Hashing

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More on implementing maps

- Arrays and Lists? Too inefficient! Linear runtimes
- Plain BSTs? Too inefficient! Linear runtimes; $O(n)$ for search, insert, delete
- RB-trees? Good choice! Logarithmic runtimes are guaranteed; $O(\log n)$ for search, insert, delete

Can we do better in practice and achieve better speed than RB-trees?

Answer

HASH TABLES

☞ With hash-tables, it is not possible to obtain a sorted sequence of the records in $O(n)$ time like BSTs by simply traversing it

Warm up (the simplest possible case)

INDEX/KEY	0	1	2	3	4	5	6	7	8	9	10
Value	D		Z			C	Q				

Assume that we have $n \leq 11$ records to maintain where the keys are in the range $[0, 10]$

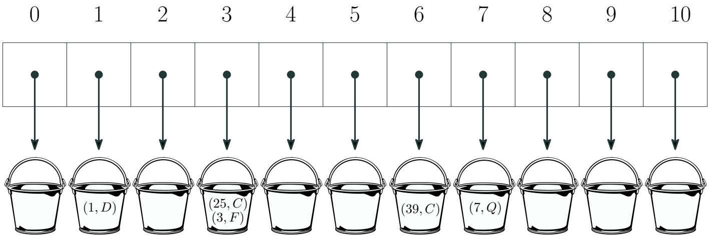
Insertions, deletions, look-ups can be executed in $O(1)$ time each since an array of length $N = 11$ can be used for implementing the table

The situation

- Let n be the number of records stored and N be the number of possible keys
- What if N is really large, say in the order of millions and n much less than N?
- **Example.** for integer keys, $N = 2^{31} - 1 = 2,147,483,647$; but, n is most cases is much less than $2^{31} - 1$. Are we still going to use an array of size $2,147,483,647$? Probably not a good idea. Space wastage may be severe. Storing such an array will require $(2^{31} - 1) \times 1$ byte ≈ 2.147 GB of space (assuming 1 byte is enough to store a character)!

A space-efficient solution

Map the keys to the set of array indices using some function (a.k.a. hash function). A bucket (singly linked-list, for instance) is present at every index that can hold more than one records. Note that In this setup, we can store more than N records inside the table since we are using buckets.



These buckets have theoretically *infinite* capacities

Let $h(\textbf{key}) = \textbf{key} \bmod 11$

For any key k, $0 \leq h(k) \leq 10$

The record (k, v) is put in the bucket at index $h(k)$

What if the keys are not integers?

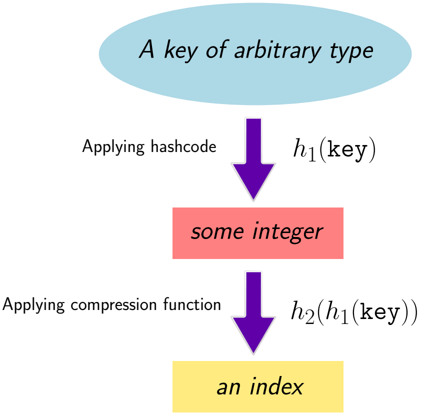
1. Convert the non-integer key to an integer using some function h_1; after applying h_1, we get an integer $h_1(k)$; the function h_1 is known as the **hashcode**
2. Next, map $h_1(k)$ to an array index using another function h_2 known as the **compression function**

The record (k, v) maps to the index $h_2(h_1(k))$

Example

$h_1(\texttt{"Doctor Strange"}) = 1938383$
$h_2(1938383) = 1938383 \bmod N$, where N is the size of the array used

☞ In the previous example, $N = 11$



Hash function

$$h = h_2(h_1(k))$$

The array plus the hash function is called **hash-table**

The main idea behind hashing

It is *unlikely* that two different records (k_i, v_i) and (k_j, v_j), where $k_i \neq k_j$ will map to the same bucket in the hash table when the size of the table N is sufficiently large and the hash function is chosen appropriately. Consequently, it is unlikely that the buckets will be crowded.

If the buckets are not crowded, *searches*, *insertions*, and *deletions* would run fast in practice.

Hashcodes

- Based on the type of keys we are using, one can design various kinds of hash-codes. Desired properties of hash codes:

☞ If two keys k_i, k_j are different, then the two corresponding outputs of hashcode should be different

$$k_i \neq k_j \implies h_1(k_i) \neq h_1(k_j)$$

☞ Should be very fast to compute

- In Java, the `Object` class (super-class of every Java class) defines the **`hashCode()`** method using the object's memory address
- This means the **`hashCode()`** method can be invoked on any object using the dot operator!
- If two objects are equal according to the optional `equals` method of a certain class, then separately invoking the **`hashCode()`** method on each of the two objects must produce the same integer result.

Illustration

Java's hashcode for Strings

Let $s = s_0 s_1 \ldots s_{n-1}$, where every s_i is a character

$$h_1(s) = (\mathtt{ASCII}(s_0) \times 31^{n-1}) + (\mathtt{ASCII}(s_1) \times 31^{n-2}) + \ldots + (\mathtt{ASCII}(s_{n-1}) \times 31^0)$$

```java
public class HashCodeDemo {
   public static void main(String[] args) {
      String s1 = "UNF is FUN";
      String s2 = "FUN is UNF";
      String s3 = "UNF iss FUN";

      System.out.print(s1.hashCode() + " ");
      System.out.print(s2.hashCode() + " ");
      System.out.print(s3.hashCode());
   }
}
```

Output

```
120001564 63052472 -499412747
```

☞ *We may get negative integers from the* `hashCode()` *method due to overflow!*

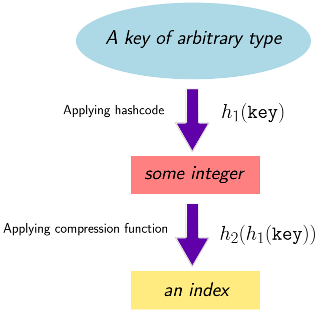
- The built-in Java classes such as `String`, `Integer`, `Double`, etc. redefine this function; see Java's documentation to see the precise mathematical functions

```
Double d1 = 101.98;
System.out.print(d1.hashCode() + " ");

d1 = 101.981;
System.out.print(d1.hashCode() + " ");

d1 = -101.981;
System.out.print(d1.hashCode() + " ");
```

Output

```
296942503 -195025192 1952458456
```

Compression function



The popular compression function

$$h_2(x) = x \bmod N$$

where N is the size of the table.

☞ Make sure that $x \geq 0$, otherwise, $h_2(x) < 0$, making it useless for array indexing!

What is a collision?

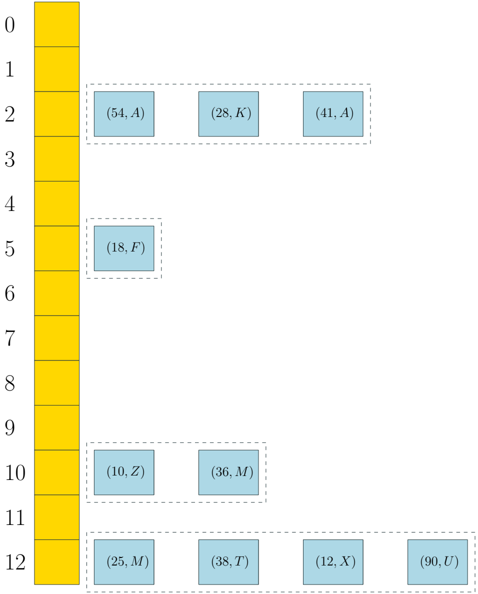
We say that two records (k_i, v_i) and (k_j, v_j) are said to have **collided** if $h(k_i) = h(k_j)$. This means both got mapped to the same array index.

What to do if multiple records map to the same index?

- ☞ Choice 1: **Separate chaining** (uses buckets, very popular); number of records stored can be much more than the size of the array
- ☞ Choice 2: **Open addressing** (does not use buckets, less popular); number of records stored cannot be more than the size of the array

Separate Chaining

Separate chaining: use a container (linked-list/ArrayList) at every index



Hash-function used in this example

$$h(x) = x \text{ mod } 13$$

Load-factor of hash-tables

$$\lambda = \frac{\text{the number of records present in the map } (n)}{\text{size of the array } (N)}$$

☞ $\lambda < 1$ is always desired. This implies no container is probably overcrowded, resulting in fast operation speeds. In Java's implementation of hash-tables, by default, load-factors are never allowed to exceed **0.75**. If the load-factor exceeds 0.75, the size of the array is increased and all the records present in the map are re-inserted (every one of them). We are allowed to choose a different value for the load-factor.

☞ In the beginning, when the table is empty, the number of buckets is set to **16** in Java's implementation of hash-tables.

put(k, v)**: inserts the record** (k, v) **into the map**

1. $i = h(k)$;
2. Let L be the container present at index i;
3. Check L to see if a record is already present in L having key k;
4. If such a record is present in L, the new record (k, v) cannot be inserted;
5. Otherwise, insert the record into L;
6. Compute the current load factor λ;
7. If λ exceeds **0.75**, execute **rehash()**;

Takes $O(1)$ time on average.

☞ After rehashing, $\lambda \leq 0.75$

`rehash`(): rehashes the map (an internal method)

1. Let N be the size of the current array A;
2. Create a new array of empty containers A' whose size is $2N$;
3. Insert every record currently present in the map into the appropriate container in A' using the new compression function $h_2(x) = x \bmod 2N$ (before rehashing, it was $h_2(x) = x \bmod N$) but using the old hashcode h_1 (since the type of keys did not change). This step requires n `put`(k, v) operations.

Rehashing illustration

```
Bucket 0: [96, Jim] -> [112, Peter]
Bucket 1:
Bucket 2:
Bucket 3: [99, Jack]
Bucket 4: [36, Rose]
Bucket 5:
Bucket 6: [22, Charles]
Bucket 7: [23, Alice]
Bucket 8: [40, Bob]
Bucket 9: [41, Matthew]
Bucket 10: [10, Tom]
Bucket 11: [11, Dorothy]
Bucket 12: [92, Eric]
Bucket 13:
Bucket 14: [62, Donald]
Bucket 15:
```

Load-factor $(\lambda) = \frac{n}{N} = \frac{12}{16} = 0.75$

```
Bucket 0: [96, Jim]
Bucket 1:
Bucket 2:
Bucket 3: [99, Jack]
Bucket 4: [36, Rose]
Bucket 5:
Bucket 6:
Bucket 7:
Bucket 8: [40, Bob]
Bucket 9: [41, Matthew]
Bucket 10: [10, Tom]
Bucket 11: [11, Dorothy]
Buckets 12 to 15:
Bucket 16: [112, Peter]
Buckets 17 to 21:
Bucket 22: [22, Charles]
Bucket 23: [23, Alice]
Buckets 24 to 27:
Bucket 28: [92, Eric]
Bucket 29:
Bucket 30: [62, Donald]
Bucket 31: [31, Toby]
```

After inserting `[31, Toby]`, $\lambda = \frac{n}{N} = \frac{13}{16} = 0.8125 > 0.75$; rehashing is required; new $\lambda = \frac{n}{N} = \frac{13}{32} \approx 0.40625 \leq 0.75$

get(k)**: returns the value part of the record whose key is** k

1. $i = h(k)$;
2. Let L be the container present at index i;
3. Check L to see if a record is already present in L having key k;
4. If such a record is present in L, return its value part;
5. Otherwise, return `null`;

Takes $O(1)$ time on average

`remove`(k): removes the record having key k, if present

1. $i = h(k)$;
2. Let L be the container present at index i;
3. Check L to see if a record is present in L having key k;
4. If it is present in L, remove and return the value part of the record (k, v);
5. Otherwise, return **`null`**;

Takes $O(1)$ time on average

Code

See the class `HashMapSeparateChaining`

In our implementation, the containers/buckets are **singly linked-lists**

Worst-case scenario of separate chaining

- In the worst case, most of the records can map to the same index!
- Thus, insertion, deletion, and searching take $O(n)$ time in the worst-case if lists are used and $O(\log n)$ time if RB-trees are used
- In such cases, we do not get any advantage out of hashing
- However, if we are using good hash function, these extreme situations will almost never happen and we get super-speedy performance in the real-world

Java's `HashMap` vs `TreeMap`: insertion time comparison

n	`java.util.HashMap`	`java.util.TreeMap`
10	1	1
100	1	1
1000	1	3
10000	1	7
100000	17	19
1000000	97	167
10000000	815	2889

☞ Times are reported in milliseconds

☞ In practice, `HashMaps` are more efficient in practice than `TreeMaps`

Open Addressing

Open addressing

- Use an array of size N
- At every index, we can store at most one record (no use of infinite capacity buckets)
- Let us look at such a popular technique known as **Linear Probing**: if the spot is already occupied by some other record (collision), we consider the next available spot in the array by wrapping around

Example

0	1	2	3	4	5	6	7	8	9	10

Insert $(13, A)$

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$								

$$13 \mod 11 = 2$$

Insert $(26, T)$

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$		$(26, T)$						

$$26 \ \mathtt{mod} \ 11 = 4$$

Insert $(5, Z)$

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$		$(26, T)$	$(5, Z)$					

$$5 \texttt{ mod } 11 = 5$$

Insert $(37, G)$; collision!

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$		$(26, T)$	$(5, Z)$	$(37, G)$				

$37 \mod 11 = 4$, linear probe and then put the record at index 6

Insert $(16, A)$; collision!

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$		$(26, T)$	$(5, Z)$	$(37, G)$	$(16, A)$			

$16 \mod 11 = 5$, linear probe and then put the record at index 7

Insert $(21, F)$

0	1	2	3	4	5	6	7	8	9	10
		$(13, A)$		$(26, T)$	$(5, Z)$	$(37, G)$	$(16, A)$			$(21, F)$

$$21 \ \texttt{mod} \ 11 = 10$$

Insert $(43, Q)$; collision!

0	1	2	3	4	5	6	7	8	9	10
$(43, Q)$		$(13, A)$		$(26, T)$	$(5, Z)$	$(37, G)$	$(16, A)$			$(21, F)$

$43 \bmod 11 = 10$, linear probe by wrapping around and then put the record at index 0

Operations

In the following, `DEFUNCT` is a special record $(\texttt{null}, \texttt{null})$ whose key and value are set to `null`

1. **Insertion.** Use the hash function on the key to obtain the target index; if the spot is taken by some other record, use wrap-around linear probing to find the next available spot; cells containing `DEFUNCT` are considered to be empty
2. **Searching.** Use the hash function on the key to obtain the target index; if the desired record is not present at the index, do a wrap-around linear probe to search the record. If an empty spot is encountered, the desired record is not present. However, skip over the cells containing **DEFUNCT**. In this case, cells containing `DEFUNCT` are not considered to be empty.
3. **Deletion.** Use the hash function on the key to obtain the target index; if the desired record is not present at the index, use wrap-around linear probing to locate the record. However, skip over the cells containing `DEFUNCT`. In this case, cells containing `DEFUNCT` are not considered to be empty. If located, delete the record from the cell and put a special symbol `DEFUNCT` in its place (see the example next to see why)

All these three operations takes $O(n)$ time in the worst-case

Why DEFUNCT is necessary?

0	1	2	3	4	5	6	7	8	9	10
$(43, Q)$		$(13, A)$		$(26, T)$	$(5, Z)$		$(16, A)$			$(21, F)$

$(37, G)$
deleted

0	1	2	3	4	5	6	7	8	9	10
$(43, Q)$		$(13, A)$		$(26, T)$	$(5, Z)$	DEFUNCT	$(16, A)$			$(21, F)$

$(37, G)$
deleted

Search for $(16, A)$ will fail if we store null at index 6. For this reason, putting a special marker **`DEFUNCT`** will help the search algorithm to find $(16, A)$ by skipping over index 6.

Observations

- If we are using open addressing, the number of records that can be present in the array cannot exceed the array size since we are not using chaining
- But it is not the case in separate chaining
- Separate chaining consumes more space to maintain the containers at every index

Reading

Chapter 15 from

https://opendsa-server.cs.vt.edu/ODSA/Books/Everything/html/index.html