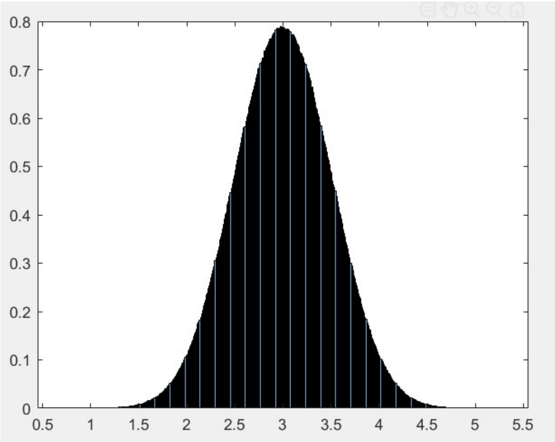
1.

(1) plot normal distribution:

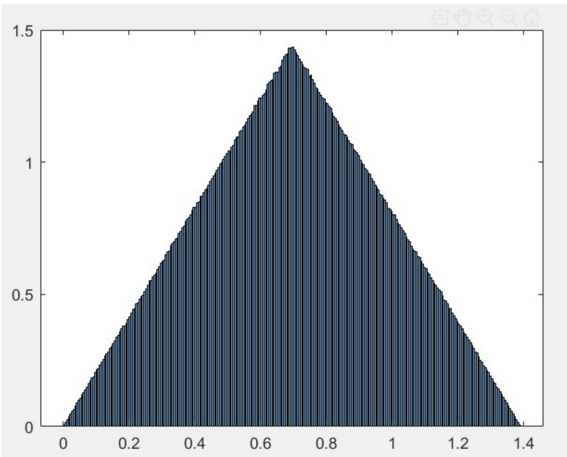
Central-limit theorem

$$x = \sum_{i} rand()$$



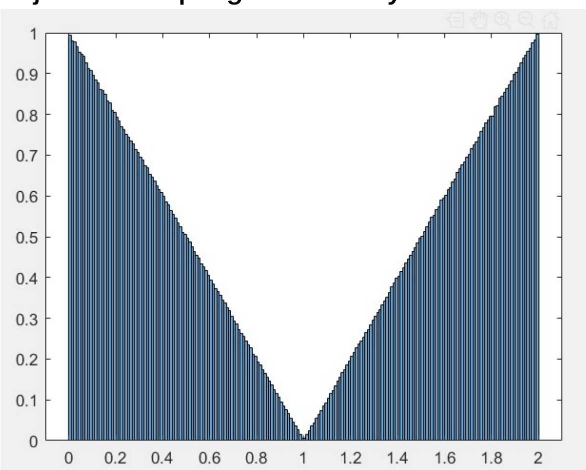
(2) plot triangle distribution

$$u_1(a,b) + u_2(a,b)$$

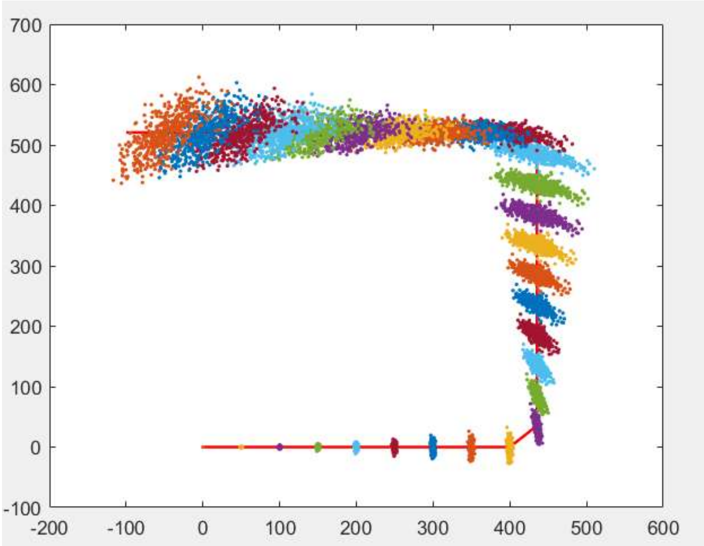


(3) plot abs distribution

Rejection sampling or **use Axisymmetric transformation of triangle sampling**

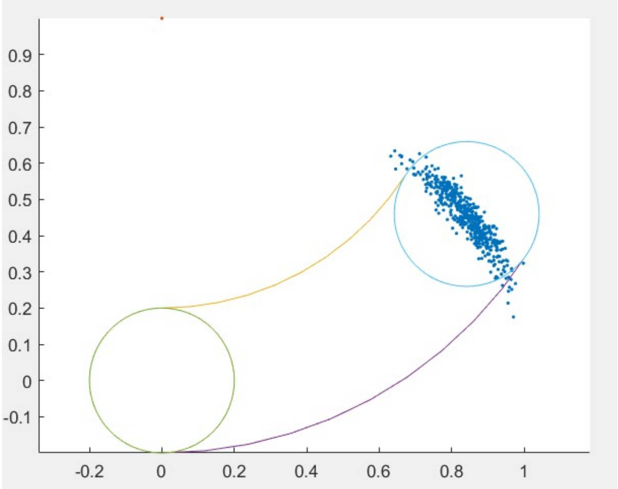


2. see the code

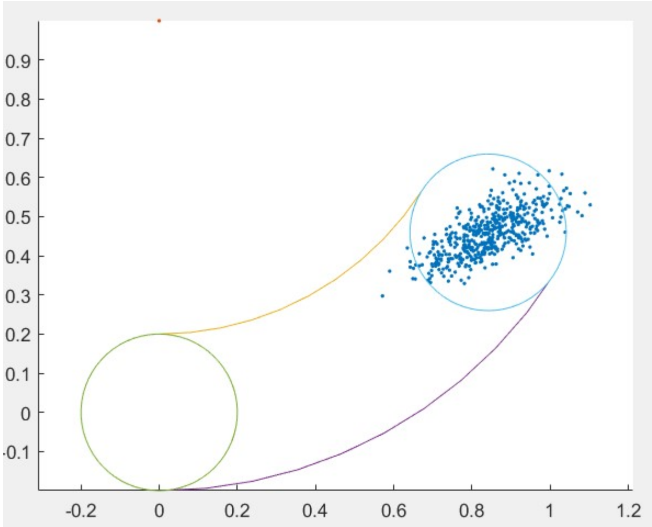


3.

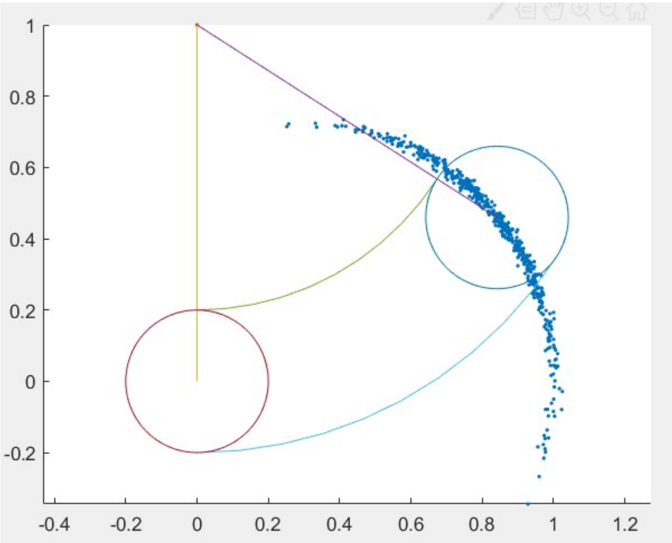
Case 1 **small velocity estimation error** and **large angular velocity estimation error**



Case 2 **large velocity estimation error** and **small angular velocity estimation error**

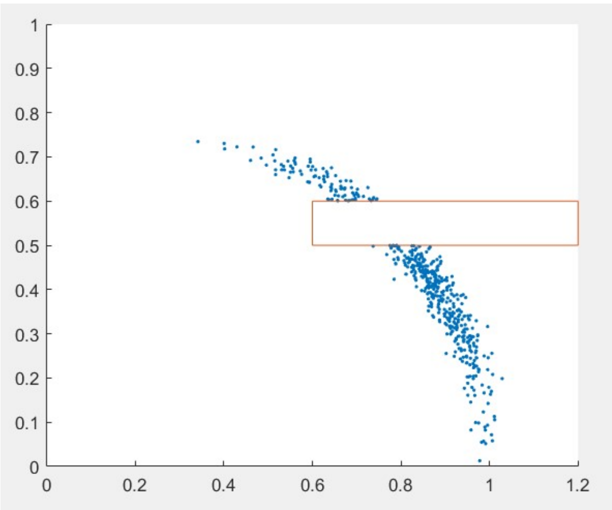


Case 3 **small velocity estimation error** and **very large angular velocity estimation error**



4.

Assume the machine driven in the obstacle is 0



Addition chapter 5 exercise 7

Assume forward speed is v_1 , orthogonal sidewards velocity is v_2, use θ as angular

$$\Delta x = \left(v_1 * \cos\theta + v_2 \cos\left(\theta + \frac{\pi}{2}\right)\right) * \Delta t = (v_1 * \cos\theta - v_2 \sin\theta\ ) * \Delta t$$

$$\Delta y = \left(v_1 * \sin\theta + v_2 \sin\left(\theta + \frac{\pi}{2}\right)\right) * \Delta t = \ (v_1 * \cos\theta + v_2 \cos\theta) * \Delta t$$

$$\Delta\theta = w * \Delta t$$

$$x_t = x_{t-1} + \Delta x$$

$$y_t = y_{t-1} + \Delta y$$

$$\theta_t = \theta_{t+1} + \Delta\theta$$

(1) State a mathematical model for such a robot, assuming that its controls are subject to independent Gaussian noise.

Replace above $[v_1, v_2, w]$ to $[v_1', v_2', w']$ such that

$$v_1' = v_1 + sample(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)$$

$$v_2' = v_2 + sample(\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)$$

$$w' = w + sample(\alpha_7 v_1 + \alpha_8 v_2 + \alpha_9 w)$$

Where $\text{sample}(a)$ means sample from $X \sim N(0, a^2)$

(2) Provide a procedure for calculating $p(x_t \mid u_t, x_{t-1})$

$$x_t = x_{t-1} + ((v_1 + sample(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)) * \cos\theta - (v_2 + sample(\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)) * \sin\theta\ ) * \Delta t$$

$$= x_{t-1} + c + \Delta t * \text{sample}\left(\sqrt{(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)^2 + (\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)^2}\ \right)$$

Where $c = \left(v_1 * \cos\theta + v_2 \cos\left(\theta + \frac{\pi}{2}\right)\right) * \Delta t$

$$x_t \sim N(x_{t-1} + c,\ \ (\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)^2 + (\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)^2)$$

$$p(x_t \mid u_t, x_{t-1}) = \ N(x_{t-1} + c,\ \ (\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)^2 + (\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)^2)$$

(3) Provide a sampling procedure for sampling $x_t \sim (x_t \mid u_t,\ x_{t-1})$

$$p(x_t \mid u_t, x_{t-1}) = \ N(x_{t-1} + c,\ \ (\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)^2 + (\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)^2)$$

$$f(x_t) = N(x_{t-1} + c,\ \ (\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 w)^2 + (\alpha_4 v_1 + \alpha_5 v_2 + \alpha_6 w)^2)$$

sample algorithm, rejection sampling:

input $\mathrm{f, b}$

1. Repeat
2. $\quad \mathrm{x = rand(-b, b)}$
3. $\quad \mathrm{Y = rand}\left(0, \frac{1}{\sigma_0\sqrt{2\pi}}\right)$
4. Until $(\mathrm{y} \leq \mathrm{f(x)})$
5. Return $\mathrm{x + x_{t-1}} + c$