



# TWO-DIMENSIONAL STELLAR EVOLUTION

## with 2DStars:

### Introduction & Applications

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Binary Stars in Cambridge  
July 27, 2016, Cambridge, UK

# Two-dimensional Stellar Evolution: **2DStars**

- ❖ science goal
- ❖ applications
- ❖ input physics
- ❖ decisions

# Science goal

The goal is to develop a **general-use 2D, adaptable to 3D, stellar evolution code** (Izzard 2015) to model a variety of multi-dimensional phenomena in the evolution of single and binary stars.

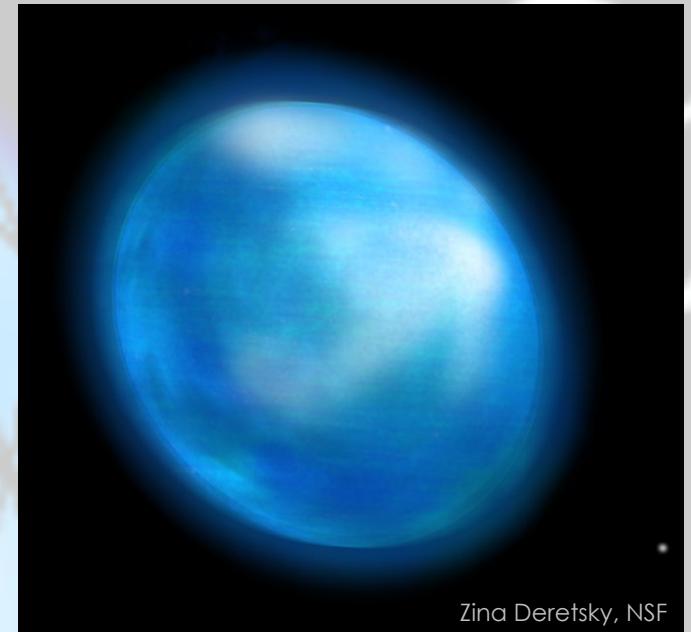
- Rotating Stars
  - Close Binaries
  - Star Formation
  - X-ray Binaries

# Applications (i)

## Rotating stars

A large fraction of stars rotate rapidly, are not spherical and exhibit surface temperature variations.

- ▶ The centrifugal force caused by rotation changes the hydrostatic balance, which alters the structure. This affects intrinsic stellar properties like luminosity (Potter + 2012), oscillation frequencies (Reese 2015) ..
- ▶ Rotation distorts the shape of the photosphere.
- ▶ Rotation introduces a brightness asymmetry due to the variation in the flux flowing through the surface as a function of latitude (**von Zeipel's theorem**: higher radiative flux at higher latitudes).



Zina Deretsky, NSF

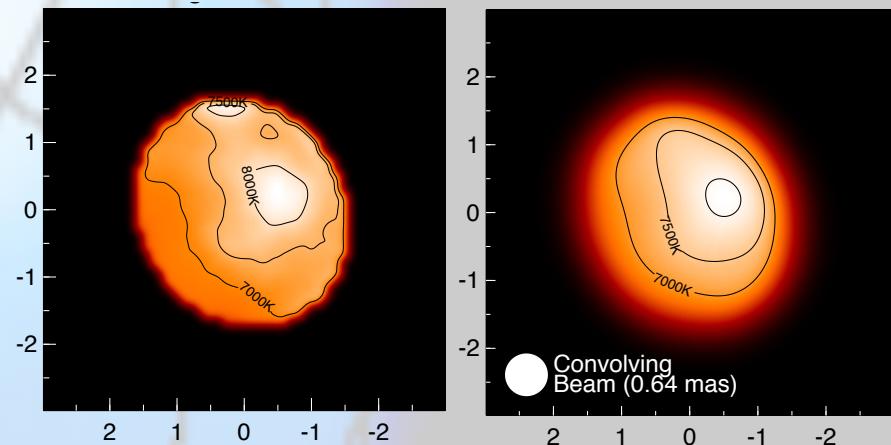


Image of **Altair** from **CHARA** interferometric array (Monnier et al. *Science*, 2007).

**Altair** rotates at 90% of its breakup velocity with a period of 9 hours (2.8 rev/day). This causes **the equator to bulge and darken (cooler)**.  $I_{eq} = 60\% I_{pole}$ .

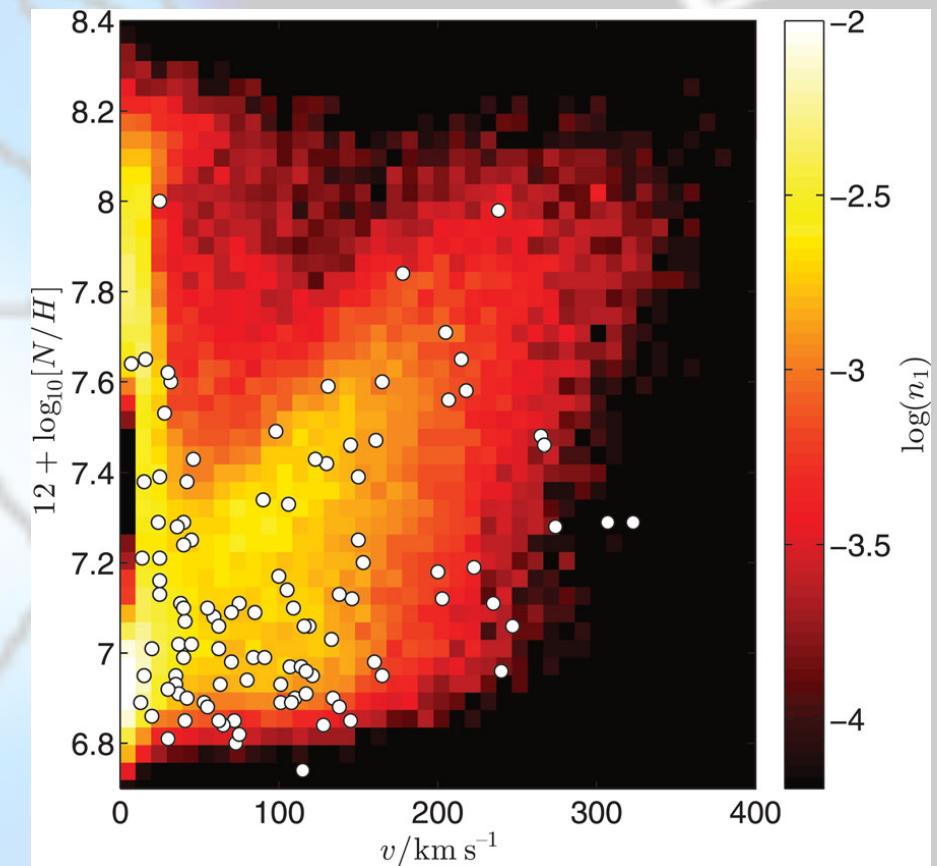
# Applications (i) – cont'd.

## Rotation:

- ▶ It alters the stellar chemistry by developing internal currents (such as the meridional Eddington-Sweet circulation)
- ▶ It couples to magnetic fields, commonly referred to as an  $\alpha - \Omega$  dynamo (Schmalz & Stix 1991, Potter, Chitre & Tout 2012).
- ▶ It may affect mass-loss or cause wind anisotropies:  $g_{\text{eff}}$  effect/  $\kappa_{\text{eff}}$  effect (Maeder & Meynet 2000).

Rotation is a key parameter besides M & Z (Maeder & Meynet 2000).

Thus, stars can only be modelled properly in multi-dimensions.



Potter, Chitre & Tout (2012)

The rotationally-driven dynamo model reproduces observed N enrichment in two distinct populations of massive stars observed in the LMC (VLT-FLAMES survey).

More massive stars: cannot support a dynamo are enriched by rotational mixing  
Lower-mass stars: spun down rapidly and are enriched by magnetic mixing.

# Applications (i) – cont'd.

## State-of-the-art

### 1. 1D codes simplifications:

- ▶ The first models assumed solid body rotation  $\Omega = \text{cnst}$ .
- ▶ Differential rotation:  $\Omega(r) = \text{cnst}$  on isobars (shellular rotation).
- ▶ modelling meridional circulation: advective or diffusive, free parameters

### 2. 2D codes:

- ▶ Roxburgh (2004): non-evolving uniformly-rotating models
- ▶ Li+ (2009): solar models but on short timescales
- ▶ ROTORC (Dupree 1990) : only models main-sequence stars on short timescales
- ▶ ESTER (Espinosa Lara & Rieutord 2013): predicts pulsation frequencies of main-sequence stars

### 3. 3D codes:

- ▶ Djehuty (Dearborn+ 2006): hydrodynamical code (ideal for rapid phenomena but not to evolve a star).

2DStars will  
fill this void

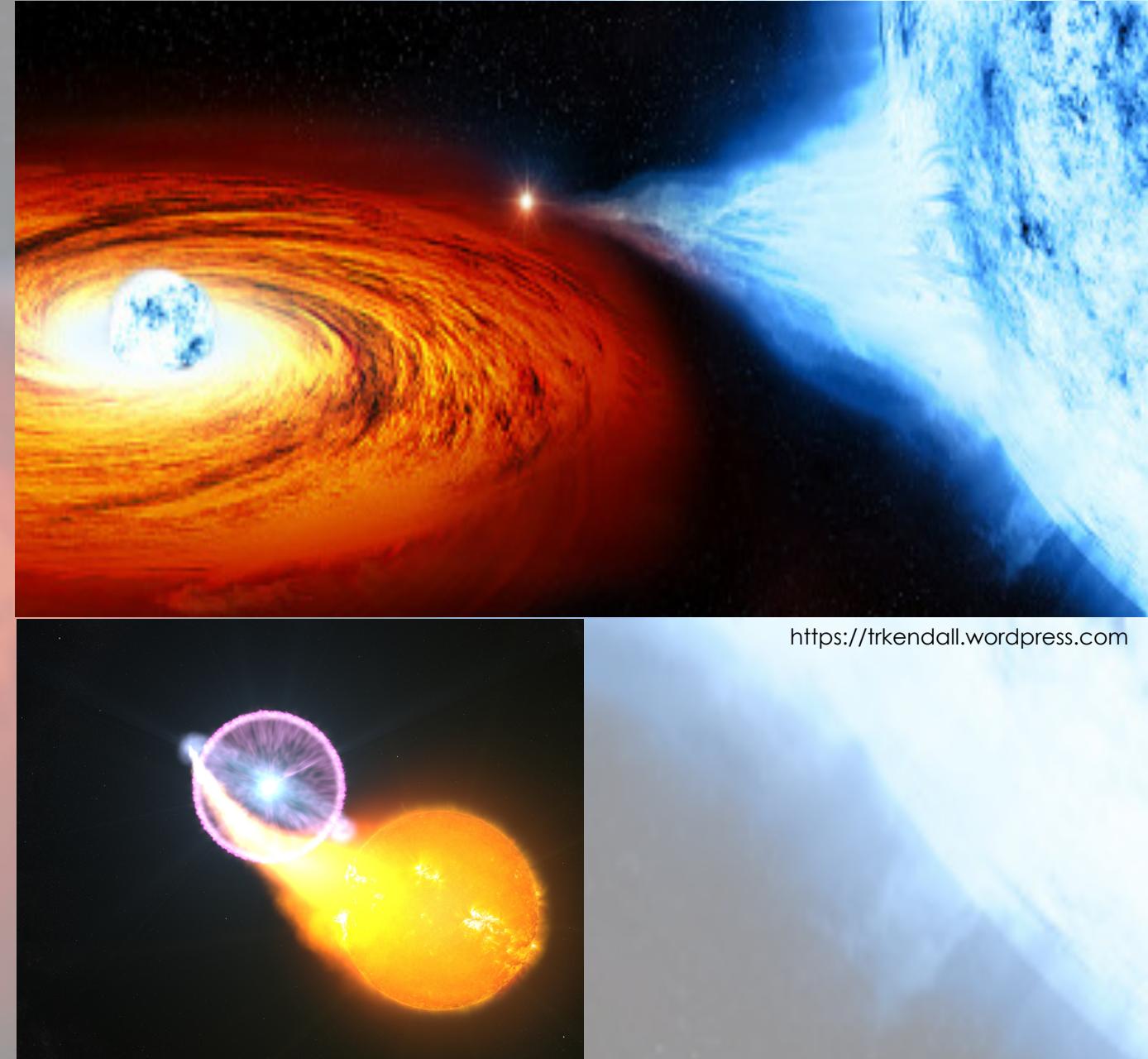
# Applications (ii)

## Mass Transfer in Close Binaries

Formation of an **accretion disc** by Roche-lobe overflow from the giant companion star.

Scott (2000) suggested that oblate distortion of rotating WDs drive **latitude-dependent** abundance gradients that may affect dust formation following a nova ejection (**prolate ejecta?**).

2D models may provide important feedback on the accretion process preceding the synthesis of C-rich dust in CO nova outbursts (**Jordi José, G. M. Halabi & Mounib El Eid 2016 A&A (accepted)**)



<https://trkendall.wordpress.com>

S. Wiessinger/Nasa Goddard Space Flight Center

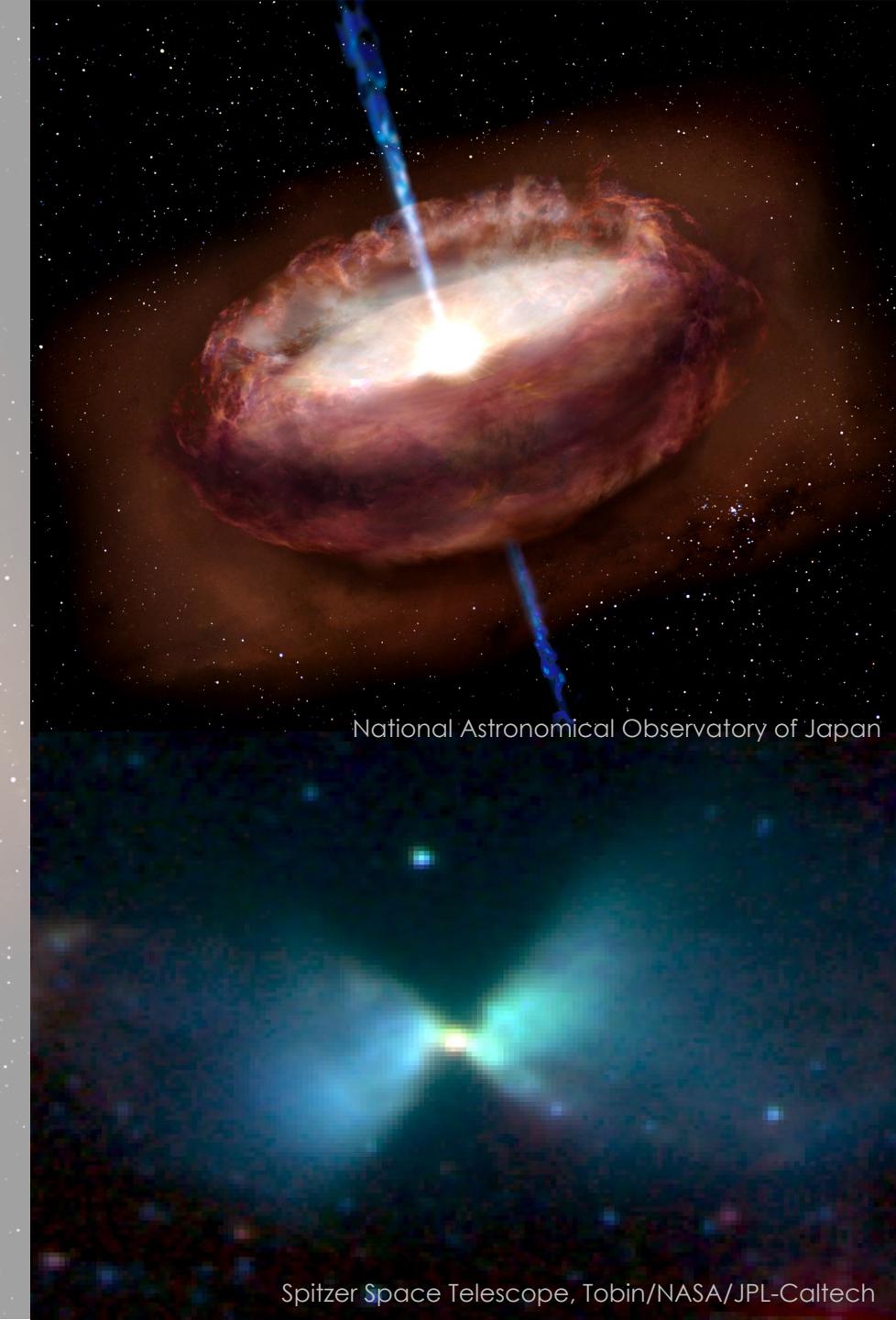
# Applications (iii)

## Star formation: Jets and Accretion Discs

A protoplanetary disc around young star *RY Tau* and infrared image of protostar *L1527*:

Both have rapidly spinning stars and discs which are magnetically connected and launch high-energy jets.

The star spins faster than the disc, twisting and shearing the magnetic field, storing up energy. Transition to a stable state unleashes stored energy in X-ray blasts.



National Astronomical Observatory of Japan

Spitzer Space Telescope, Tobin/NASA/JPL-Caltech

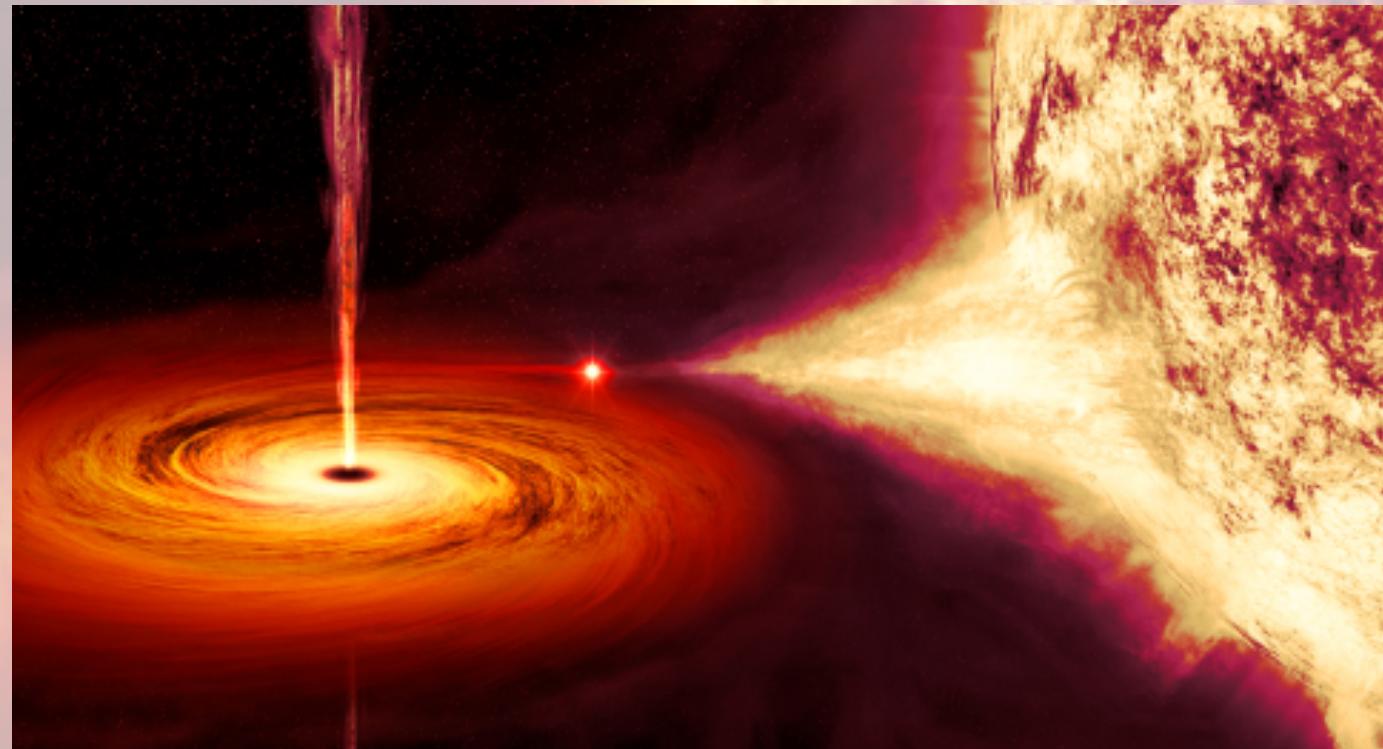
# Applications (iv)

## Tidally-Locked X-ray Binary Systems

V404 Cygni is a low-mass X-ray binary system that consists of a black hole and a star similar to our Sun.

The gas ripped from the star by tidal forces heats to millions of degrees as it falls inwards to the black hole.

A steady stream of high-energy radiation is emitted.



<http://www.cfa.harvard.edu>

# Input physics

- **Rotation:** Slow internal fluid rotation-driven flows including meridional circulation will be modelled consistently.
- **Magnetic fields:** They enforce co-rotation and couple stellar cores to their envelopes. Only in at least 2D can this be done self-consistently with angular momentum transport.
- **Chemistry:** Slow internal mixing will be fully incorporated. Fast mixing (convection, horizontal turbulence...) will be parameterized.
- **Mass transfer:** Tides and mass transfer in binaries can spin stars to their critical breakup limit. Material accretes through an accretion disc which should be modelled in 2D.

# Initial setup

- ★ A **Single rotating star** that evolves in time, for a given set of initial conditions.
- ★ We are interested in the **long term** (nuclear/thermal time scale) evolution i.e. that of the order of the stellar lifetime.
- ★ We **ignore magnetic fields** for now.
- ★ The star is **axisymmetric**. This symmetry can be broken by:
  1. Magnetic fields (e.g. O & B stars (Donati+ 2006, Grunhut+ 2012), Ap & Bp stars (Hubrig+ 2005)): Ignoring magnetic fields is not too bad for stars like Vega (Lignières+ 2009, Petit+ 2010).
  2. Non-axisymmetric deformation of the stellar surface in Roche lobe-filling systems by external irradiation of the secondary by a close compact companion (X-ray binaries, binary pulsar systems ..). This is a 3D problem (Philips & Podsiadlowski 2002).

# Specifics of the code

- Programming language: **JAVA** (portability, garbage collection, performance, build)
- Discretization scheme: **Finite difference**
- The physics
- Grid structure

$$\left. \begin{array}{l} r \\ 4 \\ l \\ l \end{array} \right\} \begin{array}{l} (m+1)(n+1) \\ (m+1)(n+1) \\ nm \\ nm \end{array}$$

$$4nm + 2n + 2m + 2$$

$$\left. \begin{array}{ll} r_{i,j} & nm \\ r_{(r_{i+1,j}):(r_{i+1,j+1})} & (m+1)(n-1) \\ r_{\text{abs. cont.}} & m+1 \\ r_{\text{ab. super.}} & m+1 \end{array} \right\} 4nm - 3 - m$$

$$\left. \begin{array}{l} \text{mom. eq:} \\ \text{parmn} \end{array} \right\} \begin{array}{l} (m-1)(n-1) \\ (m-1)(n-1) \end{array}$$

$$\left. \begin{array}{l} \frac{\partial p}{\partial r} \text{ out.} \\ \frac{\partial q}{\partial r} \text{ out.} \end{array} \right\} \begin{array}{l} m \\ m+1 \end{array}$$

$$\left. \begin{array}{l} V_p : mn \\ V_\theta : (m+1)n \\ V_r : m(n+1) \\ P \cdot (\vec{v}) = 0 : mn \end{array} \right\} \begin{array}{l} BC \\ BC \\ BC \\ BC \end{array}$$

$$\begin{array}{l} 0 \\ Zn \\ Zn \\ 2m \\ 3m \end{array}$$

# The physics

Discretized equations of stellar structure:

**Continuity equation:**

$$\operatorname{div}(\rho \vec{v}) = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho v) = 0$$

**Momentum equation:**

**r-component:**

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho uv) - \rho \frac{u^2 + w^2}{r} = -\frac{\partial P}{\partial r} - \rho \frac{\partial \psi}{\partial r}$$

**$\theta$ -component:**

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho uv) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho v^2) + \rho \frac{uv}{r} - \frac{\cot \theta \rho w^2}{r} = -\frac{1}{r} \frac{\partial P}{\partial \theta} - \frac{\rho}{r} \frac{\partial \psi}{\partial \theta}$$

**$\varphi$ -component:**

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho uw) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho vw) + \rho \frac{uw}{r} + \frac{\cot \theta \rho vw}{r} = 0$$

**Poisson equation:**

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial \psi}{\partial \theta} = 4\pi G \rho$$

# What we have so far

A Well-structured 1D code that:

1. Solves the equations of stellar structure ([hydrostatic equilibrium & Poisson equation](#)) + [polytropic equation of state](#), without considering energy generation and opacity. This is helpful since an analytical solution exists to test the code.
2. Is highly modular:
  - ❖ Integrator (Euler integrator, relaxation integrator)
  - ❖ Building models
  - ❖ Writing files
  - ❖ Constants
  - ❖ Visualizations
3. Can be easily modified to accommodate more complicated physics/solvers etc..

# Currently underway...

- ▶ Upgrading the 1D code to **2D** ( $r, \theta$ )
- ▶ Uniform mesh (in  $r$  and  $\theta$ )

## Next

- ▶ Consider a non-uniform mesh
- ▶ Adding energy transport equation. Consider fully convective case that incorporates transport coefficients in 2D (Jermyn, Tout, Chitre & Lesaffre)

# Conclusions

- ▶ Many astrophysical phenomena require multi-D approaches. **2DStars** aims to provide such a framework.
- ▶ Most model output is affected by rotation by various degrees depending on rotational velocity (tracks in the HR diagram, lifetimes, masses, chemical composition...). Stellar evolution is thus a function of **M**, **Z** and  **$\Omega$** .
- ▶ A number of serious discrepancies between current models and observations have been noticed over the past few years (the distribution of stars in the HR diagram at various metallicities, He and N abundances in massive O- and B-type stars and in giants and supergiants..).
- ▶ The proliferation of physical formulations used to describe rotation requires data to constrain the models. The VLT–FLAMES survey of massive stars (Evans+ 2005, 2006), VLT–FLAMES Tarantula Survey (Evans 2011) and the ongoing Gaia-ESO Survey make such comparisons possible.