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Analysis on the adaptive filter based on LMS algorithm

Zhen Zhu[a], Xiang Gao[a,*], Leilei Cao[a], Daoyuan Pan[b], Yumeng Cai[a], Yu Zhu[c]

[a] School of Automotive and Traffic Engineering, Jiangsu University, Zhenjiang 212013, China
[b] School of Mechanical and Automotive Engineering, Anhui polytechnic University, Wuhu 241000, China
[c] Axle Branch Company, IVECO Co. Ltd , Nanjing, 212028, China

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ABSTRACT

This article focuses on the application of adaptive filter based on the LMS algorithm. An adaptive filter of the closed-loop system is introduced, including the elimination of interference signal, the prediction of useful signal, and the approximation of expected signal. LMS (Least Mean Square) algorithm is used to meet the optimum norm of error between estimated signal and expected signal. The structure of LMS algorithm is presented and the simulation of LMS algorithm is carried out. The results indicate that the convergence performances of LMS algorithm are prefect, and the input signal can converge to the expected signal. The application of adaptive filtering technology in this article includes the correction of channel mismatch by an adaptive linear filter, the improvement of system performance by an adaptive equalizer, and the filter of frequency signal by an adaptive notch filter. The analysis on adaptive linear filter shows that the constant channel mismatch can be corrected quite well by the correction algorithm. The analysis on adaptive equalizer shows that the error rate of system with an adaptive equalizer has significant improvement gains over that of system without an adaptive equalizer. The smaller the error rate, the larger the SNR. The relationship between error rate and multi-path loss show that the error rate is largest when the loss factor is 0.5. The analysis on adaptive notch filter shows that the interference signal with two different known frequencies can be eliminated effectively by the adaptive notch filter. The filtered signals accord with the corresponding useful signals very well.

1. Introduction

Commonly used filters can be divided into three kinds: FIR (Finite Impulse Response) filter, IIR (Infinite Impulse Response) filter, and adaptive filter. FIR filter can be designed as the system with arbitrary amplitude-frequency characteristic and linear phase characteristic. The design target of FIR filter is that the requirement of filter coefficients for performance indexes should be met and the hardware resources of system can be saved. Comparing with FIR filter, IIR filter has higher efficiency and lower order under the condition of same amplitude-frequency response. However, the disadvantage of IIR filter is the non-linear phase characteristic, which restricts the application range of filter. The theoretical basis of FIR filter and IIR filter in this article is the Linear Time-Invariant System, which means the structure design and analysis method between FIR filter and IIR filter are similar. In a word, IIR filter is suitable to the condition that the amplitude-frequency characteristic is given and the phase characteristic is not considered; FIR filter is suitable to the condition that the amplitude-frequency characteristic is different from the typical filter characteristic or the linear phase characteristic is required [1–4].

The precondition of FIR filter and IIR filter is that the statistical characteristics of input signals are known. That is to say, the performance indexes of filter, such as passband, stopband, tolerance of passband, and stopband, are definite. The design target of filter is that the performance indexes of filter, which are not related to time, should be met. The premise design condition of adaptive filter is that some of the statistical characteristics of input signals are unknown. The performance indexes of filter can be replaced by the estimated value of unknown signal [5,6]. In order to analyze the adaptive filter based on LMS (Least Mean Square) algorithm, the principle and application of adaptive filter should be introduced, and the simulation results based on the statistical experimental method are presented according to the principle and structure of LMS algorithm [7–9]. The applications of adaptive filtering technology are shown by the introduction of three parts: an adaptive linear filter for the correction of channel mismatch, an adaptive equalizer for the improvement of system performance, and an adaptive notch filter for the elimination of the interference signal with known frequency.

* Corresponding author. Tel.: +86 13775531530; fax: +86 51188722660.
E-mail address: xianggaojs@163.com (X. Gao).

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Nomenclature

n	sequence length
y (n)	output signal
e (n)	error signal
N_2 (n)	Interference signal 2
s_N (n)	narrow-band signal
Δ	time-delay
k_N	sampling time of narrow-band signal
X (n)	Vector of input signal
e*(n)	complex conjugate pair of e(n)
SNR	signal-to-noise ratio
H_k (jω)	frequency response of the kth channel
C (jω)	correction factor
G_T (ω)	transfer function of transmitting filter
n (t)	Gaussian white noise signal added in the channel
G_E (ω)	transfer function of equalizer
s (t)	useful signal
ω_1/ω_2	frequency of interference signal
θ_1/θ_2	phase of interference signal
x (n)	input signal
t (n)	excepted signal
N_1 (n)	Interference signal 1
s (n)	useful signal
s_B (n)	wide-band signal
k_B	sampling time of wide-band signal
W (n)	coefficient vector of filter
μ	step factor
j (n)	Gaussian white noise signal
D_k (jω)	channel mismatch characteristic of the kth channel
H_r (jω)	frequency response of the reference channel
H (ω)	transfer function of signal system
C (ω)	transfer function of channel
G_R (ω)	transfer function of receiving filter
x (t)	input signal with the interference signal
r (t)	reference signal
A_1/A_2	amplitude of interference signal
$W_1 \sim W_4$	weight value of single-frequency signal

2. Introduction of adaptive filter and LMS algorithm

2.1. Introduction of adaptive filter

2.1.1. Adaptive filter of the closed-loop system

The structure parameters of the adaptive filter studied in this article can be adjusted automatically by the use of LMS algorithm, according to the statistical characteristics of input signal. The adaptive filter is composed of the digital filter with adjustable parameters and adaptive algorithm. This article focuses on the FIR adaptive filter of the closed-loop system, which is shown in Fig. 1.

Output signal $y(n)$ is generated with the use of FIR digital filter, and is related to input signal $x(n)$. Error signal $e(n)$ is generated with the comparison of output signal $y(n)$ and target signal $t(n)$. The filter parameters can be modified automatically, according to the optimal criterion for FIR adaptive filter and error signal. The process of adjusting parameters of FIR adaptive filter is called "learning process" when the statistical characteristics of input signals are tested; and the process of adjusting parameters of FIR adaptive filter is called "tracking process" when the statistical characteristics of input signals are unknown.

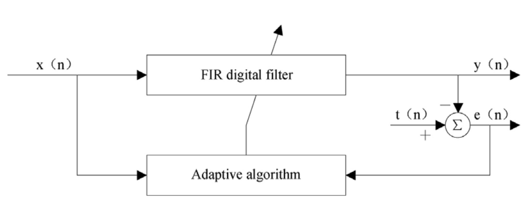


Fig. 1. FIR adaptive filter of the closed-loop system.

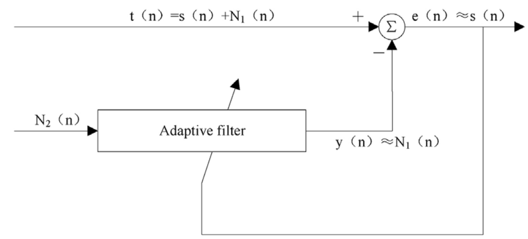


Fig. 2. Principle diagram for the elimination of interference signal.

2.1.2. Application of adaptive filter

With the development of the signal processing theory, the FIR adaptive filter has been widely used as a valuable signal-processing device to eliminate interference signal, predict useful signal and approximate expected signal.

2.1.2.1. Elimination of interference signal. The principle diagram for the elimination of interference signal is shown in Fig. 2. Expected signal $t(n)$ is the sum of useful signal $s(n)$ and noise signal $N_1(n)$. $N_2(n)$ is another noise signal relevant to $N_1(n)$. The parameters of adaptive filter should be adjusted automatically to eliminate interference signal. Namely, output signal $y(n)$ is approximate to $\hat{N}_1(n)$, the optimal estimation of $N_1(n)$, and error signal $e(n)$ is approximate to $s(n)$ [10,11].

There are two special conditions should be considered about the elimination of interference signal:

The system can realize the elimination of interference signal when $N_2(n)$ is correlated with $N_1(n)$. However, it is difficult to eliminate the interference signal, which is uncorrelated with $N_2(n)$ or superimposed on $s(n)$.

A part of useful signal will be eliminated when $s(n)$ leaks into the input terminal of adaptive filter, it is necessary to avoid this situation.

2.1.2.2. Prediction of useful signal. The principle diagram for the prediction of useful signal is shown in Fig. 3. The input signal of adaptive filter is the useful signal with time-delay, and the output signal of predictive filter is the prediction of useful signal with time-delay [12–14].

One of the applications of adaptive prediction is the separation of narrow-band and wide-band signal. A mixed signal, which is added to the input terminal of predictive filter, can be expressed as follows:

$$x(n) = s_N(n) + s_B(n) \quad (1)$$

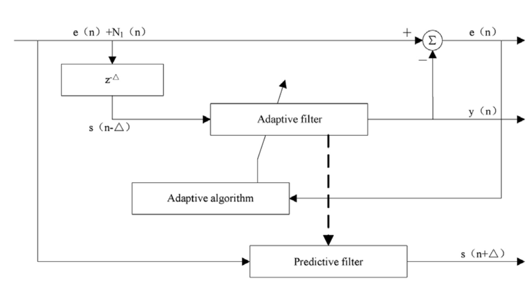


Fig. 3. Principle diagram for the prediction of useful signal.

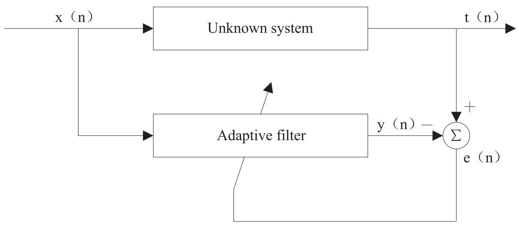


Fig. 4. Principle diagram for the approximation of expected signal.

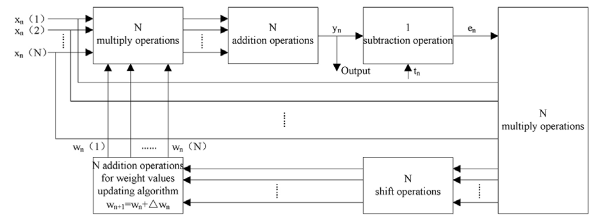


Fig. 5. Flow diagram of LMS algorithm structure.

The autocorrelation coefficient of narrow-band is shorter than that of wide-band. When the delay-time is set to $k_B < \Delta < k_N$, $s_B(n-\Delta)$ is uncorrelated with $s_B(n)$, and $s_N(n-\Delta)$ is correlated with $s_N(n)$, so the following equations can be obtained.

$$y(n) = \hat{s}_N(n) \tag{2}$$

$$\hat{s}_B(n) = s_B(n) + s_N(n) - \hat{s}_N(n) \tag{3}$$

Where $\hat{s}_N(n)$ and $\hat{s}_B(n)$ are the best estimate values of $s_N(n)$ and $s_B(n)$, respectively. Therefore, $s_B(n)$ and $s_N(n)$ can be separated to complete the prediction of useful signal.

2.1.2.3. Approximation of expected signal. The principle diagram for the approximation of expected signal is shown in Fig. 4. The weight values of adaptive filter are regulated automatically to meet requirements. Namely, when the frequency components of excitation source are unchanged, and the system noise is small enough, the adaptive filter will become an approximation model of unknown system [15,16].

2.1.3. Adaptive algorithm

An adaptive filter is mainly composed of the digital filter with the adjustable parameters and the adaptive algorithm. The final purpose of adaptive algorithm is to meet the optimal criterion of error between estimated signal and expected signal, and the optimal algorithm is the decisive factor for the performance and structure of adaptive criterion. This article adopts LMS algorithm as the optimal criterion to solve the optimal problem [17–19].

2.2. Introduction of LMS algorithm

2.2.1. LMS algorithm principle

By using the iteration operations, filter coefficients can converge to the optimal values. LMS algorithm, which is proposed by Widrow and Hoff in 1960, is one of the iteration operations.

LMS algorithm can be expressed by the following recursion equation.

$$\begin{cases} y(n) = W^H(n)X(n) \\ e(n) = t(n) - y(n) \\ W(n+1) = W(n) + 2\mu X(n)e^*(n) \end{cases} \tag{4}$$

The larger the μ, the faster the convergence rate, and the larger the error. So the performance of LMS algorithm can be adjusted by controlling the step factor. Namely, larger μ is used for the initial stage to accelerate convergence, and smaller μ is used for the algorithm convergence stage to improve stability.

2.2.2. LMS algorithm structure

According to Equation (4), $W(n)$ can be calculated by the cyclic iteration method to realize the least mean square value of $e(n)$.

$$W(n+1) = W(n) + 2\mu X(n)[t(n) - W^H(n)X(n)]^* \tag{5}$$

The flow diagram of LMS algorithm structure is shown in Fig. 5.

The updating algorithms of whole weight values include $2N$ multiply operations, $2N$ addition operations, one subtraction operation and N shift operations. LMS algorithm is a closed-loop system, so the updating algorithm of each weight value should be finished in the corresponding data cycle.

2.2.3. Simulation of LMS algorithm based on the statistical experimental method

Statistical experimental method is a stochastic analysis method, which tests the dynamic characteristics of system and concludes the statistical results by mass data of computer simulations. The basic idea of problem solving is that a similar probability model should be built, then the statistical sampling should be carried out, the estimated values of sample should be considered as the approximate solution of original problem finally. The theoretical basis of statistical experimental method is law of large numbers, and the main method is the sampling analysis of random variables [20,21].

Simulation analysis should be carried out to visually display the LMS algorithm. Input signal is the synthesis signal, which is synthesized by a sine signal and a white noise signal.

$$x(n) = \sqrt{2}s(n) + sqrt(10^{-\frac{SNR}{10}})j(n) \tag{6}$$

Where $s(n)$ is the sine signal, $j(n)$ is the Gaussian white noise signal (average value is zero, power is one). The Gaussian white noise signal can form an interference cancellation system. It is noteworthy that there is not any time-delay in the LMS algorithm, because the correlation function of Gaussian white noise signal is the impulse function, the correlativity will disappear when the time-delay exists in the expected signal and input signal.

Simulation conditions are shown in Table 1.

Simulation results are shown in Fig. 6. The results indicate that the convergence performances of LMS algorithm are prefect, the input signal can converge to the expected signal (sine signal or noise signal). Besides, the convergence speed and performances of LMS algorithm have great relationship with the length of filter and step factor.

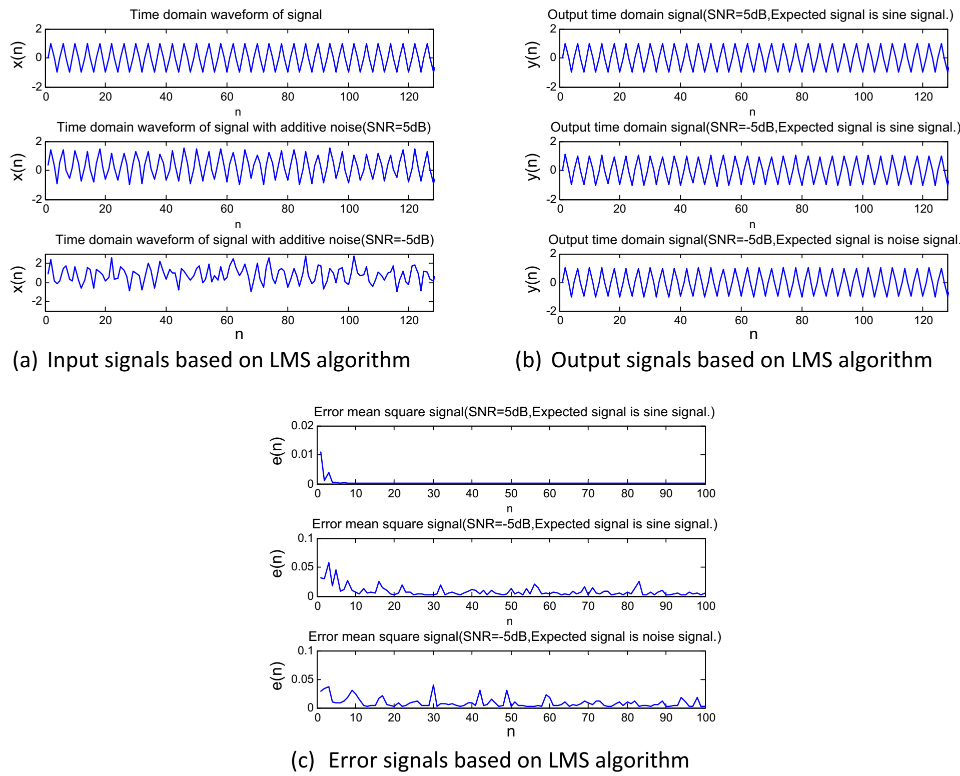
3. Application of adaptive filtering technology

The application of adaptive filtering technology in this article includes the correction of channel mismatch by an adaptive linear filter, the improvement of system performance by an adaptive equalizer, and the filter of frequency signal by an adaptive notch filter.

3.1. An adaptive linear filter for the correction of channel mismatch

FIR filter is one of the adaptive linear filters for its strict linear phase characteristic. Section 2.2.3 presents the simulation analysis of LMS algorithm, this section will carry out the channel mismatch correction for an adaptive linear filter based on LMS algorithm.

In the digital signal processing system, the compositions of analog processing units for each channel are not identical, which cause the inconsistency of amplitude and phase of frequency response



(a) Input signals based on LMS algorithm

(b) Output signals based on LMS algorithm

(c) Error signals based on LMS algorithm

Fig. 6. Signals based on LMS algorithm.

among channels. Therefore, it is necessary to correct the channel mismatch to acquire the higher performance of subsequent processing [22–25].

The channel mismatch characteristic of the kth channel can be defined as follows:

$$D_k(jw) = \frac{H_k(jw)}{H_r(jw)} \tag{7}$$

A channel, whose frequency response is $H(jw)$, can be expressed as follows:

$$D(jw) = \frac{H(jw)}{H_r(jw)} = [1 + \Delta H(jw)]e^{j\Delta\varphi(w)} \tag{8}$$

$\Delta H(jw)$ and $\Delta\varphi(w)$ are constants because the channel mismatch is not related to the frequency. The input signal can be regarded as stationary signal for the general signal system, and the main problem of the correction of channel mismatch is attributed to the determination of correction factor, which is shown in Equation (9).

$$C(jw) = \left[\frac{1}{1 + \Delta H(jw)}\right] e^{-j\Delta\varphi(w)} \tag{9}$$

This article analyzes the constant channel mismatch independent from frequency response, and the correction model of channel mismatch is shown in Fig. 7.

Input signal is transmitted to the reference channel and the predictive correction channel. The mean square deviation between the reference signal and the corrected channel should trend to the minimum value, and the value of $D(jw)C(jw)$ is approximate to one finally.

The time-delay Δ in the reference channel should make up for the time difference, which is caused by $D(jw)C(jw)$. In order to correct the amplitude and phase factor at the same time, the operation of LMS algorithm is a complex operation, and the length of filter is one. That is to say, a complex weight can realize the correction of amplitude and phase.

The simulation results can be shown in Fig. 8.

The correction result is prefect when amplitude range is 0∼1.5, and phase range is −75°∼75°. Namely, the constant channel mismatch can be corrected quite well by the correction algorithm.

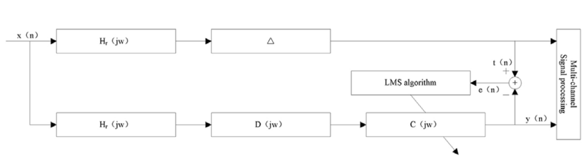


Fig. 7. Correction model of channel mismatch.

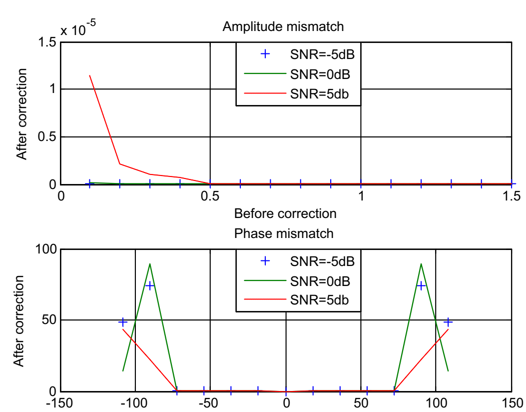


Fig. 8. Characteristic analysis of the correction algorithm of channel mismatch.

Table 1
Simulation conditions of LMS algorithm.

Simulation parameter	Data	Simulation parameter	Data
Iteration times	100	Sequence length of input signal	1024
Length of filter	128	Step factor	1/128
Useful signal	sin (0.5*pi*t)	SNR parameters of input signal	–5 dB/5 dB

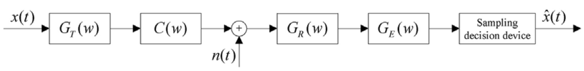


Fig. 9. Flow diagram of system model with an adaptive equalizer.

3.2. An adaptive equalizer for the improvement of system performance

Because of the multi-path effect of communication channel, the limitation of channel bandwidth and the imperfection of channel characteristics in the wireless data communication system, the intersymbol interference are inevitably occurred when the data pass through the channel. The adaptive equalizer can automatically adjust its parameters to adapt to the change of channel characteristics and eliminate interference to improve the communication quality [26–29].

Theoretical analysis and experimental results show that a tunable filter in the digital communication system can compensate system characteristics and eliminate intersymbol interference. The tunable filter is called equalizer, and the main realization way of adaptive filter is the transversal filter. The system model with an adaptive equalizer is shown in Fig. 9.

The transfer function of signal system can be expressed as follows:

$$H(w) = G_T(w)C(w)G_R(w)G_E(w) \tag{10}$$

The transmitting filter and the receiving filter are matched to each other, and the equalizer is used to compensate the channel distortion. The transfer function of equalizer should satisfy Equation (11).

$$G_E(w) = \frac{1}{C(w)} = \frac{1}{|C(w)|}e^{-j\varphi(\omega)} \tag{11}$$

The filter is used as the equalizer to compensate the distortion of frequency characteristic. The demodulated output samples obtained by the sampling decision device are the modified samples. The adaptive filter adjusts the gain of input signal continuously according to some algorithms, in order to adapt to the random variation of channels and keep the optimum operating condition of equalizer. From parameters adjustment to convergence formation, the whole working process of system is the function on the equalizer algorithm and communication change rate. The equalizer should be trained periodically to eliminate intersymbol interference, and be modified according to the same training sequence when the equalizer receives the new data. The training sequence is the expected signal of some algorithm.

The simulation conditions of LMS algorithm for an adaptive equalizer are shown in Table 2, and the error rate contrasting diagrams between the system with an adaptive equalizer and the system without an adaptive equalizer are shown in Fig. 10.

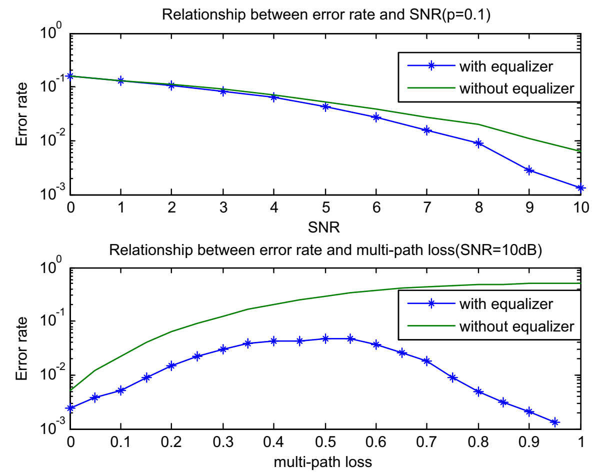


Fig. 10. Error rate contrasting diagrams between the system with an adaptive equalizer and the system without an adaptive equalizer.

Table 2
Simulation conditions of LMS algorithm for an adaptive equalizer.

Simulation parameter	Data	Simulation parameter	Data
Length of simulation data	20000	Length of training data	2000
Step factor	1/128	Length of filter	8

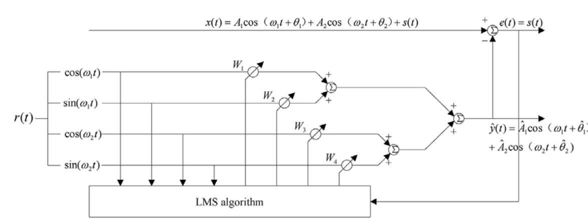
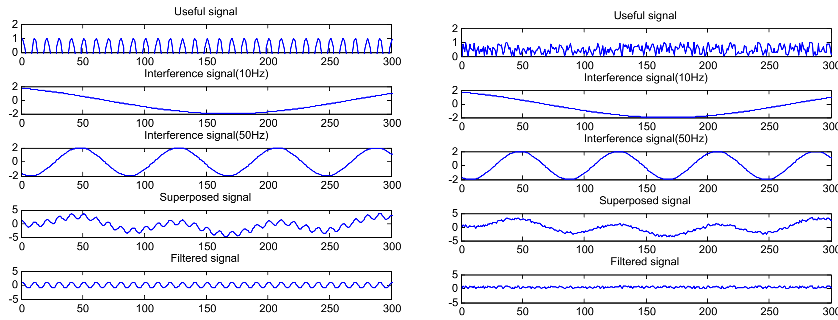


Fig. 11. The principle diagram of adaptive notch filter for the elimination of interference signal with two different known frequencies (ω_1, ω_2).

The simulation results show that the error rate of system with an adaptive equalizer has significant improvement gains over that of system without an adaptive equalizer Table 1–3. The smaller the error rate, the larger the SNR. The relationship between error rate and multi-path loss show that the error rate is largest when the loss factor is 0.5, which means the transmission paths of two signals are in the most interferential conditions. The simulation results accord well with the theoretical analysis.

Table 3
Simulation conditions of LMS algorithm for an adaptive notch filter.

Simulation parameter	Data	Simulation parameter	Data
Sampling frequency	4000 Hz	Frequency of interference signal	f_1 = 50 Hz f_2 = 10 Hz
Frequency of useful signal	(1)400 Hz; (2)Random signal.		



(a) signal frequency signal (400H_z) (b) randomsignal

Fig. 12. Simulation curves of different signals for an adaptive notch filter.

3.3. An adaptive notch filter for the elimination of the interference signal with known frequency

Notch filter is another kind of filter, which is used to eliminate the interference signal with known frequency, unknown phase, and amplitude. The adaptive notch filter based on the LMS algorithm can eliminate the interference signal with known frequency from the useful signal effectively. Fig. 11 presents the principle diagram of adaptive notch filter. A signal with two different known frequencies can be eliminated at the same time by the adaptive notch filter [30–32].

$x(t)$ is the input signal with the interference signal, the adaptive notch filter is used to eliminate the interference signal with two different known frequencies (ω_1, ω_2), $s(t)$ is the useful signal. The amplitudes of interference signal (A_1, A_2) and the phases of interference signal of (θ_1, θ_2) should be estimated by the LMS algorithm. In order to estimate the amplitude and phase of $A_1 \cos(\omega_1 t+\theta_1)$, two orthogonal single-frequency signals ($\sin(\omega_1 t)$, $\sin(\omega_2 t)$) and corresponding weight values (W_1, W_2) are used to synthesis estimated signal $\hat{A}_1 \cos(\omega_1 t + \hat{\theta}_1)$. Therefore, the interference signal with two different known frequencies can be eliminated effectively by four channel reference signals, which can be shown in Fig. 11. The interference signal with two different known frequencies can be measured by the estimated signal $\hat{y} = \hat{A}_1 \cos(\omega_1 t + \hat{\theta}_1) + \hat{A}_2 \cos(\omega_2 t + \hat{\theta}_2)$, and the error signal based on the LMS algorithm is the useful signal without the interference signal.

The simulation conditions of LMS algorithm for an adaptive notch filter are shown in Table 3.

Simulation curves of signal frequency signal for an adaptive notch filter are shown in Fig. 12 (a); simulation curves of random signal for an adaptive notch filter are shown in Fig. 12 (b).

According to the simulation results in Fig. 12, the interference signal with two different known frequencies can be eliminated effectively by the adaptive notch filter. The filtered signals accord with the corresponding useful signals very well.

4. Conclusions

(1) An adaptive filter of the closed-loop system is introduced, including the elimination of interference signal, the prediction of useful signal, and the approximation of expected signal.
(2) The application of adaptive filtering technology is presented, including the correction of channel mismatch by an adaptive linear filter, the improvement of system performance by an adaptive equalizer, and the filter of frequency signal by an adaptive notch filter.
(3) The analysis on adaptive linear filter shows that the constant channel mismatch can be corrected quite well by the correction algorithm. The analysis on adaptive equalizer shows that the error rate of system with an adaptive equalizer has significant improvement gains over that of system without an adaptive equalizer. The smaller the error rate, the larger the SNR. The relationship between error rate and multi-path loss show that the error rate is largest when the loss factor is 0.5. The analysis on adaptive notch filter shows that the interference signal with two different known frequencies can be eliminated effectively by the adaptive notch filter. The filtered signals accord with the corresponding useful signals very well.

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