Ghosh's monic quintic equation has roots expressible in radicals

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Abstract

In this paper, I describe the roots of my monic quintic equation, which are expressible in radicals.
The paper ends with "The End"

Introduction

In a previous paper, I've described my monic quintic. In this paper, I describe the roots of my monic quintic equation which are expressible in radicals.

Ghosh's monic quintic equation has roots expressible in radicals

When

$$b - aP + P^2 - Q \neq 0$$

by the right side of Ghosh's monic quintic identity,

Ghosh's monic quintic equation

$$\left(x^3 + Px^2 + Qx + \frac{e}{b - aP + P^2 - Q}\right)\left(x^2 + (a-P)x + \left(b - aP + P^2 - Q\right)\right) = 0$$

can be written as the product of a cubic equation and a quadratic equation, both of which can be solved.

The roots of Ghosh's monic quintic equation are

$$x_1 = \frac{1}{2}\left(-a + P - \sqrt{a^2 + 2aP - 4b - 3P^2 + 4Q}\right)$$

$$x_2 = \frac{1}{2}\left(-a + P + \sqrt{a^2 + 2aP - 4b - 3P^2 + 4Q}\right)$$

$$x_3 = \frac{\sqrt[3]{-\frac{27e}{-aP+b+P^2-Q} + \sqrt{\left(-\frac{27e}{-aP+b+P^2-Q} - 2P^3 + 9PQ\right)^2 + 4\left(3Q - P^2\right)^3} - 2P^3 + 9PQ}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}\left(3Q - P^2\right)}{3\sqrt[3]{-\frac{27e}{-aP+b+P^2-Q} + \sqrt{\left(-\frac{27e}{-aP+b+P^2-Q} - 2P^3 + 9PQ\right)^2 + 4\left(3Q - P^2\right)^3} - 2P^3 + 9PQ}} - \frac{P}{3}$$

$$x_4 = -\frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{-\frac{27e}{-aP+b+P^2-Q} + \sqrt{\left(-\frac{27e}{-aP+b+P^2-Q} - 2P^3 + 9PQ\right)^2 + 4\left(3Q - P^2\right)^3} - 2P^3 + 9PQ}}{6\sqrt[3]{2}} + \frac{\left(1 + i\sqrt{3}\right)\left(3Q - P^2\right)}{3\ 2^{2/3}\sqrt[3]{-\frac{27e}{-aP+b+P^2-Q} + \sqrt{\left(-\frac{27e}{-aP+b+P^2-Q} - 2P^3 + 9PQ\right)^2 + 4\left(3Q - P^2\right)^3} - 2P^3 + 9PQ}} - \frac{P}{3}$$

$$x_5 = -\frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{-\frac{27e}{-aP+b+P^2-Q} + \sqrt{\left(-\frac{27e}{-aP+b+P^2-Q} - 2P^3 + 9PQ\right)^2 + 4\left(3Q - P^2\right)^3} - 2P^3 + 9PQ}}{6\sqrt[3]{2}} + \frac{\left(1 - i\sqrt{3}\right)\left(3Q - P^2\right)}{3\ 2^{2/3}\sqrt[3]{-\frac{27e}{-aP+b+P^2-Q} + \sqrt{\left(-\frac{27e}{-aP+b+P^2-Q} - 2P^3 + 9PQ\right)^2 + 4\left(3Q - P^2\right)^3} - 2P^3 + 9PQ}} - \frac{P}{3}$$

The End