

UNIT - IV: Inverse Laplace Transforms

Author : Mathematics Department

Date : 2025

Definition of Inverse Laplace Transform

The Inverse Laplace Transform retrieves the time-domain function from the s-domain:
[$f(t) = \mathcal{L}^{-1} [F(s)]$]

Common Inverse Laplace Transforms

- $(\mathcal{L}^{-1} \left(\frac{1}{s} \right) = 1)$
- $(\mathcal{L}^{-1} \left(\frac{1}{s^2} \right) = t)$
- $(\mathcal{L}^{-1} \left(\frac{s}{s^2 + a^2} \right) = \cos(at))$
- $(\mathcal{L}^{-1} \left(\frac{a}{s^2 + a^2} \right) = \sin(at))$

Convolution Theorem

The convolution theorem states:

$$[\mathcal{L}^{-1} [F(s) G(s)] = (f * g)(t) = \int_0^t f(\tau) g(t-\tau) d\tau]$$

This is useful in solving differential equations.

Solutions of Linear Ordinary Differential Equations

Laplace transforms are useful for solving second-order linear ODEs:

- Convert the ODE into algebraic equation using Laplace transform.
- Solve for $F(s)$.
- Apply inverse Laplace transform to obtain $f(t)$.

