UNIT - I: Fourier Series

Author : Mathematics Department
Date : 2025

Dirichlet's Conditions

A function f(x) can be represented by a Fourier series if it satisfies Dirichlet's conditions:
• f(x) is periodic.
• f(x) has a finite number of discontinuities.
• f(x) has a finite number of maxima and minima.
• The integral of |f(x)| over a period is finite.

General Fourier Series

A periodic function f(x) can be represented as:
[f(x) = a0 + \sum{n=1}^{\infty} (an \cos n\omega x + bn \sin n\omega x)]
where:
• (a0) is the average value.
• (an) and (b_n) are Fourier coefficients.

Odd and Even Functions

• Even function: f(-x) = f(x) (contains only cosine terms).
• Odd function: f(-x) = -f(x) (contains only sine terms).

Half-Range Fourier Series

• Half-range sine series: Represents odd extensions.
• Half-range cosine series: Represents even extensions.
• Used when the function is defined only on half the interval.

Change of Interval

To apply Fourier series to functions defined on different intervals, the function is scaled appropriately using:
[x' = \frac{2\pi}{b-a} (x - a)]

Parseval's Identity

Parseval's theorem states that the sum of the square of a function equals the sum of the squares of its Fourier coefficients:
[\int{-\pi}^{\pi} |f(x)|^2 dx = a0^2 + \sum{n=1}^{\infty} (an^2 + b_n^2)]