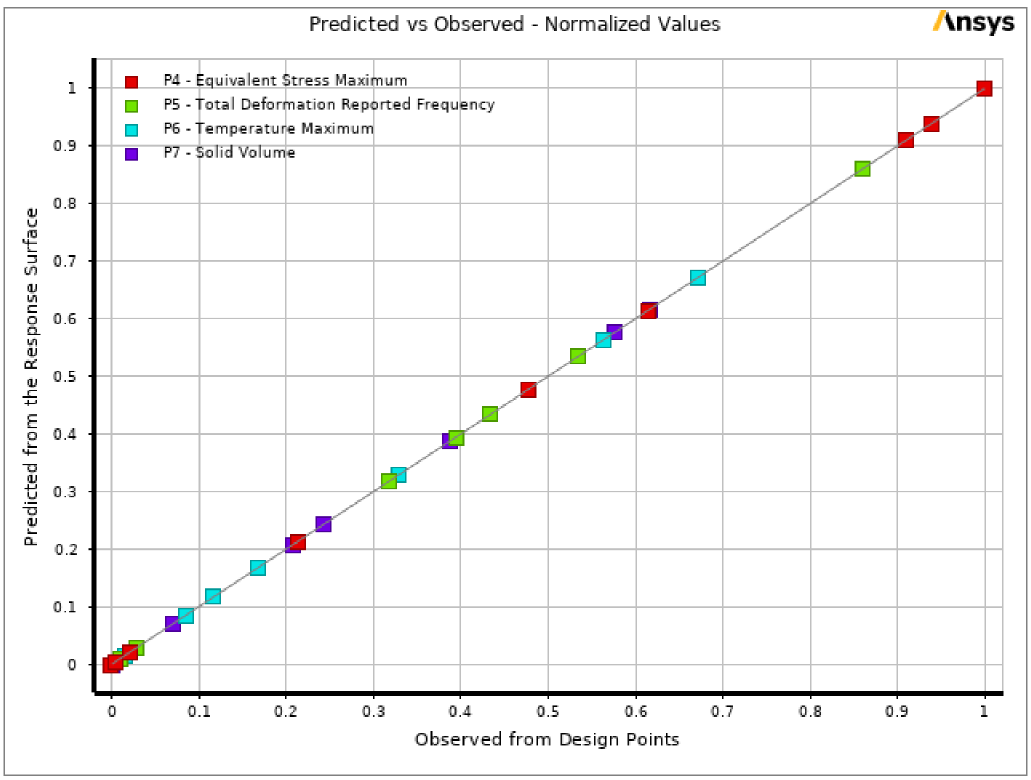

Figure 6: Goodness of Fit

Table of Outline A2: Design Points of Design of Experiments

	A	B	C	D	E	F
1	Name	P1 - rotor_thickness (mm)	P2 - rotor_OD (mm)	P3 - rotor_ID (mm)	P4 - Equivalent Stress Maximum (Pa)	P7 - Solid Volume (m^3)
2	1	20.85	123.7	70.5	1.3253E+07	0.00087277
3	3	15.65	134.1	79.5	1.1303E+07	0.00080054
4	4	23.45	128.9	68.7	1.1395E+07	0.0010665
5	5	16.95	126.3	72.3	1.0894E+07	0.00077313
6	6	27.35	131.5	74.1	1.1734E+07	0.0012222
7	7	22.15	136.7	77.7	1.174E+07	0.001101
8	10	21.5	125	75	1.1034E+07	0.00088671
*	New Design Point					

Figure 7: DOE Points

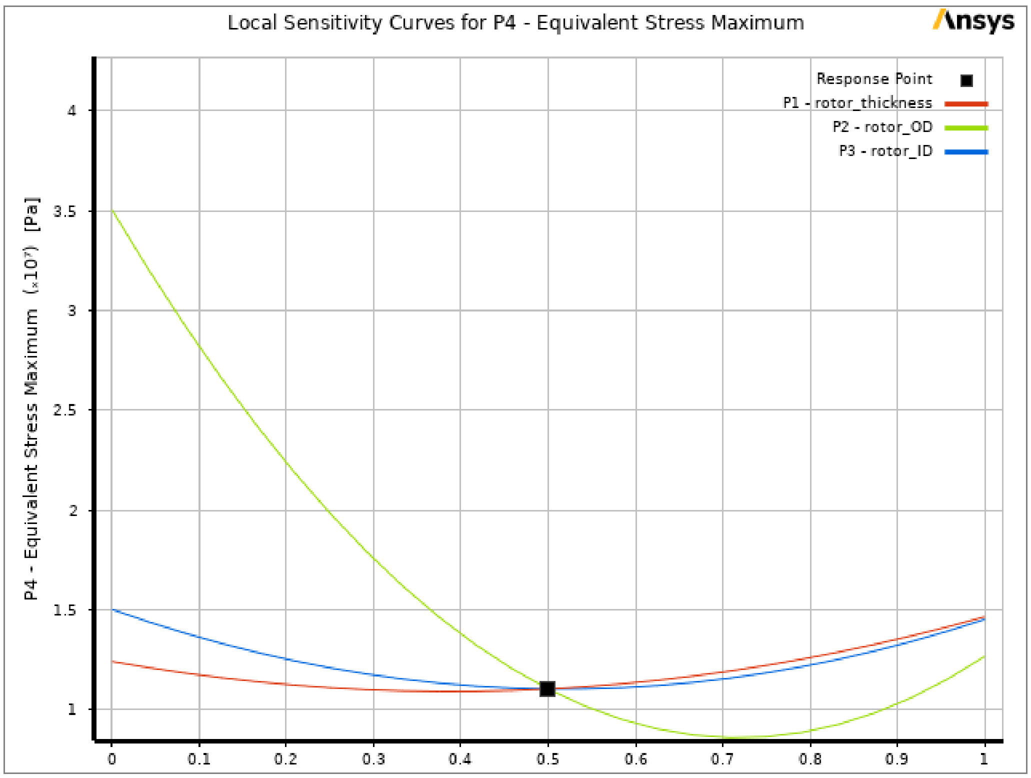

Figure 8: Local Sensitivity Curve for Equivalent Maximum Stress

The minimum stress was 1.223×10^7 Pa and the volume was 0.00088578 m^3. The initial and optimized dimension of the brake disk and their corresponding maximum stress and volume are compared in table 2. There was **11.2% reduction** in the volume of the brake disk.

The objective of any design optimization is to improve the previous design iteration probably in terms of cost saving and performance improvement. In the case of the brake disk design, this design iteration led to cost saving due to reduction in thickness of the brake disk. We did not see any change in the other two dimension. This could be further improved by increasing the parameter space. However, there are several other factors that need to be taken into consideration when carrying out Design of Experiments. Factors such as fit between assembly housing and the assembly might constrain the assembly further.

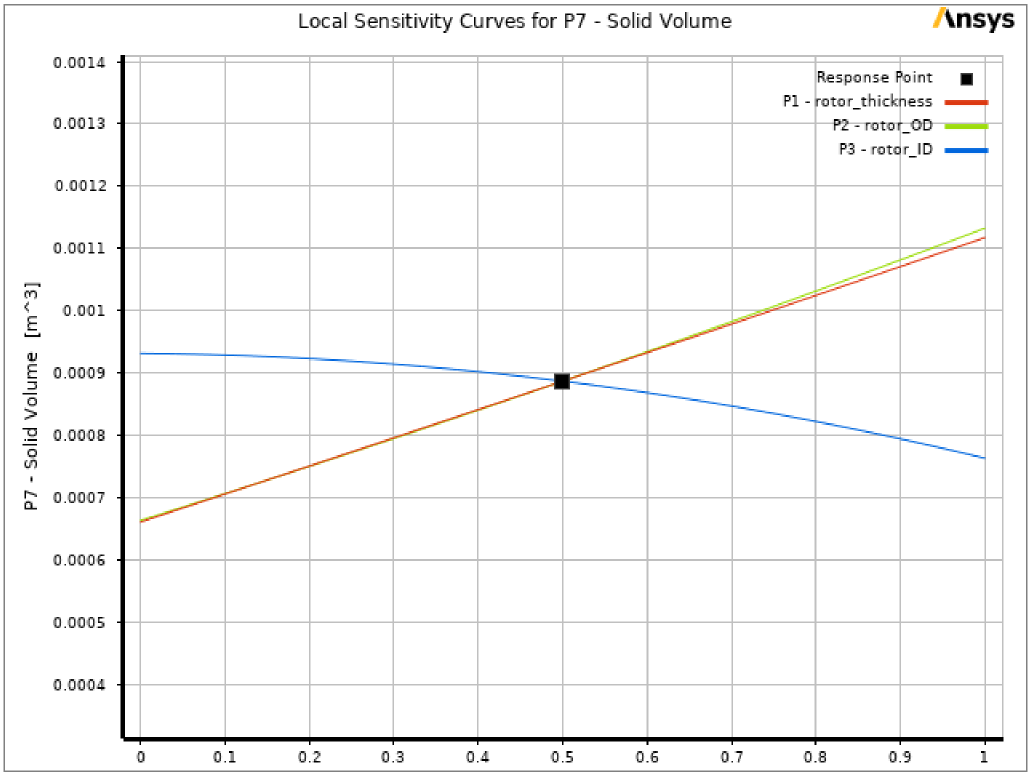

Figure 9: Local Sensitivity Curve for Volume of the Disk

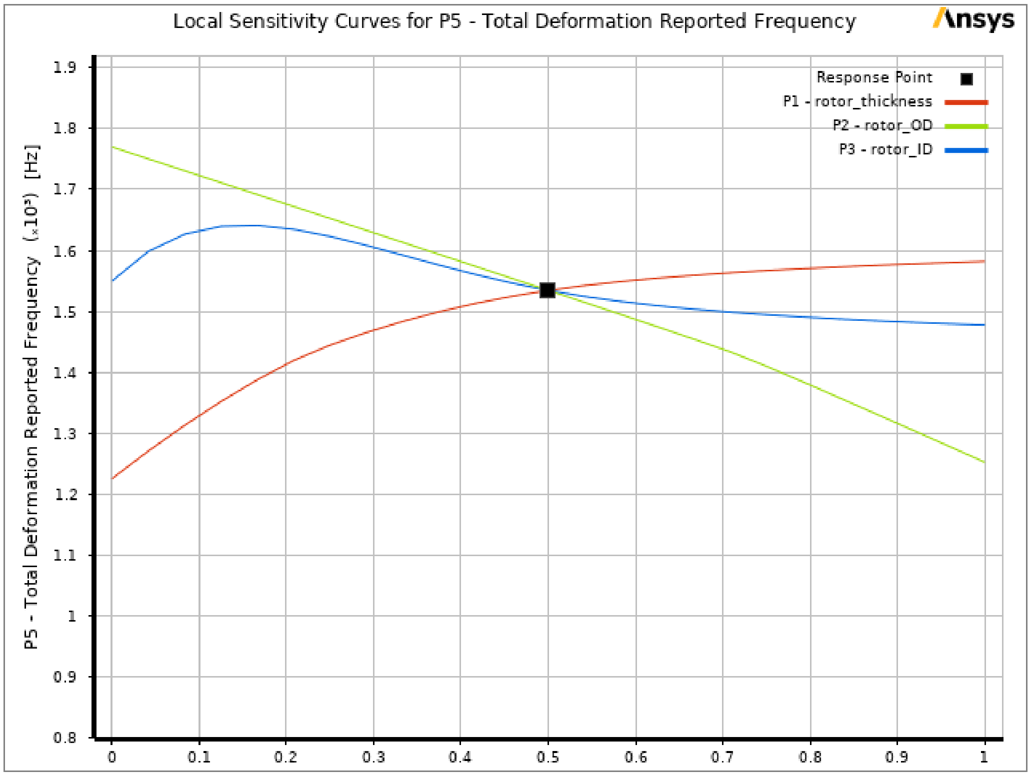

Figure 10: Local Sensitivity Curve for Total Deformation Reported Frequency

Hence, although in theory we could have much better performing Design Point, it may not

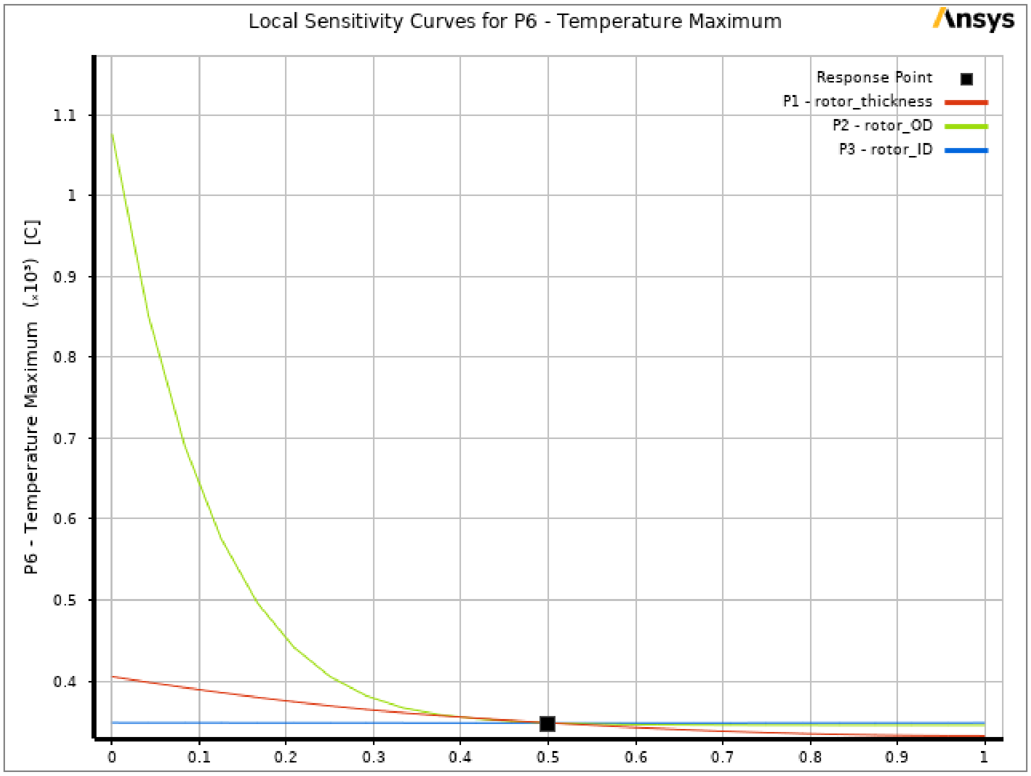

Figure 11: Local Sensitivity Curve for Maximum Temperature

Table 2: Comparision between initial and final result

	Thickness(mm)	Outer Diameter(mm)	Inner Diameter(mm)	Equivalent Maximum Stress($10^7 Pa$)	Volume (m^3)
Initial	25	125	75	1.309	0.00099667
Final	21.5	125	75	1.223	0.00088578

be applicable for the particular use case.

4 Conclusion

In this project, three different analyses – Structural, Modal, and Thermal Analysis were performed on a brake disk to determine the failure modes and maximum operating conditions. Using ANSYS to conduct Design of Experiments, an optimum brake disk design was selected. Using the suggested design, the volume of the brake disk was reduced by **11.2%**. The maximum stress experienced by the brake disk was reduced by **7%**. The reduction in brake disk occured in its thickness only.