Vector Calculus Cheat Sheet

1. Gradient, Divergence, and Curl

Gradient (f\nabla f)

- The gradient of a scalar function f(x,y,z)f(x, y, z) is given by: f=(∂f∂x,∂f∂y,∂f∂z)\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)
- Represents the direction and rate of the fastest increase of ff.

Divergence (F\nabla \cdot \mathbf{F})

- The divergence of a vector field F=(P,Q,R)\mathbf{F} = (P, Q, R) is given by: F=∂P∂x+∂Q∂y+∂R∂z\nabla \cdot \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}
- Measures the rate at which a vector field spreads out from a point.

Curl (×F\nabla \times \mathbf{F})

- The curl of a vector field F=(P,Q,R)\mathbf{F} = (P, Q, R) is given by: ×F=(∂R∂y−∂Q∂z,∂P∂z−∂R∂x,∂Q∂x−∂P∂y)\nabla \times \mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)
- Represents the rotational tendency of a vector field.

2. Directional Derivatives

- The directional derivative of a scalar function f(x,y,z)f(x, y, z) in the direction of a unit vector u\mathbf{u} is given by: Duf= f uD_\mathbf{u} f = \nabla f \cdot \mathbf{u}
- Measures the rate of change of ff in the direction of u\mathbf{u}.

3. Irrotational and Solenoidal Vector Fields

Irrotational Vector Field

- A vector field F\mathbf{F} is irrotational if ×F=0\nabla \times \mathbf{F} = 0.
- Implies the existence of a scalar potential function ϕ\phi such that F= ϕ\mathbf{F} = \nabla \phi.

Solenoidal Vector Field

- A vector field F\mathbf{F} is solenoidal if F=0\nabla \cdot \mathbf{F} = 0.
- Implies that the field has no divergence (incompressible flow).

4. Theorems

Gauss Divergence Theorem

- Relates the flux of a vector field through a closed surface to the volume integral of its divergence. SF dS=∫V(F)dV\oint_S \mathbf{F} \cdot d\mathbf{S} = \int_V (\nabla \cdot \mathbf{F}) dV
- Converts a surface integral into a volume integral.

Stokes' Theorem

- Relates the circulation of a vector field around a closed curve to the surface integral of its curl. CF dr=∫S(×F) dS\oint_C \mathbf{F} \cdot d\mathbf{r} = \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}
- Converts a line integral into a surface integral.

Properties (Statements Only)

- Gradient, divergence, and curl operators are linear.

- The curl of a gradient field is always zero: × f=0\nabla \times \nabla f = 0.
- The divergence of a curl field is always zero: (×F)=0\nabla \cdot (\nabla \times \mathbf{F}) = 0.
- Gauss and Stokes' theorems provide fundamental connections between different types of integrals in vector calculus.

This cheat sheet provides a quick reference for key vector calculus concepts.