Price of Gold in India

A Statistical Analysis

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Price of gold in India…

~a statistical analysis

Introduction:

Quite frequently in the morning newspaper, we come across the issue of rise in gold prices in India. Moreover in the past few years, there has been an abrupt increase in gold prices. Since gold behaves less like a commodity than long lived assets, such as stocks or bonds, it can be treated as a currency and thus, knowing its future value can be very useful. This has evoked in us, the curiosity of learning the nature of this increase and as well as predicting the future value of gold.

The price of gold depends on the market's psychological perception of the value of gold, which in turn depends on a myriad of interrelated variables, such as the foreign exchange rate, the amount of gold reserves in the RBI, the inflation rates, political turmoil, etc.

In this project, we have tried to forecast the weekly gold prices for 2015, using three distinct approaches:

> **Approach~I:** Classical decomposition of a time~series.
> **Approach~II:** Winter's exponential smoothing technique.
> **Approach~III:** Regression model.

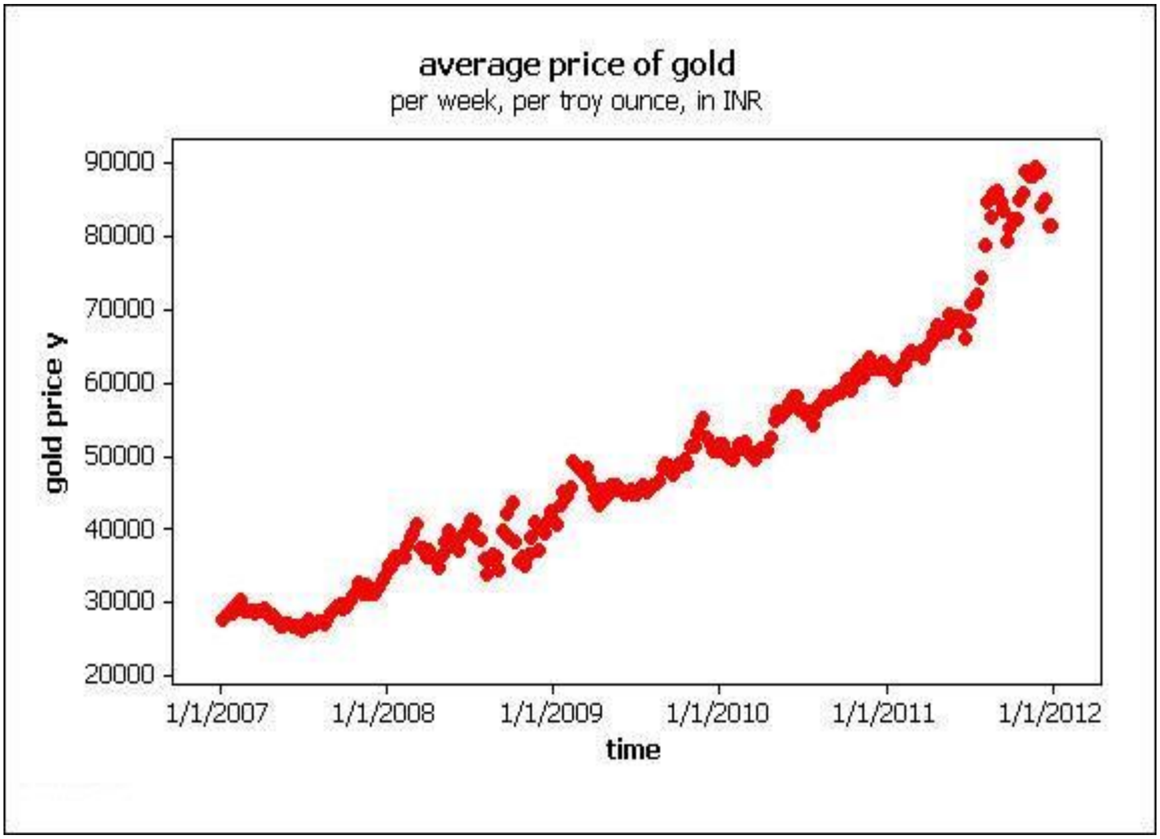
A comparative study of the performance of the three methods is presented at the end.

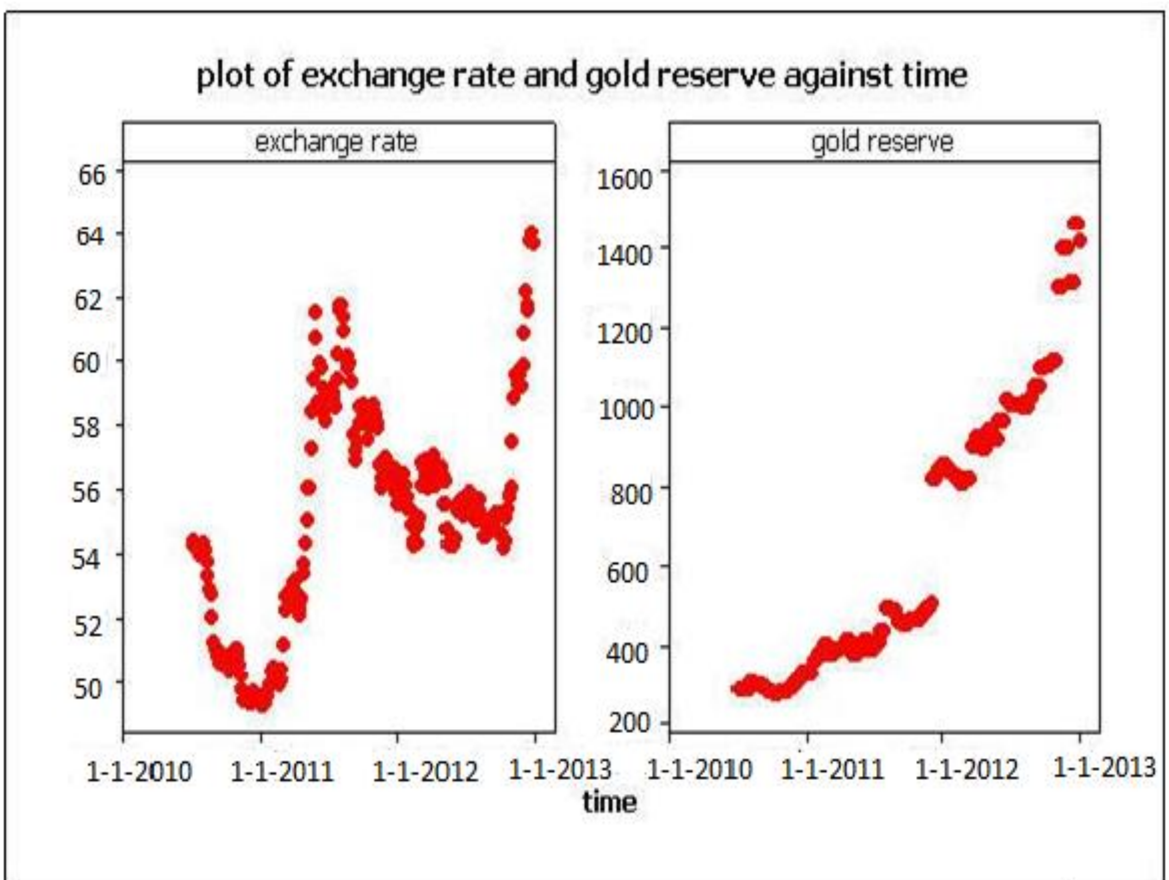
Data at hand:

- Average price of gold, per troy-ounce (31.1034768 gram), per week (cut-off points being end of the week i.e. Friday), in INR, over the period 2010-2014.
- Average USD-INR exchange rate, per week (cut-off points being end of the week i.e. Friday), over the period 2010-2014.
- Average value of gold reserve (in billion rupees), with the RBI, per week (cut-off points being end of the week i.e. Friday), in INR, over the period 2010-2014.

The entire dataset at hand is provided in **Appendix-1(a)**.

The plots of gold price, exchange rate and gold reserve against time are given below:


average price of gold
per week, per troy ounce, in INR
gold price y
90000
80000
70000
60000
50000
40000
30000
20000
1/1/2007
1/1/2008
1/1/2009
1/1/2010
1/1/2011
1/1/2012
time


plot of exchange rate and gold reserve against time
exchange rate
gold reserve
66
64
62
60
58
56
54
52
50
1600
1400
1200
1000
800
600
400
200
1-1-2010
1-1-2011
1-1-2012
1-1-2013
1-1-2010
1-1-2011
1-1-2012
1-1-2013
time

Approach-I: Classical decomposition:

Theory:

A close scrutiny of the above graph reveals that it is a complex combination of three identifiable movements, namely:

- An overall tendency (or trend),
- A regular periodic movement with periodicity less than a year (or seasonal fluctuation) and
- Some random or irregular fluctuations

The two identifiable movements i.e. trend and seasonal fluctuations constitute the systematic component or the accountable variation whereas the random or irregular fluctuations constitute the unsystematic component or the unaccountable variation in the series.

Observation:

Trend:

Clearly the inherent trend is **exponential.**

Seasonal fluctuations:

There are 52 complete weeks in a year; hence we have a total of 52 seasons.

Further, since every year has an extra day or two (according as the year being a normal or a leap year) apart from the 52 complete weeks, we have a 53rd weekly cut-off point if 1st January (of a year) or 2nd January (of a leap year) falls on a Friday (as in 2010).

For a bit of simplicity, we assume that, this “extra” time point has NO seasonal effect.

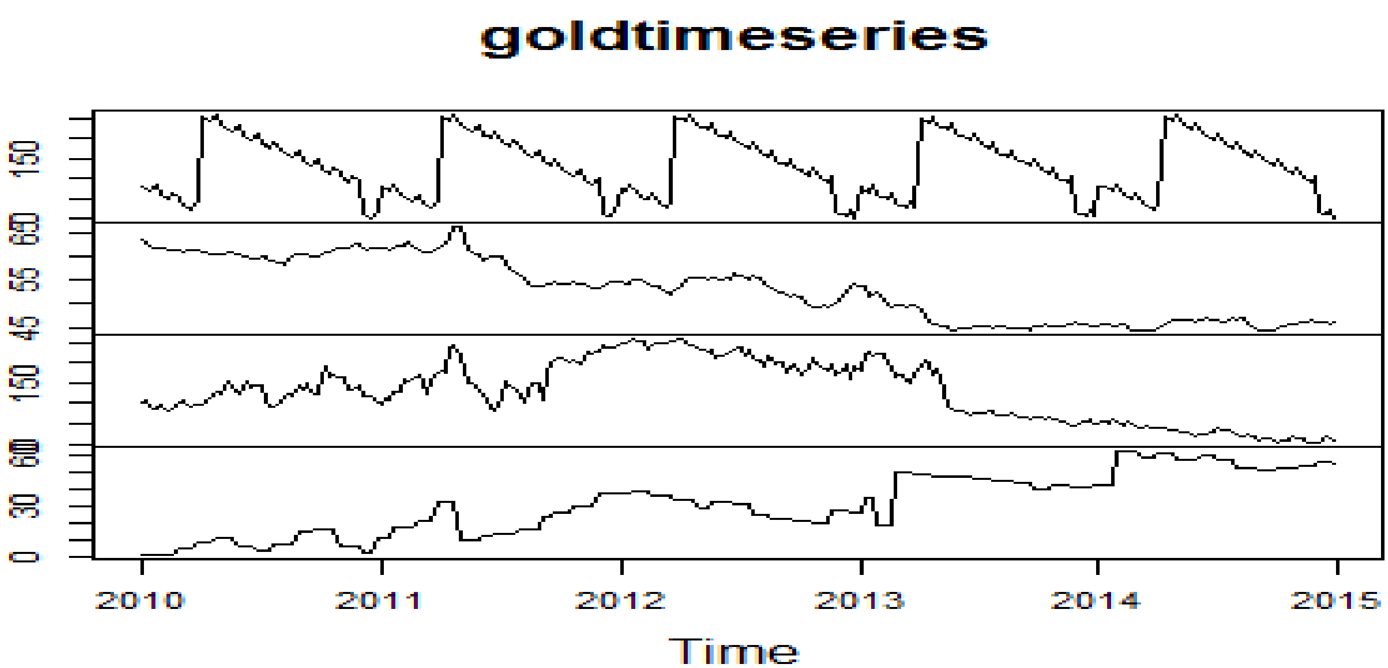
Mathematical model:

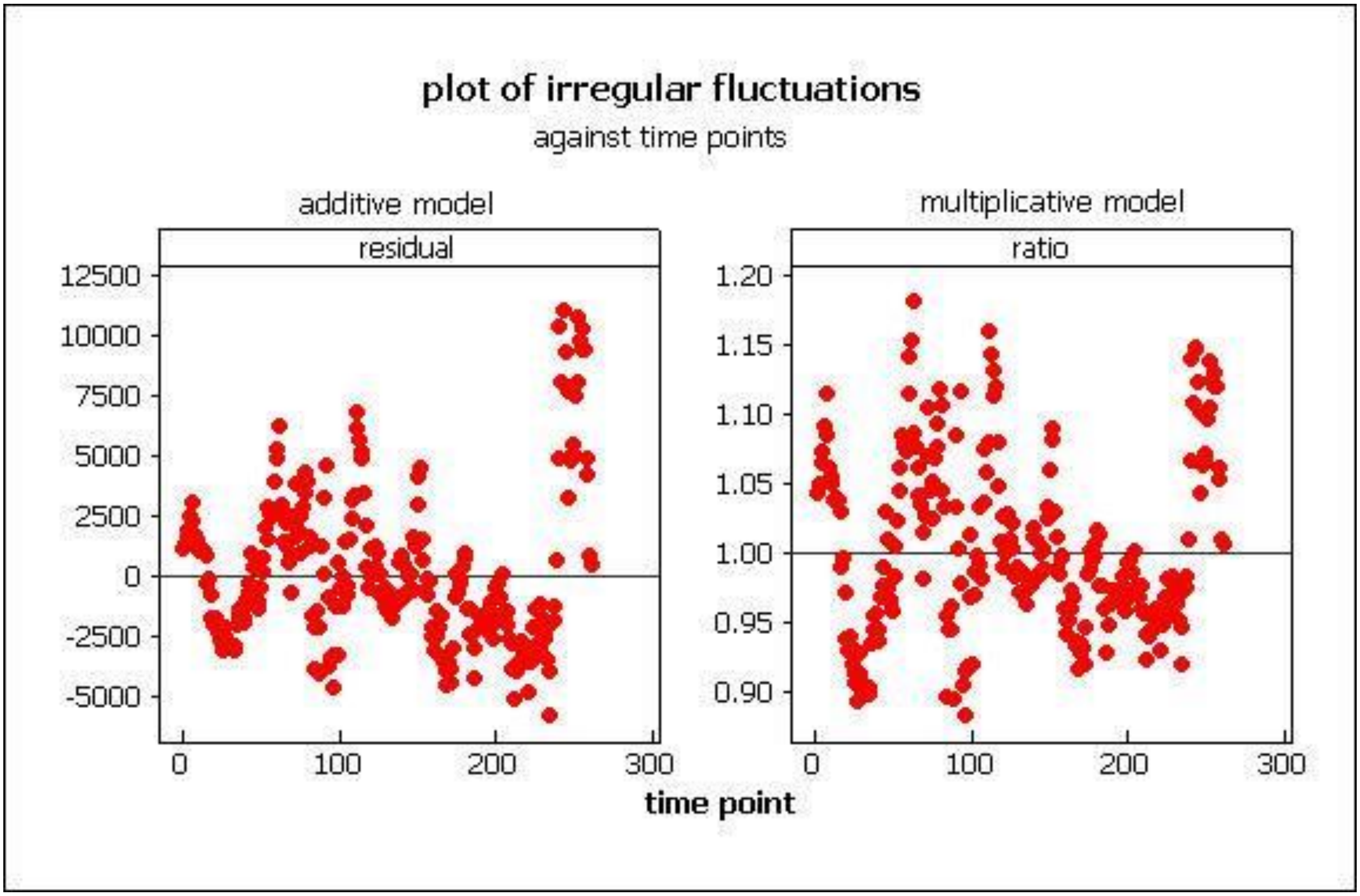
Two commonly used models are:

- **Additive model:** where it is assumed that the effects of the various components are independent and additive in nature.
- **Multiplicative model:** where it is assumed that the effects of the different components are dependent on one another and are multiplicative in nature.

The appropriate model is chosen after a look on the respective irregular fluctuations of the above mentioned models. The model which gives random irregular fluctuations is chosen to be the appropriate one.

The residuals of the two models are given below:





Both the residuals as well as the ratios fail the test of randomness with equal no. of observed runs. Thus, it would not matter which model we are taking up.

So, we proceed with the multiplicative model as it seems more practically appropriate.

Calculations and Results:

Define:

Y_t: Price of gold at the t^th time point

T_t: Trend value at the t^th time point

S_t: Seasonal variation corresponding to the t^th time point

I_t: Irregular fluctuation corresponding to the t^th time point

Thus, our model is:

$$Y_t = T_t \,.S_t \,.I_t$$

which reduces to:

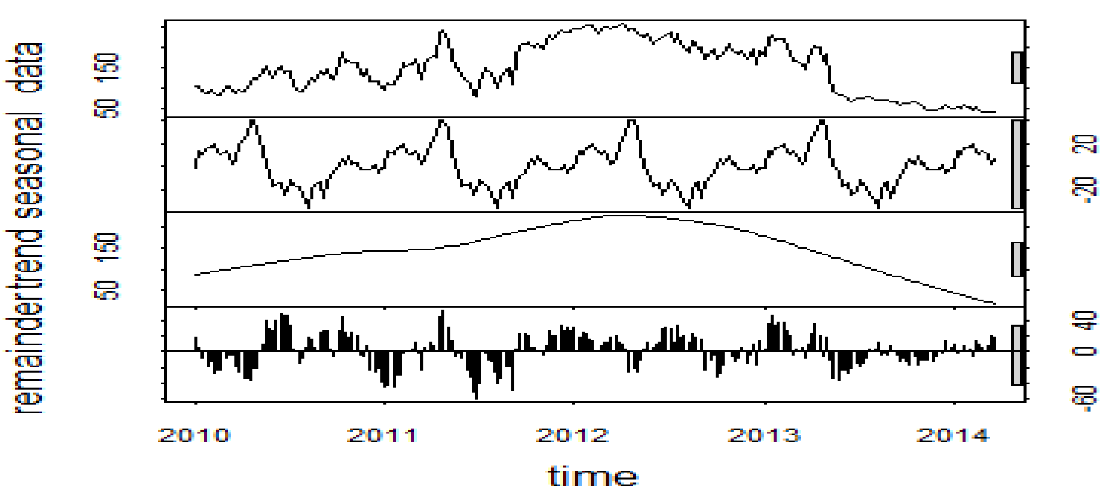
$$lnY_t = lnT_t + lnS_t + lnI_t$$

$$\Rightarrow Y'_t = T'_t + S'_t + I'_t$$

Observed Trend:

$$\boldsymbol{T'_t} = 10.1709 + 0.00431819 * t$$

Observed Seasonal variations:



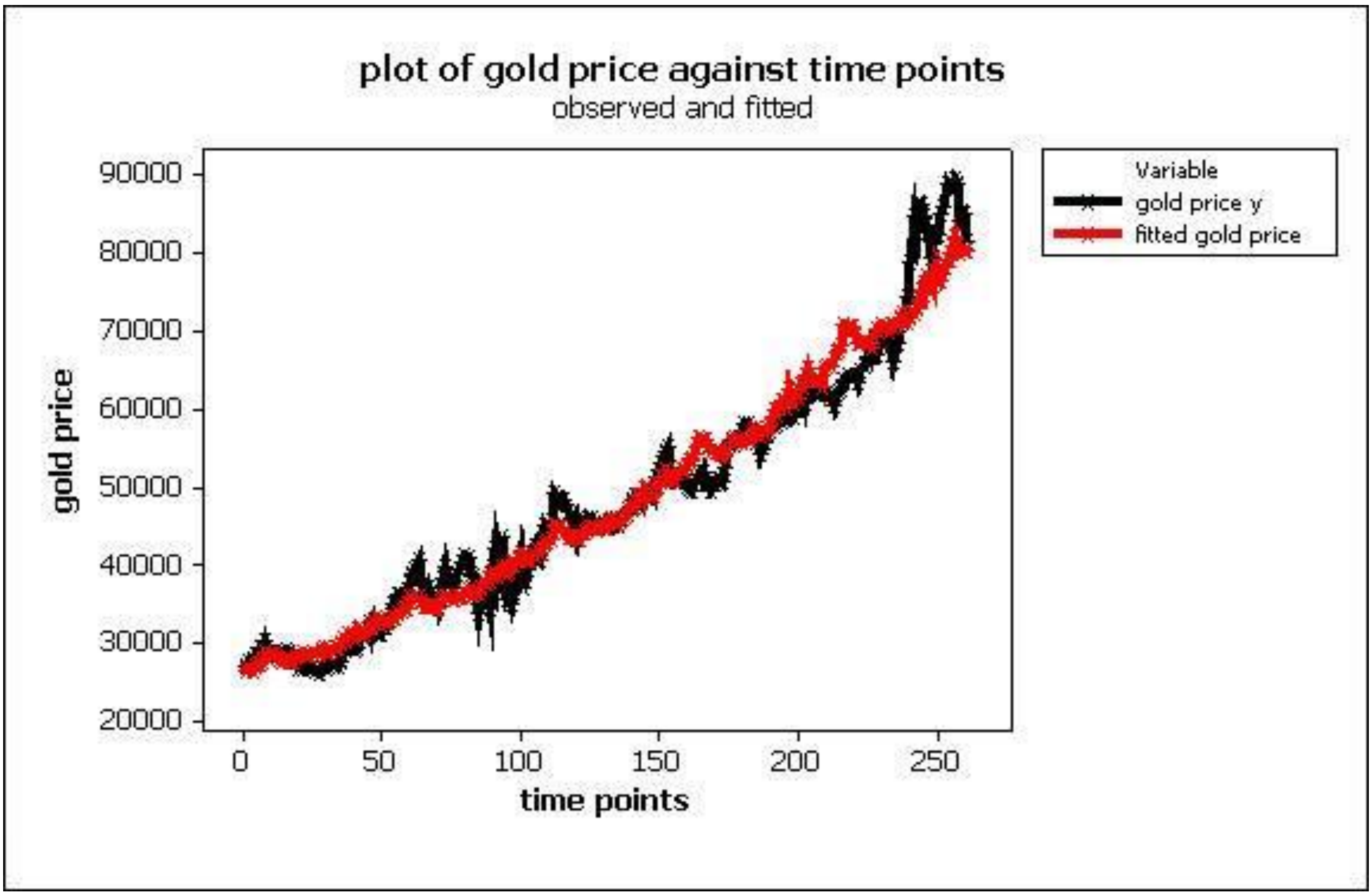
T	S'_t	t	S'_t	t	S'_t	t	S'_t
1	0.0192468	2	0.0105577	3	0.0073080	4	0.0082530
5	0.0142300	6	0.0184138	7	0.0168369	8	0.0417617
9	0.0502386	10	0.0560176	11	0.0403725	12	0.0173123
13	0.0091933	14	-0.0173563	15	-0.0140998	16	-0.0205546
17	-0.0149863	18	-0.0172565	19	0.0003084	20	0.0253324
21	0.0083920	22	0.0157096	23	0.0084017	24	-0.0152646
25	0.0069540	26	0.0108495	27	-0.0458760	28	-0.0174123
29	-0.0178689	30	-0.0171550	31	-0.0445757	32	-0.0514926
33	-0.0568896	34	-0.0526115	35	-0.0421521	36	-0.0193837
37	-0.0157075	38	0.0119115	39	0.0000176	40	-0.0087761
41	0.0139453	42	-0.0054026	43	-0.0061120	44	0.0047330
45	0.0381853	46	0.0108736	47	0.0153759	48	0.0125462
49	0.0068877	50	-0.0054262	51	-0.0002880	52	0.0064819

Also, as per assumption, $\boldsymbol{S'_{53}} = 0$…

Since, the irregular fluctuations (I'_t) are unaccountable, our fitted gold price is :

$$Y_t \text{ (fitted)} = \exp\,(T'_t + S'_t)$$

The plot of the observed along with the fitted gold price is given below:



Approach-II: Winter’s exponential smoothing technique:

Theory:

Winter’s seasonal exponential method of smoothing is applicable in situations when the seasonal pattern in the time series are constant year after year and the necessary computations for updating with new data are not a problem. The Winter’s seasonal exponential smoothing technique employs the smoothing process three times:

- To estimate the average level of the series.
- To estimate the slope component of the series.
- To estimate the seasonal component of the series.

Each of the above three steps employs its own smoothing constant, which can be adjusted as the situation demands. The exponential smoothing technique continually revises forecast by giving some data points more weights than the others. This is done with the smoothing constants.

Additive Winter’s method:

The Winter’s additive exponential smoothing technique forecasts a time series that has a linear trend and additive seasonal variation.

In our case, the data exhibits exponential trend and constant seasonal pattern year after year. Thus, we have:

$$T_t = a.b^t$$

$$\Rightarrow \ln(T_t) = \ln(a) + t.\ln(b)$$

$$\Rightarrow T_t' = a' + b'.t$$

The Winter's methodology uses the following steps:

- The model containing exponential trend is represented as:

$$lnX_t = lnT_t + lnS_t + lne_t$$

$$\Rightarrow X'_t = T'_t + S'_t + e'_t,$$

where, $T'_t = a' + b'.t$

- The basic concept of exponential smoothing is:
  Estimate = constant*(actual data) + (1-constant)*(old estimate)
- The final forecast value is then given by:
  Forecast = (level estimate) + (slope estimate) + (seasonal estimate)

Updating the results obtained via decomposition:

Winter's method is useful in updating the parameters found in the additive decomposition process with the following smoothing equations:

- $a_t = \alpha.(x_t - S_t) + (1 - \alpha).(a_{t-1} + b_{t-1})$
- $b_t = \beta.(a_t - a_{t-1}) + (1 - \beta).b_{t-1}$
- S_{t+L}(t) $= \gamma.(x_t - a_t) + (1 - \gamma).S_t$(t-L)

where,

x_t = original data at time point t.

a_t = smoothed estimate for the level at time point t.

b_t = smoothed estimate for the slope at time point t.

L = no. of seasons, (in our case, 52)

S_{t+L}(t) = smoothed seasonal estimate at time period (t+L), based on the seasonal estimate at time period t, i.e. a week earlier.

α = weighting factor for the level.

β = weighting factor for the slope.

γ = weighting factor for the seasonal component.

Clearly, $a_0, b_0, S_1, S_2, \ldots.., S_{52}$ are the actual level, slope and seasonal components as obtained from the decomposition method.

Now, after obtaining the smoothed estimates for the level, slope and seasonal effects, a one-step ahead forecast is obtained with the following equation:

$$\hat{x}_{t+1} = a_t + b_t + S_{t+1}(t+1-L)$$

Thus, the forecasted price of gold, at time point (t+1) is obtained as:

$$Y_t = e^{\hat{x}_{t+1}}$$

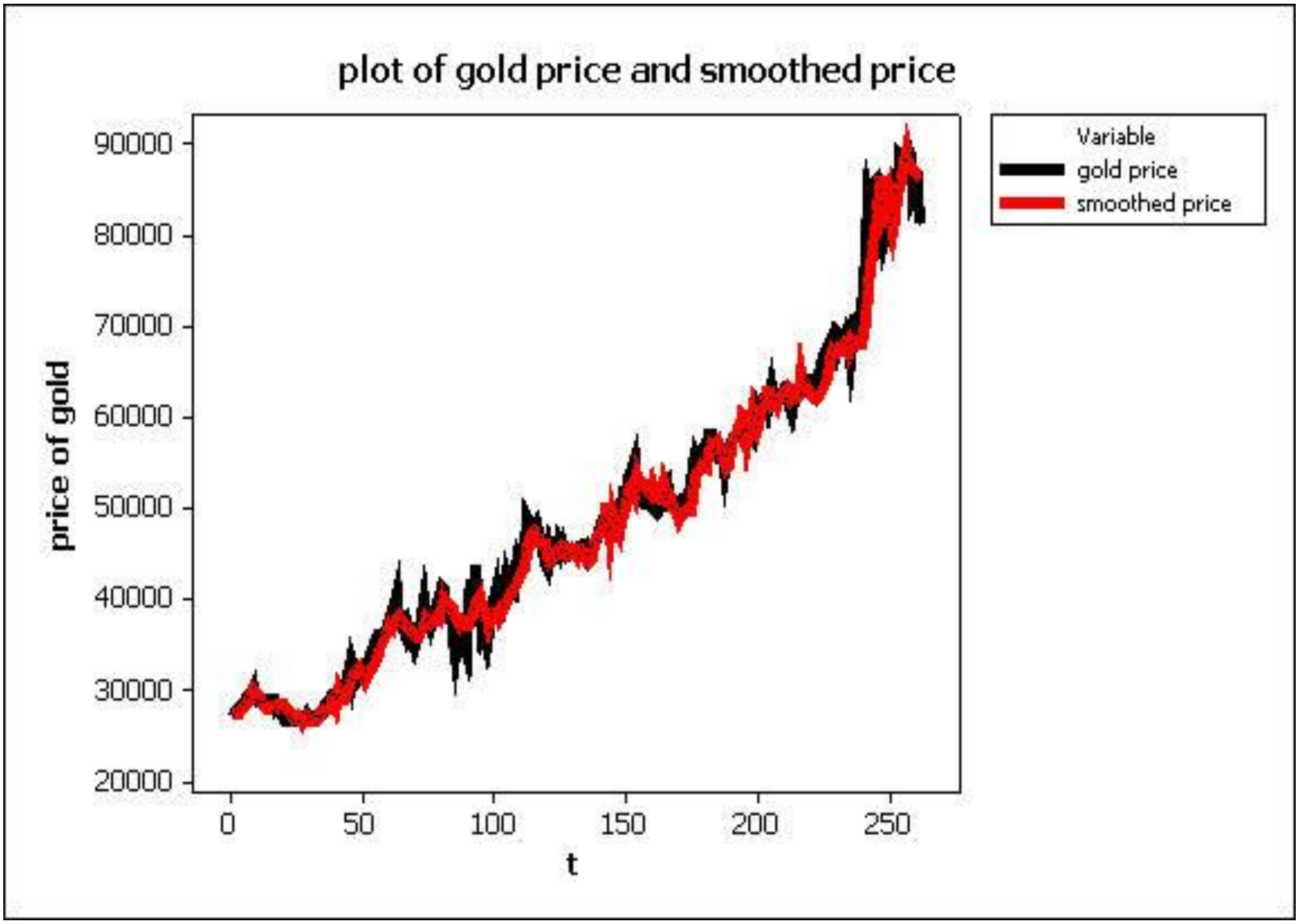
Observations, Calculations and Results:

The optimum values of α, β, γ are obtained by trial and error.

That set of values are chosen, which gives the least MSE (mean square error). The results obtained are given below:

We find $\alpha = 0.3,\ \beta = 0,\ \gamma = 0.2$ is the set of optimum smoothing constants, since it has the least MSE.

The plot of the original gold price along with the smoothed values, as obtained from the Winter's exponential smoothing technique is given below:



Approach-III:Regression model:

Ordinary least square regression:

Theory:

Define:

Y_t: Price of gold at the t[th] time point

X_{1t}: USD-INR exchange rate at the t[th] time point

X_{2t}: Value of gold reserve with the RBI at the t[th] time point

We consider a multiple linear regression equation of the form:

$$Y_t = \beta_0 + \beta_1 . X_{1t} + \beta_2 . X_{2t} + r_t$$

where β_0,β_1,β_2 are constants to be estimated and r_t is the error term.

We estimate β_0,β_1,β_2 by minimizing the error sum of squares, i.e. by minimizing

$$\sum_{t=1}^{n} {r_t}^2 = \sum_{t=1}^{n} (Y_t - \beta_0 - \beta_1 . X_{1t} - \beta_2 . X_{2t})^2$$

with respect to β_0,$\beta_1$andβ_2.

Observations, Calculations and Results:

We have,

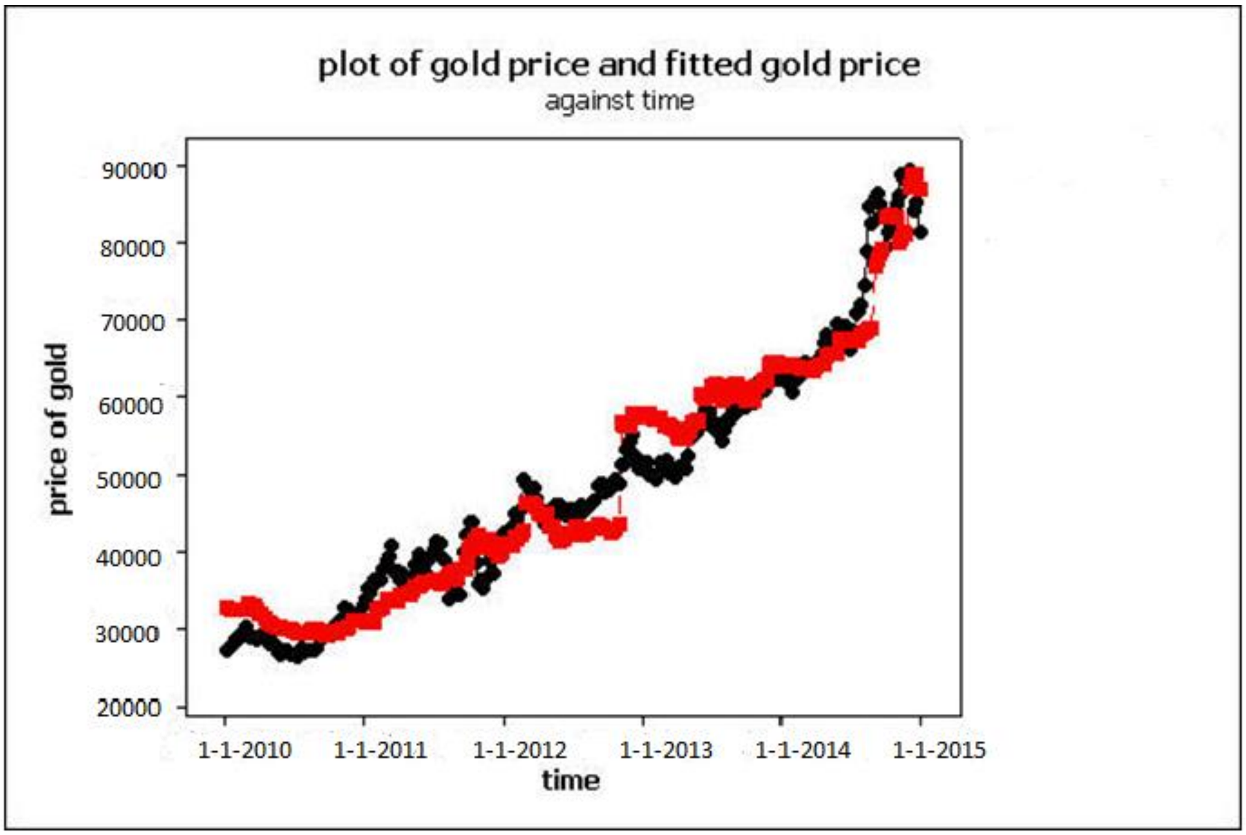
$$\widehat{\beta_0} = -9853$$

$$\widehat{\beta_1} = 678.03$$

$$\widehat{\beta_2} = 42.4556$$

The list of fitted gold prices is given in **Appendix-2** (column 6).

The plot of the observed as well as the predicted gold price is given below:



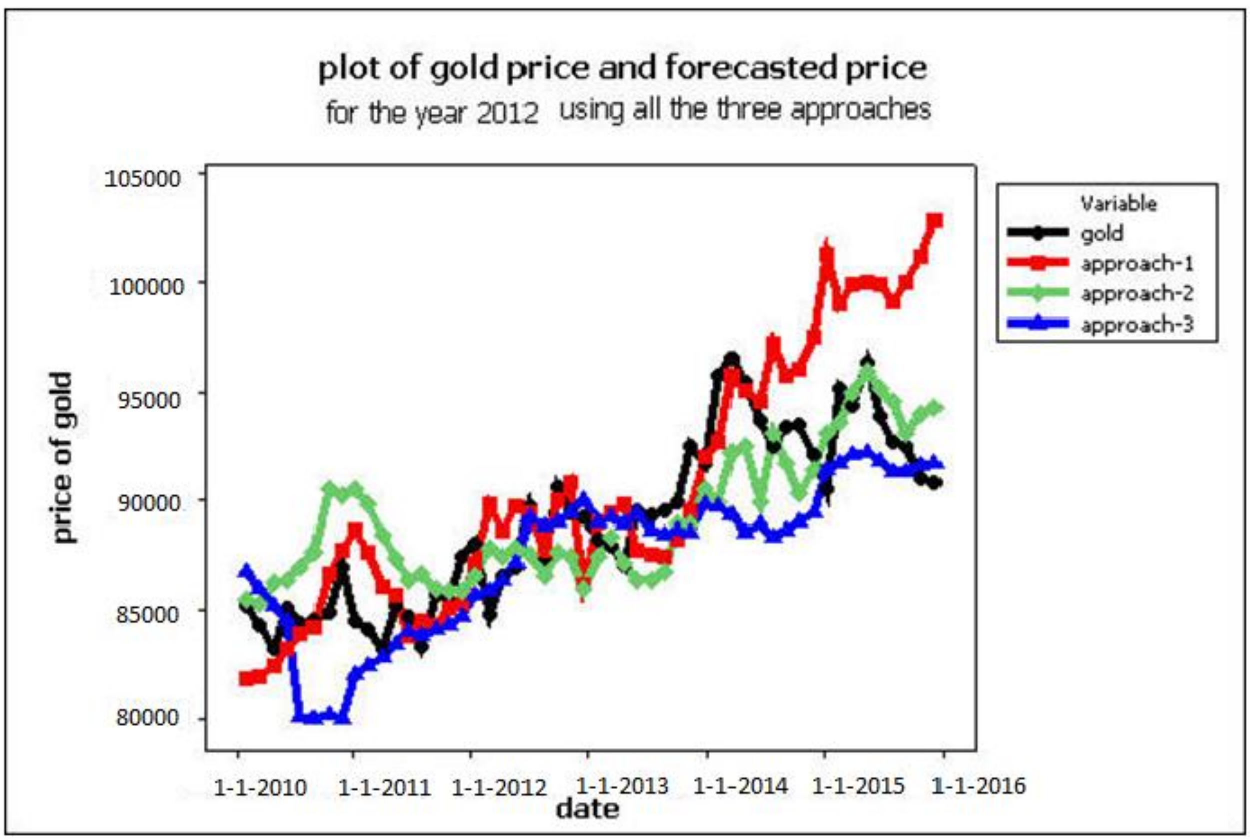
A comparative study:

Using the methods described above, we have tried to forecast the average price of gold per troy-ounce (31.1034768 gram), per week (cut-off points being end of the week i.e. Friday), in INR, for the year 2012.

Data in hand:

- Average price of gold, per troy-ounce (31.1034768 gram), per week (cut-off points being end of the week i.e. Friday), in INR, for the year 2015.
- Average USD-INR exchange rate, per week (cut-off points being end of the week i.e. Friday), for the year 2015.
- Average value of gold reserve (in billion rupees), with the RBI, per week (cut-off points being end of the week i.e. Friday), in INR, for the year 2015.

The entire data set required for forecasting is provided in **Appendix-1**.



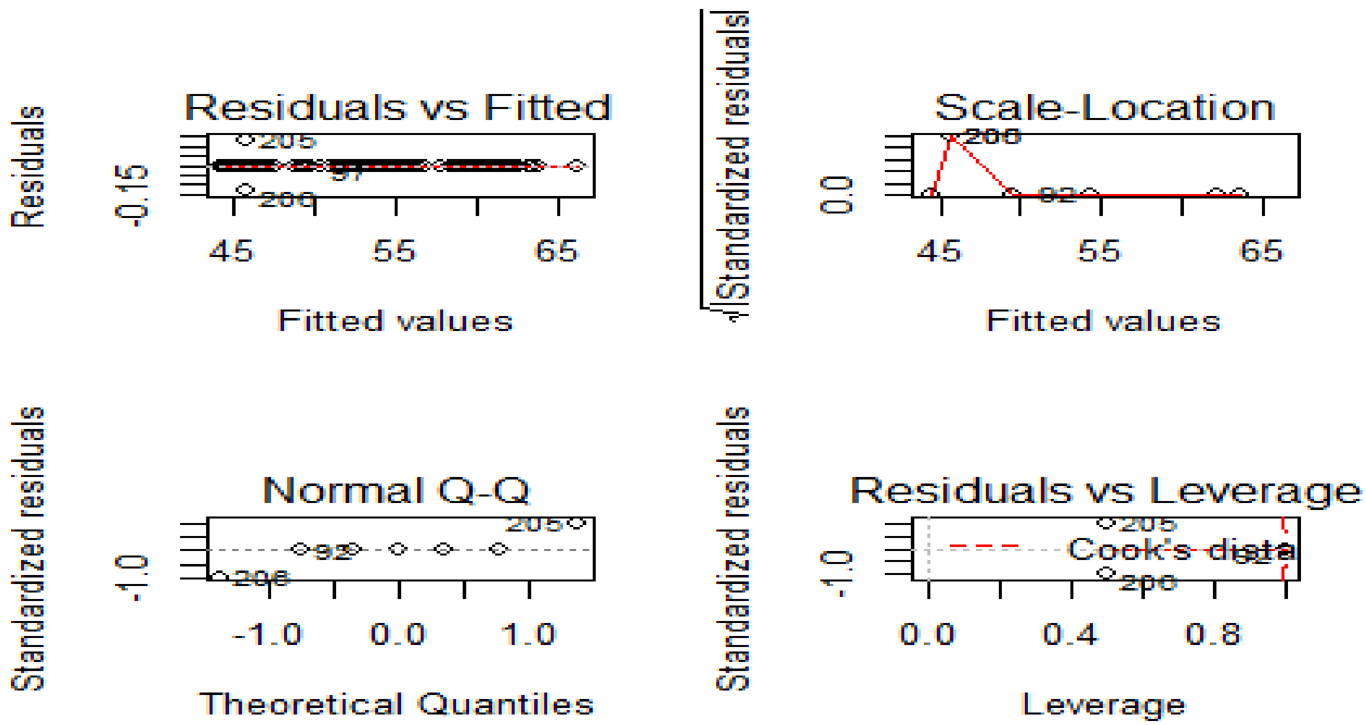
Clearly, the best approach is the one, which has the least variation from the observed values. The measure of variation used here, are:

RMSD (Root Mean Square Deviation)$=\sqrt{\frac{1}{52}\sum(gold\ price - fitted\ price)^2}$

MAD (Mean Absolute Deviation) $=\frac{1}{52}\sum|gold\ price - fitted\ price|$

MAPE (Mean Absolute Percentage Error) $= 100\%.\frac{1}{52}\sum\frac{|gold\ price - fitted\ price|}{|fitted\ price|}$

The following table shows the RMSDs, MADs and MAPEs corresponding to the three approaches:



Approach	RMSD	MAD	MAPE
Approach-1 (decomposition)	3890.28	2824.42	3.0037%
Approach-2 (Winter's smoothing)	2770.59	2283.30	2.5562%
Approach-3a (o.l.s. regression)	2960.48	2279.82	2.6266%

Conclusion:

Thus, we conclude that:

- Withrespect to RMSD and MAPE, **Approach-II**, i.e. Winter's exponential smoothing technique gives the best forecast of gold price for the year 2015, and hence may be used for further forecasting.
- But, with respect to MAD, **Approach-IIIa** and **Approach-IIIb**, i.e., the approach of regression models give better forecasted values for the year 2015.

Appendix-1

End Date	USD/INR	gold price (inr)	avg gold reseserves(in billion rupees)
12/28/2014	63.5243	75,657.20	1,176.60
12/21/2014	63.2757	75,813.00	1,176.60
12/14/2014	62.1062	73,866.80	1,176.60
12/7/2014	61.8524	73,395.50	1,212.10
11/30/2014	61.771	74,391.70	1,212.10
11/23/2014	61.6651	72,204.70	1,212.10
11/16/2014	61.4165	71,157.60	1,212.10
11/9/2014	61.2618	71,484.90	1,212.10
11/2/2014	61.1598	75,456.60	1,233.10
10/26/2014	61.0667	75,844.70	1,233.10
10/19/2014	61.2867	74,706.40	1,233.10
10/12/2014	61.292	73,641.90	1,233.10
10/5/2014	61.5475	74,239.00	1,265.90
9/28/2014	60.9839	74,237.00	1,265.90
9/21/2014	60.8893	74,773.60	1,265.90
9/14/2014	60.5993	76,477.50	1,265.90
9/7/2014	60.3913	78,019.30	1,275.60
8/31/2014	60.4076	77,305.50	1,275.60
8/24/2014	60.6035	78,816.20	1,275.60
8/17/2014	60.9697	80,254.90	1,275.60
8/10/2014	61.091	78,856.60	1,275.60
8/3/2014	60.2602	77,833.90	1,240.00
7/27/2014	59.9456	78,859.90	1,240.00
7/20/2014	60.0484	80,186.80	1,240.00
7/13/2014	59.7708	78,904.30	1,240.00
7/6/2014	59.7671	79,162.00	1,227.30
6/29/2014	60.0603	78,953.40	1,227.30
6/22/2014	59.9878	76,029.90	1,227.30
6/15/2014	59.1596	73,810.20	1,227.30
6/8/2014	59.0378	73,892.00	1,265.00
6/1/2014	58.6959	75,368.70	1,265.00
5/25/2014	58.3613	76,237.20	1,265.00
5/18/2014	59.1941	77,468.50	1,265.00
5/11/2014	59.8516	77,144.10	1,265.00
5/4/2014	60.2652	78,927.30	1,296.20
4/27/2014	60.6738	78,453.10	1,296.20
4/20/2014	60.2949	79,389.70	1,296.20
4/13/2014	60.1443	78,075.00	1,296.20
4/6/2014	59.8838	77,325.70	1,302.10
3/30/2014	60.2809	81,549.40	1,302.10
3/23/2014	61.102	84,713.50	1,302.10
3/16/2014	61.0494	81,577.10	1,302.10
3/9/2014	61.4308	82,216.50	1,302.10

3/2/2014	61.7845	82,140.70	1,254.30
2/23/2014	61.8346	81,958.80	1,254.30
2/16/2014	61.8797	78,526.80	1,254.30
2/9/2014	62.2494	78,381.40	1,254.30
2/2/2014	62.5924	79,174.80	1,254.30
1/26/2014	61.7514	76,843.70	1,220.90
1/19/2014	61.2131	77,075.10	1,220.90
1/12/2014	61.8434	76,795.10	1,220.90
1/5/2014	61.8383	75,207.90	1,285.50
12/29/2013	61.7156	74,272.80	1,285.50
12/22/2013	61.8413	76,587.30	1,285.50
12/15/2013	61.3249	75,992.90	1,285.50
12/8/2013	61.7101	78,300.00	1,303.60
12/1/2013	62.2	78,376.70	1,303.60
11/24/2013	62.4164	81,431.40	1,303.60
11/17/2013	63.1996	80,639.40	1,303.60
11/10/2013	62.2597	80,815.90	1,303.60
11/3/2013	61.318	83,014.60	1,366.40
10/27/2013	61.0625	80,629.00	1,366.40
10/20/2013	60.8807	77,366.30	1,366.40
10/13/2013	61.1324	80,798.50	1,366.40
10/6/2013	61.957	83,752.10	1,446.30
9/29/2013	62.2961	84,105.50	1,446.30
9/22/2013	62.4969	83,583.00	1,446.30
9/15/2013	63.7853	90,578.00	1,446.30
9/8/2013	66.295	92,151.10	1,446.30
9/1/2013	66.2109	88,173.80	1,267.90
8/25/2013	63.9435	84,496.40	1,267.90
8/18/2013	61.4754	79,672.30	1,267.90
8/11/2013	61.0785	79,746.40	1,267.90
8/4/2013	60.3919	78,502.40	1,267.90
7/28/2013	59.3225	77,272.00	1,286.80
7/21/2013	59.5591	76,689.00	1,286.80
7/14/2013	60.2108	73,244.00	1,286.80
7/7/2013	60.0215	70,834.60	1,290.00
6/30/2013	59.8532	76,646.40	1,290.00
6/23/2013	58.8081	80,073.40	1,290.00
6/16/2013	57.7807	79,105.90	1,290.00
6/9/2013	56.7529	78,768.30	1,290.00
6/2/2013	56.1101	77,304.80	1,299.90
5/26/2013	55.3427	75,110.20	1,299.90
5/19/2013	54.6527	78,008.10	1,299.90
5/12/2013	53.9477	79,280.70	1,299.90
5/5/2013	53.6235	80,042.20	1,299.90
4/28/2013	53.9379	76,002.40	1,397.40
4/21/2013	53.9721	83,792.20	1,397.40

4/14/2013	54.403	85,981.30	1,397.40
4/7/2013	54.4981	86,864.90	1,413.80
3/31/2013	54.2911	87,357.10	1,413.80
3/24/2013	54.2015	86,228.80	1,413.80
3/17/2013	54.1553	85,912.70	1,413.80
3/10/2013	54.6927	86,873.40	1,437.50
3/3/2013	54.2806	85,446.30	1,437.50
2/24/2013	54.2603	87,335.60	1,437.50
2/17/2013	53.8499	89,326.40	1,437.50
2/10/2013	53.232	88,774.10	1,437.50
2/3/2013	53.3997	89,133.70	1,491.00
1/27/2013	53.5884	90,866.60	1,491.00
1/20/2013	54.2962	90,814.40	1,491.00
1/13/2013	54.7967	90,829.50	1,491.00
1/6/2013	54.6209	90,847.60	1,516.00
12/23/2012	54.7357	92,530.40	1,516.00
12/16/2012	54.2587	92,689.20	1,516.00
12/9/2012	54.3415	93,885.80	1,516.00
12/2/2012	55.0641	96,412.20	1,525.50
11/25/2012	55.0601	94,396.70	1,525.50
11/18/2012	54.823	95,177.90	1,525.50
11/11/2012	54.2546	90,577.20	1,525.50
11/4/2012	53.7774	92,106.30	1,482.50
10/28/2012	53.6269	93,511.40	1,482.50
10/21/2012	53.0265	93,434.60	1,482.50
10/14/2012	52.3692	92,562.80	1,482.50
10/7/2012	51.9961	93,675.10	1,482.50
9/30/2012	52.8487	95,497.50	1,462.10
9/23/2012	53.6	96,613.80	1,462.10
9/16/2012	54.7987	95,765.80	1,462.10
9/9/2012	55.3563	91,697.80	1,462.10
9/2/2012	55.4489	92,476.80	1,462.10
8/26/2012	55.3088	89,901.20	1,435.10
8/19/2012	55.4692	89,543.50	1,435.10
8/12/2012	55.0772	89,319.50	1,435.10
8/5/2012	55.3909	89,553.90	1,450.60
7/29/2012	55.5665	87,032.60	1,450.60
7/22/2012	54.8922	88,015.70	1,450.60
7/15/2012	55.3222	88,229.20	1,450.60
7/8/2012	55.0331	89,252.20	1,450.60
7/1/2012	56.4733	89,499.60	1,443.50
6/24/2012	56.0784	90,597.10	1,443.50
6/17/2012	55.426	87,503.60	1,443.50
6/10/2012	55.2694	89,759.30	1,443.50
6/3/2012	55.7777	86,958.10	1,398.00
5/27/2012	55.5356	86,588.00	1,398.00

5/20/2012	54.3689	84,777.60	1,398.00
5/13/2012	53.6705	88,047.40	1,398.00
5/6/2012	53.3225	87,433.60	1,382.50
4/29/2012	52.9627	85,513.90	1,382.50
4/22/2012	52.2978	85,708.10	1,382.50
4/15/2012	51.9698	83,303.30	1,382.50
4/8/2012	51.5895	84,696.00	1,382.50
4/1/2012	51.8276	85,155.20	1,376.60
3/25/2012	51.4097	83,206.70	1,376.60
3/18/2012	50.4955	84,113.40	1,376.60
3/11/2012	49.9095	84,505.00	1,376.60
3/4/2012	49.2776	86,982.00	1,327.80
2/26/2012	49.3183	84,900.80	1,327.80
2/19/2012	49.5844	84,556.70	1,327.80
2/12/2012	49.3958	84,428.40	1,327.80
2/5/2012	49.5438	85,109.10	1,418.10
1/29/2012	50.3484	83,187.20	1,418.10
1/22/2012	51.3519	84,285.50	1,418.10
1/15/2012	52.5851	85,221.90	1,418.10
1/8/2012	53.6889	81,303.70	1,418
1/1/2012	54.0741	85,086.30	1,418
12/25/2011	53.8011	84,019.70	1,418
12/18/2011	53.7742	88,927.80	1,463
12/11/2011	51.61	89,448.60	1,463
12/4/2011	51.7814	88,198.80	1,463
11/27/2011	52.2188	88,253.50	1,314
11/20/2011	50.919	88,871.60	1,314
11/13/2011	49.9113	85,884.60	1,314
11/6/2011	49.238	84,882.40	1,314
10/30/2011	49.5906	82,166.10	1402.6
10/23/2011	49.7209	82,247.20	1402.6
10/16/2011	49.3298	81,204.10	1402.6
10/9/2011	49.5766	79,339.50	1402.6
10/2/2011	49.5735	83,495.70	1303.23
9/25/2011	48.905	84,793.40	1303.23
9/18/2011	47.5335	86,187.20	1303.23
9/11/2011	46.079	85,858.30	1303.23
9/4/2011	45.8099	82,534.10	1119.4
8/28/2011	45.8564	84,536.80	1119.4
8/21/2011	45.4412	78,718.90	1119.4
8/14/2011	45.1301	74,204.20	1119.4
8/7/2011	44.4445	71,963.40	1119.4
7/31/2011	44.2406	71,056.70	1103.17
7/24/2011	44.596	70,645.30	1103.17
7/17/2011	44.6599	68,334.70	1103.17
7/10/2011	44.6217	66,115.80	1103.17

7/3/2011	45.1197	68,156.20	1098.32
6/26/2011	45.3115	68,979.90	1098.32
6/19/2011	45.2643	68,395.70	1098.32
6/12/2011	45.1187	68,999.70	1098.32
6/5/2011	44.8471	69,237.90	1055.82
5/29/2011	45.1265	67,068.80	1055.82
5/22/2011	45.0212	67,555.50	1055.82
5/15/2011	44.826	66,580.30	1055.82
5/8/2011	44.653	67,892.10	1055.82
5/1/2011	44.5723	66,747.50	1025.72
4/24/2011	44.6208	65,464.30	1025.72
4/17/2011	44.5876	64,760.90	1025.72
4/10/2011	44.5398	63,235.70	1025.72
4/3/2011	45.0521	64,146.10	1000.41
3/27/2011	45.3587	64,070.40	1000.41
3/20/2011	45.7133	63,863.30	1000.41
3/13/2011	45.4339	64,193.60	1000.41
3/6/2011	45.0568	63,568.30	1007.39
2/27/2011	45.2451	62,541.10	1007.39
2/20/2011	45.4297	62,314.30	1007.39
2/13/2011	45.6098	61,781.20	1007.39
2/6/2011	45.8765	60,364.00	1006.86
1/30/2011	45.9235	61,290.50	1006.86
1/23/2011	45.8355	62,041.30	1006.86
1/16/2011	45.5697	62,041.30	1006.86
1/9/2011	45.42	62,846.90	1006.86
1/2/2011	45.2351	61,992.90	1018.57
12/31/2010	45.35	62,038.00	1018.57
12/26/2010	45.59	62,054.60	1018.57
12/19/2010	45.6991	61,961.90	1018.57
12/12/2010	45.2653	63,311.90	1018.57
12/5/2010	45.405	62,120.00	965.1
11/28/2010	45.5798	60,808.50	965.1
11/21/2010	45.328	62,125.00	965.1
11/14/2010	44.5237	61,702.00	965.1
11/7/2010	44.3004	59,829.40	965.1
10/31/2010	44.4551	58,970.30	921.57
10/24/2010	44.34	60,327.20	921.57
10/17/2010	44.2592	59,596.10	921.57
10/10/2010	44.3303	58,540.20	921.57
10/3/2010	44.8066	58,695.70	941.99
9/26/2010	45.549	58,457.50	941.99
9/19/2010	46.2915	57,918.60	941.99
9/12/2010	46.4312	57,801.10	941.99
9/5/2010	46.6328	57,933.80	895.64
8/29/2010	46.7387	57,106.80	895.64

8/22/2010	46.5911	56,784.40	895.64
8/15/2010	46.3809	55,743.70	895.64
8/8/2010	46.1402	54,247.40	895.64
8/1/2010	46.7412	55,876.10	927.04
7/25/2010	47.1188	55,633.10	927.04
7/18/2010	46.7605	56,406.30	927.04
7/11/2010	46.7873	56,188.10	927.04
7/4/2010	46.4655	58,028.80	902.2
6/27/2010	46.0922	57,958.10	902.2
6/20/2010	46.4123	57,132.60	902.2
6/13/2010	46.9616	56,390.00	902.2
6/6/2010	46.7538	55,961.60	823.77
5/30/2010	46.882	55,383.40	823.77
5/23/2010	46.1863	55,908.30	823.77
5/16/2010	45.1535	54,678.30	823.77
5/9/2010	44.8791	52,311.50	823.77
5/2/2010	44.3836	50,628.00	811.89
4/25/2010	44.4419	51,034.50	811.89
4/18/2010	44.3165	51,038.40	811.89
4/11/2010	44.4556	50,439.50	811.89
4/4/2010	44.9031	49,600.20	828.45
3/28/2010	45.3659	50,305.80	828.45
3/21/2010	45.4487	50,262.50	828.45
3/14/2010	45.4706	51,761.70	828.45
3/7/2010	45.8126	51,095.90	828.45
2/28/2010	46.1756	51,525.90	837.24
2/21/2010	46.1658	50,231.80	837.24
2/14/2010	46.5235	49,435.00	837.24
2/7/2010	46.2871	49,794.30	837.24
1/31/2010	46.2862	49,972.40	853.87
1/24/2010	45.961	51,645.50	853.87
1/8/2010	45.5629	51,554.40	853.87
1/1/2010	45.9555	50,606.80	84508

R Console Printout

```
> goldprice<-read.csv(file="goldanalysis.csv",head=TRUE,sep=",")
> head(goldprice)
     End.Date USD.INR gold.price..inr. avg.gold.reseserves.in.billion.rupees.
1  12/28/2014 63.5243        75,657.20                               1,176.60
2  12/21/2014 63.2757        75,813.00                               1,176.60
3  12/14/2014 62.1062        73,866.80                               1,176.60
4   12/7/2014 61.8524        73,395.50                               1,212.10
5  11/30/2014 61.7710        74,391.70                               1,212.10
6  11/23/2014 61.6651        72,204.70                               1,212.10
7  11/16/2014 61.4165        71,157.60                               1,212.10
8   11/9/2014 61.2618        71,484.90                               1,212.10
9   11/2/2014 61.1598        75,456.60                               1,233.10
10 10/26/2014 61.0667        75,844.70                               1,233.10
11 10/19/2014 61.2867        74,706.40                               1,233.10
12 10/12/2014 61.2920        73,641.90                               1,233.10
13  10/5/2014 61.5475        74,239.00                               1,265.90
14  9/28/2014 60.9839        74,237.00                               1,265.90
15  9/21/2014 60.8893        74,773.60                               1,265.90
16  9/14/2014 60.5993        76,477.50                               1,265.90
17   9/7/2014 60.3913        78,019.30                               1,275.60
18  8/31/2014 60.4076        77,305.50                               1,275.60
19  8/24/2014 60.6035        78,816.20                               1,275.60
20  8/17/2014 60.9697        80,254.90                               1,275.60
```

```
> summary(goldprice)
      End.Date       USD.INR       gold.price..inr. avg.gold.reseserves.in.billion.rupees.
 1/1/2010 :  1   Min.   :44.24   62,041.30:  2     1,212.10:  5
 1/1/2012 :  1   1st Qu.:46.05   49,435.00:  1     1,254.30:  5
 1/12/2014:  1   Median :53.68   49,600.20:  1     1,265.00:  5
 1/13/2013:  1   Mean   :53.03   49,794.30:  1     1,267.90:  5
 1/15/2012:  1   3rd Qu.:60.00   49,972.40:  1     1,275.60:  5
 1/16/2011:  1   Max.   :66.30   50,231.80:  1     1,290.00:  5
 (Other)  :254                   (Other)  :253     (Other) :230
> names(goldprice)
[1] "End.Date"                               "USD.INR"
[3] "gold.price..inr."                       "avg.gold.reseserves.in.billion.rupees."
> goldtimeseries <- ts(goldprice, frequency=52, start=c(2010,1))
> head(goldtimeseries)
Time Series:
Start = c(2010, 1)
End = c(2014, 52)
Frequency = 52
         End.Date USD.INR gold.price..inr. avg.gold.reseserves.in.billion.rupees.
2010.000       80 63.5243              105                                      1
2010.019       75 63.2757              106                                      1
2010.038       69 62.1062               92                                      1
2010.058       85 61.8524               89                                      2
2010.077       61 61.7710               98                                      2
2010.096       55 61.6651               87                                      2
2010.115       49 61.4165               84                                      2
2010.135       65 61.2618               85                                      2
2010.154       52 61.1598              104                                      5
2010.173       35 61.0667              107                                      5
2010.192       29 61.2867               99                                      5
2010.212       24 61.2920               90                                      5
2010.231       41 61.5475               96                                      9
2010.250      253 60.9839               95                                      9
2010.269      248 60.8893              100                                      9
2010.288      242 60.5993              112                                      9
2010.308      258 60.3913              128                                     11
2010.327      234 60.4076              122                                     11
2010.346      228 60.6035              138                                     11
2010.365      223 60.9697              155                                     11
2010.385      218 61.0910              139                                     11
2010.404      233 60.2602              126                                      6
2010.423      208 59.9456              140                                      6
2010.442      203 60.0484              154                                      6

> plot.ts(goldtimeseries)
> loggoldprice<-log(goldprice)
> myts <- ts(goldprice$gold.price..inr., start=c(2010, 1), end=c(2014, 12), frequency=52)
> plot(myts)
> fit <- stl(myts, s.window="period")
> plot(fit)
> fit <- HoltWinters(myts, beta=FALSE, gamma=FALSE)
> fit
Holt-Winters exponential smoothing without trend and without seasonal component.

Call:
HoltWinters(x = myts, beta = FALSE, gamma = FALSE)

Smoothing parameters:
 alpha: 0.9659133
 beta : FALSE
 gamma: FALSE

Coefficients:
      [,1]
a 42.06477
```

```
>install.packages("ggplot2")
library(ggplot2)
ggplot(aes(w = time, t = variable), data = data) + geom_point()
plot(data, xaxt="n")
axis.POSIXct(side=1, at=cut(data$time, "days"), format="%m/%d")
ggplot(aes(x = time, y = variable), data = data) + geom_line()
p<-ggplot(data(dat), aes(w=months, t=value, colour=value)) +geom_line()
print(p)
p<-ggplot(data(dat), aes(w=months, t=value, colour=value)) +geom_line()
print(p)
ggplot(data, mapping = aes(), ...,
       environment = globalenv())


> accuracy(fit)
> install.packages("forecast")
> library(forecast)
Loading required package: timeDate
> install.packages("timeDate")
Installing package into ‘C:/Users/moy/Documents/R/win-library/3.1’
(as ‘lib’ is unspecified
> library(forecast)
Loading required package: timeDate
> goldpricecomponents
> plot(goldpricecomponents)
> goldprice.mean<-HoltWinters(goldprice$gold.price..inr.,alpha=0.3,beta=0,gamma=0.2)
decompose(ts(x[1L:wind], start = start(x), frequency = f), seasonal) :
> goldprice.pred<-predict(goldprice.mean,n.ahead=10,prediction.interval=TRUE)
> plot.ts(goldprice$gold.price..inr.,xlim=c(200,370))
> lines(goldprice.mean$fitted[,1],col="black")
> lines(goldprice.pred[,1],col="blue")
> lines(goldprice.pred[,2],col="red")
> lines(goldprices.pred[,3],col="green")
> names(goldprice)
[1] "End.Date"                              "USD.INR"
[3] "gold.price..inr."                      "avg.gold.reseserves.in.billion.rupees."
> g<-goldprice$USD.INR
> i<-goldprice$gold.price..inr.
> j<-goldprice$avg.gold.reseserves.in.billion.rupees.
> lm(g~i+j)

> head(predict(Model, newdata= goldtimeseries, interval='confidence'))
       fit      lwr      upr
1 28.19952 26.14755 30.25150
2 24.45839 23.01617 25.90062
3 18.04503 16.86172 19.22834

Call:

lm(formula = g ~ i + j)

Coefficients:
(Intercept)   i49,600.20   i49,794.30   i49,972.40   i50,231.80   i50,262.50   i50,305.80   i50,
    46.5235      -1.6204      -0.2364      -0.2373      -0.3577      -1.0748      -1.1576      -
 i50,606.80   i50,628.00   i51,034.50   i51,038.40   i51,095.90   i51,525.90   i51,554.40   i51,
    -0.5680      -2.1399      -2.0816      -2.2070      -0.7109      -0.3479      -0.9606      -
 i51,761.70   i52,311.50   i54,247.40   i54,678.30   i55,383.40   i55,633.10   i55,743.70   i55,
    -1.0529      -1.6444      -0.3833      -1.3700       0.3585       0.5953      -0.1426
 i55,908.30   i55,961.60   i56,188.10   i56,390.00   i56,406.30   i56,784.40   i57,106.80   i57,
    -0.3372       0.2303       0.2638       0.4381       0.2370       0.0676       0.2152      -
 i57,801.10   i57,918.60   i57,933.80   i57,958.10   i58,028.80   i58,457.50   i58,540.20   i58,
    -0.0923      -0.2320       0.1093      -0.4313      -0.0580      -0.9745      -2.1932      -
 i58,970.30   i59,596.10   i59,829.40   i60,327.20   i60,364.00   i60,808.50   i61,290.50   i61,
    -2.0684      -2.2643      -2.2231      -2.1835      -0.6470      -0.9437      -0.6000      -
 i61,781.20   i61,961.90   i61,992.90   i62,038.00   i62,041.30   i62,054.60   i62,120.00   i62,
    -0.9137      -0.8244      -1.2884      -1.1735      -0.8209      -0.9335      -1.1185      -
 i62,314.30   i62,541.10   i62,846.90   i63,235.70   i63,311.90   i63,568.30   i63,863.30   i64,
    -1.0938      -1.2784      -1.1035      -1.9837      -1.2582      -1.4667      -0.8102      -
 i64,146.10   i64,193.60   i64,760.90   i65,464.30   i66,115.80   i66,580.30   i66,747.50   i67,
    -1.4714      -1.0896      -1.9359      -1.9027      -1.9018      -1.6975      -1.9512      -
 i67,555.50   i67,892.10   i68,156.20   i68,334.70   i68,395.70   i68,979.90   i68,999.70   i69,
```

```
     -1.5023      -1.8705      -1.4038      -1.8636      -1.2592      -1.2120      -1.4048      -
  i70,645.30   i70,834.60   i71,056.70   i71,157.60   i71,484.90   i71,963.40   i72,204.70   i73,
     -1.9275      13.4980      -2.2829      14.8930      14.7383      -2.0790      15.1416      1
  i73,395.50   i73,641.90   i73,810.20   i73,866.80   i73,892.00   i74,204.20   i74,237.00   i74,
     15.3289      14.7685      12.6361      15.5827      12.5143      -1.3934      14.4604      1
  i74,272.80   i74,391.70   i74,706.40   i74,773.60   i75,110.20   i75,207.90   i75,368.70   i75,
     15.1921      15.2475      14.7632      14.3658       8.8192      15.3148      12.1724      1
  i75,657.20   i75,813.00   i75,844.70   i75,992.90   i76,002.40   i76,029.90   i76,237.20   i76,
     17.0008      16.7522      14.5432      14.8014       7.4144      13.4643      11.8378      1
  i76,587.30   i76,646.40   i76,689.00   i76,795.10   i76,843.70   i77,075.10   i77,144.10   i77,
     15.3178      13.3297      13.0356      15.3199      15.2279      14.6896      13.3281
          NA

> lmr=lm(g~i+j)
> summary(lmr)

Call:
lm(formula = g ~ i + j)

Residuals:
          1          2          3          4          5          6          7          8
  1.476e-18  1.052e-19  3.176e-20 -4.379e-20  9.298e-20 -4.521e-20 -4.658e-20 -4.415e-20  2.816e-
         10         11         12         13         14         15         16         17
  1.224e-20  1.032e-19 -4.189e-20  1.726e-20  2.410e-20  7.793e-21  8.635e-20  3.316e-20  6.065e-
         19         20         21         22         23         24         25         26
  2.659e-19 -5.063e-20  6.582e-20  3.017e-20  8.178e-20 -3.989e-20  9.810e-20  1.233e-19  3.324e-
         28         29         30         31         32         33         34         35
 -6.936e-20 -5.967e-20 -4.759e-20 -2.423e-20  1.020e-19  1.489e-19  6.630e-20 -6.315e-20  1.194e-
         37         38         39         40         41         42         43         44
  2.039e-19  1.391e-19 -1.658e-20  8.673e-20 -6.961e-20  1.037e-19  1.365e-19 -8.657e-20  2.707e-
         46         47         48         49         50         51         52         53
 -3.922e-20  2.718e-19  4.728e-20 -5.238e-20  5.772e-20  2.099e-19  2.759e-19  5.082e-19  6.485e-
         55         56         57         58         59         60         61         62
  2.182e-20  1.256e-20  6.136e-20  6.162e-20  2.338e-20  7.800e-21  9.702e-20  8.950e-20 -1.857e-
         64         65         66         67         68         69         70         71
  8.506e-21  8.583e-21  1.171e-19  3.635e-20 -2.617e-19  1.669e-17 -1.410e-18  7.497e-20  4.804e-
         73         74         75         76         77         78         79         80
 -1.146e-19 -3.388e-20  7.433e-20 -6.808e-21  2.018e-20 -6.937e-21  7.431e-20  1.017e-19  7.496e-
         82         83         84         85         86         87         88         89
  6.492e-20  7.572e-20 -1.006e-20  2.878e-19 -4.102e-21  5.251e-20 -5.163e-20  2.031e-19  3.514e-
         91         92         93         94         95         96         97         98
 -6.269e-19 -2.054e-18  6.996e-17  1.764e-19 -7.828e-19  1.390e-19 -1.663e-16 -3.851e-18 -1.721e-
        100        101        102        103        104        105        106        107         1
 -9.762e-19 -2.673e-18 -5.909e-18  8.632e-17 -4.459e-18 -1.334e-17 -5.254e-18  7.379e-18 -7.156e-
        109        110        111        112        113        114        115        116         1
 -2.816e-17  4.283e-18 -6.488e-18 -8.326e-18  5.367e-17 -8.556e-18 -4.592e-18  6.930e-18  8.876e-
        118        119        120        121        122        123        124        125         1
 -1.531e-18  1.140e-17  2.938e-18 -1.847e-18 -6.184e-18  3.083e-17  3.605e-17 -1.253e-17 -7.333e-
        127        128        129        130        131        132        133        134         1
  3.697e-20  2.365e-19  1.116e-18  2.170e-17 -8.687e-18  3.754e-18 -4.190e-18 -9.189e-19 -3.666e-
        136        137        138        139        140        141        142        143         1
  7.206e-20  1.324e-18 -1.444e-18  9.315e-21  2.930e-20  1.913e-20  1.576e-19 -4.456e-19 -4.870e-
        145        146        147        148        149        150        151        152         1
 -8.049e-20  3.914e-20  3.735e-18 -1.896e-19 -2.107e-19 -6.253e-20  2.622e-19  8.952e-20  2.188e-
        154        155        156        157        158        159        160        161         1
  6.049e-20 -6.271e-20 -2.606e-21 -6.460e-21 -1.121e-19  3.328e-19 -1.607e-18  1.450e-18  3.034e-
        163        164        165        166        167        168        169        170         1
  4.995e-20 -1.924e-20 -2.079e-19  4.205e-19  2.456e-22  9.499e-21 -3.186e-19  1.419e-19  8.257e-
        172        173        174        175        176        177        178        179         1
  5.057e-20  9.393e-21 -5.263e-20 -9.721e-21  7.393e-23  9.904e-21 -4.804e-22  5.019e-21 -7.793e-
        181        182        183        184        185        186        187        188         1
  9.631e-21 -6.816e-21 -3.081e-21 -1.289e-20  6.226e-21 -6.200e-21 -5.953e-21  2.484e-21 -5.295e-
        190        191        192        193        194        195        196        197         1
  7.356e-21  8.250e-21  4.426e-21 -1.616e-20  3.542e-21 -5.726e-21 -1.322e-21 -1.539e-20 -1.176e-
        199        200        201        202        203        204        205        206         2
 -9.613e-21 -1.066e-20 -3.192e-21 -3.177e-20 -2.851e-20 -4.609e-20  1.329e-01 -1.329e-01 -3.699e-
        208        209        210        211        212        213        214        215         2
 -2.188e-21 -1.270e-21 -9.976e-22 -2.727e-21 -4.489e-21 -1.599e-21 -1.575e-21 -2.257e-21 -1.664e-
        217        218        219        220        221        222        223        224         2
  1.945e-22  9.902e-23 -9.898e-22  5.152e-23  1.320e-21  1.176e-21  2.077e-21  2.286e-21  2.530e-
        226        227        228        229        230        231        232        233         2
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 1.263e-21  2.295e-21  3.074e-21  5.419e-21  7.104e-21  4.557e-21  5.538e-21  3.995e-21  6.042e-
       235        236        237        238        239        240        241        242         2
 1.902e-21  1.332e-21  2.274e-21  4.989e-21  4.658e-21  5.970e-21  4.533e-21  6.181e-21  6.083e-
       244        245        246        247        248        249        250        251         2
 9.094e-21  9.138e-21  8.280e-21  1.003e-20  1.394e-20  1.090e-20  1.179e-20  6.316e-21  7.788e-
       253        254        255        256        257        258        259        260
 7.338e-21  1.267e-20 -7.531e-17  1.355e-20  1.268e-20  6.716e-21  6.786e-21  9.613e-21

Coefficients: (61 not defined because of singularities)
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  46.5235     0.1880 247.533  0.00257 **
i49,600.20   -1.6204     0.2658  -6.096  0.10351
i49,794.30   -0.2364     0.2658  -0.889  0.53723
i49,972.40   -0.2373     0.2658  -0.893  0.53603
i50,231.80   -0.3577     0.2658  -1.346  0.40684
i50,262.50   -1.0748     0.2658  -4.044  0.15434
i50,305.80   -1.1576     0.2658  -4.355  0.14369
i50,439.50   -2.0679     0.2658  -7.780  0.08138 .
i50,606.80   -0.5680     0.2658  -2.137  0.27864
i50,628.00   -2.1399     0.2658  -8.051  0.07867 .
i51,034.50   -2.0816     0.2658  -7.831  0.08085 .
i51,038.40   -2.2070     0.2658  -8.303  0.07630 .
i51,095.90   -0.7109     0.2658  -2.675  0.22778
i51,525.90   -0.3479     0.2658  -1.309  0.41534
i51,554.40   -0.9606     0.2658  -3.614  0.17185
i51,645.50   -0.5625     0.2658  -2.116  0.28103
i51,761.70   -1.0529     0.2658  -3.961  0.15742
i52,311.50   -1.6444     0.2658  -6.187  0.10202
i54,247.40   -0.3833     0.2658  -1.442  0.38599
i54,678.30   -1.3700     0.2658  -5.154  0.12200
i55,383.40    0.3585     0.2658   1.349  0.40616
i55,633.10    0.5953     0.2658   2.240  0.26734
i55,743.70   -0.1426     0.2658  -0.536  0.68652
i55,876.10    0.2177     0.2658   0.819  0.56313
i55,908.30   -0.3372     0.2658  -1.269  0.42497
i55,961.60    0.2303     0.2658   0.866  0.54548
i56,188.10    0.2638     0.2658   0.992  0.50240
i56,390.00    0.4381     0.2658   1.648  0.34717
i56,406.30    0.2370     0.2658   0.892  0.53643
i56,784.40    0.0676     0.2658   0.254  0.84145
i57,106.80    0.2152     0.2658   0.810  0.56673
i57,132.60   -0.1112     0.2658  -0.418  0.74775
i57,801.10   -0.0923     0.2658  -0.347  0.78723
i57,918.60   -0.2320     0.2658  -0.873  0.54316
i57,933.80    0.1093     0.2658   0.411  0.75163
i57,958.10   -0.4313     0.2658  -1.623  0.35161
i58,028.80   -0.0580     0.2658  -0.218  0.86323
i58,457.50   -0.9745     0.2658  -3.666  0.16952
i58,540.20   -2.1932     0.2658  -8.251  0.07678 .
i58,695.70   -1.7169     0.2658  -6.459  0.09778 .
i58,970.30   -2.0684     0.2658  -7.782  0.08136 .
i59,596.10   -2.2643     0.2658  -8.519  0.07439 .
i59,829.40   -2.2231     0.2658  -8.364  0.07576 .
i60,327.20   -2.1835     0.2658  -8.215  0.07712 .
i60,364.00   -0.6470     0.2658  -2.434  0.24815
i60,808.50   -0.9437     0.2658  -3.550  0.17478
i61,290.50   -0.6000     0.2658  -2.257  0.26548
i61,702.00   -1.9998     0.2658  -7.524  0.08412 .
i61,781.20   -0.9137     0.2658  -3.438  0.18022
i61,961.90   -0.8244     0.2658  -3.102  0.19856
i61,992.90   -1.2884     0.2658  -4.847  0.12952
i62,038.00   -1.1735     0.2658  -4.415  0.14180
i62,041.30   -0.8209     0.2302  -3.566  0.17405
i62,054.60   -0.9335     0.2658  -3.512  0.17659
i62,120.00   -1.1185     0.2658  -4.208  0.14853
i62,125.00   -1.1955     0.2658  -4.498  0.13928
i62,314.30   -1.0938     0.2658  -4.115  0.15176
i62,541.10   -1.2784     0.2658  -4.810  0.13050
i62,846.90   -1.1035     0.2658  -4.152  0.15048
i63,235.70   -1.9837     0.2658  -7.463  0.08480 .
i63,311.90   -1.2582     0.2658  -4.734  0.13254
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i63,568.30   -1.4667     0.2658  -5.518  0.11413
i63,863.30   -0.8102     0.2658  -3.048  0.20181
i64,070.40   -1.1648     0.2658  -4.382  0.14283
i64,146.10   -1.4714     0.2658  -5.536  0.11377
i64,193.60   -1.0896     0.2658  -4.099  0.15232
i64,760.90   -1.9359     0.2658  -7.283  0.08687 .
i65,464.30   -1.9027     0.2658  -7.158  0.08836 .
i66,115.80   -1.9018     0.2658  -7.155  0.08840 .
i66,580.30   -1.6975     0.2658  -6.386  0.09888 .
i66,747.50   -1.9512     0.2658  -7.341  0.08619 .
i67,068.80   -1.3970     0.2658  -5.256  0.11970
i67,555.50   -1.5023     0.2658  -5.652  0.11148
i67,892.10   -1.8705     0.2658  -7.037  0.08986 .
i68,156.20   -1.4038     0.2658  -5.281  0.11913
i68,334.70   -1.8636     0.2658  -7.011  0.09019 .
i68,395.70   -1.2592     0.2658  -4.737  0.13244
i68,979.90   -1.2120     0.2658  -4.560  0.13744
i68,999.70   -1.4048     0.2658  -5.285  0.11905
i69,237.90   -1.6764     0.2658  -6.307  0.10011
i70,645.30   -1.9275     0.2658  -7.252  0.08724 .
i70,834.60   13.4980     0.2658  50.783  0.01253 *
i71,056.70   -2.2829     0.2658  -8.589  0.07379 .
i71,157.60   14.8930     0.2658  56.031  0.01136 *
i71,484.90   14.7383     0.2658  55.449  0.01148 *
i71,963.40   -2.0790     0.2658  -7.822  0.08095 .
i72,204.70   15.1416     0.2658  56.966  0.01117 *
i73,244.00   13.6873     0.2658  51.495  0.01236 *
i73,395.50   15.3289     0.2658  57.671  0.01104 *
i73,641.90   14.7685     0.2658  55.562  0.01146 *
i73,810.20   12.6361     0.2658  47.540  0.01339 *
i73,866.80   15.5827     0.2658  58.626  0.01086 *
i73,892.00   12.5143     0.2658  47.082  0.01352 *
i74,204.20   -1.3934     0.2658  -5.242  0.12000
i74,237.00   14.4604     0.2658  54.403  0.01170 *
i74,239.00   15.0240     0.2658  56.524  0.01126 *
i74,272.80   15.1921     0.2658  57.156  0.01114 *
i74,391.70   15.2475     0.2658  57.365  0.01110 *
i74,706.40   14.7632     0.2658  55.543  0.01146 *
i74,773.60   14.3658     0.2658  54.047  0.01178 *
i75,110.20    8.8192     0.2658  33.180  0.01918 *
i75,207.90   15.3148     0.2658  57.618  0.01105 *
i75,368.70   12.1724     0.2658  45.795  0.01390 *
i75,456.60   14.6363     0.2658  55.065  0.01156 *
i75,657.20   17.0008     0.2658  63.961  0.00995 **
i75,813.00   16.7522     0.2658  63.026  0.01010 *
i75,844.70   14.5432     0.2658  54.715  0.01163 *
i75,992.90   14.8014     0.2658  55.686  0.01143 *
i76,002.40    7.4144     0.2658  27.895  0.02281 *
i76,029.90   13.4643     0.2658  50.656  0.01257 *
i76,237.20   11.8378     0.2658  44.536  0.01429 *
i76,477.50   14.0758     0.2658  52.956  0.01202 *
i76,587.30   15.3178     0.2658  57.629  0.01105 *
i76,646.40   13.3297     0.2658  50.149  0.01269 *
i76,689.00   13.0356     0.2658  49.043  0.01298 *
i76,795.10   15.3199     0.2658  57.637  0.01104 *


Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1879 on 1 degrees of freedom
Multiple R-squared:      1,   Adjusted R-squared:  0.9992
F-statistic:  1211 on 258 and 1 DF,  p-value: 0.0229

> head(coef(lmr))
(Intercept)  i49,600.20  i49,794.30  i49,972.40  i50,231.80  i50,262.50  i50,305.80  i50,439.50
    46.5235     -1.6204     -0.2364     -0.2373     -0.3577     -1.0748     -1.1576     -2.0679
 i50,628.00  i51,034.50  i51,038.40  i51,095.90  i51,525.90  i51,554.40  i51,645.50  i51,761.70
    -2.1399     -2.0816     -2.2070     -0.7109     -0.3479     -0.9606     -0.5625     -1.0529
 i54,247.40  i54,678.30  i55,383.40  i55,633.10  i55,743.70  i55,876.10  i55,908.30  i55,961.60
    -0.3833     -1.3700      0.3585      0.5953     -0.1426      0.2177     -0.3372      0.2303
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 i56,390.00  i56,406.30  i56,784.40  i57,106.80  i57,132.60  i57,801.10  i57,918.60  i57,933.80
     0.4381      0.2370      0.0676      0.2152     -0.1112     -0.0923     -0.2320      0.1093
 i58,028.80  i58,457.50  i58,540.20  i58,695.70  i58,970.30  i59,596.10  i59,829.40  i60,327.20
    -0.0580     -0.9745     -2.1932     -1.7169     -2.0684     -2.2643     -2.2231     -2.1835
 i60,808.50  i61,290.50  i61,702.00  i61,781.20  i61,961.90  i61,992.90  i62,038.00  i62,041.30
    -0.9437     -0.6000     -1.9998     -0.9137     -0.8244     -1.2884     -1.1735     -0.8209
 i62,120.00  i62,125.00  i62,314.30  i62,541.10  i62,846.90  i63,235.70  i63,311.90  i63,568.30
    -1.1185     -1.1955     -1.0938     -1.2784     -1.1035     -1.9837     -1.2582     -1.4667
 i64,070.40  i64,146.10  i64,193.60  i64,760.90  i65,464.30  i66,115.80  i66,580.30  i66,747.50
    -1.1648     -1.4714     -1.0896     -1.9359     -1.9027     -1.9018     -1.6975     -1.9512
 i67,555.50  i67,892.10  i68,156.20  i68,334.70  i68,395.70  i68,979.90  i68,999.70  i69,237.90
    -1.5023     -1.8705     -1.4038     -1.8636     -1.2592     -1.2120     -1.4048     -1.6764
 i70,834.60  i71,056.70  i71,157.60  i71,484.90  i71,963.40  i72,204.70  i73,244.00  i73,395.50
    13.4980     -2.2829     14.8930     14.7383     -2.0790     15.1416     13.6873     15.3289
 i73,810.20  i73,866.80  i73,892.00  i74,204.20  i74,237.00  i74,239.00  i74,272.80  i74,391.70
    12.6361     15.5827     12.5143     -1.3934     14.4604     15.0240     15.1921     15.2475
 i74,773.60  i75,110.20  i75,207.90  i75,368.70  i75,456.60  i75,657.20  i75,813.00  i75,844.70
    14.3658      8.8192     15.3148     12.1724     14.6363     17.0008     16.7522     14.5432
 i76,002.40  i76,029.90  i76,237.20  i76,477.50  i76,587.30  i76,646.40  i76,689.00  i76,795.10
     7.4144     13.4643     11.8378     14.0758     15.3178     13.3297     13.0356     15.3199
 i77,075.10  i77,144.10  i77,272.00  i77,304.80  i77,305.50  i77,325.70  i77,366.30  i77,468.50
    14.6896     13.3281     12.7990      9.5866     13.8841     13.3603     14.3572     12.6706
 i78,008.10  i78,019.30  i78,075.00  i78,300.00  i78,376.70  i78,381.40  i78,453.10  i78,502.40
     8.1292     13.8678     13.6208     15.1866     15.6765     15.7259     14.1503     13.8684
 i78,718.90  i78,768.30  i78,816.20  i78,856.60  i78,859.90  i78,904.30  i78,927.30  i78,953.40
    -1.0823     10.2294     14.0800     14.5675     13.4221     13.2473     13.7417     13.5368
 i79,162.00  i79,174.80  i79,280.70  i79,339.50  i79,389.70  i79,672.30  i79,746.40  i80,042.20
    13.2436     16.0689      7.4242      3.0531     13.7714     14.9519     14.5550      7.1000
 i80,186.80  i80,254.90  i80,629.00  i80,639.40  i80,798.50  i80,815.90  i81,204.10  i81,303.70
    13.5249     14.4462     14.5390     16.6761     14.6089     15.7362      2.8063      7.1654
 i81,549.40  i81,577.10  i81,958.80  i82,140.70  i82,166.10  i82,216.50  i82,247.20  i82,534.10
    13.7574     14.5259     15.3111     15.2610      3.0671     14.9073      3.1974     -0.7136
```

```
> resid(lmr)
            1             2             3             4             5             6
 1.476455e-18  1.052150e-19  3.176432e-20 -4.379135e-20  9.298337e-20 -4.520556e-20 -4.658195e-2
            8             9            10            11            12            13            1
-4.415103e-20  2.816404e-20  1.224034e-20  1.031604e-19 -4.189318e-20  1.726431e-20  2.409982e-2
           15            16            17            18            19            20            2
 7.793352e-21  8.634643e-20  3.316482e-20  6.064620e-20  2.658900e-19 -5.062882e-20  6.582456e-2
           22            23            24            25            26            27            2
 3.017209e-20  8.177764e-20 -3.988689e-20  9.809932e-20  1.232538e-19  3.323578e-19 -6.935504e-2
           29            30            31            32            33            34            3
-5.966976e-20 -4.759218e-20 -2.423122e-20  1.020282e-19  1.489459e-19  6.630357e-20 -6.315194e-2
           36            37            38            39            40            41            4
 1.194108e-19  2.038878e-19  1.391164e-19 -1.657549e-20  8.672862e-20 -6.961287e-20  1.036603e-1
           43            44            45            46            47            48            4
 1.365273e-19 -8.657294e-20  2.706921e-19 -3.922126e-20  2.718058e-19  4.727713e-20 -5.237553e-2
           50            51            52            53            54            55            5
 5.772492e-20  2.099368e-19  2.759263e-19  5.081905e-19  6.485034e-20  2.181907e-20  1.256091e-2
           57            58            59            60            61            62            6
 6.136162e-20  6.161756e-20  2.337690e-20  7.799830e-21  9.702254e-20  8.950006e-20 -1.856556e-2
           64            65            66            67            68            69            7
 8.506129e-21  8.583380e-21  1.171477e-19  3.635334e-20 -2.617446e-19  1.668710e-17 -1.410282e-1
           71            72            73            74            75            76            7
 7.497119e-20  4.803533e-20 -1.146106e-19 -3.388223e-20  7.433206e-20 -6.807812e-21  2.018331e-2
           78            79            80            81            82            83            8
-6.937057e-21  7.431295e-20  1.016930e-19  7.496331e-20  6.491667e-20  7.572242e-20 -1.006064e-2
           85            86            87            88            89            90            9
 2.878167e-19 -4.101934e-21  5.251344e-20 -5.163377e-20  2.031411e-19  3.514297e-19 -6.268547e-1
           92            93            94            95            96            97            9
-2.053737e-18  6.996180e-17  1.764196e-19 -7.828066e-19  1.390362e-19 -1.662525e-16 -3.851421e-1
           99           100           101           102           103           104           10
-1.721137e-18 -9.761713e-19 -2.672723e-18 -5.908756e-18  8.631510e-17 -4.459206e-18 -1.333944e-1
          106           107           108           109           110           111           11

> fitted(lmr)
      1       2       3       4       5       6       7       8       9      10      11      12
63.5243 63.2757 62.1062 61.8524 61.7710 61.6651 61.4165 61.2618 61.1598 61.0667 61.2867 61.2920
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     14      15      16      17      18      19      20      21      22      23      24      25
60.9839 60.8893 60.5993 60.3913 60.4076 60.6035 60.9697 61.0910 60.2602 59.9456 60.0484 59.7708
     27      28      29      30      31      32      33      34      35      36      37      38
60.0603 59.9878 59.1596 59.0378 58.6959 58.3613 59.1941 59.8516 60.2652 60.6738 60.2949 60.1443
     40      41      42      43      44      45      46      47      48      49      50      51
60.2809 61.1020 61.0494 61.4308 61.7845 61.8346 61.8797 62.2494 62.5924 61.7514 61.2131 61.8434
     53      54      55      56      57      58      59      60      61      62      63      64
61.7156 61.8413 61.3249 61.7101 62.2000 62.4164 63.1996 62.2597 61.3180 61.0625 60.8807 61.1324
     66      67      68      69      70      71      72      73      74      75      76      77
62.2961 62.4969 63.7853 66.2950 66.2109 63.9435 61.4754 61.0785 60.3919 59.3225 59.5591 60.2108
     79      80      81      82      83      84      85      86      87      88      89      90
59.8532 58.8081 57.7807 56.7529 56.1101 55.3427 54.6527 53.9477 53.6235 53.9379 53.9721 54.4030
     92      93      94      95      96      97      98      99     100     101     102     103
54.2911 54.2015 54.1553 54.6927 54.2806 54.2603 53.8499 53.2320 53.3997 53.5884 54.2962 54.7967
    105     106     107     108     109     110     111     112     113     114     115     116
54.7357 54.2587 54.3415 55.0641 55.0601 54.8230 54.2546 53.7774 53.6269 53.0265 52.3692 51.9961
    118     119     120     121     122     123     124     125     126     127     128     129
53.6000 54.7987 55.3563 55.4489 55.3088 55.4692 55.0772 55.3909 55.5665 54.8922 55.3222 55.0331
    131     132     133     134     135     136     137     138     139     140     141     142
56.0784 55.4260 55.2694 55.7777 55.5356 54.3689 53.6705 53.3225 52.9627 52.2978 51.9698 51.5895
    144     145     146     147     148     149     150     151     152     153     154     155
51.4097 50.4955 49.9095 49.2776 49.3183 49.5844 49.3958 49.5438 50.3484 51.3519 52.5851 53.6889
    157     158     159     160     161     162     163     164     165     166     167     168
53.8011 53.7742 51.6100 51.7814 52.2188 50.9190 49.9113 49.2380 49.5906 49.7209 49.3298 49.5766
    170     171     172     173     174     175     176     177     178     179     180     181
48.9050 47.5335 46.0790 45.8099 45.8564 45.4412 45.1301 44.4445 44.2406 44.5960 44.6599 44.6217
    183     184     185     186     187     188     189     190     191     192     193     194
45.3115 45.2643 45.1187 44.8471 45.1265 45.0212 44.8260 44.6530 44.5723 44.6208 44.5876 44.5398
    196     197     198     199     200     201     202     203     204     205     206     207
45.3587 45.7133 45.4339 45.0568 45.2451 45.4297 45.6098 45.8765 45.9235 45.7026 45.7026 45.4200
    209     210     211     212     213     214     215     216     217     218     219     220
45.3500 45.5900 45.6991 45.2653 45.4050 45.5798 45.3280 44.5237 44.3004 44.4551 44.3400 44.2592
    222     223     224     225     226     227     228     229     230     231     232     233
44.8066 45.5490 46.2915 46.4312 46.6328 46.7387 46.5911 46.3809 46.1402 46.7412 47.1188 46.7605
    235     236     237     238     239     240     241     242     243     244     245     246
```

```
> fit
ETS(M,Ad,M)
Smoothing parameters:
alpha = 0.0267
beta = 0.0232
gamma = 0.025
phi = 0.98
Initial states:
l = 162.5752
b = -0.1598
s = 1.1979 1.2246 1.1452 0.9354 0.9754 0.9068
0.8523 0.9296 0.9342 1.016 0.9131 0.9696
sigma: 0.0578
AIC AICc BIC
499.0295 515.1347 533.4604
```

```
> fit2
ETS(M,N,M)
Smoothing parameters:
alpha = 0.247
gamma = 0.01
Initial states:
l = 168.1208
s = 1.2417 1.2148 1.1388 0.9217 0.9667 0.8934
0.8506 0.9182 0.9262 1.049 0.9047 0.9743
sigma: 0.0604
AIC AICc BIC
500.0439 510.2878 528.3988
```

```
> layout(matrix(1:4,2,2))
```

```
> plot(lmr)
```

```
1: not plotting observations with leverage one:
  2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27
31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55,
59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82,
86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108,
111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 1
133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 1
155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 1
179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 1
201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 21 [... truncate
```

```
2: not plotting observations with leverage one:
  2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27
31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55,
59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82,
86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108,
111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 1
133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 1
155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 1
179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 1
201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 21 [... truncate
3: In sqrt(crit * p * (1 - hh)/hh) : NaNs produced
4: In sqrt(crit * p * (1 - hh)/hh) : NaNs produced
```

```
> accuracy(fit)

ME     RMSE    MAE      MPE    MAPE    MASE

0.0774 3890.28 2824.42 -0.2915 3.0037% 0.4351

> accuracy(fit2)

ME     RMSE     MAE     MPE    MAPE    MASE

-1.3884 2770.59  2283.30 -1.1945  2.5562%  0.4535

> accuracy(fit3)

ME     RMSE     MAE     MPE    MAPE    MASE

-1.4557 2960.48 2279.82  -1.2345  2.6266%  0.4612
```