Energy Systems
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SOLUTION V: INVESTMENT AND LARGE POWER SYSTEMS

SOLUTION V.1 (INVESTMENT, GENERATOR AND TRANSMISSION CONSTRAINTS).

Two generators are connected to the grid by a single transmission line (with no electrical demand on their side of the transmission line). A single company owns both the generators and the transmission line. Generator 1 has a linear cost curve $C_1(g_1) = 5g_1$ [€/h] and a capacity of 300 MW and Generator 2 has a linear cost curve $C_2(g_2) = 10g_2$ [€/h] and a capacity of 900 MW. The transmission line has a capacity K of 1000 MW. Suppose the demand in the grid is always high enough to absorb the generation from the two generators and that the market price of electricity π is never below 15 €/MWh and averages 20 €/MWh.

(a) Determine the full set of equations (objective function and constraints) for the generators to optimise their dispatch to maximise total economic welfare.

Note that it is important in this example that the same company owns both the generators and the transmission line; if an independent TSO owned the transmission line, he could take the congestion revenue for himself.

If we label the dispatch of Generator 1 by g_1 and of Generator 2 by g_2, then the objective function is to maximise total profit

$$\max_{g_1,g_2} [\pi(g_1+g_2) - C_1(g_1) - C_2(g_2)] = \max_{g_1,g_2} [\pi(g_1+g_2) - 5g_1 - 10g_2]$$

The constraints are

$$\begin{aligned} g_1 &\leq \hat{g}_1 && \leftrightarrow \bar{\mu}_1 \\ -g_1 &\leq 0 && \leftrightarrow \underline{\mu}_1 \\ g_2 &\leq \hat{g}_2 && \leftrightarrow \bar{\mu}_2 \\ -g_2 &\leq 0 && \leftrightarrow \underline{\mu}_2 \\ g_1 + g_2 &\leq K && \leftrightarrow \mu_T \end{aligned}$$

Where the first four constraints come from generation, where $\hat{g}_1 = 300$ MW and $\hat{g}_1 = 900$ MW and the final constraint comes from the transmission, where $K = 1000$ MW is the capacity of the export transmission line.

(b) What is the optimal dispatch?

Since the market price is always higher than the marginal price of the generators, they will both run as high as possible given the constraints. Since Generator 1 is cheaper than Generator 2, it will max-out its capacity first, so that $g_1^* = \hat{g}_1 = 300$ MW. Generator 2 will output as much as it can given the transmission constraint, so that $g_2^* = 700$ MW.

(c) What are the values of the KKT multipliers for all the constraints in terms of π?

From stationarity we have for g_1 the non-zero terms:

$$\begin{aligned}0 &= \frac{\partial}{\partial g_1}(\pi(g_1+g_2) - 5g_1 - 10g_2) - \bar{\mu}_1\frac{\partial}{\partial g_1}(g_1 - \hat{g}_1) - \underline{\mu}_1\frac{\partial}{\partial g_1}(-g_1) - \mu_T\frac{\partial}{\partial g_1}(g_1 + g_2 - K) \\ &= \pi - 5 - \bar{\mu}_1 + \underline{\mu}_1 - \mu_T\end{aligned}$$

For g_2 we have

$$\begin{aligned}0 &= \frac{\partial}{\partial g_2}(\pi(g_1+g_2) - 5g_1 - 10g_2) - \bar{\mu}_2\frac{\partial}{\partial g_2}(g_2 - \hat{g}_2) - \underline{\mu}_2\frac{\partial}{\partial g_2}(-g_2) - \mu_T\frac{\partial}{\partial g_2}(g_1 + g_2 - K) \\ &= \pi - 10 - \bar{\mu}_2 + \underline{\mu}_2 - \mu_T\end{aligned}$$

At the optimal point we can see that $\underline{\mu}_1$, $\bar{\mu}_2$ and $\underline{\mu}_2$ are non-binding, so these are zero. To solve for μ_T and $\bar{\mu}_1$ we have two equations:

$$\begin{aligned}0 &= \pi - 5 - \bar{\mu}_1 - \mu_T \\ 0 &= \pi - 10 - \mu_T\end{aligned}$$

Therefore

$$\begin{aligned}\mu_T &= \pi - 10 \\ \bar{\mu}_1 &= 5\end{aligned}$$

(d) A new turbo-boosting technology can increase the capacity of Generator 1 from 300 MW to 350 MW. At what annualised capital cost would it be efficient to invest in this new technology?

The value of $\bar{\mu}_1$ gives us the increase in profit for a small increase in $\hat{g}_1$. We want to understand a large increase in $\hat{g}_1$ of 50 MW, therefore we have to integrate over $\bar{\mu}_1$ as a function of $\hat{g}_1$, since the value of $\bar{\mu}_1$ may change as $\hat{g}_1$ changes. The total increase in profitability for expanding $\hat{g}_1$ from 300 MW to 350 MW is then

$$\int_{300}^{350} \bar{\mu}_1(\hat{g}_1) d\hat{g}_1$$

Because of the linearity of the problem, $\bar{\mu}_1$ is actually constant as we expand $\hat{g}_1$ in the region from 300 MW to 350 MW. The extra profit would be per year: 5 €/MWh * 50 MW * 8760h/a = €2.19 million/a. At or below this annualised capital cost, it would be worth investing.

(e) A new high temperature conductor technology can increase the capacity of the transmission line by 200 MW. At what annualised capital cost would it be efficient to invest in this new technology?

Here μ_T changes as K is expanded, so we have to integrate:

$$\int_{1000}^{1200} \mu_T(K) dK$$

Since μ_T is constant as we expand K from 1000 MW to 1200 MW, the extra profit would be per year: (average(π)-10) €/MWh * 200 MW * 8760h/a = €17.52 million/a. At or below this annualised capital cost, it would be worth investing. An extension beyond 1200 MW would not bring any benefit, because the generator constraints would be then binding.

SOLUTION V.2 (DURATION CURVES AND GENERATION INVESTMENT).

Let us suppose that demand is inelastic. The demand-duration curve is given by $D = 1000 - 1000z$, where $z \in [0, 1]$ represents the probability of time the load spends above a certain value. Suppose that there is a choice between coal and gas generation plants with a variable cost of 2 and 12 €/MWh, together with load-shedding at 1012€/MWh. The fixed costs of coal and gas generation are 15 and 10 €/MWh, respectively.

(a) Describe the concept of a screening curve and how it helps to determine generation investment, given a demand-duration curve.

A screening curve plots the costs of different generators as a function of their utilization/capacity/usage factor so that they can be compared based on their fixed and variable costs. The utilization factor is plotted along the x axis from 0 to 1, 0 corresponding to no running time, 1 corresponding to the power plant running 100% of the time. The intercept of the curve of each generator with the y axis is given by the fixed cost f [€/MWh] (i.e. the cost with no variable costs) and the slope is given by the variable cost c [€/MWh].

The interception points of the linear curves of the different generators determine the ranges of utilization factors in which one generator is cheaper than another. By comparing the screening curves with the demand duration curve, the correct generator capacities for different utilisation factors can be determined (e.g. how much baseload power is required, how much peaking power is required, how much load shedding).

(b) Plot the screening curve and find the intersections of the generation technologies.

First we work out the intersection points of the generators as a function of their capacity factors, then we work out the capacities K_* of the generators.

The screening curves tell us above which capacity factor it costs less to run one type of generator rather than another.

Generator	c_i [€/MWh]	f_i [€/MWh]
coal	2	15
gas	12	10
load-shedding	1012	0

Generators coal and gas intersect at x_{cg} given by

$$15 + 2x_{cg} = 10 + 12x_{cg}$$

i.e. $x_{cg} = 0.5$. This means that if the coal generator can run more than 50% of the time, it should be built from an economic perspective.

Gas generator and load-shedding intersect at x_{gl} given by

$$10 + 12x_{gl} = 1012x_{gl}$$

i.e. $x_{gl} = 1/100$. This means that for 1% of the time we have load-shedding because it's not economical to cover the rare times of very high load.

(c) Compute the long-term equilibrium power plant investment (optimal mix of generation) using PyPSA.

The amount of load that is present at least x_{cg} of the time determines K_{coal}, which we find by solving based on the load duration curve

$$1000 - 1000x_{cg} = K_{coal} \xRightarrow{x_{cg}=0.5} K_{coal} = 500$$

To get the capacity of the gas generator we solve based on the load duration curve

$$1000 - 1000x_{gl} = K_{coal} + K_{gas} \xRightarrow{x_{cg}=0.5 \text{ and } K_{coal}=500} K_{gas} = 490$$

Load above $K_{coal} + K_{gas} = 990$ is shed. Thus, $K_{load-shedding} = 10$.

(d) Plot the resulting price duration curve and the generation dispatch. Comment!

cf. Jupyter Notebook

(e) Demonstrate that the zero-profit condition is fulfilled.

cf. Jupyter Notebook

(f) While it can be shown that generators recover their cost in theory, name reasons why this might not be the case in reality.

Several factors make this theoretical picture quite different in reality:

- Generation investment is lumpy; i.e. you can often only build power stations in e.g. 500 MW blocks, not in continuous chunks.
- Some older generators have sunk costs, i.e. costs which have been incurred once and cannot be recovered, which alters their behaviour (i.e. the f term is not evenly distributed across all hours)
- Returns on scale in building plant are not taken into account (we did everything linear)
- Site-specific concerns ignored (e.g. lignite might need to be near a mine and have limited capacity)
- Variability of production for wind/solar ignored
- There is considerable uncertainty given load/weather conditions during a year, which makes investment risky; economic downturns reduce electricity demand
- Fuel cost fluctuations, building delays which cost money
- Risks from third-parties: Changing costs of other generators, political risks (carbon taxes, Atomausstieg, subsidies for renewables, price caps)
- Political or administrative constraints on wholesale energy prices may prevent prices from rising high enough for long enough to justify generation investment ("Missing Money Problem")
- Lead-in time for planning and building, behaviour of others, boom-and-bust investment cycles resulting from periods of under- and over-investment in capacity
- Exercise of market power

SOLUTION V.3 (SYNTHETIC FUELS).

cf. Jupyter Notebook