Project-II: Design of Experiments

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1 Introduction

The primary objective of this project is to get familiar with the "Design of Experiments", which is a powerful tool used for optimization. We take the problem of designing brake disc for a four-wheeled vehicle. By the end of the project, we aim to determine the optimum dimension of the brake disk such that the volume of the brake disk is minimized all while minimizing the stress on the disk, maximizing its natural frequency and minimizing the temperature. ANSYS is used for modeling and optimization. The steps taken to achieve the optimum dimension are discussed in this report.

1.1 ANSYS Schematics

Fig. 1 shows the project schematics in the ANSYS software package after setting up all the subsystems and Design of Experiments.

2 Analyses

The 3-D model of the brake disk is shown in Fig. 2. It consists of a disk and two brake pads. The materials for brake pads and brake disk are structural steel and gray cast iron respectively.

Before beginning the DOE (Design of Experiments), we perform three different analysis on the brake disk – Structural, Modal, and Thermal analysis.

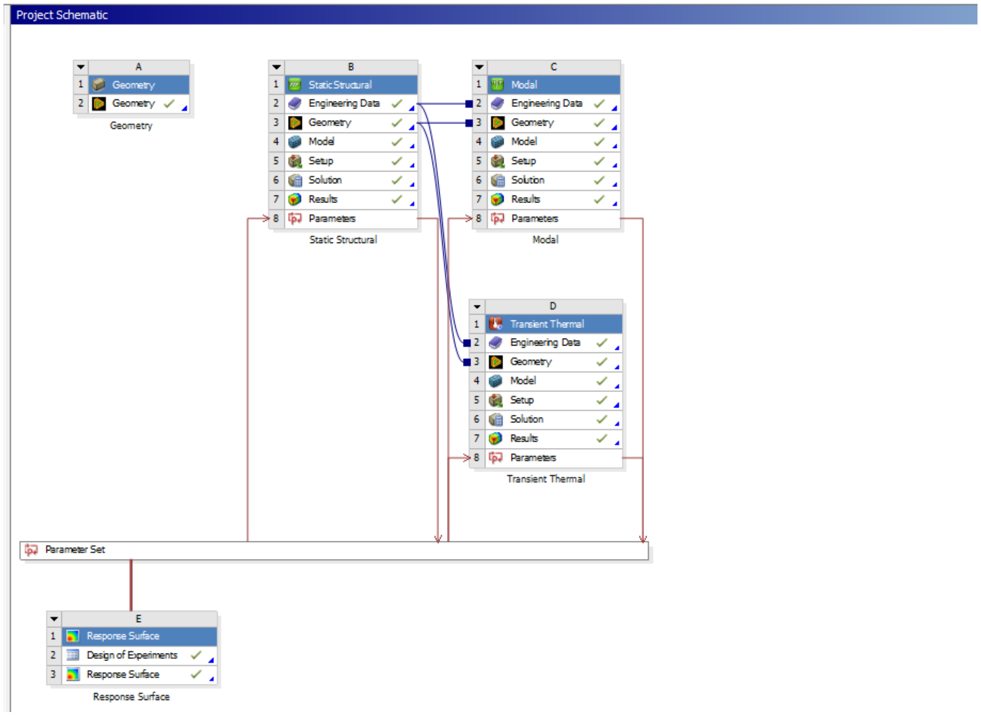

Figure 1: Project Schematics

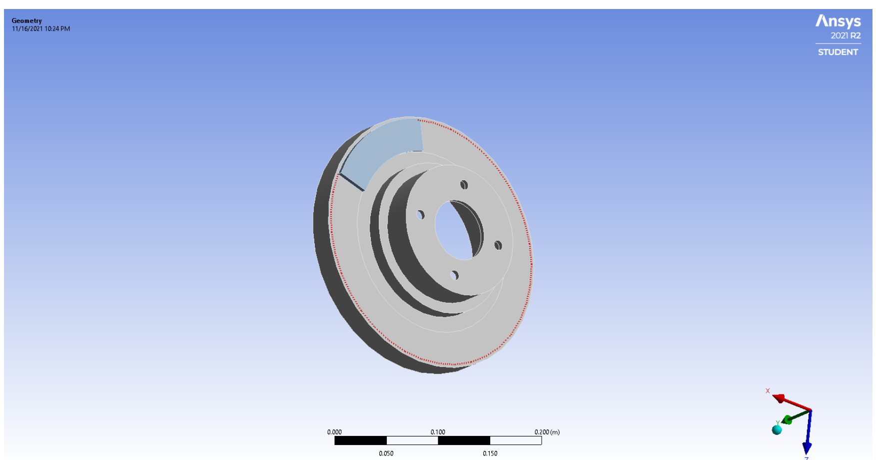

Figure 2: Disk Brake Model

2.1 Structural Analysis

In structural analysis, we evaluate the maximum stress that the brake disk assembly goes through when subjected to some loading. We subject the brake disk to an angular velocity of 250 rad/s in the y-direction. We assume the pressure applied on the brakes is $1.0495 \times 10^7 Pa$. Furthermore, we fix the displacement of the brake pads in x and z direction and allow the displacement in the y-direction. The stress distribution obtained after solving for the equivalent von-Misses stress in shown in Fig. 8.

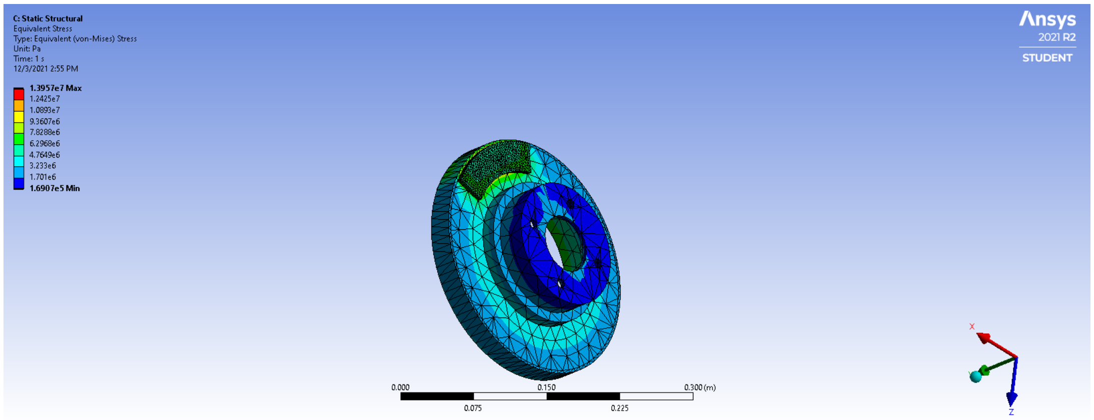

Figure 3: Stress Distribution on the Brake Disk

As expected, the stress is maximum in the contact region between the brake pads and the brake disk. The maximum stress is $1.3957 \times 10^7 Pa$.

2.2 Modal Analysis

Modal analysis is performed to determine the natural frequencies of the brake disk as the maximum deformation occurs at the natural frequency due to resonance. We want the disk's first natural frequency to be higher than the operating frequency of the disk. The result of the modal analysis is shown in Fig. 4. The natural frequency of the brake disk was found to be 1590.4 Hz and the maximum deformation experienced by the assembly was 0.80965 m.

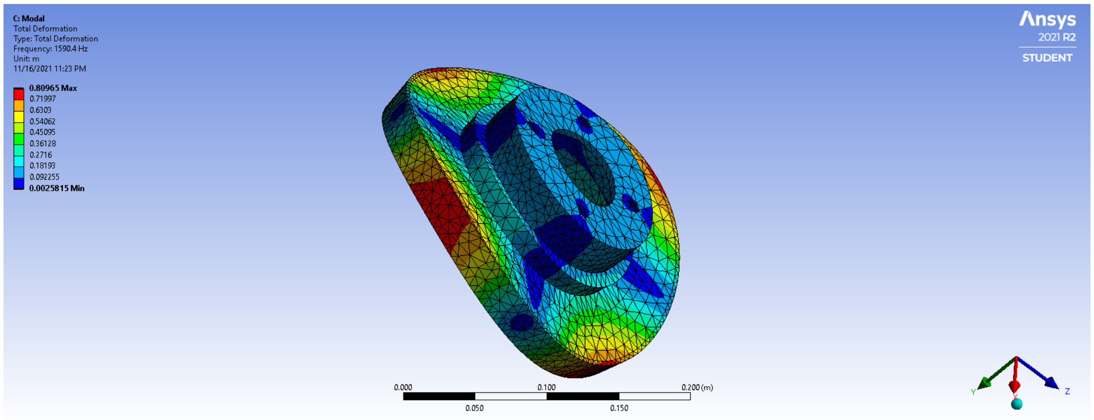

Figure 4: Modal Analysis Results

2.3 Thermal Analysis

Transient Thermal Analysis is done on the brake disk to evaluate the maximum temperature that the disk experienced as a result of braking. Heat flux of $1.5395 \times 10^6 \frac{W}{m^2}$ was applied on the two surfaces of the brake disk. The initial temperature of the disk was set to $35°C$. Convection of $5 \frac{W}{m^2K}$ was applied on all surfaces of the disk. The braking time was assumed to be 5 s.The result of the thermal analysis is shown in Fig. 5.

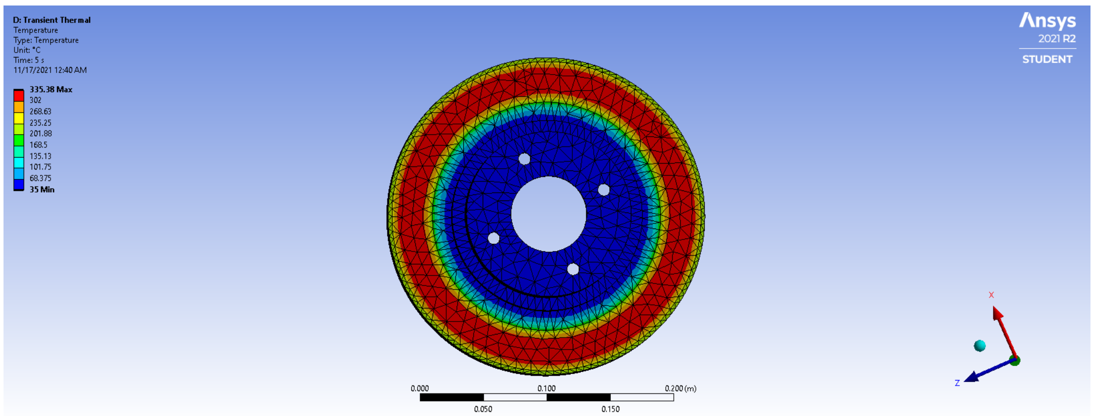

Figure 5: Thermal Analysis Result

The maximum temperature was observed at the contact surface between the brake pads and the disk, as expected. The maximum temperature was found out to be 335.38 $°C$.

3 Design of Experiments

After performing initial system analyses, design of experiments was carried out. In order to find the parameters (dimension of the brake disk), it is necessary to sample range of points from the upper and lower bounds of the design parameters. The bounds of the parameters of the brake disk are tabulated in table 1.

Table 1: Upper and Lower Bounds (mm) of the Brake Disk for DOE

Parameter	Lower Bound	Upper Bound
Thickness	15	28
Outer Diameter	112	138
Inner Diameter	66	84

In order to sample the points for the DOE, Latin Hypercube Sampling (LHS) method with user defined sample points was used to create a response surface. 10 points were used to determine the response surface. The design points are shown in Fig. 7. Kriging method was used to calculate the response surface.

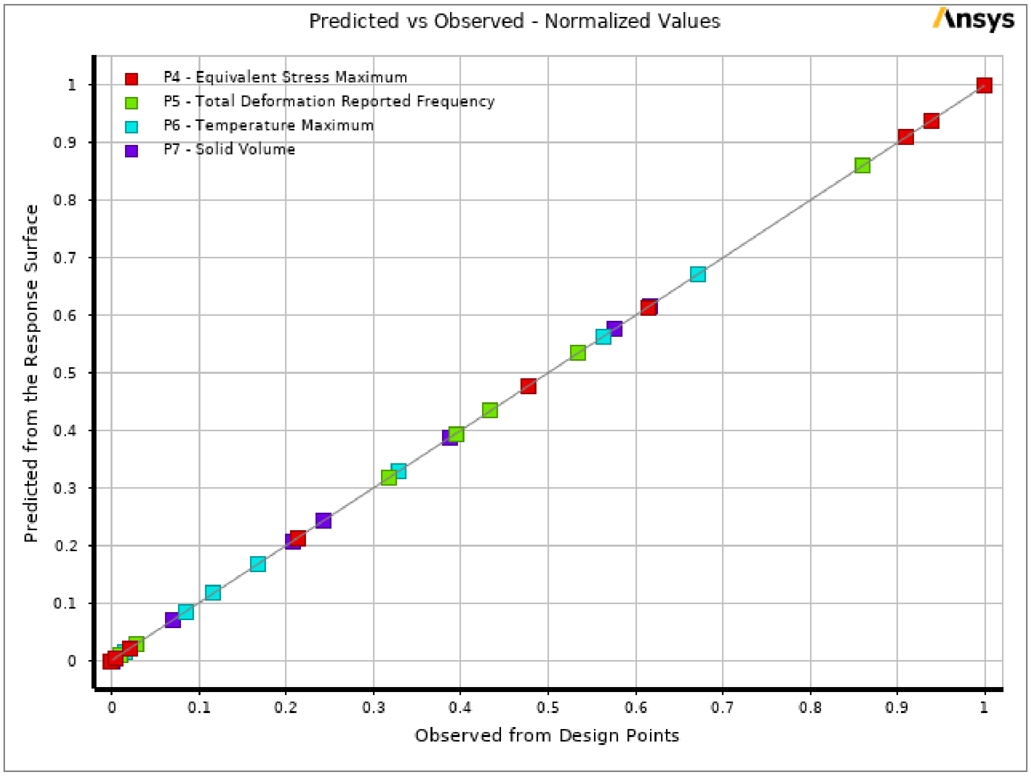

Figure 6: Goodness of Fit

Table of Outline A2: Design Points of Design of Experiments

	A	B	C	D	E	F
1	Name	P1 - rotor_thickness (mm)	P2 - rotor_OD (mm)	P3 - rotor_ID (mm)	P4 - Equivalent Stress Maximum (Pa)	P7 - Solid Volume (m^3)
2	1	20.85	123.7	70.5	1.3253E+07	0.00087277
3	3	15.65	134.1	79.5	1.1303E+07	0.00080054
4	4	23.45	128.9	68.7	1.1395E+07	0.0010665
5	5	16.95	126.3	72.3	1.0894E+07	0.00077313
6	6	27.35	131.5	74.1	1.1734E+07	0.0012222
7	7	22.15	136.7	77.7	1.174E+07	0.001101
8	10	21.5	125	75	1.1034E+07	0.00088671
*	New Design Point					

Figure 7: DOE Points

4 Optimization

4.1 Structural Optimization

After determining the response surface, the model was first optimized to minimize the stress on the brake disk. The optimization was done using NLPQL (Non Linear Programming by Quadratic Lagrangian) method. The top three candidate points are picked from the over 80 design points. The best points are shown in Fig. 8 and the convergence plot is shown in Fig. 9.

Candidate Points	Starting Point DP 0	Candidate Point 1 DP 146	Candidate Point 2 DP 187	Candidate Point 3 DP 130
P2 - rotor_OD (mm)	125	124.13	124.97	124.42
P3 - rotor_ID (mm)	75	75.949	75.889	75.919
P4 - rotor_thickness (mm)	25	25.457	25.37	25.426
P5 - Equivalent Stress Maximum (Pa)	1.4522E+07	1.3183E+07	1.3408E+07	1.3499E+07
P6 - Total Deformation Reported Frequency (Hz)	1620.1	1623.7	1610.3	1620.3

Figure 8: Candidate Points for Static Structural Optimization

Figure 9: Convergence Criteria for Static Structural Optimization

The minimum stress among the candidate points was 1.318×10^7 Pa. The dimension corresponding to this best point is compared with the initial dimension of the brake disk in Fig. 2

Table 2: Comparision between initial and final result (Structural Optimization)

	Thickness(mm)	Outer Diameter(mm)	Inner Diameter(mm)	Equivalent Maximum Stress($10^7 Pa$)
Initial	25	125	75	1.396
Final	25.457	124.13	75.949	1.318

4.2 Modal Optimization

The goal of the Modal Optimization, in this project, is to maximize the natural frequency of the brake disk. Since the brake disk usually has high operating frequency, it is necessary that it has a relatively higher natural frequency so as to avoid resonance. Same settings from the Structural Optimization were used to perform Modal Optimization. The top three candidate points are shown in Fig. 10.

Candidate Points

	Starting Point DP 0	Candidate Point 1 DP 111	Candidate Point 2 DP 44	Candidate Point 3 DP 36
P2 - rotor_OD (mm)	125	122.01	122.89	123.95
P3 - rotor_ID (mm)	75	68.578	67.5	70.087
P4 - rotor_thickness (mm)	25	26.843	26.918	26.069
P5 - Equivalent Stress Maximum (Pa)	1.4522E+07	1.3648E+07	1.6318E+07	1.3345E+07
P6 - Total Deformation Reported Frequency (Hz)	1620.1	1814.4	1787.1	1724.7

Figure 10: Candidate Points for Modal Optimization

From the above figure, we can observe that the best candidate point resulted in much higher natural frequency of 1814.4 Hz, as compared to the initial natural frequency of 1590.4 Hz. The dimension of the optimized brake disk is compared to the initial dimension in table 3.

Based on the necessity, the user could select one of the two optimization results to optimize the disk brake. Furthermore, we could also perform a multi-objective optimization so as to obtain optimum brake disk design that optimizes both frequency parameter and the

Table 3: Comparison between initial and final result (Modal Optimization)

	Thickness(mm)	Outer Diameter(mm)	Inner Diameter(mm)	Frequency(Hz)
Initial	25	125	75	1590.4
Final	26.843	122.01	68.578	1814.4

stress parameter. Due to lack of computing resources, multi-objective optimization was not feasible in this project.

5 Conclusion

In this project, three different analyses – Structural, Modal, and Thermal Analysis were performed on a brake disk to determine the failure modes and maximum operating conditions. Using ANSYS to conduct Design of Experiments and generate response surfaces, which were used to perform direct optimization on the brake disk. Two separate optimizations were performed – Static Structural Optimization, and Modal Optimization. In Static Structural Optimization, the brake disk was optimized so as to minimize the stress experienced by the disk, whereas in the Modal Optimization, the brake disk's natural frequency was maximized. From the structural optimization, stress on the brake disk was reduced by **6**% and from the modal optimization, the natural frequency was increased by **14**%. Future work could incorporate optimizing brake disk so that it has both high natural frequency and low stress distribution. Multi-objective optimization could be used to achieve such result.