working with time series

Moving Average Models: MA(q)

note: Moving Average Models are **not** the same as the Moving Average Smoothing discussed earlier

Using the Price of Microsoft Stock as a working example:

Day1: Microsoft makes an announcement that affects the stock by ε on that day

Day2: Fulle impact of the announcement affects the stock by $\theta_1\varepsilon$ on that day

Day3: lingering effects of the announcements affects the stock by $\theta_2\varepsilon$ on that day with no further effects

Assuming that Microsoft makes an announcement everyday, what is the effect on stock at day t?

The answer is comprised of the summartion of the current market effect, plus the prior days:

$X_t = \mathcal{E}_t + \theta_1 \mathcal{E}_{t-1} + \theta_2 \mathcal{E}_{t-2}$ ← exactly a moving average model of order 2

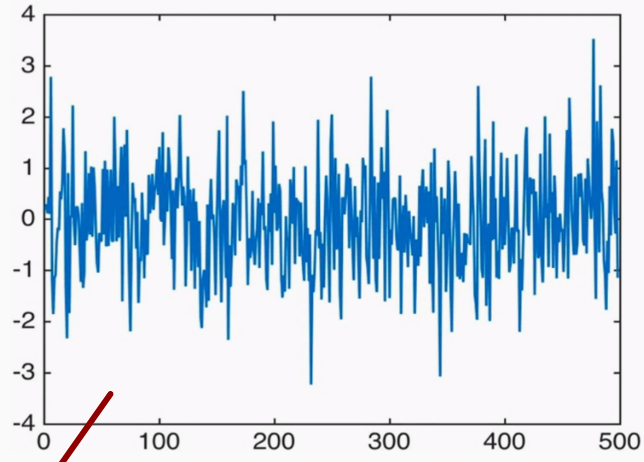
Generally, the value at time t for a moving average model derives from the mean $\mu +$ noise in the form of today's announcement $\mathcal{E}_t$ and yesterday's announcement $\mathcal{E}_{t-1}$, etc:

$X_t = \mu + \mathcal{E}_t + \theta_1 \mathcal{E}_{t-1} + \cdots + \theta_q \mathcal{E}_{t-q}$ ← weighted average of current/previous noise terms

Moving average models are always stationary (the above is "order q)

The models look up to q days ago and does not consider noise outside of the recent ($>q$ days ago) looking at data from a moving average model:

$X_t = \cancel{\mu} + \mathcal{E}_t$



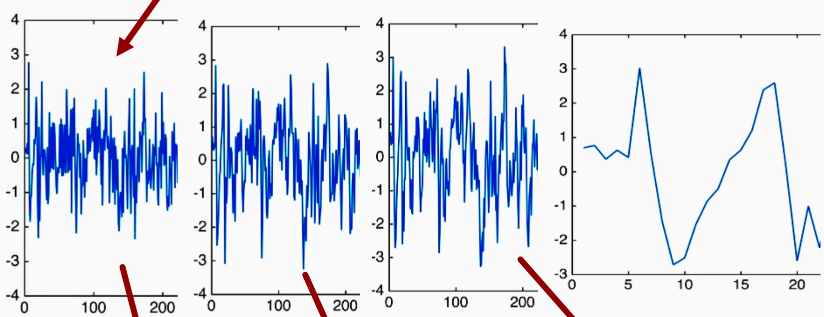
The example is a basic model assuming $\mu = 0$

(assuming $\mu = 0$ in examples going foreward)

If $\mu \neq 0$ a trend results and will subsequently be subtracted out as illustrated in decomposition

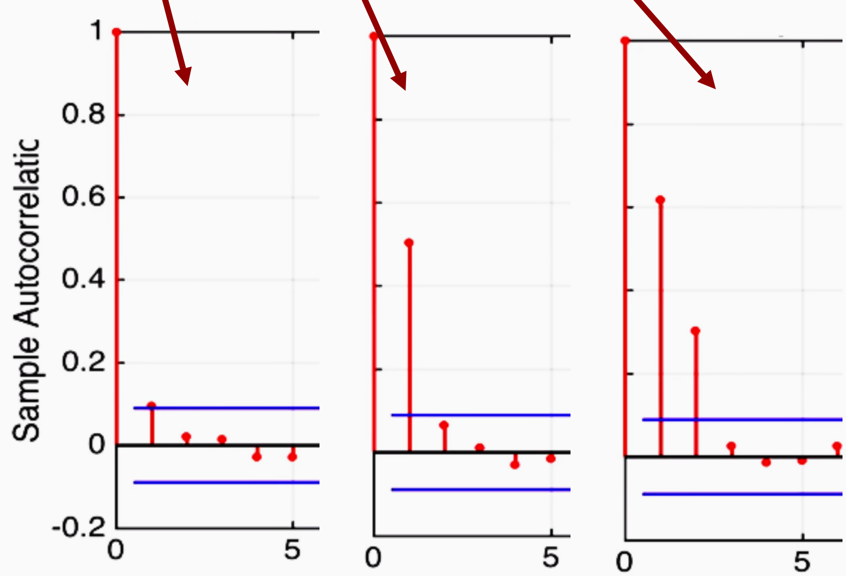
When the model is increased to the order of 1 and succinctly the order of 2, the noice remains indecipherable between the models as seen below:

Each figure to the right illustrates X_t as a function of just $\mathcal{E}_t$ (1st), the order of 1 (2nd), and the order of 2 (3rd); each being generally indistinguishable from the other.



Zooming into the data frame clarifies the noise but continues to lack analytical value. Such is solved through the use of an Autocorrelation plot:

The illustratons depict the Autocorrelation function derived earlier. For the noise series $X_t = \mathcal{E}_t$, the Autocorrelation function is expectedly 0 (independent noise is not correlated with the past). When additional orders are included in the model, non-zero terms are represented appropriately.



Given a set of timeseries data, plotting a sample autocorrelation is an appropriate way to determine if the data can be modeled as opposed to visually determining the latter.

If data arises from an MA model of order q (MA(q)), the autocorrelation function will sharply drop past q; making an $\mathrm{MA}(g)$ model a valuble way to check if timeseries data can/should be modeled well.

Subsequent to identifying data to be modeled using a Moving Average Model of order q, the parameters $\theta_1, \dots \theta_q$ of the model must be determined. This is achieved by fitting the coeficients to the data using standard least squares regression.

$$X_t = \mu + \mathcal{E}_t + \theta_1 \mathcal{E}_{t-1} + \cdots + \theta_q \mathcal{E}_{t-q}$$