Derived equations for the polarization/sensitive FP microscopy

The electric fields passing through an anisotropic sample encounter polarization and the magnitude and phase will be changed. This phenomenon can be interpreted as:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \text{ (1)}$$

The Jones matrix includes the corresponding matrices for polarizer, retarder and rotator

$$J' = J(-\theta)\, J_R\, J(\theta) \text{ (2)}$$

$$J' = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix} \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \text{ (3)}$$

J has been defined for linear polarizer here.

Accordingly, the Jones matrix for a rotated polarizer is:

$$J_p = \begin{pmatrix} p_x cos^2\theta + p_y sin^2\theta & (p_x - p_y) sin\theta cos\theta \\ (p_x - p_y) sin\theta cos\theta & p_x sin^2\theta + p_y cos^2\theta \end{pmatrix} \text{ (4)}$$

Therefore, for an ideal linear polarizer (each channel in this paper), it simplifies as:

$$J_p(\theta) = \begin{pmatrix} cos^2\theta & sin\theta cos\theta \\ sin\theta cos\theta & sin^2\theta \end{pmatrix} \text{ (5)}$$

By considering a retardation phase as δ, the Jones matrix of a retarder in the left/handed circular polarization satate, will be:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} e^{\frac{-j\delta}{2}} & 0 \\ 0 & e^{\frac{j\delta}{2}} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \text{ (5)}$$

$$J_R(\delta) = \begin{pmatrix} e^{\frac{-j\delta}{2}} & 0 \\ 0 & e^{\frac{j\delta}{2}} \end{pmatrix}$$

It means that the total phase shift δ is halve into two orthogonally polarized components by assigning a phase advance of $\delta/2$ to the fast y axis and phase retardation of $-\delta/2$ to the slow x axis. It should be considered that these values have inverse sign in the case of right-handed circular polarization.

According to (2),

$$J(\delta,\theta) = \begin{pmatrix} \cos\left(\frac{\delta}{2}\right) - jsin\left(\frac{\delta}{2}\right)\cos(2\theta) & -jsin\left(\frac{\delta}{2}\right)\sin(2\theta) \\ -jsin\left(\frac{\delta}{2}\right)\sin(2\theta) & \cos\left(\frac{\delta}{2}\right) + jsin\left(\frac{\delta}{2}\right)\cos(2\theta) \end{pmatrix} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix}$$

Intensity is generally defined as $I' = E^{*}.E$

And hence, we have:

Derived equations for the polarization/sensitive FP microscopy

Based upon the paper, $E_{in} = \sqrt{\frac{I}{2}}[1\ i]^T$ that for the LH-CP should be $E_{in} = \sqrt{\frac{I}{2}}[1\quad -i]^T$. Otherwise equations of 5-8 are not verified.

For 0 deg:

$$E_x = J_{xx} + j\,J_{xy}\,, E_y = 0$$

$$I_0 = \left(\frac{I}{2}\right).|E_x|^2 = (I/2)(1 - sin\delta \sin(2\theta))$$

Similarly for the other polarization states, we have:

For 45 deg:

$$I_{45} = \left(\frac{I}{2\sqrt{2}}\right)\left|\begin{pmatrix} J_{xx} + J_{yx} - j(J_{xy} + J_{yy}) \\ J_{xx} + J_{yx} - j(J_{xy} + J_{yy}) \end{pmatrix}\right|^2 = (I/2)(1 + sin\delta \cos(2\theta))$$

For 90 deg:

$$E_x = 0\,, E_y = J_{yx} - j\,J_{yy}$$

$$I_{90} = \left(\frac{I}{2}\right).\left|E_y\right|^2 = (I/2)(1 + sin\delta \sin(2\theta))$$

For 135 deg:

$$I_{135} = \left(\frac{I}{2\sqrt{2}}\right)\left|\begin{pmatrix} (J_{xx} - J_{yx}) + j(J_{xy} - J_{yy}) \\ -(J_{xx} - J_{yx}) - j(J_{xy} - J_{yy}) \end{pmatrix}\right|^2 = (I/2)(1 - sin\delta \cos(2\theta))$$

Until now, we do not have information about retardation (δ) and rotation (θ). Furthermore, the intensity of incoming wave (I) has not been defined. To remove the dependency to I, two auxiliary variables are defined as v_1 and v_2 .

$$I_{90} + I_0 = I$$

$$I_{135} + I_{45} = I$$

$$I_{90} - I_0 = I(sin\delta \sin(2\theta))$$

$$I_{45} - I_{135} = I(sin\delta \cos(2\theta))$$

$$v_1 = (I_{90} - I_0)/(I_{90} + I_0)\,, \qquad v_2 = (I_{45} - I_{135})/(I_{45} + I_{135})$$

$v_1^2 + v_2^2 = sin\delta^2$

$$\frac{v_1}{v_2} = \tan(2\theta)$$

Derived equations for the polarization/sensitive FP microscopy