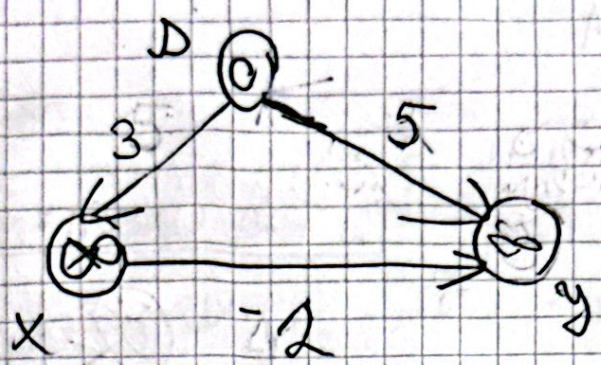
Homework 13

Problem 13.1

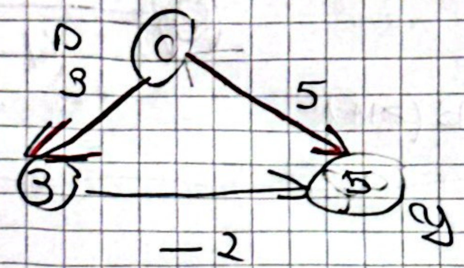
The algorithm in question is not a correct one. By adding a large constant to each edge weight does not lead to correct computations of the shortest path in graphs with negative edge weights. We can disprove the correctness of the algorithm by providing one valid counter example.

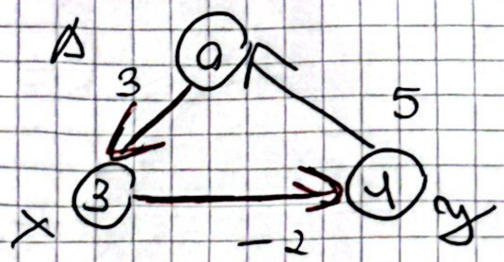
Suppose we have the following graph with the following ~~w~~ weights:

Simple graph with 3 nodes



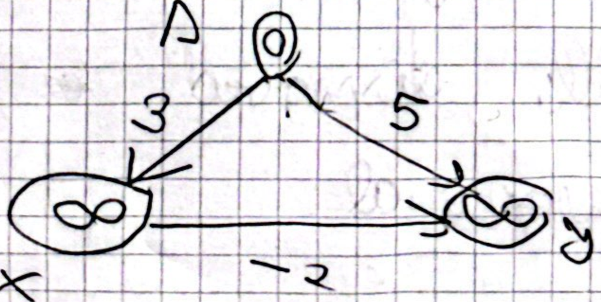
shortest path from s to y would be:



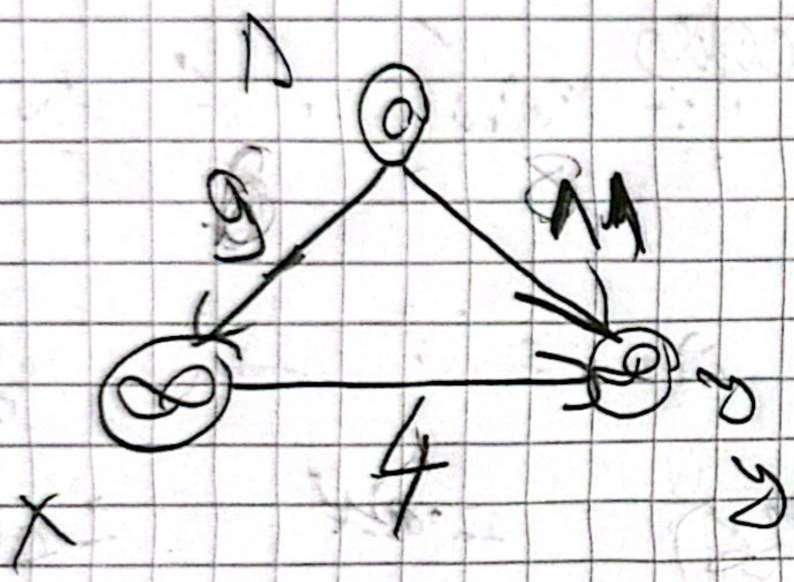


$s \to x \to y = 3 - 2 = 1$

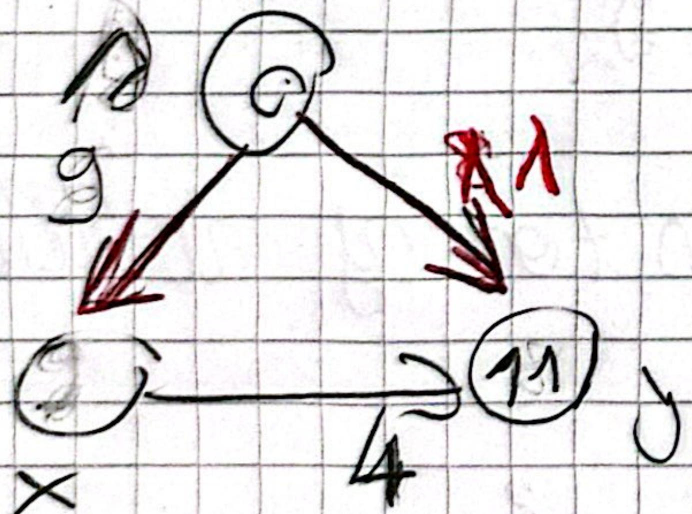
The friend is sugesting adding a large constant to make all edges positive.

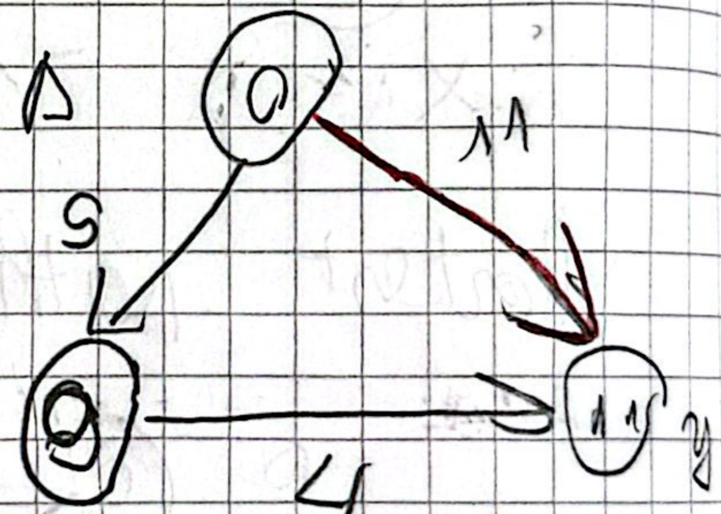


Adding a large constant (+6) to make each edge positive



The path chosen :





$9+4=13\ (>11)$

So, the chosen path will now be

$o \to y = 11$ which is incorrect in the

original ~~graph~~ graph.

$\Rightarrow$ We successfully disproved the

friend's proposal.

Problem 3 a) to formalize the puzzle into a graph problem, we need to define the nodes, the edges and also finding corelations between the puzzle rules and graph terms;

We can easily associate the vertices to a cell in the $n \times n$ grid.

Each node can be uniquely identified by its (i, j) coordinates $(i = \overline{0, n} ; j = \overline{0, n})$

$\Rightarrow$ We have $n \cdot n$ nodes in total $= n^2$;

From any given cell (i, j) with a given value $val = x[i][j]$, the player can move exactly val steps in one of the four cardinal directions (north (up), south (down), east (right), west (left)), only if the resulting position stays within the grid.

So, for a node (i, j), edges go to:

$(i + val, j)$ if $i + val < n$ (moving down, closer to n)

$(i - val, j)$ if $i - val \geq 0$ (moving up, closer to 0).

$(i, j + val)$ if $j + val < m$ (move right)

$(i, j - val)$ if $j - val \geq 0$ (moving left)

<u>Ex</u> $m = 4$

$\Rightarrow 4 \times 4$

$i \backslash j =$	0	1	2	3
0				
1				
2				
3				

<u>left to right</u> (0 to 3)

up to down ↓ (0 to 3)

each valid move (e.g. $(i+val, j)$, etc...) corresponds to a valid directed edge from node (i, j) to the resulting coordinate.

Formalizing the given problem into a graph problem:

- Graph: directed graph
- Nodes: cells in the $n \times n$ grid
- Edges: Directed edges from (i, j) to any valid coordinates that can be reached by moving $X[i][j]$ ($=$ val) steps in any direction
- Starting point: $(0, 0) \rightarrow$ top-left corner
- Edge Weight: Edges are not assigned a weight as each move is one step in the graph (regardless of val).

Objective: Finding a path from $(0, 0)$ to $(n-1, n-1)$ using just valid moves $\Longleftrightarrow$ Finding a path in the graph from the start node to the node that was chosen as an ending point.