Multiple Integrals – Cheat Sheet

1. Double Integrals

Definition:

The double integral of a function f(x,y)f(x, y)f(x,y) over a region RRR is:
Rf(x,y) dx dy\iint_R f(x, y) \,dx\,dy Rf(x,y)dxdy

Evaluation:

1st
2nd**Iterated Integral (Cartesian Form):**
3rd∫ab∫cdf(x,y) dy dx\int_{a}^{b} \int_{c}^{d} f(x, y) \,dy\,dx∫ab∫cdf(x,y)dydx

- Integrate **inner integral first** (fix outer variable).
- Then evaluate the **outer integral**.
-

4th **Limits Based on Region:**

-
- **Rectangular Region:** Limits are constants.
- **General Region:** Limits depend on one variable.
-

---

2. Change of Order of Integration (Cartesian Form)

When to Change Order?

- When the given order is difficult to evaluate.
- Convert ∫ab∫y1(x)y2(x)f(x,y) dy dx\int_{a}^{b} \int_{y_1(x)}^{y_2(x)} f(x, y) \,dy\,dx ∫ab∫y1(x)y2(x)f(x,y)dydx into
∫cd∫x1(y)x2(y)f(x,y) dx dy\int_{c}^{d} \int_{x_1(y)}^{x_2(y)} f(x, y) \,dx\,dy∫cd∫x1(y)x2(y)f(x,y)dxdy.

Steps to Change Order:

1st Identify **region RRR** from given limits.
2ndSketch the **integration region**.
3rdRewrite limits in terms of the other order.
4th Solve the new integral.

---

3. Applications of Double Integrals

Area of a Region (Cartesian Form):

A= RdA= R1 dx dyA = \iint_R dA = \iint_R 1 \,dx\,dyA= RdA= R1dxdy

- Set f(x,y)=1f(x, y) = 1f(x,y)=1 and integrate over region RRR.

---

4. Triple Integrals

Definition:

The **triple integral** of f(x,y,z)f(x, y, z)f(x,y,z) over a volume VVV is:

Vf(x,y,z) dx dy dz\iiint_V f(x, y, z) \,dx\,dy\,dz   Vf(x,y,z)dxdydz

Evaluation:

1st **Iterated Form (Cartesian Coordinates):** ∫ab∫cd∫eff(x,y,z) dz dy dx\int_{a}^{b} \int_{c}^{d} \int_{e}^{f} f(x, y, z) \,dz\,dy\,dx∫ab∫cd∫eff(x,y,z)dzdydx
2nd**Follow order of integration:**

- Integrate the **innermost** integral first.
- Then evaluate the **middle** integral.
- Finally, evaluate the **outer** integral.

---

5. Applications of Triple Integrals

Volume of a Region (Cartesian Form):

V=   VdV=   V1 dx dy dzV = \iiint_V dV = \iiint_V 1 \,dx\,dy\,dzV=   VdV=   V1dxdydz

- Set f(x,y,z)=1f(x, y, z) = 1f(x,y,z)=1 and integrate over volume VVV.

Quick Summary Table

Topic	Key Formula/Steps
Double Integral	Rf(x,y) dx dy\iint_R f(x, y) \,dx\,dy Rf(x,y)dxdy
Change of Order	Sketch region, rewrite limits, swap order.
Area using Double Integral	A= R1 dx dyA = \iint_R 1 \,dx\,dyA= R1dxdy
Triple Integral	Vf(x,y,z) dx dy dz\iiint_V f(x, y, z) \,dx\,dy\,dz V f(x,y,z)dxdydz
Volume using Triple Integral	V= V1 dx dy dzV = \iiint_V 1 \,dx\,dy\,dzV= V 1dxdydz