# TD 11: Petri Nets

**Exercise 1** (Reachability in Petri nets)**.** Let $N = \langle P, T, F, W, m_0 \rangle$ be a Petri net. We say that $N$ is *acyclic* if the directed graph $\langle P \cup T, F \rangle$ does not contain any cycles. Let $\mathcal{A}$ denote the class of Petri nets that are (i) 1-safe and (ii) acyclic.

(a) Assuming that every transition is connected to at least one place, show that in no firing sequence of a net $N \in \mathcal{A}$ can contain the same transition twice.

(b) Show that the reachability problem for the class $\mathcal{A}$ is in NP.

(c) Show that the reachability problem for the class $\mathcal{A}$ is NP-hard, by reduction from the SAT problem. (Without loss of generality, one can assume that a SAT formula is given in conjunctive normal form.)

**Exercise 2** (Traffic Lights)**.** Consider again the traffic lights example from the lecture notes:

(a) How can you correct this Petri net to avert unwanted behaviours (like $r \to ry \to rr$) in a 1-safe manner?

(b) Extend your Petri net to model two traffic lights handling a street intersection.

**Exercise 3** (Producer/Consumer)**.** A producer/consumer system gathers two types of processes:

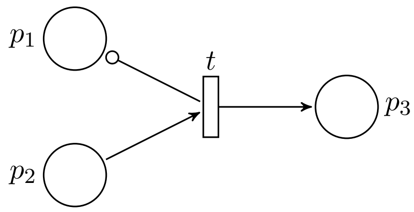
**producers** who can make the actions *produce* ($p$) or *deliver* ($d$), and

**consumers** with the actions *receive* ($r$) and *consume* ($c$).

All the producers and consumers communicate through a single unordered channel.

(a) Model a producer/consumer system with two producers and three consumers. How can you modify this system to enforce a maximal capacity of ten simultaneous items in the channel?

(b) An *inhibitor arc* between a place $p$ and a transition $t$ makes $t$ firable only if the current marking at $p$ is zero. In the following example, there is such an inhibitor arc between $p_1$ and $t$. A marking $(0, 2, 1)$ allows to fire $t$ to reach $(0, 1, 2)$, but $(1, 1, 1)$ does not allow to fire $t$.




Using inhibitor arcs, enforce a priority for the first producer and the first consumer on the channel: the other processes can use the channel only if it is not currently used by the first producer and the first consumer.

**Exercise 4** (Model Checking Petri Nets)**.** Let us fix a Petri net $\mathcal{N}$ with inhibitor arcs. We consider propositional LTL, with a set of atomic propositions AP equal to $P$, the set of places of the Petri net. We define proposition $p$ to hold in a marking $m$ in $\mathbb{N}^P$ if $m(p) > 0$.

The models of our LTL formulæ are *computations* $m_0 m_1 \cdots$ in $(\mathbb{N}^P)^\omega$ such that, for all $i \in \mathbb{N}$, $m_i \rightarrow_{\mathcal{N}} m_{i+1}$ is a transition step of the Petri net $\mathcal{N}$.

(a) Prove that state-based LTL model checking can be performed in polynomial space for 1-safe nets. For this, prove that one can construct an exponential-sized Büchi automaton $\mathcal{B}_{\mathcal{N}}$ from a 1-safe Petri net that recognizes all the infinite computations of $\mathcal{N}$ starting in $m_0$.

(b) In the general case, state-based LTL model checking is undecidable for this model. Prove it for Petri nets with at least two unbounded places, by a reduction from the halting problem for 2-counter Minsky machines.