Feynman Path Integral

The Feynman path integral is an alternative equation to the Schrodinger equation to give the position of a particle with time-evolution.
We will explore the probability distribution of a ground state wave function in one dimension using the Feynman Integral

1. LEAST POSSIBLE ACTION

In classical mechanics, the trajectory of a particle would be the path where the **classical action**, given in Eq. (1), is a minimum, where *L* is the Lagrangian.

In quantum mechanics, a particle does not travel along one defined path between two points. Therefore, we consider all possible paths a particle can take. Feynman used a '**propagator**' to describe the state of a particle with time-evolution. All paths contribute equally to the propagator; however, each path is weighted differently dependent on its phase. This phase is proportional to the least possible action [1].

To compute the integral for the **least possible action**, we discretise the continuous path into time intervals. Therefore, the equation for the least possible action used in the propagator is Eq. (2).

$$S[x] \equiv \int_{t_i}^{t_f} dt\, L(x,\dot{x}) \equiv \int_{t_i}^{t_f} dt \left[\frac{m\dot{x}(t)^2}{2} + V\big(x(t)\big)\right] \quad (1)$$

$$S_{lat}[x] = \sum_{j=0}^{N-1}\left[\frac{m}{2a}\left(x_{j+1}-x_j\right)^2 + aV(x_j)\right] \quad (2)$$

REFERENCES

Feynman R. & Hibbs A. R., 1965 *Quantum Mechanics and Path Integrals, Emended Edition,* 2010

Lapage, P. G., 2005 'Lattice QCD for Novices'. http://arxiv.org/pdf/hep-lat/0506036v1.pdf

Perepelitsa D. V., "Path Integrals in Quantum Mechanics", 2007

2. DISCRETISED TIME

Continuous paths of the particle is approximated by considering a particle moving in **discretised time intervals** between points. This is visualised through Fig. (1) where 4 different random paths are shown. The particle moves to a new position every discretised time step.

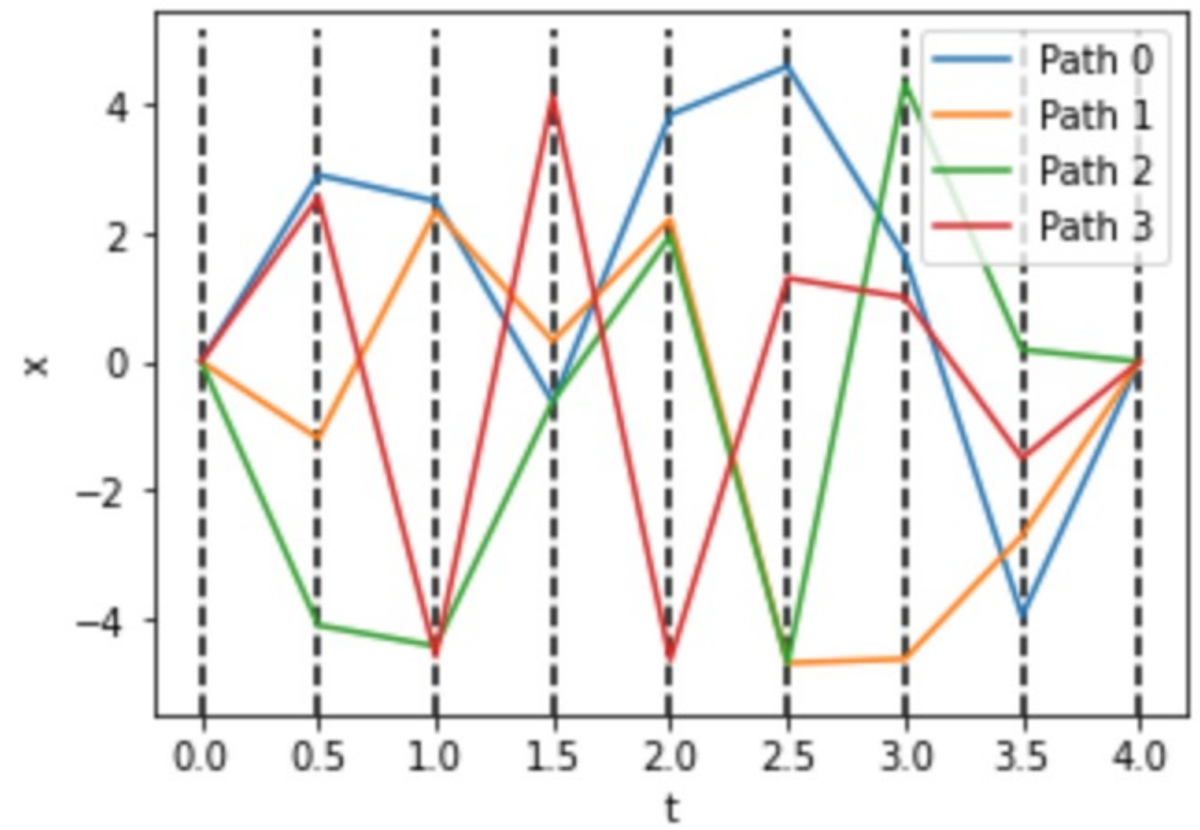

Fig. 1: Plot of 4 different random paths a particle can travel. Each path has 8 points it travels through discretised time

3. THE PROPAGATOR

The **amplitude** of all possible paths are given by the propagator in Eq. (3) where *A* is the **normalisation factor**. To compute the multidimensional integral, we use the **Monte Carlo method** using a module is Python called Vegas. [2]

$$\hat{G}(x,T,a) = A\int_{-\infty}^{\infty} dx_1 dx_2 \dots dx_{N-1} e^{-S[x(t)]} \quad (3)$$

4. PROBABILITY DISTRIBUTION OF A GROUND STATE WAVE FUNCTION

Consider a **particle in the ground state** moving in one-dimension. We can calculate its probability distribution using the Feynman integral [3]. The **probability distribution** is given by Eq. (4). We also define the starting and final position to equal each other in the propagator. The probability distribution of the exact and computed result is shown in Fig. 2. The time period is defined as *T=4*, and the particle travels through 8 random points for each path between this time interval giving a grid spacing of *a=4/8=1/2.*

$$\left|\Psi_{E_0}(x)\right|^2 \approx \frac{\hat{G}(x, T, a)}{\int dx \hat{G}(x, T, a)} \quad (4)$$

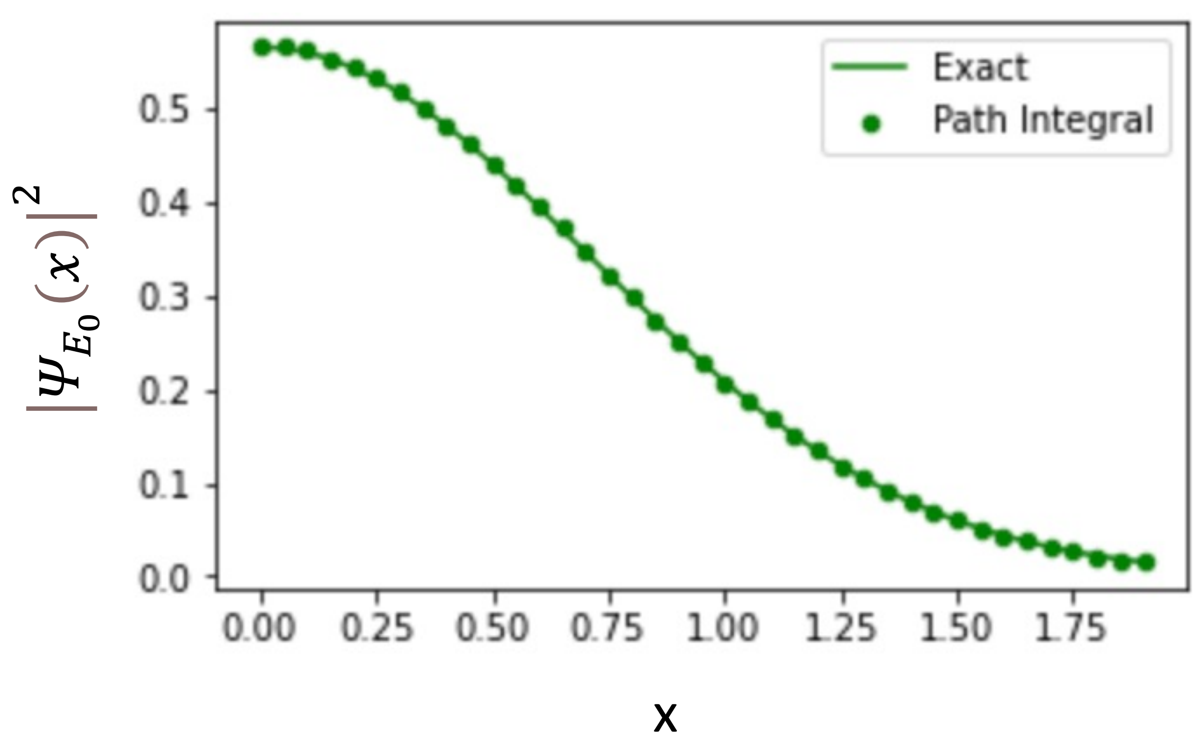

Fig. 2: Plot of the probability distribution of a particle in the ground state in one dimension between the positions 0 and 2.