Department of Statistics
North Maharashtra University, Jalgaon
Course: M.Sc. Statistics (II)
Practical Sheet: ST-305 (Based on ST-301)
Topic: Consistency-I

1. Based on the random samples generated from $Cauchy(\mu, 1)$. Check the consistency of the estimators $T_1 = \overline{X}$ and $T_2 = Sample\ Median$ for the parameter μ .Construct the following table.

Sample size (n)	$P(\lvert T_1 - \mu\rvert < 0.1)$	$P(\lvert T_2 - \mu\rvert < 0.1)$

2. Check the consistency of $T_1 = X_{(n)}, T_2 = \overline{X}$ and $T_3 = 2\overline{X}$ in the case of $Uniform(0, \theta)$ based on the random sample. Suggest one more consistent estimator for θ. Compare T_1 , T_2 and T_3 in terms of MSE.

3. Demonstrate the consistency of sample mean and variance for the population mean and variance respectively of Normal distribution with parameters μ and σ^2 based on random samples.

4. Based on the random samples, check the consistency of empirical distribution function at t=2 for the $F_X(t)$, where, F(.) is cdf of Exponential distribution with parameter θ.

5. Let X be a random variable having Poisson distribution with parameter λ. Suggest one consistent estimator for $e^{-\lambda}$. Based on the random samples, demonstrate the consistency of the estimator suggested by you.

Department of Statistics
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Course: M.Sc. Statistics (II)
Practical Sheet: ST-305 (Based on ST-301)
Topic: Moment Estimators and Consistency

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1. Obtain method of moment estimates of n and p of $B(n,p)$. Based on the samples drawn from $B(n,p)$, demonstrate the consistency of the estimators and obtain asymptotic distribution of the same.

2. Consider Gamma distribution, with pdf,

$$f(x,\alpha,\beta) = \begin{cases} \frac{1}{\Gamma\alpha\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} & ,x,\alpha,\beta > 0 \\ 0 & ,o.w. \end{cases}$$

Obtain moment estimates of α and β. Demonstrate the consistency of the estimators and obtain asymptotic distribution of the same.

3. Consider a Beta distribution of first kind, with pdf,

$$f(x,a,b) = \begin{cases} \frac{1}{\beta(a,b)} x^{a-1}(1-x)^{b-1} & ,0 < x < 1 \\ 0 & ,o.w. \end{cases}$$

Obtain moment estimates of a and b. Demonstrate the consistency of the estimators and obtain asymptotic distribution of the same.

Department of Statistics
North Maharashtra University, Jalgaon
Course: M.Sc. Statistics (II)
Practical Sheet: ST-305 (Based on ST-301)
Topic: Consistency and CAN

1. Demonstrate the consistency of sample mean and variance for the population mean and variance respectively of Normal distribution with parameters μ and σ^2 based on random samples and check whether both estimators are asymptotic normally distributed.

2. Let X be a random variable having Poisson distribution with parameter λ. Suggest two consistent estimators for λ. Based on the random samples, demonstrate the consistency and check asymptotic normality of the estimators suggested by you.

3. Let $f(x,\theta) = (1/2\theta) * \exp(-|x|/\theta)$, $-\infty < x < \infty$. Based on the random samples from this distribution,
   a) Show that $\overline{X}$ is not consistant for θ
   b) Show that $\frac{1}{n}\sum x_i^2$ is CAN.
   c) Show that $\frac{1}{n}\sum |x_i|$ is CAN.

   Compare estimators in sub question b and c

4. Let X be a random variable having $E(\mu, \sigma)$. Based on the random samples from this distribution, check the consistency and CAN property of following estimators:
   a) $\overline{X}$ for μ
   b) $X_{(1)}$ for μ
   c) Sample variance for σ
   d) $\frac{1}{n}\sum_{i=1}^{n}(X_i - X_{(1)})$ for σ

Department of Statistics
North Maharashtra University, Jalgaon
Course: M.Sc. Statistics (II)
Practical Sheet: ST-305 (Based on ST-301)
Topic: Invariance property of consistent, CAN and percentile estimators

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1. Let X be a random variable having $N(\mu, \sigma^2)$ distribution. Based on the samples of size 25, 50, 100 and 500, check following estimators are consistent and CAN for corresponding parametric functions:
   a) $\bar{X}^2$ for μ^2
   b) $\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2\right)^2$ for σ^4
   c) $\bar{X}^2 + \left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2\right)^2$ for $\mu^2 + \sigma^4$
2. Let X be a random variable having $Poisson\ (\lambda)$. Based on the samples of size n obtain CAN estimator of
   a) $P(X = 0)$
   b) $P(X = 1)$
3. Let X be a random variable having $Binomial\ (n, p)$. Based on the samples of size n obtain CAN estimator of
   a) $P(X = 0)$
   b) $P(X = 1)$
   c) $p(1 - p)$
   d) Demonstrate the consistency and asymptotic normality of the estimators suggested by you.
4. Let X be a random variable having $Binomial\ (n, p)$. Based on the samples of size n obtain CAN estimator of
   a) $P(X = 0)$
   b) $P(X = 1)$
   c) $P(X = 2)$
   d) $\left(\frac{q}{p}\right)^2$
5. Obtain percentile estimator of θ in double exponential $(\theta, 1)$. Check whether it is CAN for θ.

Department of Statistics
North Maharashtra University, Jalgaon
Course: M.Sc. Statistics (II)
Practical Sheet: ST-305 (Based on ST-301)
Topic: Method of Scoring

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1. Let X be a random variable having $N(\mu, 1)$ distribution. Generate a random sample of size 25 from $N(4,1)$.Starting with initial value T_1=sample median, use method of scoring and Newton-Raphson method to obtain MLE of μ. Compare MLE's obtained by both the methods.
2. Let X be a random variable having $C(\mu, 1)$ distribution. Generate a random sample of size 25 from $C(0, 1)$.Starting with initial value T_1—sample median, use method of Scoring to obtain MLE of μ.
3. A biologist obtained a sample of number of eggs in the unopened flower heads of the black Knapweed by the Knapweed fly. The flower heads in which no eggs were laid were not included in the sample. A truncated Poisson distribution(λ) with zero class missing was used as a model since the event of a flower head without any egg could not be ascribed to the event $X = 0$ under the complete Poisson model. Obtain MLE of λ.

No. of eggs per flower head	1	2	3	4	5	≥ 6
No. of flower heads	96	32	9	7	1	0

4. Let X be a random variable having truncated normal distribution left truncated at $X = 0$ with location parameter distribution μ and scale parameter 1. Based on the random sample of size 25, $\mu = 0$ and starting with initial value T_1=sample mean and T_2=sample median, use method of scoring to obtain MLE of μ. Compare the number of iterations to obtain MLE in both the cases.

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Course: M.Sc. Statistics (II)

Practical Sheet: ST-305 (Based on ST-301)

Topic: Asymptotic Confidence Intervals (ACI)

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1. Let X be a random variable having $Poisson\ (\lambda)$. Based on the random sample of size $n = 20$ from $Poisson(2)$, obtain 95% ACI for λ. using:

   a) VST b) Pivotal method.

2. Let X be a random variable having $B(1, \theta)$. Based on the random sample of size $n = 25$ from $B(1, 0.4)$, obtain 95% ACI for θ using:

   a) VST b) Pivotal method.

3. Let X be a random variable having $C(\theta, 1)$. Based on the random sample of size $n = 25$ from $C(4, 1)$, obtain 95% ACI for θ using Pivotal method.

4. Let X be a random variable having $Exp(mean = \theta)$. Based on the random sample of size $n = 25$ from $Exp\ (3)$, obtain 95% ACI for θ using:

   a) VST b) Pivotal method.

5. Let X be a random variable having $Laplace(\theta, 1)$. Based on the random sample of size $n = 25$ from $Laplace(2.5,1)$, obtain 95% ACI for θ using Pivotal method.
