

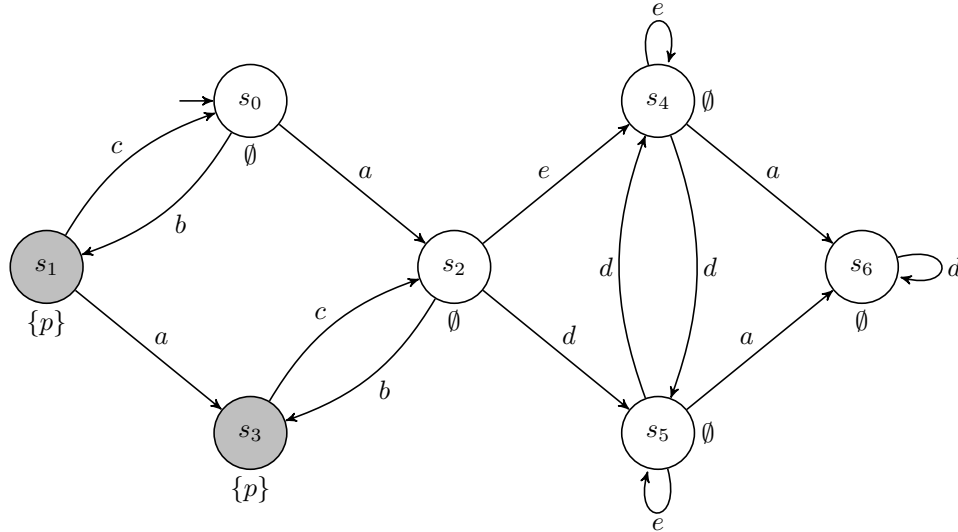
## Homework 6

Answers can be written in french or in english.

### Reminder (conditions for ample sets):

- (C0)  $red(s) = \emptyset$  iff  $en(s) = \emptyset$ .
- (C1) For every path  $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_n} s_n \xrightarrow{a} t$  in  $\mathcal{K}$  (for any  $n \geq 0$ ), if  $a \notin red(s)$  and  $a$  depends on some action in  $red(s)$  (i.e. there exists  $b \in red(s)$  such that  $(a, b) \notin I$ ), then there exists  $1 \leq i \leq n$  such that  $a_i \in red(s)$ .
- (C2) If  $red(s) \neq en(s)$ , then all actions in  $red(s)$  are invisible.
- (C3) For all cycles in the reduced system  $\mathcal{K}'$ , the following holds: if  $a \in en(s)$  for some state  $s$  in the cycle, then  $a \in red(s')$  for some (possibly other) state  $s'$  in the cycle.

**Exercise 1.** Let  $\mathcal{K} = (S, A, \rightarrow, r, AP, \nu)$  be the Kripke structure below, with set of actions  $A = \{a, b, c, d, e\}$  and atomic propositions  $AP = \{p\}$ .



1. Compute the maximal independence set  $I$  and the maximal set of invisible actions  $U$ . Justify your answers.
2. Give sets  $red(s) \subseteq en(s)$  satisfying conditions C0 to C3, and such that for all states  $s$ , no action can be removed from  $red(s)$  without breaking one of conditions C0 to C3. Justify your answer.
3. Draw the reduced system  $\mathcal{K}'$  associated with your assignment  $red$ , after removing unreachable states. Is there a smaller system  $\mathcal{K}''$ , obtained by removing additional transitions, that is stuttering equivalent to  $\mathcal{K}$ ?