Unit - V

DESIGN OF EXPERIMENTS

Introduction

Design of experiments may be defined as the logical construction of the experiment in which the degree uncertainty with which the inference is drawn may be well defined

Basic principles of Experimental design

The three basic principle of experimental design are replication, randomization and local control.

Replication

Replication mean the repetition of an experiment. It means the repetition of the treatment under investigation. It helps us in estimating the experimental error. Thus, the repetition of treatments results in more reliable estimate than is possible with single observation.

Randomization

Randomization is a technique by which two or more treatments are applied to different group of the sample, the treatments to be applied to any group being decided by random sapling technique.

Local Control

The process of reducing the experimental error by dividing the relatively the heterogeneous experimental area into homogeneous is known local control.

Analysis of Variance

Analysis of variance is a technique, which is used to test the equality of three or more population means by comparing the sample variances using F-distribution. This technique split up the variance into its various components, usually into two parts

i. Variance between the samples.

ii. Variance within the samples.

Assumption for Analysis of Variance Test

i. The sampled population is normal
ii. The treatment effects are additive
iii. The individuals have been randomly selected from the population
iv. The Variance between the sample is constant
v. Experimental errors should be homogeneous in their variance and are independent.

Techniques of Analysis of Variance

The technique of ANOVA is discussed under three types

i. One way classification [Completely Randomized Design]
ii. Two way classification [Randomized Block Design]
iii. Three way classification [Latin Square Design]

Analysis of variance for one-way classification

In this case observations (or data) are classified according to one factor (or) criterion.

Testing of Hypothesis by ANOVA

Let H_0 : $\mu_1 = \mu_2 = \dots\dots\dots = \mu_k$

H_1 : $\mu_1 \neq \mu_2 \neq \dots\dots\dots \neq \mu_k$

where $\mu_1, \mu_2, \dots\dots\dots \mu_k$ are the means of K samples

Calculation of total variation

$$T = \sum X_1 + \sum X_2 + \dots\dots\dots \sum X_K$$

Correction factor $\frac{T^2}{N}$ where N is total number of observations.

Total sum of squares =

$$SST = \sum X_1^2 + \sum X_2^2 + \dots\dots\dots + \sum X_K^2 - \frac{T^2}{N}$$

$$\left.\begin{array}{r}\text{Sum of Squares}\\ \text{between the samples}\end{array}\right\} = SSB = \frac{(\sum x_1)^2}{n_1} + \frac{(\sum x_2)^2}{n_2} + \ldots\ldots\ldots + \frac{(\sum x_K)^2}{n_K} - \frac{T^2}{N}$$

$$\left.\begin{array}{r}\text{Sum of Squares}\\ \text{with in the samples}\end{array}\right\} = SSW = SST - SSB$$

$$V_1 = K - 1 \; ; \; V_2 = N - K \text{ degrees of freedom}$$

$$\left.\begin{array}{r}\text{Mean Sum of squares}\\ \text{Between the sample}\end{array}\right\} = MSB = \frac{SSB}{V_1}$$

$$\left.\begin{array}{r}\text{Mean sum of squares}\\ \text{with in the samples}\end{array}\right\} = MSW = \frac{SSW}{V_2}$$

Case (i)

Test statistic $F = \frac{MSB}{MSW}$, if MSB > MSW to find the table value of F at 5% level of significance for (V_1, V_2) degrees of freedom.

If Calculated value of F is lessthan table value of F then H_0 is accepted at 5% level of significance, otherwise H_0 is rejected.

Case (ii)

Test statistic $F = \frac{MSW}{MSB}$, if MSW > MSB.

To find the table value of F at 5% level of significance for (V_2, V_1) degrees of freedom.

If calculated value of F is lessthan table value of F than H_0 is accepted at 5% level of significance, otherwise H_0 is rejected.

ANOVA Table for One Way Classification

Sources of Variation	Sum of Squares	Degrees of freedom	Mean sum of squares	Test Statistic
Between the sample	SSB	$V_1 = K - 1$	$MSB = \frac{SSB}{V_1}$	$F = \frac{MSB}{MSW}$
Within the samples	SSW	$V_2 = N - K$	$MSW = \frac{SSW}{V_2}$	or
Total	SST	$V_3 = N - 1$		$F = \frac{MSW}{MSB}$

Note

Coding method simplifies the calculations. The value of F being the variance ratio does not change if all the values are coded. i.e., are either multiplied or divided by a common factor, or if a common number is either subtracted from or added to each of the given values. By coding the given values, big figures are reduced in magnitude and work is simplified without altering the value of F.

Example 1

A manager of a Mercantile firm wishes to test whether its three salesman A, B , C tend to make sales of the same size or differ in their selling abilities. During a week there have been 14 sale calls. A made 5 calls, B made 4 Calls and C made 5 calls. Following are the sales data for the week of the three salesman.

A	500	400	700	800	600
B	300	700	400	600	-
C	500	300	500	400	300

Perform the analysis of variance and draw you conclusion

Solution

Let H_0 : the three salesmen tend to make sales of the same size

H_1 : A, B, C differ in their selling abilities

As the sales data have a common factor 100, we divide the given values by 100 and write the coded data and their squares in the following table.

Calculations for Analysis of Variance

	Salesman A		Salesman B		Salesman C	
	X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
	5	25	3	9	5	25
	4	16	7	49	3	9
	7	49	4	16	5	25
	8	64	6	36	4	16
	6	36	-	-	3	9
Total	$\Sigma X_1 = 30$	$\Sigma X_1^2 = 190$	$\Sigma X_2 = 20$	$\Sigma X_2^2 = 110$	$\Sigma X_3 = 20$	$\Sigma X_3^2 = 84$

$$T = \sum X_1 + \sum X_2 + \sum X_3$$

$$= 30 + 20 + 20 = 70$$

$N = 5 + 4 + 5 = 14$

$$\text{Correction factor} = \frac{T^2}{N} = \frac{(70)^2}{14} = 350$$

$$SST = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$$

$$= 190 + 110 + 84 - 350 = 34$$

$$SSB = \frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_1} + \frac{(\sum X_3)^2}{n_1} - \frac{T^2}{N}$$

$$= \frac{(30)^2}{5} + \frac{(20)^2}{4} + \frac{(20)^2}{4} - 350 = 10$$

$SSW = SST - SSB = 34 - 10 = 24$

$V_1 = K - 1 = 3 - 1 = 2$

$V_2 = N - K = 14 - 3 = 11$

$V_3 = N - 1 = 14 - 1 = 13$

$$MSB = \frac{SSB}{V_1} = \frac{10}{2} = 5$$

$$MSW = \frac{SSW}{V_2} = \frac{24}{11} = 2.18$$

The table value of F at 5% level of significance for ($V_1 = 2$, $V_2 = 11$) degrees of freedom is 3.98

ANOVA Table

Source of Variation	Sum of Squares	d.f.	Mean sum of squares	Test Statistic
Bsetween the Samples	SSB = 10	$V_1 = 2$	MSB = 5	$F = \frac{MSB}{MSW}$
Within the samples	SSW = 24	$V_2 = 11$	MSW = 2.18	= 2.29
Total	SST = 34	$V_3 = 13$		

Here calculated value of F is lessthan table value of F at 5% level of significance

∴ H_0 is accepted

i.e., There is no difference in the sales ability of the salesmen A, B, C.

Example 2

The following data give the yields on 14 plots of land in three samples of 5, 5, 4 plots under three varieties of seeds A, B, C.

A	35	40	33	36	31
B	30	25	34	28	33
C	28	24	30	26	-

To construct the ANOVA table and use it to test whether difference in the average yields under the three varieties of seeds is significant or not.

Solution

H_0 : Three is no significant difference in average yields under the three varieties of seeds.

H_1 : The difference is significant

Subtracting 30 from each of the given value, the coded data are given in the following table.

Sample A		Sample B		Sample C	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
5	25	0	0	– 2	4
10	100	–5	25	– 6	36
3	9	4	16	0	0
6	36	–2	4	– 4	16
1	1	3	9	-	-
$\Sigma X_1 = 25$	$\Sigma X_1^2 = 171$	$\Sigma X_2 = 0$	$\Sigma X_2^2 = 54$	$\Sigma X_3 = -12$	$\Sigma X_3^2 = 56$

$$T = \sum X_1 + \sum X_2 + \sum X_3$$

$$= 25 + 0 - 12 = 13$$

$$N = 5 + 5 + 4 = 14$$

$$\text{Correction factor} = \frac{T^2}{N} = \frac{(13)^2}{14} = 12.07$$

$$SST = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$$

$$= 171 + 54 + 56 - 12.07 = 268.93$$

$$SSB = \frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \frac{(\sum X_3)^2}{n_3} - \frac{T^2}{N}$$

$$= \frac{(25)^2}{5} + \frac{(0)^2}{5} + \frac{(-12)^2}{4} - 12.07 = 148.93$$

$$SSW = SST - SSB = 268.93 - 148.93 = 120$$

$$V_1 = K - 1 = 3 - 1 = 2$$

$V_2 = N - K = 14 - 3 = 11$

$V_3 = N - 1 = 14 - 1 = 13$

$MSB = \frac{SSB}{V^1} = 74.465$

$MSW = \frac{SSW}{V_2} = 10.9$

ANOVA Table

Source of Variation	Sum of Squares	d.f.	Mean sum of squares	Test Statistic
Bsetween the Samples	SSB = 148.93	$V_1 = 2$	MSB = 74.465	$F = \frac{MSB}{MSW}$
Within the samples	SSW = 120.00	$V_2 = 11$	MSW = 10.900	= 6.83
Total	SST = 268.93	$V_3 = 13$		

Table value of F at 5% level of significance for ($V_1 = 2$, $V_2 = 11$) degrees of freedom = 3.98

Here the calculated value of F is greater than the table value of F

$\therefore H_0$ is rejected.

i.e., There is a significant difference in the average yields under three varieties of seeds.

Exercises

1. There are 3 typists working in an office. The times [in minutes] they spend for the tea-break in addition to the allowed lunch break are observed and noted below.

A	25	18	30	32	35	37	19	-	-	-
B	24	22	26	28	30	32	28	26	-	-
C	28	20	27	19	29	35	30	23	27	32

Can the difference in average times that the 3 typists spend for the tea break be attributed to chance variation

The three samples given below have been obtained from three normal populations with equal variances. Test the hypothesis that the population means are equal at 5% level of significance.

Sample 1	6	8	5	12	9
Sample 2	5	3	8	7	7
Sample 2	10	7	11	10	12

3. The three samples below have been obtained from normal population with equal variances. Test the hypothesis at 5% level that the population means are equal.

Sample A	8	10	7	14	11
Sample B	7	5	10	9	9
Sample C	12	9	13	12	14

Two Way Classification (RBD)

[Randomized Block Design]

In this case, observations (or data) are classified according to two different factors or criteria.

Here SST = SSC + SSR + SSE

where SSC - The sum of squares between the columns

SSR - The sum of squares between the rows

and SSE - The sum of squares due to errors

If c be the number of columns and r be the number of rows

$V_1 = c - 1$; $V_2 = r - 1$; $V_3 = (c - 1)(r - 1)$; $V_4 = N - 1$

ANOVA table for two-way classification is shown below

Source of variation	Sum of Squares	Degrees of freedom	Mean sum of squares	Test Statistic
Between columns	SSC	$V_1 = c - 1$	$MSC = \frac{SSC}{V_1}$	$F_1 = \frac{MSC}{MSE}$
Between Rows	SSR	$V_2 = r - 1$	$MSR = \frac{SSR}{V_2}$	$F_2 = \frac{MSR}{MSE}$
Residual (or) Error	SSE	$V_3 = (c - 1)(r - 1)$	$MSE = \frac{SSE}{V_3}$	-
Total	SST	$V_4 = N - 1$	-	-

If the calculated value of F_1 is less than table value of F at 5% level of significance for (V_1, V_3) degrees of freedom and the calculated value of F_2 is lessthan table value of F at 5% levels of significance for (V_2, V_3) degrees of freedom then H_0 is accepted.

Example 1

To study the performance of three detergents and three different water temperatures the following whiteness readings were obtained with specially designed equipment:

Water Temperature	Detergent A	Detergent B	Detergent C
Cold Water	57	55	67
Warm Water	49	52	68
Hot Water	54	46	58

Perform a two way analysis of variance using 5% level of significance.

Solution

Let H_0 : There is no significant difference in whiteness due to the three varieties of Detergents / three different water temperatures.

H_1 : There is a significant difference.

Subtracting 50 from all the data. we get

Calculations for ANOVA

	Detergent A	Detergent B	Detergent C	Row Total
Cold Water	7	5	17	29
Warm Water	−1	2	18	19
Hot Water	4	−4	8	8
Column Total	10	3	43	T = 56

$$\text{Correction factor} = \frac{T^2}{N} = \frac{(56)^2}{9} = 348.44$$

$$SSC = \frac{(10)^2}{3} + \frac{(3)^2}{3} + \frac{(43)^2}{3} - \frac{T^2}{N} = 304.22$$

$$V_1 = c - 1 = 3 - 1 = 2 \quad MSC = \frac{SSC}{V_1} = 152.11$$

$$SSR = \frac{(29)^2}{3} + \frac{(19)^2}{3} + \frac{(8)^2}{3} = \frac{T^2}{N} = 73.55$$

$$V_2 = r - 1 = 3 - 1 = 2 \quad MSR = \frac{SSR}{V_1} = 36.78$$

$$SST = 7^2 + (-1)^2 + 4^2 + 5^2 + 2^2 + (-4)^2 + (17)^2 + (18)^2 + 8^2 - \frac{T^2}{N}$$

$$= 788 - 348.44$$

$$= 439.56$$

$$\therefore SSE = SST - SSC - SSR$$

$$= 439.56 - 304.22 - 73.55 = 61.79$$

$$V_3 = (c-1)(r-1) = 2 \times 2 = 4$$

$$MSE = \frac{SSE}{V_3} = 15.45$$

The ANOVA table of the given data is shown below :-

Source of variation	Sum of Squares	Degrees of freedom	Mean sum of squares	Test Statistic
Between columns	SSC = 304.22	$V_1 = 2$	$MSC = \frac{SSC}{V_1}$ = 152.11	$F_1 = \frac{MSC}{MSE}$ = 9.85
Between Rows	SSR = 73.55	$V_2 = 2$	$MSR = \frac{SSR}{V_2}$ = 36.78	$F_2 = \frac{MSR}{MSE}$ = 2.38
Residual (or) Error	SSE = 61.79	$V_3 = 4$	$MSE = \frac{SSE}{V_3}$ = 15.45	-
Total	SST = 439.56	$V_4 = 8$	-	-

The table value of F at 5% level of significance for

$(V_1 = 2, V_3 = 4)$ d.o.f. = 6.94

and $(V_2 = 2, V_3 = 4)$ d.o.f = 6.94

Here the calculated value of F_1 is greater than the table value of F.

∴ There is significant difference in whiteness due to the three varieties of detergents.

Also the calculated value of F_2 is lessthan the table value of F.

∴ There is no significant difference in whiteness due to the three different temperatures.

Example 2

The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machines.

Workers	Machine Type			
	A	B	C	D
1	44	38	47	36
2	46	40	52	43
3	34	36	44	32
4	43	38	46	33
5	38	42	49	39

a. Test whether the mean productivity is the same for the different machine types.

b .Test whether the 5 men differ respect to mean productivity

Solution

Let H_0 : The mean productivity is the same for the different machines different mean

H_1 : The mean productivity will differ

subtracting 40 from all the data. then the given table becomes.

Workers	Machines				
	A	B	C	D	Row Total
1	4	–2	7	–4	5
2	6	0	12	3	21
3	–6	–4	4	–8	–14
4	3	–2	6	–7	0
5	–2	2	9	–1	8
Column Total	5	–6	38	–17	20

$$\text{Correction factor} = \frac{T^2}{N} = \frac{(20)^2}{20} = 20$$

$$\text{SSC (Machines)} = \frac{(5)^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - \frac{T^2}{N}$$

$$= \frac{1}{5}[25 + 36 + 1444 + 289] - 20$$

$$= 358.8 - 20 = 338.8$$

$$V_1 = c - 1 = 4 - 1 = 3$$

$$\text{SSR (workers)} = \frac{(5)^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - \frac{T^2}{N}$$

$$= \frac{1}{4}[25 + 441 + 196 + 0 + 64] - 20$$

$$= 181.5 - 20 = 161.5$$

$$V_2 = r - 1 = 5 - 1 = 4$$

$$\text{SST} = \begin{bmatrix} 4^2 + (-2)^2 + 7^2 + (-4)^2 + 6^2 + 0^2 + 12^2 \\ +3^2 + (-6)^2 + (-4)^2 + 4^2 + (-8)^2 + 3^2 + (-2)^2 \\ + 6^2 + (-7)^2 + (-2)^2 + 2^2 + 9^2 + (-1)^2 \end{bmatrix} - \frac{T^2}{2}$$

$$= 594 - 20$$

$$= 574$$

$$V_3 = (c - 1)(r - 1) = 3 \times 4 = 12$$

$$V_4 = N - 1 = 19$$

$$\therefore \quad \text{SSE} = \text{SST} - [\text{SSC} + \text{SSR}]$$

$$= 574 - 338.8 - 161.5 = 73.7$$

Two Way - ANOVA Table

Source of variation	Sum of Squares	Degrees of freedom	Mean sum of squares	Test Statistic
Between columns (Machine)	SSC = 338.8	$V_1 = 3$	$MSC = \frac{SSC}{V_1}$ = 112.933	$F_1 = \frac{MSC}{MSE}$ = 18.387
Between Rows (Workers)	SSR = 161.5	$V_2 = 4$	$MSR = \frac{SSR}{V_2}$ = 40.375	$F_2 = \frac{MSR}{MSF}$ = 6.574
Residnal (or) Error	SSE = 73.7	$V_3 = 12$	$MSE = \frac{SSE}{V_3}$ = 6.142	-
Total	SST = 594	$V_4 = 19$	-	-

Conclusion

a. for d.f. = (V_1 = 3, V_3 = 12), the value of F at 5% l.o.s. = 3.49

Here calculated value of F_1 is greater than table value of F.

b. for d.f. = (V_2 = 4, V_3 = 12), the value of F at 5% l.o.s. = 3.26

Here calculated value of F_2 is greater than table value of F.

∴ The mean productivity is not the same for the four different types of machines and five different workers.

Exercise

1. The following data represent the number of units of production per day turned out by four randomly chosen operators using three milling machines.

Operators	Machines		
	M_1	M_2	M_3
1	150	151	156
2	147	159	155
3	141	146	153
4	154	152	159

Perform ANOVA and test the hypothesis that

a. The machines are not significantly different

b. The operators are not Significantly different

2. Set up a two-way ANOVA table for the data given below

Pieces of Field	Treatment			
	A	B	C	D
P	45	40	38	37
Q	43	41	45	38
R	39	39	41	41

3. Performa a two-way ANOVA table on the data given below

Treatment II	Treatment I		
	i	ii	iii
A	30	26	38
B	24	29	28
C	33	24	35
D	36	31	30
E	27	35	33

Answers

1. Accepted for both (a) & (b)
2. Subtracting 40 form all the data, we get
   SSC = 22.92 ; SSR 8.17 ; SSE = 45.83 ; SST = 76.92
   MSC = 7.64 ; MSR = 4.08 ; MSE = 7.64
3. Subtracting 30 from all the data we get
   SSC = 38.80 ; SSR 52.93 ; SSE = 173.87 ; SST = 265.60
   MSC = 19.40 ; MSR = 13.23 ; MSE = 21.73

Three Way Classification of ANOVA

(Latin Square Design)

We consider an agricultural experiment, in which n^2 plots are taken and arranged in the form of an nxn square, such that the plots in each row will be homogeneous as for as possible with respect to one factor of classification, say, soil fertility and plots in each column will be homogeneous as for as possible with respect to another faith classification say seed quality.

The n treatment are given to these plots such that each treatment occurs only once in each row and only once in each column. The various possible arrangements obtained in this manner are known as Latin squares of order n. This design of experiment is called the Latin Square Design.

The ANOVA table for the Three factors of Classification

Source of variation	Sum of Squares	Degrees of freedom	Mean sum of squares	Test Statistic
Between columns	SSC	$n-1$	$MSC = \frac{SSC}{n-1}$	$F_1 = \frac{MSC}{MSE}$
Between Rows	SSR	$n-1$	$MSR = \frac{SSR}{n-1}$	$F_2 = \frac{MSR}{MSE}$
Between Letters	SSL	$n-1$	$MSL = \frac{SSL}{n-1}$	$F_3 = \frac{MSL}{MSE}$
Residnal (or) Error	SSE	$(n-1)(n-2)$	$MSE = \frac{SSE}{(n-1)(n-2)}$	-
Total	SST	n^2-1	-	-

Where $SST = \sum_i \sum_j x_{ji}^2 - \frac{T^2}{N}$

$$SSC = \frac{1}{n}\sum_i C_i^2 - \frac{T^2}{N}$$

$$SSR = \frac{1}{n}\sum_j R_j^2 - \frac{T^2}{N}$$

$$SSL = \frac{1}{n}\sum_k L_k^2 - \frac{T^2}{N}$$

$$SSE = SST - SSC - SSR - SSL$$

Comparison of LSD and RBD

i. In LSD, the number of rows and columns are equal and hence the number of replications is equal to the number of treatments. There is no such restriction in R.B.D.

ii. LSD is suitable for the case when the number of treatment is between 5 and 12, where as RBD can be used for any number of treatments and replication.

iii. The main advantage of LSD is that it removes the variations between rows and columns from that within the rows resulting in the reduction of experiment error to a large extent.

iv. The RBD can performed equally on rectangular of square plots but for LSD, a more or less a square field is required. Due to the condition (iii) LSD is preferred over RBD.

Note

A 2 × 2 LSD is not possible. The degree of freedom for error in a mxm LSD is $(m - 1)(m - 2)$.

For m = 2, we get the d.o.f of error is zero hence comparisons are not possible.

Example 1

Analyse the variance in the following LSD of yields (in kgs) of paddy where A, B, C, D denote the different methods of cultivation

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields.

Solution

Let H_0 : There is no significant difference between the methods of Cultivation

H_1 : There is a significant difference.

Subtracting 120 form the given values and work out with the new value of x_{ij}

i→ j↓	1	2	3	4	Row total
1	D2	A1	C3	B2	8
2	B4	C3	A2	D5	14
3	A0	B(−1)	D0	C1	0
4	C2	D3	B1	A2	8
Column total	8	6	6	10	T=30

$$\text{Correction factor} = \frac{T^2}{N} = \frac{(30)^2}{16} = 56.25$$

$$\text{SSC} = \frac{1}{n}\sum_i C_i^2 - \frac{T^2}{N}$$

$$= \frac{1}{4}(8^2 + 6^2 + 6^2 + 10^2) - 56.25$$

$$= 59 - 56.25 = 2.75$$

$$\text{SSR} = \frac{1}{n}\sum_j R_j^2 - \frac{T^2}{N}$$

$$= \frac{1}{4}(8^2 + 14^2 + 0^2 + 8^2) - 56.25$$

$$= 81 - 56.25 = 24.75$$

$$\text{SST} = \sum_i\sum_j x_{ij}^2 - \frac{T^2}{N}$$

$$= \left[\begin{array}{l} 2^2 + 1^2 + 3^2 + 2^2 + 4^2 + 3^2 \\ + 2^2 + 5^2 + 0^2 + (-1)^2 + 0^2 + 1^2 \\ + 2^2 + 3^2 + 1^2 + 2^2 \end{array}\right] - 56.25$$

$$= 35.75$$

Rearranging the data according to letters

Letter					Total
A	1	2	0	2	5
B	2	4	−1	1	6
C	3	3	1	2	9
D	2	5	0	3	10

$$SSL = \frac{1}{n}\sum_{k} L_k^2 - \frac{T^2}{N}$$

$$= \frac{1}{4}(5^2 + 6^2 + 9^2 + 10^2) - 56.25$$

$$= 4.25$$

$$SSE = SST - SSC - SSR - SSL$$

$$= 35.75 - 2.75 - 24.75 - 4.24$$

$$= 4.0$$

ANOVA Table

Source of variation	Sum of Squares	Degrees of freedom	Mean sum of squares	Test Statistic
Between columns	SSC = 2.75	$V_1 = n-1$ = 3	$MSC = \frac{SSC}{n-1}$ = 0.92	
Between Rows	SSR = 24.75	$V_2 = n-1$ = 3	$MSR = \frac{SSR}{n-1}$ = 8.25	
Between Letters	SSL = 4.25	$V_3 = n-1$ = 3	$MSL = \frac{SSL}{n-1}$ = 1.42	$F_3 = \frac{MSL}{MSE}$ = 2.12
Residnal (or) Error	SSE = 4.00	$V_4 = (n-1)(n-2) = 6$	$MSE = \frac{SSE}{(n-1)(n-2)}$ = 0.67	
Total	SST = 35.75	$V_5 = n^2 - 1$ = 15		

From the F tables $F_{5\%}(V_3 = 3, V_4 = 6) = 4.76$

Calculated value of F_3 is less than table value of F

$\therefore$ There is no significant difference between the methods of Cultivation.