# Stacks and Queues

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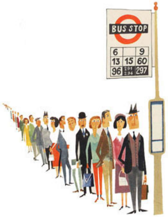
# Stacks and Queues



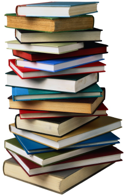

- **Stacks** are **Queues** are two popular easy-to-implement **abstract data types** (ADTs) used in algorithm design
- For an abstract data type, we **describe** the operations that can be executed on the data, but we do not define them
- For instance, the stack ADT can be implemented using a linked-list and an array as well

# Stack

**Definition**

A stack is an ADT where items are inserted (**pushed**) and removed (**popped**) according to the **last-in, first-out** (LIFO) principle

# Operations on a stack



Let us denote a stack by `S`.

1. `S.push(e)`: **adds** element `e` to the top of `S`
2. `S.pop()`: **removes** and **returns** the top element from `S`; returns **`null`** if `S` is empty
3. `S.top()`: **returns** the top element from `S` but does not remove it
4. `S.size()`: **returns** the number of elements stored in `S` currently
5. `S.isEmpty()`: **returns** **`true`** if `S` is empty otherwise returns **`false`**

# push(55)

$55$

$$55, 3$$

# push(69)

$$55, 3, 69$$

$$55, 3, 69, -88$$

# push(-17)

$$55, 3, 69, -88, -17$$

## `isEmpty()` returns `false`

$$55, 3, 69, -88, -17$$

## `size()` returns 5

$$55, 3, 69, -88, -17$$

# `pop()` removes and returns –17

$$55, 3, 69, -88$$

# `pop()` removes and returns `-88`

$$55, 3, 69$$

# `top()` returns 69

$$55, 3, 69$$

# Applications

- Can be used to implement the **FORWARD** and **BACK** buttons in a browser.
- Used by text editors to implement the **UNDO** operation. When you change something, the current state is pushed onto the stack before applying the new change. Hitting UNDO will simply revert the document to the previous state
- Method calls, recursion, and compiler design
- Used as an auxiliary data structure for many cornerstone algorithms

# An `interface` for stacks

```java
public interface StackADT<E> {
   int size();
   boolean isEmpty();
   void push(E e);
   E pop();
   E top();
}
```

To use a stack, one should create a class by implementing this interface

# So how to implement a stack using the `StackADT` interface?

- Use an array (stack size must be known in advance, speedy in practice)
  See the class `ArrayStack`
- Use a linked-list (can grow arbitrarily, a bit slower than the array-based stack)
  See the class `LinkedStack`

Irrespective of the implementation, every one of these five stack operations, `size()`, `isEmpty()`, `push()`, `pop()`, `top()`, takes $O(1)$ time

# An application: matching parentheses

- Given an algebraic expression, how to check if the parentheses, braces, and brackets in it are properly **balanced** or not
- Some **balanced** ones
  - `(a + b) + (66 * (s / t) - 12) * {p + (z - [2 + 99.1 / (m - x)])}`
    Sequence: `( ) ( ( ) ) { ( [ ( ) ] ) }`
  - `30 + (40 + (9 * (6 - 10) * (1 - (17 + 19 + 99) - 1) / {5 + (8 - [6 + 2 * (1 + (-99)) - 1] - 4) - 1} - 1) - 1) * 88`
    Sequence: `( ( ( ) ( ( ) ) { ( [ ( ) ] ) } ) )`
- Some **unbalanced** sequences
  - `) ( ( ) ) { ( [ ( ) ] ) }`
  - `( { [ ] ) }`
  - `(`

# How to solve it?

- Recall the three pairs we are interested in:

  `( ),    { },    [ ]`

- Here is a stack-based algorithm for this problem:
  1. Declare a **`Character`** stack `S`
  2. Scan the input algebraic expression from left to right
  3. If a left symbol `(`, `{`, or `[` is encountered, **push** it onto `S`
  4. If a right symbol `)`, `}`, or `]` is encountered, check the top element in `S`; if these two elements form a matching pair, then pop from `S` otherwise report **INVALID**
  5. At the end, when the string is fully scanned, if `S` is empty, then report **VALID**, else **INVALID**

# Sample run

| Current symbol scanned | Stack (right is the stack top) | Action taken |
|---|---|---|
| ( ) ( ( ) ) { ( [ ( ) ] ) } | ( | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | | pop |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | ( | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | ( ( | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | ( | pop |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | | pop |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { ( | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { ( [ | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { ( [ ( | push |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { ( [ | pop |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { ( | pop |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | { | pop |
| ( ) ( ( ) ) { ( [ ( ) ] ) } | | pop |

Result: **VALID**

# Another sample run

| Current symbol scanned | Stack (right is the stack top) | Action taken |
|---|---|---|
| ( { [ ] ) } | ( | push |
| ( { [ ] ) } | ( { | push |
| ( { [ ] ) } | ( { [ | push |
| ( { [ ] ) } | ( { | pop |
| ( { [ ] ) } | ( { | **mismatch** |

Result: **INVALID**

# Code

```java
import java.util.Scanner;
public class ExpresssionChecker {
    public static void main(String[] args) {
        Scanner input = new Scanner(System.in);
        String expression = input.nextLine();
        LinkedStack<Character> S = new LinkedStack<>();
        int pos;

        for(pos = 0; pos < expression.length(); pos++) {
            char current = expression.charAt(pos);

            if( current == '(' || current == '{' || current == '[')        S.push(current);
            else if( current == ')' && !S.isEmpty() && S.top() == '(' )   S.pop();
            else if( current == ')' && !S.isEmpty() && S.top() != '(' )   break;
            else if( current == '}' && !S.isEmpty() && S.top() == '{' )   S.pop();
            else if( current == '}' && !S.isEmpty() && S.top() != '{' )   break;
            else if( current == ']' && !S.isEmpty() && S.top() == '[' )   S.pop();
            else if( current == ']' && !S.isEmpty() && S.top() != '[' )   break;
        }

        if( S.isEmpty() && pos == expression.length() )   System.out.println("VALID");
        else System.out.println("INVALID");

        input.close();
    }
}
```

# Food for thought

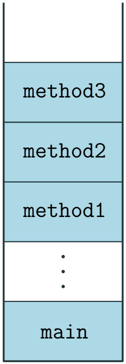
```
<html>
   <head>
      <title>Data Structure</title>
   </head>

   <body>
      <h1>
         <p>Stacks are fun...</p>
      </h1>

      <p>Queues are also fun...</p>
   </body>
</html>
```

Write a program that can check if an HTML file is correctly formatted. This means every opening tag must have its closing tag at an appropriate location in the file.

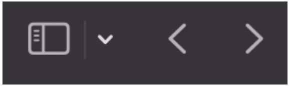
# Method calls and use of stack

- Ever wondered how the control returns to the **caller** method after the **callee** method is done with its execution?

```java
public char method1() {
   // ...
   method2();
   counter++; // control returns here
   i = j + 10;
   //...
}
public double method2() {
   // ...
    s = p + s;
    method3();
    arr[q] = arr[q] / (10 * m); // control returns here
   // ...
}
public void method3() {
   // ...
    s = p + s;
   // ...
   return;
}
```




- A stack of **activation records** is used by the system; push a record when a method starts execution; pop its record when it is done



- For every tab, we get to see the *go forward* and *go backward* buttons
- How to implement this button
- Use 2 **stacks**: one for the *go backward* button and one for the *go forward* button

# Code

```java
public class BrowserTab {
    String currentPage = "";
    private final LinkedStack<String> backPages = new LinkedStack<>(), forwardPages = new LinkedStack<>();
    public static String getTimeStamp(){ return "[" + new Date() + "] "; }

    public BrowserTab() { System.out.println(getTimeStamp() + "New tab opened."); }

    public void typeAndGoNewSite(String newPage) {
        if( !currentPage.isEmpty() ) // some page is being viewed currently
        backPages.push(currentPage); // send the current page to the previousPages DLL

        currentPage = newPage;
        System.out.println(getTimeStamp() + "Currently viewing: " + currentlyViewing());
    }
    // contd. on the next slide
}
```

# Code

```java
public class BrowserTab {
   public void clickOnGoBackButton() {
      if( backPages.isEmpty() ) {
         System.out.println(getTimeStamp() + "Back button is greyed out (unavailable).");
         return;
      }
      else {
         System.out.print(getTimeStamp() + "Back button is clicked. ");
         forwardPages.push(currentPage);
         currentPage = backPages.top();
         backPages.pop();
      }
      System.out.println("Currently viewing: " + currentlyViewing());
   }

   public void clickOnGoForward() {
      if( forwardPages.isEmpty() ) {
         System.out.println(getTimeStamp() + "Forward button is greyed out (unavailable).");
         return;
      }
      else {
         backPages.push(currentPage);
         currentPage = forwardPages.pop();
      }
      System.out.println(getTimeStamp() + "Forward button is clicked. Currently viewing: " + currentlyViewing());
   }   // contd. on the next slide
}
```

# Code

```java
public class BrowserTab {

   public String currentlyViewing() { return currentPage; }

   public String toString() {
      StringBuilder prettyString = new StringBuilder();

      for (String site : previousPages)
         prettyString.append(site).append(" ");

      prettyString.append(" ***").append(currentPage).append("*** ");

      for (String site : nextPages)
         prettyString.append(site).append(" ");

      return prettyString.toString();
   }
}
```

For an usage refer to the class **`TestBrowserTab`**

# Queue

## Definition

A queue is an ADT where items are inserted (**enequed**) and removed (**dequeued**) according to the **first-in, first-out** (FIFO) principle


# Operations on a queue

Let us denote a queue by `Q`.

1. **`Q.enqueue(e)`**: **adds** an element `e` to the back of `Q`
2. **`Q.dequeue()`**: **removes** and **returns** the first element from `Q`; returns **`null`** if `Q` is empty
3. **`Q.first()`**: **returns** the first element of `Q`, without removing it; returns **`null`** is `Q` is empty
4. **`Q.size()`**: **returns** the number of elements stored in `Q` currently
5. **`Q.isEmpty()`**: **returns** **`true`** if `Q` is empty, otherwise returns **`false`**

## enqueue(55)

$55$

# enqueue(3)

$$55, 3$$

## enqueue(69)

$$55, 3, 69$$

# enqueue(-88)

$$55, 3, 69, -88$$

## `enqueue(-17)`

$$55, 3, 69, -88, -17$$

# `isEmpty()` returns `false`

$$55, 3, 69, -88, -17$$

# `size()` returns 5

$$55, 3, 69, -88, -17$$

# `dequeue()` removes and returns 55

$$3, 69, -88, -17$$

## `dequeue()` removes and returns 3

$$69, -88, -17$$

# `first()` returns 69

$$69, -88, -17$$

# Applications

- Simulation of real-world queues (airlines, ticket counter, etc.)
- Graph algorithms
- Resource sharing in multi-user systems
- Operating systems
- Computer networks

  ...

# An `interface` for queues

```java
public interface QueueADT<E> {
   void enqueue(E e);
   E dequeue();
   int size();
   boolean isEmpty();
   E first();
}
```

To use a queue, one should create a class by implementing this interface

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $Z$ | $N$ | $Z$ | $T$ | $P$ | $A$ | $Y$ | $\bullet\ \bullet\ \bullet$ | $S$ | $C$ | `empty` | `empty` |
| front | | | | | | | | | | back | |

After 3 dequeues and 2 enqueues the situation is: we have space but we cannot enqueue anymore!

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| `empty` | `empty` | `empty` | $T$ | $P$ | $A$ | $Y$ | $\bullet\ \bullet\ \bullet$ | $S$ | $C$ | $K$ | $X$ |
| | | | front | | | | | | | | |

# What to do then?

| $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| `empty` | `empty` | `empty` | $T$ | $P$ | $A$ | $Y$ | $\bullet\ \bullet\ \bullet$ | $S$ | $C$ | $K$ | $X$ |
| | | | front | | | | | | | | |

- Trivial solution: shift the content to the left and add new stuff
- Downside? This is very expensive! Takes $O(n)$ time
- We need to implement `enqueue` faster than this
- Solution: wrap around

| $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ |

$$i = 0$$

Next index: $(i + 1)$ mod $5 = 1$ mod $5 = 1$

# Using the mod operator for wrapping around

| $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ |

$$i = 1$$

Next index: $(i+1)$ mod $5 = 2$ mod $5 = 2$

# Using the mod operator for wrapping around

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ |

$$i = 2$$

Next index: $(i+1) \bmod 5 = 3 \bmod 5 = 3$

# Using the mod operator for wrapping around

| $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ |

$$i = 3$$

Next index: $(i + 1)$ mod $5 = 4$ mod $5 = 4$

# Using the mod operator for wrapping around

| $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ |

$$i = 4$$

Next index: $(i + 1)$ mod $5 = 5$ mod $5 = 0$

# Using the mod operator for wrapping around

| $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|
| $A$ | $B$ | $C$ | $D$ | $E$ |

$$i = 0$$

Next index: $(i + 1) \bmod 5 = 1 \bmod 5 = 1$

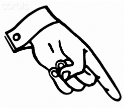
# Using this wrap-around approach for array-based queues

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $Z$ | $N$ | $Z$ | $T$ | $P$ | $A$ | $Y$ | $\cdots$ | $S$ | $C$ | `empty` | `empty` |

front back

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| `empty` | `empty` | `empty` | $T$ | $P$ | $A$ | $Y$ | $\cdots$ | $S$ | $C$ | $K$ | $X$ |

front



| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| `empty` | `empty` | `empty` | $T$ | $P$ | $A$ | $Y$ | $\cdots$ | $S$ | $C$ | $K$ | $X$ |

back front

# Using this wrap-around approach for array-based queues

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| empty | empty | empty | $T$ | $P$ | $A$ | $Y$ | $\bullet\ \bullet\ \bullet$ | $S$ | $C$ | $K$ | $X$ |
| back | | | front | | | | | | | | |

After **`Q.enqueue('A');`**, we get

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | empty | empty | $T$ | $P$ | $A$ | $Y$ | $\bullet\ \bullet\ \bullet$ | $S$ | $C$ | $K$ | $X$ |
| | back | | front | | | | | | | | |

# So how to implement a queue using the `QueueADT` interface?

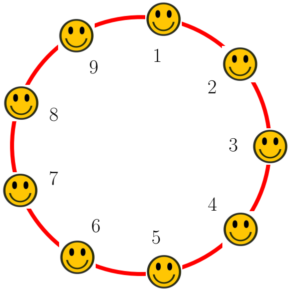
- Use an array (queue size must be known in advance, speedy in practice)
  See the class `ArrayQueue`
- Use a linked-list (can grow arbitrarily, a bit slower than the array-based queue)
  See the class `LinkedQueue`

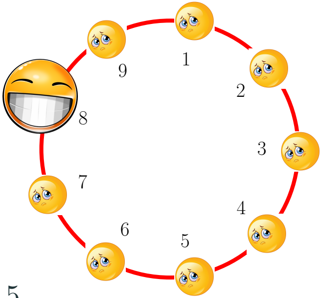
Irrespective of the implementation, every one of these five queue operations, `size()`, `isEmpty()`, `enqueue()`, `dequeue()`, `first()`, takes $O(1)$ time

## The problem

A group of $n$ people agree to play the following fun game. They arrange themselves on a circle (at positions numbered from $1$ to $n$) and proceed around the circle clockwise, eliminating every $m$th person until only one person is left.

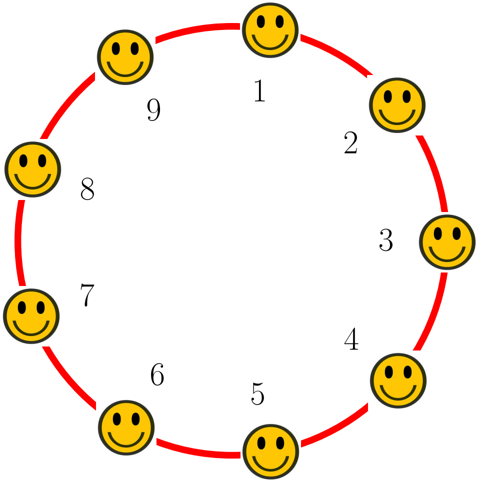
The last remaining person wins the game. How to figure out the winning position?



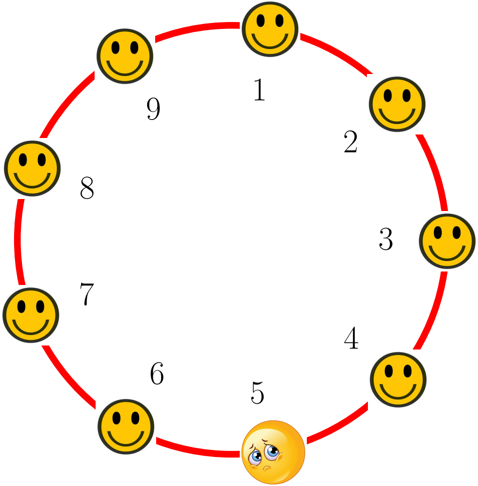




$n = 9, m = 5$

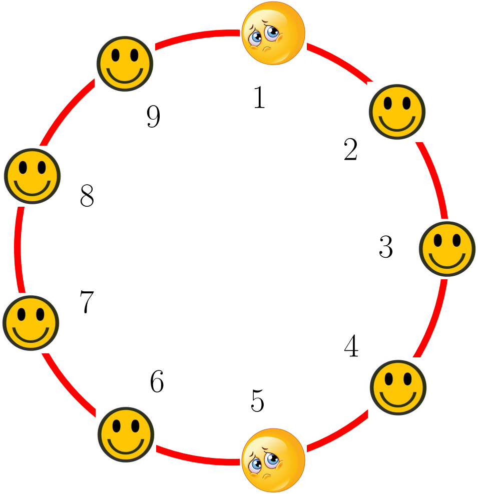
# Example: $n = 9, m = 5$



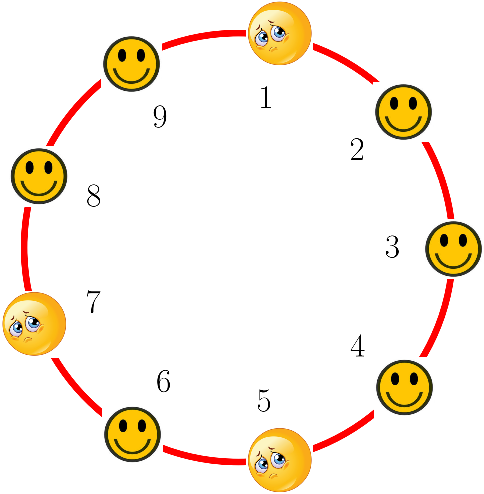
# **Example:** $n = 9, m = 5$



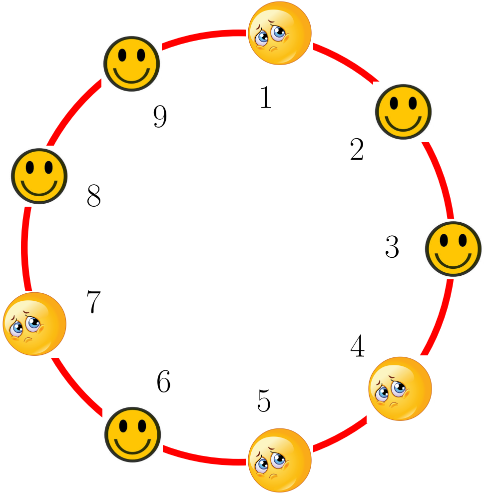
# Example: $n = 9, m = 5$



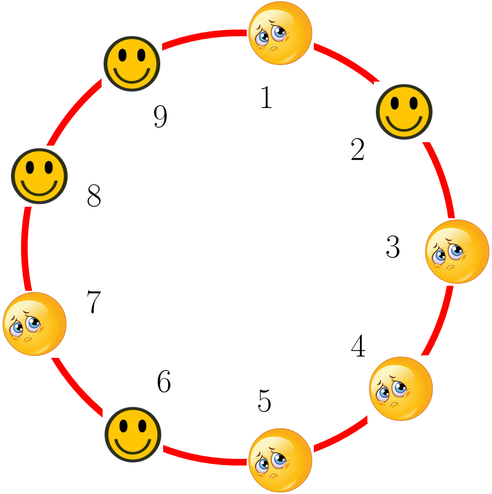
## **Example:** $n = 9, m = 5$



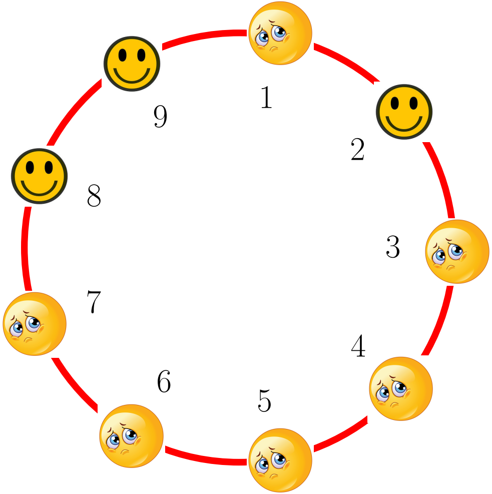
## **Example:** $n = 9, m = 5$



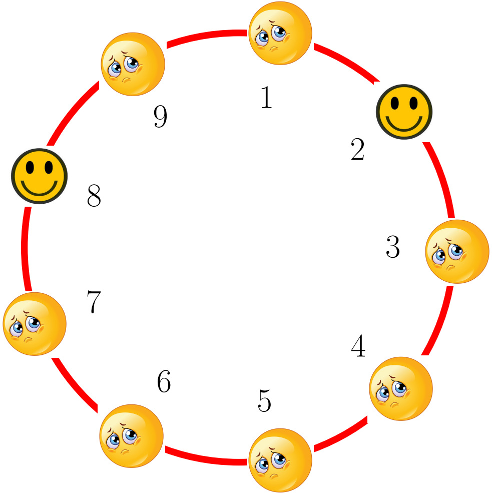
# **Example:** $n = 9, m = 5$



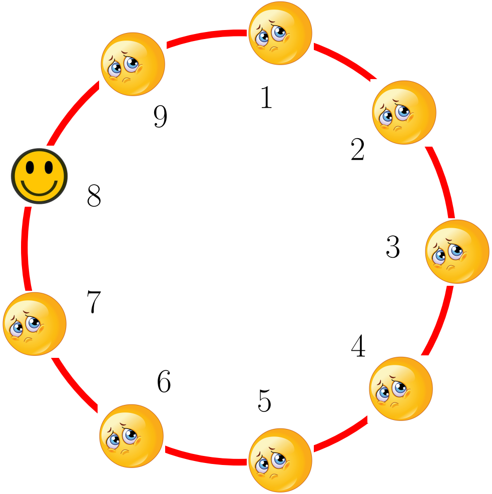
# Example: $n = 9, m = 5$

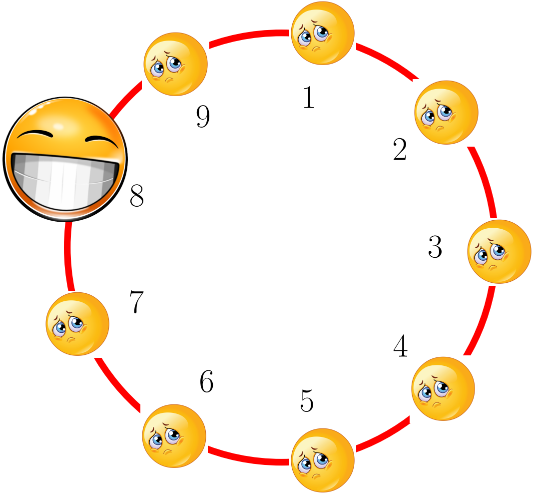


# Example: $n = 9, m = 5$



# Example: $n = 9, m = 5$

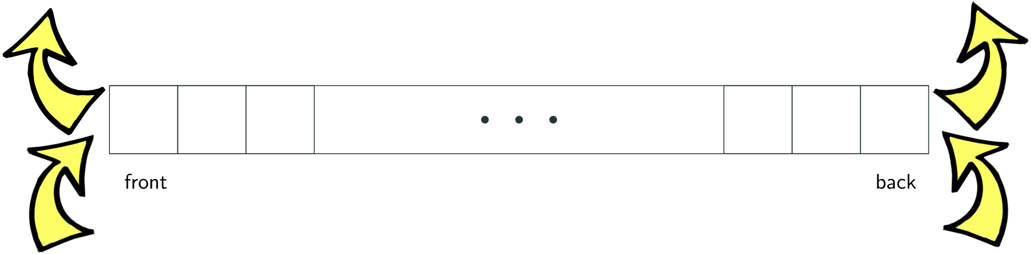



1
2
3
4
5
6
7
8
9

# Code

```java
import java.util.Scanner;

public class JosephusSolver {
   public static void main(String[] args) {
      int n = 9, m = 5;

      LinkedQueue<Integer> Q = new LinkedQueue<>();

      for (int i = 1; i <= n; i++) // populate the queue with the integers 1,2,...,n
         Q.enqueue(i);

      while ( (Q.size() > 1) ) {
         for (int i = 0; i < m-1; i++) // send the first m-1 elements to the back of the queue
            Q.enqueue(Q.dequeue());

         System.out.println("Eliminating player: " + Q.dequeue() + " ");
      }

      System.out.print("\nWinning position: " + Q.first()); // the sole integer in the queue is the winner
   }
}
```

# Double ended queues: Deque (pronounced as DECK)




Supports insertion and removal at both the front and the back

Deque can be used as a stack or a queue

# Deque ADT

```java
public interface DequeADT<E> {
   int size();
   boolean isEmpty();
   E first();
   E last();
   void addFirst(E e);
   void addLast(E e);
   E removeFirst();
   E removeLast();
}
```

**An easy way to implement a deque**

Doubly linked-list (using a singly linked-list will make the `removeLast` operation run in $O(n)$ time)

☞ Refer to the class `LinkedDeque` for an implementation

**Time complexity**

Just like stacks and queues, all operations on deques run in $O(1)$ time

# Suggested exercise

Create an array-based iterable deque class

# Reading

Stacks and queues from **Chapter 5**

https://opendsa-server.cs.vt.edu/OpenDSA/Books/CS3/html/index.html