hw2

October 3, 2017

1 ML Homework 2

Gede Ria Ghosalya - 1001841

```
In [1]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt

        import theano
        import theano.tensor as T
```

```
In [2]: csv='https://www.dropbox.com/s/oqoyy9p849ewzt2/linear.csv?dl=1'
        data = np.genfromtxt(csv, delimiter=',')
```

1.0.1 Q1. Ridge Regression with Offset

---

a) Let the first 10 entries of the data set be the validation set, and the last 40 entries to be the training set. Concatenate their features into matrices vX and tX, and their responses into vectors vY and tY. Print the shapes of vX, tX, vY, tY.

```
In [3]: vdata = data[0:10,:]
        tdata = data[10:,:]
        vX = vdata[:,1:]
        vY = vdata[:,0]
        tX = tdata[:,1:]
        tY = tdata[:,0]

        print("""
        vX shape:{vx}
        vY shape:{vy}
        tX shape:{tx}
        tY shape:{ty}
        """\
        .format(vx=vX.shape,
                vy=vY.shape,
                tx=tX.shape,
                ty=tY.shape))
```

```
vX shape:(10, 4)
vY shape:(10,)
tX shape:(40, 4)
tY shape:(40,)
```

---

b) Write a program in Theano that performs ridge regression by using a regularization penalty of $0.5\lambda||w||\hat{}2$ with λ=0.15. You may use the source codes from Homework 1. Print the resulting value of w. Which feature may we assume to be irrelevant?

```
In [4]: def ridge_regression(X, Y, lamb=0.15, learn_rate=0.5):
            d = X.shape[1]
            n = X.shape[0]
            x = T.matrix(name='x')
            y = T.vector(name='y')
            w = theano.shared(np.zeros((d,1)),name='w')
            regularizer = 0.5*lamb*(T.dot(w[:-1].T,w[:-1])[0,0])
            point_loss = 0.5*(T.dot(x,w).T-y)**2
            risk = (T.sum(point_loss))/n + regularizer
            grad_risk = T.grad(risk, wrt=w)
            train_model = theano.function(inputs=[],
                                          outputs=risk,
                                          updates=[(w, w-learn_rate*grad_risk)],
                                          givens={x:X,y:Y})
            training = [train_model() for i in range(100)]
            return w.get_value()

        w = ridge_regression(tX, tY)
        wp = ridge_regression(tX, tY, 0.5)
        print('with lambda=0.15, w:\n\n',w,'\n\nwith lambda=0.5, w:\n\n', wp)
```

```
with lambda=0.15, w:

 [[-0.53590673]
 [ 1.20293332]
 [ 0.04334568]
 [-1.85492455]]

with lambda=0.5, w:

 [[-0.4577457 ]
 [ 0.9361236 ]
 [ 0.0534599 ]
 [-1.81257696]]
```

At this point, it is hard to tell which feature is irrelevant

---

c) Compute the optimal solution using BFGS optimizer from `scipy`.

```
In [5]: from scipy.optimize import fmin_l_bfgs_b as minimize

        def ridgecost(w,X,Y,lamb):
            n = X.shape[0]
            regularizer = 0.5*lamb*np.dot(w[:-1].T,w[:-1])
            point_loss = 0.5*(np.dot(X,w).T-Y)**2
            risk = (np.sum(point_loss))/n + regularizer
            j = np.identity(X.shape[1])
            j[-1,-1] = 0
            grad = lamb*np.dot(j,w) + np.dot(np.dot(X.T,X),w)/n - np.dot(X.T,Y)/n
            return risk, grad

        d = tX.shape[1]
        w = np.zeros((d,1))

        optx, cost, messages = minimize(ridgecost, w, args=[tX,tY,0.15])
        print(optx)
```

```
[-0.53591305  1.20293347  0.04332715 -1.85492236]
```

---

d) Write a function `ridge_regression(tX, tY, l)` that takes the training features, training responses and regularizing parameter λ, and output the exact solution w for ridge regression with offset. Print the resulting value of w for λ=0.15.

```
In [6]: def ridge_regression(X, Y, l):
            n, d = X.shape
            j = np.identity(d)
            j[-1,-1] = 0
            j_matrix = n*l*j
            xtx = np.dot(X.T, X)
            term = j_matrix + xtx
            in_xtx = np.linalg.inv(term)
            xty = np.dot(X.T, Y)
            exact = np.dot(in_xtx, xty)
            return exact


        we = ridge_regression(tX, tY, 0.15)
        print(we)
```

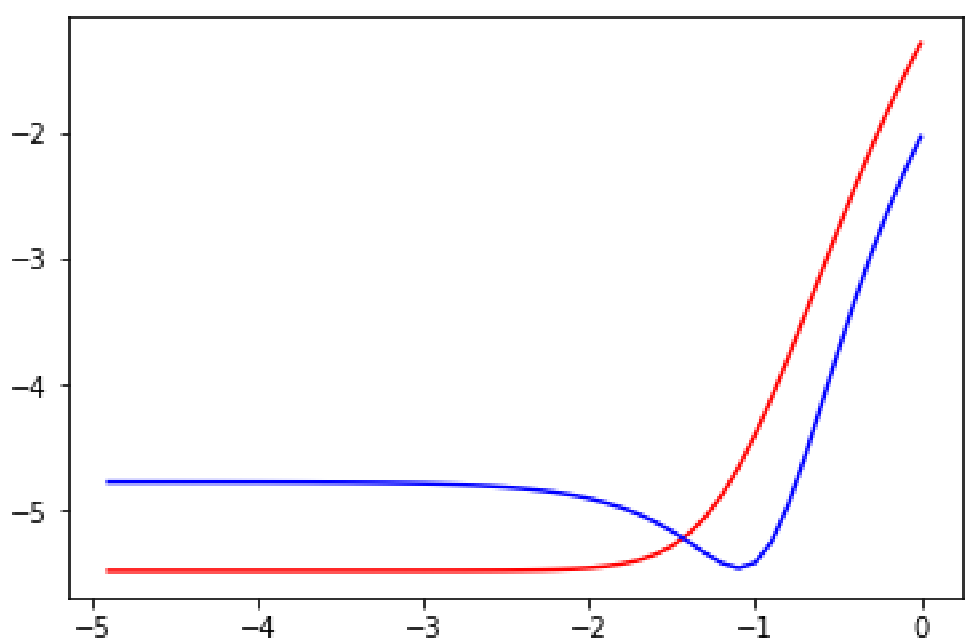
```
[-0.53590673  1.20293332  0.04334568 -1.85492455]
```

---

e) Use the following code to plot graphs of the validation loss and training loss as λ varies on a logarithmic scale from λ=10e-5 to λ=10e0.

```
In [7]: tn = tX.shape[0]
        vn = vX.shape[0]
        tloss = []
        vloss = []
        index = -np.arange(0,5,0.1)
        for i in index:
            w = ridge_regression(tX, tY, 10**i)
            tloss = tloss+[np.sum((np.dot(tX,w)-tY)**2)/(2*tn)]
            vloss = vloss+[np.sum((np.dot(vX,w)-vY)**2)/(2*vn)]

        import matplotlib.pyplot as plt
        plt.plot(index, np.log(tloss),'r')
        plt.plot(index, np.log(vloss),'b')
```

```
Out[7]: [<matplotlib.lines.Line2D at 0x7fba4ff34588>]
```



Looking at the validation line (blue), it can be seen that the validation loss is lowest when λ**=10e-5**.

1.0.2 Q2. Clustering

```
In [8]: import numpy.random as rng
        import matplotlib.pyplot as plt
        import matplotlib.image as mpimg
        from sklearn.cluster import KMeans
        from sklearn.metrics import pairwise_distances_argmin

        n_colors = 32
        pic = 'http://www.dropbox.com/s/bmwwfct2qxjfje4/sutd.png?dl=1'
        img = mpimg.imread(pic)
        img = img[:,:,:3]

        w, h, d = tuple(img.shape)
        image_array = np.reshape(img, (w * h, d))
```

```
In [9]: def recreate_image(palette, labels, w, h):
            d = palette.shape[1]
            image = np.zeros((w,h,d))
            label_idx = 0
            for i in range(w):
                for j in range(h):
                    image[i][j] = palette[labels[label_idx]]
                    label_idx += 1
            return image
```

---

a) Sampling 1000 pixels at random

```
In [10]: rand_index = np.random.randint(0, w*h, 1000)
         rand_img_sample = image_array[rand_index]

         rand_cluster = KMeans(n_clusters=32).fit(rand_img_sample)
         kmeans_palette = rand_cluster.cluster_centers_
         kmeans_labels = rand_cluster.predict(image_array)
```

---

b) Sampling 32 pixels at random

```
In [11]: rand_index = np.random.randint(0, w*h, 1000)
         random_palette = image_array[rand_index]
         pdam = pairwise_distances_argmin
         random_labels = pdam(image_array, random_palette)
```

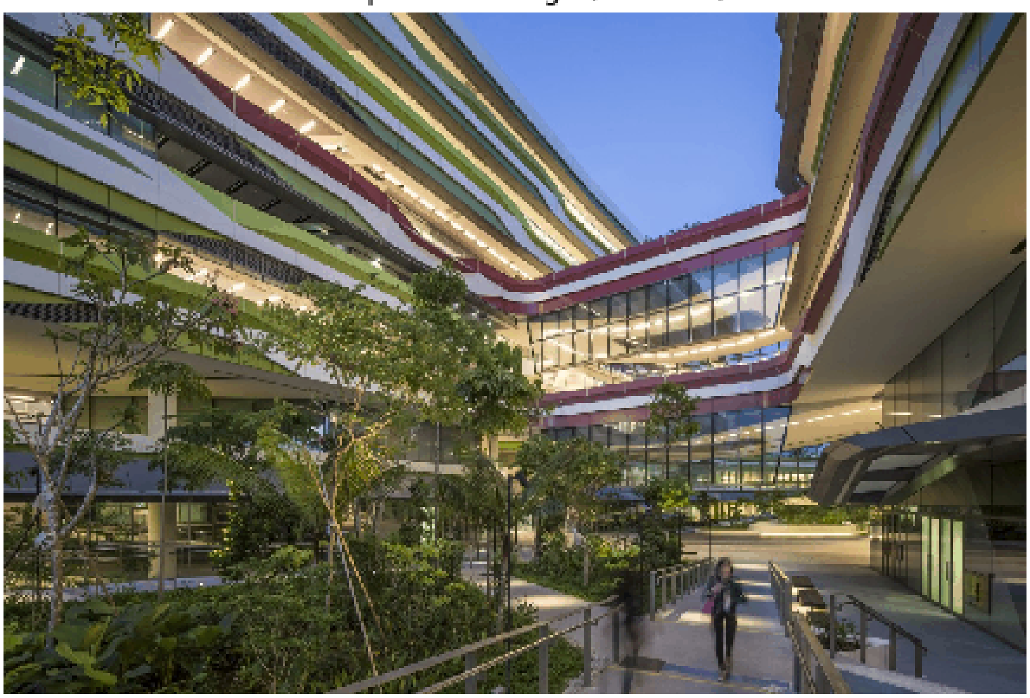
Resulting plot:

In [12]:
```
plt.figure(1)
plt.clf()
ax = plt.axes([0,0,1,1])
plt.axis('off')
plt.title('Original Image (16.8 million colors)')
plt.imshow(img)

plt.figure(2)
plt.clf()
ax = plt.axes([0,0,1,1])
plt.axis('off')
plt.title('Compressed Image (K-Means)')
plt.imshow(recreate_image(kmeans_palette, kmeans_labels, w, h))

plt.figure(3)
plt.clf()
ax = plt.axes([0,0,1,1])
plt.axis('off')
plt.title('Compressed Image (Random)')
plt.imshow(recreate_image(random_palette, random_labels, w, h))

plt.show()
```

Original Image (16.8 million colors)

Compressed Image (K-Means)

Compressed Image (Random)



c) See end of document.

1.0.3 Q3. Logistic Regression

```
In [13]: import numpy as np
         import pandas as pd

         X_data = pd.read_csv('kaggle_train.csv')
         X_test = pd.read_csv('kaggle_test.csv')
         X_valid = X_data.sample(frac=0.2, random_state=200)
         X_train = X_data.drop(X_valid.index)
         Y_data = X_data['Survived']
         Y_train = X_train['Survived']
         Y_valid = X_valid['Survived']
         ID_test = X_test['PassengerId']
```

```
In [14]: from IPython.display import display
         print("\nKaggle's train.csv")
         display(X_data.head())
         display(X_data.describe())
         print("\nKaggle's test.csv")
         display(X_test.head())
         display(X_test.describe())
```

```
Kaggle's train.csv


   PassengerId  Survived  Pclass  \
0            1         0       3
1            2         1       1
2            3         1       3
3            4         1       1
4            5         0       3

                                                Name     Sex   Age  SibSp  \
0                            Braund, Mr. Owen Harris    male  22.0      1
1  Cumings, Mrs. John Bradley (Florence Briggs Th...  female  38.0      1
2                             Heikkinen, Miss. Laina  female  26.0      0
3       Futrelle, Mrs. Jacques Heath (Lily May Peel)  female  35.0      1
4                           Allen, Mr. William Henry    male  35.0      0

   Parch            Ticket     Fare Cabin Embarked
0      0         A/5 21171   7.2500   NaN        S
```

```
1      0          PC 17599  71.2833   C85        C
2      0  STON/O2. 3101282   7.9250   NaN        S
3      0            113803  53.1000  C123        S
4      0            373450   8.0500   NaN        S

       PassengerId    Survived      Pclass         Age       SibSp  \
count   891.000000  891.000000  891.000000  714.000000  891.000000
mean    446.000000    0.383838    2.308642   29.699118    0.523008
std     257.353842    0.486592    0.836071   14.526497    1.102743
min       1.000000    0.000000    1.000000    0.420000    0.000000
25%     223.500000    0.000000    2.000000   20.125000    0.000000
50%     446.000000    0.000000    3.000000   28.000000    0.000000
75%     668.500000    1.000000    3.000000   38.000000    1.000000
max     891.000000    1.000000    3.000000   80.000000    8.000000

            Parch        Fare
count  891.000000  891.000000
mean     0.381594   32.204208
std      0.806057   49.693429
min      0.000000    0.000000
25%      0.000000    7.910400
50%      0.000000   14.454200
75%      0.000000   31.000000
max      6.000000  512.329200
```

```
Kaggle's test.csv
```

```
   PassengerId  Pclass                                          Name     Sex  \
0          892       3                              Kelly, Mr. James    male
1          893       3              Wilkes, Mrs. James (Ellen Needs)  female
2          894       2                     Myles, Mr. Thomas Francis    male
3          895       3                              Wirz, Mr. Albert    male
4          896       3  Hirvonen, Mrs. Alexander (Helga E Lindqvist)  female

    Age  SibSp  Parch   Ticket     Fare Cabin Embarked
0  34.5      0      0   330911   7.8292   NaN        Q
1  47.0      1      0   363272   7.0000   NaN        S
2  62.0      0      0   240276   9.6875   NaN        Q
3  27.0      0      0   315154   8.6625   NaN        S
4  22.0      1      1  3101298  12.2875   NaN        S

       PassengerId      Pclass         Age       SibSp       Parch        Fare
count   418.000000  418.000000  332.000000  418.000000  418.000000  417.000000
mean   1100.500000    2.265550   30.272590    0.447368    0.392344   35.627188
```

std	120.810458	0.841838	14.181209	0.896760	0.981429	55.907576
min	892.000000	1.000000	0.170000	0.000000	0.000000	0.000000
25%	996.250000	1.000000	21.000000	0.000000	0.000000	7.895800
50%	1100.500000	3.000000	27.000000	0.000000	0.000000	14.454200
75%	1204.750000	3.000000	39.000000	1.000000	0.000000	31.500000
max	1309.000000	3.000000	76.000000	8.000000	9.000000	512.329200

In [15]:
```
dropped_columns = ["PassengerId","Name","Ticket","Cabin"]
df = X_train.drop(dropped_columns, axis=1)
df.drop(["Survived"], axis=1, errors="ignore", inplace=True)
#imputing
df["Embarked"].fillna(df["Embarked"].mode()[0], inplace=True)
df["Fare"].fillna(df["Fare"].median(), inplace=True)
df["Age"].fillna(df["Age"].mean(), inplace=True)
```

In [16]:
```
# one-hot encoding
df = df.join(pd.get_dummies(df["Embarked"]))
df.drop(["Embarked"], axis=1, inplace=True)
df = df.join(pd.get_dummies(df["Sex"]))
df.drop(["Sex"], axis=1, inplace=True)
df = df.join(pd.get_dummies(df["Pclass"]))
df.drop(["Pclass"], axis=1, inplace=True)
```

In [17]:
```
# feature engineering
df.loc[:,"Family"] = (df["SibSp"]+df["Parch"] > 0).astype(int)
df.loc[:,"Child"] = (df["Age"] < 16).astype(int)
```

In [18]:
```
def preprocess(df):
    dropped_columns = ["PassengerId","Name","Ticket","Cabin"]
    df = df.drop(dropped_columns, axis=1)
    df.drop(["Survived"], axis=1, errors="ignore", inplace=True)
    #imputing
    df["Embarked"].fillna(df["Embarked"].mode()[0], inplace=True)
    df["Fare"].fillna(df["Fare"].median(), inplace=True)
    df["Age"].fillna(df["Age"].mean(), inplace=True)
    # one-hot encoding
    df = df.join(pd.get_dummies(df["Embarked"]))
    df.drop(["Embarked"], axis=1, inplace=True)
    df = df.join(pd.get_dummies(df["Sex"]))
    df.drop(["Sex"], axis=1, inplace=True)
    df = df.join(pd.get_dummies(df["Pclass"]))
    df.drop(["Pclass"], axis=1, inplace=True)
    # feature engineering
    df.loc[:,"Family"] = (df["SibSp"]+df["Parch"] > 0).astype(int)
    df.loc[:,"Child"] = (df["Age"] < 16).astype(int)
    return df
```

a) Preprocessing all dataframes & displaying.

```
In [19]: X_train = preprocess(X_train)
         X_valid = preprocess(X_valid)
         X_data = preprocess(X_data)
         X_test = preprocess(X_test)

         print('\nTrain')
         display(X_train.head())
         print('\nValidation')
         display(X_valid.head())
         print('\nData')
         display(X_data.head())
         print('\nTest')
         display(X_test.head())
```

```
Train
```

```
         Age  SibSp  Parch     Fare  C  Q  S  female  male  1  2  3  Family  \
0  22.000000      1      0   7.2500  0  0  1       0     1  0  0  1       1
2  26.000000      0      0   7.9250  0  0  1       1     0  0  0  1       0
3  35.000000      1      0  53.1000  0  0  1       1     0  1  0  0       1
4  35.000000      0      0   8.0500  0  0  1       0     1  0  0  1       0
5  29.449243      0      0   8.4583  0  1  0       0     1  0  0  1       0

   Child
0      0
2      0
3      0
4      0
5      0
```

```
Validation
```

```
           Age  SibSp  Parch      Fare  C  Q  S  female  male  1  2  3  \
659  58.000000      0      2  113.2750  1  0  0       0     1  1  0  0
525  40.500000      0      0    7.7500  0  1  0       0     1  0  0  1
828  30.671233      0      0    7.7500  0  1  0       0     1  0  0  1
753  23.000000      0      0    7.8958  0  0  1       0     1  0  0  1
518  36.000000      1      0   26.0000  0  0  1       1     0  0  1  0

     Family  Child
659       1      0
525       0      0
```

```
828          0        0
753          0        0
518          1        0
```

```
Data
```

	Age	SibSp	Parch	Fare	C	Q	S	female	male	1	2	3	Family	Child
0	22.0	1	0	7.2500	0	0	1	0	1	0	0	1	1	0
1	38.0	1	0	71.2833	1	0	0	1	0	1	0	0	1	0
2	26.0	0	0	7.9250	0	0	1	1	0	0	0	1	0	0
3	35.0	1	0	53.1000	0	0	1	1	0	1	0	0	1	0
4	35.0	0	0	8.0500	0	0	1	0	1	0	0	1	0	0

```
Test
```

	Age	SibSp	Parch	Fare	C	Q	S	female	male	1	2	3	Family	Child
0	34.5	0	0	7.8292	0	1	0	0	1	0	0	1	0	0
1	47.0	1	0	7.0000	0	0	1	1	0	0	0	1	1	0
2	62.0	0	0	9.6875	0	1	0	0	1	0	1	0	0	0
3	27.0	0	0	8.6625	0	0	1	0	1	0	0	1	0	0
4	22.0	1	1	12.2875	0	0	1	1	0	0	0	1	1	0

---

b) Using `LogisticRegression` from the `sklearn.linear_model` module, fit a classifier to the training set `X_train` and `Y_train`. Evaluate the accuracy of the classifier via the validation set `X_valid` and `Y_valid`. What is the score?

```
In [20]: from sklearn.linear_model import LogisticRegression as logres
         logres_model = logres().fit(X_train, Y_train)
         logres_prediction = logres_model.predict(X_valid)
         logres_score = logres_model.score(X_valid, Y_valid)
         print('Accuracy: ',logres_score)
```

```
Accuracy:  0.792134831461
```

---

c) Fit a classifier to the data `X_data` and `Y_data`. What is the value of the parameter?

```
In [21]: alldata_model = logres().fit(X_data, Y_data)
         parameters = alldata_model.get_params()
         print('Parameters: ',parameters)
```

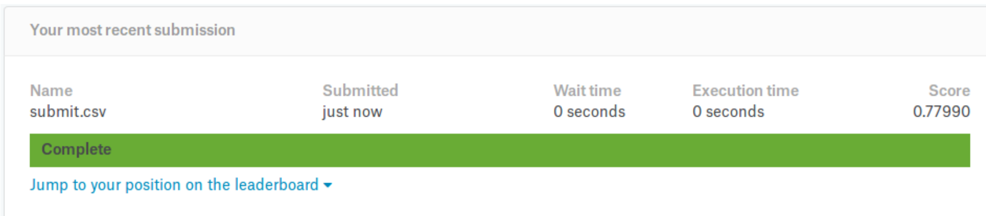
```
Parameters:  {'C': 1.0, 'class_weight': None, 'dual': False, 'fit_intercept': True, 'intercept_s
```

---

d) Apply classifier in (c) to the test data `X_test`. Let `Y_test` denote the predicted labels. You are welcome to tweak the code to improve your model. Use the following code to prepare a submission file for Kaggle.

```
In [22]: X_data = pd.read_csv('kaggle_train.csv')
         X_test = pd.read_csv('kaggle_test.csv')
         X_valid = X_data.sample(frac=0.2, random_state=200)
         X_train = X_data.drop(X_valid.index)
         Y_data = X_data['Survived']
         Y_train = X_train['Survived']
         Y_valid = X_valid['Survived']


         def tweaked_preprocess(df):
             dropped_columns = ["PassengerId","Name","Ticket","Cabin"]
             df = df.drop(dropped_columns, axis=1)
             df.drop(["Survived"], axis=1, errors="ignore", inplace=True)
             #imputing
             df["Embarked"].fillna(df["Embarked"].mode()[0], inplace=True)
             df["Fare"].fillna(df["Fare"].median(), inplace=True)
             df["Age"].fillna(df["Age"].mean(), inplace=True)
             # one-hot encoding
             df = df.join(pd.get_dummies(df["Embarked"]))
             df.drop(["Embarked"], axis=1, inplace=True)
             df = df.join(pd.get_dummies(df["Sex"]))
             df.drop(["Sex"], axis=1, inplace=True)
             # feature engineering
             df.loc[:,"Family"] = (df["SibSp"]+df["Parch"] > 0).astype(int)
             df.loc[:,"Child"] = (df["Age"] < 16).astype(int)
             return df


         X_train = tweaked_preprocess(X_train)
         X_valid = tweaked_preprocess(X_valid)
         X_data = tweaked_preprocess(X_data)
         X_test = tweaked_preprocess(X_test)

         tweaked_model = logres().fit(X_train, Y_train)
         accuracy = tweaked_model.score(X_valid, Y_valid)

         print("Accuracy: ",accuracy)

Accuracy:  0.803370786517
```

```python
In [23]: Y_test = tweaked_model.predict(X_test)

         ans = pd.DataFrame({"PassengerId":ID_test,"Survived":Y_test})
         ans.to_csv("submit.csv", index=False)
```



score

Q2c) Given a cluster {x(1), x(2), ..., x(m)} of points, prove that the point *z* minimizing SUM(|x(i) -z|^2) is the centroid.

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Date No.

2c) Prove that the point z minimizing

$$\sum_{i=1}^{m} \| x^{(i)} - z \|^2$$

is the centroid

$$z = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}.$$

Answer:

$$\mathcal{L}_n(z; x) = \sum_{i=1}^{m} \| x^{(i)} - z \|^2$$

$$= \frac{1}{m} \sum_{i=1}^{m} m \| x^{(i)} - z \|^2$$

$$\mathcal{L}_1(z; x) = m \| x^{(i)} - z \|^2$$

$$= m \left((x_1^{(i)} - z_1)^2 + (x_2^{(i)} - z_2)^2 + \dots + (x_n^{(i)} + z_n)^2 \right)$$

$$\nabla \mathcal{L}_1(z, x) = \begin{pmatrix} \frac{\partial}{\partial z_1} m \left((x_1^{(i)} - z_1)^2 + (x_2^{(i)} - z_2)^2 + \dots + (x_n^{(i)} + z_n)^2 \right) \\ \frac{\partial}{\partial z_2} m \left((x_1^{(i)} - z_1)^2 + (x_2^{(i)} - z_2)^2 + \dots \right. \\ \vdots \end{pmatrix}$$

$$\nabla \mathcal{L}_1(z, x) = \begin{pmatrix} -2m (x_1^{(i)} - z_1) \\ -2m (x_2^{(i)} - z_2) \\ \vdots \\ -2m (x_n^{(i)} - z_n) \end{pmatrix}$$

$$= -2m \begin{pmatrix} x_1^{(i)} - z_1 \\ x_2^{(i)} - z_2 \\ \vdots \\ x_n^{(i)} - z_n \end{pmatrix}$$

$$= -2m (x^{(i)} - z)$$

$$\nabla \mathcal{L}_n(z, x) = \cancel{-2m} \sum_{i=1}^{m} (x^{(i)} - z) = 0$$

$$\sum_{i=1}^{m} x^{(i)} - mz = 0$$

$$\sum_{i=1}^{m} x^{(i)} = mz$$

$$\therefore z = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}$$

Answer