Szemerédi Regularity Lemma

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Let G be a graph and let A and B be disjoint set of vertices of G. Let $e(A,B)$ be the number of edges from A to B. Define $d(A,B) = \frac{e(A,B)}{|A||B|}$ [1]

[1] Note that $d(A,B)$ is a somewhat normalised measure of how well two vertex set is connected with each other. The value of d will range from 0(i.e no connection) to 1(i.e all possible edges are present and very well connected)

Lemma 0.1 *If $X \subset A$ and $Y \subset B$ with $\frac{|X|}{|A|} \geq 1-\delta$ and $\frac{|Y|}{|B|} \geq 1-\delta$ where $0 \leq \delta \leq \frac{1}{2}$. Then:*

$$|d(X,Y) - d(A,B)| \leq 2\delta \tag{1}$$

$$|d^2(X,Y) - d^2(A,B)| \leq 4\delta \tag{2}$$

References