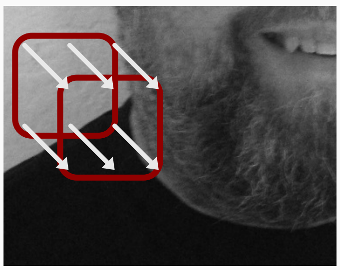
working with images

Corner Detection

Edge detectors do not typically classify corners well because they register linear gradient changes. In corners, pixel intensities occur in multiple directions at a time. The **Harris Corner Detector** registers changes in pixel intensities as it travels around the image.

The following function neasures the change in intensity across the region when δ_x and δ_y are shifted:

$$E(\delta_x, \delta_y) = \sum_{x,y} w(x,y)\left[I(x+\delta_x, y+\delta_y) - I(x,y)\right]^2$$



The x and y represents the directions within the image. The subtraction within the brackets represents the intensity from shifting. Each starting point (x,y) is shifted by (δ_x, δ_y). The function is squared because the significance of differences in any direction is relevant. Approaching a corner will output a high intensity, regardless of the direction approached. If the intensity is ~constant then the function $\left[I(x+\delta_x, y+\delta_y) - I(x,y)\right]^2$ will output ~0. Each pixel will then be weighted by $w(x,y)$ as chosen by the scientist. $w(x,y) = 1$ inside the box and 0 outside. $w(x,y) =$ larger value near the center of the box (similar to a **Gaussian Kernel**). The weighted differences are then summed within the entire box, computing function $E(\delta_x, \delta_y)$.

A first order **Taylor Expansion** is use to approximate the intensity at the shifted position:

$$I(x+\delta_x, y+\delta_y) \approx I(x,y) + I_x(x,y)\delta_x + I_y(x,y)\delta_y$$ *use discrete derivatives

Plug the **Taylor Expansion** into the orginal function:

$$E(\delta_x, \delta_y) \approx \sum_{x,y} w(x,y)\left[I_x(x,y)\delta_x + I_y(x,y)\delta_y\right]^2$$

Explaining the intution behind the above expressions:

$$E(\delta_x, \delta_y) \approx \sum_{x,y} w(x,y)\left[I_x(x,y)\delta_x + I_y(x,y)\delta_y\right]^2$$

$$= [\delta_x \quad \delta_y]\; Q \begin{bmatrix}\delta_x \\ \delta_y\end{bmatrix}$$

$$\text{where Q} = \sum_{\delta_x}\sum_{\delta_y} w(\delta_x, \delta_y)\begin{bmatrix} I_x^2(x,y) & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y^2(x,y)\end{bmatrix}$$

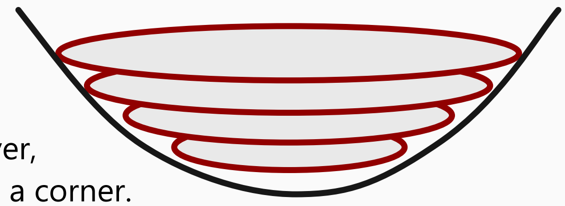
$$Q = \begin{bmatrix} \sum_{\delta_x}\sum_{\delta_y} w(\delta_x,\delta_y)\, I_x^2(x,y) & \sum_{\delta_x}\sum_{\delta_y} w(\delta_x,\delta_y)\, I_x(x,y)I_y(x,y) \\ \sum_{\delta_x}\sum_{\delta_y} w(\delta_x,\delta_y)\, I_x(x,y)I_y(x,y) & \sum_{\delta_x}\sum_{\delta_y} w(\delta_x,\delta_y)\, I_y^2(x,y) \end{bmatrix}$$

The expression can be written out as the vector of deltas, times a matrix, times the vector of deltas.

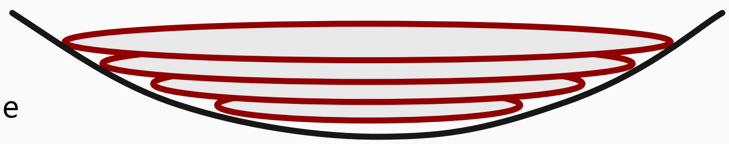
The summations are distributed within the matrix to illustrate 4 seperately derivable numbers. If the function $E(\delta_x, \delta_y)$ is equal to a constant, the result is just the formula for an ellipse:

$$= [\delta_x \quad \delta_y]\, Q \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = \text{Constant} \quad \rightarrow \quad \text{formula for an ellipse}$$

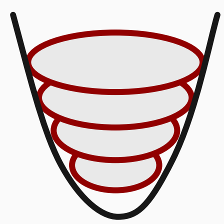
Therefore, the level sets of the function $E(\delta_x, \delta_y)$ are ellipses.

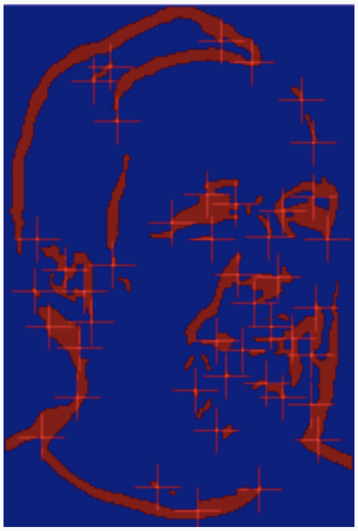


A notable curve in the ellipses indicate an approaching corner in the image. When the ellipses are broad as the image is passed over, the function $E(\delta_x, \delta_y)$ remains stationary in value not indicicating a corner.



In constrast, if the function $E(\delta_x, \delta_y)$ experiences drastic changes, meaning the intensities are varying significantly as the image is passed over, corners in the image are being detected throughout.





The determination of whether or not the function $E(\delta_x, \delta_y)$ is changing can be seen through examining the ellipses. However, determining whether or not the ellipses are wide (narrow) is seen through evaluating the **eigenvalues** of the Q matrix. The Eigenvalues of $Q\begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}$ are related to $\frac{1}{\text{principal axes of the ellipse}}$. Therefore, if both **eigenvalues** are large, there exists intensity variance in multiple directions ($\geq$ two directions), indicating a corner.

In summary, compute (approximate) $\boldsymbol{Q}$ for each $(\boldsymbol{x}, \boldsymbol{y})$. If $\boldsymbol{Q}$ has **2** large **eigenvalues** → **corner**.

The right image illustrates the **Harris Corner Detector's** output of the image.