UNIT - III: Laplace Transforms

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Laplace Transform Definition

The Laplace Transform converts a time-domain function into an s-domain function:

$$F(s) = \int_{0}^{\infty} e^{-st} f(t) dt$$

Laplace Transform of Elementary Functions

- $L(1) = \frac{1}{s}$
- $L(t) = \frac{1}{s^2}$
- $L(e^{at}) = \frac{1}{s-a}$
- $L(\sin at) = \frac{a}{s^2 + a^2}$
- $L(\cos at) = \frac{s}{s^2 + a^2}$

Laplace Transform of Periodic Functions

For a periodic function f(t) with period T:

$$L(f(t)) = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$

Basic Properties of Laplace Transform

- Linearity: $L(a f(t) + b g(t)) = a L(f) + b L(g)$
- First shifting: $L(e^{at} f(t)) = F(s-a)$
- Differentiation: $L(f') = sF(s) - f(0)$
- Integration: $L \left(\int_0^t f(u) du \right) = \frac{F(s)}{s}$

Laplace Transform of Derivatives and Integrals

- $L(f'(t)) = sF(s) - f(0)$
- $L(f''(t)) = s^2F(s) - sf(0) - f'(0)$
- $L \left(\int f(t) dt \right) = \frac{F(s)}{s}$

Initial and Final Value Theorems

- Initial Value Theorem:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

- Final Value Theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$