## TUTORIAL I: TIME SERIES ANALYSIS

Problems which are discussed in the tutorial are marked with  $\begin{center} \begin{center} \be$ 

Exercises to study at home are marked with

Questions on both kind of tasks are answered in the tutorial and **all exercises are** relevant for the exam.

## Problem I.1 (programming) – Data Analysis 👺

The following data are made available to you on the course home page<sup>1</sup> and the github repository <sup>2</sup>:

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de_data.csv, gb_data.csv, eu_data.csv, (wind.csv, solar.csv, load.csv).
```

They describe (quasi-real) time series for wind power generation W(t), solar power generation S(t) and load L(t) in Great Britain (GB), Germany (DE) and Europe (EU). The time step is 1 h and the time series are several years long.

- (a) Check that the wind and solar time series are normalized to 'per-unit of installed capacity', and that the load time series is normalized to MW.
- (b) For all three regions, calculate the maximum, mean, and variance of the time series.
- (c) For all three regions, plot the time series W(t), S(t), L(t) for a winter month (January) and a summer month (July).
- (d) Resample the time series to daily, weekly and monthly data points and visualise them in plots. Can you identify some recurring patterns?
- (e) For all three regions, plot the duration curve for W(t), S(t), L(t).
- (f) For all three regions, plot the probability density function of W(t), S(t), L(t).
- (g) Recurring patterns of time series can also be visualised more rigorously by applying a Fourier Transform. Apply a (Fast) Fourier Transform to the three time series  $X \in W(t)$ , S(t), L(t):

$$\tilde{X}(\omega) = \int_0^T X(t)e^{\mathrm{i}\omega t} \,\mathrm{d}t$$
.

For all three regions, plot the energy spectrum  $|\tilde{X}(\omega)|^2$  as a function of  $\omega$ . Discuss the relationship of these results with the findings obtained in (b)-(f).

<sup>1</sup>https://nworbmot.org/courses/esm-2020/

<sup>2</sup>https://github.com/lisazeyen/ESM\_tutorial/tree/master/tutorial/01-tutorial-04. 06.2020/notebooks/data

(h) Normalize the time series to one, so that  $\langle W \rangle = \langle S \rangle = \langle L \rangle = 1$ . Now, for all three regions, plot the mismatch time series

$$\Delta(t) = \gamma \alpha W(t) + \gamma (1 - \alpha) S(t) - L(t)$$

for the same winter and summer months as in (c). Choose  $\alpha \in \{0.0, 0.5, 0.75, 1.0\}$  with  $\gamma = 1$ , and  $\gamma \in \{0.5, 0.75, 1.0, 1.25, 1.5\}$  with  $\alpha = 0.75$ .

Which configuration entails the lowest mismatch on average and in extremes?

(i) For all three regions, repeat (b)-(g) for the mismatch time series.

## PROBLEM I.2 (ANALYTICAL) – EFFECT OF SEASONALITY A

Figure 1 shows approximations to the seasonal variations of wind and solar power generation W(t) and S(t) and load L(t):

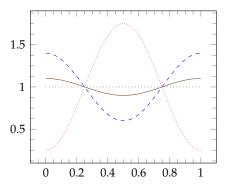
$$W(t) = 1 + A_W \cos \omega t$$

$$S(t) = 1 - A_S \cos \omega t$$

$$L(t) = 1 + A_L \cos \omega t$$

The time series are normalized to  $\langle W \rangle = \langle S \rangle = \langle L \rangle := \frac{1}{T} \int_0^T L(t) dt = 1$ , and the constants have the values

$$\omega = \frac{2\pi}{T}$$
  $T = 1 \text{ year}$   $A_W = 0.4$   $A_S = 0.75$   $A_L = 0.1$ 



**Figure 1:** Seasonal variations of wind and solar power generation W(t) - - and S(t) —, and load L(t) — around the mean 1 —.

(a) What is the seasonal optimal mix  $\alpha$ , which minimizes

$$\langle [\alpha W(t) + (1 - \alpha)S(t) - L(t)]^2 \rangle = \frac{1}{T} \int_0^T [\alpha W(t) + (1 - \alpha)S(t) - L(t)]^2 dt,$$

- (b) How does the optimal mix change if we replace  $A_L \rightarrow -A_L$ ?
- (c) Now assume that there is a seasonal shift in the wind signal

$$W(t) = 1 + A_W \cos(\omega t - \phi).$$

Express the optimal mix  $\alpha$  as a function of  $\phi$ .

(d) A constant conventional power source  $C(t) = 1 - \gamma$  is now introduced. The mismatch then becomes

$$\Delta(t) = \gamma \left[ \alpha W(t) + (1 - \alpha)S(t) \right] + C(t) - L(t).$$

Analogously to (a), find the optimal mix  $\alpha$  as a function of  $0 \le \gamma \le 1$ , which minimizes  $\langle \Delta^2 \rangle$ .