

# Matrices - Important Topics

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## Rank of a Matrix

The rank of a matrix is the maximum number of linearly independent row or column vectors in the matrix. It determines the dimension of the column space.  
Formula:  $\text{Rank}(A) = \text{number of nonzero rows in its row echelon form.}$

## Systems of Linear Equations

A system of linear equations consists of multiple linear equations involving the same set of variables.

Solution methods: Gaussian elimination, Cramer's Rule, Matrix inversion method.

## Characteristic Equation

The characteristic equation of a square matrix  $A$  is given by  $\det(A - \lambda I) = 0$ , where  $\lambda$  represents the eigenvalues of  $A$ .

## Cayley-Hamilton Theorem

This theorem states that every square matrix satisfies its own characteristic equation.

## Eigenvalues and Eigenvectors

For a square matrix  $A$ , an eigenvector  $v$  and its corresponding eigenvalue  $\lambda$  satisfy  $Av = \lambda v$ .

## Diagonalization of Matrices

A matrix  $A$  is diagonalizable if there exists an invertible matrix  $P$  such that  $P^{-1}AP$  is a diagonal matrix.

