ANALYSIS OF VARIANCE

(A case study demonstrating the applications of one-way, two-way and incomplete three-way ANOVA, one-way and two-way ANOCOVA, and Factorial Experiments for some industrial and agricultural data.)

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Instructor's Remarks:

4. ANALYSIS OF VARIANCE (10 CASES)

4.1 ONE-WAY ANOVA

1. COMPLETELY RANDOMIZED DESIGN – ANALYZING PERFORMANCE OF 4 DIFFERENT DRUGS OVER PLACEBO

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3. LATIN SQUARE DESIGN (LSD) – COMPARING THE EFFECTS OF 5 DIFFERENT CHEMICALS WHILE CONTROLLING FOR THE EFFECT OF DAY AND BATCH OF EXPERIMENT

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3. 2^3 FACTORIAL EXPERIMENT WITH PARTIAL CONFOUNDING - ANALYZING THE EFFECT OF NITROGEN, PHOSPHORUS AND POTASSIUM AND THEIR ALL POSSIBLE INTERACTIONS ON POTATO CROP YIELD WHILE CONFOUNDING DIFFERENT INTERACTIONS IN DIFFERENT REPLICATES

4. 3^2 FACTORIAL EXPERIMENT – ANALYZING THE EFFECT OF DEVELOPER STRENGTH AND DEVELOPMENT TIME ON THE DENSITY OF PHOTOGRAPHIC FILM

4. Analysis of Variance

Analysis Of Variance(ANOVA) is the separation of variance ascribable to one group of causes from the variance ascribable to other group. The ANOVA consists in the estimation of the amount of variation due to each of the independent factors(causes) separately and then comparing these estimates due to assignable factors with the estimate due to chance factor.

Assumptions for ANOVA Test: ANOVA test is based on the test statistics F (or variance ratio). For validity of the F-test in ANOVA, the following assumptions are made:

1) The observations are independent,
2) Parent population from which observations are taken is normal.
3) Various treatment and environmental effects are additive in nature.

Here we will discuss the analysis of variance for:

(a) One way classification and (b) Two way classification.

4.6 One-Way ANOVA

One-way analysis of variance (abbreviated one-way ANOVA) is a technique used to compare means of three or more samples (using the F distribution). This technique can be used only for numerical data.

4.1.1 Completely Randomized Design

It is a linear model which describes treatment groups that are identically normally distributed with different means. Thus fitting this model requires not only the means of each treatment groups but also an average variance within the groups.

Model :

If the factor levels under consideration are the only levels of interest, then the fixed effect or parametric model given below is used:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}; (i = 1,2, \dots, k; j = 1,2, \dots, n_i)$$

Where α_i's are fixed unknown constants, y_{ij} is the yield from the jth row fed on the ith ration, μ is the general mean effect and ϵ_{ij} is the error effect due to chance.

ASSUMPTIONS IN MODEL:

a) All the observations y_{ij} are independent and $y_{ij} \sim N(\mu_i, \sigma_e^2)$.

b) Different effects are additive in nature.

c) ϵ_{ij} are i.i.d $N(0, \sigma_0^2)$ i.e. $E(\epsilon_{ij}) = 0$ and $V(\epsilon_{ij}) = 0 \ \forall$ i and j.

Hypothesis Testing :

H_0: Drugs do not differ significantly as regards their effect on performance level i.e $\alpha_i = 0$ v/s

H_1 : Drugs differ significantly. $\alpha_i \neq 0$ for all i

Test Statistic :

$$F = \frac{\text{between groups variability}}{\text{within groups variability}} = \frac{SS(B)/(k-1)}{SS(W)/(n-k)} \sim F(k-1)(N-k)$$

where, k = no of groups of treatments to be compared

N = sample size

Test Criteria :

Reject the null hypothesis at $100(1-\alpha)\%$ if $F > F(k-1)(N-k)$

ANOVA Table :

ANOVA

Observations					
	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	**361.480**	**4**	**90.370**	**9.678**	**.000**
Within Groups	**420.200**	**45**	**9.338**		
Total	**781.680**	**49**			

Interpretation :

From the above table, it can be observed that the significant value of the F statistic is 0.000 i.e less than 0.05. This implies that the null hypothesis is rejected i.e. mean of all the treatment groups are not equal.

Post Hoc Analysis :

Since it has been observed that mean of all the treatment groups are not equal, so now the purpose is to know which treatment group is effective in terms of its mean

Multiple Comparisons

Observations

Tukey HSD

(I) Flag1	(J) Flag1	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	.100	1.367	1.000	-3.78	3.98

	3	-4.000*	1.367	.041	-7.88	-.12
	4	-6.600*	1.367	.000	-10.48	-2.72
	5	-4.900*	1.367	.007	-8.78	-1.02
2	1	-.100	1.367	1.000	-3.98	3.78
	3	-4.100*	1.367	.034	-7.98	-.22
	4	-6.700*	1.367	.000	-10.58	-2.82
	5	-5.000*	1.367	.006	-8.88	-1.12
3	1	4.000*	1.367	.041	.12	7.88
	2	4.100*	1.367	.034	.22	7.98
	4	-2.600	1.367	.331	-6.48	1.28
	5	-.900	1.367	.964	-4.78	2.98
4	1	6.600*	1.367	.000	2.72	10.48
	2	6.700*	1.367	.000	2.82	10.58
	3	2.600	1.367	.331	-1.28	6.48
	5	1.700	1.367	.726	-2.18	5.58
5	1	4.900*	1.367	.007	1.02	8.78
	2	5.000*	1.367	.006	1.12	8.88
	3	.900	1.367	.964	-2.98	4.78
	4	-1.700	1.367	.726	-5.58	2.18

*. The mean difference is significant at the 0.05 level.

Here 1 refers to placebo and 2,3,4,5 denotes drug A to drug D respectively

We observe that the pairs (Drug A, Drug B), (Drug A , Drug C), (Drug A , Drug D), (Drug B, placebo), (Drug C, placebo), (Drug D, placebo) differ significantly.

Observations

Tukey HSD

Flag1	N	Subset for alpha = 0.05	
		1	2
2	10	7.90	
1	10	8.00	
3	10		12.00
5	10		12.90
4	10		14.60
Sig.		1.000	.331

Means for groups in homogeneous subsets are displayed.

From the above tables it can be seen that the 4^{th} treatment is the most effective out of 4 treatments.

4.7 Two-Way ANOVA

The two-way analysis of variance is an extension to the one-way analysis of variance. There are two independent variables (hence the name two-way).

Assumptions

- The populations from which the samples were obtained must be normally or approximately normally distributed.
- The samples must be independent.

- The variances of the populations must be equal.
- The groups must have the same sample size.

Testing Hypotheses:

There are three sets of hypothesis with the two-way ANOVA:

- The population means of the first factor are equal. This is like the one-way ANOVA for the row factor.
- The population means of the second factor are equal. This is like the one-way ANOVA for the column factor.
- There is no interaction between the two factors. This is similar to performing a test for independence with contingency tables.

4.2.1 Randomized Block Design (RBD) with 1 observation per cell

Model :

If in an RBD a single observation is made on each of the experimental units, then its analysis is analogous to ANOVA for fixed effect model for a two-way classified data with one observation per cell and linear model becomes:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk}$$

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over group of blocks

k = 1, 2, ……. , K is an index over experimental units

y_{ijk} = observation corresponding to the i^{th} treatment and j^{th} block

μ = overall mean of all the observations

α_i = additional fixed effect of the i^{th} treatment

β_j = additional fixed effect of the j^{th} block

$\varepsilon \sim N(0, \sigma^2)$, ε_{ijk} are normally distributed zero mean random errors

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over group of blocks

k = 1, 2, ……. , K is an index over experimental units

y_{ijk} = observation corresponding to the i^{th} treatment and j^{th} block

μ = overall mean of all the observations

α_i = additional fixed effect of the i^{th} treatment

β_j = additional fixed effect of the j^{th} block

$\varepsilon \sim N(0, \sigma^2)$, ε_{ijk} are normally distributed zero mean random errors

Assumptions :

1. Samples are independent.
2. Response variables are normally distributed.
3. Error term has a constant variance.
4. Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).

Hypothesis Testing :

When we want to compare t independent treatment groups and b groups of block, then the hypothesis of interest become :

H_{01} : All methods of analysis are equally effective i.e $\alpha_i = 0$ v/s H_{11} : At least 2 α_i's are different i.e $\alpha_i \neq 0$ for all i

H_{02} : There is no significant difference between analysts i.e $\beta_j = 0$ v/s
H_{12} : At least 2 β_j's are different. $\beta_j \neq 0$ for all

Test Statistic :

$$F_1 = \frac{\text{mean square of group of treatments}}{\text{within groups variability}}$$

$$= \frac{SST/(t-1)}{SSE/(t-1)(b-1)} \sim F(t-1), (t-1)(b-1)$$

$$F_2 = \frac{\text{mean square of group of blocks}}{\text{within groups variability}}$$

$$= \frac{SSB/(b-1)}{SSE/(t-1)(b-1)} \sim F(b-1), (t-1)(b-1)$$

where, t = no of groups of treatments to be compared

b = no of groups of blocks to be compared

Test Criteria :

Reject the null hypothesis (H_{01}) at $100(1-\alpha)\%$

if $F_1 > F(t-1)(t-1)(b-1)$

Reject the null hypothesis (H_{02}) at $100(1-\alpha)\%$

if $F_2 > F(b-1)(t-1)(b-1)$

Part (a)

Anova Table :

Tests of Between-Subjects Effects

Dependent Variable: Observation

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.839[a]	6	.140	8.142	.005
Intercept	764.694	1	764.694	4.455E4	.000
Analyst	.043	4	.011	.621	.660
Method	.796	2	.398	23.184	.000
Error	.137	8	.017		
Total	765.670	15			
Corrected Total	.976	14			

a. R Squared = .859 (Adjusted R Squared = .754)

INFERENCE: Since p-value for treatment (0.000)<0.05, therefore we reject H_{01} at 5% l.o.s and conclude that all methods of analysis are not equally effective.

Since p-value for blocks (0.660)>0.05, therefore we may accept H'_{02} at 5% l.o.s and conclude that there is no significant difference between analysts.

Part (b) Analysis for the data with shifted origin

Hypothesis:

H_{01}: All methods of analysis are equally effective i.e. $\tau_1=\tau_2=\tau_3$.

H_{11}: At least 2 τ_i's are different.

and

H_{02}: There is no significant difference between analysts i.e. $b_1=b_2=b_3=b_4=b_5$

H_{12}: At least 2 b_i's are different.

Tests of Between-Subjects Effects

Dependent Variable: obsv

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	.839[a]	6	.140	8.142	.005
Intercept	764.694	1	764.694	44545.282	.000
analyst	.043	4	.011	.621	.660
method	.796	2	.398	23.184	.000
Error	.137	8	.017		
Total	765.670	15			
Corrected Total	.976	14			

a. R Squared = .859 (Adjusted R Squared = .754)

INFERENCE:

Since p-value for treatment (0.000)<0.05, therefore we reject H_{01} at 5% l.o.s and conclude that all methods of analysis are not equally effective.

Since p-value for blocks (0.660)>0.05, therefore we may accept H'_{02} at 5% l.o.s and conclude that there is no significant difference between analysts.

We observe that the ANOVA performed on Original data and that on the transformed data gives the same results for treatment effects as well as for block effects.

Since H_o is rejected for treatment effects in this case, we proceed to Post-Hoc analysis to find out which of the treatment pairs differ significantly from the rest.

POST-HOC ANALYSIS:

We set the hypothesis: H_o: $\tau i, \tau j$ do not differ significantly. Where $i \neq j$ and $\tau i, \tau j$ is any pair of treatments.

H_1: $\tau i, \tau j$ differ significantly. Where $i \neq j$ and $\tau i, \tau j$ is any pair of treatments.

Multiple Comparisons

Observation

Tukey HSD

(I)	(J)	Mean Difference	Std. Error	Sig.	95% Confidence Interval

Method	Method	(I-J)			Lower Bound	Upper Bound
1	2	.3400*	.08287	.009	.1032	.5768
	3	.5600*	.08287	.000	.3232	.7968
2	1	-.3400*	.08287	.009	-.5768	-.1032
	3	.2200	.08287	.067	-.0168	.4568
3	1	-.5600*	.08287	.000	-.7968	-.3232
	2	-.2200	.08287	.067	-.4568	.0168

Based on observed means.

The error term is Mean Square(Error) = .017.

*. The mean difference is significant at the 0.05 level.

Observation

Tukey HSD

Method	N	Subset	
		1	2
3	5	6.8800	
2	5	7.1000	
1	5		7.4400
Sig.		.067	1.000

From the above tables it can be observed that 1st treatment is the most effective.

INFERENCE: For post hoc analysis, we see that the treatment pairs for which the p value < 0.05 are the treatment pairs (method effect) that differ significantly. Thus we reject the null hypothesis at 5% l.o.s. for such treatment pairs.

We observed that 1 method differs significantly from 2 and 3 method.

4.2.1 Randomized Block Design (RBD) with m observations per cell

This is same as the randomized block design described above the main difference being that there are m observations per cell and with the main effects we need to study the interaction effects between the cell members or observations.

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$$

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over group of blocks

k = 1, 2, ……. , K is an index over experimental units

y_{ijk} = observation corresponding to the i^{th} treatment and j^{th} block

μ = overall mean of all the observations

α_i = additional mean effect of the i^{th} treatment

β_j = additional mean effect of the j^{th} block

γ_{ij} = interaction effect between the i^{th} treatment and the j^{th} block

$\varepsilon \sim N(0, \sigma^2)$, ε_{ijk} are normally distributed zero mean random errors

Assumptions :

5. Samples are independent.
1. Response variables are normally distributed.
2. Error term has a constant variance.
3. Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).

Hypothesis Testing :

H_{01}: There is no significant difference between the effects of varieties of grains $\alpha_i = 0$ v/s

H_{11} : At least 2 bi's are different. $\alpha_i \neq 0$ for all i

H_{02}: There is no significant difference between the effects of varieties of grains i.e $\beta_j = 0$

v/s H_{12} : At least 2 bi's are different i.e $\beta_j \neq 0$ for all j

H_{03}: Different varieties of grains behave differently at different spacing i.e. there is no dependence between variety of grain used and spacing pattern adopted.

v/s H_{13}: There is dependence between variety of grain used and spacing pattern adopted.

Test Statistic :

$$F_1 = \frac{\text{mean square of group of treatments}}{\text{within groups variability}}$$

$$= \frac{SST/(t-1)}{SSE/(tb(m-1)-1)} \sim F(t-1),(tb(m-1))$$

$$F_2 = \frac{\text{mean square of group of blocks}}{\text{within groups variability}}$$

$$= \frac{SST/(b-1)}{SSE/(tb(m-1)-1)} \sim F(b-1),(tb(m-1)-1)$$

$$F_3 = \frac{\text{mean square of the interaction effect}}{\text{within groups variability}}$$

$$= \frac{SSTB/(t-1)(b-1)}{SSE/(tb(m-1)-1)} \sim F(t-1)(b-1),(tb(m-1)-1)$$

where, t = no of groups of treatments to be compared

b = no of groups of blocks to be compared

m = no of observations per cell

Test Criteria :

Reject the null hypothesis (H_{01}) at $100(1-\alpha)\%$

if $F_1 > F(t-1),(tb(m-1)-1)$

Reject the null hypothesis (H_{02}) at $100(1-\alpha)\%$

if $F_2 > F(b-1),(tb(m-1)-1)$

Reject the null hypothesis (H_{03}) at $100(1-\alpha)\%$

if $F_2 > F(t-1)(b-1),(tb(m-1)-1)$

ANOVA Table :

Tests of Between-Subjects Effects

Dependent Variable:Observation

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2073.500[a]	14	148.107	5.694	.000
Intercept	194940.000	1	194940.000	7.494E3	.000
Variety	1089.167	4	272.292	10.468	.000
Spacing	109.200	2	54.600	2.099	.134
Variety * Spacing	875.133	8	109.392	4.206	.001

Error	1170.500	45	26.011		
Total	198184.000	60			
Corrected Total	3244.000	59			

a. R Squared = .639 (Adjusted R Squared = .527)

INFERENCE:

Since p-value for block effect (0.134)>0.05, therefore we may accept H_{01} at 5 % l.o.s and conclude that all patterns of spacing are equally effective.

Since p-value for treatment effect (0.000)<0.05, therefore we reject H_{02} at 5 % l.o.s and conclude that all varieties of grains are not equally effective.

Since p-value for interaction effect (0.001)<0.05, therefore we reject H_{03} at 5 % l.o.s and conclude that there is dependence (interaction) between variety of grain used and spacing pattern adopted

Post Hoc Analysis :

Since it has been observed that mean of all the variety are not the same, so now the purpose is to know which variety of grain is effective in terms of its mean. For this post hoc is done

Hypothesis: H_o: $\tau i, \tau j$ do not differ significantly. Where $i \neq j$ and $\tau i, \tau j$ is any pair of treatments.

H_1: $\tau i, \tau j$ differ significantly. Where $i \neq j$ and $\tau i, \tau j$ is any pair of treatments.

Multiple Comparisons

Observation

Tukey HSD

(I) Variety	(J) Variety	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-4.8333	2.08211	.157	-10.7495	1.0829
	3	-4.0833	2.08211	.301	-9.9995	1.8329
	4	-6.3333*	2.08211	.030	-12.2495	-.4171
	5	-13.0833*	2.08211	.000	-18.9995	-7.1671
2	1	4.8333	2.08211	.157	-1.0829	10.7495
	3	.7500	2.08211	.996	-5.1662	6.6662
	4	-1.5000	2.08211	.951	-7.4162	4.4162
	5	-8.2500*	2.08211	.002	-14.1662	-2.3338

3	1	4.0833	2.08211	.301	-1.8329	9.9995
	2	-.7500	2.08211	.996	-6.6662	5.1662
	4	-2.2500	2.08211	.815	-8.1662	3.6662
	5	-9.0000*	2.08211	.001	-14.9162	-3.0838
4	1	6.3333*	2.08211	.030	.4171	12.2495
	2	1.5000	2.08211	.951	-4.4162	7.4162
	3	2.2500	2.08211	.815	-3.6662	8.1662
	5	-6.7500*	2.08211	.018	-12.6662	-.8338
5	1	13.0833*	2.08211	.000	7.1671	18.9995
	2	8.2500*	2.08211	.002	2.3338	14.1662
	3	9.0000*	2.08211	.001	3.0838	14.9162
	4	6.7500*	2.08211	.018	.8338	12.6662

Based on observed means.

The error term is Mean Square(Error) = 26.011.

*. The mean difference is significant at the 0.05 level.

Observation

Tukey HSD

Variety	N	Subset		
		1	2	3
1	12	51.3333		
3	12	55.4167	55.4167	
2	12	56.1667	56.1667	
4	12		57.6667	
5	12			64.4167
Sig.		.157	.815	1.000

Means for groups in homogeneous subsets are displayed.

Based on observed means.

The error term is Mean Square(Error) = 26.011.

From the above tables it can be inferred that 5th treatment is most effective.

INFERENCE: For post hoc analysis, we see that the treatment pairs for which the p value < 0.05 are the treatment pairs (variety effect) that differ significantly. Thus we reject the null hypothesis at 5% l.o.s. for such treatment pairs.

We observe that the variety 5 differs significantly from all the other varieties (1, 2, 3, 4). Also 1 differs significantly from 2 and 4.

4.2.2 Latin Square Design (LSD)

An experimental design that can be used to control the random variation of two factors. Such a layout is a Latin Square Design. The design is arranged with an equal number of rows and columns, so that all combinations of possible values for the two variables can be tested multiple times. This design is used to reduce the effect of random or nuisance factors.

Let $y_{ijk}\,(i,j,k = 1,2,\ldots,m)$ denote the response from the unit in the i^{th} row, j^{th} column and receiving the k^{th} treatment. The triple (i,j,k) assumes only m^2 of the possible m^3 values of an LS selected by the experiment. If S represents the set of m^2 values, then symbolically (i, j, k) Є S. If a single observation is made per experimental unit, then the linear additive model is:

$$y_{ijk} = \alpha_i + \beta_j + \tau_k + \epsilon_{ijk}\ (i,j,k)\ \epsilon\ S$$

Where μ is the constant mean effect; $\alpha_i, \beta_j, \tau_k$ are the constant effects due to the i^{th} row, j^{th} column and k^{th} treatment respectively and ϵ_{ijk} is error effect due to random component assumed to be normally distributed with mean zero and **variance σ_e^2.**

Hypothesis Testing:

$$H_0 :\ \rho_i = 0 \qquad \text{v/s} \qquad H_1 :\ \rho_i \neq 0$$

Test Statistic :

$$F_1 = \frac{\text{mean square of group of treatments}}{\text{within groups variability}}$$

$$= \frac{SST/(t-1)}{SSE/(t-1)(t-2)} \sim F(t-1),(t-1)(t-2)$$

Test Criteria :

Reject the null hypothesis (H_0) at $100(1-\alpha)\%$

if $F_1 > F(t-1)(t-1)(t-2)$

ANOVA TABLE

Tests of Between-Subjects Effects

Dependent Variable:Observations

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	170.396[a]	12	14.200	4.701	.006
Intercept	864.360	1	864.360	286.182	.000
Row	15.440	4	3.860	1.278	.332
Column	8.716	4	2.179	.721	.594
Treatments	142.716	4	35.679	11.813	.000
Error	36.244	12	3.020		
Total	1071.000	25			
Corrected Total	206.640	24			

a. R Squared = .825 (Adjusted R Squared = .649)

INFERNCE:

Since 0.000<0.05, therefore we reject H_0 at 5% level of significance. Hence we conclude that all chemicals are not equally effective.

Since H_o is rejected for treatment effects in this case, we proceed to Post-Hoc analysis to find out which of the treatment pairs differ significantly from the rest.

POST-HOC ANALYSIS:

We set the hypothesis:

H_o: $\tau i, \tau j$ do not differ significantly. Where $i \neq j$ and $\tau i, \tau j$ is any pair of treatments.

H_1: $\tau i, \tau j$ differ significantly. Where $i \neq j$ and $\tau i, \tau j$ is any pair of treatments.

Multiple Comparisons

Dependent Variable: Observations

Tukey HSD

(I) Treatments	(J) Treatments	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
A	B	2.80	1.099	.119	-.49	6.09
	C	-.40	1.099	.996	-3.69	2.89
	D	5.80*	1.099	.000	2.51	9.09
	E	4.40*	1.099	.006	1.11	7.69
B	A	-2.80	1.099	.119	-6.09	.49
	C	-3.20	1.099	.059	-6.49	.09
	D	3.00	1.099	.085	-.29	6.29
	E	1.60	1.099	.601	-1.69	4.89
C	A	.40	1.099	.996	-2.89	3.69
	B	3.20	1.099	.059	-.09	6.49
	D	6.20*	1.099	.000	2.91	9.49
	E	4.80*	1.099	.002	1.51	8.09
D	A	-5.80*	1.099	.000	-9.09	-2.51
	B	-3.00	1.099	.085	-6.29	.29
	C	-6.20*	1.099	.000	-9.49	-2.91
	E	-1.40	1.099	.710	-4.69	1.89
E	A	-4.40*	1.099	.006	-7.69	-1.11
	B	-1.60	1.099	.601	-4.89	1.69
	C	-4.80*	1.099	.002	-8.09	-1.51
	D	1.40	1.099	.710	-1.89	4.69

Based on observed means.

The error term is Mean Square(Error) = 3.020.

*. The mean difference is significant at the 0.05 level.

Observations

Tukey HSD[a,b]

Treatments	N	Subset	
		1	2
D	5	2.60	
E	5	4.00	
B	5	5.60	5.60
A	5		8.40
C	5		8.80
Sig.		.085	.059

Means for groups in homogeneous subsets are displayed.

Based on observed means.

The error term is Mean Square(Error) = 3.020.

a. Uses Harmonic Mean Sample Size = 5.000.

b. Alpha = 0.05.

INFERENCE: For post hoc analysis, we see that the treatment pairs for which the p value < 0.05 are the treatment pairs (chemical effect) that differ significantly. Thus we reject the null hypothesis at 5% l.o.s. for such treatment pairs.

We observe that the chemical pairs **(A,D), (A,E), (C,D) and (C,E)** differ significantly from each other.

4.3 One Way ANOCOVA

Model :

It is a general linear model that blends ANOVA and regression. It evaluates whether the population means of a dependent variable are equal across levels of a treatment while statistically controlling for the effects of another continuous variables known as covariates.

$$y_{ij} = \mu + \alpha_i + B(x_{ij} - \bar{x_i}) + \varepsilon_{ij}$$

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over experimental units

y_{ij} = observation corresponding to the i^{th} treatment and j^{th} experimental unit

μ = overall mean of all the observations

α_i = additional fixed effect of the i^{th} treatment

x_{ij} = j^{th} observation of the covariate under i^{th} treatment

$\bar{x_i}$ = i^{th} group mean

$\varepsilon \sim N(0, \sigma^2)$, ε_{ij} are normally distributed zero mean random errors

Assumptions :

1. Samples are independent.
2. The regression relationship between the dependent variable and the covariate must be linear.
3. Response variables are normally distributed.
4. Error term has a constant variance.
5. Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).

Hypothesis Testing :

When we want to know whether the covariate is significant or not, then the hypothesis of interest becomes :

H_0 : All machines are equally effective i.e. $\alpha_i = 0$ v/s

H_1 : At least 2 αi's are different: $\alpha_i \neq 0$ for all i

Test Statistic :

$$F = \frac{SST/(k-1)}{SSE/(n-k-1)} \sim F(k-1)(n-k-1)$$

where, k = no of groups of treatments to be compared

N = sample size

Test Criteria :

Reject the null hypothesis at $100(1-\alpha)\%$ if $F > F(k-1)(n-k-1)$

ANCOVA Table :

Tests of Between-Subjects Effects

Dependent Variable:Y

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	240.152[a]	3	80.051	14.299	.000
Intercept	183.952	1	183.952	32.858	.000
Machine_Flag	55.802	2	27.901	4.984	.029
X	105.618	1	105.618	18.866	.001
Error	61.582	11	5.598		
Total	24462.000	15			
Corrected Total	301.733	14			

a. R Squared = .796 (Adjusted R Squared = .740)

INFERENCE: Since 0.05 > 0.029, therefore we reject H_0 at 5% level of significance. Hence we conclude that all machines are not equally effective.

4.5 Two way ANCOVA

Model :

It is a general linear model that blends ANOVA and regression. It evaluates whether the population means of a dependent variable are equal across levels of a treatment while statistically controlling for the effects of two continuous variables known as covariates.

$$y_{ijk} = \mu + \alpha_i + \beta_j + B(x_{ijk} - \bar{x_i}) + \varepsilon_{ijk}$$

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over groups of block

k = 1,2, …….. , K is an index over experimental units

y_{ijk} = observation corresponding to the i^{th} treatment and j^{th} block

μ = overall mean of all the observations

α_i = additional fixed effect of the i^{th} treatment

x_{ik} = k^{th} observation of the covariate under i^{th} treatment

$\bar{x_i}$ = i^{th} group mean

β_j = mean effect of the j^{th} block

$\varepsilon \sim N(0, \sigma^2)$, ε_{ij} are normally distributed zero mean random errors

Assumptions :

1. Samples are independent.
2. The regression relationship between the dependent variable and the covariates must be linear.
3. Response variables are normally distributed.
4. Error term has a constant variance.
5. Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).

Hypothesis Testing :

When we want to compare t independent treatment groups and b groups of block, then the hypothesis of interest become :

H_{01} : All manure treatments are equally effective $\alpha_i = 0$ v/s

H_{11} : : At least 2 αi's are different i. e $\alpha_i \neq 0$ for all i

Test Statistic : $F_1 = \frac{mean\ square\ of\ group\ of\ treatments}{within\ groups\ variability}$

$$= \frac{SST/(t-1)}{SSE/(t-1)(b-1)} \sim F(t-1), ((t-1)(b-1)-1)$$

$$F_2 = \frac{mean\ square\ of\ group\ of\ blocks}{within\ groups\ variability}$$

$$= \frac{SSB/(b-1)}{SSE/(t-1)(b-1)} \sim F(b-1), ((t-1)(b-1)-1)$$

where, t = no of groups of treatments to be compared

b = no of groups of blocks to be compared

Test Criteria :

Reject the null hypothesis (H_{01}) at $100(1-\alpha)\%$

if $F_1 > F(t-1)((t-1)(b-1)-1)$

Reject the null hypothesis (H_{02}) at $100(1-\alpha)\%$

if $F_2 > F(b-1)((t-1)(b-1)-1)$

ANCOVA Table :

Tests of Between-Subjects Effects

Dependent Variable:Yield

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	741.185[a]	8	92.648	33.019	.000
Intercept	37.995	1	37.995	13.541	.004
Treatment	551.957	4	137.989	49.178	.000
Block	26.911	3	8.970	3.197	.066
No_Plants	92.635	1	92.635	33.014	.000
Error	30.865	11	2.806		
Total	8652.500	20			
Corrected Total	772.050	19			

a. R Squared = .960 (Adjusted R Squared = .931)

INFERENCE: Since p-value for treatment (0.000)<0.05, therefore we reject H_0 at 5% l.o.s and conclude that all treatments are not equally effectiv

Factorial Experiment

In statistics, a full factorial experiment is an experiment whose design consists of two or more factors, each with discrete possible values or "levels", and whose experimental units take on all possible combinations of these levels across all such factors. A full factorial design may also be called a fully crossed design. Such an experiment allows the investigator to study the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable.

4.5.1 2^3 Factorial Experiment

Model

If y_{ijkl} is the response observed at the i^{th} level of A, j^{th} level of B, k^{th} level of C in the l^{th} replicate then the linear model for a 2^3-experiment becomes:

$$y_{ijkl} = \mu + \alpha_i + \beta_j + y_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \rho_l + \varepsilon_{ijkl}$$

$$(i,j,k) = 0,1; l = 1,2 \ldots r$$

Where μ is the general mean, α_i, β_j, γ_k are the effects of the i^{th} level of A, j^{th} level of B, k^{th} level of C; $(\alpha\beta)_{ij}$ and $(\alpha\gamma)_{ik}$ are the interaction effect of i^{th} level of A with j^{th} level of B and k^{th} level of C; $(\beta\gamma)_{jk}$ is the interaction effect of j^{th} level of B and k^{th} level of C, ρ_l is the effect due to l^{th} replicate; ε_{ijkl} is the error due to chance.

Assumptions:

- all the observations are independent & $y_{ijkl} \sim N(\mu_i, \sigma^2)$
- Different effects are additive in nature.
- $\varepsilon_{ijkl} \xrightarrow{i.i.d} N(0, \sigma_e^2)$
- $\sum_{i=0}^{1} \alpha_i = 0 \qquad \sum_{j=0}^{1} \beta_j = 0 \qquad \sum_{k=0}^{1} \gamma_k = 0$
- $\sum(\alpha\beta)_{ij} \quad \sum(\alpha\gamma)_{ik} \quad \sum(\beta\gamma)_{jk} \quad \sum(\alpha\beta\gamma)_{ijk}$ are equal to zero respectively, when summed over either subscript for all the values of the remaining subscripts.

In the analysis of 2^3 design we split the treatment SS with 7 d.f. into 7 mutually orthogonal components corresponding to 7 factorial effects, each carrying 1 d.f. Factorial effect totals are obtained using yates technique. The SS due to any factorial effect is given by:

$$\frac{[.]^2}{8r}$$

Hypothesis Testing :

When we want to compare t independent treatment groups and r groups of replicates, then the hypothesis of interest become :

$$H_{01}: \text{treatments effects are not significant.} \alpha_i = 0 \quad \text{v/s}$$
$$H_{11}: \text{treatments effects are significant } \alpha_i \neq 0 \quad \text{for all } i$$

$$H_{02}: \text{replicates are homogeneous } \beta_j = 0 \quad \text{v/s}$$
$$H_{12}: \text{replicates are not homogeneous } \beta_j \neq 0 \quad \text{for all } j$$

$$H_{03}: \text{interaction effects are not significant.} \sigma_k = 0 \quad \text{v/s}$$
$$H_{13}: \text{interaction effects are significant } \sigma_k \neq 0 \quad \text{for all } k$$

Test Statistic :

$$F_1 = \frac{\text{mean square of group of treatments}}{\text{within groups variability}}$$

$$= \frac{SST/(t-1)}{SSE/(t-1)(r-1)} \sim F(t-1),(t-1)(r-1)$$

$$F_2 = \frac{mean\ square\ of\ group\ of\ replicates}{within\ groups\ variability}$$

$$= \frac{SSB/(r-1)}{SSE/(t-1)(r-1)} \sim F(b-1),(t-1)(r-1)$$

where, t = no of groups of treatments to be compared

r = no of groups of replicates to be compared

Test Criteria :

Reject the null hypothesis (H_{01}) at $100(1-\alpha)\%$

if $F_1 > F(t-1)(t-1)(b-1)$

Reject the null hypothesis (H_{02}) at $100(1-\alpha)\%$

if $F_2 > F(b-1)(t-1)(b-1)$

Tests of Between-Subjects Effects

Dependent Variable:Obs

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	349171.625[a]	9	38796.847	147.905	.000
Intercept	2012025.042	1	2012025.042	7.670E3	.000
Repetition	520.333	2	260.167	.992	.396
N	5673.375	1	5673.375	21.629	.000
P	205535.042	1	205535.042	783.559	.000
K	124272.042	1	124272.042	473.761	.000
N * K	1053.375	1	1053.375	4.016	.065
P * K	11837.042	1	11837.042	45.126	.000
N * P	273.375	1	273.375	1.042	.325
N * P * K	7.042	1	7.042	.027	.872
Error	3672.333	14	262.310		
Total	2364869.000	24			
Corrected Total	352843.958	23			

Tests of Between-Subjects Effects

Dependent Variable:Obs

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	349171.625[a]	9	38796.847	147.905	.000
Intercept	2012025.042	1	2012025.042	7.670E3	.000
Repetition	520.333	2	260.167	.992	.396
N	5673.375	1	5673.375	21.629	.000
P	205535.042	1	205535.042	783.559	.000
K	124272.042	1	124272.042	473.761	.000
N * K	1053.375	1	1053.375	4.016	.065
P * K	11837.042	1	11837.042	45.126	.000
N * P	273.375	1	273.375	1.042	.325
N * P * K	7.042	1	7.042	.027	.872
Error	3672.333	14	262.310		
Total	2364869.000	24			

a. R Squared = .990 (Adjusted R Squared = .983)

INFERENCE: **Replicates:** Since 0.396>0.05, therefore we may accept H_0 at 5% level of significance and conclude that the replicates are homogeneous.

Main Effects: Since for each of the main effects, the p value 0.000 < 0.05, therefore we reject H_0 at 5% level of significance and conclude that the treatments effects are significant.

Interaction effects: Since p value for the interaction PK is less than 0.05, therefore we reject H_0 at 5% level of significance and conclude that interaction PK is significant for the potato yield.

4.5.2 2^3 Factorial Experiment (Total Confounding)

When there are two or more replications, if the same set of interactions is confounded in all the replications, **confounding** is called **complete** and if different sets of interaction are confounded in different replications, **confounding** is called **partial**. In complete confounding all the information on confounded interactions are lost. But in partial confounding, the confounded interactions can be recovered from those replications in which they are not confounded.

In a 2^3-factorial experiment, let the three factors be A, B & C each at 2 levels. The 8 treatment combinations require 8 units of homogeneous material each to form a block.

Each replicate is divided into two blocks. When we look at the principal block of each replicate we observe that the confounded effect is same. Since one effect is confounded, thus we will not be able to estimate it. So df corresponding to it will be 0.

The analysis will be done using two way ANOVA technique.

$$y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk}$$

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over group of blocks

k = 1, 2, ……. , K is an index over experimental units

y_{ijk} = observation corresponding to the i^{th} treatment and j^{th} block

μ = overall mean of all the observations

α_i = additional fixed effect of the i^{th} treatment

β_j = additional fixed effect of the j^{th} block

$\varepsilon \sim N(0,\ \sigma^2),\ \varepsilon_{ijk}$ are normally distributed zero mean random errors

Assumptions :

1. Samples are independent.
2. Response variables are normally distributed.
3. Error term has a constant variance.
4. Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).

Hypothesis Testing :

When we want to compare t independent treatment groups and b groups of block, then the hypothesis of interest become :

$$H_{01}: \text{effects are not significant } \alpha_i = 0 \quad \text{v/s}$$
$$H_{11}: \text{effects are significant } \alpha_i \neq 0 \quad \text{for all } i$$

$$H_{02}: \text{replicates are homogeneous } \beta_j = 0 \quad \text{v/s}$$
$$H_{12}: \text{replicates are not homogeneous } \beta_j \neq 0 \quad \text{for all } j$$

Test Statistic :

$$F_1 = \frac{\text{mean square of group of treatments}}{\text{within groups variability}}$$

$$= \frac{SST/(t-2)}{SSE/(N-t-b+1)} \sim F(t-2), (N-t-b+1)$$

$$F_2 = \frac{\text{mean square of group of blocks}}{\text{within groups variability}}$$

$$= \frac{SSB/(b-1)}{SSE/(N-t-b+1)} \sim F(b-1), (N-t-b+1)$$

where, t = no of groups of treatments to be compared

b = no of groups of blocks to be compared

N = total no of observations

Test Criteria :

Reject the null hypothesis (H_{01}) at $100(1-\alpha)\%$

if $F_1 > F(t-1)(t-1)(b-1)$

Reject the null hypothesis (H_{02}) at $100(1-\alpha)\%$

if $F_2 > F(b-1)(t-1)(b-1)$

ANOVA Table

Tests of Between-Subjects Effects

Dependent Variable:Obs

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	398297.875[a]	7	56899.696	50.409	.000
Intercept	2570778.125	1	2570778.125	2.278E3	.000

Block	.000	0	.	.	.
N	7381.125	1	7381.125	6.539	.017
P	232221.125	1	232221.125	205.733	.000
K	126253.125	1	126253.125	111.852	.000
N * K	2080.125	1	2080.125	1.843	.187
P * K	27028.125	1	27028.125	23.945	.000
N * P	3081.125	1	3081.125	2.730	.112
N * P * K	.000	0	.	.	.
Error	27090.000	24	1128.750		
Total	2996166.000	32			
Corrected Total	425387.875	31			

a. R Squared = .936 (Adjusted R Squared = .918)

Interpretation :

From the above table, we see that the treatment combination NPK is being confounded.

So we will remove NPK and again do the same analysis dropping NPK.

Tests of Between-Subjects Effects

Dependent Variable:Obs

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	398297.875[a]	7	56899.696	50.409	.000
Intercept	2570778.125	1	2570778.125	2.278E3	.000
Block	253.125	1	253.125	.224pc	.640
N	7381.125	1	7381.125	6.539	.017
P	232221.125	1	232221.125	205.733	.000
K	126253.125	1	126253.125	111.852	.000
N * K	2080.125	1	2080.125	1.843	.187
P * K	27028.125	1	27028.125	23.945	.000

N * P	3081.125	1	3081.125	2.730	.112
Error	27090.000	24	1128.750		
Total	2996166.000	32			
Corrected Total	425387.875	31			

a. R Squared = .936 (Adjusted R Squared = .918)

Inference:

From the ANOVA table, it can be inferred that:

Replicates: Since 0.729>0.05, therefore we may accept H_0 at 5% level of significance and conclude that the replicates are homogenous.

Main Effects: Since for each of the main effects, the p value < 0.05, therefore we reject H_0 at 5% level of significance and conclude that the treatments are significant w.r.t their effects on potato yield.

Interaction effects: Since p value for the interaction PK is less than 0.05, therefore we reject H_0 at 5% level of significance and conclude that interaction PK is significant w.r.t its effect on potato yield.

4.5.3 2^3 Factorial Experiment with Partial Confounding

For partial confounding everything remains same as complete confounding the main difference being that different treatments are confounded in different replicates and in this case none of the information is lost as the confounded treatment in a block can be estimated from the other block where it is not replicated.

ANOVA Table

Tests of Between-Subjects Effects

Dependent Variable:Observations

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	552546.500[a]	14	39467.607	4.040	.004
Intercept	1928648.000	1	1928648.000	197.422	.000

Block	134798.000	7	19256.857	1.971	.120
N	90100.125	1	90100.125	9.223	.007
P	38088.000	1	38088.000	3.899	.065
K	203203.125	1	203203.125	20.800	.000
N * K	.042	1	.042	.000	.998
P * K	322.667	1	322.667	.033	.858
N * P	15913.500	1	15913.500	1.629	.219
N * P * K	51987.042	1	51987.042	5.322	.034
Error	166075.500	17	9769.147		
Total	2647270.000	32			
Corrected Total	718622.000	31			

a. R Squared = .769 (Adjusted R Squared = .579)

INFERENCE:

From the ANOVA table, it can be inferred that:

Replicates: Since 0.120>0.05, therefore we may accept H_0 at 5% level of significance and conclude that the replicates are homogenous.

Main Effects: Since for the main effects due to factor N & K, the p value< 0.05, therefore we reject H_0 at 5% level of significance and conclude that the treatments N & K are significant w.r.t their effects on potato yield.

Interaction effects: Since p value for the interaction NPK is less than 0.05, therefore we reject H_0 at 5% level of significance and conclude that interaction NPK is significant w.r.t its effect on potato yield.

4.5.4 3^2 Factorial Experiment

This is the simplest three-level design. It has two factors, each at three levels. These are (usually) referred to as low, intermediate and high levels. These levels are numerically expressed as 0, 1, and 2.

Here we have 2 treatments each at three levels thereby obtaining 9 treatment combinations.

Each replicate is divided into two blocks. When we look at the principal block of each replicate we observe that the confounded effect is different for each replicate. The analysis will be done using two way ANOVA technique.

$$y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk}$$

where,

i = 1, 2, ……. , I is an index over treatment groups

j = 1, 2, ……. , J is an index over group of replicates

k = 1, 2, ……. , K is an index over experimental units

y_{ijk} = observation corresponding to the i[th] treatment and j[th] replicate

μ = overall mean of all the observations

α_i = additional fixed effect of the i[th] treatment

β_j = additional fixed effect of the j[th] replicate

$\varepsilon \sim N(0,\ \sigma^2),\ \varepsilon_{ijk}$ are normally distributed zero mean random errors

Assumptions :

1. Samples are independent.
2. Response variables are normally distributed.
3. Error term has a constant variance.
4. Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).

Hypothesis Testing :

When we want to compare t independent treatment groups and r groups of replicates, then the hypothesis of interest become :

$$H_{01}:\ \alpha_i = 0 \quad \text{v/s} \quad H_{11}:\ \alpha_i \neq 0 \quad for\ all\ i$$

$$H_{02}:\ \beta_j = 0 \quad \text{v/s} \quad H_{12}:\ \beta_j \neq 0 \quad for\ all\ j$$

Test Statistic :

$$F_1 = \frac{mean\ square\ of\ group\ of\ treatments}{within\ groups\ variability}$$

$$= \frac{SST/(t-1)}{SSE/(8(r-1))} \sim F(t-1), (8(r-1))$$

$$F_2 = \frac{mean\ square\ of\ group\ of\ replicates}{within\ \text{groups variability}}$$

$$= \frac{\text{SSB}/(\text{b}-1)}{\text{SSE}/(8(\text{r}-1))} \sim \text{F}(\text{b}-1), (8(\text{r}-1))$$

where, t = no of groups of treatments to be compared

r = no of groups of replicates to be compared

Test Criteria :

Reject the null hypothesis (H_{01}) at $100(1-\alpha)\%$

if $F_1 > F(t-1),(8(r-1))$

Reject the null hypothesis (H_{02}) at $100(1-\alpha)\%$

if $F_2 > F(b-1)(8(r-1))$

ANOVA Table

Tests of Between-Subjects Effects

Dependent Variable:Observations

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	238.556[a]	11	21.687	9.185	.000
Intercept	1418.778	1	1418.778	600.894	.000
Replicate	14.333	3	4.778	2.024	.137
DS	198.222	2	99.111	41.976	.000
DT	22.722	2	11.361	4.812	.017
DS * DT	3.278	4	.819	.347	.843
Error	56.667	24	2.361		
Total	1714.000	36			
Corrected Total	295.222	35			

a. R Squared = .808 (Adjusted R Squared = .720)

Inference:

Main Effects: Since for the main effects due to factor DS & DT, the p value is less that the level of significance 0.05, therefore we reject H_0 at 5% level of significance and conclude that the treatments DT & DS are significant w.r.t their effects on density of photographic film.

Interaction effects: Since p value for the interaction DS * DT is greater than 0.05, therefore we may accept H_0 at 5% level of significance and conclude that interaction DS * DT is not significant w.r.t its effect on density of photographic film.