working with time series

Exponentially Weighted Moving Averages (EWMA)

aka Simple Exponential Smoothing Model (SES)

EWMA is a special case of an ARIMA Model; ARIMA(0,1,1); which is essentially an ARMA Model considering $d = 0$ and 1 AR term and 1 MA term

EWMA makes predictions as follows:

$$\hat{X}_t = \alpha X_{t-1} + (1-\alpha)X_{t-1}$$

The above is a mixture of what is being predicted and the previous value

To expand for intuition:

$$\begin{aligned}\hat{X}_t &= \alpha X_{t-1} + (1-\alpha)\hat{X}_{t-1} \\ &= \alpha X_{t-1} + \hat{X}_{t-1} - \alpha\hat{X}_{t-1} \\ &= \hat{X}_{t-1} + \alpha\left(X_{t-1} - \hat{X}_{t-1}\right)\end{aligned}$$

Using $e_{t-1} = X_{t-1} - \hat{X}_{t-1}$ to define the prediction error X at time $t-1$ (actual value – predicted value):

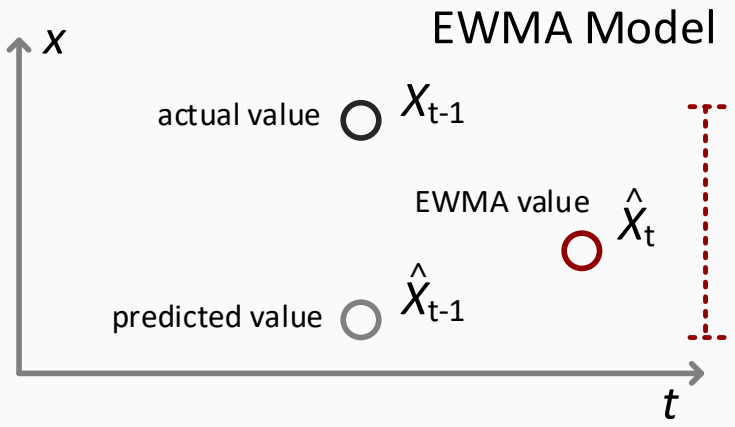
$$\hat{X}_t = \hat{X}_{t-1} - \alpha e_{t-1} \quad \rightarrow \text{the last forecast is adjusted by its error}$$

The prediction of is simply the prediction plus some adjustment times the prediction error

(The prediction of time t is a mixture of what occurred yesterday and a measure of what occurred today)

Assuming the model was fitted or some other method of obtaining alpha:

the prediction $\hat{X}_{t-1}$ turned out to be much lower than its actual value X_{t-1}; the EWMA Model in turn takes the combination of the two through the adjusting to compute the final prediction $\hat{X}_t$ (forecast)



Returning to the EWMA Model for Intuition:

$$\begin{aligned}\hat{X}_t &= \alpha X_{t-1} \\ &\quad + (1-\alpha)\hat{X}_{t-1} \\ &= \alpha X_{t-1} + (1-\alpha)X_{t-1} - (1-\alpha)X_{t-1} + (1-\alpha)\hat{X}_{t-1} \\ &= X_{t-1} - (1-\alpha)\left(X_{t-1} - \hat{X}_{t-1}\right)\end{aligned}$$

using $e_{t-1} = X_{t-1} - \hat{X}_{t-1}$...

$$\hat{X}_t = \hat{X}_{t-1} - (1-\alpha)e_{t-1} \rightarrow \text{ARMA Model with 1 AR and 1 Ma term (ARIMA(0,1,1)}$$