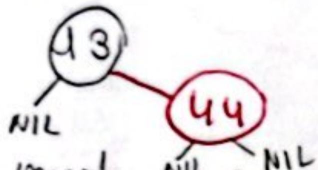
Homework 10

Problem 10.1.

(a) 1st insertion

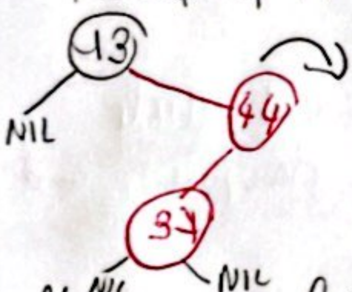
1) Base case: we inserted the root => we recolor it black

2) we insert 44 on the right side (44 > 13

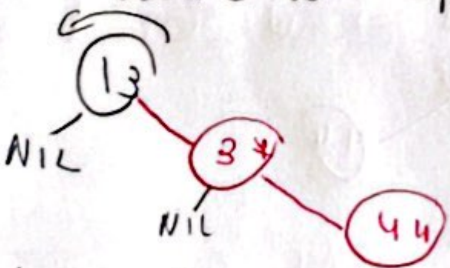


This insertion does not result in a violation

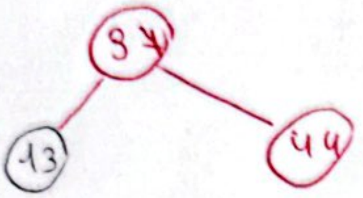
3) 37 on the left of 44



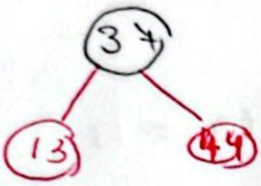
This results in a violation, as there are 2 red Nodes in a row. The uncle of the node we just inserted is black (the NIL node) and z (the node we just inserted) is a left child => case 3 => we rotate z's parent in the opposite direction of z



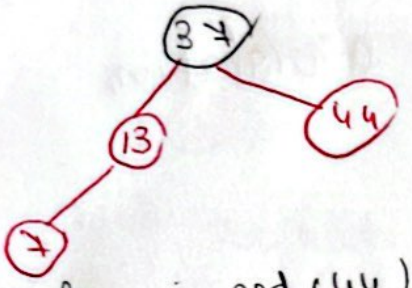
Now, we find ourselves in case 2. z is a right child and it has a black uncle => we rotate z's grandparent (13) in the opposite direction of z(44).



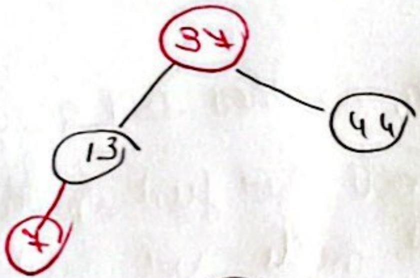
Finally, we recolor the previous parent (37) and the previous grand grandparent (13) of z(44):



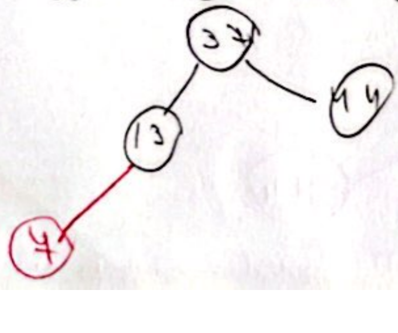
4) z on the left of 13



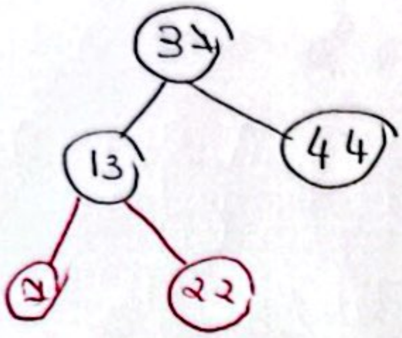
z's uncle is red (44) => Case 1: Red uncle
SOLUTION => Recolor z's parent, grandparent, and uncle:



We set z to 37 as it is the violating node this time (root must be black
when z is the root => we color it black !!

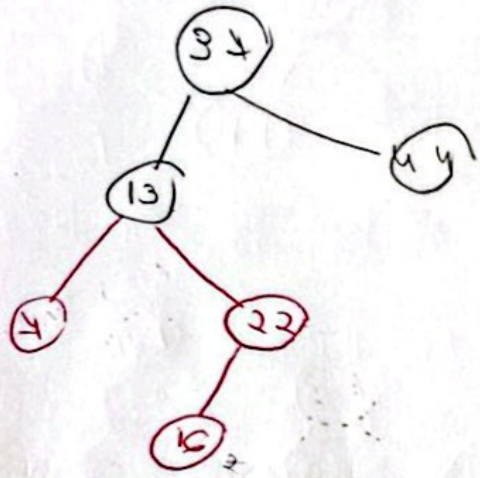


5/ We insert 22



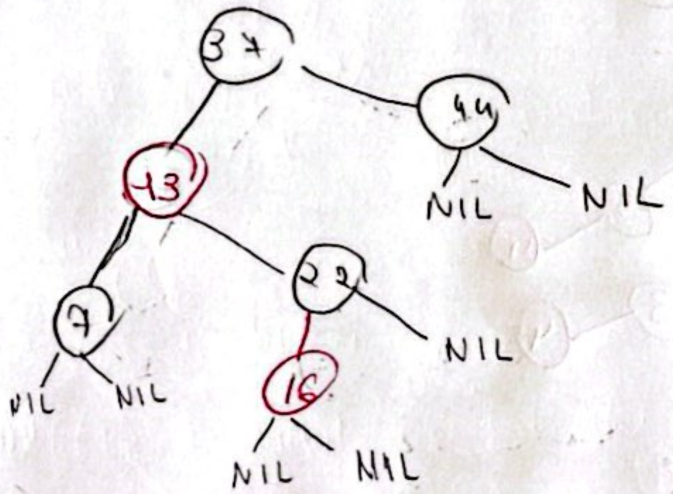
No rule is violated with this insertion;

6) We insert 16



We find ourselves in case 1 (z's uncle is red)

=> We recolor z's parent, grandparent and uncle

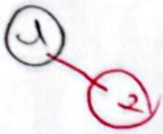


(b) Choosin 1 as the root

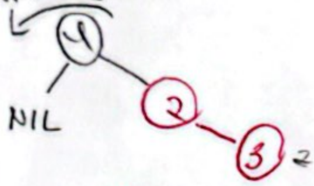
1) Insert 1

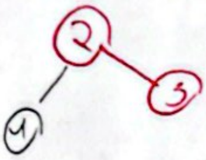
We recolor it black

2) Insert 2

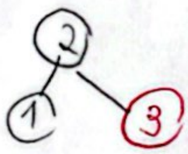


3) Insert 3

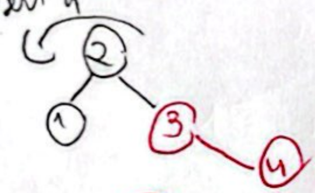


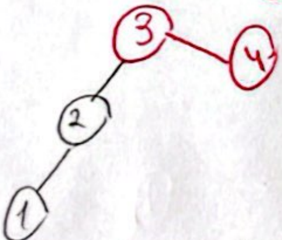


• We point z to the root (2) cause that is causing a violation

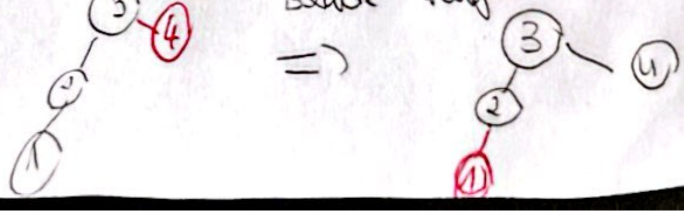


4) Insert 4

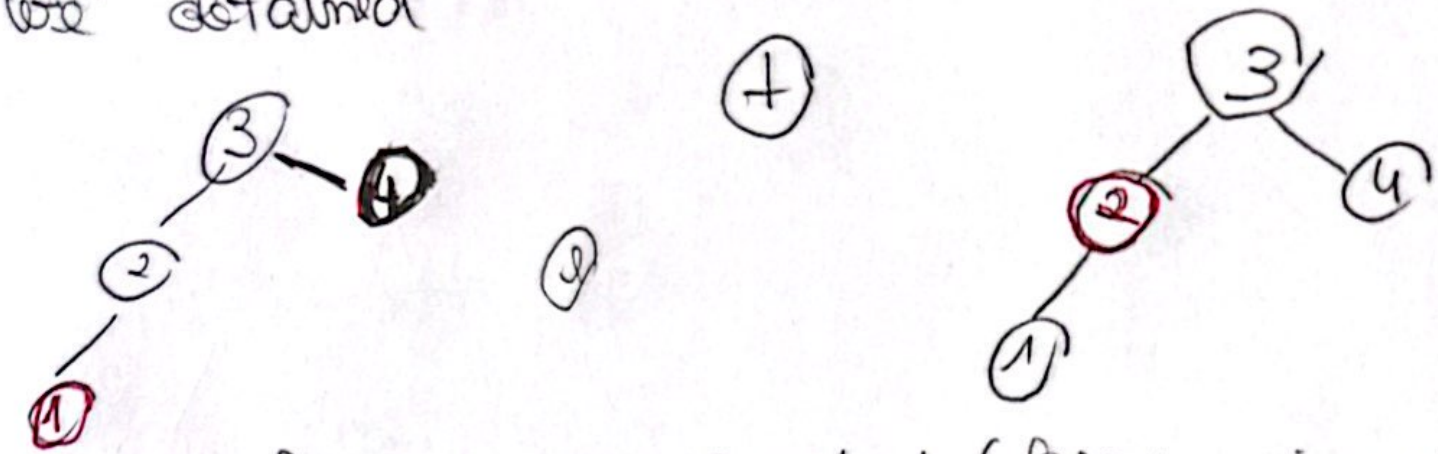




We point z to 3 cause that is causing the violation
black height is unbalanced



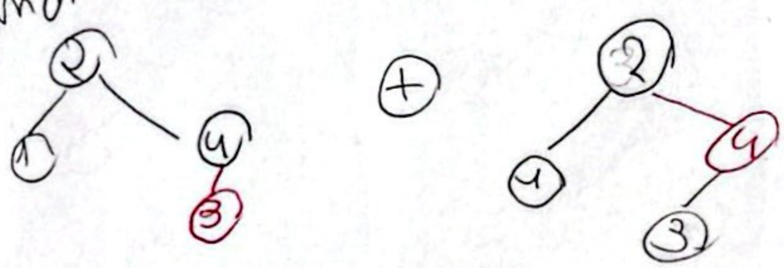
So we obtained



we can also reverse 2 and 1 (keeping in mind that e should be on the rightside of 1

We can notice that the extremes (1 and 4) are not valid roots. By following the same algorithm and bearing this in mind, we can build :

and



1 and 4 are not valid roots as the tree would be unbalanced :

(1) 1 as the root => all elements on it's right
(2) 4 as the root => all elements on it's left