Sudden stops and asset prices: the role of financial market participation [1]

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Abstract

This paper studies the role of household heterogeneity in the severity of sudden stop crises and its implications for prudential capital control policy. I use data on sudden stop events and financial market participation to document that a lower level of financial market participation is associated with a higher drop in asset prices. To explain the role that financial market participation plays in the drop in asset prices, I build an equilibrium business cycle model with a collateral constraint and with limited financial market participation. The heterogeneity in access to the financial market generates income and consumption inequality in the model. The extent to which the limited financial market participation amplifies the drop in the asset price depends on the cyclicality of consumption inequality. Consistent with my empirical findings using household survey data from Mexico, the model generates a drop in consumption inequality during the financial crisis that amplifies the drop in asset prices, output, and consumption. I show that the optimal time-consistent debt tax should be higher in a limited financial market participation economy, which rationalizes the use of capital control in emerging markets. Finally, my findings suggest it is possible to address financial instability without raising inequality.

Keywords: Financial crisis, asset prices, inequality, optimal policy, capital controls.

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1 Introduction

Output and consumption are more volatile in emerging markets than in advanced economies. Since the 1980s, emerging markets have on average faced a higher drop in the asset price and output relative to advanced economies during a sudden stop crisis[3]. Several studies have been done to understand sudden stops and how macroprudential policy or capital control can reduce their severity, but accounting for household heterogeneity in the financial market is a topic that remains less investigated. This paper studies the role that limited financial market participation plays in the severity of sudden stop crises and its implications for prudential capital control. I show that the effect of household heterogeneity in access to the financial market depends crucially on the cyclicality of consumption inequality.

My paper is motivated by three main empirical facts that I document in Section 2. First, across countries, there is heterogeneity in financial market participation. In emerging markets, the proportion of households that have access to financial markets is small. Those households consume their available labor income and do not hold any assets. Second, there is a negative correlation between the level of financial market participation and the drop in the asset price during sudden stops. Historically, countries with a low level of financial market participation faced a higher drop in the asset price during sudden stops. Third, consumption inequality is procyclical during Mexico's 1995 sudden stop crisis. Indeed, during Mexico's 1995 crisis, households who participated in the financial market faced a higher drop in their consumption compared to those who did not participate in the financial market.

To quantity the importance of inequality in the severity of sudden stops, I enrich a standard dynamic stochastic general equilibrium model that features an occasionally binding collateral constraint with limited household heterogeneity. The model features two types of households. The first type comprises households who participate in the financial market and have access to the capital and bond market. These households are called *asset holders*. The second type of household comprises those who do not participate in either the capital or bond market. These households, called *"hand-to-mouth"* consumers, consume all of their labor income plus any additional transfers.

[3] A sudden stop is an economic crisis that features current account reversal (i.e., massive capital outflows).

The small open economy faces shocks to its productivity, the real interest rate, and the price of imported inputs. The model economy nests the model in Mendoza (2010) with a fixed supply of capital.

The collateral constraint limits the total private debt to a fraction of the market value of capital. Total private debt is composed of private debt with one-year maturity plus a within-period working capital loan. The within-period working capital loan generates a contemporaneous drop in output when the collateral constraint binds, as it does when successive negative aggregate shocks hit the economy. If the constraint binds, the economy faces a financial crisis called a *sudden stop*. Here two main credit channels are in place. The first is the endogenous financing premium on debt, equity, and working capital as borrowing costs rise when the collateral constraint binds. The second is the Fisher (1933) debt-deflation mechanism. When the collateral constraint binds, asset holders subject their assets to a fire sale to smooth consumption and meet their obligations. The fire sale of assets leads to a decline in the capital price, which further tightens the collateral constraint. This non-linear feedback between the price of capital and the collateral value (borrowing capacity) exacerbates a financial crisis.

Qualitatively, the credit channel I describe is the same in a model with or without household heterogeneity. Quantitatively, I show that the severity of the drop in the asset price depend on the cyclicality of consumption inequality[4]. If consumption inequality is constant, then the level of financial market participation does not affect the severity or the drop in the asset price during the sudden stop. This shows that the pro(cyclicality), not the level of inequality, exacerbates the severity of a sudden stop crisis. If consumption inequality decreases during the crisis, the asset price drops more, and vice versa. Since consumption inequality decreased during Mexico's 1995 sudden stop episode, limited financial market participation exacerbated the severity of the crisis.

In the model, macroeconomic policy plays a role because of two externalities: a pecuniary externality and an aggregate demand externality. First, asset holders take the asset price as given. Their choice of debt affects, in the aggregate, the asset price, which determines the value of the collateral. This general equilibrium effect is called a pecuniary externality. The social planner who

[4] A crisis is more (less) severe when the drop in output and consumption is high (low).

takes this pecuniary externality into account may substantially reduce the severity of a financial crisis because of a binding collateral constraint. Second, the aggregate demand externality arises because asset holders take as given the choice of labor supply of hand-to-mouth consumers. Note that this aggregate demand externality is only present in the case of limited financial market participation. In the optimal policy, I choose to use a tax on foreign debt — a capital control — to decentralize the planner solution.

The quantitative results show that limited financial market participation amplifies the drop in output and asset holders' consumption. The fall in the asset price is 23% larger in a limited financial market participation economy compared to a full financial market participation economy. The model is consistent with three main business cycle fact in emerging markets. First, consumption is more volatile than output. Second, the trade balance is countercyclical. Third, the real interest rate is countercyclical.

To study the optimal policy, I introduce a financial shock. A financial shock — a drop to the loan-to-value ratio — consists of a drop during a financial crisis in the fraction of the total value of physical assets that households can pledge as collateral. The introduction of a financial shock to study the optimal policy is consistent with the data. The loan-to-value ratio is consistently low during a sudden stop crisis. With a financial shock, the asset price drops by 57% and asset holders' consumption drops by 25% in a limited financial market participation economy, whereas the asset price drops by 40% and the asset holders' consumption drops by 15% in a full financial market participation economy. The fall in the asset price is now 42% higher in a limited financial market participation economy compared to a full financial market participation economy. As expected, the results suggest that the financial shock exacerbates the financial crisis.

The optimal time-consistent constrained efficient allocation suggests three main points. First, the optimal time-consistent solution effectively reduces the frequency and severity of the financial crisis in both the full and limited financial market participation economies. Second, the average tax on foreign debt needed to decentralize the optimal time-consistent solution is higher in a limited financial market participation economy. This suggests that more capital control is needed in emerging markets, which have a low level of financial market participation relative to advanced

economies. This second lesson rationalizes the prevalent use of capital control in emerging markets. In fact, data on capital controls suggest that emerging markets control more capital flows than advanced economies. Third, the optimal time-consistent solution suggests that there is no trade-off between financial stability and consumption inequality in the case of limited financial market participation. While in the very short run (at the time of the financial crisis), the social planner may tolerate a slight increase in consumption inequality, average consumption inequality is no higher in the optimal time-consistent equilibrium than it is in the competitive economy.

My paper mainly relates to the literature that studies the aggregate effects of a sudden stop (see, for example, Arellano and Mendoza (2002), Chari et al. (2005), Mendoza (2006), Calvo et al. (2006). Mendoza (2010), and Korinek and Mendoza (2014)). My paper is closely related to Mendoza (2010), who studies how an endogenous binding collateral can trigger the economy within standard business cycle moments. My contribution to this literature is twofold. First, I introduce limited financial market participation where a fixed share of households do not participate in the financial market. This characterization of the economy is closer to that of emerging markets and helps us to explain the observed gap in the decline in the asset price during sudden stops between emerging markets and advanced economies. In addition, my work studies the optimal time-consistent solution and rationalizes the prevalent use of capital control in emerging markets.

My work is also related to recent literature that studies the optimal policy in a financial crisis model. These papers include Caballero and Krishnamurthy (2004), Bianchi (2011), Bengui (2014), Bengui and Bianchi (2018), Bianchi and Mendoza (2018), and Arce et al. (2019). I contribute to this literature by taking into account household heterogeneity in the financial market and show that it is possible to address financial instability without raising inequality.

The rest of the paper is organized as follows. In Section 2, I present the data and the empirical facts. Section 3 presents the model. Sections 4 and 5 present my findings and discuss the results, and Section 6 concludes.

2 Data and empirical facts related to asset prices and inequality

In this section, I present the data and the empirical facts related to financial market participation, sudden stops, and inequality. I use three sources of macro and micro data. The first is panel data on financial market participation, which cover low-income countries, emerging markets, and advanced economies. The second is aggregate data on the drop in asset prices during sudden stops. The third is micro survey data on household consumption, income, and wealth in Mexico.

Financial market participation. I use the IMF's Financial Development Index Database. The index database provides nine indexes for 180 countries for every year since 1980. I focus on two indexes that measure the ability of individuals and firms to access financial services: the Financial Institution Access index (FIA) and the Financial Market Access index (FMA). FIA measures the number of bank branches per 100,000 adults and the number of ATMs per 100,000 adults. FMA measures the percentage of market capitalization outside of the top 10 largest companies and the total number of issuers of debt (domestic and external, non-financial corporations) per 100,000 adults. All indexes are between 0 and 1 where 1 means full access to financial services.

Sudden stops. I use the sudden stops data constructed by Korinek and Mendoza (2014). A sudden stop is defined as a large capital outflow as measured by a year-over-year increase in the current account/GDP ratio by more than two standard deviations above the average change in this ratio. I use the stock market index provided in the data as a measure of asset prices. The data include emerging markets and advanced economies over the period 1980-2012.

Consumption, income, and wealth. I use Mexico's National Survey of Household Income and Expenditure (ENIGH). This is a representative household survey that covers rural and urban areas and has been conducted every two years since 1992. More than 10,000 households are interviewed at each survey. The survey has detailed information about household consumption items as well as household income and wealth. I define "hand-to-mouth" consumers as households who hold zero liquid wealth. I define consumption inequality as the ratio of *asset holders'* consumption to the consumption of hand-to-mouth households.

I document three facts:

Fact 1: There is heterogeneity in financial market participation across countries. Also, on average, emer-

ging markets have a lower level of financial market participation relative to advanced economies. Figure 1 displays the median of the financial market access index from 1980 to 2017 for each country. Figure 2 displays the median of the financial institution access index from 1980 to 2017 for each country. Darker red areas indicate higher financial market participation. North American countries, western European countries, Japan, Australia, and a few other countries have relatively high access to financial services.

Fact 2: There is a negative correlation between the level of financial market participation and the drop in asset prices during a sudden stop episode. Table 1 presents the results from a panel regression of asset prices on a dummy variable that takes the value of 1 if a given country in a given year is in a sudden stop crisis, financial market access, and the cross-product between the dummy and financial market access. The cross-product captures the marginal effect of the level of financial market participation on the drop in asset prices during sudden stops. The first column considers advanced economies and emerging markets. The second column considers only advanced economies, and the third column considers only emerging markets. All regressions include country and year fixed effects. I control for capital flows. The estimated coefficient on the cross-product is positive, which means that a country with a higher level of financial market participation has a lower drop in the asset price during a sudden stop episode.

Fact 3: Consumption inequality is procyclical during Mexico's 1995 sudden stop crisis. Figure 3 plots the consumption inequality dynamics in Mexico from 1992 to 2000. Consumption inequality is defined as the ratio of *asset holders'* consumption to the consumption of "hand-to-mouth" households. Prior to the sudden stop crisis in 1995, consumption inequality increases. It then decreases during the crisis from 1994 to 1996 and starts to increase again beginning in 1996, evidence that asset holder consumers, are hit relatively harder by the sudden stop crisis than "hand-to-mouth" consumers.

I use these three facts to discipline my model. In my model, the financial crisis and consumption inequality are endogenous, whereas the level of financial market participation is exogenous. I present the model in the next section.

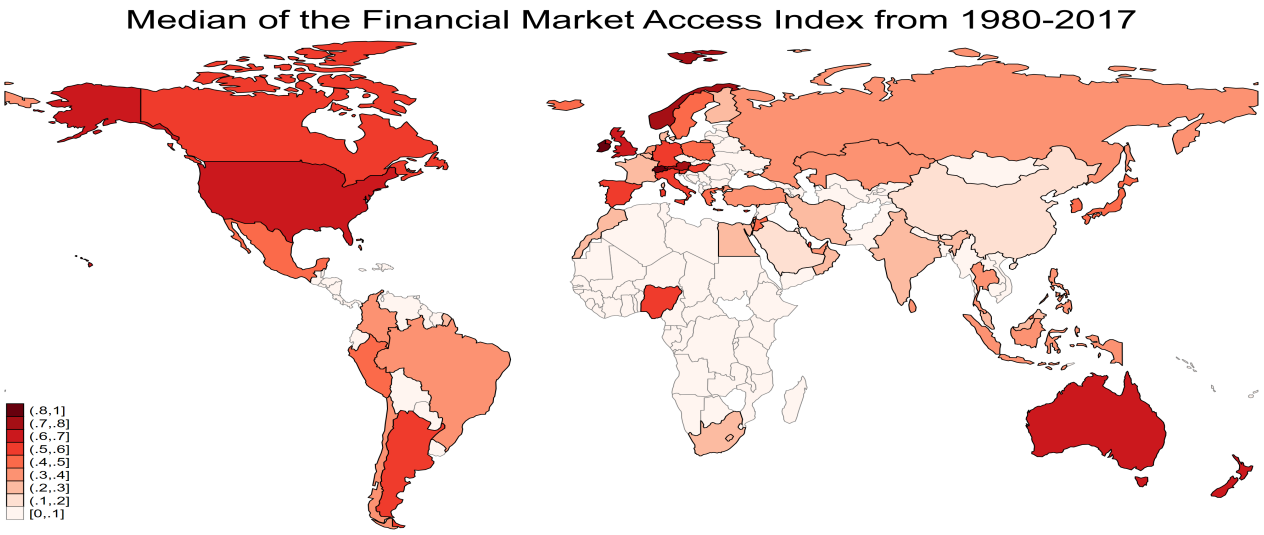


Figure 1: Financial market accessibility across countries

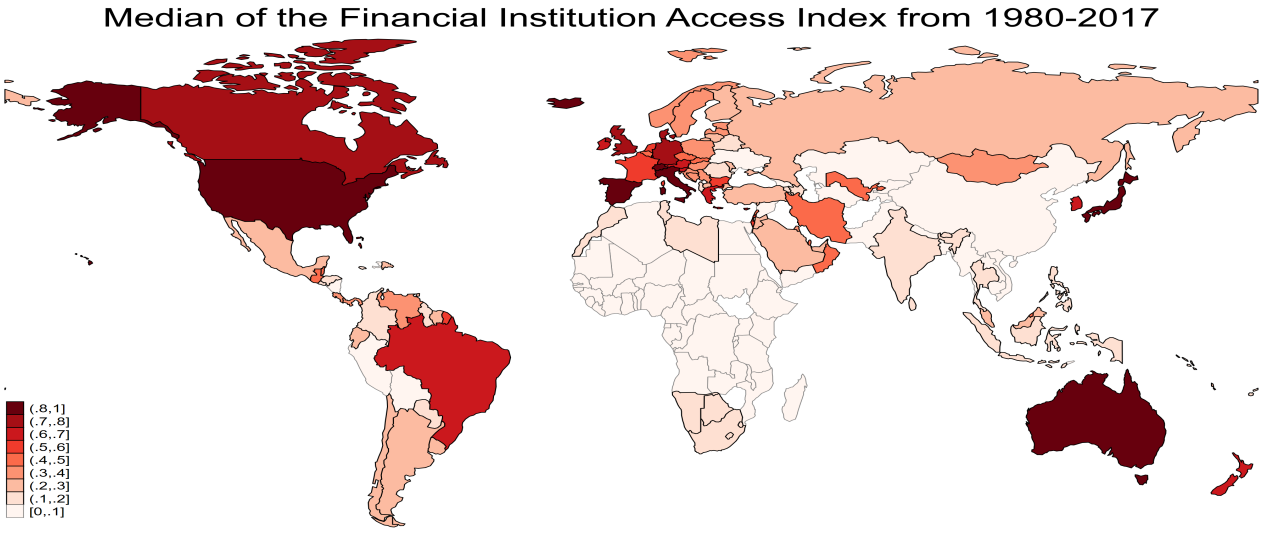


Figure 2: Financial institution accessibility across countries

Table 1: Panel regression of equity price growth on sudden stops

Asset price growth	(1) Aggregate	(2) Advanced Economies	(3) Emerging Markets
Sudden stops (SS)	-0.438***	-0.334***	-0.649***
Financial market participation (FMA)	-0.110	0.0104	-0.269
FMAxSS	0.416**	0.295*	0.985**
Observations	631	366	265
R-squared	0.352	0.533	0.421
Number of countries	29	15	14

Note: Regressions are done with country and year fixed effects. SS is a dummy variable that takes the value of 1 if a country is in a sudden stop for a given year. Data on sudden stops are from Korinek and Mendoza (2014). FMA is the Financial Market Access index from the IMF. FMAxSS is a cross-product of FMA and SS. I control for capital flows. The data cover the period 1980-2012. I drop the sudden stop events that have an increase in asset prices. *** p<0.01, ** p<0.05, * p<0.1

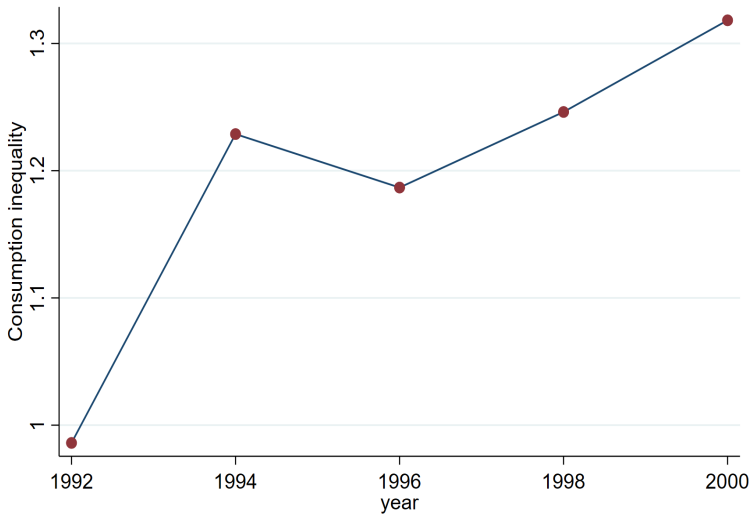


Figure 3: Consumption inequality in Mexico: consumption inequality is defined as the ratio of *asset holder* consumers' consumption to "hand-to-mouth" consumers' consumption.

3 Model with collateral constraint and household heterogeneity

I build a small open economy model with household heterogeneity and a collateral constraint. The sudden stop crisis is driven by an occasionally binding collateral constraint. There are two types of households. The first type comprises asset holder consumers who have access to the financial market through their holding of both physical assets and foreign bonds. The second type are "hand-to-mouth" consumers who do not hold any assets — neither physical assets nor foreign bonds. They consume all of their labor income plus any additional transfers from the government. In this section, I assume that asset holder consumers make production and consumption decisions.[5]

3.1 Firm and asset holder households' optimization problem

There is a continuum of identical *asset holder* households of measure $1-\theta \in (0,1]$. The preferences of an *asset holder* consumer indexed by 1 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{1t} - G(L_{1t})), \tag{1}$$

where $\mathbb{E}_0$ is the expectations operator; β is the discount factor; C_{1t} is consumption, and L_{1t} is labor supply; $u(.)$ is the utility function which is a standard concave, twice continuously differentiable function that satisfies the Inada condition; and G(L) is a convex, strictly increasing, and continuously differentiable function that measures the disutility of labor. These preferences (known as GHH preferences due to Greenwood et al. (1988)) remove the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.

Households produce final goods using three inputs, which are physical assets k_t, intermediate goods v_t, and labor L_t. Total labor L_t in the economy is given by $(1-\theta)L_{1t} + \theta L_{2t}$, where L_{2t} is the labor supply of a "hand-to-mouth" consumer. The production technology is such that $y = A_t F(k_t, L_t, v_t)$, where F is a twice continuously differentiable, concave production function and $A_t = A\exp(\epsilon_t^A)$ is TFP subject to a random shock ϵ_t^A. This shock follows a stationary Markov process. Intermediate goods are traded in competitive world markets at a price p_t^v. The price

[5] In appendix A, I show that a separate problem between firm and asset holder consumers has the same outcome.

$p_t^v = p\exp(\epsilon_t^v)$ is subject to a random shock ϵ_t^v that follows a stationary Markov process. *Asset holder* households borrow on the foreign bond market at the real interest rate $R_t^v = R\exp(\epsilon_t^r)$, where ϵ_t^r is a random shock that follows a stationary Markov process. The budget constraint of *asset holder* households is given by

$$(1-\theta)\,C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} = F\left(k_t, L_t, v_t\right) - p_t^v v_t - \theta w_t L_{2t} - \phi r_t\left(w_t L_t + p_t^v v_t\right) + b_t + q_t k_t - T_t. \quad (2)$$

In equation 2, q_t is the price of the physical asset k_t, $r_t = R_t - 1$ is the net real interest rate, and w_t is the real wage. On the right-hand side of (2), the term $\phi r_t\left(w_t L_t + p_t^v v_t\right)$ represents the interest payment abroad on the working capital loan. The working capital loan is a fraction ϕ of the total cost of intermediate inputs and labor in advance of sales. The term $\theta w_t L_{2t}$ represents the total labor income paid to "hand-to-mouth" households. The term T_t is the total lump-sum taxes paid by all *asset holder* households. Lump-sum taxes are used to calibrate the average consumption inequality.

The total private debt in the economy is restrained to a fraction κ of the market value of the end-of-period physical asset given by

$$\frac{b_{t+1}}{R_t} - \phi R_t\left(w_t L_t + p_t^v v_t\right) \geq -\kappa q_t k_{t+1}. \quad (3)$$

On the left-hand side of (3), total private debt (in negative terms) is the sum of private debt with one-year maturity and the within-period working capital loan. On the right-hand side of (3), the term $\kappa q_t k_{t+1}$ represents a fraction κ of the market value of the end-of-period physical asset. Only *asset holder* households who borrow in the foreign bond market face this collateral constraint. Although I do not derive the collateral constraint from an optimization problem, Bianchi and Mendoza (2018) show that this type of constraint could be obtained as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the market value of an asset owned by a defaulting debtor.

The *asset holder* households choose consumption, borrowing, capital, labor, and intermediate inputs to maximize their utility (1) subject to their budget constraint (2) and their borrowing con-

straint (3), taking prices as given. Their optimality conditions are given by

$$
\begin{aligned}
u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + \mu_t, && (4)\\
q_t u'(t) &= \beta \mathbb{E}_t \left[\left(d_{t+1} + q_{t+1} \right) u'(t+1) \right] + k q_t \mu_t, && (5)\\
A_t F_l \left(k_t, L_t, v_t \right) &= G'(L_{1t}) + \phi \left(r_t + R_t \frac{\mu_t}{u'(t)} \right) w_t, && (6)\\
A_t F_v \left(k_t, L_t, v_t \right) &= p_t^v + \phi \left(r_t + R_t \frac{\mu_t}{u'(t)} \right) p_t^v, && (7)
\end{aligned}
$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the borrowing constraint, $u'(t)$ is the partial derivative of $u(C_{1t} - G(L_{1t}))$ with respect to C_{1t}, and $d_{t+1} = A_{t+1} F_k \left(k_{t+1}, L_{t+1}, v_{t+1} \right)$.

The first two optimality conditions are the Euler equations for bonds and physical assets, respectively. The last two optimality conditions are the intratemporel conditions on the labor market and intermediate good market, respectively.

Condition (4) states that if the collateral constraint is not binding ($\mu_t = 0$), the marginal benefit of borrowing to increase today's consumption is equal to the expected marginal cost of repaying back tomorrow. If the collateral costraint binds, the shadow price of relaxing the collateral constraint is positive ($\mu_t > 0$), so the marginal benefit of borrowing is greater than its expected marginal cost. Condition (5) states that the marginal cost of buying one additional unit of physical asset at price q_t is equal to its expected marginal benefit. If the collateral constraint binds, the marginal cost exceeds the marginal benefit by $k q_t \mu_t$.

Condition (6) states that the marginal productivity of labor demand is equal to the marginal disutility of labor supply plus the financing cost of labor from the working capital loan. The financing cost is higher when the collateral constraint binds. Condition (7) states that the marginal productivity of the intermediate input is equal to its price plus the financing cost of the intermediate input from the working capital loan. The financing cost of the intermediate input is higher when the collateral constraint binds.

3.2 Hand-to-mouth households' optimization problem

There is a continuum of identical "hand-to-mouth" households of measure $\theta \in [0, 1)$. The preferences of a "hand-to-mouth" consumer indexed by 2 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{2t} - G(L_{2t})), \tag{8}$$

where C_{2t} is consumption, L_{2t} is labor supply, and $u(.)$ is the same utility function as in section 3.1. The budget constraint of "hand-to-mouth" households is given by

$$\theta C_{2t} = \theta w_t L_{2t} + T_t. \tag{9}$$

The *hand-to-mouth* households choose consumption and labor to maximize their utility (8) subject to their budget constraint (9), taking prices as given. Their optimality condition is given by

$$G'(L_{2t}) = w_t. \tag{10}$$

Condition (10) states that the marginal disutility of labor supply for *asset holder* consumers is equal to the real wage rate.

3.3 Competitive equilibrium

In this this section, I define the competitive equilibrium and the main credit channel through which sudden stops arise in this type of framework. The aggregate resource of the economy is given by

$$C_t + \frac{b_{t+1}}{R_t} - b_t + \phi r_t \left(w_t L_t + p_t^v v_t\right) = F\left(1, L_t, v_t\right) - p_t^v v_t, \tag{11}$$

where $C_t = (1-\theta) C_{1t} + \theta C_{2t}$ is aggregate consumption, the term $\frac{b_{t+1}}{R_t} - b_t + \phi r_t \left(w_t L_t + p_t^v v_t\right)$ represents the trade balance, and the term $F\left(1, L_t, v_t\right) - p_t^v v_t$ represents GDP.

A competitive equilibrium in this model is a stochastic sequence $Q_t = \{C_{1t}, C_{2t}, L_{1t}, L_{2t}, v_t, b_{t+1}\}_{t \geq 0}$ and prices $P_t = \{q_t, w_t\}_{t \geq 0}$ such that:

1. Given P_t, Q_t solves households' and firms' problems;

2. w_t and q_t are determined competitively that is: $G'(L_t) = w_t$ and q_t solves equation 5;

3. markets clear:

   (a) labor market: $L_t = L_{1t} = L_{2t}$,

   (b) capital market: $K_t = 1$,

   (c) aggregate resource: equation (11) is satisfied.

3.4 Equity premium and consumption inequality wedge

In this section, I characterize the equity premium and show how limited financial market participation distorts the equity premium.

Let $\frac{\lambda_t^R}{E_t[\beta\lambda_{t+1}^R]}$ be the inverse of the stochastic discount factor in the economy with full financial market participation ($\theta = 0$) where $\lambda_t^R = \beta R_t \mathbb{E}_t \lambda_{t+1}^R + \mu_t^R$. Let $\frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$ be the inverse of the stochastic discount factor in the economy with limited financial market participation ($\theta > 0$) where $\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t$. Using the definition of asset returns and conditions 4 and 5, the expected excess returns of bonds $E_t\left[R_{t+1}^q - R_t\right]$ can be decomposed into a liquidity premium, an inequality wedge, and the risk premium as follows:

$$E_t\left[R_{t+1}^q - R_t\right] = (1-\kappa) \underbrace{\frac{\mu_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]}}_{\textbf{liquidity premium}} - (1-\kappa) \underbrace{\left(\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]} - \frac{\lambda_t}{E_t\left[\beta\lambda_{t+1}\right]}\right)}_{\textbf{consumption inequality wedge}} - \underbrace{\frac{Cov\left(\lambda_{t+1}, R_{t+1}^q\right)}{E_t\left[\lambda_{t+1}\right]}}_{\textbf{risk premium}} \tag{12}$$

The term $\frac{\lambda_t^R}{E_t[\beta\lambda_{t+1}^R]} - \frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$, which I call the *consumption inequality wedge*, is the difference of the inverse of the stochastic discount factor between the economy with full financial market participation $\theta = 0$ and the economy with limited financial market participation $\theta > 0$. For $\kappa = 1$, the liquidity premium and the consumption inequality wedge do not affect the equity premium. If the agents can pledge all of their assets as collateral ($\kappa = 1$), when the collateral constraint binds, the agents can always offset it by increasing their physical assets by one unit. The relevant case is when the agents cannot pledge all of their assets as collateral ($\kappa < 1$).

The liquidity premium raises the equity premium when the collateral constraint binds for $\kappa < 1$. The rise in the equity premium decreases asset prices and the collateral constraint tightens even more. This mechanism is known as the debt-deflation mechanism and is at the core of the financial crisis generated by the model I present. In the model, the collateral constraint binds endogenously when the leverage is relatively high, and an exogenous negative aggregate shock (high real interest rate, low productivity shock, and/or high imported price) hits the economy.

The limited financial market participation add a new term to the equity premium in equation (12), which is the consumption inequality wedge. When the consumption inequality wedge is negative, it raises the equity premium and the asset price decreases more, leading to a high decline in the asset price. Inversely, if the consumption inequality wedge is positive, the equity premium decreases, leading to a low decline in the asset price. The effect of the consumption inequality wedge depends on the cyclicality of consumption inequality.

I now characterize how the cyclicality of consumption inequality affects the consumption inequality wedge. Suppose that $u(C_t - G(L_t)) = \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}}{1-\sigma}$ where ω is the labor elasticity. Then $\lambda_t = a_t^{\sigma} \lambda_t^R$ with $a_t = \frac{(1-\theta)\omega + (\omega\theta - 1)\frac{c_{2t}}{c_{1t}}}{\omega - \frac{c_{2t}}{c_{1t}}}$. The ratio $\frac{c_{1t}}{c_{2t}}$ is defined as consumption inequality.

Claim 1: If consumption inequality is constant over time then financial market participation does not matter for sudden stop crises. The drop in the asset price is the same in the economy with full financial market participation as in the economy with limited financial market participation.

The proof follows from the definition of the consumption inequality wedge, which is zero for a constant consumption inequality since $\frac{\lambda_t^R}{E_t\left[\beta \lambda_{t+1}^R\right]} = \frac{\lambda_t}{E_t[\beta \lambda_{t+1}]}$. The consequence of claim 1 is that the level of consumption inequality does not affect the drop in the asset price, hence the severity of the crisis. The next claim completes this claim by showing that the cyclicality of the consumption inequality indeed matters.

Claim 2: Let's suppose perfect foresight (no uncertainty); that is, $\mathbb{E}_t[X_{t+1}] = X_{t+1}$.
If the consumption inequality is lower (higher) during the financial crisis, the economy will generate a higher (lower) amplification effect.

Under perfect foresight, the consumption inequality can then be rewritten as $\frac{\lambda_t^R}{\beta \lambda_{t+1}^R}\left(1 - \left(\frac{a_t}{a_{t+1}}\right)^{\sigma}\right)$. Suppose now that at time t, the collateral constraint binds and $\frac{c_{1t}}{c_{2t}} < \frac{c_{1,t+1}}{c_{2,t+1}}$ (that is lower consump-

tion inequality) this implies that $a_t > a_{t+1}$. It follows that the consumption inequality wedge is negative, leading to a high equity premium.

Claims 1 and 2 have shown that, given the limited financial market participation, what matters is the cyclicality of consumption inequality. As shown in fact 3 of Section 2, the consumption inequality is procyclical, leading quantitatively to a higher drop in asset prices. Werning (2015) and Acharya and Dogra (2020) have argued that the cyclicality of income inequality could affect the aggregate outcome variables in a monetary policy with a household heterogeneity framework. In my framework, where there are no nominal rigidities, I find that the cyclicality of consumption inequality does affect the severity of a financial crisis. In the next section, I present the quantitative results.

4 Quantitative results

This section studies the model's quantitative implications using numerical simulation. First, I present the calibration and then discuss the results.

4.1 Calibration

A period in the model represents a year. The calibration uses data from Mexico. The results are presented in Table 2. The functions forms for preference and technology are the following:

$$\begin{aligned} u(C_t - G(L_t)) &= \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}, \quad \omega > 1 \\ F\left(k_t, L_t, v_t\right) &= A_t k_t^{\gamma} L_t^{\alpha} v_t^{\eta}. \end{aligned}$$

The preference parameters for risk aversion and the elasticity of substitution are set to standard values from the literature: $\sigma = 2$. The average real interest rate is set to 4%, also standard in the literature.

Labor supply elasticity ω is set equal to 1.846, as in Mendoza (2010). Mendoza (2010) uses data for the period 1993:1-2005:11 and finds that the annualized average ratio of GDP to gross output

(gdp/y) is 0.896 and the ratio of imported inputs to GDP (pv/gdp) is 0.114. The average share of imported inputs in gross output is 0.102, which implies that η= 0.102. The labor share on GDP for Mexico is 0.66, which implies that $\alpha = 0.592$. The value of $\gamma = 0.042$ is set so that the equity premium is zero at the deterministic steady state. The steady state asset price is set to 1.

The shocks are modeled as a joint discrete Markov process that approximates the statistical moments of their actual time-series processes. The Markov process is defined by a set E of all combinations of realizations of the shocks, each combination given by a triple $e = (\epsilon^A, \epsilon^R, \epsilon^P)$ and by a matrix of transition probabilities of moving from e_t to e_{t+1}. I closely follow Mendoza (2010) to set the transition probability between the different states. In the data, $\epsilon^A, \epsilon^R, \epsilon^P$ are AR(1) processes with standard deviations and first-order autocorrelations,respectively, 0.537, 0.572, and 0.737. Since the three shocks are nearly independent, except for a statistically significant correlation between ϵ^A and ϵ^R of about -0.67, the Markov process is constructed using the parsimonious structure of the two-point symmetric simple persistence rule as in Mendoza (2010) . Each shock has two realizations equal to plus/minus one standard deviation of each shock in the data ($\epsilon^A_1 = -\epsilon^A_2 = 0.0134$, $\epsilon^R_1 = -\epsilon^R_2 = 0.0196$, $\epsilon^P_1 = -\epsilon^P_2 = 0.0335$), so E contains eight triples. The simple persistence rule produces an 8x8 matrix, which yields autocorrelations of the shocks and a correlation between ϵ^A and ϵ^R that match those in the data. Mendoza (2010) points out, however, that the procedure requires that the AR(1) coefficients of the shocks that are correlated with each other (ϵ^A and ϵ^R) be the same, which is in line with the data where $\rho(eR) = 0.572$ and $\rho(eA) = 0.537$.

The two parameters remaining are β and κ. The value of β is set to 0.92 to match the average net foreign asset of 20% of GDP. The value of κ is set to 0.43 to match the frequency of the financial crisis of 4%. Average private debt is 19.7 % of GDP, and the frequency of the financial crisis is 4.6%.

4.2 Sudden stops: the dynamics of asset prices, output, debt, and consumption

This section presents the quantitative results. I first describe the difference in the debt policy function between the economies with limited and full financial market participation. Second, I show the dynamics of some aggregate variables when the collateral constraint binds. I finish this section by showing some long-run moments.

Table 2: Parameter values

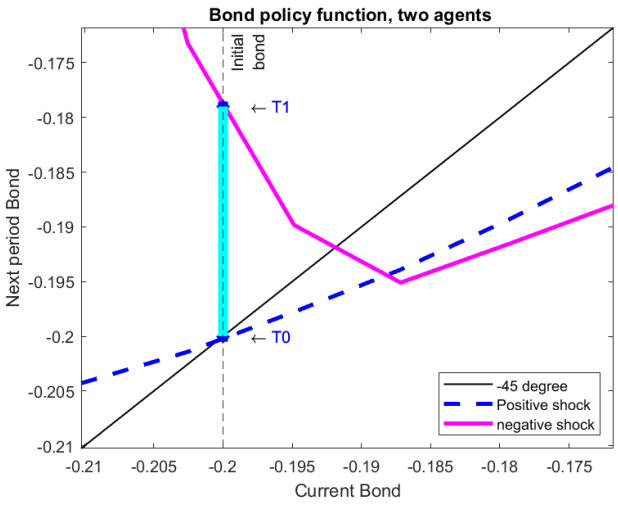
Parameters set independent	Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Share of labor in gross output	$\alpha = 0.592$	Mexico GDP labor share 0.66
Share of input in gross output	$\eta = 0.10229$	Mexico data
Share of asset in output	$\gamma = 0.043$	steady state asset return
Frisch elasticity	$\omega = 1.846$	Mendoza (2010)
Working capital coefficient	$\phi = 0.13$	Working capital/ GDP ratio = 10%
Share of Hand-to-Mouth	$\theta = 0.5$	Mexico data
Transfer	$T_t = 0.14$	Avr cons ineq of 1.25
Parameters set simulation	Value	Target
Discount factor	$\beta = 0.920$	Net foreign asset of 20%
Fraction of collateral value	$\kappa = 0.43$	Financial crisis of 4%

Policy functions for gross private debt. Figure 4 presents the next period private debt b_{t+1} as a function of current private debt b_t. The solid magenta line represents the policy function for a negative aggregate shock that has a high real interest rate and low productivity. The dash blue line represents the policy function for a positive aggregate shock that has a low real interest rate and high productivity. In panel (a), the economy with limited financial market participation ($\theta = 0.5$) is shown. In panel (b), the economy with full financial market participation ($\theta = 0$) is shown .

In both panels, the next period debt as a function of the current debt for a negative aggregate shock has a V-shape. This V-shape is due to the collateral constraint, which is more likely to bind for a high debt and negative aggregate shock. If the debt level is high (that is, the bond is more negative), households are forced to deleverage when a negative shock hits the economy. For a positive shock, the policy function is almost linear since the collateral constraint is less likely to bind.

To understand the quantitative difference between the two economies (panel (a) and panel (b)), consider the cyan line (T0,T1) and (O0,O1). Suppose that both economies start at the same bond, which is equal to -0.2. In a limited financial market participation economy, for a positive shock, households will choose T0, otherwise they will choose T1. Households then expect to reduce the debt level by 0.02 from a positive shock to a negative shock. In the same way, in a full

(a) Limited financial market participation



(b) Full financial market participation

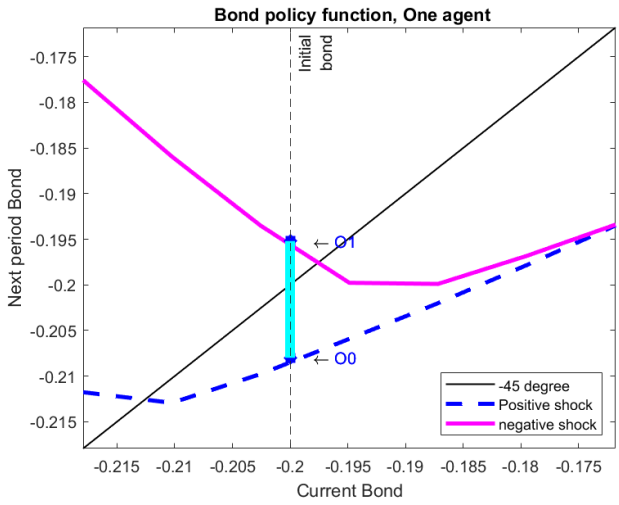


Figure 4: Policy function for private debt

financial market participation economy, households expect to reduce the debt by 0.015 from a positive shock to a negative shock. The limited financial market participation economy will generate a higher amplification because households are expected to reduce their debt more when the collateral constraint binds.

Financial crises. I now analyze the ability of the model to generate financial crises and the role of limited financial market participation in generating a higher drop in the asset price. For that purpose, I simulated the model for 100,000 periods and constructed a nine-year event windows centered at the crisis year. A financial crisis is defined as when the collateral constraint binds and the trade balance is two standard deviations above its mean. Using the current account instead of the trade balance gives the same result.

Figure 5 shows the average of output, consumption of asset holders, consumption of hand-to-mouth consumers, private debt, asset prices, and exogenous shock across the nine-year event windows for the two economies. I normalize the average of the variables to 1 at t-1. For panels (a) to (e), the solid blue line represents the limited financial market participation economy where the share of hand-to-mouth consumers is set to 50%. The dashed magenta line represents the economy with full financial market participation; that is, the share of hand-to-mouth consumers is set to

0. Panel (f) shows the percentage differences relative to the unconditional averages of aggregate exogenous shocks that hit the economy.

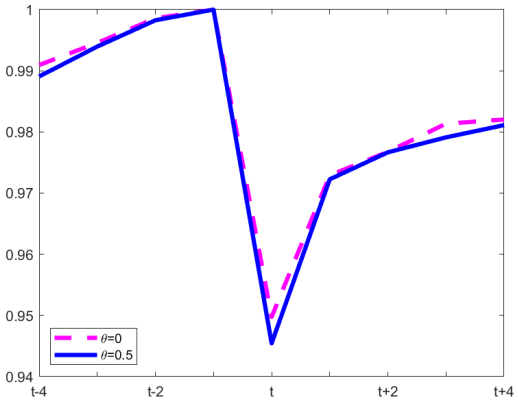
I follow Bianchi and Mendoza (2018) to construct comparable event windows for the two economies. First, I simulate the limited financial market economy for 100,000 periods and identify financial crises using the definition described above. Second, I construct nine-year event windows centered at the crisis year, denoted date t, by computing averages for each variable across the cross section of crisis events at each date. The result of this procedure is the solid blue line in Figure 5. Third, I take the initial bond position at t-4 of the limited financial market participation economy crisis and the sequences of aggregate exogenous shocks in the 9-year window in this economy, and I pass them through the policy functions of the full financial market participation economy. Finally, I compute the average, as in the previous case. The result of this procedure is the dashed magenta line in Figure 5 .

Panels (a) , (b), and (e) show that output, consumption of the asset holders, asset prices fall more in the economy with limited financial market participation relative to the economy with full financial market participation. In the full financial market participation economy, output falls short by 0.5 percentage points (5% vs. 5.5%), consumption of asset holders falls short by 2 percentage points (9% vs. 11%), and asset prices fall short by 3 percentage points (13.5% vs. 16.75%). The fall in the asset price is then 23% higher in a limited financial market participation economy than in a full financial market participation economy.

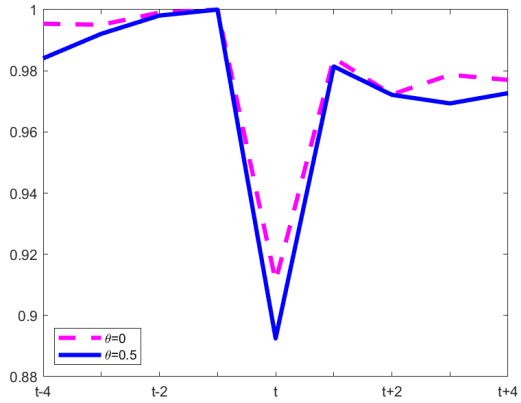
Panel (f) shows that prior to the crisis, the real interest rate is below the average real interest rate by almost 200 basis points, which corresponds to one standard deviation. At date t of the crisis, the real interest rate rises sharply to almost 200 basis points above its average. It then decreases slowly to converge to the average value four years after the crisis. Contrary to the real interest rate, TFP rises prior to the crisis and decreases to one standard deviation below its average. The input import price shows small fluctuations along the financial crisis dynamics.

To summarize, the financial crises show a higher amplification effect in a limited financial market participation economy relative to a full financial market participation economy. The asset prices fall more and the asset holders face higher burdens. I next present why the asset price falls more

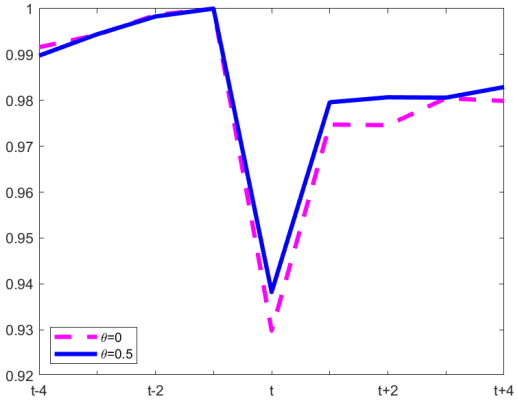
(a) Output



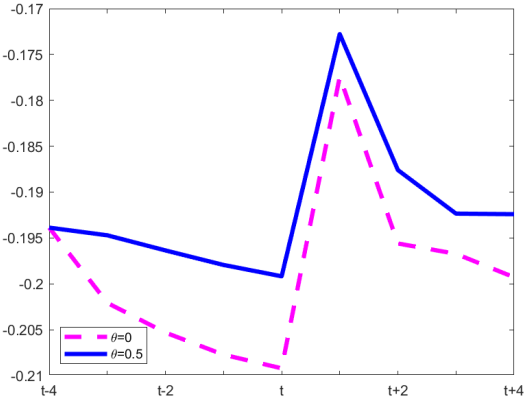
(b) Consumption of Asset holders



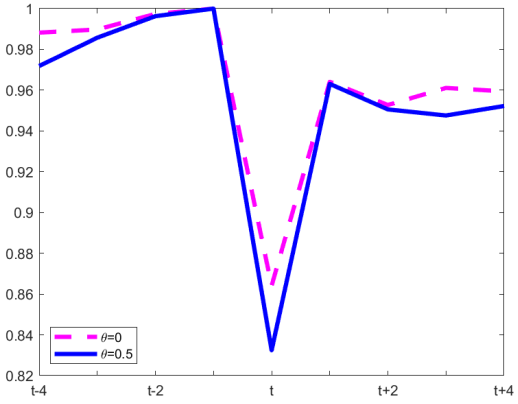
(c) Consumption of Hand-to-Mouth Consumers



(d) Private Debt



(e) Asset prices



(f) Exogenous shocks

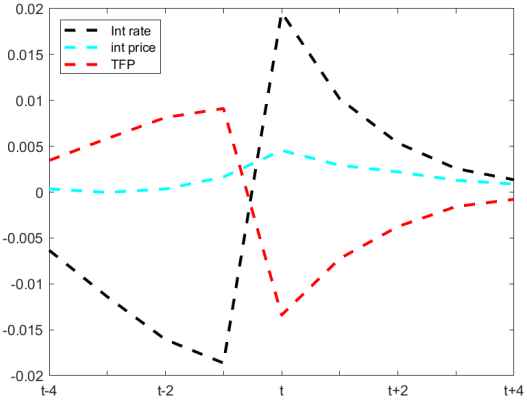


Figure 5: Financial crisis dynamics

when every household is not participating in the financial market.

Asset prices and inequality. In Section 3.4, I show that, given the share of hand-to-mouth consumers, the cyclicality of consumption inequality is important to determine the relative amplification effect in a limited financial market participation economy. As shown by Figure 6, a higher drop in the asset price in a limited financial market participation economy is followed by a drop in consumption inequality during the crisis year. Second, the overall pattern of the asset prices is qualitatively similar to the consumption inequality dynamics. A drop in consumption inequality is associated with a higher drop in the asset price in a limited financial market participation economy because of the burden on asset holders. Indeed, when asset holders face a higher burden following an aggregate shock, they will be willing to sell more of their assets to meet their obligations. By doing so, they increase the supply of the asset, leading to a decrease in its prices.

It is worth noting that, in the model, I did not calibrate the drop in the consumption inequality. Instead, I use a constant transfer (a constant lump-sum tax on asset holders) to hand-to-mouth consumers to calibrate the average consumption inequality of 1.27 in the economy. Consistent with Mexico's data, the model endogenously generates the drop in consumption inequality during the financial crisis. The model is able to generate a drop in consumption inequality during the financial

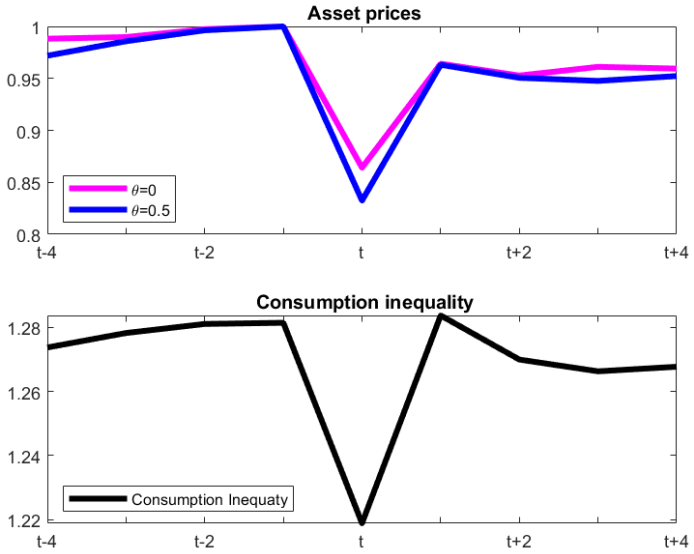


Figure 6: Asset prices and inequality

Table 3: Business cycle moments

	Standard deviation			Correlation with output		
	$\theta = 0$	$\theta = 0.5$	data	$\theta = 0$	$\theta = 0.5$	data
GDP	2.68	2.73	2.72	1.00	1.00	1.00
Consumption	3.55	3.51	3.39	0.94	0.95	0.89
Trade balance/GDP	1.33	1.26	2.1	-0.51	-0.49	-0.68
Asset prices	5.63	6.68	14.64	0.89	0.89	0.57
Interest rate	1.95	1.95	1.95	-0.64	-0.65	-0.59

crisis because the labor supply, which determines the labor income of hand-to-mouth consumers, has shown a small decline. Mendoza (2010) suggests that the labor decline is not the main cause of the decline in GDP during Mexico's 1995 sudden stop crisis.

Long-run moments. Table 3 presents the business cycle moments. The column with $\theta = 0$ represents the economy with full financial market participation. The column with $\theta = 0.5$ represents the economy with limited financial market participation where the share of hand-to-mouth consumers is 50% of the population. The statistics in the data column come from Mendoza (2010). In the table, only the standard deviation of the real interest rate has been used to calibrate the exogenous process of the real interest rate. The results show that the models do a good job of matching the standard deviation and the correlation of key aggregate variables such as output and consumption.

The emerging market is characterized by three main business cycles. First, consumption is more volatile than output. The models with full and limited financial market participation replicate well the standard deviation of output and consumption quantitatively. Second, the trade balance is countercyclical. In fact, the correlation between output and the trade balance ratio to GDP is negative. Third, the real interest rate is countercyclical. In the limited financial market participation economy, the correlation between the real interest rate and output is -0.65, which is comparable to what is observed in the data: -0.59.

The models underestimate the volatility of asset prices. While the data suggest that the volatility of asset prices in Mexico is six times the volatility of output (in the data, the volatility of asset prices and output is 14.64 and 2.72, respectively), the estimates for both economies are only about

two times the volatility. By introducing a financial shock in the model where the parameter κ is not constant over time, one can significantly increase the volatility of asset prices. The results also suggest that the economy with limited financial market participation displays more volatility in the asset price relative to the economy with full financial market participation.

Even though the model falls short in displaying the volatility of asset prices relative to the volatility of output, as is observed in the data, it is, however, able to show that asset prices are more volatile than output. In contrast, Mendoza (2010) has found that asset prices are less volatile than output because in his framework, expectation does not play a direct role in the determination of asset prices. Indeed, with an investment and capital adjustment cost in his framework, the asset price is equal to one plus the marginal adjustment cost. So, without a change in the stock of capital, the asset price does not change. Mendoza (2010) has then excluded any direct role for expectation where a change in the expectation of the supply of assets could substantially affect the asset price even though at the equilibrium, the stock of the asset is constant.

5 Financial crisis with a financial shock

The framework presented in Section 3 does not have a financial shock. In this section, I introduce a financial shock, as in Bianchi and Mendoza (2018) to analyze the optimal policy. The parameter κ that represents the fraction of the total value of physical assets the households can pledge as collateral is not constant anymore. But it takes two values: a high value κ_h regime and a low value κ_l regime (time of crisis) with a switching probability between both regimes. This is consistent with the data, which suggest that the loan-to-value ratio decreases during a financial crisis. According to the loan-to-value ratio in Mexico in the 1990s, I set $\kappa_h = 0.7$ and $\kappa_l = 0.55$. The probability of staying in the low regime is set to zero to reflect the fact that the average duration of a sudden stop is one year. I then use the probability of staying in the high regime to calibrate the frequency of the financial crisis. In addition to the financial shock, a small change is made to make the model comparable to Bianchi and Mendoza (2018), who analyze the optimal time-consistent problem with a representative agent. I assume there is no shock on the imported input price. The beginning-of-period asset K_t is used as collateral instead of the end-of-period asset K_{t+1}, and there is no labor

in the working capital loan. See in Appendix B.1 for the full model.

5.1 Optimal prudential capital control and financial crisis

In this section, I analyze the optimal time-consistent policy presented in Appendix B.2. In the optimal policy, I choose to use a tax on foreign debt — a capital control — to decentralize the planner's solution. The taxes collected are redistributed in the form of lump-sum transfers to asset households. This section answers two main questions. First, how effective is an optimal tax on debt in reducing the severity and frequency of a financial crisis in a limited financial market participation economy? Second, can we rationalize the prevalent use of a capital control in emerging market characterized by a low level of financial markets participation?

The optimal time-consistent solution suggests two main lessons. First, the optimal time- consistent solution effectively reduces the frequency and severity of the financial crisis in both the full and limited financial market participation economies. Second, the average tax on foreign debt needed to decentralize the optimal time-consistent solution is higher in a limited financial market participation economy (1.2% vs. 6%). This suggests that more capital control is needed in emerging markets, which have a low level of financial market participation relative to advanced economies. This second lesson rationalizes the use of capital control in the world. In fact, data on capital control suggest that emerging markets control more capital flows than advanced economies.

Figure 7 shows the average of output, consumption of asset holders, consumption of hand-to-mouth consumers, bonds, asset prices, and the exogenous shock across the nine-year event windows for the competitive equilibrium with a limited financial market participation economy and the optimal policy. I normalize the average of the variables to 1 at t-1. In the first five panels, the solid blue line represents the limited financial market participation economy where the share of hand-to-mouth consumers is set to 50%. The dashed magenta line represents the optimal solution economy with limited financial market participation with the same share of hand-to-mouth consumers. The last panel shows the percentage differences relative to the unconditional averages of aggregate exogenous shocks that hit the economy.

The top three panels shows that the decline in output, asset holder consumption, and hand-to-mouth consumption is substantially higher in the competitive equilibrium relative to the social planner's solution during the financial crisis. Indeed, output drops from 4.5% to 2.5%, asset holder consumption drops from 24% vs. 2.3%, and hand-to-mouth consumption drops from 6% to 3.3%. Therefore, the planner's solution is effective in reducing the severity of the financial crisis.

The bottom left panel shows the debt dynamics around the financial crisis. At t-4, both the competitive equilibrium (CE) and the social planner (SP) start with the same stock of debt. But at t-3, while the SP reduces debt by 4 percentage points, the CE builds up the debt. This trend continues until t-1, where the difference in the debt between the CE and SP is more than 3 percentage points. Therefore, the debt dynamics suggest that the household overborrows in the competitive equilibrium. Because of this overborrowing, the CE experiences a larger adjustment in the debt when a financial crisis hits the economy.

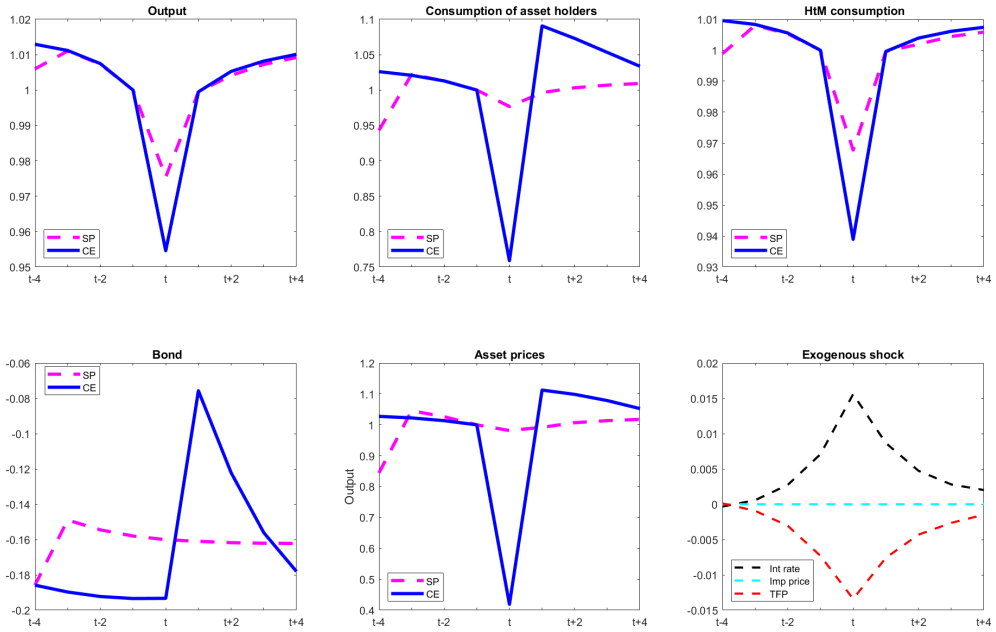


Figure 7: Financial crisis with the optimal policy. SP stands for the social planner and CE for the competitive equilibrium.

The bottom middle panel shows the asset prices dynamics around the financial crisis. The CE

experiences a larger decline in the asset price relative to the SP. The decline in the asset price is 59% in the CE, and only 2% in the SP. The SP effectively reduces the decline in the asset price substantially because it does not reduce the debt too much when the financial crisis hits the economy. By taking into account the pecuniary externality, the SP did not experience a sharp decline in the asset price. The sharp decline observed in the asset price for the CE is because asset holders substantially deleverage when a financial crisis hits the economy and therefore are more willing to sell their physical assets to meet their obligations. The excess supply of assets leads to a sharp decline in the asset price.

At t-4, the real interest rate and TFP are at their average. During the financial crisis, the real interest rate increases by 150 basis points, which reflects the scarcity of the availability of the foreign asset. The real interest rate is slightly below its average level four years after the crisis. TFP decreases by one standard deviation during the financial crisis, and as with the real interest rate, it is slightly below its average level four years after the crisis.

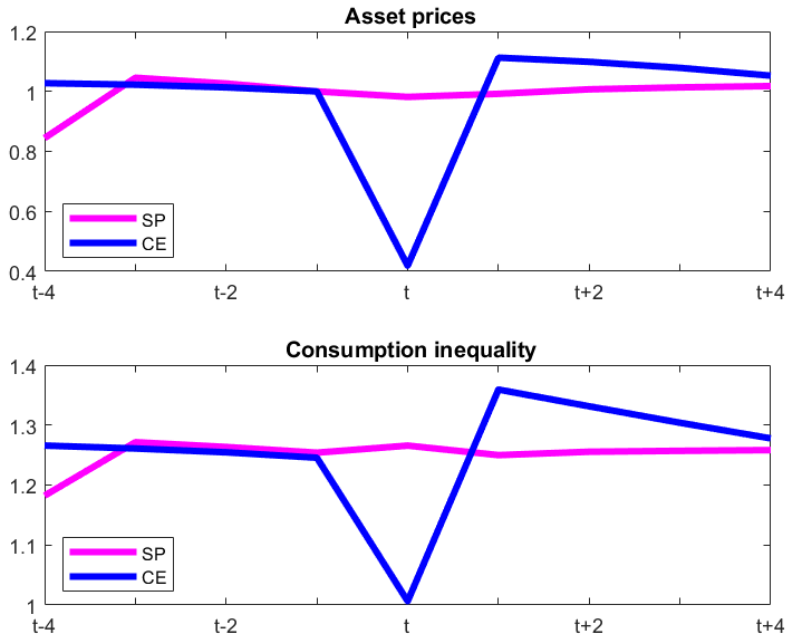


Figure 8: Inequality and asset prices. SP stands for the social planner and CE for the competitive equilibrium

5.2 Optimal prudential capital control and inequality

Section 5.1 shows that prudential capital control is very effective at reducing the severity of a financial crisis. This section answers the following question: Does the inequality increase when using a prudential capital control (a tax on foreign debt)? The optimal time-consistent solution suggests that it is possible to address financial stability without raising (consumption) inequality (see Figure 8). While in the very short run (at the time of the financial crisis), the social planner may accept a slight increase in consumption inequality, it appears that long-run (average) consumption inequality is lower in the optimal time-consistent equilibrium relative to the competitive economy.

6 Conclusion

This paper presents a financial crisis model in a limited financial market participation economy with a collateral constraint. Participation in the financial market is limited because a fixed share of households do not hold any liquid wealth. That is, they do not participate in either the bond market or the stock market. When negative aggregate shocks hit the economy, the collateral constraint binds, and households are forced to deleverage. The extent to which limited financial market participation amplifies or dampens the economy's response to the aggregate shocks depends on the cyclicality of the consumption inequality. In the model, the consumption inequality is endogenous.

Consistent with the empirical evidence I document using Mexico's household survey data, the model generates a decline in consumption inequality during the financial crisis. This decline in consumption inequality amplifies the economy's response to the aggregate shocks during the financial crisis. Moreover, the optimal time-consistent solution shows that the average tax on foreign assets is higher in a limited financial market participation economy than in a full financial market participation economy. This finding rationalizes the prevalent use of capital control in emerging markets.

References

Acharya, Sushant and Keshav Dogra, "Understanding HANK: Insights from a PRANK," *Econometrica*, 2020, *88* (3), 1113–1158.

Arce, Fernando, Julien Bengui, and Javier Bianchi, "A macroprudential theory of foreign reserve accumulation," Technical Report, National Bureau of Economic Research 2019.

Arellano, Cristina and Enrique G Mendoza, "Credit frictions and'sudden stops' in small open economies: An equilibrium business cycle framework for emerging markets crises," 2002.

Bengui, Julien, "Macro-prudential policy coordination," *Unpublished Manuscript, University of Montreal*). *https://www. julienbengui. com/uploads/5/4/9/2/54926627/globalmp. pdf*, 2014.

__ **and Javier Bianchi**, "Macroprudential policy with leakages," Technical Report, National Bureau of Economic Research 2018.

Bianchi, Javier, "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

__ **and Enrique G Mendoza**, "Optimal time-consistent macroprudential policy," *Journal of Political Economy*, 2018, *126* (2), 588–634.

Caballero, Ricardo J and Arvind Krishnamurthy, "Smoothing sudden stops," *Journal of Economic Theory*, 2004, *119* (1), 104–127.

Calvo, Guillermo A, Alejandro Izquierdo, and Ernesto Talvi, "Sudden stops and phoenix miracles in emerging markets," *American Economic Review*, 2006, *96* (2), 405–410.

Chari, Varadarajan V, Patrick J Kehoe, and Ellen R McGrattan, "Sudden stops and output drops," *American Economic Review*, 2005, *95* (2), 381–387.

Fisher, Irving, "The debt-deflation theory of great depressions," *Econometrica: Journal of the Econometric Society*, 1933, pp. 337–357.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, 1988, pp. 402–417.

Korinek, Anton and Enrique G Mendoza, "From sudden stops to fisherian deflation: Quantitative theory and policy," 2014.

Mendoza, Enrique G, "Lessons from the debt-deflation theory of sudden stops," *American Economic Review*, 2006, *96* (2), 411–416.

_ , "Sudden stops, financial crises, and leverage," *American Economic Review*, 2010, *100* (5), 1941–66.

Werning, Iván, "Incomplete markets and aggregate demand," Technical Report, National Bureau of Economic Research 2015.

A Separate problem

A.1 Problem of households who are not hand-to-mouth consumers

$$\begin{aligned}
\max_{C_{1t},b_{1t+1},s_{t+1},N_{1t}} \quad & \mathbb{E} \quad \sum_{t=0}^{\infty} \beta^t U(C_{1t} - G(N_{1t})) \\
C_{1t} + \frac{b_{1t+1}}{R_t} + q_t s_{t+1} \quad & = \quad w_t N_{1t} + b_{1t} + (d_t + q_t) s_t \\
\frac{b_{1t+1}}{R_t} \quad & \geq \quad -\kappa q_t s_{t+1}
\end{aligned}$$

$$EE1: \quad \overbrace{U'(t)}^{\text{marginal benefit of borr.}} = \overbrace{\beta R_t E_t U'(t+1)}^{\text{marginal cost of borr.}} + \overbrace{\mu_t^{nh}}^{\text{shadow price of relaxing the constr.}}$$

$$EE2: \quad \underbrace{q_t U'(t)}_{\text{marginal cost of buy.}} = \underbrace{\beta E_t \left[(d_{t+1} + q_{t+1}) U'(t+1) \right]}_{\text{marginal benef. of buy.}} + \underbrace{k q_t \mu_t^{nh}}_{\text{gain of relax. the constr.}}$$

$$Lab: \quad \underbrace{G'(N_{1t})}_{\text{marginal disutility of labor}} = \underbrace{w_t}_{\text{real wage}}$$

A.2 Firm's problem

$$\begin{aligned}
\max_{d_t,k_{t+1}^f,b_{t+1}^f,v_t,L_t} \quad & \mathbb{E} \quad \sum_{t=0}^{\infty} \beta^t U'(C_{1t} - G(N_{1t})) d_t \\
d_t + \frac{b_{t+1}^f}{R_t} + i_t \quad & = \quad F\left(k_t^f, L_t, v_t\right) - (1 + \theta r_t)\left(w_t L_t + p_t^v v_t\right) + b_t^f \\
i_t \quad & = \quad k_{t+1}^f - k_t^f + \delta k_t^f + \left(k_{t+1}^f - k_t^f\right) \psi\left(\frac{k_{t+1}^f - k_t^f}{k_t^f}\right) \\
\frac{b_{t+1}^f}{R_t} - \theta R_t \left(w_t L_t + p_t^v v_t\right) \quad & \geq \quad -\kappa^f q_t k_{t+1}^f
\end{aligned}$$

Optimality conditions for firm

$$
\begin{aligned}
\left[b_{t+1}^{f}\right] &:: U'(t) = R_t E_t\left[U'(t+1)\right] + U'(t)\mu_t^{f} \\
\left[k_{t+1}^{f}\right] &:: U'(t)\frac{\partial i_t}{\partial k_{t+1}} = E_t\left[U'(t+1)\left\{F_k\left(k_t^{f}, L_t, v_t\right) - \frac{\partial i_{t+1}}{\partial k_{t+1}}\right\}\right] \\
+k^f q_t U'(t)\mu_t^{f} & \\
[L_t] &:: F_l\left(k_t^{f}, L_t, v_t\right) = \left(1 + \phi r_t + \phi R_t \mu_t^{f}\right) w_t \\
[v_t] &:: F_v\left(k_t^{f}, L_t, v_t\right) = \left(1 + \phi r_t + \phi R_t \mu_t^{f}\right) p_t^{v} \\
KT &:: \mu_t^{f}\left(\frac{b_{t+1}^{f}}{R_t} - \phi R_t\left(w_t L_t + p_t v_t\right) + k^f q_t k_{t+1}^{f}\right) \\
\mu_t^{f} &\geq 0
\end{aligned}
$$

A.3 Market equilibrium for the separate problem

Labor market: $L_t = (1-\lambda)\, N_{1t} + N_{2t}$

Stock market : $s_t = \frac{1}{1-\lambda}$

Good market : $C_t = (1-\lambda)\, C_{1t} + \lambda C_{2t}$ Bond market:$b_{t+1} = (1-\lambda)\, b_{1t+1} + b_{t+1}^{f}$

Aggregate capital: $k_{t+1} =, k_{t+1}^{f}$

Definition : A competitive equilibrium is a set of allocations

$Q_t = \left\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, s_{t+1}, b_{1t+1}, b_{t+1}^{f}, k_{t+1}^{f}, d_t\right\}$ and prices $P_t = \{p_t, w_t, R_t, q_t\}$ such that:

1. Given P_t, Q_t solves households' and firms' problem ;
2. w_t and q_t are determined competitively G'(L_t) $= w_t$; $\frac{\partial i_t}{\partial k_{t+1}} = q_t$
3. Markets are clear.

A.4 Equivalence result: Separate versus Non-separate firm household problem

If the allocation $\left\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, s_{t+1}, b_{1t+1}, b_{t+1}^{f}, k_{t+1}^{f}, d_t, w_t, q_t, \mu_t^{nh}\right\}$ is a competitive equilibrium in the economy with separate asset holder consumers and firm problems, then $\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, b_{t+}$ is a competitive equilibrium in the economy with non-separate firm and asset holders consumer problem with $b_{t+1} = (1-\lambda)\, b_{1t+1} + b_{t+1}^{f}$ and $k_{t+1} = k_{t+1}^{f}$. (The converse is also true.)

The proof follows Bianchi and Mendoza (2018), who show it in the representative agent eco-

nomy. The equivalence result still holds because there is no heterogeneity among firm owners.

B Model with financial shock

B.1 Firm-Asset holder households' optimization problem

There is a continuum of identical *asset holder* households of measure $1-\theta \in (0,1]$. The preferences of an *asset holder* consumer indexed by 1 are given by

$$\begin{aligned}
\max_{C_{1t},b_{t+1},k_{t+1},v_t,L_{1t}} \quad & \mathbb{E}_0 \quad \sum_{t=0}^{\infty} \beta^t u(C_{1t} - G(L_{1t})) \\
(1-\theta)\, C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} \quad &= \quad F\left(k_t, L_t, v_t\right) - p_t^v v_t - \theta w_t L_{2t} + b_t + q_t k_t - T_t \\
\frac{b_{t+1}}{R_t} - \phi\left(p_t^v v_t\right) \quad &\geq \quad -\kappa_t q_t k_t.
\end{aligned}$$

The optimal solution gives

$$\begin{aligned}
u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + \mu_t, \\
q_t u'(t) &= \beta \mathbb{E}_t \left[\left(d_{t+1} + q_{t+1}\right) u'(t+1) + \kappa_{t+1} q_{t+1} \mu_{t+1}\right], \\
A_t F_l\left(k_t, L_t, v_t\right) &= G'(L_{1t}) \\
A_t F_v\left(k_t, L_t, v_t\right) &= p_t^v + \phi\left(\frac{\mu_t}{u'(t)}\right) p_t^v.
\end{aligned}$$

B.2 Time-consistent Planner's Problem

$$
\begin{aligned}
V(b,s) &= \max_{c_1,c_2,b',L,v,q} \left\{ \theta u(c_2 - G(L) + (1-\theta)u(c_1 - G(L)) + \beta \mathbb{E}_{s',s} V(b',s') \right\} \\
(1-\theta)\, c_1 + \frac{b'}{R} &= F\left(1, L, v\right) - p^v v - \theta w L + b - T. \\
\theta c_2 &= \theta w L + T \\
-\phi\left(p^v v\right) + \frac{b'}{R} &\geq -\kappa q \\
AF_v\left(1, L, v\right) &= p^v + \phi\left(\frac{\mu}{u'(c_1 - G(L))}\right) p^v \\
AF_l\left(1, L, v\right) &= G'(L) \\
w &= G'(L) \\
q u'(c_1 - G(L)) &= \beta \mathbb{E}\left[\left(\mathbb{D}(b',s') + \mathbb{Q}(b',s')\right) u'(\mathbb{C}(b',s')) + \kappa' \mathbb{Q}(b',s') \mu(b',s')\right]
\end{aligned}
$$