Inverse Laplace Transforms & Differential Equations – Cheat Sheet (Theory Only)

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1. Definition of Inverse Laplace Transform

The **Inverse Laplace Transform** converts a function from the **Laplace domain (frequency domain)** back to the **time domain**.
Helps solve **differential equations, control system problems, and signal processing applications**.
Used to **find original time-domain functions** from their Laplace-transformed counterparts.

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2. Convolution Theorem (Without Proof)

The **Laplace transform of the convolution of two functions** is the **product of their individual Laplace transforms**.
Helps solve **integral equations** and analyze **system responses**.
Used in **engineering, physics, and signal processing applications**.

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3. Solutions of Linear Ordinary Differential Equations (ODEs)

Definition of Second-Order ODE with Constant Coefficients

A second-order **linear ordinary differential equation (ODE)** has the general form:

- **$a\, d^2y/dt^2 + b\, dy/dt + c\, y = f(t)$**
  Where:
- **a, b, c** are constants.
- **y(t)** is the unknown function.
- **f(t)** is a given function (forcing function).
  Used in **mechanical vibrations, electrical circuits, and control systems**.

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4. Solving Second-Order ODEs Using Laplace Transforms

Step 1: **Apply Laplace Transform** to the given equation.
Step 2: **Use Laplace properties** (differentiation property for derivatives).
Step 3: **Convert the equation into an algebraic equation** in terms of the Laplace variable.
Step 4: **Solve for Y(s)** (Laplace-transformed solution).
Step 5: **Apply the Inverse Laplace Transform** to obtain the time-domain solution.

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5. Applications of Laplace Transform in ODE Solutions

Mechanical Systems → Spring-mass-damper systems, oscillations.
Electrical Circuits → RLC circuits, step response analysis.
Control Systems → Stability analysis and system response.

This **Inverse Laplace Transform Cheat Sheet** covers **definition, convolution theorem (without proof), and solving second-order ODEs with constant coefficients** using Laplace Transforms. Let me know if you need further explanations!