Fourier Transforms Cheat Sheet (Theory Only)

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1. Fourier Transform & Its Inverse

Fourier Transform

Converts a **time-domain function** into its corresponding **frequency-domain representation**.
Useful in **signal processing, image processing, physics, and engineering**.
Helps analyze signals by breaking them down into their **frequency components**.

Inverse Fourier Transform

Converts the **frequency-domain representation** back to the **time-domain function**.
Ensures that the original function can be **reconstructed** from its transformed version.

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2. Properties of Fourier Transform

1. Linearity Property

The Fourier Transform of a linear combination of functions is the same linear combination of their transforms.
Used in **superposition of signals**.

2. Time Shifting Property

Shifting a function in the time domain **introduces a phase shift** in the frequency domain.
Used in **modulation and communication systems**.

3. Frequency Shifting Property

A multiplication in the time domain corresponds to a shift in the frequency domain.
Used in **filtering and signal processing applications**.

4. Scaling (Dilation) Property

If a function is compressed in time, its Fourier Transform expands in frequency and vice versa.
Helps in **analyzing high-frequency signals and wavelets**.

5. Convolution Theorem

The Fourier Transform of the **convolution of two functions** is the **product of their Fourier Transforms**.
Used in **signal filtering and image processing**.

6. Differentiation Property

Differentiating a function in the time domain corresponds to **multiplication by a factor in the frequency domain**.
Helps in **solving differential equations** using Fourier methods.

7. Integration Property

The Fourier Transform of an integral of a function is related to its transform divided by frequency.
Useful in **solving integral equations and analyzing low-frequency components**.

8. Parseval's Theorem

The **total energy** of a function in the **time domain** is equal to the total energy in the **frequency domain**.
Used in **signal energy calculations**.

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3. Fourier Sine & Cosine Transforms

Fourier Sine Transform

Represents a function using only **sine terms**, which is useful for **odd functions**.
Used in solving **boundary value problems and heat equations**.

Fourier Cosine Transform

Represents a function using only **cosine terms**, which is useful for **even functions**.
Applied in **wave equations and symmetrical systems**.

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4. Properties of Fourier Sine & Cosine Transforms

1. Linearity Property

The sine and cosine transforms of a sum of functions are the sum of their individual transforms.

2. Shifting Property

A shift in the time domain introduces a **phase shift** in the sine/cosine transform.

3. Scaling Property

Compressing a function in time expands its Fourier transform in frequency.

4. Convolution Property

The Fourier sine/cosine transform of a convolution is the **product of individual transforms**.

This **Fourier Transform Cheat Sheet** covers **Fourier Transforms, its inverse, properties, Fourier sine & cosine transforms, and their properties** without proofs. Let me know if you need more explanations!