Fourier Series & Dirichlet's Conditions – Cheat Sheet (Theory Only)

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1. Dirichlet's Conditions for Fourier Series

A function can be represented by a **Fourier series** if it satisfies the following conditions:

Periodicity → The function must be **periodic** over a certain interval.
Single-valued & Finite → The function should have a **finite value** everywhere.
Finite Discontinuities → The function can have **a finite number of discontinuities** in one period.
Finite Number of Maxima & Minima → The function must have **a limited number of peaks and troughs** in a given interval.

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2. General Fourier Series

- A **Fourier series** represents a **periodic function** as an **infinite sum of sine and cosine terms**.
- It provides a way to analyze and express complex waveforms using **simpler trigonometric components**.
- It is used in **signal processing, electrical engineering, and physics**.

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3. Odd & Even Functions in Fourier Series

Odd Functions

Symmetry about the origin → $f(-x)=-f(x)f(-x) = -f(x)$.
Contains **only sine terms** in its Fourier series expansion.
Example: **Sine wave, x^3, x^5, etc.**

Even Functions

Symmetry about the y-axis → $f(-x)=f(x)f(-x) = f(x)$.
Contains **only cosine terms** in its Fourier series expansion.
Example: **Cosine wave, x^2, x^4, etc.**

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4. Half-Range Fourier Series

Half-Range Sine Series

Used when a function is **defined only over half of its normal range**.
Expands the function using **sine terms only**, making it **odd**.
Useful in problems with **antisymmetric behavior**.

Half-Range Cosine Series

Expands the function using **cosine terms only**, making it **even**.
Useful when the given function is **naturally symmetric about the y-axis**.

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5. Change of Interval in Fourier Series

A function originally defined over one interval **can be transformed** to another interval while maintaining its Fourier representation.
This is useful for **scaling and normalizing** problems across different domains.
Applied in **engineering problems, physics wave equations, and heat conduction analysis**.

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6. Parseval’s Identity

Parseval’s theorem states that the **total energy (or power) of a function** in the **time domain** is equal to the sum of the **squares of its Fourier coefficients** in the **frequency domain**.

It is widely used in **signal processing, electrical circuits, and communication systems** to analyze energy distribution.

This **Fourier Series Cheat Sheet** covers **Dirichlet’s conditions, general Fourier series, odd/even functions, half-range expansions, change of interval, and Parseval’s identity** without formulas. Let me know if you need more explanations!