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Problem 1

$y = Cx + n$

$\Rightarrow x = C^{-1}y - C^{-1}n$

and also because n~$Gauss(0, Q)$

$\Rightarrow$

$$\begin{cases} \Sigma_{X|Y} = C^{-1}Q(C^{-1})^T \\ \mu_{X|Y} = C^{-1}y - \mu_n = C^{-1}y \end{cases}$$

Problem 2

$$\begin{cases} x(t) = Ax(t-1) + m(t) \\ x(t-1)\ Gauss(\mu_{t-1}, \Sigma_{t-1}) \\ m\ Gauss(0, R) \end{cases}$$

$\Rightarrow$

$$\begin{cases} \Sigma_t = A\Sigma_{t-1}A^T + \Sigma_m = A\Sigma_{t-1}A^T + R \\ \mu_t = A\mu_{t-1} + \mu_m = A\mu_{t-1} + 0 = A\mu_{t-1} \end{cases}$$

Problem 3

from problem1, problem2, we get $\Sigma_{X|Y}, \mu_{X|Y}, \Sigma_t, \mu_t$

$\Sigma^{-1} = \Sigma_{X|Y}^{-1} + \Sigma_t^{-1} = C^TQ^{-1}C + \Sigma_t^{-1}$

$\Sigma^{-1}\mu = \Sigma_{X|Y}^{-1}\mu_{X|Y} + \Sigma_t^{-1}\mu_t = C^TQ^{-1}C + \Sigma_t^{-1}\mu_t$

$\mu = \Sigma(C^TQ^{-1}y + \Sigma_t^{-1}\mu_t)$

without $K(t)$:

$$\begin{cases} \Sigma^{-1} = C^TQ^{-1}C + \Sigma_t^{-1} \\ \mu = \Sigma(C^TQ^{-1}y + \Sigma_t^{-1}\mu_t) \end{cases}$$

with $K(t)$:

$$\begin{cases} K(t) = \Sigma_t A^T(A\Sigma_t A^T + Q)^{-1} \\ \mu = \mu_t + K(t)(y - Au_t)\Sigma = (I - K(t)A)\Sigma_t \end{cases}$$