



DATE:

NO:

2-d space

•  $(2,6) \rightarrow x^{(2)} = \begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \end{pmatrix}$

$$L_1 \text{ Norm} = \|x^{(1)} - x^{(2)}\|_1 = \sum_{i=1}^2 (x_i^{(1)} - x_i^{(2)})$$

$$= (1) + (4) = 5$$

$(1,2) \rightarrow x^{(1)} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix}$

$$L_2 \text{ Norm} = \|x^{(1)} - x^{(2)}\|_2 = \left( \sum_{i=1}^2 (x_i^{(1)} - x_i^{(2)})^2 \right)^{1/2}$$

$$= \sqrt{1^2 + 4^2}$$

$$= \sqrt{17}$$

We first look at distances in 2-dimensional space, we can then generalize to  $n$ -dimensional space.

$L_1$  - Norm Manhattan

Let  $\vec{p}, \vec{q}$  be two vectors in the  $n$ -dimension space  $\mathbb{R}^n$ . Then the  $L_1$ -norm

$$d_1(\vec{p}, \vec{q}) = \|\vec{p} - \vec{q}\|_1 = \sum_{i=1}^n |p_i - q_i|$$

where  $\vec{p}, \vec{q}$  are vectors of the form

$$\vec{p} = (p_1, p_2, \dots, p_n)$$

$$\vec{q} = (q_1, q_2, \dots, q_n)$$

$L_2$  - Norm Euclidean

Let  $\vec{p}, \vec{q}$  be defined similarly as  $L_1$ -Norm. Then the  $L_2$ -norm is

$$d_2(\vec{p}, \vec{q}) = \|\vec{p} - \vec{q}\|_2 = \left[ \sum_{i=1}^n (p_i - q_i)^2 \right]^{1/2}$$

$L_p$  - Norm Minkowski

Let  $\vec{p}, \vec{q}$  be defined similarly as  $L_1$ -Norm, then the  $L_p$  Norm is

$$d_p(\vec{p}, \vec{q}) = \|\vec{p} - \vec{q}\|_p = \left[ \sum_{i=1}^n (p_i - q_i)^p \right]^{1/p}$$