Sudden stops and asset prices: the role of financial market participation [1]

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Abstract

This paper studies the role of household heterogeneity for the severity of sudden stop crisis and its implication for prudential capital control policy. I use data on sudden stop events and on financial market participation to document that a lower financial market participation is associated with a higher drop in asset prices. To explain the role of financial market participation in the drop in asset prices, I build an equilibrium business cycle model with a collateral constraint, and with limited financial market participation. The heterogeneity in access to the financial market generates income and consumption inequality in the model. The extent to which the limited financial market participation amplifies the drop in the asset price depends on the cyclicality of consumption inequality. Consistent with my empirical findings using a household survey data from Mexico, the model generates a drop in consumption inequality during the financial crisis which amplifies the drop in asset prices, output, and consumption. I show that the optimal time-consistent debt tax should be higher in a limited financial market participation economy, which rationalizes the use of capital control in emerging markets. Finally, my findings suggest it is possible to address financial instability without raising inequality.

Keywords: Financial crisis, asset prices, inequality, optimal policy, capital controls.

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1 Introduction

Output and consumption are more volatile in emerging markets than in advanced economies. Since the 1980s, emerging markets have on average faced a higher drop in the asset price and output than advanced economies during a sudden stop crisis. A sudden stop is an economic crisis that features current account reversal (i.e., massive capital outflows). Several studies have been done to understand sudden stops and how macroprudential policy or capital control can reduce its severity, but accounting for household heterogeneity in the financial market remains less investigated. This paper studies the role of limited financial market participation in the severity of sudden stops crises and its implication for prudential capital control. I show that the effect of household heterogeneity in access to the financial market depends crucially on the cyclicality of consumption inequality.

My paper is motivated by three main empirical facts that I document in section 2. First, across countries, there is heterogeneity in financial market participation. In emerging markets, a low proportion of households has access to financial markets. Those households consume their available labor income and do not hold any assets. Second, there is a negative correlation between the level of financial market participation and the drop in the asset price during sudden stops. Countries with low financial market participation faced a higher drop in the asset price during sudden stops. Third, the consumption inequality is procyclical during Mexico's 1995 sudden stops crisis. Indeed, during Mexico's 1995 crisis, households who participated in the financial market faced a higher drop in their consumption compare to those who did not participate in the financial market.

To quantity the importance of inequality on the severity of sudden stops, I enrich a standard dynamic stochastic general equilibrium model that features an occasionally binding collateral constraint with limited household heterogeneity. The model features two types of households. The first type comprises households who participate in the financial market and have access to the capital and bond market. This type of household is called *asset holders*. The second type of household comprises those who participate neither in the capital nor the bond markets. This type of household, called *"hand-to-mouth"* consumers, consumes all their labor income plus any additional transfer. The small open economy faces shocks to its productivity, the real interest rate and the price of

imported inputs. The model economy nests the model in Mendoza (2010) with a fixed supply of capital.

The collateral constraint limits the total private debt to a fraction of the market value of capital. The total private debt is composed of private debt with one-year maturity plus a within-period working capital loan. The within-period working capital loan generates a contemporaneous drop in output when the collateral constraint binds,. When successive negative aggregate shocks hit the economy, the collateral constraint binds. If the constraint binds, the economy faces a financial crisis called a *sudden stop*. Two mains credit channels are in place. The first is the endogenous financing premium on debt, equity, and working capital as borrowing costs rise when the collateral constraint binds. The second is the Fisher (1933) debt-deflation mechanism. When the collateral constraint binds, asset holders fire-sale assets to smooth their consumption and meet their obligations. The fire-sale of assets leads to a decline in capital price, which further tightens the collateral constraint. This non-linear feedback between the price of capital and the collateral value (borrowing capacity) exacerbates a financial crisis.

Qualitatively, the credit channel I describe is the same in a model with or without household heterogeneity. Quantitatively, I show that the severity[3] and the drop in the asset price depend on the cyclicality of consumption inequality. If the consumption inequality is constant, then the level of financial market participation does not affect the severity and the drop in the asset price during the sudden stops. This shows that the pro(cyclicality), not the level of inequality, exacerbates the severity of a sudden stop crisis. If the consumption inequality decreases during the crisis, the asset price drop more, and vice-versa. Since consumption inequality decreased during Mexico's 1995 sudden stops episode, limited financial cmarket participation exacerbated the severity of the crisis.

In the model, macroeconomic policy plays a role because of two externalities: a pecuniary externality and an aggregate demand externality. First, asset holders take the asset price as given. Their choice of debt affects, in the aggregate, the asset price, which determine the value of the collateral. This general equilibrium effect is called a pecuniary externality. The social planner who

[3] A crisis is more (less) severe when the drop in output and consumption is high (low).

takes this pecuniary externality into account may substantially reduce the severity of a financial crisis due to a binding collateral constraint). Second, there is an aggregate demand externality. This externality arises because the asset holders take as given the choice of labor supply of hand-to-mouth consumers. Note that this aggregate demand externality is only present in the case of limited financial market participation. In the optimal policy, I choose to use a tax on foreign debt-a capital control- to decentralize the planner solution.

The quantitative results show that limited financial market participation amplifies the drop in output and asset holders' consumption. The fall in the asset price is 23 % higher in a limited financial market participation economy compared to a full financial market participation economy. The model is consistent with three main business cycles facts in emerging markets. First, consumption is more volatile than output. Second, the trade balance is counter-cyclical. Third, the real interest rate is counter-cyclical.

To study the optimal policy, I introduce a financial shock. A financial shock-a drop to a loan-to-value ratio- consists of a drop during a financial crisis in the fraction of the total value of physical asset that the households can pledge as collateral. The introduction of a financial shock to study the optimal policy is consistent with data. Loan to value ratio is consistently low during a sudden stops crisis. With a financial shock, the asset price drops by 57 % and the asset holders consumption drops by 25% in a limited financial market participation economy, whereas the asset price drops by 40 % and the asset holders consumption drop by 15 % in a full financial market participation economy participation. The fall in the asset price is now 42 % higher in a limited financial market participation economy compared to a full financial market participation economy. As expected, the results suggest that the financial shock exacerbates the financial crisis.

The optimal time-consistent constrained efficient allocation suggests three main points. First, the optimal time-consistent solution effectively reduces the frequency and severity of the financial crisis in both the full and the limited financial market participation economies. Second, the average tax on foreign debt needed to decentralize the optimal time-consistent solution is higher in a limited financial market participation economy. This suggests that more capital control is needed in emerging markets which have a low level of financial market participation than advanced econom-

ies. This second lesson rationalizes the prevalent use of capital control in emerging markets. In fact, data on capital controls suggest that emerging markets control more capital flows than advanced economies. Third, the optimal time-consistent solution suggests that there is no trade-off between financial stability and consumption inequality in the case of limited financial market participation. While in the very short run (at the time of the financial crisis), the social planner may tolerate a slight increase in consumption inequality, the average consumption inequality is not higher in the optimal time-consistent equilibrium than the competitive economy.

My paper mainly relates to the literature which studies the aggregate effects of a sudden stop (see for example Arellano and Mendoza (2002), Chari et al. (2005) Mendoza (2006), Calvo et al. (2006) Mendoza (2010), Korinek and Mendoza (2014)). My paper is closely related to Mendoza (2010) who studied how an endogenous binding collateral can trigger the economy within a standard business cycle moments. My contribution to this literature is two folds. First, I introduce limited financial market participation where a fixed share of households do not participate to financial market. This economy characterization is more closer to emerging markets and helps us to explain the observed gap in the decline in the asset price during sudden stops between emerging markets and advanced economy. In addition, my work studies the optimal time-consistent solution and rationalizes the prevalent use of capital control in emerging markets.

My work is also related to recent literature which studies the optimal policy in a financial crisis model. These papers include Caballero and Krishnamurthy (2004), Bianchi (2011), Bengui (2014), Bengui and Bianchi (2018), Bianchi and Mendoza (2018), Arce et al. (2019). I contribute to this literature by taking into account household heterogeneity in the financial market and show that it is possible to address financial instability without raising inequality.

The rest of the paper is organized as follows. In Section 2, I present the data and the empirical facts. Section 3 presents the model. Section 4 and 5 present my findings and discuss the results and Section 6 concludes.

2 Data and empirical facts related to asset prices and inequality

In this section, I present the data and the empirical facts related to financial market participation, sudden stops, and inequality. I use three sources of macro and micro data. The first is a panel data on financial market participation which covers low-income countries, emerging markets, and advanced economies. The second is aggregate data on the drop in asset prices during the sudden stops. The third is a micro survey data on household consumption, income, and wealth in Mexico.

Financial market participation. I use the IMF's Financial Development Index Database. The index database provides nine index for 180 countries every year since 1980. I focus on two indexes that measures the ability of individuals and firms to access the financial services which are called the Financial Institution Access index (FIA) and the Financial Market Access index (FMA). FIA measures the number of bank branches per 100.000 adults and the number of ATMs per 100.000 adults. FMA measures the percent of market capitalization outside of the top 10 largest companies and the total numbers of issuers of debt (domestic and external, non financial corporations) per 100.000 adults. All indexes are between 0 and 1 where 1 means full access to financial services.

Sudden stops. I use the sudden stops data constructed by Korinek and Mendoza (2014). A sudden stops is defined as a large capital outflows as measured by a year-over-year increase in the current account/GDP ratio by more than two standard deviations above the average change in this ratio. I use the stock market index provided in the data as a measure of asset prices. The data includes emerging markets and advanced economies over the period 1980-2012.

Consumption, Income, and Wealth. I use Mexico's National Household Consumption and Income (ENIGH) survey. It is a representative households survey which covers rural and urban areas. The survey is conducted every two years since 1992. More than 10 000 households are interviewed at each survey. The survey has detailed information about household consumption items, their income and wealth. I define "hand-to-mouth" consumers as household who holds zero liquid wealth. I define consumption inequality as the ratio of *asset holders* consumption to the consumption of "hand-to-mouth".

I document three facts:

Fact 1: There is heterogeneity in the financial market participation across countries. Also, on average,

emerging markets have lower financial market participation than advanced economies. Figure 1 displays the median of the financial market access index from 1980 to 2017 for each country. Figure 2 displays the median of the financial institution access index from 1980 to 2017 for each country. Darker red means higher financial market participation. North-American countries, western European countries, Japan, Australia, and a few other countries have relatively high access to financial services.

Fact 2: There is a negative correlation between the level of financial market participation and the drop in the asset prices during a sudden stop episode. Table 1 presents the results from a panel regression of asset prices on a dummy variable, which takes value one if a given country in a given year is in a sudden stop crisis, the financial market access, and the cross-product between the dummy and the financial market access. The cross product captures the marginal effect of the level of the financial market participation on the drop in asset prices during sudden stops. The first column considers advanced economies and emerging markets. The second column considers only the advanced economies, and the third column considers only emerging markets. All regressions include country and year fixed effect. I control for capital flows. The estimated coefficient on the cross-product is positive, which means that a country with a higher level of financial market participation has a lower drop in the asset price during a sudden stop episode.

Fact 3: The consumption inequality is procyclical during Mexico's 1995 sudden stops crisis. Figure 3 plots the consumption inequality dynamics in Mexico from 1992 to 2000. Consumption inequality is defined as the ratio of *asset holders* consumption to the consumption of "hand-to-mouth"households. Before the sudden stops crises in 1995, the consumption inequality increases then decreases during the crisis from 1994 to 1996. It starts to increase from 1996. This is an evidence that the sudden stop crisis hits relatively hard the asset holders, which are *asset holders* consumers, than the "hand-to-mouth" consumers.

I use the three facts to discipline my model. In my model, the financial crisis and the consumption inequality are endogenous, whereas the level of financial market participation is exogenous. I present the model in the next section.

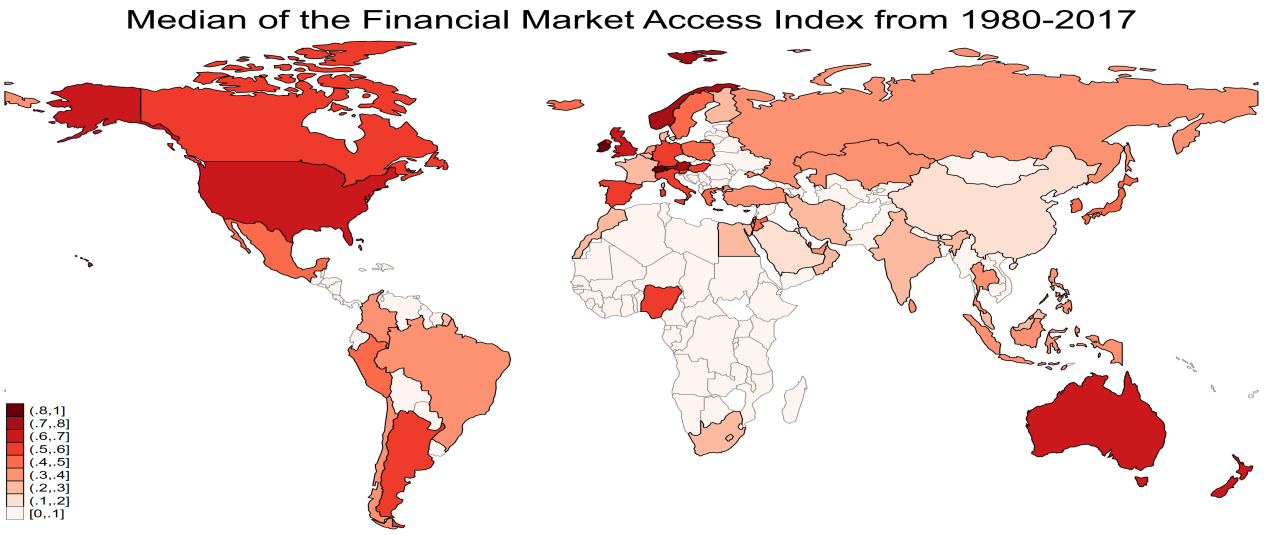


Figure 1: Financial market accessibility across countries

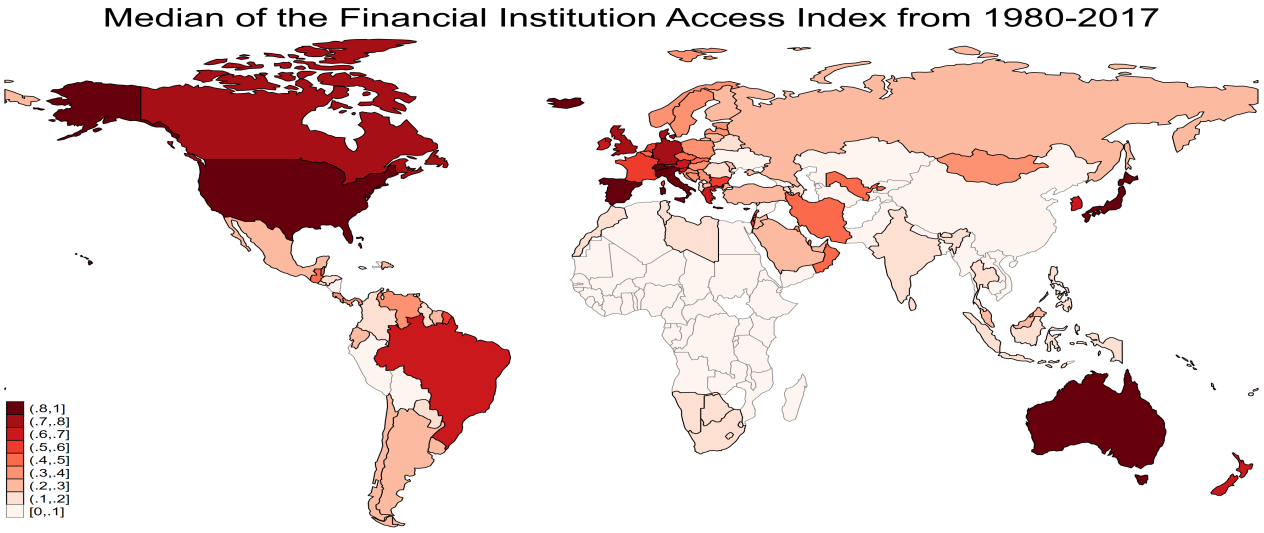


Figure 2: Financial institution accessibility across countries

Table 1: Panel regression of equity price growth on sudden stops

Asset price growth	(1) Aggregate	(2) Advanced Economies	(3) Emerging Market
Sudden stops (SS)	-0.438***	-0.334***	-0.649***
Financial market participation (FMA)	-0.110	0.0104	-0.269
FMAxSS	0.416**	0.295*	0.985**
Observations	631	366	265
R-squared	0.352	0.533	0.421
Number of countries	29	15	14

*** p<0.01, ** p<0.05, * p<0.1

Regression is done with country and year fixed effect. SS is a dummy variable which takes 1 if a country is in a sudden stop for a given year. Data on sudden stops are from Korinek and Mendoza (2014). **FMA is the Financial Market Access index from IMF**. FMA.SS is a cross-product of FMA and SS. We control for capital flow. The data covers 1980-2012. We drop the sudden stops events with an increase in asset prices.

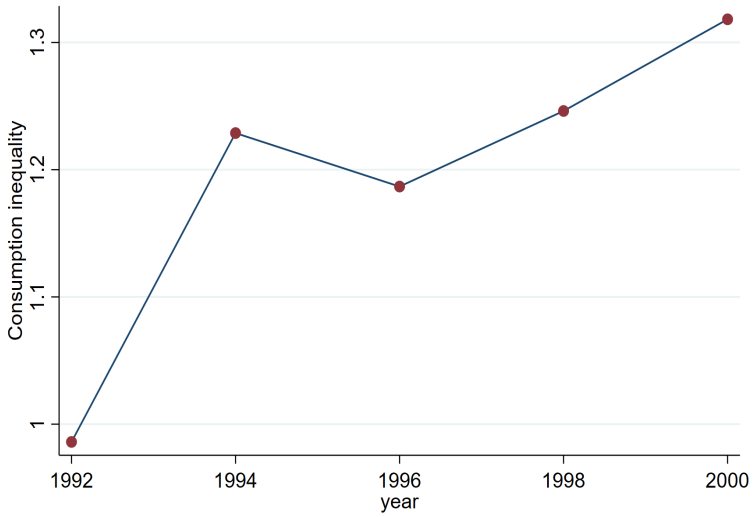


Figure 3: Consumption inequality in Mexico: the consumption inequality is defined as the ratio of *asset holder* consumers' consumption to "hand-to-mouth" consumers's consumption.

3 Model with collateral constraint and household heterogeneity

I build a small-open-economy model with household heterogeneity and collateral constraint. The sudden stops crisis is driven by an occasionally binding collateral constraint. There are two types of households. The first are *asset holder* consumers who have access to financial market by holding both physical asset and foreign bond. The second are "hand-to-mouth" consumers who do not hold any asset neither the physical asset nor the foreign bond. They consume all their labor income plus any additional transfer from the government. In this section, I assume that Non hand-to-mouth Households make production and consumption decision.[4]

3.1 Firm-Asset holder households optimization problem

There is a continuum of identical *asset holder* households of measure $1-\theta \in (0,1]$. The preferences of a *asset holder* consumer indexed by 1 is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{1t} - G(L_{1t})), \tag{1}$$

where $\mathbb{E}_0$ is the expectations operator; β is the discount factor; C_{1t} is consumption, and L_{1t} is labor supply; $u(.)$ is the utility function which is a standard concave, twice continuously differentiable function that satisfies the Inada condition; G(L) is a convex, strictly increasing, and continuously differentiable function that measures the disutility of labor. This type of preferences (known as GHH preference due to the Greenwood et al. (1988)) removes the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.

The households produce the final goods using three inputs, which are the physical assets k_t, the intermediate goods v_t, and the labor L_t. The total labor L_t in the economy is given by $(1-\theta)L_{1t} + \theta L_{2t}$, where L_{2t} is the labor supply by a "hand-to-mouth" consumer. The production technology is such that $y = A_t F(k_t, L_t, v_t)$, where F is a twice continuously differentiable, concave production function and $A_t = A\exp(\epsilon_t^A)$ is the TFP subject to a random shock ϵ_t^A. This shock follows a stationary Markov process. Intermediate goods are traded in competitive world markets at

[4] In the appendix A I show that a separate problem between firm and non hand-to-mouth Households reaches the same outcome

a price p_t^v. The price $p_t^v = p\exp(\epsilon_t^v)$ is subject a to a random shock ϵ_t^v, which follows a stationary Markov process. *Asset holder* households borrow on the foreign bond market at the real interest rate $R_t^v = R\exp(\epsilon_t^r)$, where ϵ_t^r is a random shock, which follows a stationary Markov process. The budget constraint of *asset holder* households is given by

$$(1-\theta)\,C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} = F\left(k_t, L_t, v_t\right) - p_t^v v_t - \theta w_t L_{2t} - \phi r_t\left(w_t L_t + p_t^v v_t\right) + b_t + q_t k_t - T_t. \quad (2)$$

In the equation 2, q_t is the price of the physical asset k_t; $r_t = R_t - 1$ is the net real interest rate; w_t is the real wage. In the right-hand side of (2), the term $\phi r_t\left(w_t L_t + p_t^v v_t\right)$ represents the interest payment to abroad on the working capital loan. Working capital loan is a fraction ϕ of total cost of intermediate inputs and labor in advance of sales. The term $\theta w_t L_{2t}$ represents the total labor income paid to "hand-to-mouth" households. The term T_t is the total lump-sum taxes paid by all *asset holder* households. The lump-sum taxes is used to calibrate the consumption inequality.

The total private debt in the economy is restrained to a fraction κ of the market value of the end-period physical asset given by

$$\frac{b_{t+1}}{R_t} - \phi R_t\left(w_t L_t + p_t^v v_t\right) \geq -\kappa q_t k_{t+1}. \quad (3)$$

In the left-hand side of (3), the total private debt (in negative term) which is the sum of the private debt with one year maturity and the within-period working capital loan. In the righ-hand side of (3), a fraction κ of the market value of the end-period physical asset (in negative term). Only *asset holder* households who borrow on the foreign bond market face this collateral constraint. Although, I do not derive the collateral constraint from an optimization problem, Bianchi and Mendoza (2018) shows that this type of constraint could be obtained as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the market value of asset owned by a defaulting debtor.

The *asset holder* households choose consumption, borrowing, capital, labor and intermediate inputs to maximize their utility (1) subject to their budget constraint (2) and their borrowing con-

straint (3), taken prices as given. Their optimality conditions are given by

$$
\begin{aligned}
u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + \mu_t, && (4)\\
q_t u'(t) &= \beta \mathbb{E}_t \left[(d_{t+1} + q_{t+1}) u'(t+1) \right] + k q_t \mu_t, && (5)\\
A_t F_l (k_t, L_t, v_t) &= G'(L_{1t}) + \phi \left(r_t + R_t \frac{\mu_t}{u'(t)} \right) w_t, && (6)\\
A_t F_v (k_t, L_t, v_t) &= p_t^v + \phi \left(r_t + R_t \frac{\mu_t}{u'(t)} \right) p_t^v, && (7)
\end{aligned}
$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the borrowing constraint; $u'(t)$ is the partial derivative of $u(C_{1t} - G(L_{1t}))$ with respect to C_{1t} and $d_{t+1} = A_{t+1} F_k (k_{t+1}, L_{t+1}, v_{t+1})$.

The first two optimality conditions are the Euler equations for bond and physical asset, respectively. The last two optimality conditions are the intra-temporel conditions on the labor market and intermediate good market, respectively.

Condition (4) states that if the collateral constraint is not binding ($\mu_t = 0$), the marginal benefit of borrowing to increase today's consumption is equal to the expected marginal cost of repaying back tomorrow. If the colloteral binds, the shadow price of relaxing the collateral constraint is positive ($\mu_t > 0$) then the marginal benefit of borrowing is greater than its expected marginal cost. Condition (5) states that the marginal cost of buying one additional unit of physical asset at price q_t is equal to its expected marginal benefit. If the collateral constraint binds, the marginal cost exeeceds the marginal benefit by $k q_t \mu_t$.

Condition (6) states that the marginal productivity of labor demand is equal to the marginal disutility of labor supply plus the financing cost of labor from the working capital loan. The financing cost is higher when the collateral constraint binds. Condition (7) states that the marginal productivity of the intermediate input is equal to its price plus the financing cost of the intermediate input from the working capital loan. The financing cost of the intermediate input is higher when the collateral constraint binds.

3.2 Hand-to-mouth households optimization problem

There is a continuum of identical “hand-to-mouth" households of measure $\theta \in [0, 1)$. The preferences of a “hand-to-mouth" consumer indexed by 2 is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{2t} - G(L_{2t})), \tag{8}$$

where C_{2t} is consumption, and L_{2t} is labor supply; $u(.)$ is the same utility function as in section 3.1. The budget constraint of “hand-to-mouth" households is given by

$$\theta C_{2t} = \theta w_t L_{2t} + T_t. \tag{9}$$

The *asset holder* households choose consumption and labor to maximize their utility (8) subject to their budget constraint (9) taken prices as given. Their optimality condition is given by

$$G'(L_{2t}) = w_t. \tag{10}$$

Condition (10) states that the marginal disutility of labor supply for *asset holder* consumers is equal to the real wage rate.

3.3 Competitive equilibrium

In this this section, I define the competitive equilibrium and the main credit channel through which, the sudden stops arise in this type of framework. The aggregate resource of the the economy is given by:

$$C_t + \frac{b_{t+1}}{R_t} - b_t + \phi r_t \left(w_t L_t + p_t^v v_t\right) = F\left(1, L_t, v_t\right) - p_t^v v_t, \tag{11}$$

where $C_t = (1 - \theta) C_{1t} + \theta C_{2t}$ is the aggregate consumption; the term $\frac{b_{t+1}}{R_t} - b_t + \phi r_t \left(w_t L_t + p_t^v v_t\right)$ represents the trade balance and the term $F\left(1, L_t, v_t\right) - p_t^v v_t$ represents the Gross Domestic Product (GDP).

A competitive equilibrium in this model is a stochastic sequence of allocations $\{C_{1t}, C_{2t}, L_{1t}, L_{2t}, v_t, b_{t+1}\}_{t \geq 0}$

and prices $\{q_t, w_t\}_{t\geq 0}$ such that:

1. Given P_t, Q_t solves Households and firms problems;
2. w_t and q_t are determined competitively $G'(L_t) = w_t$;
3. Markets clear.
   (a) labor market: $L_t = L_{1t} = L_{2t}$,
   (b) capital market: $K_t = 1$,
   (c) Aggregate ressource: Equation (11) is satisfied.

3.4 Equity premium and consumption inequality wedge

In this section, I characterised the equity premium and show how the limited financial market participation distort the equity premium.

Let $\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]}$ be the inverse of the stochastic discount factor in the economy with full financial market participation ($\theta = 0$) where $\lambda_t^R = \beta R_t \mathbb{E}_t \lambda_{t+1}^R + \mu_t^R$. Let $\frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$ be the inverse of the stochastic discount factor in the economy with limited financial market participation ($\theta > 0$) where $\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t$. Using the definition of asset returns and the conditions 4 and 5, the expected excess returns of bonds $E_t\left[R_{t+1}^q - R_t\right]$ can be decomposed into a liquidity premium, inequality wedge and the risk premium as follows:

$$E_t\left[R_{t+1}^q - R_t\right] = (1-\kappa)\underbrace{\frac{\mu_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]}}_{\text{liquidity premium}} - (1-\kappa)\underbrace{\left(\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]} - \frac{\lambda_t}{E_t\left[\beta\lambda_{t+1}\right]}\right)}_{\text{consumption inequality wedge}} - \underbrace{\frac{Cov\left(\lambda_{t+1}, R_{t+1}^q\right)}{E_t\left[\lambda_{t+1}\right]}}_{\text{risk premium}} \tag{12}$$

The term $\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]} - \frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$, I called the *consumption inequality wedge* is the difference of the inverse of the stochastic discount factor between the economy with full financial market participation $\theta = 0$ and the economy with limited financial market participation $\theta > 0$. For $\kappa = 1$ the liquity premium and the consumption inequality wedge do not affect the equity premium. If the agents can pledge all his asset as collateral ($\kappa = 1$), when the collateral binds, the agents can always offset

it by increasing his physical asset by one unit. The relevant case is when the agents cannot pledge all his asset as collateral ($\kappa < 1$)

The liquidity premium raises the equity premium when the collateral constraints binds for $\kappa < 1$. The rise in the equity premium decreases asset prices and the collateral tightens more. This mechanism is known as the debt deflation mechanism and is the core of the financial crisis generated by the model I presented. In the model, the collateral binds endogenously when the leverage is relatively high and an exogenous negative aggregate shocks (high real interest rate, low productivity shock and/or high imported price) hits the economy.

The limited financial market participation add a new term to the equity premium in equation (12) which is the consumption inequality wedge. When the consumption inequality wedge is negative, it raises the equity premium an the asset prices decreases more leading to a high decline in the asset price. Inversely, if the consumption inequality wedge is positive, the equity premium decreases leading to a low declines in the asset price. The effect of the consumption inequality wedge depends on the cyclicality of consumption inequality.

I now characterize how the cyclicality of consumption inequality affects the consumption inequality wedge. Let Suppose that $u(C_t - G(L_t)) = \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}}{1-\sigma}$ where ω is the labor elasticity then $\lambda_t = a_t^{\sigma} \lambda_t^R$ with $a_t = \frac{(1-\theta)\omega + (\omega\theta - 1)\frac{c_{2t}}{c_{1t}}}{\omega - \frac{c_{2t}}{c_{1t}}}$. The ratio $\frac{c_{1t}}{c_{2t}}$ is defined as consumption inequality.

Claim 1: If consumption inequality is constant over time then the financial market participation does not matter for the sudden stops crises. The drop of the asset prices is the same in the economy with full financial market participation as the economy with limited financial market participation.

The proof follows from the definition of the consumption inequality wedge which is zero for a constant consumption inequality since $\frac{\lambda_t^R}{E_t\left[\beta \lambda_{t+1}^R\right]} = \frac{\lambda_t}{E_t[\beta \lambda_{t+1}]}$. The consequence of claim 1 is that the level of consumption inequality does not affect the drop in the asset price, hence the severity of the crisis. The next claim completes this claim by showing that indeed the cyclicality of the consumption inequality matters.

Claim 2: Let's suppose perfect foresight (no uncertainty) that is $\mathbb{E}_t[X_{t+1}] = X_{t+1}$.
If the consumption inequality is lower (higher) during the financial crises, the economy will generate higher (lower) amplification effect.

Under perfect foresight, the consumption inequality can then be rewritten $\frac{\lambda_t^R}{\beta\lambda_{t+1}^R}\left(1-\left(\frac{a_t}{a_{t+1}}\right)^{\sigma}\right)$. Suppose now that at time t the collateral constraint binds and $\frac{c_{1t}}{c_{2t}} < \frac{c_{1,t+1}}{c_{2,t+1}}$ (that is lower consumption inequality) this implies that $a_t > a_{t+1}$. It follows that the consumption inequality wedge is negative leading to a high equity premium.

Claim 1 and claim 2 have shown that, given the limited financial market participation, what matters is cyclicality of consumption inequality. As shown in fact 3 of section 2, the consumption inequality is procyclical leading quantitatively to a higher drop in asset prices. Werning (2015) and Acharya and Dogra (2020) have argued that the cyclicality of income inequality could affect the aggregate outcome variables in a monetary policy with household heterogeneity framework. In my framework where there is no nominal rigidities, I found that the cyclicality of consumption inequality does affect the severity of a financial crisis. In next section, I present the quantitative results.

4 Quantitative results

This section studies the model's quantitative implications using numerical simulation. First, I present the calibration and second I discuss the results.

4.1 Calibration

A period in the model represents a year. The calibration uses Mexico data. The result is presented in table 2 The functions forms for preference and technology are the following:

$$\begin{aligned} u(C_t - G(L_t)) &= \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}, \quad \omega > 1 \\ F\left(k_t, L_t, v_t\right) &= A_t k_t^{\gamma} L_t^{\alpha} v_t^{\eta} \end{aligned}$$

The preference parameters for risk aversion and the elasticity of substitution are set to standard values from the literature: $\sigma = 2$. The average real interest rate is set to 4%, also standard in the literature.

The labor supply elasticity ω is set equal to 1.846 as in Mendoza (2010). Mendoza (2010) used data for the period 1993:1-2005:11 and found that the annualized average ratio of GDP to gross output (gdp/y) is 0.896 and the ratio of imported inputs to GDP (pv/gdp) is 0.114. The average share of imported inputs in gross output is 0.102, which implies that η= 0.102. The labor share on GDP for Mexico is 0.66 which implies that $\alpha = 0.592$. The value of $\gamma = 0.042$ is set so that the equity premium is zero at the deterministic steady state. The steady asset price is set to 1.

The shocks are modeled as a joint discrete Markov process that approximates the statistical moments of their actual time-series processes. The Markov process is defined by a set E of all combinations of realizations of the shocks, each combination given by a triple $e = (\epsilon^A, \epsilon^R, \epsilon^P)$, and by a matrix of transition probabilities of moving from e_t to e_{t+1}. I closely follow Mendoza (2010) to set the transition probability between the differents states. In the data, $\epsilon^A, \epsilon^R, \epsilon^P$ are AR(1) processes with standard deviations and first-order autocorrelations respectively 0.537; 0.572 and 0.737. Since the three shocks are nearly independent, except for a statistically significant correlation between ϵ^A and ϵ^R , of about -0.67, the Markov process is con- structed using the parsimonious structure of the two-point, symmetric simple persistence rule as in Mendoza (1995). Each shock has two realizations equal to plus/minus one standard deviation of each shock in the data ($\epsilon_1^A = -\epsilon_2^A = 0.0134$, $\epsilon_1^R = -\epsilon_2^R = 0.0196$, $\epsilon_1^P = -\epsilon_2^P = 0.0335$), so E contains eight triples. The simple persistence rule produces an 8x8 matrix which yields autocorrelations of the shocks and a correlation between ϵ^A, and ϵ^R, that match those in the data. Mendoza (2010) pointed out that the procedure requires, however, that the AR(1) coefficients of the shocks that are correlated with each other (ϵ^A, and ϵ^R) be the same - which is in line with the data where $\rho(eR) = 0.572$ and $\rho(eA) = 0.537$.

The two parameters remaining are β and κ. The value of β is set to 0.92 to match the average net foreign asset of 20% of GDP. The value of κ is set to 0.43 to match the frequency of the financial crisis of 4%. The average private debt is 19.7 % of GDP and the frequency of the financial crisis is 4.6%.

Table 2: Parameter values

Parameters set Independently	Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Share of labor in gross output	$\alpha = 0.592$	Mexico GDP labor share 0.66
Share of input in gross output	$\eta = 0.10229$	Mexico data
Share of asset in output	$\gamma = 0.043$	steady state asset return
Frisch elasticity	$\omega = 1.846$	Mendoza (2010)
Working capital coefficient	$\phi = 0.13$	Working capital/ GDP ratio $= 10\%$
Share of HtM	$\theta = 0.5$	Mexico data
Transfer	$T_t = 0.14$	Avr cons ineq of 1.25
Parameters set by Simulation	Value	Target
Discount factor	$\beta = 0.920$	Net foreign asset of 20%
Fraction of collateral value	$\kappa = 0.43$	Financial crisis of 4 %

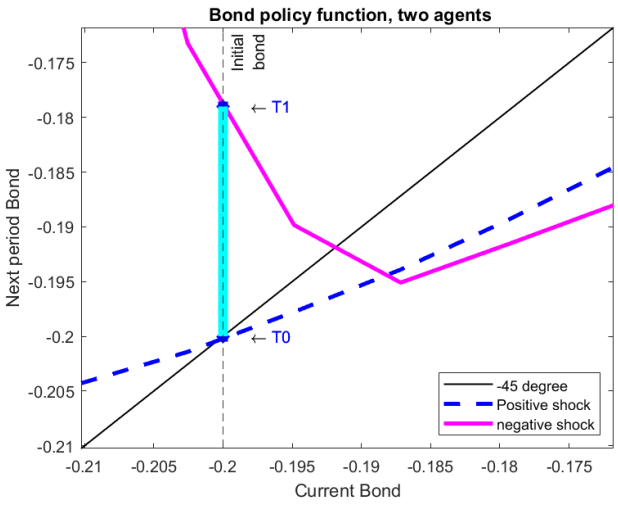
4.2 Sudden stops: the dynamics of asset prices, output, debt, and consumption

This section presents the quantitative results. I, first, describe the difference in the debt policy function between the economy with limited/full financial market participation. Second, I show the dynamics of some aggregates variables when the collateral constrained binds. I finish this section by showing some long run moments.

Policy functions for gross private debt. Figure 4 presents the next period private debt b_{t+1} as a function of current private debt b_t. The solid magenta line represents the policy function for a negative aggregate shock that is a high real interest rate and a low productivity. The dash blue line represents the policy function for a positive aggregate shock that is a low real interest rate and a high productivity. In panel (a), the economy with limited financial market participation ($\theta = 0.5$) is shown. In panel (b), the economy with limited financial market participation is shown ($\theta = 0$).

In both panel, the next period debt as a function of the current debt for a negative aggregate shock has a V-shape. This V-shape is due to the collateral constraint that is more likely to bind for a high debt and negative aggregate shock. If the debt level is high (that is the bond is more negative), households are forced to deleverage when a negative shocks hit the economy. For a positive shock, the policy function is almost linear since the collateral constraint is less likely to bind.

(a) Limited financial market participation



(b) Full financial market participation

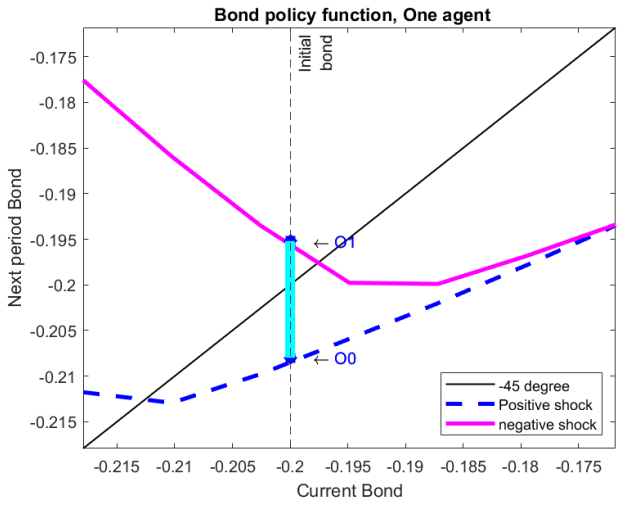


Figure 4: Policy function for private debt

To understand the quantitative difference between both economy (panel (a) and panel (b)), consider the cyan line (T0,T1) and (O0,O1). Suppose that both economy start at the same bond which is equal to -0.2. In a limited financial market participation economy, for a positive shock, households will choose T0 otherwise they will choose T1. Households then expect to reduce the debt level by 0.02 from a positive shock to a negative shock. In the same way, in a full financial market participation, households expect to reduce the debt by 0.015 from a positive shock to a negative shock. The limited financial market participation economy will generate a higher amplification because households are expected to reduce more their debt when the collateral binds.

Financial crises. I now analyze the ability of the model to generate financial crises and the role of a limited financial market participation to generate a higher drop in the asset price. For that purpose, I simulated the model for 100 000 periods and construct a 9-years events windows centered at the crisis year. The financial crisis is defined as when the collateral binds and the trade balance is two standard deviations above its mean. Using the current account instead of the trade balance gives the same result.

Figure 5 shows the average of output, consumption of asset holders, consumption of hand to mouth consumers, bond, asset price, and the exogenous shock across the 9-year events windows

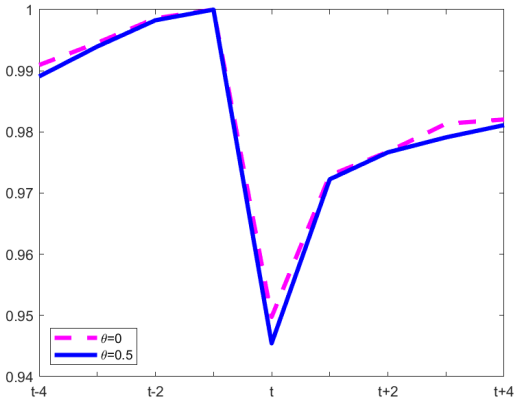
for the two economies. I normalize the average of the variables to 1 at t-1. For panel (a) to (e), the solid blue line represents the limited financial market participation economy where the share of Hand-to-Mouth consumers is set to 50 percent. The dash magenta line represents the economy with full financial market participation that is the share of Hand-to-Mouth consumers is set to 0. Panel (f) shows the percentage differences relative to the unconditional averages of aggregate exogenous shocks that hit the economy.

I follow Bianchi and Mendoza (2018) to construct comparable event windows for the two economies. First, I simulate the Limited financial market economy for 100,000 periods and identify financial crises using the definition described above. Second, I construct 9-year event windows centered at the crisis year, denoted date t, by computing averages for each variable across the cross section of crisis events at each date. The result of this procedure is the solid blue line in figure 5 . Third, I take the initial bond position at t-4 of the limited financial market participation economy crisis and the sequences of aggregate exogenous shocks in the 9-year window in this economy, and I pass them through the policy functions of the full financial market participation economy. Finally, I compute the average as in the previous case. The result of this procedure is the dash magenta line in figure 5 .

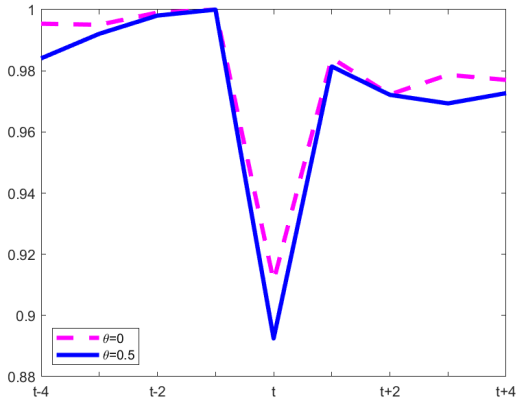
Panel (a) , (b), and (c) show that the output, the consumption of the asset holders, and the asset prices fall more in the economy with limited financial market participation than the economy with full financial market participation. In full market participation economy, output falls short by 0.5 percentage point (5 % vs 5.5%), consumption of asset holders falls short by 2 percentage point (9 % vs 11%), and asset prices fall short by 3 percent point (13.5 % vs 16.75 %). The fall in the asset price is then 23 % higher in a limited financial market participation economy than in a full financial market participation economy.

Panel (f) shows that prior to the crisis, the real interest rate is below the average real interest rate by almost 200 basis point which correspond to one standard deviation. At the date t of the crisis, the real interest rate sharply raises to almost 200 basis points above its average. It then decreases slowly to converge to the average value four years after the crisis. Contrary to the real interest rate, the TFP raises prior to the crisis and decreases to one standard deviation below its average. The

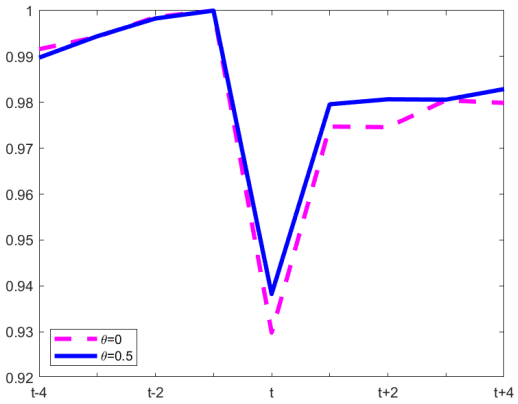
(a) Output



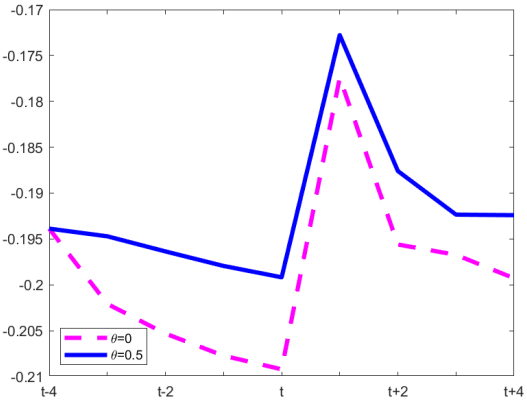
(b) Consumption of Asset holders



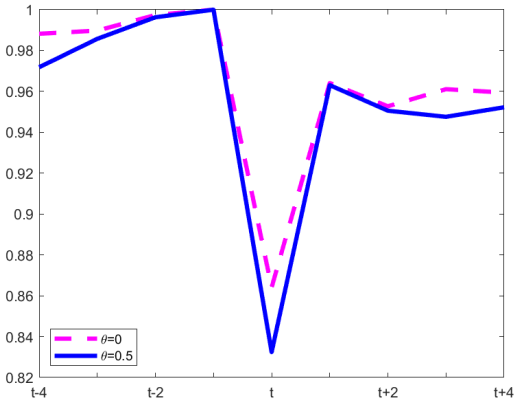
(c) Consumption of Hand-to-Mouth



(d) Private Debt



(e) Asset prices



(f) Exogenous shocks

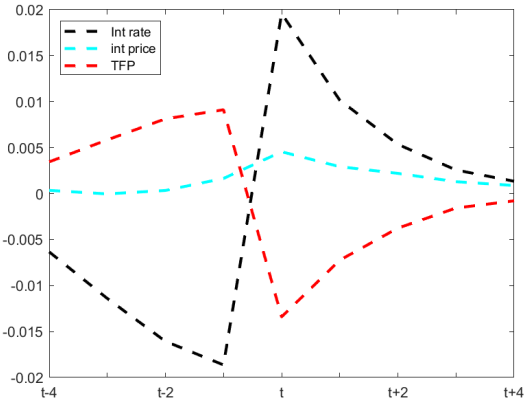


Figure 5: Financial crisis dynamics

input import price shows small fluctuations along the financial crises dynamics.

To summarize, the financial crises show a higher amplification effect in a limited financial market participation than in a full limited financial market participation. The asset prices fall more and the asset holders face higher burdens. I next present why the asset price falls more when every households are not participating on the financial market.

Asset prices and inequality. In section 3.4, I show that, given the share of hand-to-mouth consumers, the cyclicality of consumption inequality is important to determine the relative amplification effect in a limited financial market participation economy. As shown by figure by 6, higher drop in the asset price in a limited financial market participation is followed by a drop in consumption inequality during the crisis year. Second , the overall pattern of the asset prices is qualitatively similar to the consumption inequality dynamics. A drop in consumption inequality is associated with a higher drop in the asset price in a limited financial market participation economy because of the burden on asset holders. Indeed, when an asset holder faces a higher burden following an aggregate shock, he will be willing to sell more his asset to meet his obligation. By doing so, he increases the supply of the asset leading to a decrease on its prices.

It is worth noting that, in the model, I did not calibrate the drop in the consumption inequality. Instead, I use a constant transfer (a constant lump -sum tax from on asset holders) to hand-to-mouth consumers to calibrate the average consumption inequality of 1.27 in the economy. Consistently with Mexico's data, the model generates endogenously the drop in consumption inequality during the financial crises. The model is able to generate a drop in consumption inequality during the financial crisis because the labor supply, which determines the labor income of hand-to-mouth, has shown a small decline. Mendoza (2010) suggested that the labor decline is not the main cause in the decline of GDP during the mexico's 1995 sudden stops crisis.

Long run moments. The table 3 presents the business cycle moments. The column with $\theta = 0$ represents the economy with full financial market participation. The column with $\theta = 0.5$ represents the economy with limited financial market participation where the share of hand to mouth consumers is 50 % of the population. The statistics in the data's column comes from Mendoza

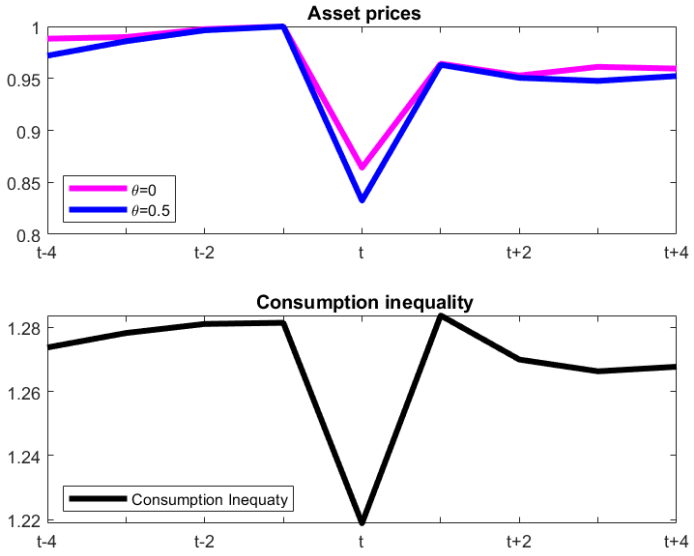


Figure 6: Asset prices and inequality

Table 3: Business cycle moments

	Standard deviation			Correlation with output		
	$\theta = 0$	$\theta = 0.5$	data	$\theta = 0$	$\theta = 0.5$	data
GDP	2.68	2.73	2.72	1.00	1.00	1.00
Consumption	3.55	3.51	3.39	0.94	0.95	0.89
Trade balance/gdp	1.33	1.26	2.1	-0.51	-0.49	-0.68
Asset prices	5.63	6.68	14.64	0.89	0.89	0.57
Interest rate	1.95	1.95	1.95	-0.64	-0.65	-0.59

(2010) . In the table, only the standard deviation of the real interest rate has been used to calibrate the exogenous process of the real interest rate. The results show that the models do a good job at matching the standard deviation and the correlation of key aggregate variables like output and consumption.

The emerging market is characterized by three main business cycles. First, consumption is more volatile than output. The model with full/limited financial market participation replicate well quantitatively the standard deviation of output and consumption. Second, the trade balance is counter-cyclical. In fact, the correlation with output of the trade balance ratio to GDP is negative. Third, the real interest rate in counter-cyclical. In the limited financial market participation

economy, the correlation of the real interest rate with output is -0.65, which is comparable to what is observed in the data -0.59.

The models under-estimate the volatility of the asset prices. While the data suggests that the volatility of the asset prices in Mexico is 6 times the volatility of output (in the data the volatility of the asset prices is 14.64 and 2.72 for output), both economy estimates' are only about 2 times. By introducing a financial shock in the model where the parameters κ is not constant over time, one can significantly increases the volatility of the asset prices. The results also suggests that the economy with limited financial market participation displays more volatility in the asset price than the economy with full financial market participation.

Despite the model falls short as observed in data the volatility of the asset prices relative to the one output, it is able to display that the asset prices is more volatile than the output. By contrary, Mendoza (2010) has found that the asset prices is less volatile than the output. The reason is that in their framework there is no direct role of expectation in the determination of asset prices. Indeed, with investment and capital adjustment cost in their framework, the asset price is equal to one plus the marginal adjustment cost. So, without a change in the stock of capital, the asset price does not change. Mendoza (2010) has then excluded any direct role of expectation where a change in the expectation of the supply of assets could substantially affect the asset price even though at the equilibrium, the stock of the asset is constant.

5 Financial crisis with a financial shock

The framework presented in section 3 does not have a financial shock. In this section, I introduce financial shock as in Bianchi and Mendoza (2018). The parameters κ that represents the fraction of the total value of physical assets the households can pledge as collateral is not constant anymore. But it takes two values: a high value κ_h regime and low value κ_l regime (time of crisis) with a switching probability between both regimes. This is consistent with data, which suggest that the loan-to-value ratio decreases during a financial crisis. The high value According to the loan-to-value ratio in mexico in the 90's, I set $\kappa_H = 0.7$ and $\kappa_L = 0.55$. The probability of staying in the low regime is set to zero to reflect the fact that the average duration of a sudden stop is one year. I then

use the probability of staying in the high regime to calibrate the frequency of the financial crisis. In addition to the financial shock, a small change is made to make the model comparable to Bianchi and Mendoza (2018) who analyse the optimal time-consistent problem with a representative agent. I assume there is no shock on the imported input price. The begin-period asset K_t is used as collateral instead of the end-period asset K_{t+1}, and there is no labor in the working capital loan. See in appendix B.1 for the full model.

5.1 Optimal prudential capital control and financial crisis

In this section, I analyze the optimal time-consistent policy presented in the appendix B.2. In the optimal policy, I choose to use a tax on foreign debt- a capital control- to decentralize the planner solution. The tax collected are redistributed in the form of lump-sum transfers to asset households. This section answers to answer two main questions. First, How effective is an optimal tax on debt to reduce the severity and the frequency of a financial crisis in a limited financial market participation economy? Second, can we rationalize the prevent use of a capital control in emerging market characterized by a low level of financial market participation?

The optimal time-consistent solution suggests three main lessons. First, the optimal time-consistency solution effectively reduces the frequency and severity of the financial crisis both in a full/limited financial market participation economy. Second, the average tax on foreign debt needed to decentralize the optimal time-consistent solution is higher in a limited financial market participation economy (1.2% vs 6%) . This suggests that more capital control is needed in emerging markets which have a low level of financial market participation than advanced economies. This second lesson rationalizes the use of capital control in the world. In fact, data on capital control suggests that emerging markets control more capital flows than advanced economies.

Figure 7 shows the average of output, consumption of asset holders, consumption of hand-to-mouth consumers, bond, asset price, and the exogenous shock across the 9-year events windows for the competitive equilibrium with a limited financial market participation economy and the optimal policy. I normalize the average of the variables to 1 at t-1. For the first six panel, the solid blue line represents the limited financial market participation economy where the share of hand-to-mouth

consumers is set to 50 percent. The dash magenta line represents the optimal solution economy with limited financial market participation with the same share of Hand-to-Mouth consumers. The last Panel shows the percentage differences relative to the unconditional averages of aggregate exogenous shocks that hit the economy.

The top three panel shows that the decline in output, asset holder consumption, and hand-to-mouth consumption is substantially higher in the competitive equilibrium than in the social planner during the financial crisis. Indeed, output drops by 4.5 vs. 2.5 percent, asset holder consumption drops by 24 vs. 2.3 percent, and hand-to-mouth consumption drops by 6 vs. 3.3 percent. Therefore the planner solution is effective in reducing the severity of the financial crisis.

The bottom-left panel shows the debt dynamics around the financial crisis. At t-4, both CE and SP start with the same stock of debt. But At t-3, while the SP reduces debt by four percentage points, the CE builds up the debt. This trend continues until t-1, where the difference in the debt between the CE and SP is more than three percentage points. Therefore, The debt dynamics suggests that household overborrows in the competitive equilibrium. Because of this overborrowing, the CE experiences a larger adjustment in the debt when a financial crisis hits the economy.

The bottom-middle panel shows the asset prices dynamics around the financial crisis. The CE experiences a larger decline in the asset price than in the SP. That decline in the asset price is 59 percent in the CE, while it is only 2 percent in the SP. The SP effectively reduces the decline in the asset price substantially because it does not reduce the debt too much when the financial crisis hits the economy. By taking into account the pecuniary externality, the SP did not experience a sharp decline in the asset price. The sharp decline observed in the asset price for the CE is because asset holders substantially deleverage when a financial crisis hits the economy and therefore are more willing to sell their physical assets to meet their obligations. The excess supply of assets leads to a sharp decline in the asset price.

At t-4, the real interest and the TFP are at their average. During the financial crisis, the real interest increases by 150 basis point which reflects the scarcity of the availability of the foreign asset. The real interest rate is slightly below its average level 4 years after the crisis. The Total Factor Productivity decreases by one standard deviation during the financial crisis and as the real

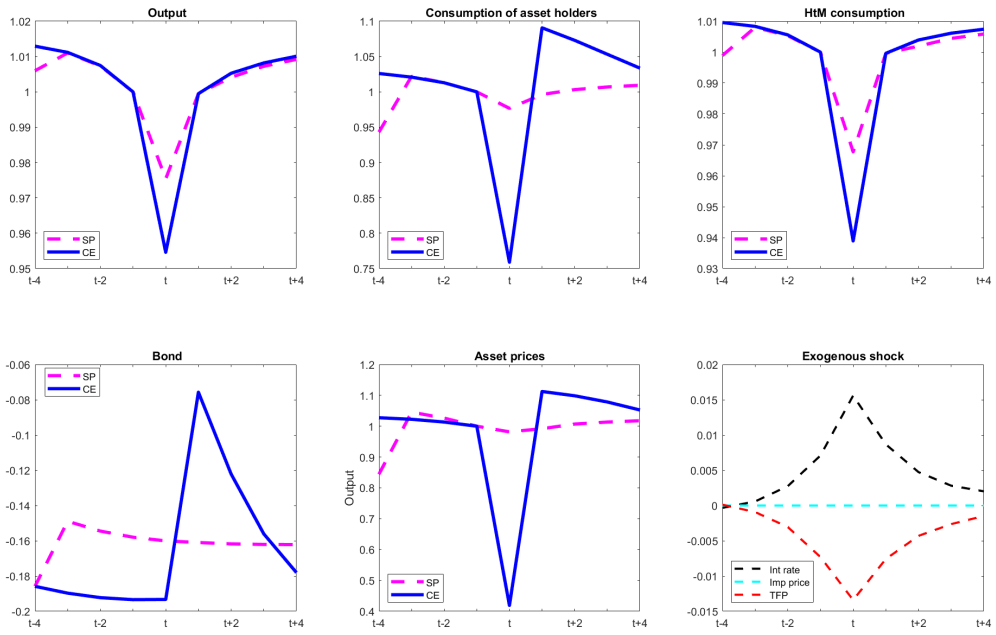


Figure 7: Financial crisis with the optimal policy. SP stands for the social planner and CE for the competitive equilibrium.

interest rate, it is slightly below its average level 4 years after the crisis.

5.2 Optimal prudential capital control and inequality

The section 5.1 shows that prudential capital control is very effective at reducing the severity of a financial crisis. This section answers the following question. Does the inequality increase when using a prudential capital control (a tax on foreign debt)? The optimal time-consistent solution suggests that it is possible to address financial stability without raising (consumption) inequality (see Figure 8). While at the very short run (at the time of the financial crisis), the social planner may accept a slightly increase in the consumption inequality, it appears that the long-run (average) consumption inequality is lower in the optimal time-consistent equilibrium than in the competitive economy.

6 Conclusion

This paper presents a financial crisis model in a limited financial market participation economy with a collateral constraint. Participation in the financial market is limited because a fixed share of households do not hold any liquid wealth. That is, they do not participate either in the bond market or the stock market. When negative aggregate shocks hit the economy, the collateral constraint binds, and households are forced to deleverage. The extent to which the limited financial market participation amplifies or dampens the economy's response to the aggregate shocks depends on the cyclicality of the consumption inequality. In the model, the consumption inequality is endogenous.

Consistent with the empirical evidence I document using Mexico's household survey data, the model generates a decline in consumption inequality during the financial crisis. This decline in consumption inequality amplifies the economy's response to the aggregate shocks during the financial crisis. Moreover, the optimal time-consistent solution shows that the average tax on foreign assets is higher in a limited financial market participation economy than in a full financial market participation economy. This finding rationalizes the prevalent use of capital control in emerging markets.

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A Separate problem

A.1 Non Hand to Mouth households' Problem

$$\begin{aligned}
\max_{C_{1t},b_{1t+1},s_{t+1},N_{1t}} \quad & \mathbb{E} \quad \sum_{t=0}^{\infty} \beta^t U(C_{1t} - G(N_{1t})) \\
C_{1t} + \frac{b_{1t+1}}{R_t} + q_t s_{t+1} \quad & = \quad w_t N_{1t} + b_{1t} + (d_t + q_t) s_t \\
\frac{b_{1t+1}}{R_t} \quad & \geq \quad -\kappa q_t s_{t+1}
\end{aligned}$$

$$EE1: \quad \overbrace{U'(t)}^{\text{marginal benefit of borr.}} = \overbrace{\beta R_t E_t U'(t+1)}^{\text{marginal cost of borr.}} + \overbrace{\mu_t^{nh}}^{\text{shadow price of relaxing the constr.}}$$

$$EE2: \quad \underbrace{q_t U'(t)}_{\text{marginal cost of buy.}} = \underbrace{\beta E_t \left[(d_{t+1} + q_{t+1}) U'(t+1) \right]}_{\text{marginal benef. of buy.}} + \underbrace{k q_t \mu_t^{nh}}_{\text{gain of relax. the constr.}}$$

$$Lab: \quad \underbrace{G'(N_{1t})}_{\text{marginal disutility of labor}} = \underbrace{w_t}_{\text{real wage}}$$

A.2 Firm's problem

$$\begin{aligned}
\max_{d_t,k_{t+1}^f,b_{t+1}^f,v_t,L_t} \quad & \mathbb{E} \quad \sum_{t=0}^{\infty} \beta^t U'(C_{1t} - G(N_{1t})) d_t \\
d_t + \frac{b_{t+1}^f}{R_t} + i_t \quad & = \quad F\left(k_t^f, L_t, v_t\right) - (1 + \theta r_t)\left(w_t L_t + p_t^v v_t\right) + b_t^f \\
i_t \quad & = \quad k_{t+1}^f - k_t^f + \delta k_t^f + \left(k_{t+1}^f - k_t^f\right) \psi\left(\frac{k_{t+1}^f - k_t^f}{k_t^f}\right) \\
\frac{b_{t+1}^f}{R_t} - \theta R_t \left(w_t L_t + p_t^v v_t\right) \quad & \geq \quad -\kappa^f q_t k_{t+1}^f
\end{aligned}$$

Optimality conditions for firm

$$
\begin{aligned}
\left[b_{t+1}^{f}\right] &:: U'(t) = R_t E_t\left[U'(t+1)\right] + U'(t)\mu_t^{f} \\
\left[k_{t+1}^{f}\right] &:: U'(t)\frac{\partial i_t}{\partial k_{t+1}} = E_t\left[U'(t+1)\left\{F_k\left(k_t^{f}, L_t, v_t\right) - \frac{\partial i_{t+1}}{\partial k_{t+1}}\right\}\right] \\
+k^f q_t U'(t)\mu_t^{f} & \\
[L_t] &:: F_l\left(k_t^{f}, L_t, v_t\right) = \left(1 + \phi r_t + \phi R_t \mu_t^{f}\right) w_t \\
[v_t] &:: F_v\left(k_t^{f}, L_t, v_t\right) = \left(1 + \phi r_t + \phi R_t \mu_t^{f}\right) p_t^{v} \\
KT &:: \mu_t^{f}\left(\frac{b_{t+1}^{f}}{R_t} - \phi R_t\left(w_t L_t + p_t v_t\right) + k^f q_t k_{t+1}^{f}\right) \\
\mu_t^{f} &\geq 0
\end{aligned}
$$

A.3 Market equilibrium for the separate problem

Labor market: $L_t = (1-\lambda) N_{1t} + N_{2t}$

Stock market : $s_t = \frac{1}{1-\lambda}$

Good market : $C_t = (1-\lambda) C_{1t} + \lambda C_{2t}$ Bond market:$b_{t+1} = (1-\lambda) b_{1t+1} + b_{t+1}^{f}$

Aggreagte capital: $k_{t+1} =, k_{t+1}^{f}$

Definition : A competitive equilibrium is a set of allocations

$Q_t = \left\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, s_{t+1}, b_{1t+1}, b_{t+1}^{f}, k_{t+1}^{f}, d_t\right\}$ and prices $P_t = \{p_t, w_t, R_t, q_t\}$ such that :

1. Given P_t, Q_t solves Households and firms problems
2. w_t and q_t are determined competitively G'(L_t) $= w_t$; $\frac{\partial i_t}{\partial k_{t+1}} = q_t$
3. Markets are clear

A.4 Equivalence result: Separate vs Non separate firm household problem

If the allocation $\left\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, s_{t+1}, b_{1t+1}, b_{t+1}^{f}, k_{t+1}^{f}, d_t, w_t, q_t, \mu_t^{nh}\right\}$ is a competitive equilibrium in the economy with separate Non-Hand to Mouth Consumer and firm problems then $\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, b_{t+1}, k_{t+1}, w_t, q_t, \mu_t\}$ is a competitive equilibrium in the economy with non separate firm-Non Hand to Mouth consumer problem with $b_{t+1} = (1-\lambda) b_{1t+1} + b_{t+1}^{f}$ and $k_{t+1} = k_{t+1}^{f}$. (The converse is also true)

The proof follows Bianchi and Mendoza (JPE, 2018) who show it in the representative agent economy. The equivalence result still holds because there is no heterogeneity within firm owners.

B Model with financial shock

B.1 Firm-Non hand-to-mouth households optimization problem

There is a continuum of identical *asset holder* households of measure $1-\theta \in (0,1]$. The preferences of a *asset holder* consumer indexed by 1 is given by

$$\begin{aligned} \max_{C_{1t},b_{t+1},k_{t+1},v_t,L_{1t}} \quad & \mathbb{E}_0 \quad \sum_{t=0}^{\infty} \beta^t u(C_{1t} - G(L_{1t})) \\ (1-\theta)\, C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} \quad &= \quad F\left(k_t, L_t, v_t\right) - p_t^v v_t - \theta w_t L_{2t} + b_t + q_t k_t - T_t \\ \frac{b_{t+1}}{R_t} - \phi\left(p_t^v v_t\right) \quad &\geq \quad -\kappa_t q_t k_t. \end{aligned}$$

Optimal solution gives:

$$\begin{aligned} u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + \mu_t, \\ q_t u'(t) &= \beta \mathbb{E}_t \left[\left(d_{t+1} + q_{t+1}\right) u'(t+1) + \kappa_{t+1} q_{t+1} \mu_{t+1} \right], \\ A_t F_l\left(k_t, L_t, v_t\right) &= G'(L_{1t}) \\ A_t F_v\left(k_t, L_t, v_t\right) &= p_t^v + \phi\left(\frac{\mu_t}{u'(t)}\right) p_t^v, \end{aligned}$$

B.2 Time-consistent Planner's Problem

$$
\begin{aligned}
V(b,s) &= \max_{c_1,c_2,b',L,v,q} \left\{ \theta u(c_2 - G(L) + (1-\theta)u(c_1 - G(L)) + \beta \mathbb{E}_{s',s} V(b',s') \right\} \\
(1-\theta)\, c_1 + \frac{b'}{R} &= F\left(1, L, v\right) - p^v v - \theta w L + b - T. \\
\theta c_2 &= \theta w L + T \\
-\phi\left(p^v v\right) + \frac{b'}{R} &\geq -\kappa q \\
AF_v\left(1, L, v\right) &= p^v + \phi\left(\frac{\mu}{u'(c_1 - G(L))}\right) p^v \\
AF_l\left(1, L, v\right) &= G'(L) \\
w &= G'(L) \\
q u'(c_1 - G(L)) &= \beta \mathbb{E}\left[\left(\mathbb{D}(b',s') + \mathbb{Q}(b',s')\right) u'(\mathbb{C}(b',s')) + \kappa' \mathbb{Q}(b',s') \mu(b',s')\right]
\end{aligned}
$$

C Quantitative results

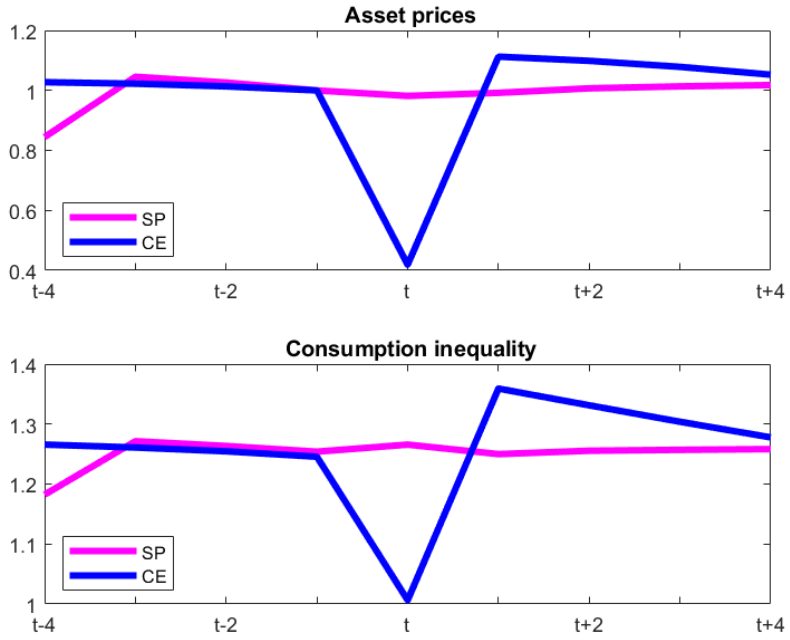

Figure 8: Inequality and asset prices. SP for the social planner and CE for the competitive equilibrium