Edges of Gaps

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1 Purpose

For a slab of BLG in an STM, this document describes how to fit the tip-sample distance d_1 and backgate-sample distance d_2.

2 Variable Definitions

The top and bottom gate voltages are V_T and V_B respectively. The top and bottom potentials on the layers of BLG are V_1 and V_2 respectively. We define $V_\pm = \frac{1}{2}(V_1 \pm V_2)$.

The distance between the BLG layers is $d \sim 0.3$ nm. The distance and permittivity between the the top gate and sample are d_1 and ϵ_1. Between the bottom gate and sample, these are d_2 and ϵ_2.

3 Shadow Gap

The 'shadow gap' is the gap caused by the Fermi Level $\epsilon_F = |e|V_+ = |e|(V_1 + V_2)$ of the BLG being pushed in to the gap. This means that the total charge $n = 0$.

An expression from the band structure of BLG yields.

$$\epsilon_F^2 = \frac{(\hbar^2 v_F^2 n\pi)^2 + \gamma_1^2 u^2}{4(\gamma_1^2 + u^2)}$$

where $u = -|e|(V_1 - V_2) = -2|e|V_-$ is the interlayer potential energy difference. We therefore find that at the edge of the shadow gap,

$$\epsilon_F = \pm\frac{\gamma_1 u}{2\sqrt{\gamma_1^2 + u^2}}$$

$$|e|V_+ = \pm\frac{\gamma_1(-2|e|V_-)}{2\sqrt{\gamma_1^2+(2|e|V_-)^2}}$$

$$V_+ = \pm\frac{\gamma_1 V_-}{\sqrt{\gamma_1^2+(2|e|V_-)^2}}$$

where $\gamma_1 \sim 0.4$ eV. This relates the Fermi Level directly to the gap. For most practical purposes $|e|V_- << \gamma_1$, so $V_+ \sim \pm V_-$.

In this case then if,

- $V_+ \sim +V_- \implies V_1 \sim 2V_- \sim 2V_+$ and $V_2 \sim 0$
- $V_+ \sim -V_- \implies V_1 \sim 0$ while $V_2 \sim -2V_+ \sim 2V_-$.

Another expression relating the Fermi Level and gap comes from the geometry and electrostatic equations. The first order potential energy difference is given by

$$u = |e|\frac{d}{2}\left(\frac{V_B - V_2}{d_2} - \frac{V_T - V_1}{d_1}\right)$$

$$V_- = \frac{d}{4}\left(\frac{V_T - V_1}{d_1} - \frac{V_B - V_2}{d_2}\right) \tag{1}$$

Lets assume the Fermi level is at the edge of the gap, so that $V_+ \sim \pm V_-$. This implies

$$V_+ \sim +V_- \implies V_T = (2 + \frac{4d_1}{d})V_- + \frac{d_1}{d_2}V_B$$

$$V_+ \sim -V_- \implies V_T = (\frac{4d_1}{d} - \frac{2d_1}{d_2})V_- + \frac{d_1}{d_2}V_B$$

Double check my algebra before using the above expresssions.

By considering the electrodes and BLG layers as parallel plate capacitors, the total charge is given by

$$\sigma = -\left(\frac{(V_T - V_1)\epsilon_1}{d_1} + \frac{(V_B - V_2)\epsilon_2}{d_2}\right)$$

or when the Fermi level is at the edge of the gap

$$\frac{(V_T - V_1)\epsilon_1}{d_1} = \frac{(V_2 - V_B)\epsilon_2}{d_2} \tag{2}$$

therefore

$$V_+ \sim +V_- \implies V_T = -V_B \frac{d_1 \epsilon_2}{\epsilon_1 d_2} + 2V_-$$

$$V_+ \sim -V_-, \implies V_T = (2V_- - V_B) \frac{d_1 \epsilon_2}{\epsilon_1 d_2}$$

Double check my algebra before using the above expressions.

Combining the two linear expressions for V_T as a function of V_B,

$$V_+ \sim +V_- \implies V_T = -\frac{d}{2d_2} \left[1 + (1 + \frac{2d_1}{d}) \frac{\epsilon_1}{\epsilon_2} \right] V_B$$

$$V_+ \sim -V_- \implies V_B = \frac{d_2}{d_1} \left[1 - (\frac{2d_2}{d} - 1) \frac{\epsilon_1}{\epsilon_2} \right] V_T$$

Again, double check this before using