UNIT-1
DC CIRCUITS

1.1 Electrical Circuit Elements

- Resistors, Inductors, Capacitors
- Definition for Voltage, Current, Power & Energy

1.2 Electric circuit laws

- Ohm’s law
- Kirchoff’s Voltage & Current law

1.3 Analysis of simple circuits with DC voltage

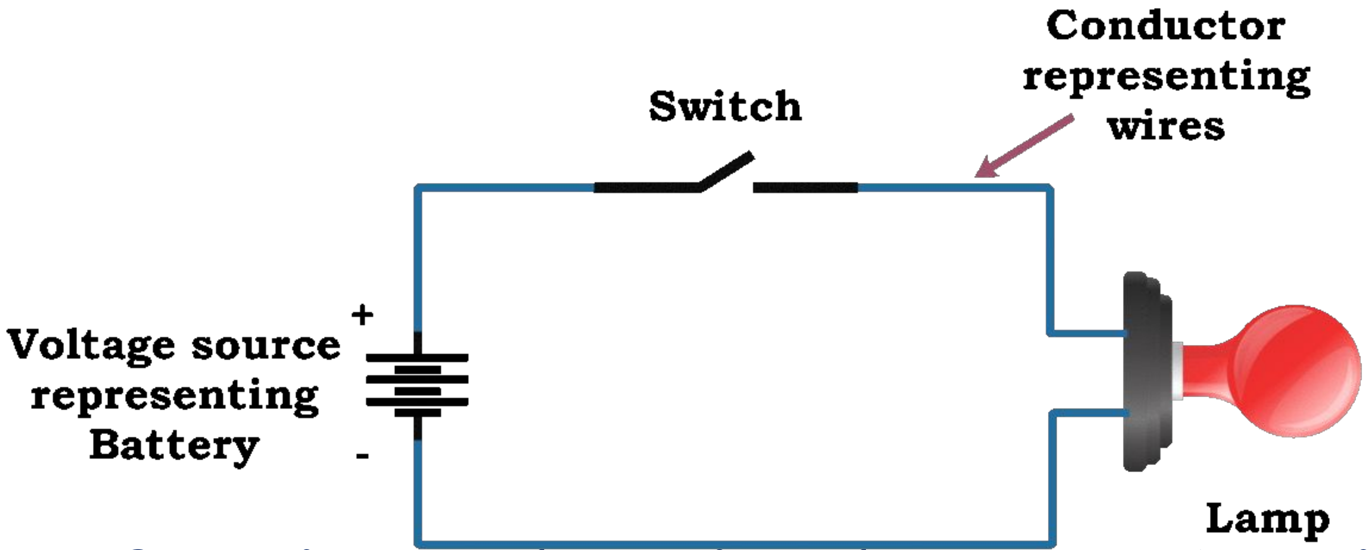
- Voltage and Current division in series and parallel circuits
- Star-Delta Conversion
- Node and Mesh method

1.4 Network Theorems

- Thevenin
- Norton
- Superposition
- Maximum Power Transfer

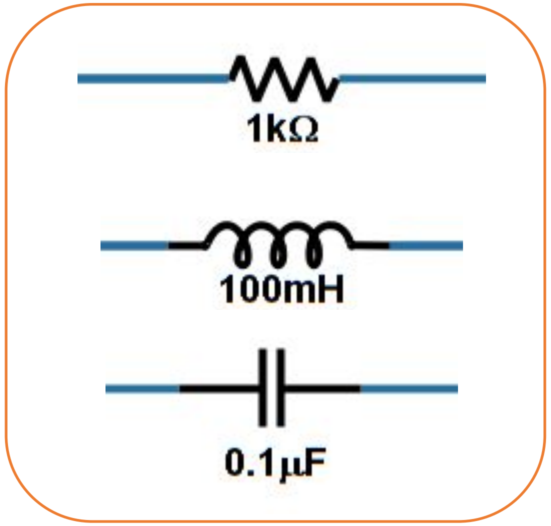
1.1 Electric Networks

- ***An electric circuit*** **is an interconnection of electrical elements.**

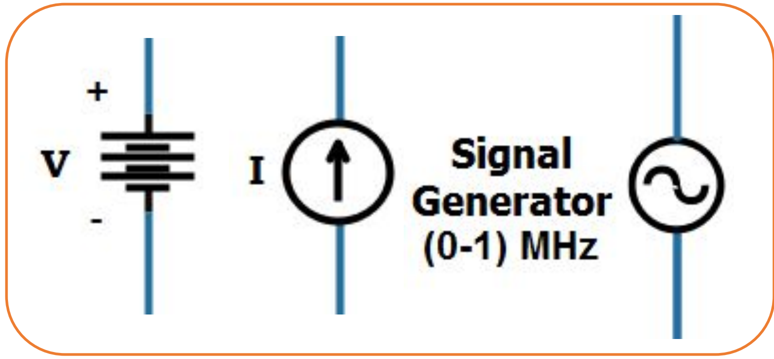


- **A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner is called** ***an electrical network.***

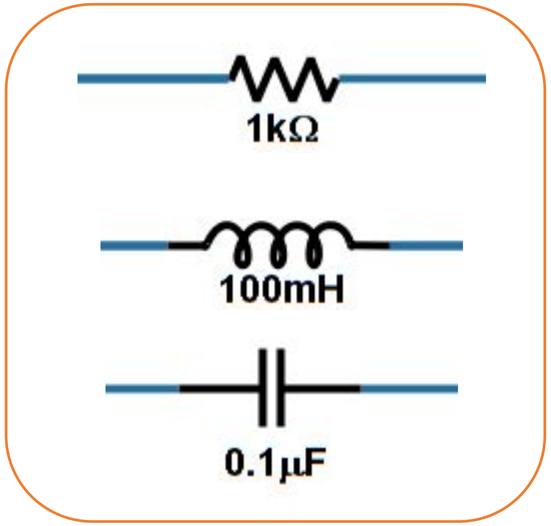
- The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called ***passive element.***



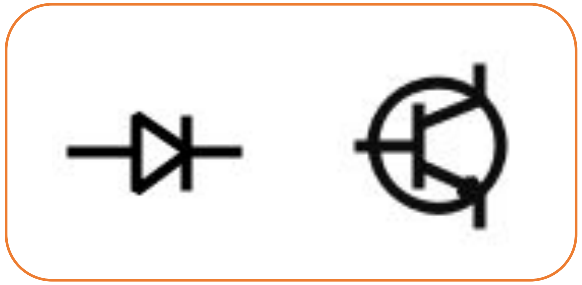
- The elements that supply energy to the circuit is called ***active element***. ***(Ex: voltage & current sources, generators, transistor etc. ~~Transformer~~)***



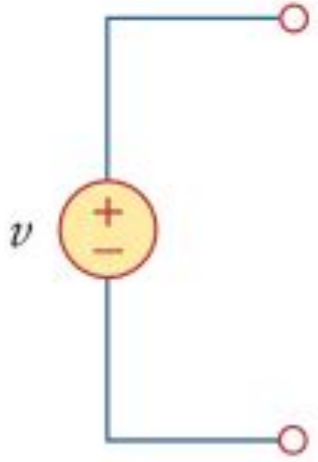
- **Conduction of current in both directions in an element with same magnitude is termed as *bilateral element*. *(ex: Resistance, Inductance, Capacitance)***

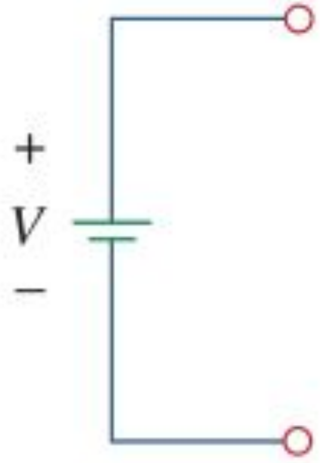


- **Conduction of current in one direction is termed as *unilateral element*. *(ex: Diode, Transistor)***

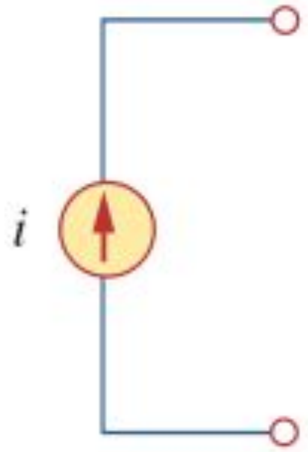


- ***An ideal independent source*** **is an active element that provides a specified voltage or current that is completely independent of other circuit elements.**



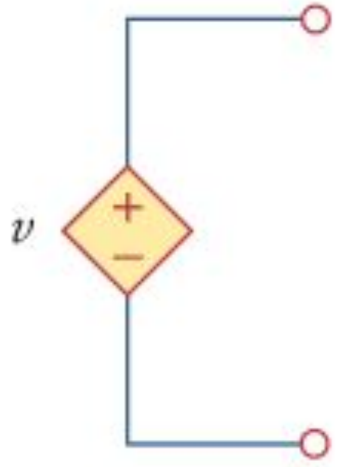


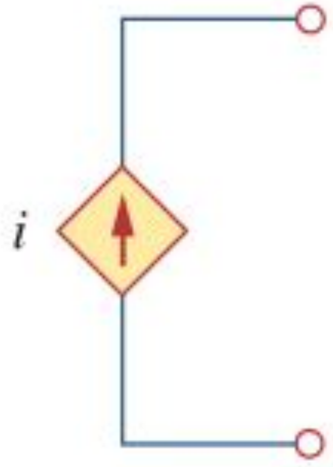
- ***An ideal voltage source*** **produces a specific voltage across its terminals and independent of load current.**



- ***An ideal current source*** **supplies a specific current through its terminals irrespective of load resistor.**

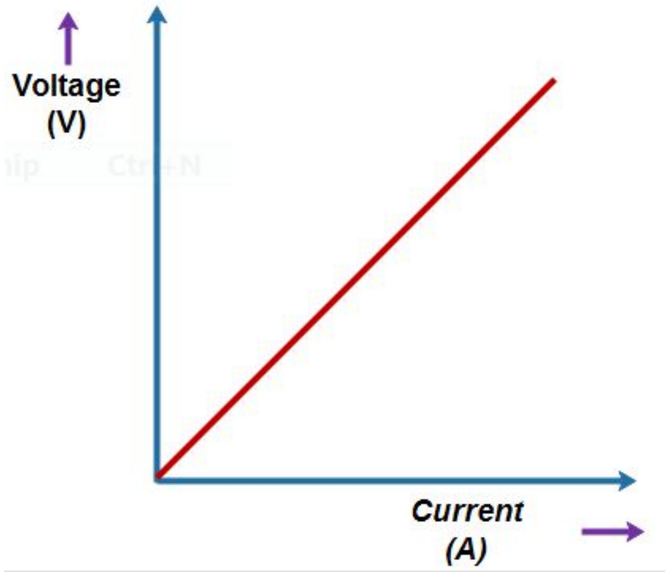
- ***An ideal dependent (or controlled) source*** **is an active element in which the source quantity is controlled by another voltage or current.**



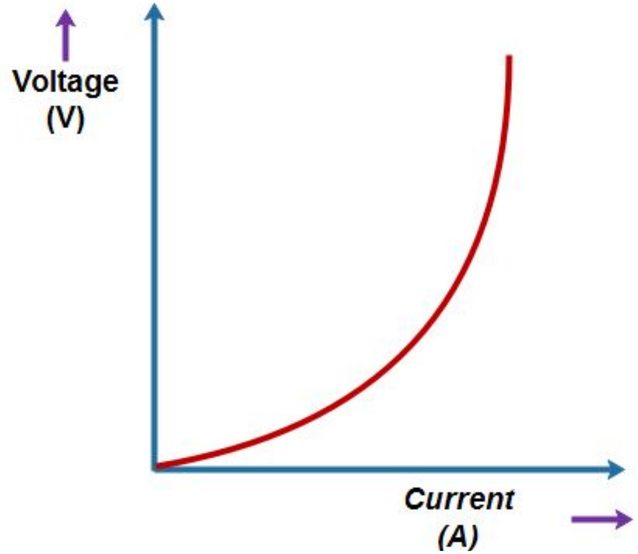


1. *A voltage-controlled voltage source (VCVS).*
2. *A current-controlled voltage source (CCVS).*
3. *A voltage-controlled current source (VCCS).*
4. *A current-controlled current source (CCCS).*

- **A linear circuit** is one whose parameters do not change with voltage or current. More specifically, a linear system is one that satisfies homogeneity and additive property.

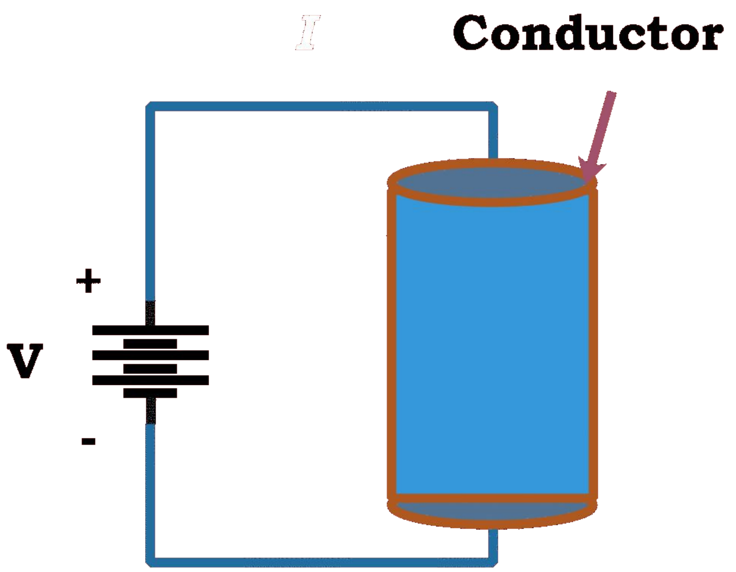


- ***A non-linear circuit*** is that whose parameters change with voltage or current. More specifically, non-linear circuit does not obey the homogeneity and additive properties.



- **Charge** is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).
- **Electric current** is the time rate of change of charge in a conductor, measured in amperes (A).

$$I = \frac{dq}{dt}$$



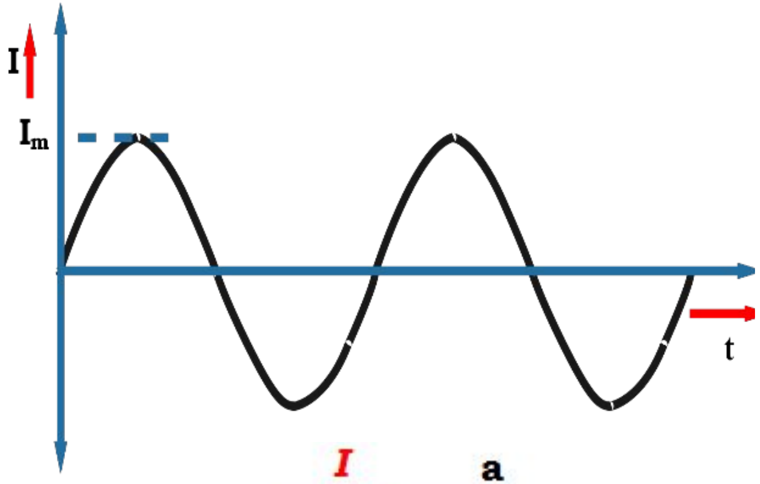
1 ampere = 1 coulomb/second

- **The law of conservation of charge** states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

Note:
Current flows from a higher potential to a lower potential in a resistor.

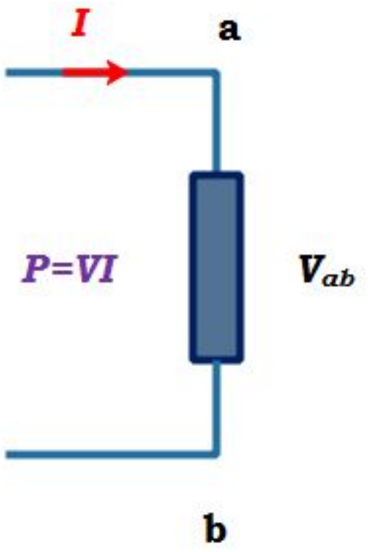
❑ ***A direct current (dc)*** **is a current that remains constant with time.**

❑ ***An alternating current (ac)*** **is a current that varies sinusoidally with time.**



❑ ***Voltage (or potential difference)*** **is the energy required to move a unit charge through an element, measured in volts (V).**

$$V = \frac{dw}{dq}$$



- ***Power*** is the time rate of expending or absorbing energy, measured in watts (W).

$$P = \frac{dw}{dt}$$

- The law of conservation of energy must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero.

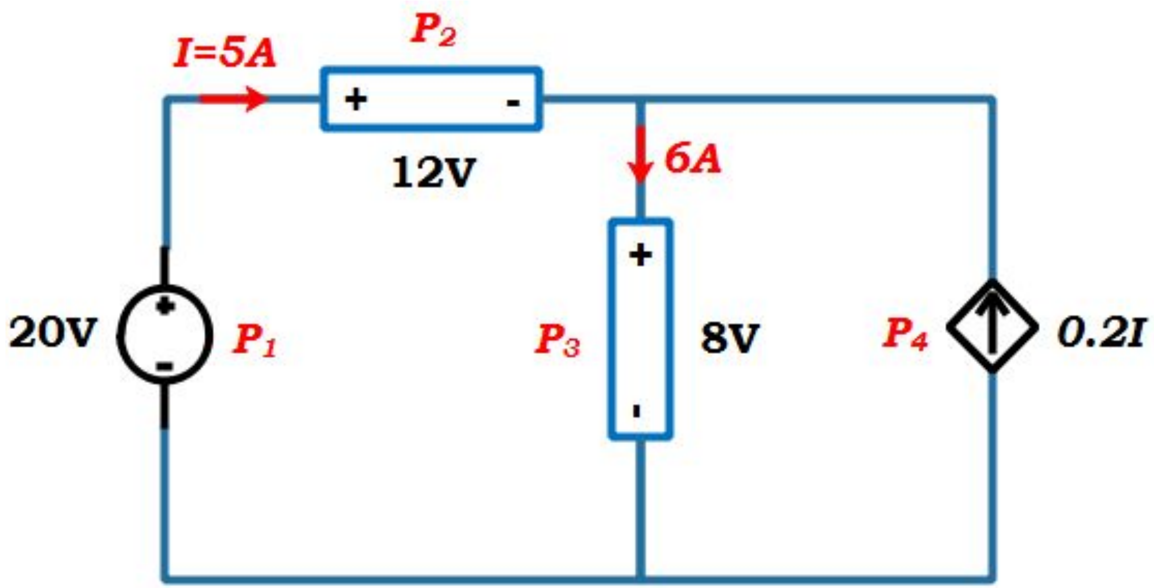
$$\sum P = 0$$

Total power supplied to the circuit = Total power absorbed by the elements

- ***Energy*** is the capacity to do work, measured in joules (J).

$$W = Pt$$

- **Current direction and voltage polarity play a major role in determining the sign of power. The voltage polarity and current direction must conform with those shown in Fig. in order for the power to have a positive sign. This is known as the passive sign convention.**
- ***Passive sign convention*** **is satisfied when the current enters through the positive terminal of an element and** $P = +VI$**. If the current enters through the negative terminal,** $P = -VI$



Note:

P_1 = -VI (Supplying power)
P_2 = +VI (Absorbing power)
P_3 = +VI (Absorbing power)
P_4 = -VI (Supplying power)

Summary

Electrical quantity	Symbol	Related equation	Units
Charge		-	Coulomb (C)
Current			Coulomb/sec Ampere (A)
Voltage			Joule/Coulomb Volts (V)
Power			Joule/sec Watt (W)
Energy			Joule (J)

SI prefixes

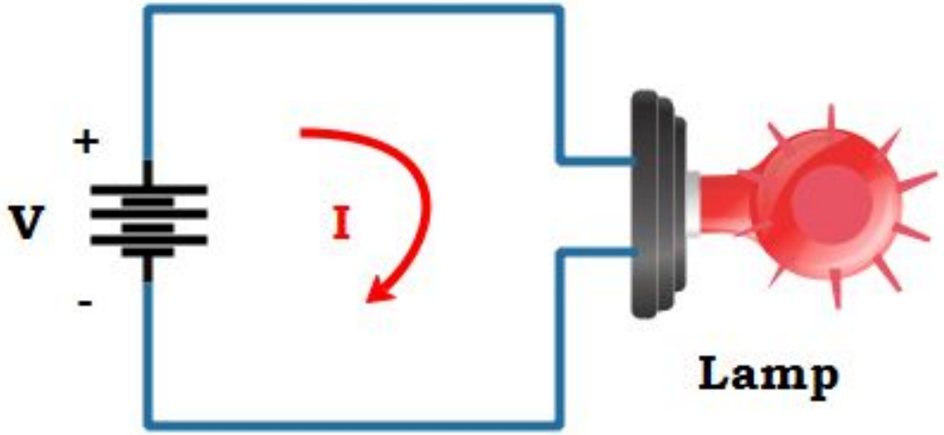
Multiplier	Prefix	Symbol
	exa	E
	peta	P
	tera	T
	giga	G
	mega	M
	kilo	k
	deca	da
	milli	m
	micro	μ
	nano	n
	pico	p
	femto	f
	atto	a

Problem-1.1

The total charge entering a terminal given by 300t mc. If 30J is given in the form of light and heat energy, calculate current at t=0.5s, voltage drop and power.

Sol:

Q=300t mc; t=0.5s; W=30J



$$Current\ (I) = \frac{dq}{dt}$$

$$Voltage\ drop\ (V) = \frac{dw}{dq}$$

$$P = V \times I$$

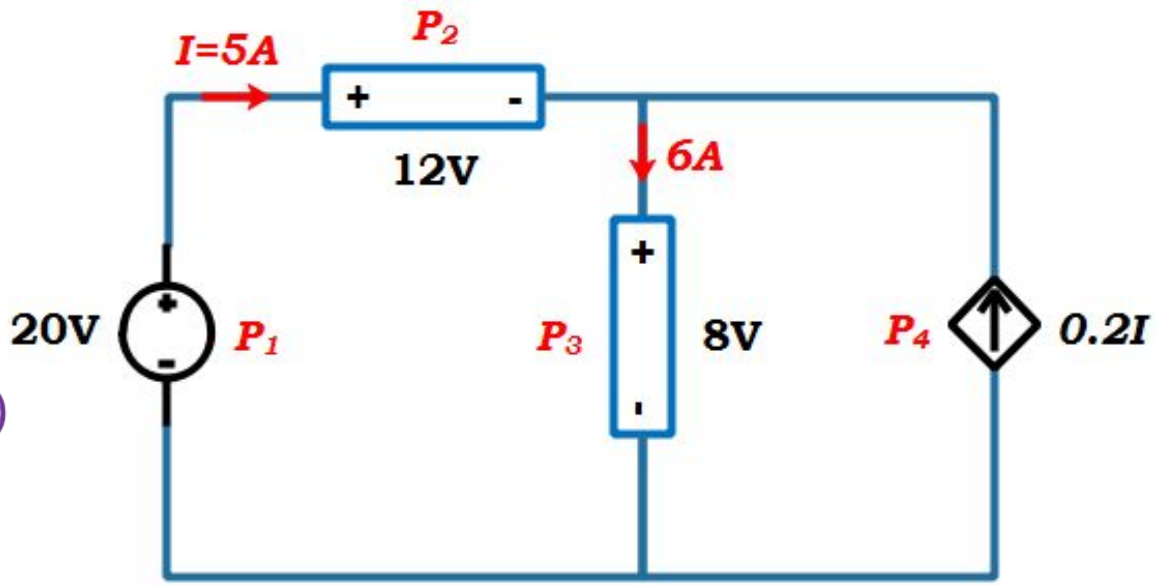
$$Current\ (I) = \frac{dq}{dt} = \frac{300 \times 10^{-3}}{0.5} = 0.6\ A$$

$$Voltage\ (V) = \frac{dw}{dq} = \frac{30}{300 \times 10^{-3}} = 100\ V$$

$$P = V \times I = 100 \times 0.6 = 60\ W$$

Problem-1.2

Calculate the power supplied or absorbed by each element in Fig



Sol:

$$P = V \times I$$

$$P_1 = 20 \times -5 \quad = -100\ W (supplied\ power)$$

$$P_2 = 12 \times 5 \quad = \quad 60\ W (absorbed\ power)$$

$$P_3 = 8 \times 6 \quad = \quad 48\ W (absorbed\ power)$$

$$P_4 = 8 \times (-0.2I)$$

$$= 8 \times (-0.2 \times 5) = \ -8\ W (supplied\ power)$$

$$P_1 + P_2 + P_3 + P_4 \quad = \quad 0\ W$$

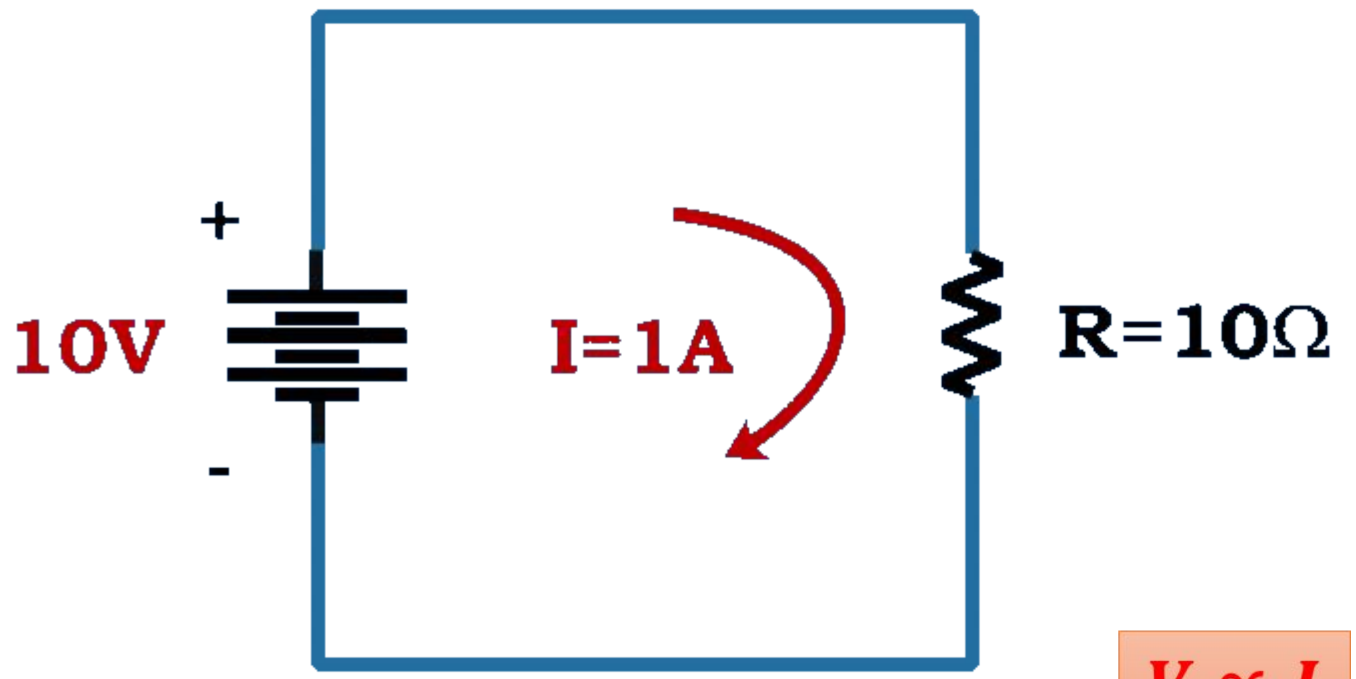
Hence $\sum P = 0$

100 + 8 = 60 + 48

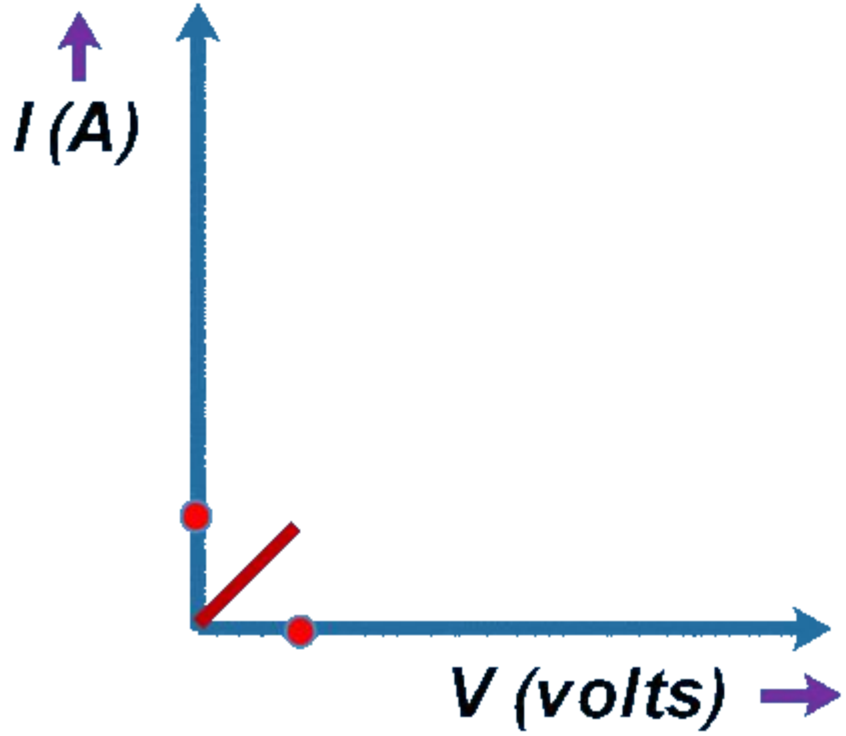
Total supplied power = Total absorbed power

1.2. Electric circuit laws

Ohm's law *(Relation between voltage and current)*



$V \propto I$



A linear circuit **is one whose output is linearly related (or directly proportional) to its input.**

Ohm's law

- "The voltage *(V)* across a resistor is directly proportional to the current *(I)* flowing through the resistor." $V \propto I$

$$V = IR$$

- The resistance *(R)* of an element denotes its ability to resist the flow of electric current; it is measured in Ω.

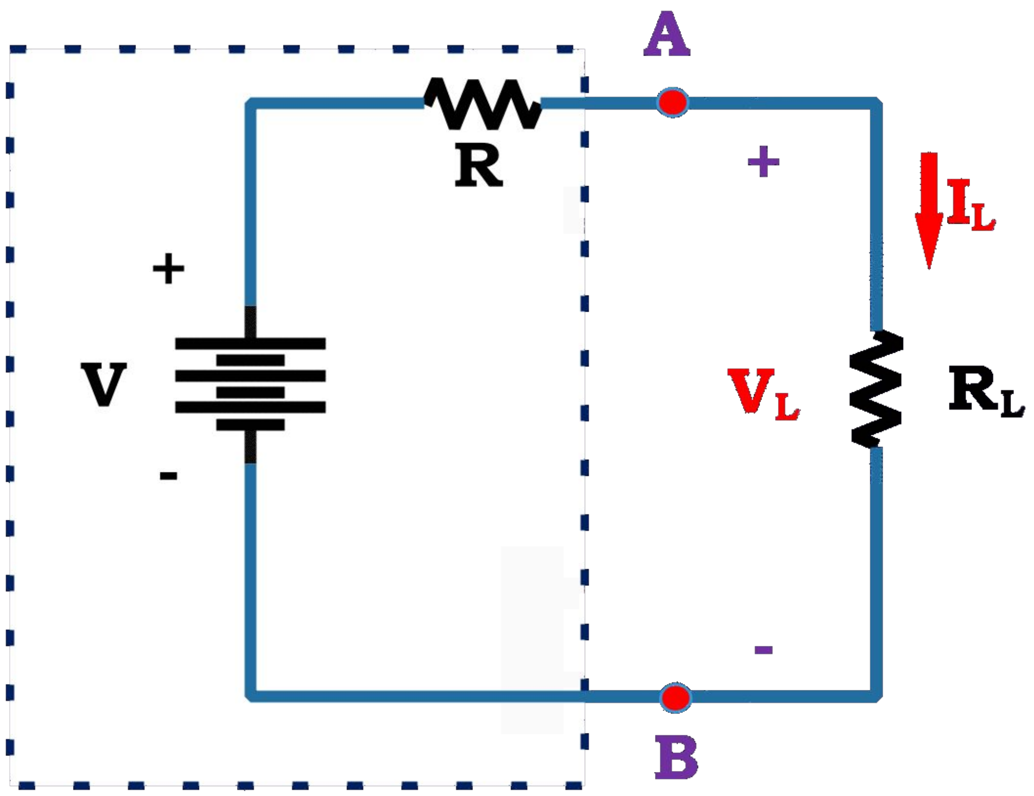
$$R = \frac{V}{I}$$

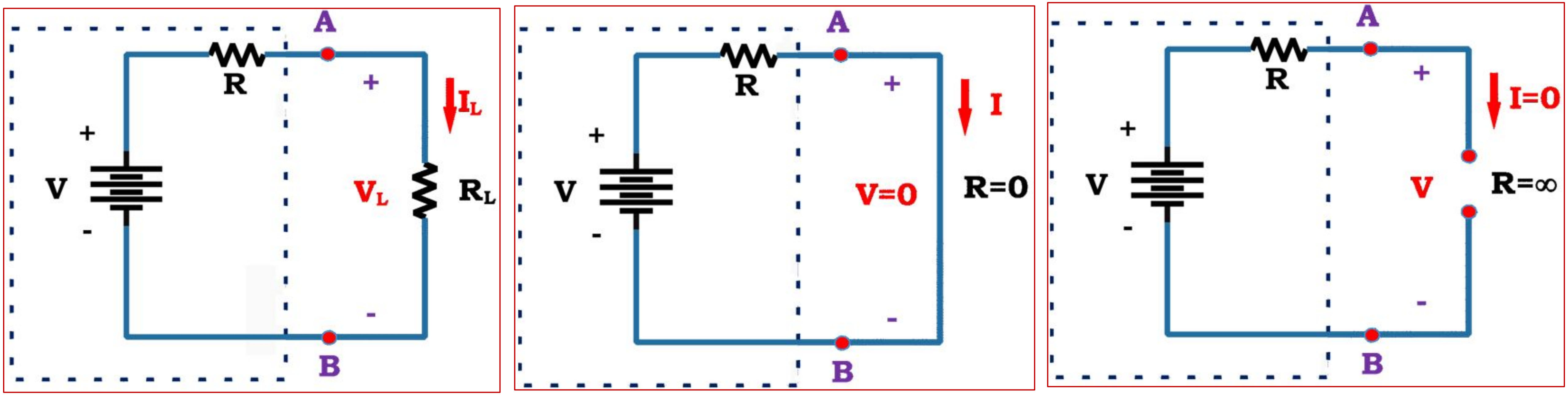
- Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{V_{higher} - V_{lower}}{R}$$

Note: The conductance *(G)* of a resistor is the reciprocal of its resistance: $G = \frac{1}{R}$

- ***A short circuit*** **is a resistor (*a perfectly conducting wire*) with zero resistance.**
- ***An open circuit*** **is a resistor with infinite resistance.**

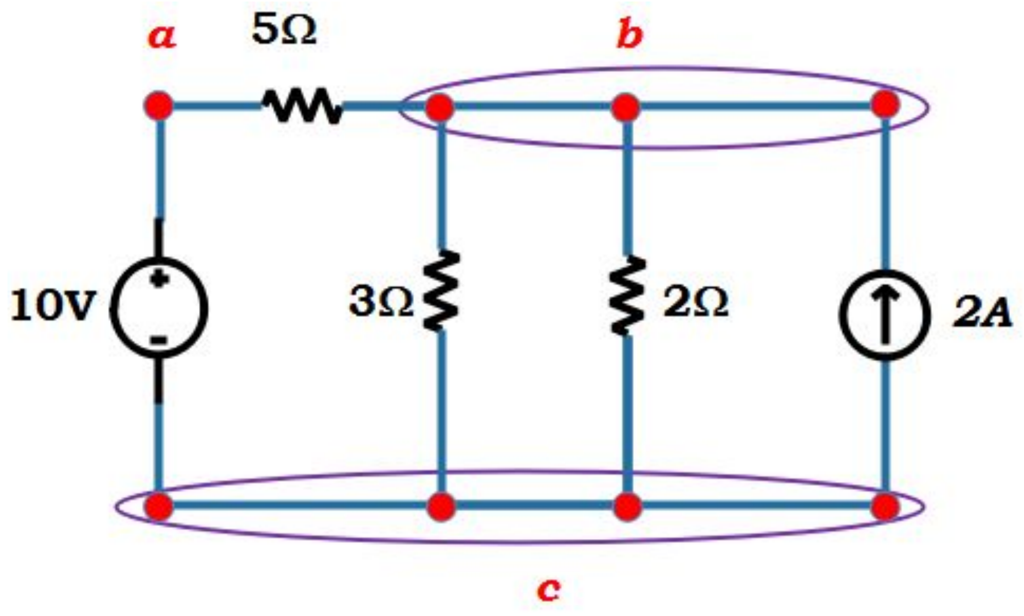


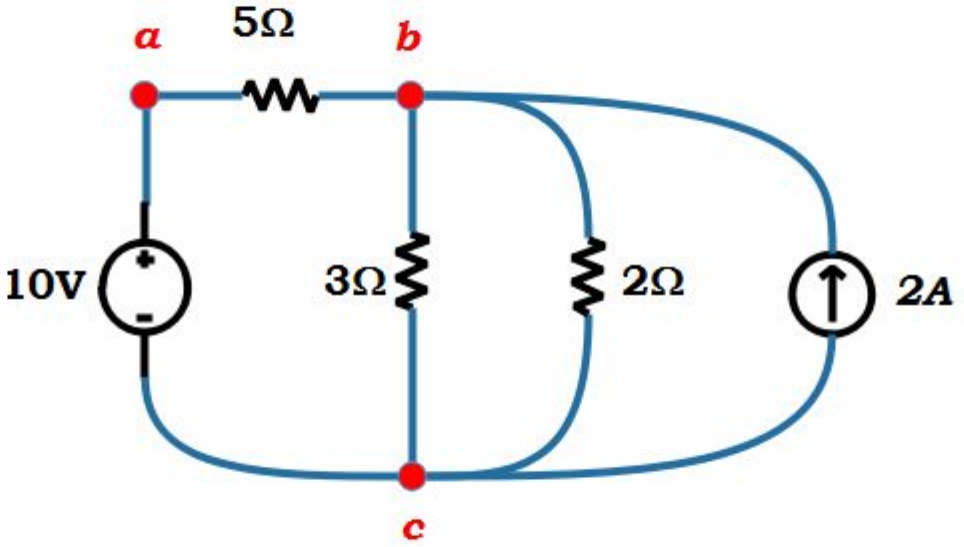


- ***A short circuit*** **is a resistor (*a perfectly conducting wire*) with zero resistance.**
- ***An open circuit*** **is a resistor with infinite resistance.**

Nodes, Branches, and Loops

- ***A branch*** **represents a single element such as a voltage source or a resistor.**
  (b=5), [a-b, b-c, b-c, b-c, c-a].
- ***A node*** **is the point of connection between two or more branches.**
  (n=3), [a,b,c].
- ***A loop*** **is any closed path in a circuit.**
  (l=3), [a-b-c-a].
- ***A mesh*** **is a loop which does not contain any other loops within it.**

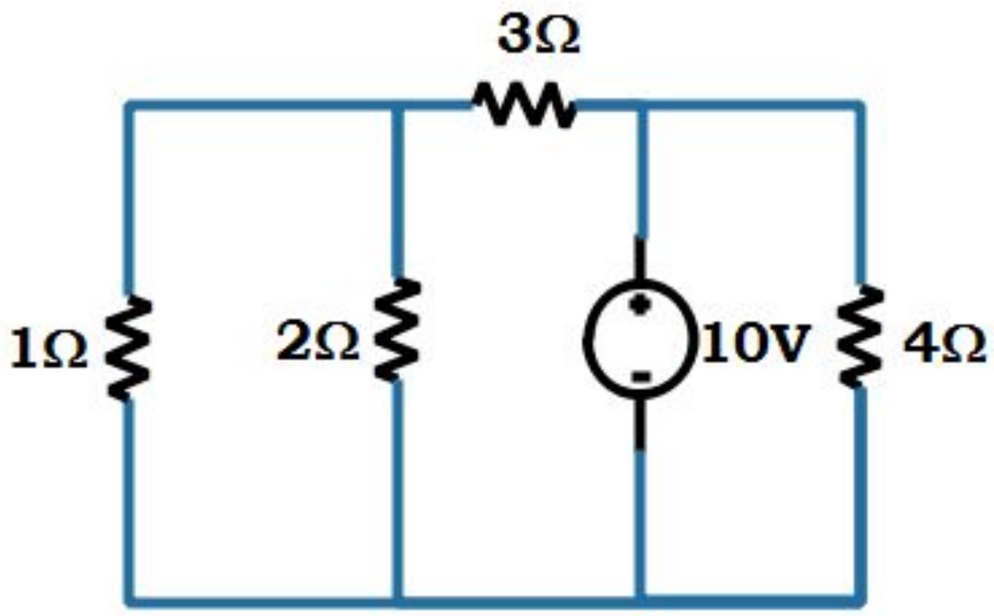




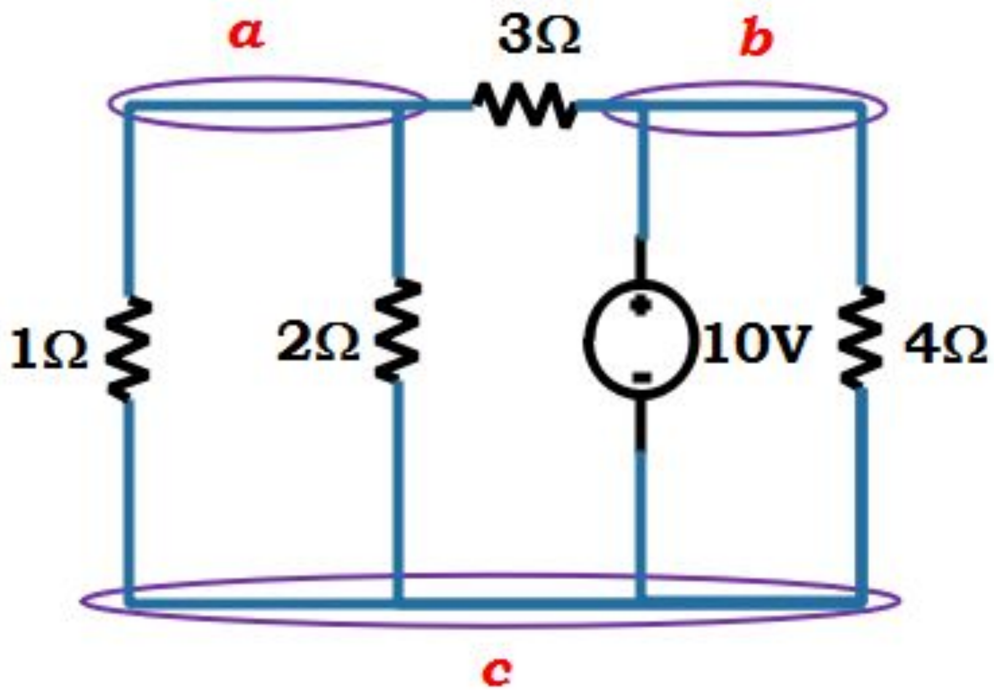
$$b = l + n - 1$$

Problem-1.3

How many branches and nodes does the circuit in Fig. have? Identify the elements that are in series and in parallel.



Sol:



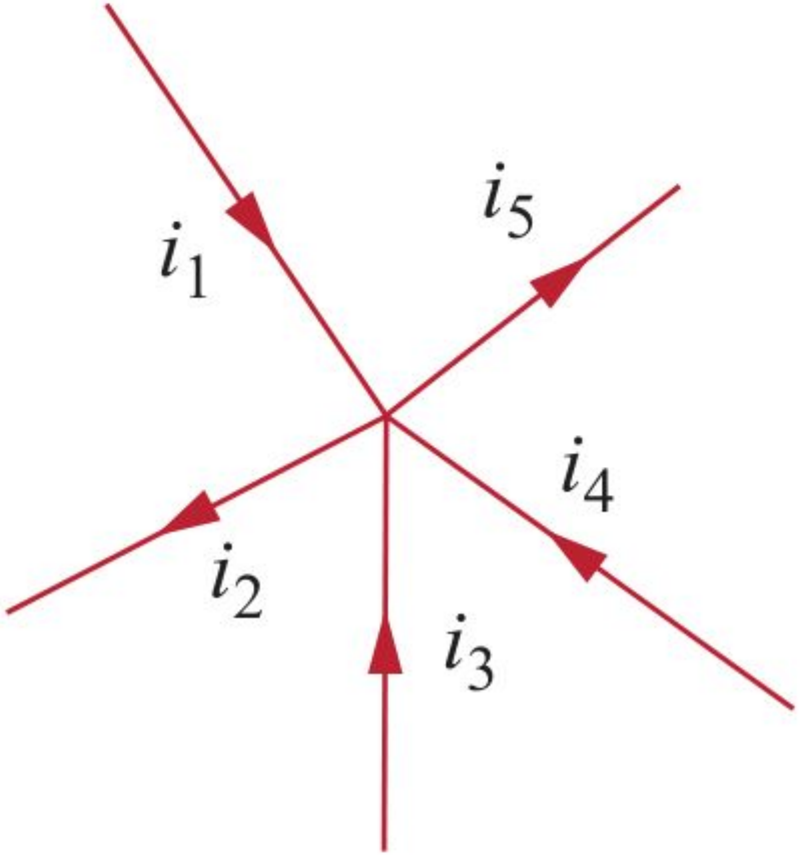
Five branches and three nodes.

The 1Ω and 2Ω resistors are in parallel.

The 4Ω resistor and 10V source are also in paral

Kirchoff’s current law

- “At any node (junction) in a circuit, the algebraic sum of currents entering and leaving a node at any instant of time must be equal to zero”.



$$\boxed{\sum i_n = 0}$$

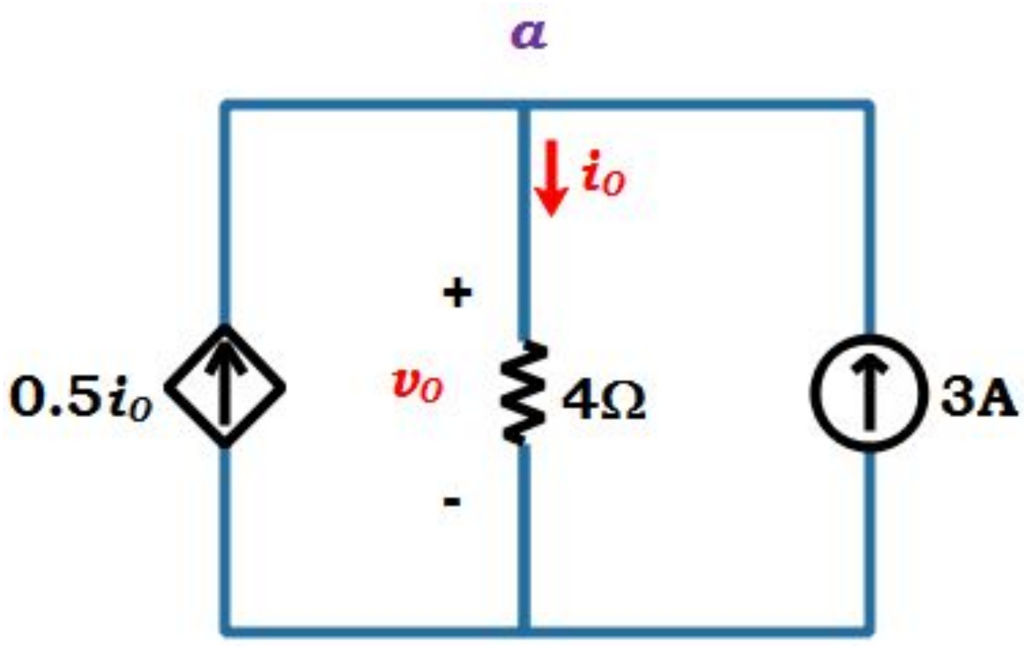
$$I_1 + (-I_2) + I_3 + I_4 + (-I_5) = 0$$

Sum of the entering currents = Sum of the leaving currents

$$I_1 + I_3 + I_4 = I_2 + I_5$$

Problem-1.4

Find current (i_o) and voltage (v_o) in the circuit shown in Fig.



Sol:

Applying KCL to node a,

$$3 + 0.5i_o = i_o$$

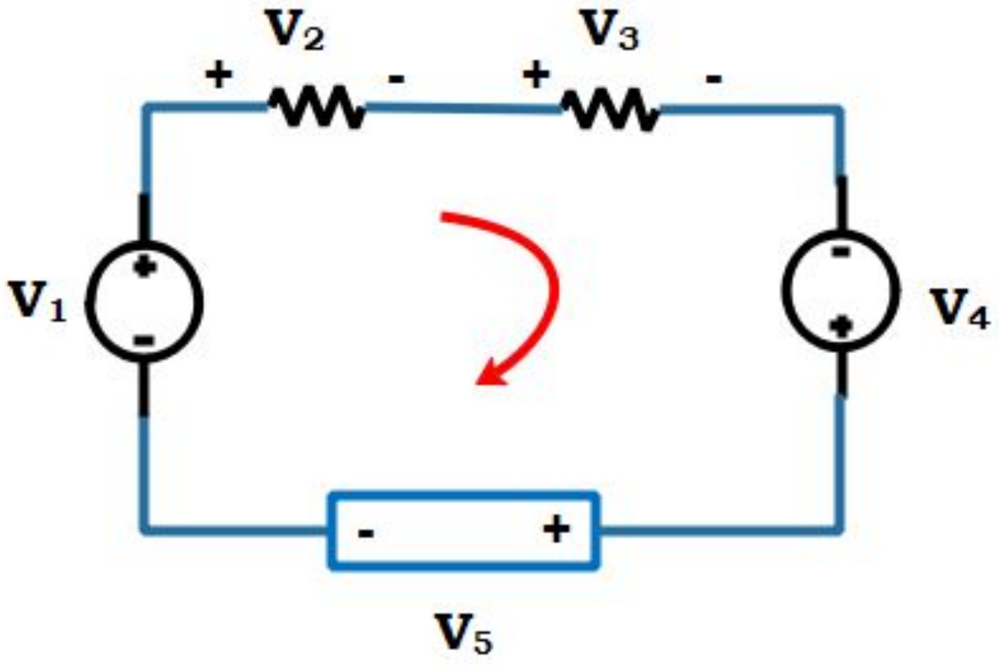
$$i_o = 6A$$

For the 4Ω resistor, Ohm's law gives,

$$v_o = 4i_o = 24V$$

Kirchoff's voltage law

- "In a closed circuit, the algebraic sum of all voltages around a closed path (or loop) is zero".



$$\sum V_n = 0$$

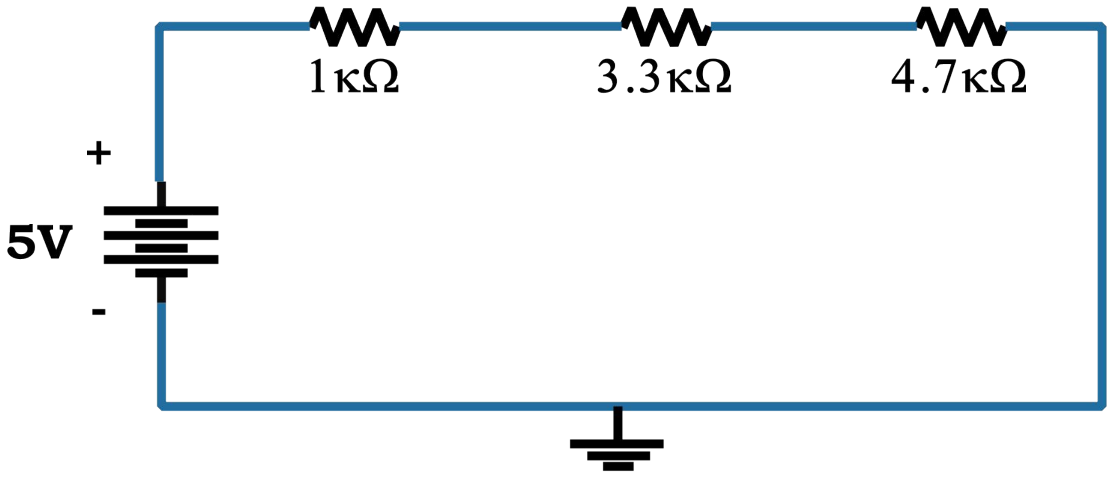
$$(-V_1) + V_2 + V_3 + (-V_4) + V_5 = 0$$

Sum of voltage sources = Sum of voltage drops

$$V_1 + V_4 = V_2 + V_3 + V_5$$

Problem-1.5

Find the voltage across $1k\Omega, 3.3k\Omega$ & $4.7k\Omega$ resistors as the circuit shown in Fig.



Sol:

Applying KVL to the circuit,

$$V = V_{1k} + V_{3.3k} + V_{4.7k}$$

$5 = I(1k + 3.3k + 4.7k)$ (*Ohm's law*)

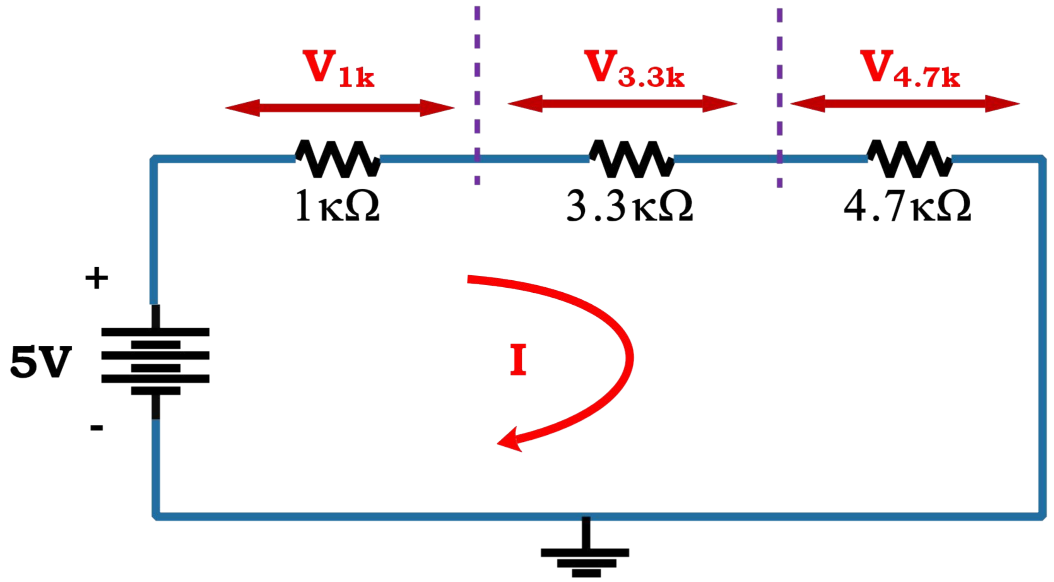
$\Rightarrow I = 0.56\ mA$

$V_{1k} = 0.56m \times 1k = 0.55\ V$ (*Ohm's law*)

$V_{3.3k} = 0.56m \times 3.3k = 1.85\ V$ (*Ohm's law*)

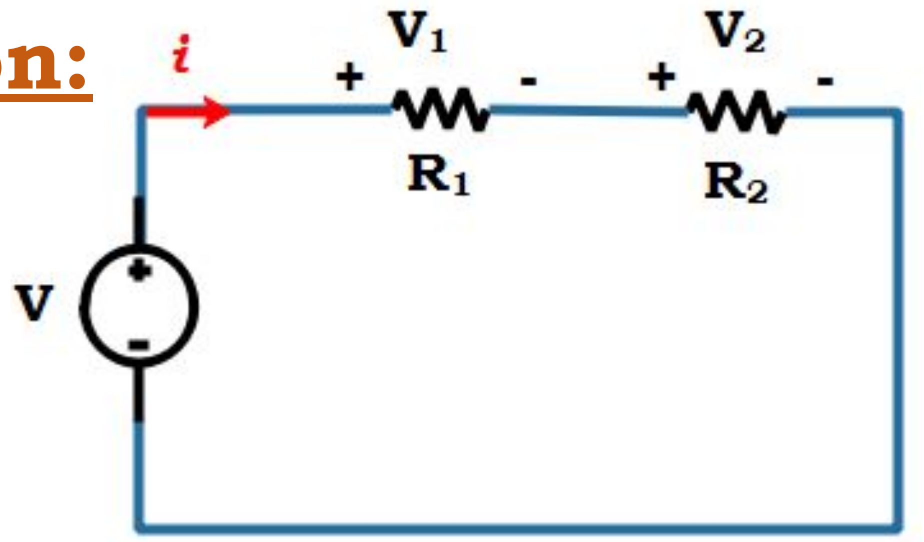
$V_{4.7k} = 0.56m \times 4.7k = 2.63\ V$ (*Ohm's law*)

$V_{1k} + V_{3.3k} + V_{4.7k} = 5.03\ V$ (*KVL verified*)



1.3. Analysis of simple circuits with DC voltage

Series Resistors and Voltage Division:



- *Applying Ohm's law to each of the resistors,*

$$V_1 = iR_1 \quad ; \quad V_2 = iR_2 \quad \rightarrow 1$$

KVL to the loop,

$$V = V_1 + V_2 \quad \rightarrow 2$$

subtitute eqn 1 in 2,

$$V = i(R_1 + R_2)$$

$$i = \frac{V}{(R_1+R_2)} = \frac{V}{R_{eq}} \quad \rightarrow 3$$

subtitute eqn 3 in 1,

$$V_1 = \frac{R_1}{(R_1+R_2)}V \; ; V_2 = \frac{R_2}{(R_1+R_2)}V$$

$$R_{eq} = R_1 + R_2$$

$$i = \frac{V}{(R_1+R_2)}$$

$$V_1 = \frac{R_1}{(R_1+R_2)}V$$

$$V_2 = \frac{R_2}{(R_1+R_2)}V$$

Parallel Resistors and Current Division:

Applying Ohm's law to each of the resistors,

$V = i_1 R_1 = i_2 R_2$

$i_1 = \frac{V}{R_1} \quad ; i_2 = \frac{V}{R_2} \rightarrow 1$

KCL at node a,

$i = i_1 + i_2 \quad \rightarrow 2$

subtitute eqn 1 in 2,

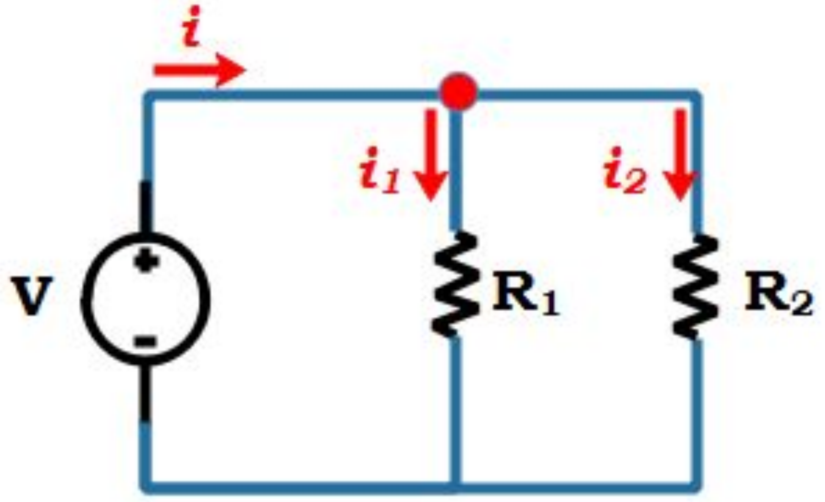
$$i = \frac{V}{R_1} + \frac{V}{R_2}$$

$$i = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}$$

$$V = iR_{eq} = \frac{iR_1R_2}{(R_1+R_2)} \rightarrow 3$$

subtitute eqn 3 in 1,

$$i_1 = \frac{R_2}{(R_1+R_2)} i \; ; i_2 = \frac{R_1}{(R_1+R_2)}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$i = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

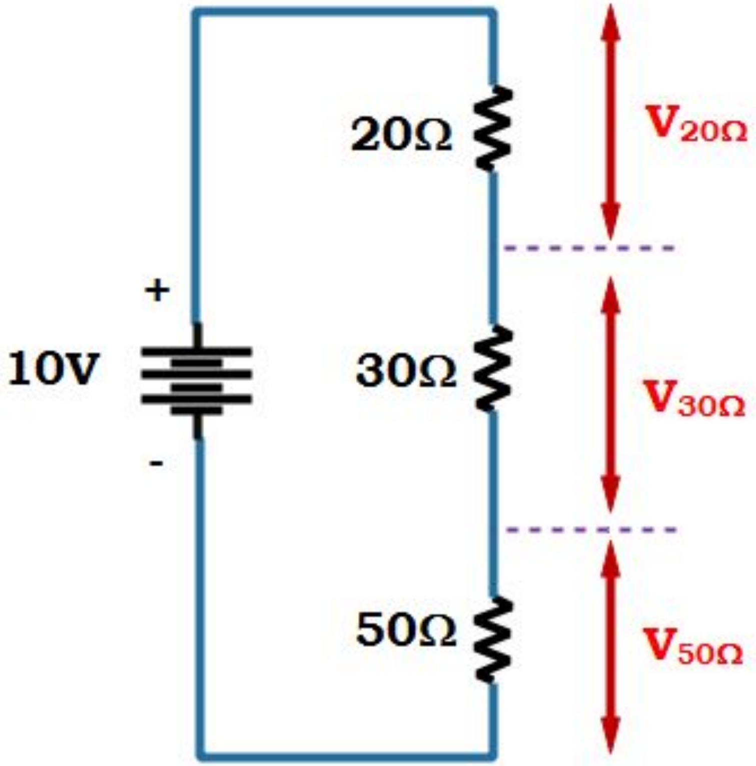
$$i_1 = \frac{R_2}{(R_1+R_2)} i$$

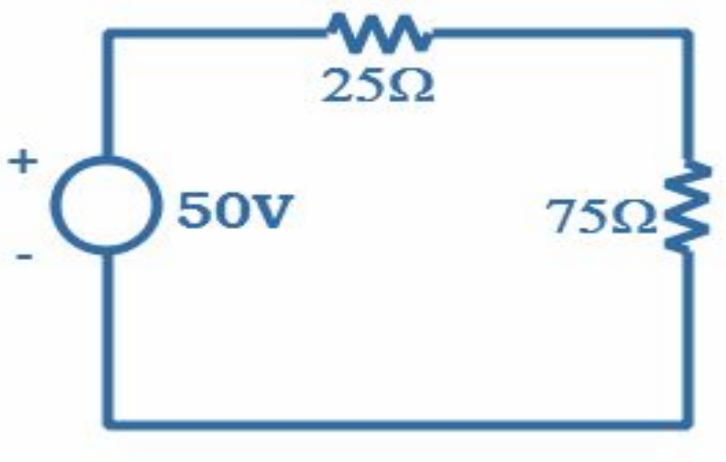
$$i_2 = \frac{R_1}{(R_1+R_2)} i$$

Problem-1.6

Find the voltage across resistors for the following circuit shown in Figs.,

Sol:





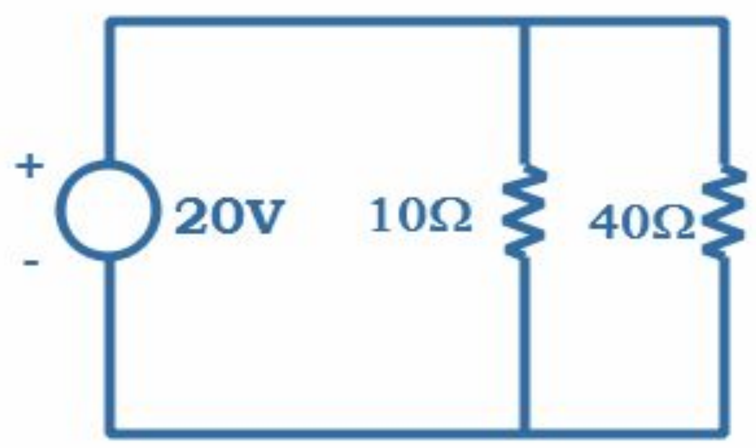
$$V_{20\Omega} = 2V$$
$$V_{30\Omega} = 3V$$
$$V_{50\Omega} = 5V$$

$$V_{25\Omega} = 12.5V$$
$$V_{75\Omega} = 37.5V$$

Problem-1.7

Find the current through resistors for the following circuit shown in Fig.,

Sol:



$$R_{eq} = \frac{10 \times 40}{10 + 40} = \frac{400}{50} = 8\Omega$$

$$I = \frac{20}{8} = 2.5A$$

$$I_{10\Omega} = 2A$$

$$I_{40\Omega} = 0.5A$$

$$R_{eq} = \frac{60 \times 40}{60 + 40} = \frac{2400}{100} = 24\Omega$$

$$I = \frac{80}{24} = 3.33A$$

$$I_{60\Omega} = 1.33A$$

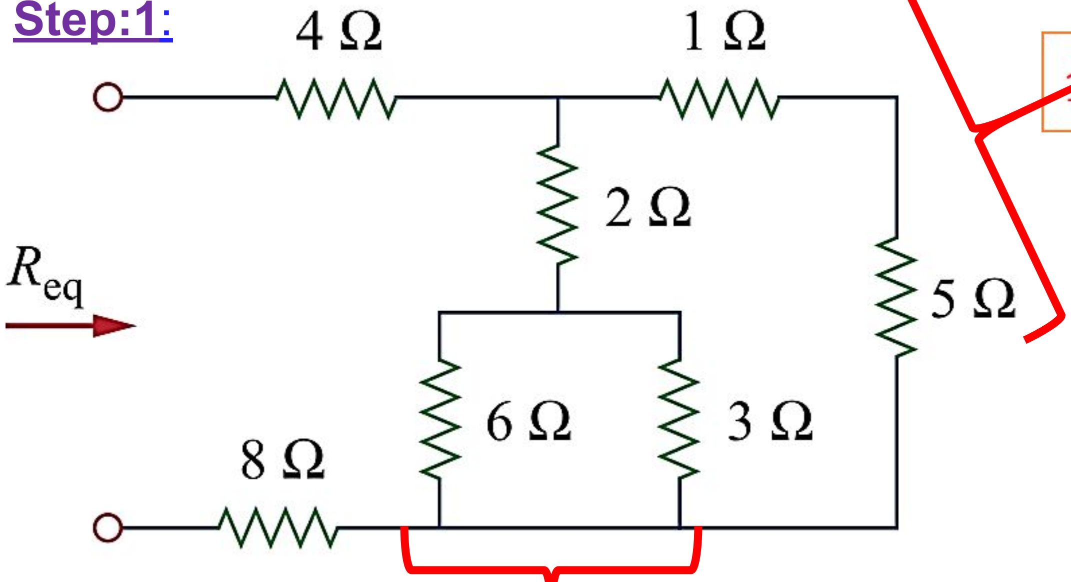
$$I_{40\Omega} = 2A$$

Problem-1.8

Find (R_{eq}) for the circuit shown in Fig.

Sol:

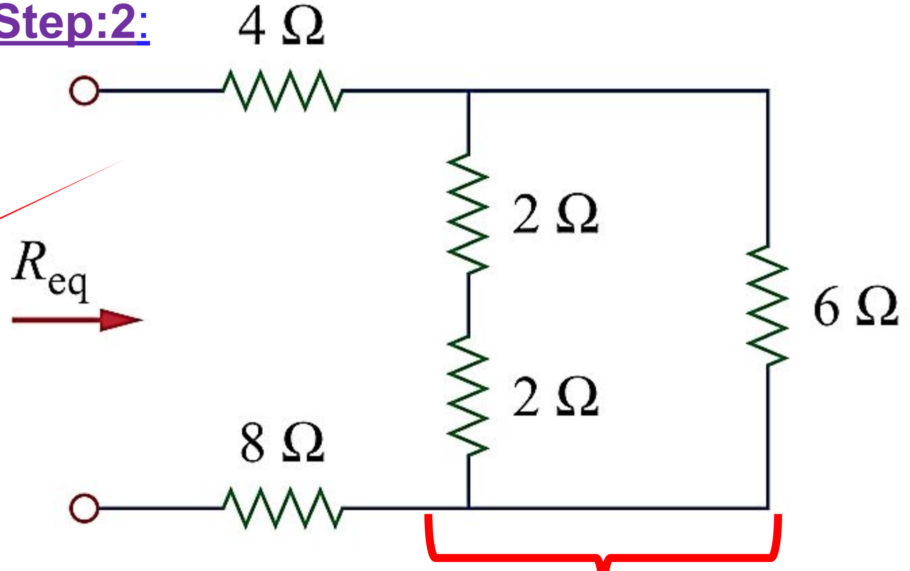
Step:1:



$1\Omega + 5\Omega = 6\Omega$

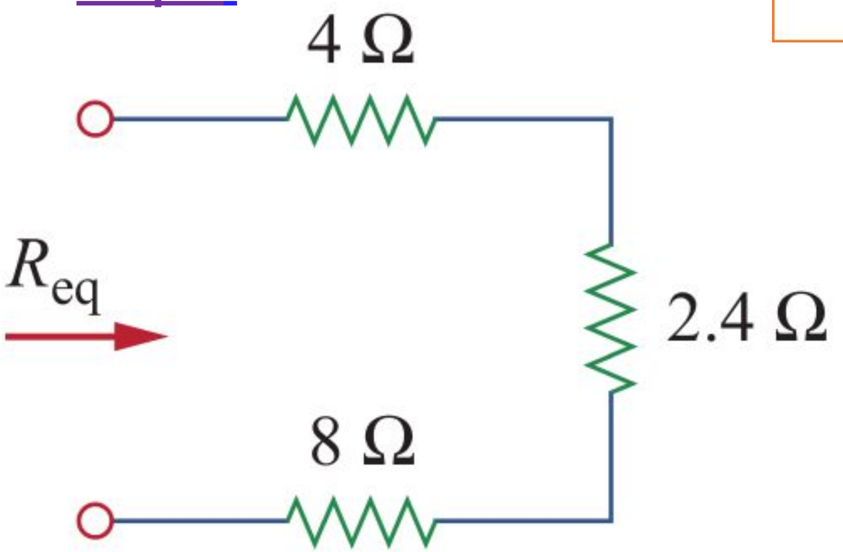
$$6\Omega||3\Omega = \frac{6\times 3}{6+3} = 2\Omega$$

Step:2:



$$6\Omega||4\Omega = \frac{6\times 4}{6+4} = 2.4\Omega$$

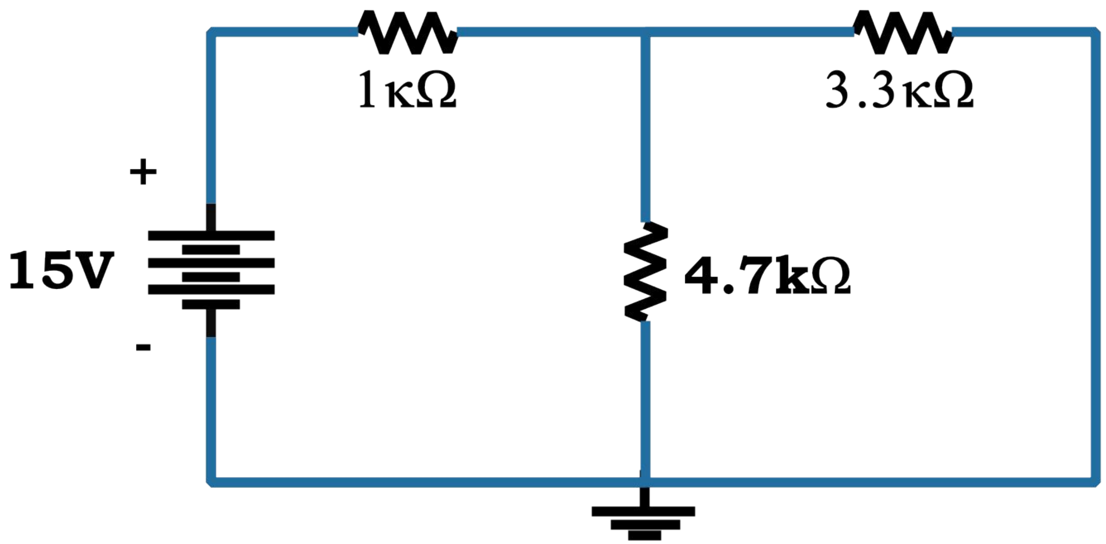
Step:3:



$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

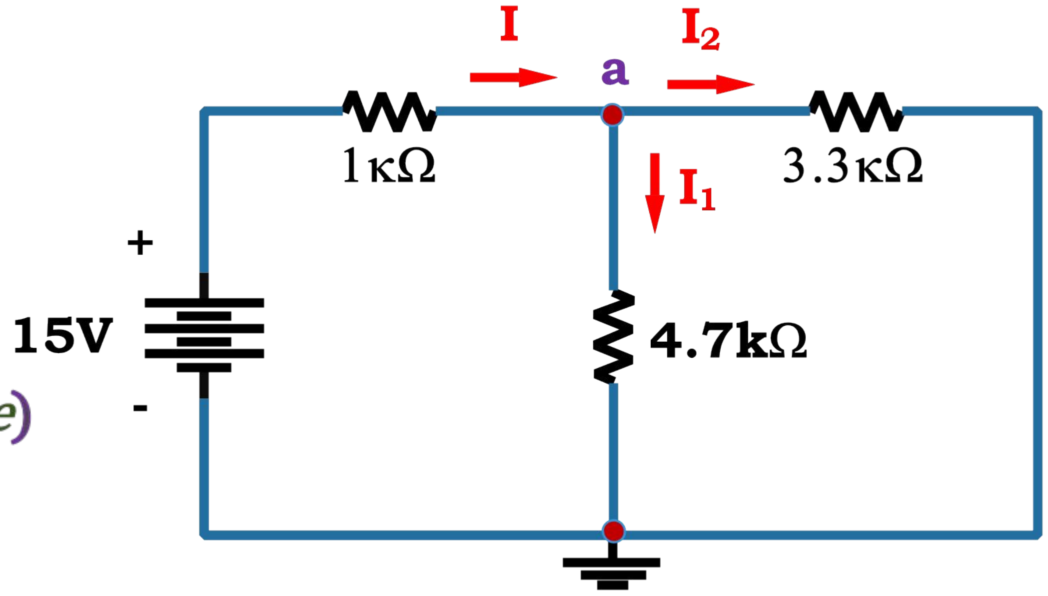
Problem-1.9

Find the current flowing through $1k\Omega, 4.7k\Omega$ & $3.3k\Omega$ resistors as the circuit shown in Fig.



Sol:

Applying KCL to node a,



$$I = I_1 + I_2$$

$$I = \frac{15}{\left(1k + \frac{(3.3k \times 4.7k)}{(3.3k + 4.7k)}\right)} = 5.1mA \quad (Ohm's\ law)$$

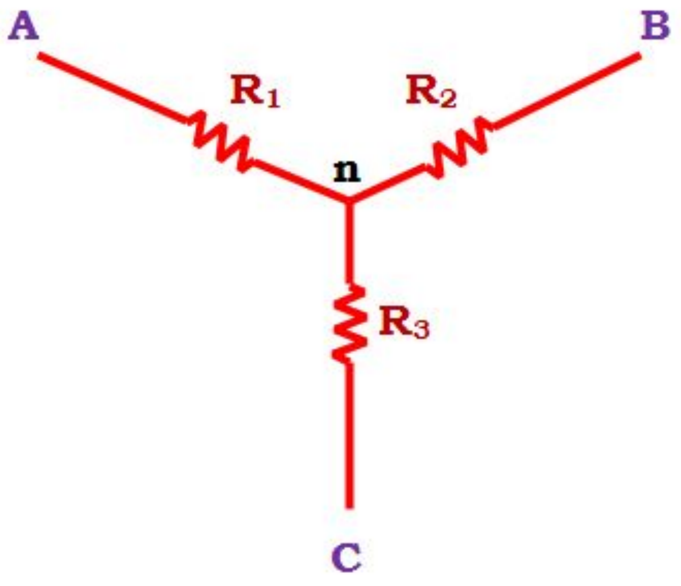
$$I_1 = \frac{3.3k}{(3.3 + 4.7)k} \times 5.1m = 2.1mA \quad (current\ division\ rule)$$

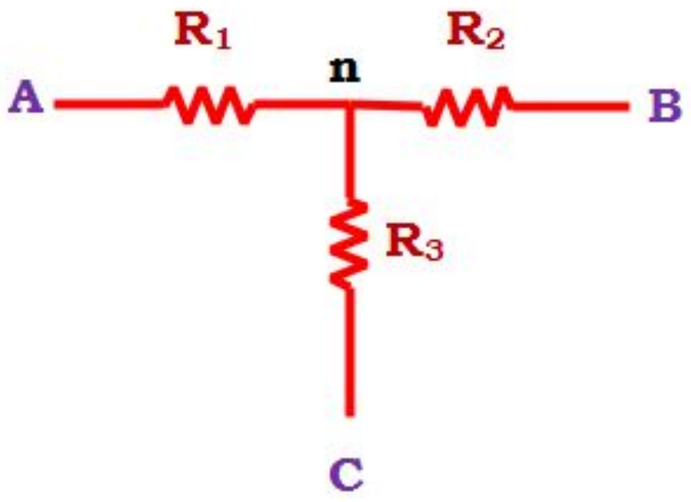
$$I_2 = \frac{4.7k}{(3.3 + 4.7)k} \times 5.1m = 3mA \quad (current\ division\ rule)$$

$$I = I_1 + I_2 = 5.1mA \quad (KCL\ verified)$$

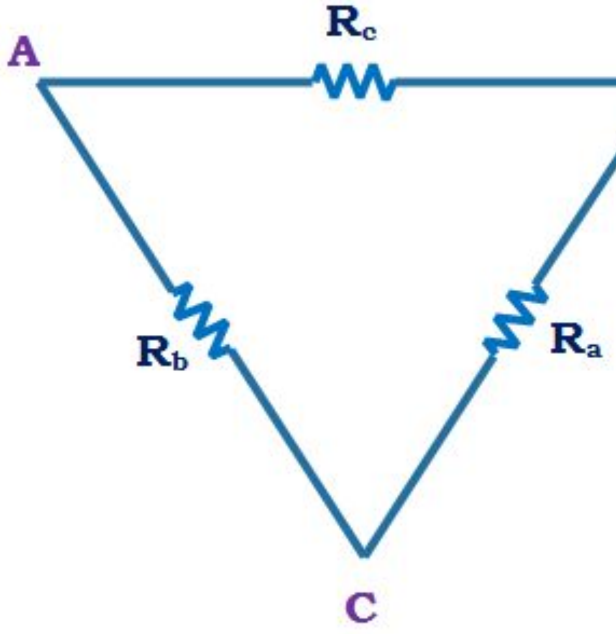
Star-Delta Network:

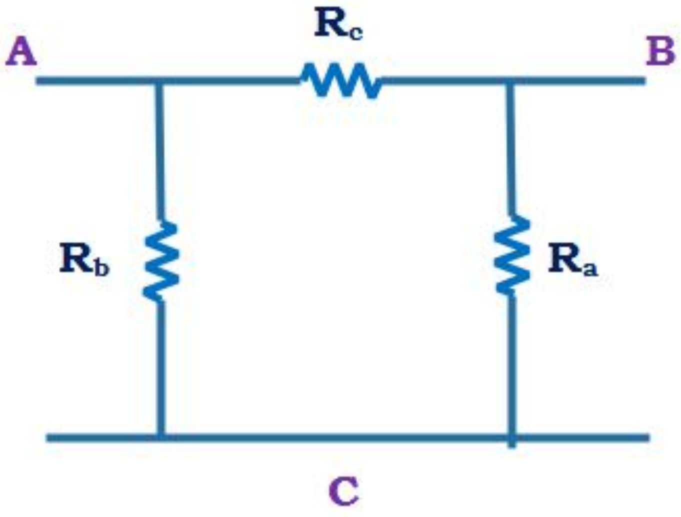
Y (or) T Network





Δ (or) Π Network





Star-Delta / Delta-star Conversion:

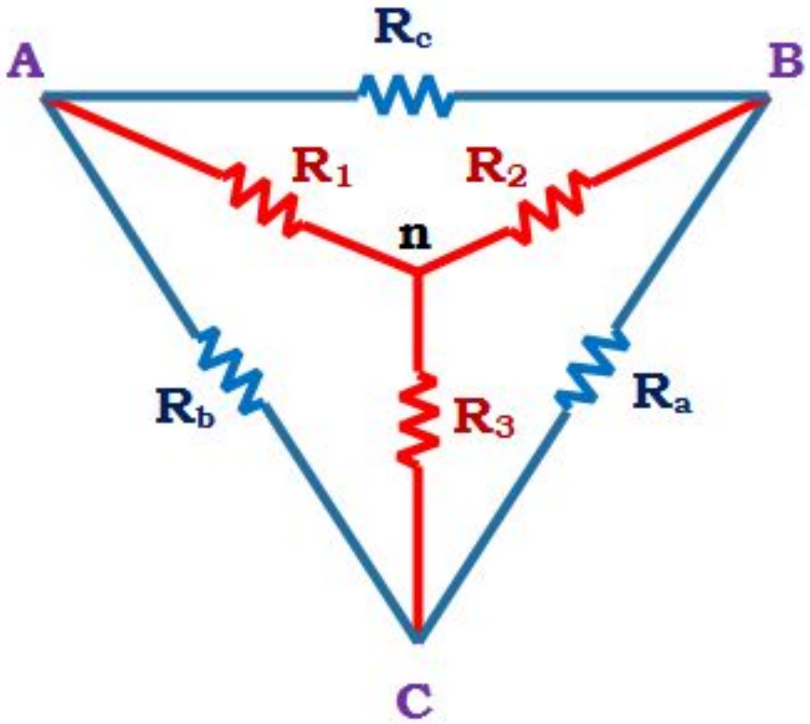
Y-Network	Δ-Network
"Each resistor in the Y network is the product of the resistors in the two adjacent branches, divided by the sum of the three resistors".	**"Each resistor in the network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor".**

R_a, R_b, R_c *are given,*

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$



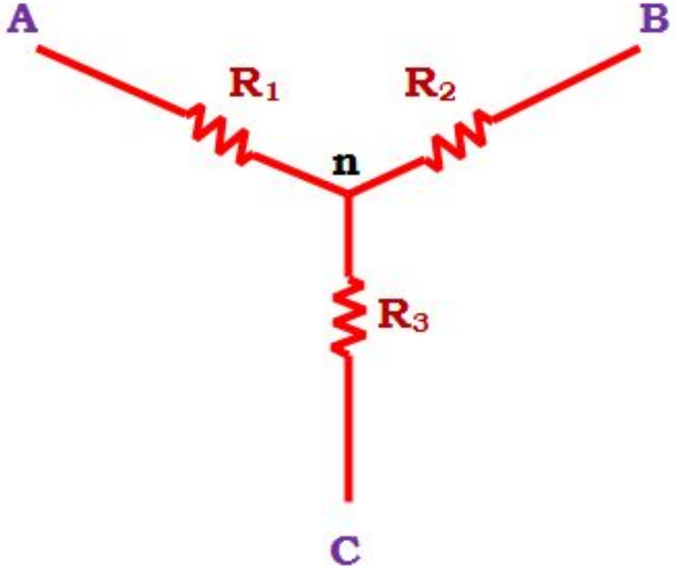
R_1, R_2, R_3 *are given,*

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

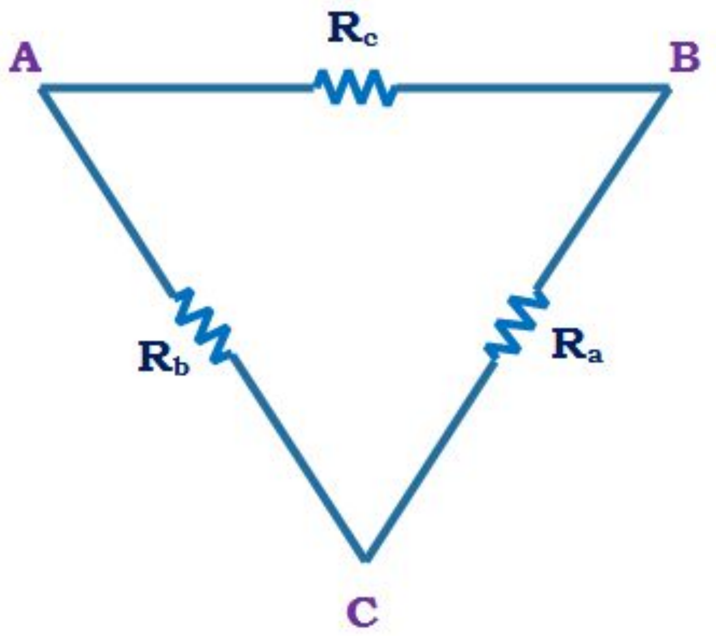
Star-Delta / Delta-star Conversion:



$$R_\Delta = 3R_Y$$

$$R_1 = R_2 = R_3$$
$$R_a = R_b = R_c$$

$$R_Y = \frac{R_\Delta}{3}$$



R_a, R_b, R_c *are given,*

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

R_1, R_2, R_3 *are given,*

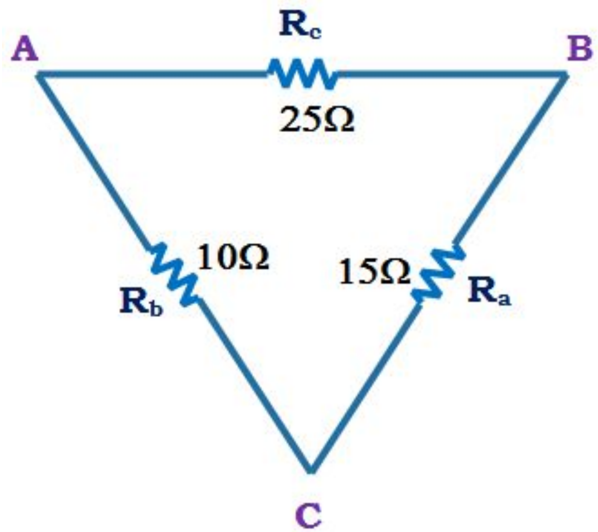
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

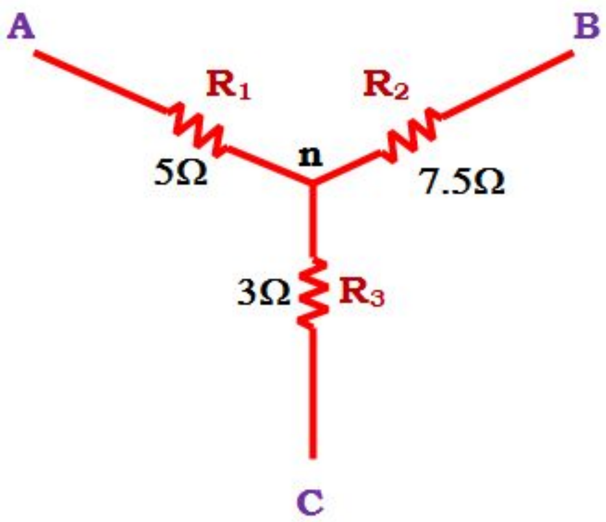
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Problem-1.10

Convert the Δ network in Fig. to an equivalent Y network





Ans:

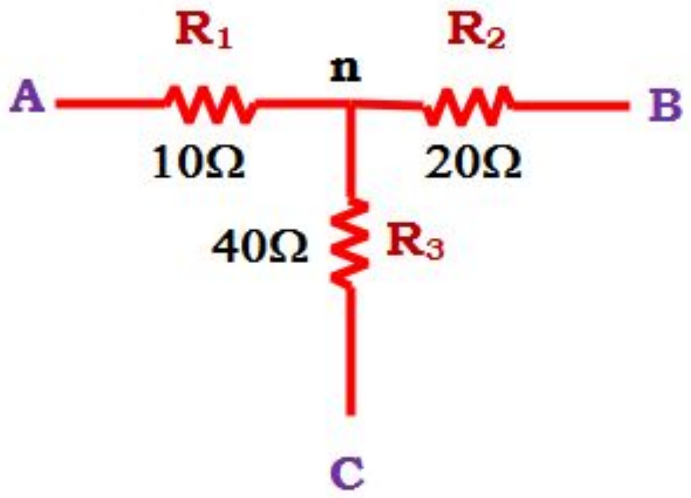
$R_1 = 5\Omega$

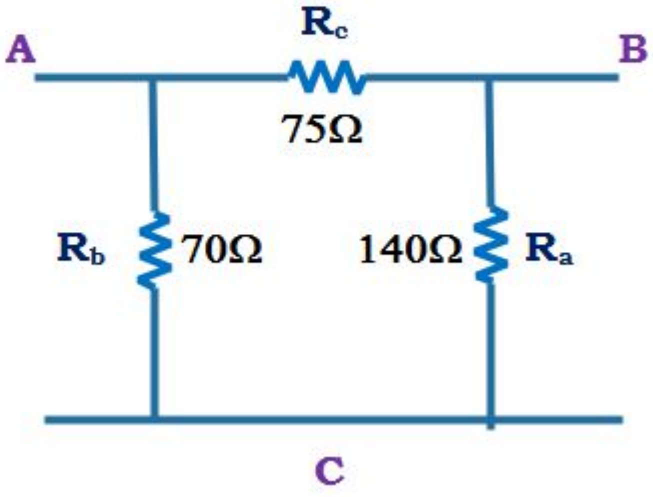
$R_2 = 7.5\Omega$

$R_3 = 3\Omega$

Problem-1.11

Transform the wye network in Fig. to a delta network.





Ans:

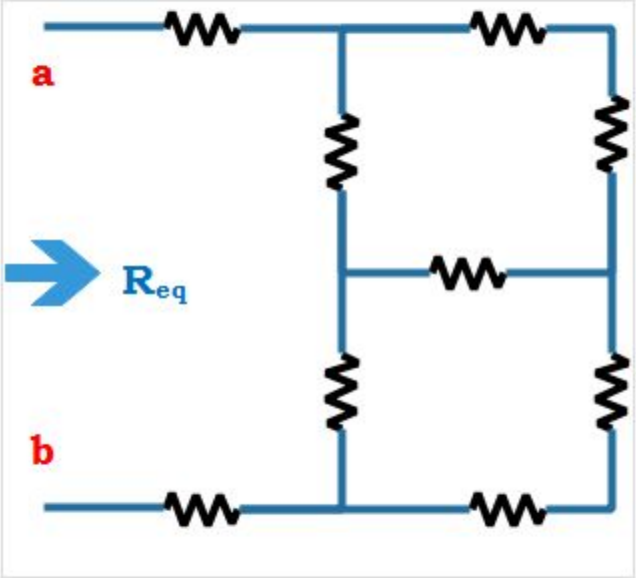
$R_a = 140\Omega$

$R_b = 70\Omega$

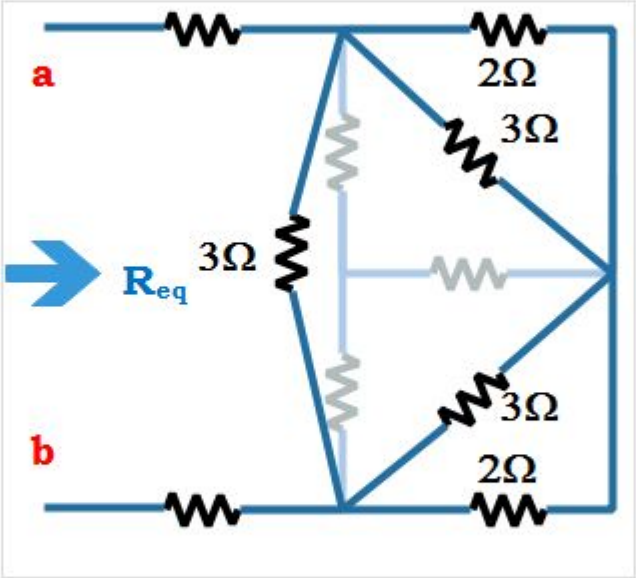
$R_c = 35\Omega$

Problem-1.12

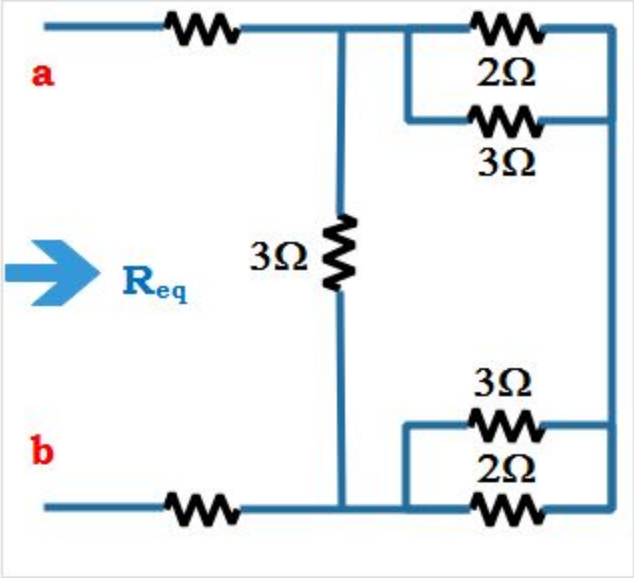
Find equivalent resistance (R_{eq}) between the terminals ‘a’ & ‘b’ and assume all resistors values are 1Ω.



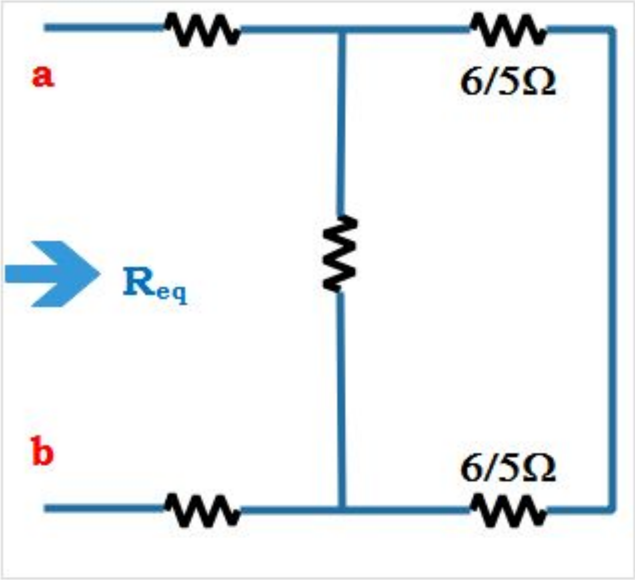
Step-1:



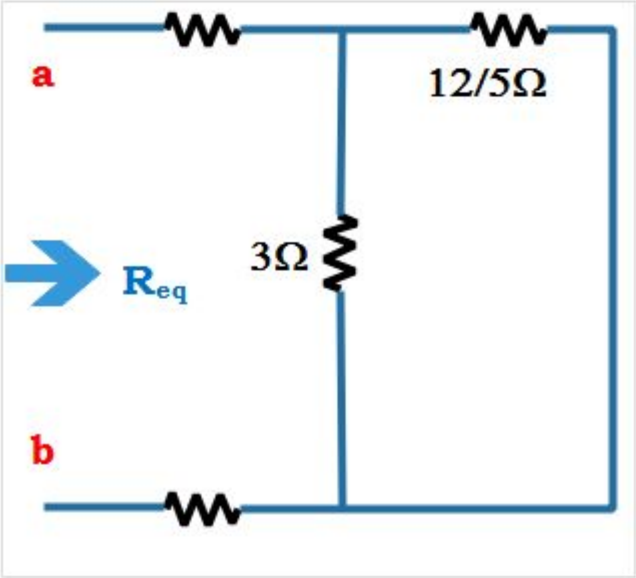
Step-2:



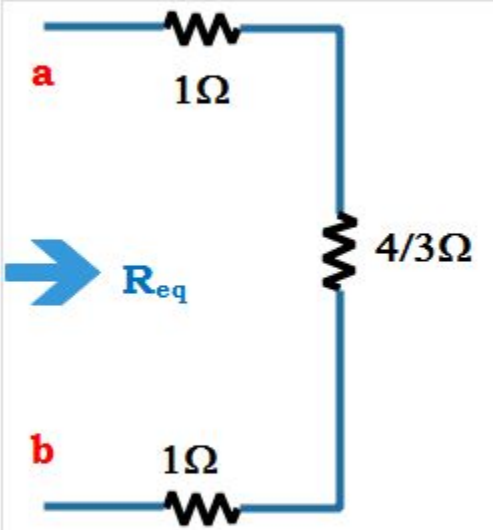
Step-3:



Step-4:



Step-5:



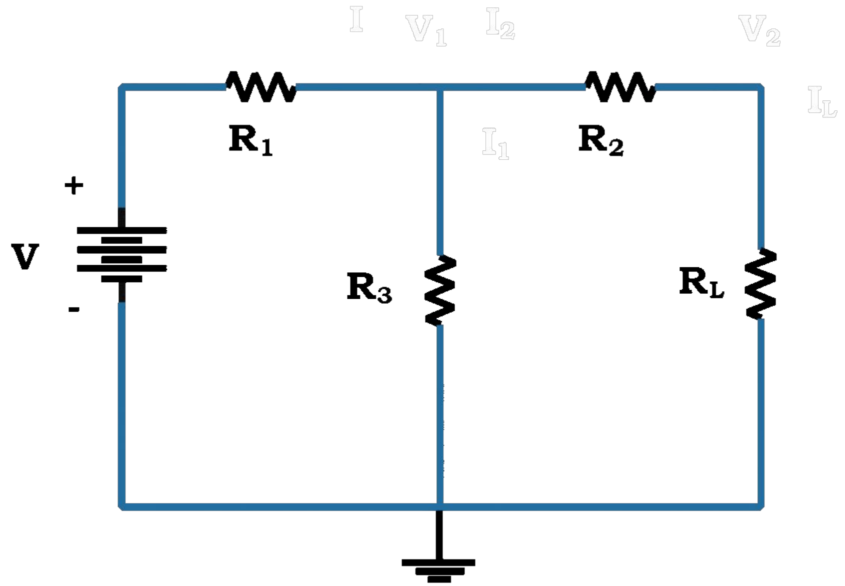
$R_{eq} = 3.33\Omega$

Node voltage analysis of DC resistive circuit:

Steps to Determine Node Voltages:

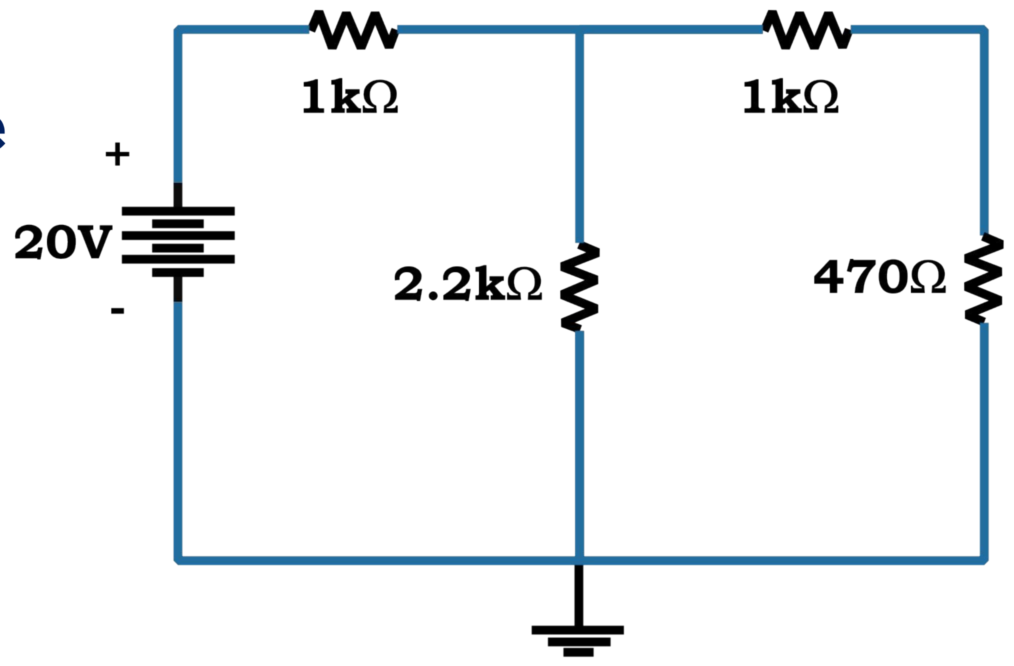
1. Select a node as the reference node. Assign voltages $(V_1, V_2, \ldots . V_{n-1})$ to the remaining nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Step-0



Problem-1.13

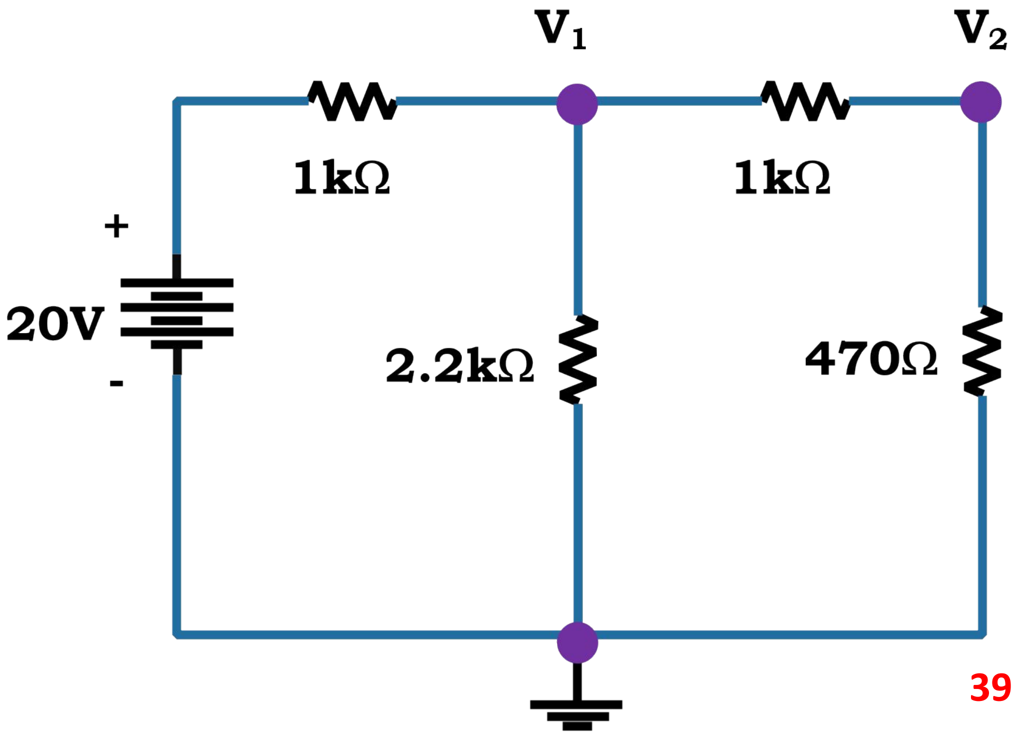
Calculate the node voltages in the circuit shown in Fig.



Sol:

Step-1

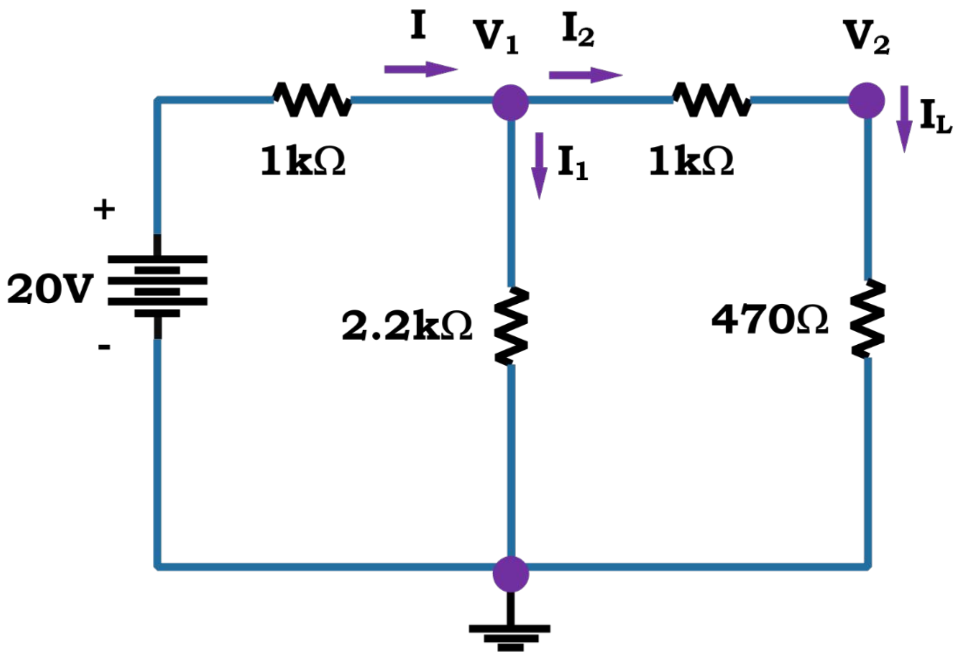
Select a node as the reference node. Assign voltages $(V_1, V_2, \ldots . V_{n-1})$ to the remaining nodes. The voltages are referenced with respect to the reference node.



Step-2

Apply KCL to each of the non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

KCL to Node-1	KCL to Node-2

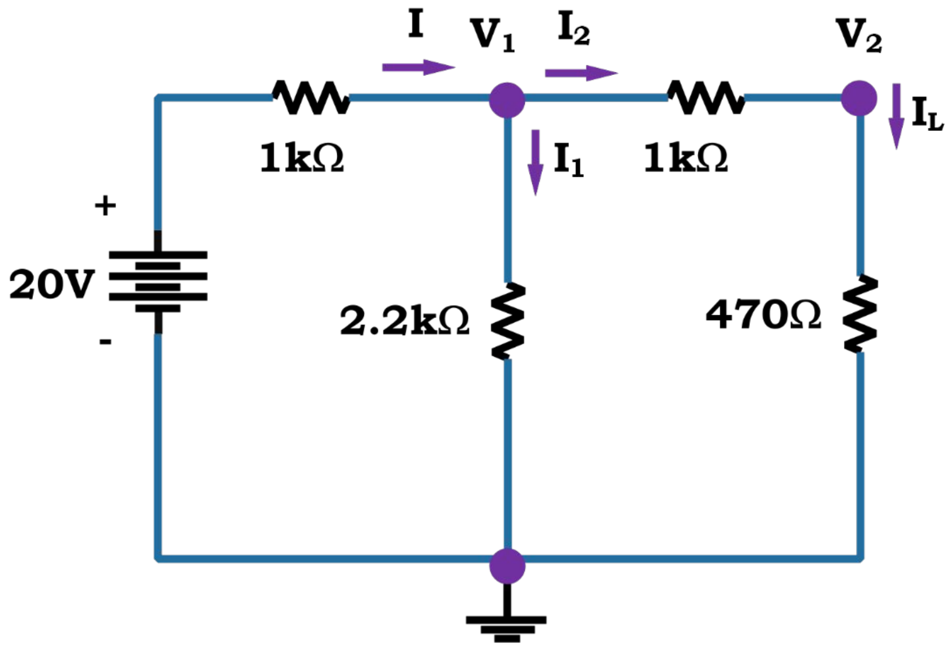


Step-3

Solve the resulting simultaneous equations to obtain the unknown node voltages.

$$2.45V_1 - V_2 = 20$$
$$V_1 - 3.127V_2 = 0$$

$$V_1 = 9.4V$$
$$V_2 = 3V$$



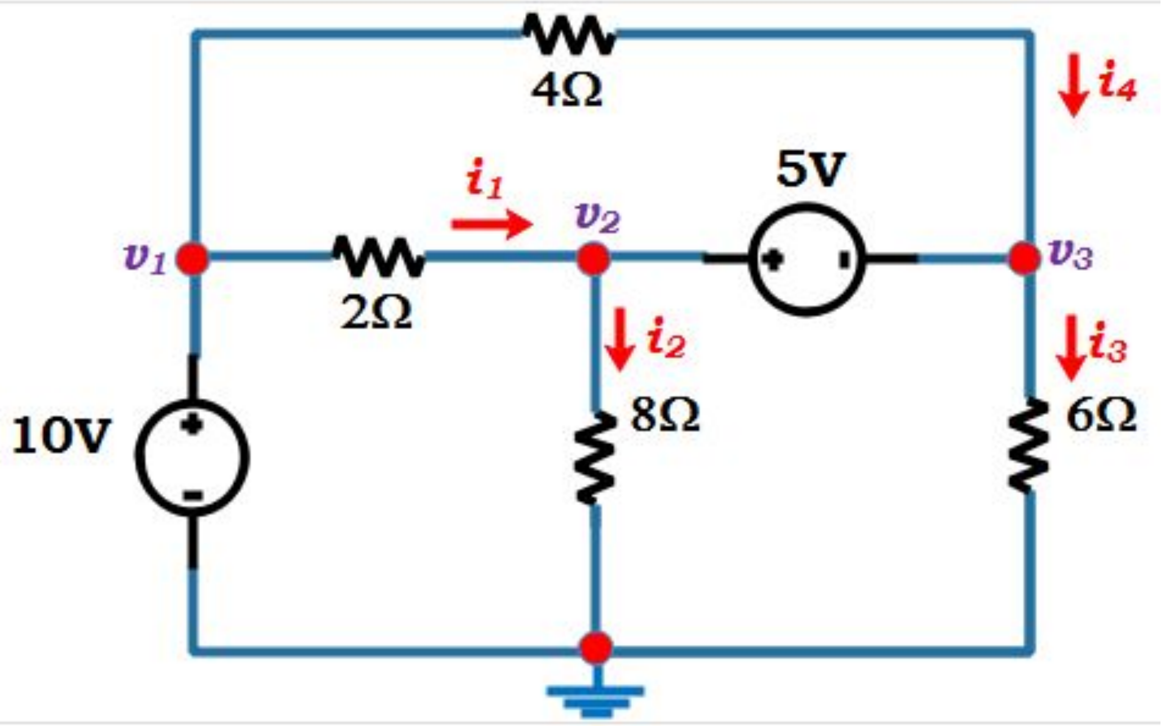
Nodal Analysis with Voltage Sources:

- "*Node voltage* is defined as the voltage rise from the reference node to the corresponding nonreference node"

Case:1

- If a voltage source is connected between the *reference node and a non-reference node*, we simply set the voltage at the non-reference node equal to the voltage of the voltage source.

$$V_1 = 10V$$



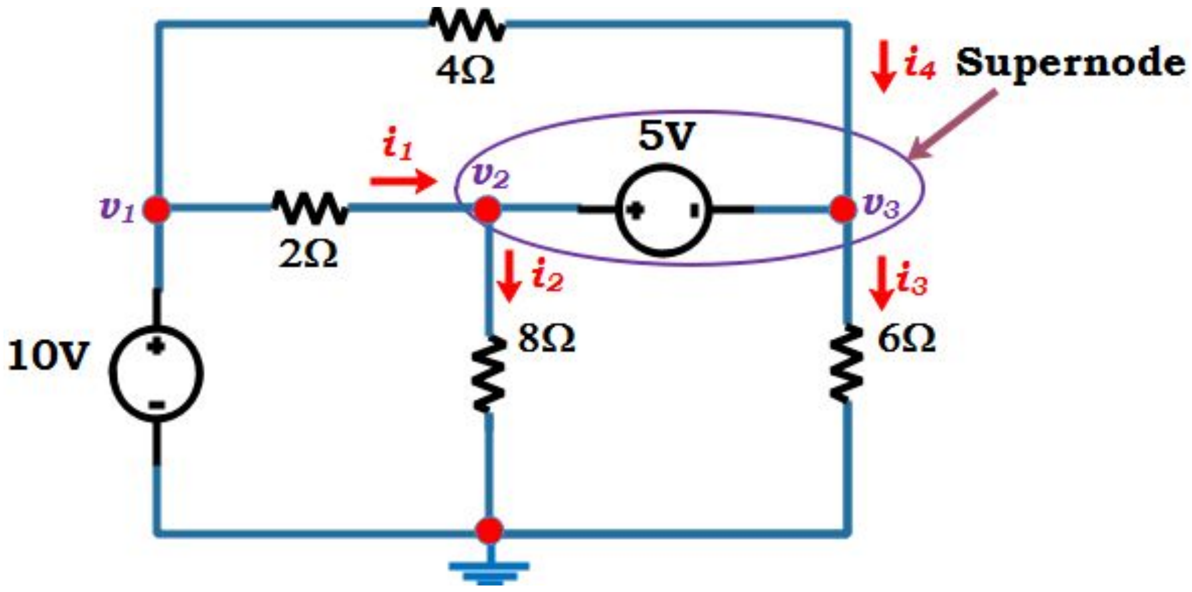
Note:

- *Reference node is ground.*
- *Non-reference nodes are V_1, V_2 & V_3.*

Nodal Analysis with Voltage Sources:

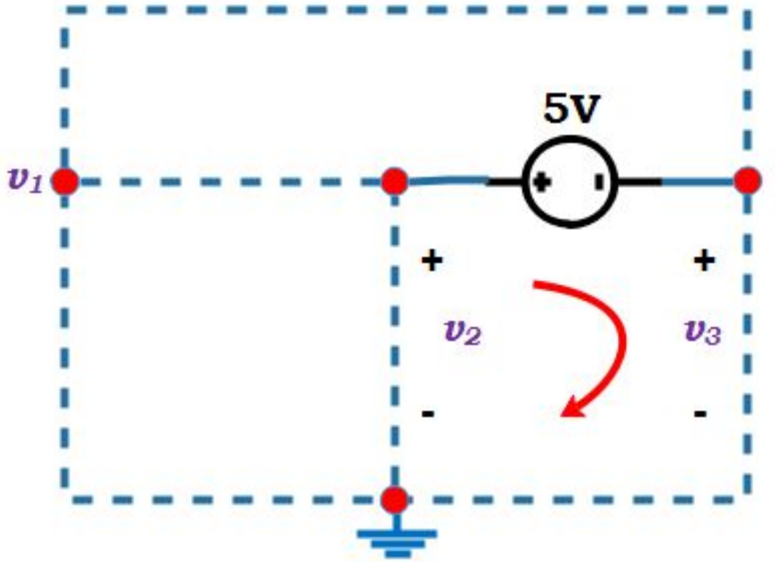
Case:2

- **If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a *generalized node or supernode*; we apply both KCL and KVL to determine the node voltages.**



$$V_2 - V_3 = 5V$$

- **A *supernode* is formed by enclosing a voltage source connected *between two non-reference nodes* and any elements connected in parallel with it.**

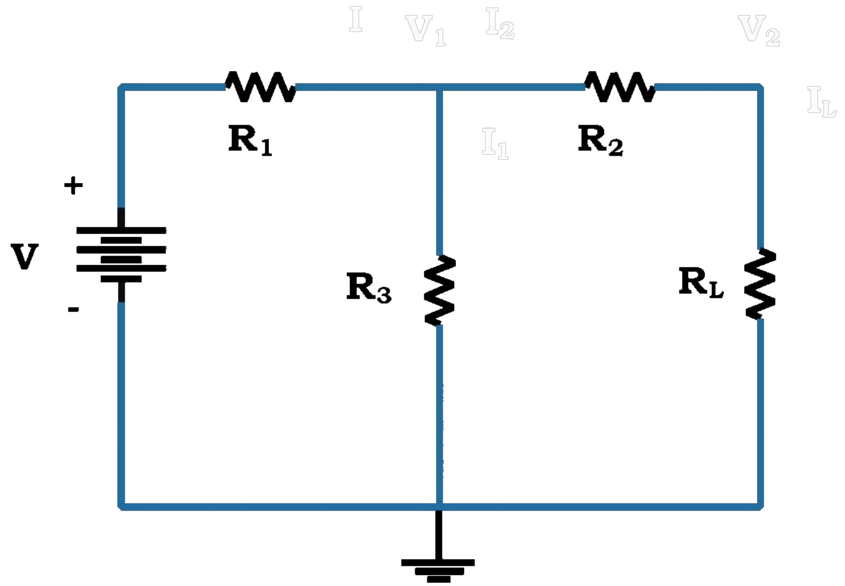


Mesh current analysis of DC resistive circuit:

Steps to Determine Mesh Current:

1. Assign mesh currents $(i_1, i_2, \dots . i_{n-1})$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting simultaneous equations to obtain the mesh currents.

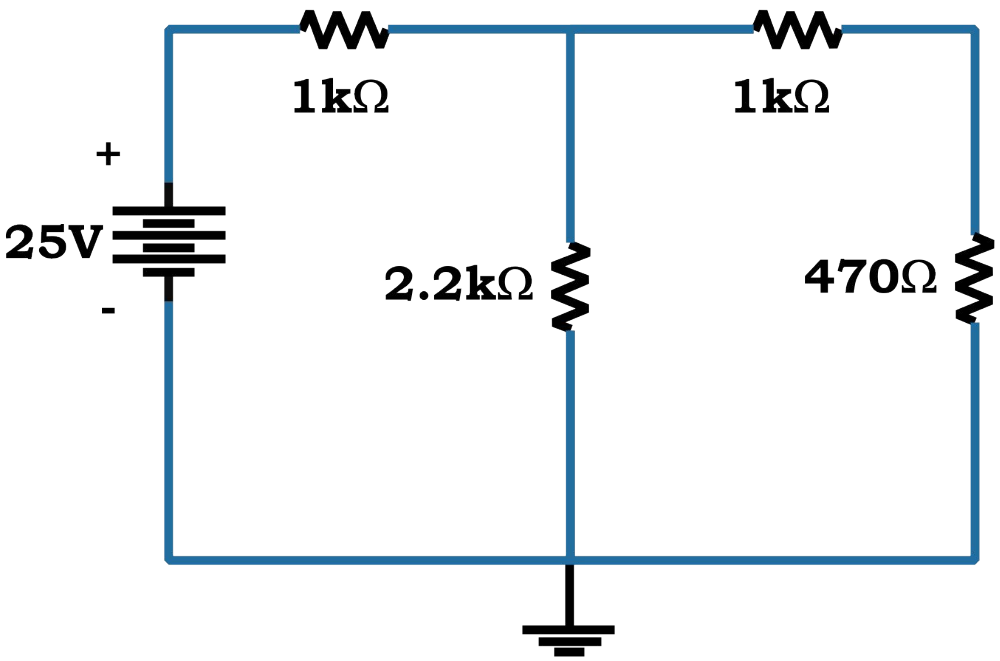
Step-0



Problem-1.14

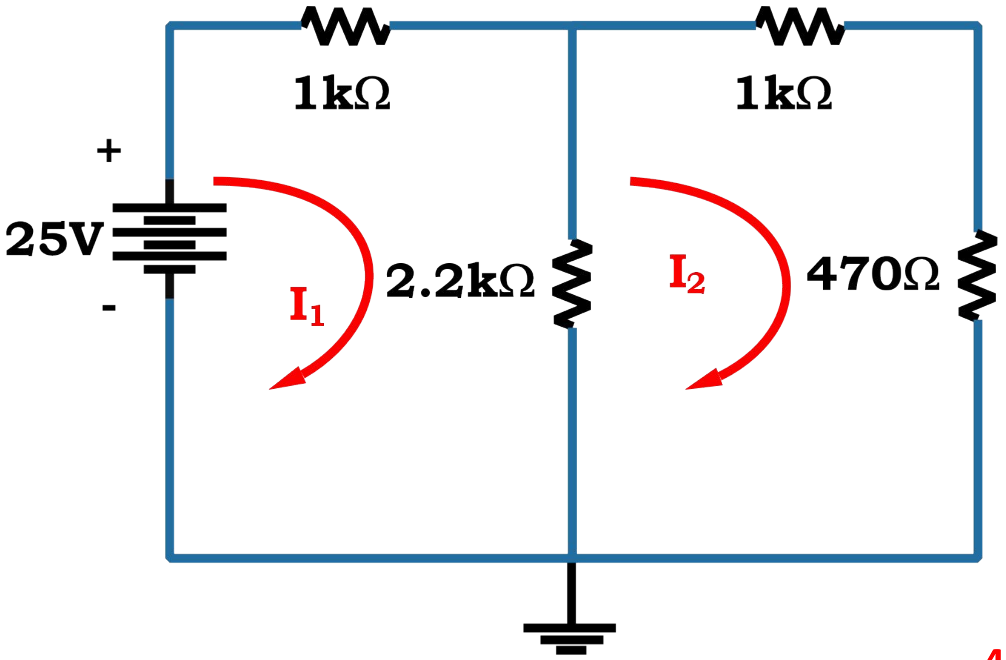
Calculate the mesh currents in the circuit shown in Fig.

Sol:



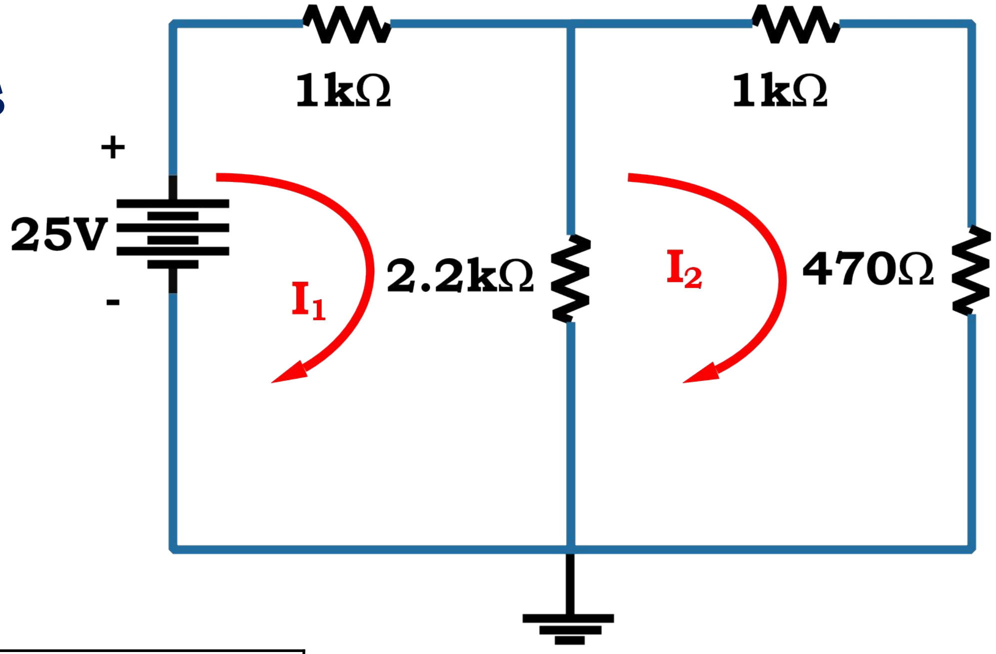
Step-1

Assign mesh currents $(i_1, i_2, \dots . i_{n-1})$ to the n meshes.



Step-2

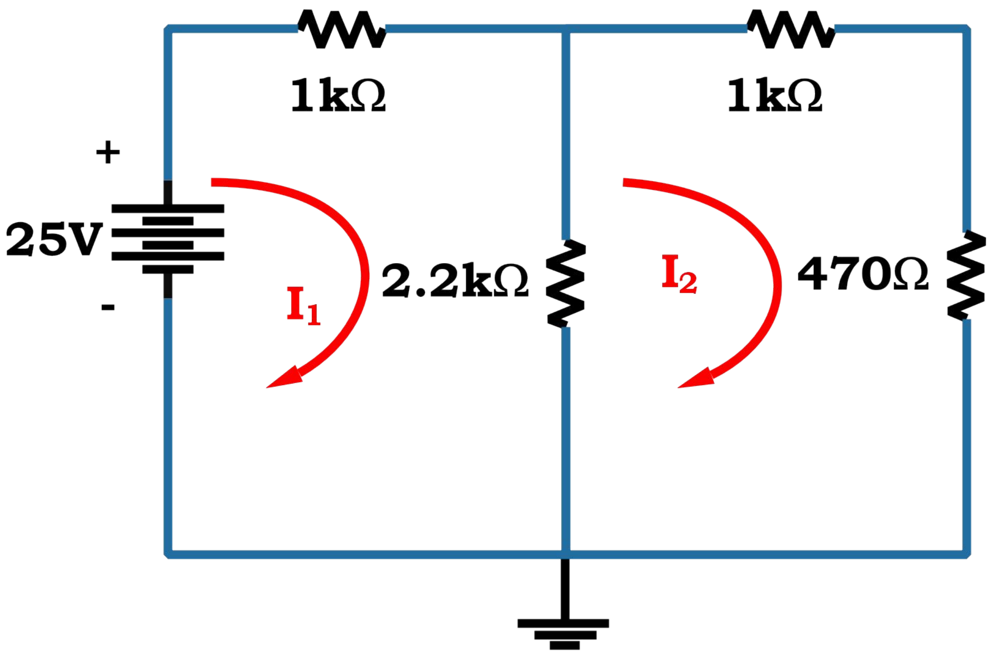
Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.



KVL to loop-1	KVL to loop-2

Step-3

Solve the resulting simultaneous equations to obtain the mesh currents.



$$3.2k\, I_1 - 2.2k\, I_2 = 25$$

$$2.2k\, I_1 - 3.67k\, I_2 = 0$$

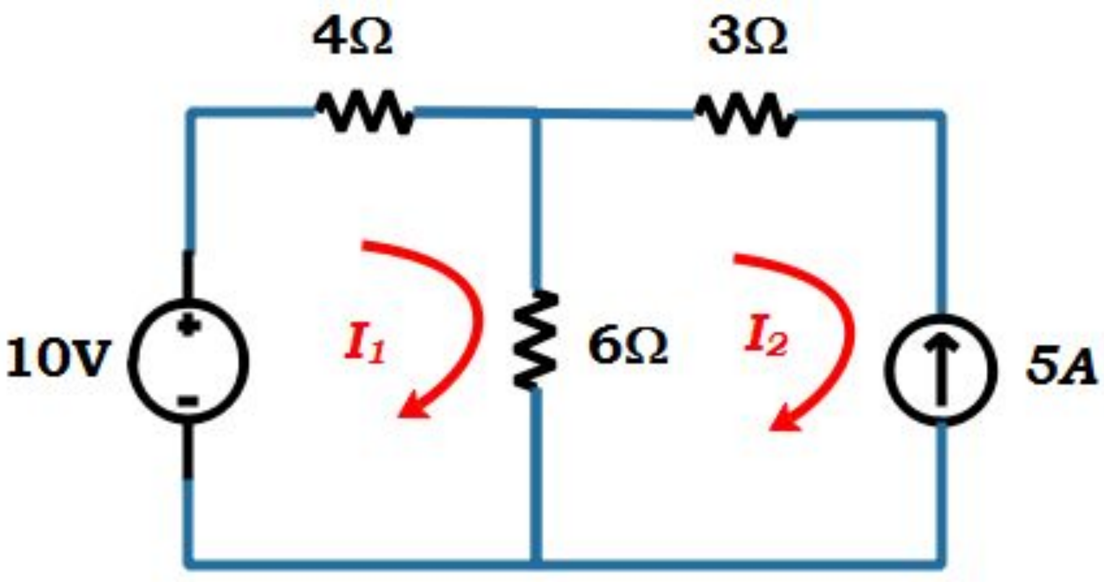
$$I_1 = 13mA$$
$$I_2 = 8mA$$

Mesh Analysis with Current Sources:

- "*Mesh current* is defined as a current through a mesh"

Case:1

- When a current source exists only in one mesh: Consider the circuit in Fig.,



- For example, We set $I_2 = -5A$ and write a mesh equation for the other mesh in the usual way; that is,.

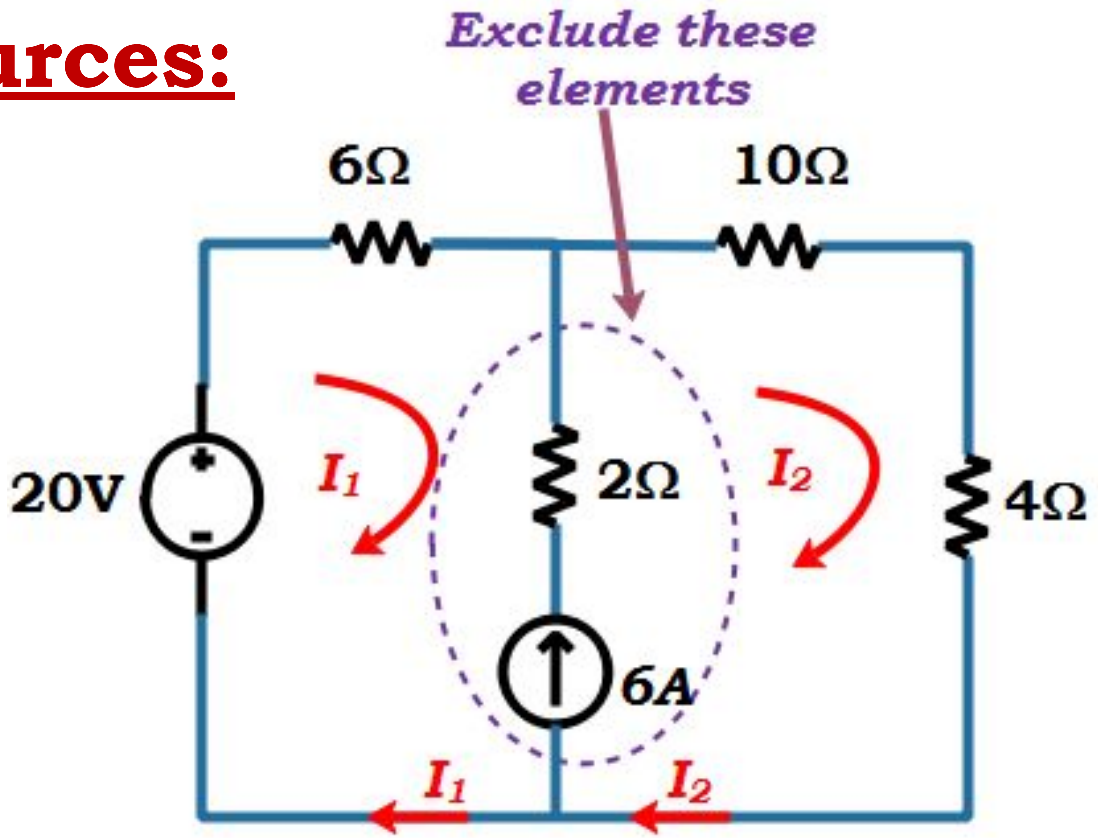
$$10 = 4I_1 + 6(I_1 - I_2)$$

$$I_1 = -2A$$

Mesh Analysis with Current Sources:

Case:2

- When a current source exists between two meshes: Consider the circuit in Fig (a),
- For example, We create a supermesh by excluding the current source and any elements connected in series with it, as shown in Fig(b).
- A *supermesh* results when two meshes have a current source in common.



Applying KCL in Fig. (a),

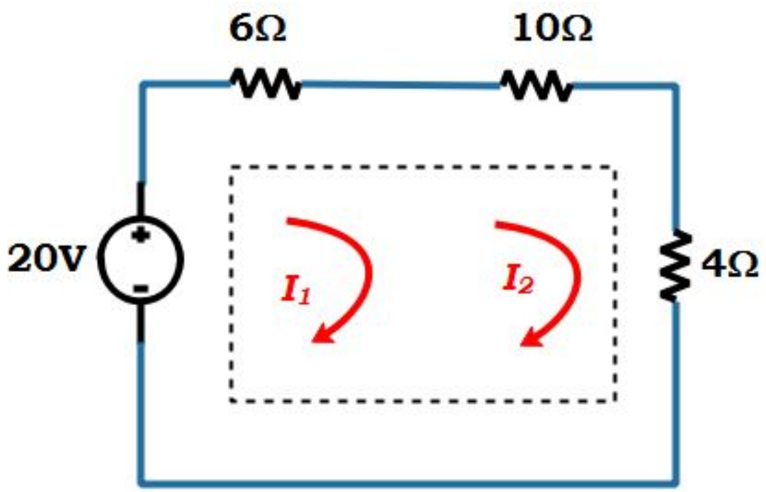
$$i_2 = i_1 + 6$$

$$i_1 - i_2 = -6 \rightarrow 1$$

Applying KVL to supermesh in Fig. (b),

$$20 = 6i_1 + 10i_2 + 4i_2$$

$$6i_1 + 14i_2 = 0 \rightarrow 2$$



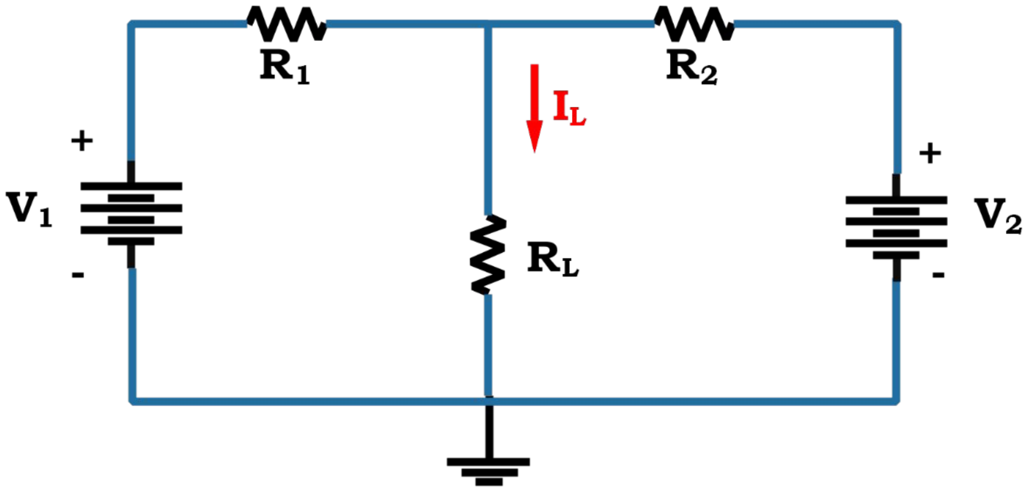
Solving equation 1 & 2,

$$i_1 = -3.2A; \quad i_2 = 2.8A$$

1.4. Network Theorems

Superposition Theorem:

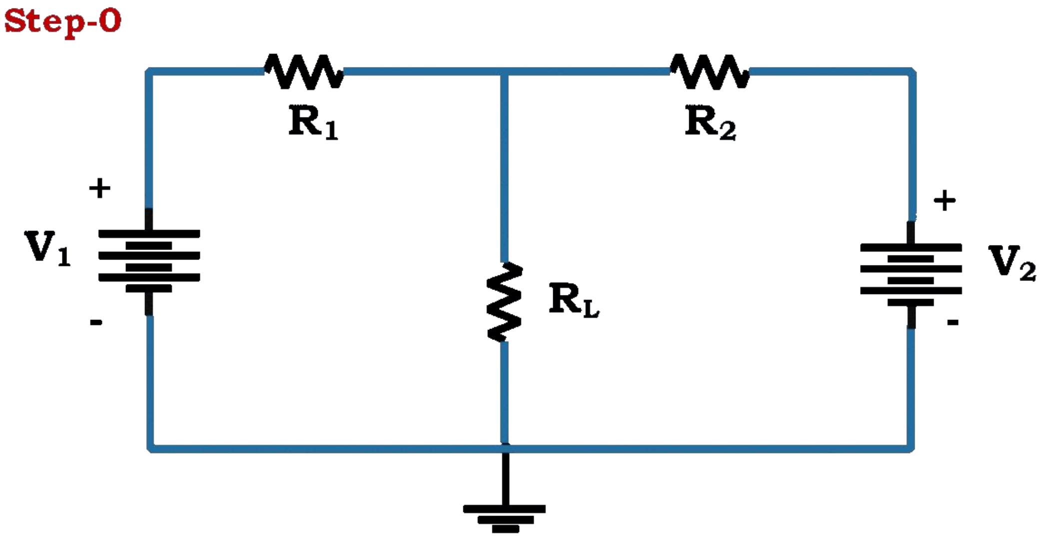
"The voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone".



Note: Independent sources are voltage or current sources or combination of voltage and current sources

Procedure to find a current (I_L) through the load resistance (R_L) as shown in fig, using Superposition theorem:

1. Retain one source at a time in the circuit and replace all other sources with their internal resistances.
2. Determine the output (current or voltage) due to the single source acting alone using Voltage or current division / mesh or node analysis methods.
3. Retain another source at a time in the circuit and replace all other sources with their internal resistances.
4. Determine the output (current or voltage) due to the single source acting alone using Voltage or current division / mesh or node analysis methods.
5. Find the total contribution by adding algebraically all the contributions due to the independent sources.



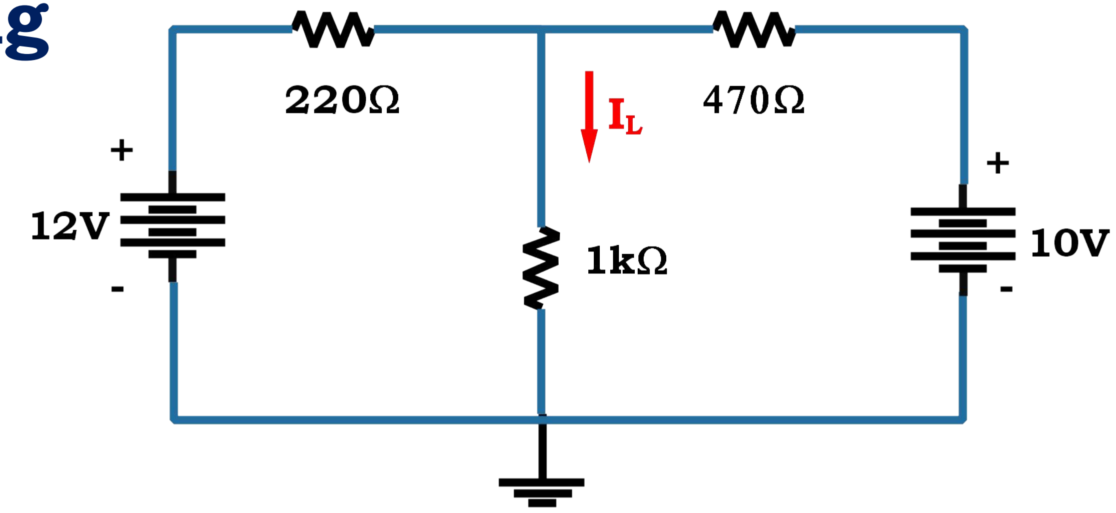
$$I_L = I_1 + I_2$$

Note:

- *Voltage sources should be short-circuited.*
- *Current sources should be open-circuited*

Problem-1.15

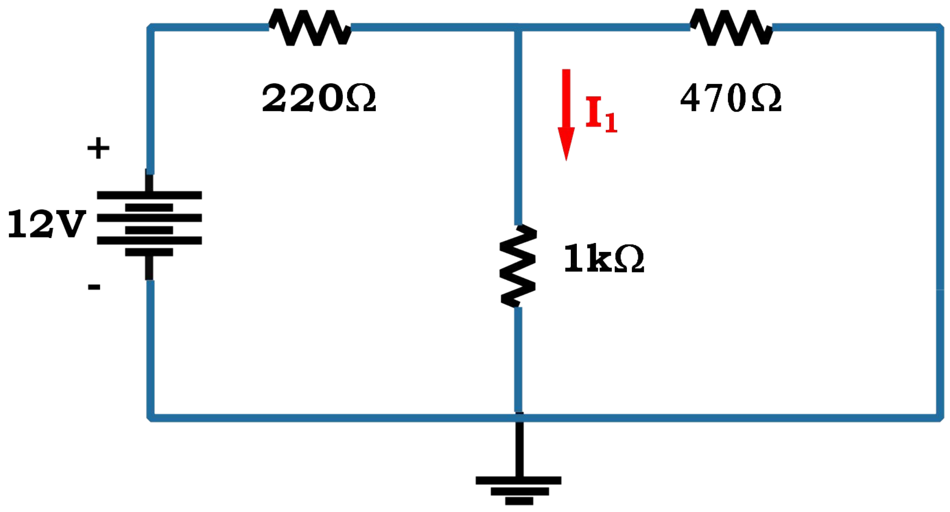
Find I_L in the circuit of Fig. using superposition.



Sol:

Step-1

- **Retain V_1 source at a time in the circuit and replace all other sources with their internal resistances.**



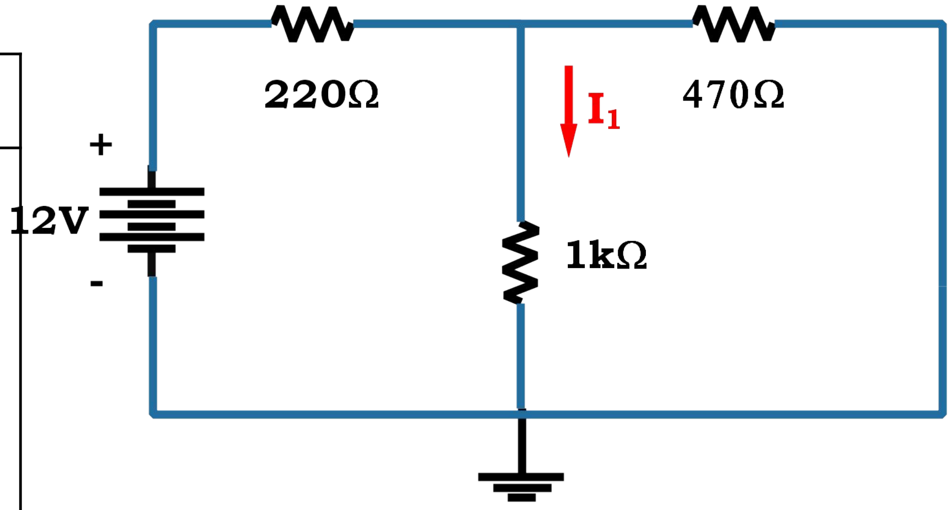
Note:

- Voltage sources should be short-circuited.
- Current sources should be open-circuited.

Step-2

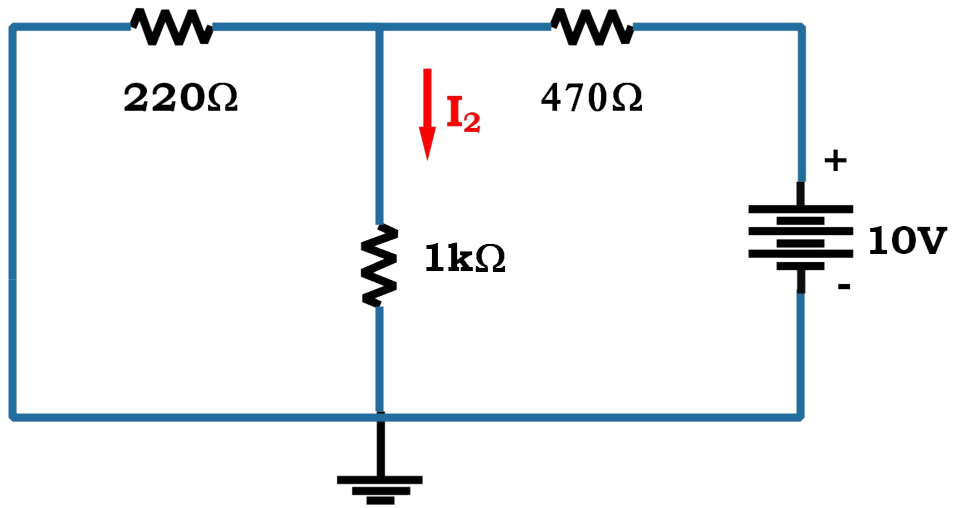
Determine the output current I_1 due to the V_1 source acting alone.

To calculate current through 1kΩ resistor



Step-3

- Retain V_2 source at a time in the circuit and replace all other sources with their internal resistances.



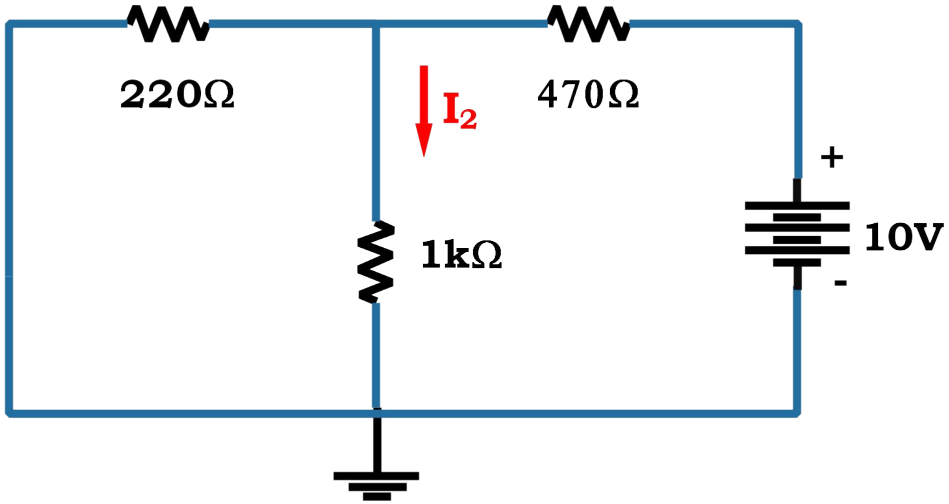
Note:

- Voltage sources should be short-circuited.
- Current sources should be open-circuited.

Step-4

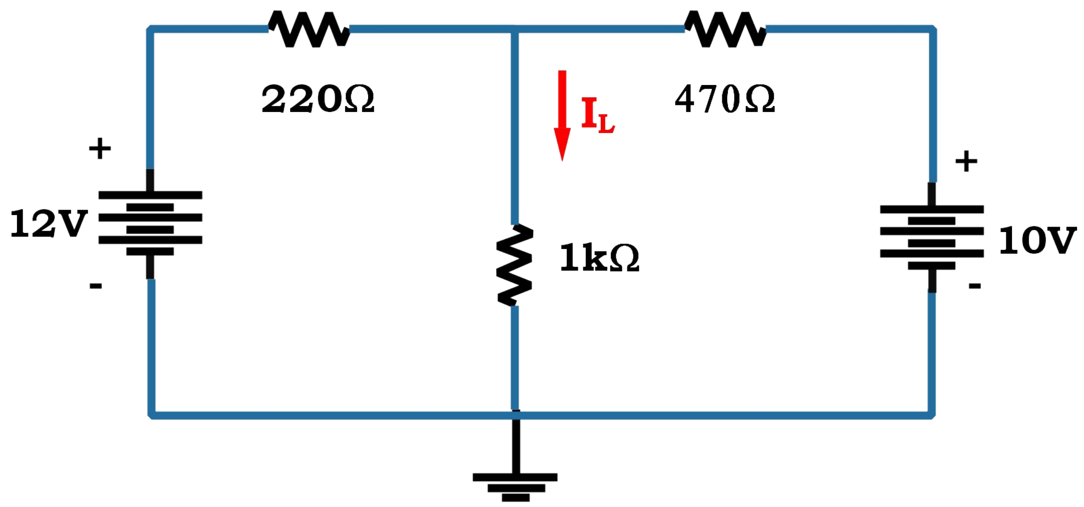
Determine the output current I_2 due to the V_2 source acting alone.

To calculate current through 1kΩ resistor



Step-5

Find the total contribution by adding algebraically all the contributions due to the independent sources.

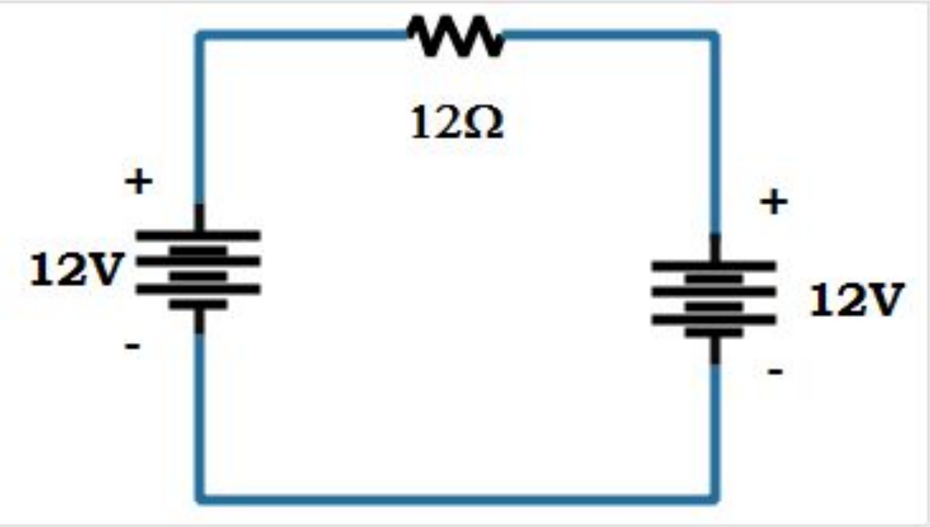


The total current (I_L) flowing through $1k\Omega$ load as shown in Fig. is,

$$I_L = I_1 + I_2 = 9.8mA$$

Limitations of superposition theorem:

- **Superposition theorem doesn't work for power calculation. Because power calculations involve either the product of voltage and current, the square of current or the square of the voltage, they are not linear operations.**
- **Superposition theorem can not be applied for non linear circuit (Diodes or Transistors).**



$$P_{W1} = 12\ watts$$

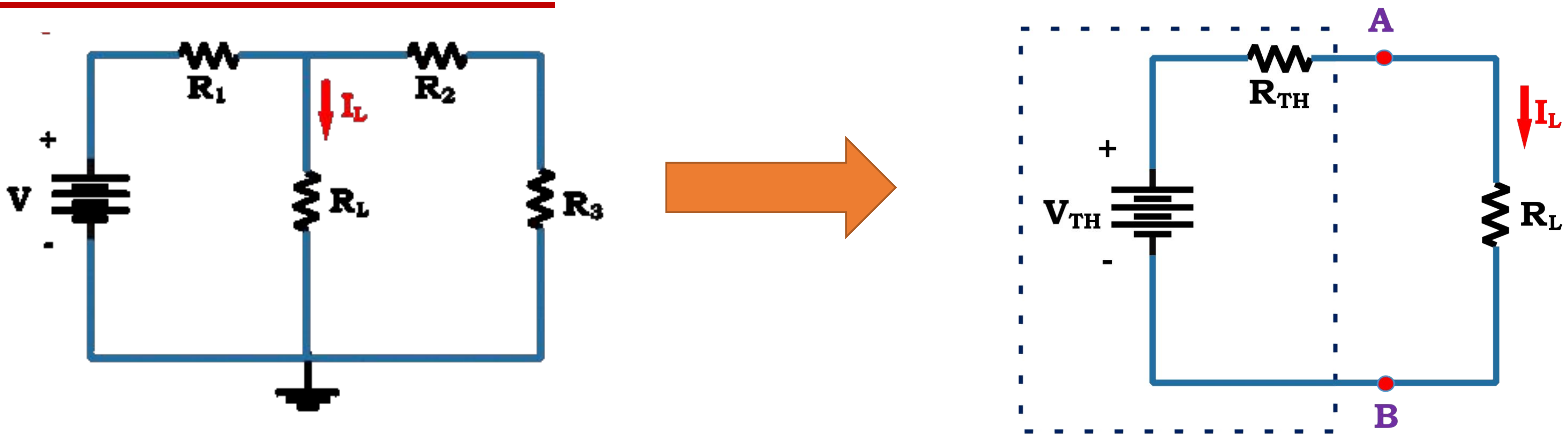
$$P_{W2} = 12\ watts$$

Then, According to superposition theorem, Total power consumed by 12 Ω

$$P_W = 24\ watts$$

This result is conceptually wrong.!

Thevenin’s Theorem:



“A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source (V_{Th}) in series with a resistor (R_{Th}).”

where,

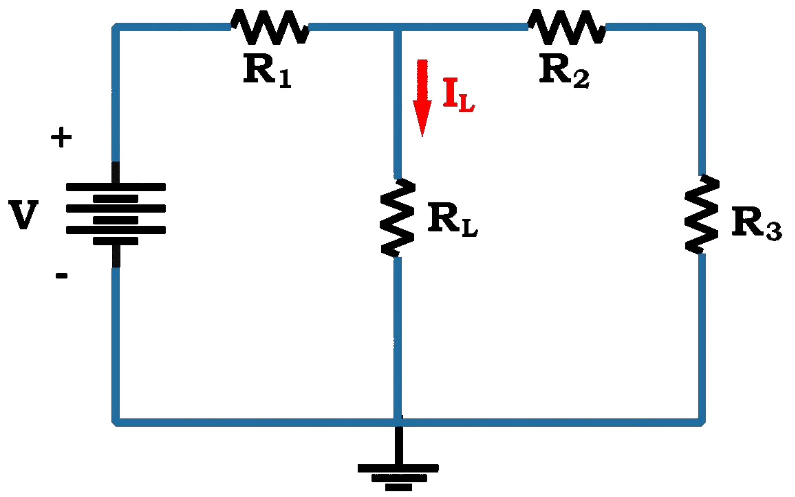
$V_{Th} = V_{oc}$ □ **Open circuit voltage at terminals**

R_{Th} □ **Equivalent resistant of network**

Procedure to find a current (I_L) through the load resistance (R_L) as shown in fig, using Thevenin's theorem:

1. Disconnect the load resistance (R_L) from the circuit, as indicated in fig.
2. Calculate the open-circuit voltage (V_{Th}) at the load terminals using mesh-current (or) node-voltage method.
3. Redraw the circuit with each practical source replaced by its internal resistance.
4. Calculate the equivalent resistance (R_{Th}) that would exist between the load terminals.
5. Place (R_{Th}) in series with (V_{Th}) to form the Thevenin's equivalent circuit.
6. Reconnect the original load to the Thevenin voltage circuit. The load's voltage, current and power may be calculated by

Step-0



$$I_L = \frac{V_{th}}{R_{th}+R_L}$$

$$V_L = I_L \times R_L$$

$$P_L = I_L^2 \times R_L$$

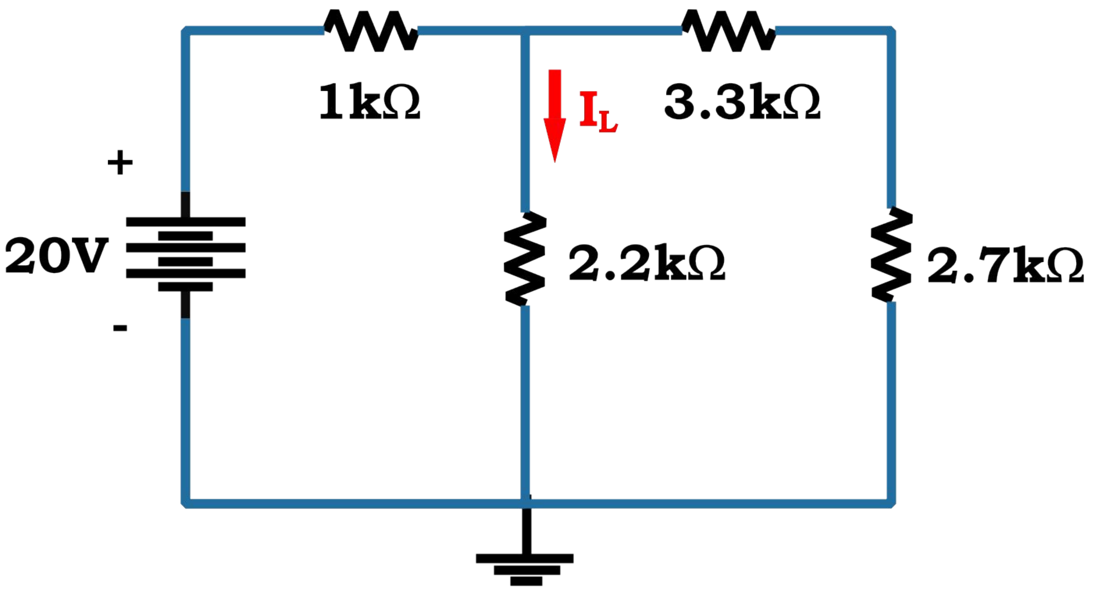
Note in step-3:

- *Voltage sources should be short-circuited.*
- *Current sources should be open-circuited*

Problem-1.16

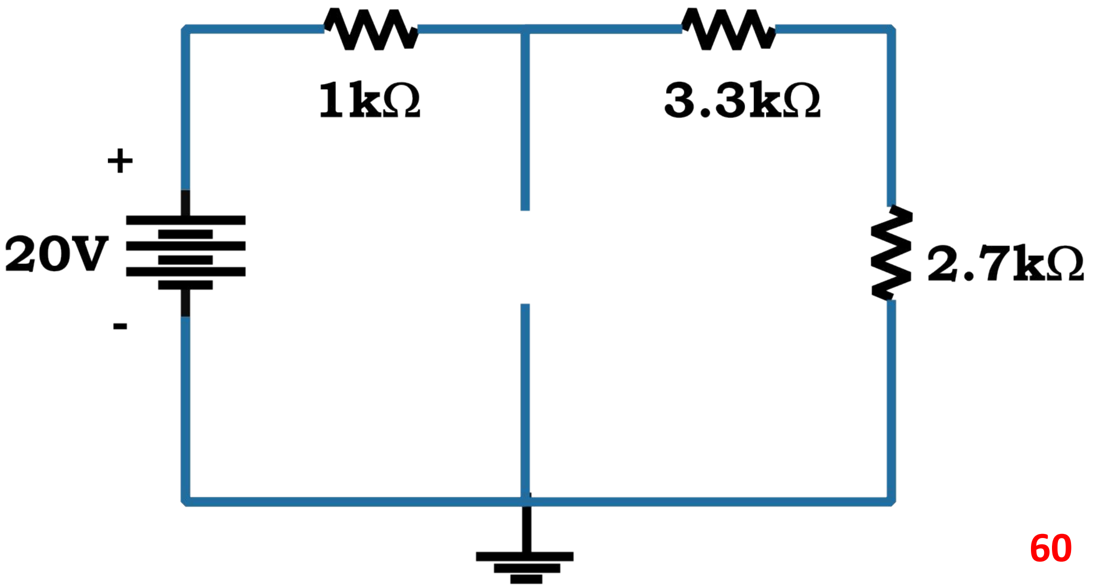
Find the Thevenin equivalent circuit for the circuit shown in Fig., at load terminals. Then find I_L

Sol:



Step-1

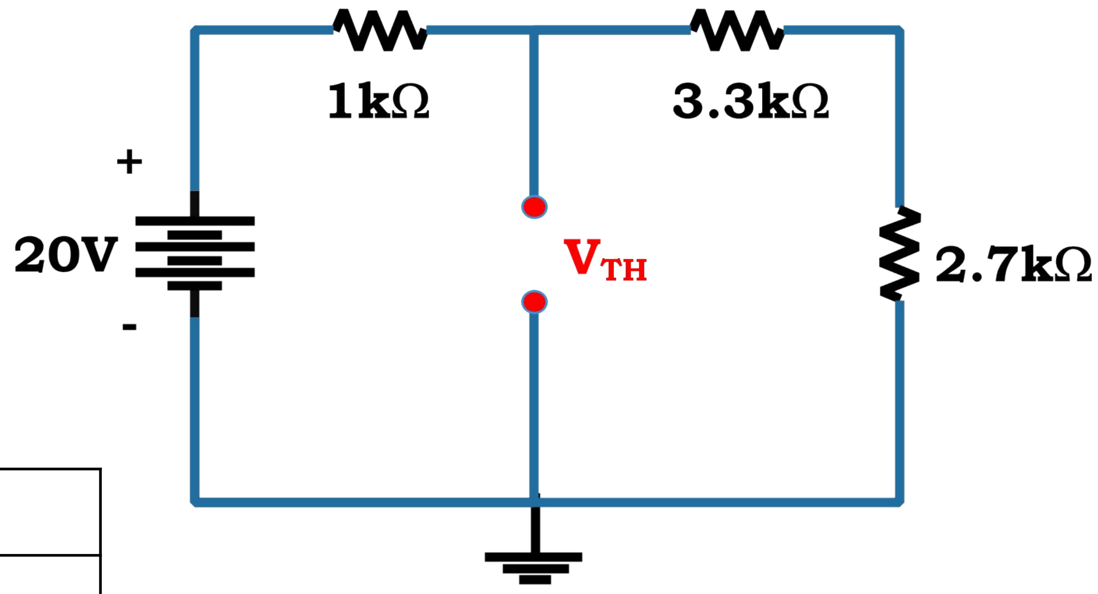
- **Disconnect the load resistance (R_L) from the circuit, as indicated in fig.**



Step-2

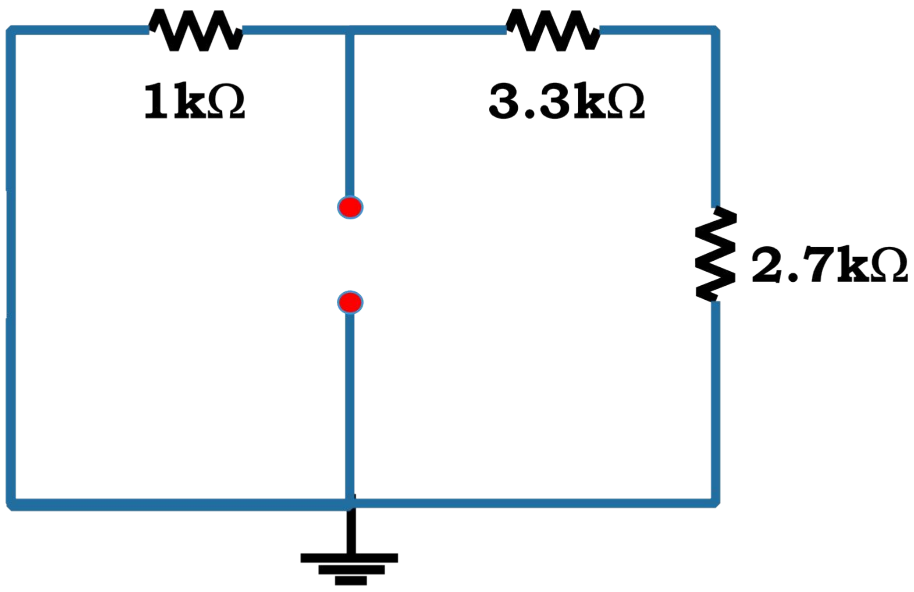
Calculate the open-circuit voltage (V_{Th}) at the load terminals using mesh-current (or) node-voltage method.

Method-1	Method-2



Step-3

Redraw the circuit with each practical source replaced by its internal resistance.

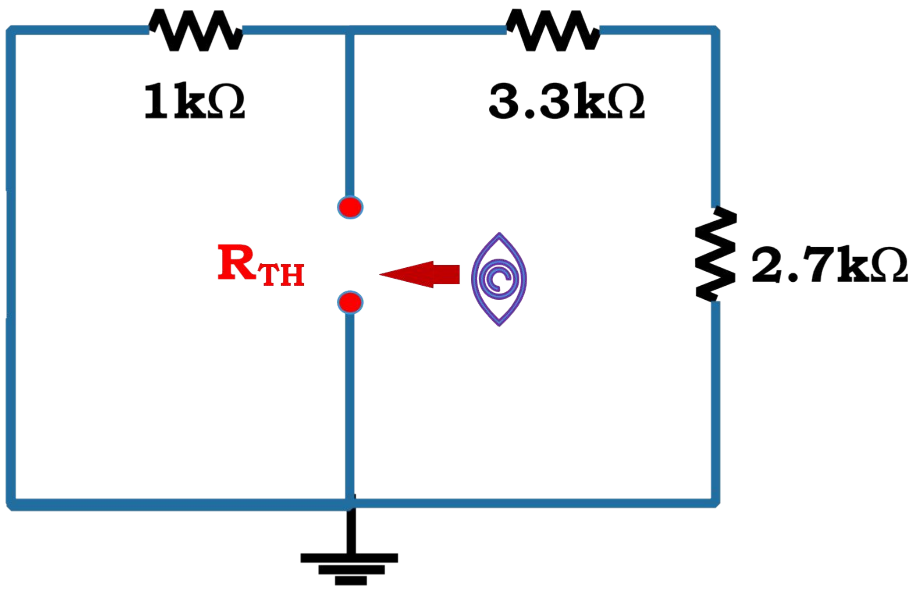


Note:

- **Voltage sources should be short-circuited.**
- **Current sources should be open-circuited.**

Step-4

Calculate the equivalent resistance (R_{Th}) that would exist between the load terminals.

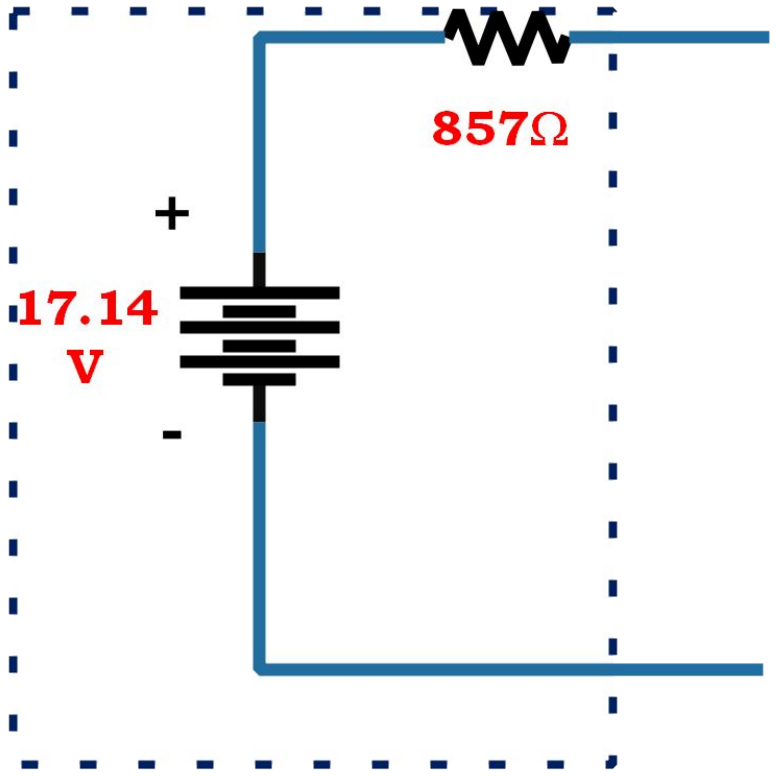


$$R_{th} = \frac{R_1 \times (R_2 + R_3)}{R_1 + R_2 + R_3}$$

$$R_{th} = \frac{1k \times 6k}{3.3k + 2.7k + 1k} = 857\Omega$$

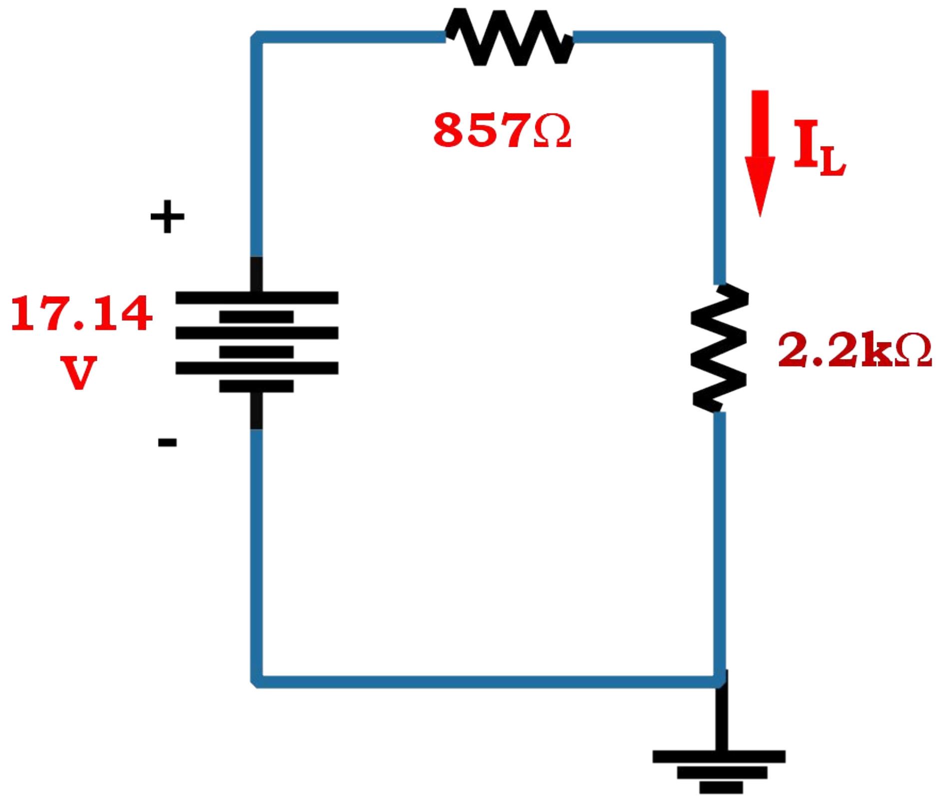
Step-5

Place (R_{Th}) in series with (V_{Th}) to form the Thevenin's equivalent circuit.



Step-6

Reconnect the original load to the Thevenin voltage circuit. The load's voltage, current and power may be calculated by

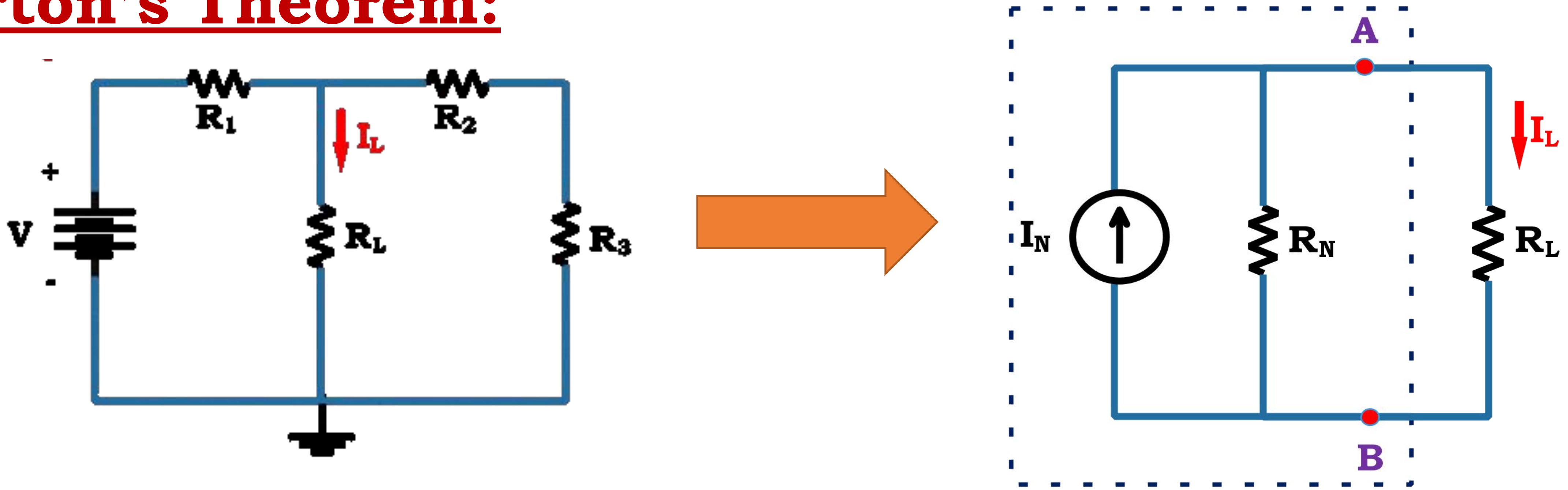


$$I_L = \frac{V_{th}}{R_{th}+R_L} = 5.607mA$$

$$V_L = I_L \times R_L = 12.3V$$

$$P_L = I_L^2 \times R_L = 69.2mW$$

Norton's Theorem:



"A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source *(I_N)* in parallel with a resistor *(R_N)*."

Where,

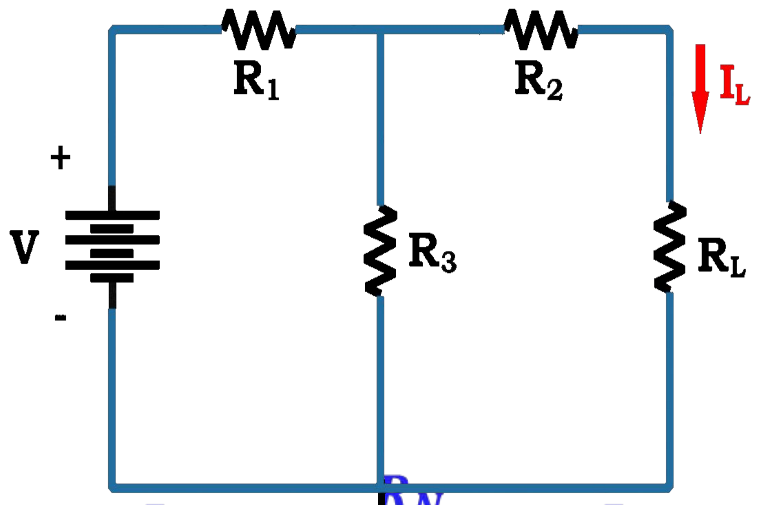
I_N=I_{sc} **Short circuit current through the terminals**

R_N **Equivalent resistant of network**

Procedure to find a current *(I_L)* through the load resistance *(R_L)* as shown in fig, using Norton's theorem:

1. Disconnect the load resistance *(R_L)* from the circuit, as indicated in fig.
2. Calculate the short-circuit current *(I_N)* at the load terminals using mesh-current (or) node-voltage method.
3. Redraw the circuit with each practical source replaced by its internal resistance.
4. Calculate the equivalent resistance *(R_N)* that would exist between the load terminals.
5. Place *(R_N)* in parallel with *(I_N)* to form the Norton's equivalent circuit.
6. Reconnect the original load to the Norton current circuit. The load's voltage, current and power may be calculated by

Step-0



$$I_L = \frac{R_N}{R_{th}+R_L} \times I_N$$

$$V_L = I_L \times R_L$$

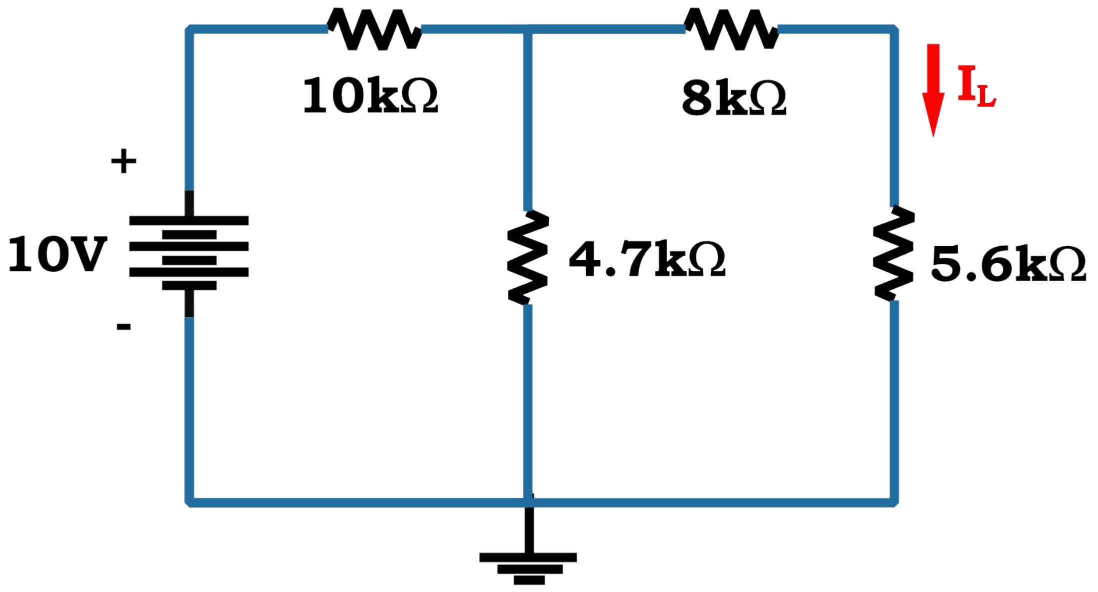
$$P_L = I_L^2 \times R_L$$

Note in step-3:

- *Voltage sources should be short-circuited.*
- *Current sources should be open-circuited*

Problem-1.17

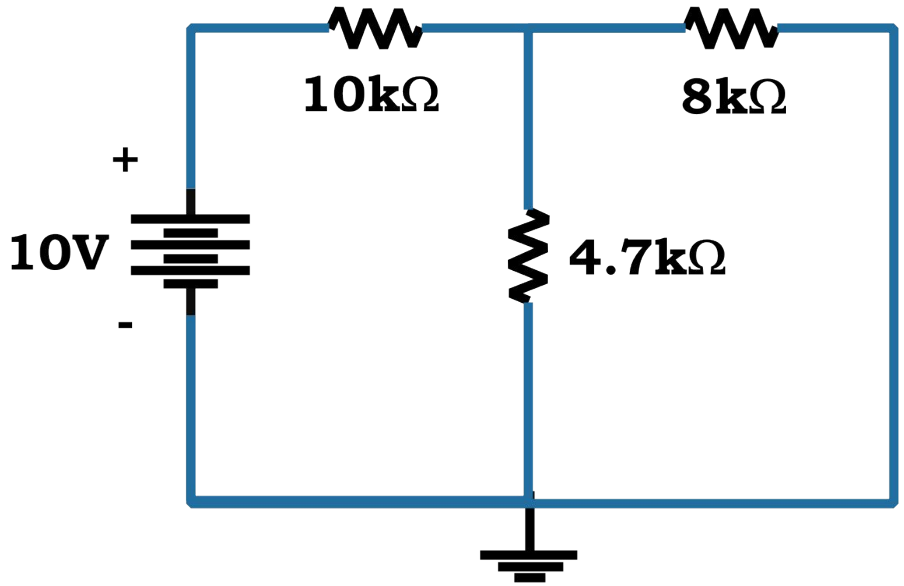
Find the Norton equivalent circuit for the circuit shown in Fig., at load terminals. Then find I_L



Sol:

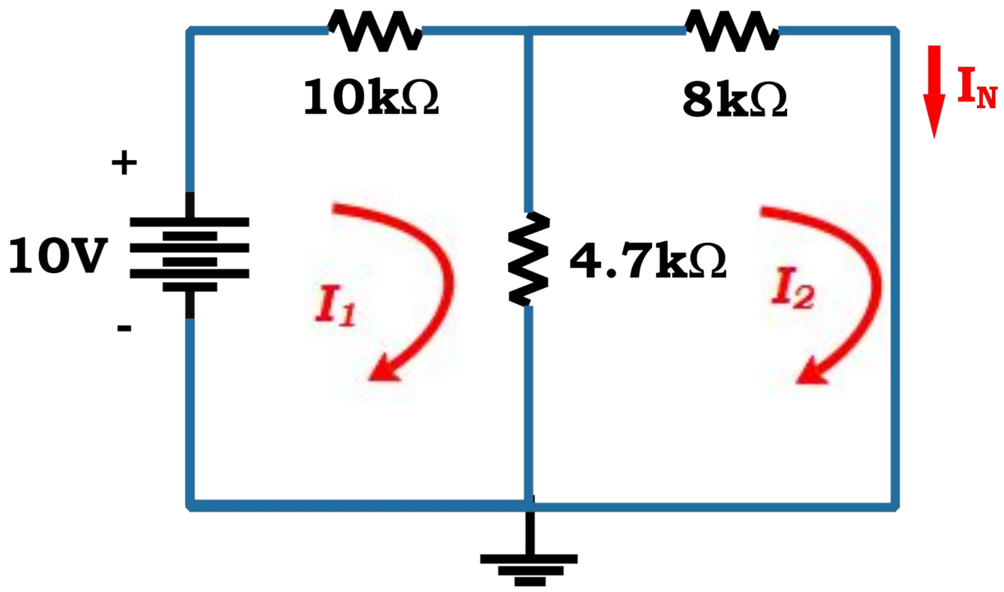
Step-1

- **Disconnect the load resistance (R_L) from the circuit, as indicated in fig.**



Step-2

Calculate the short-circuit current (I_N) at the load terminals using mesh-current (or) node-voltage method.



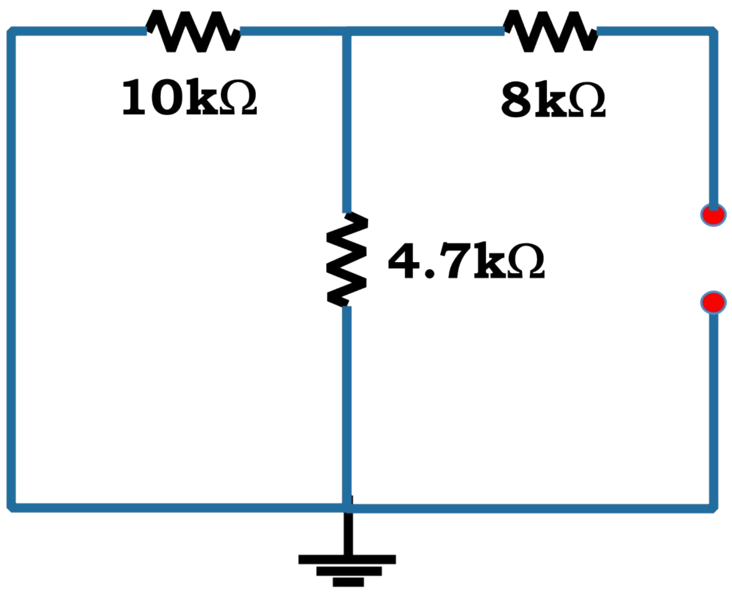
Mesh current Method

Step-3

Redraw the circuit with each practical source replaced by its internal resistance.

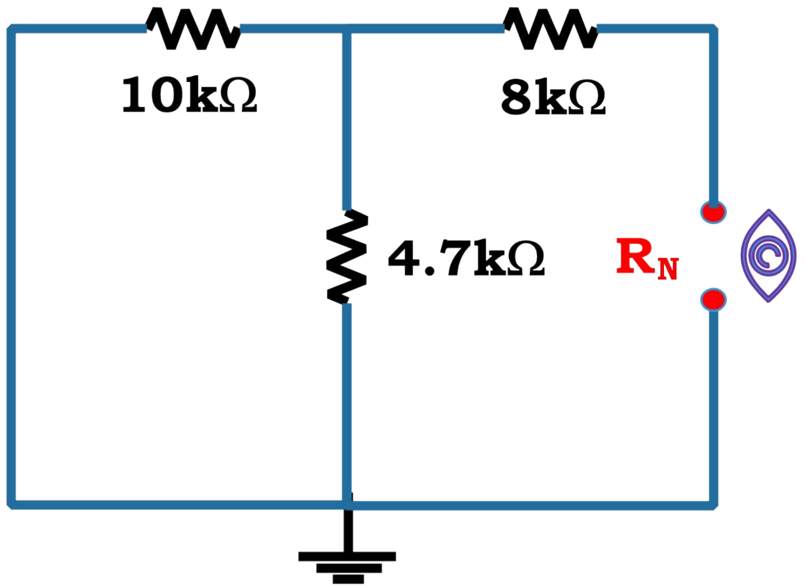
Note:

- **Voltage sources should be short-circuited.**
- **Current sources should be open-circuited.**



Step-4

Calculate the equivalent resistance (R_N) that would exist between the load terminals.

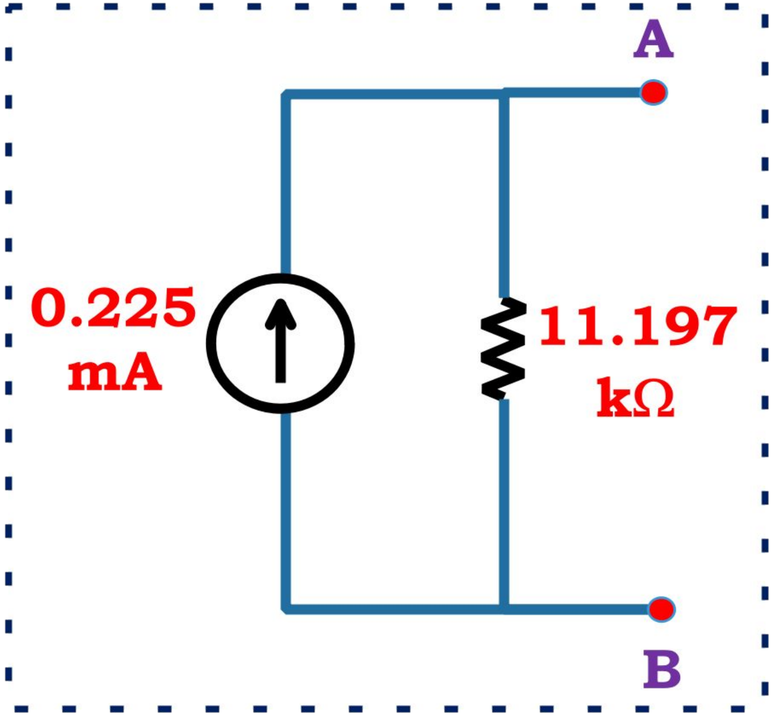


$$R_N = \mathrm{R}_2 + \frac{(R_1 \times R_3)}{R_1 + R_3}$$

$$R_N = 8\mathrm{k} + \frac{(10k \times 4.7k)}{10k + 4.7k} = 11.197\mathrm{k}\Omega$$

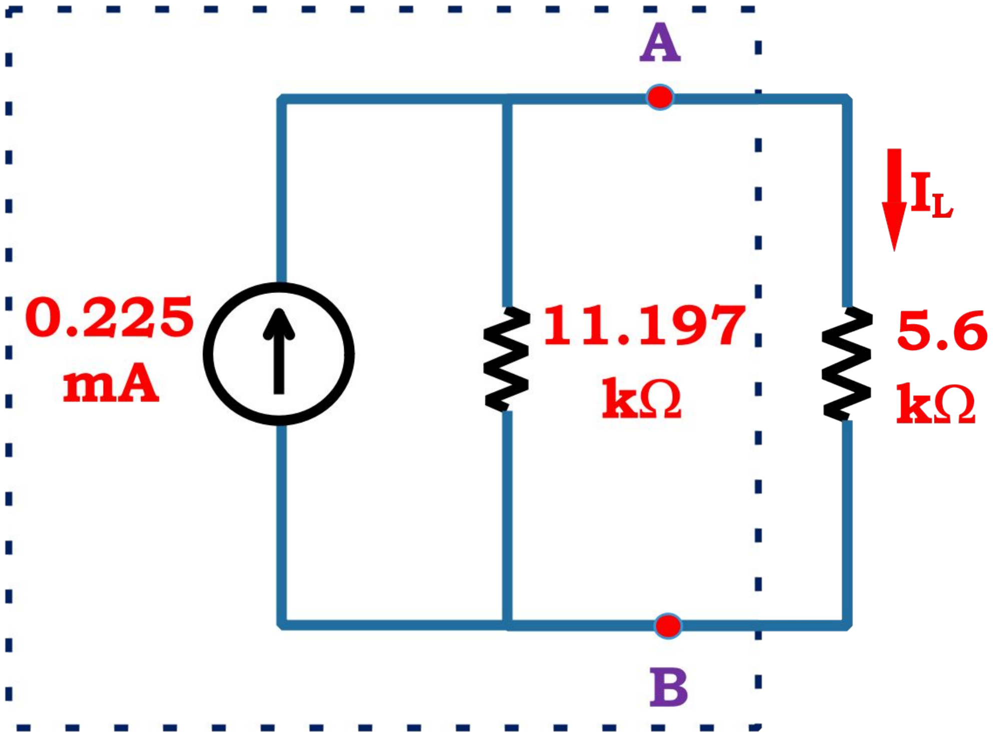
Step-5

Place (R_N) in parallel with (I_N) to form the Norton's equivalent circuit.



Step-6

Reconnect the original load to the Norton current circuit. The load's voltage, current and power may be calculated by



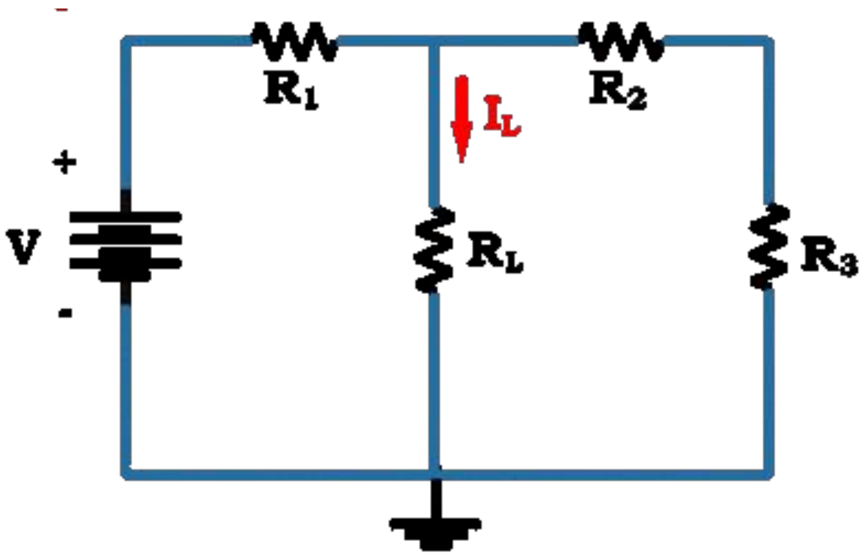
$$I_L = \frac{R_N}{R_N+R_L} \times I_N = \frac{11.2k}{11.2k+5.6k} \times 0.227m = 0.15mA$$

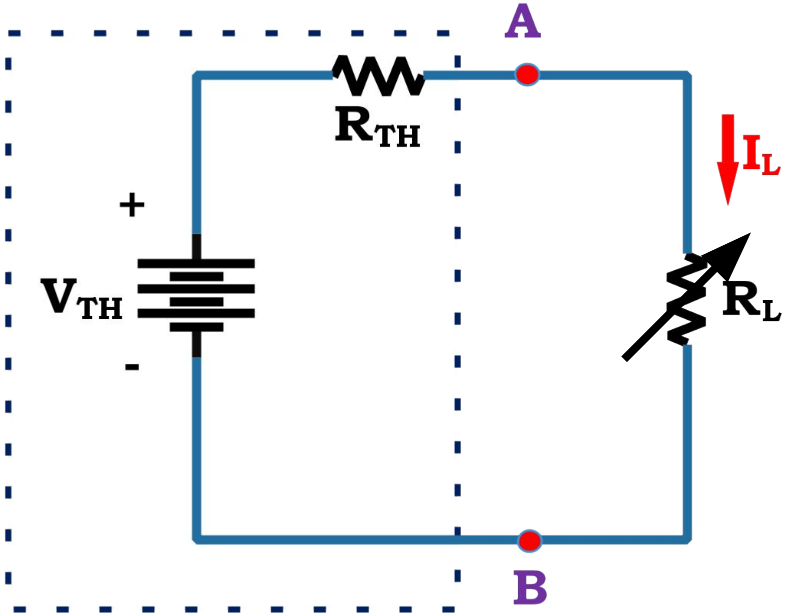
$$V_L = I_L \times R_L = 0.84V$$

$$P_L = I_L^2 \times R_L = 0.126W$$

Maximum Power Transfer Theorem:

$R_L = R_{Th}$





Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

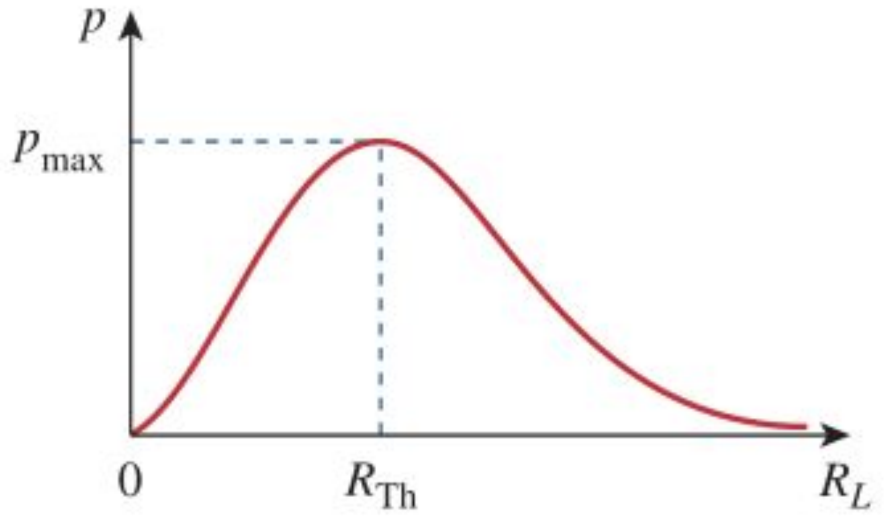
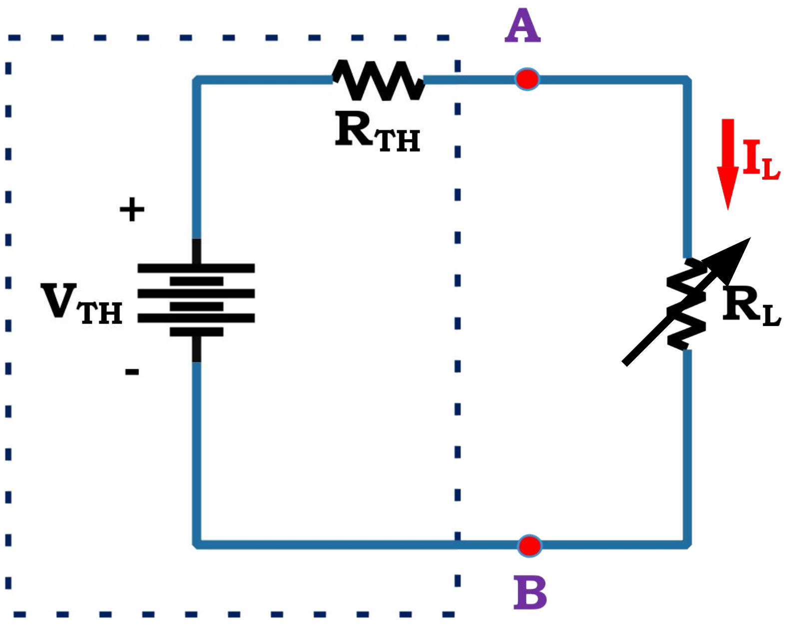


Fig. Power delivered to the load as a function of R_L

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig, the power delivered to the load is



$$P = I^2 R_L = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L \rightarrow 1$$

To prove the maximum power transfer theorem, we differentiate Eq. (1) with respect to R_L and set the result equal to zero.

$$\frac{dP}{dR_L} = \left[\frac{(R_{th}+R_L)^2 - 2R_L(R_{th}+R_L)}{(R_{th}+R_L)^4}\right] V_{th}^2$$

$$= \left[\frac{R_{th}+R_L-2R_L}{(R_{th}+R_L)^3}\right] V_{th}^2 = 0$$

$$R_{th} + R_L - 2R_L = 0$$

$$R_L = R_{th} \qquad \rightarrow 2$$

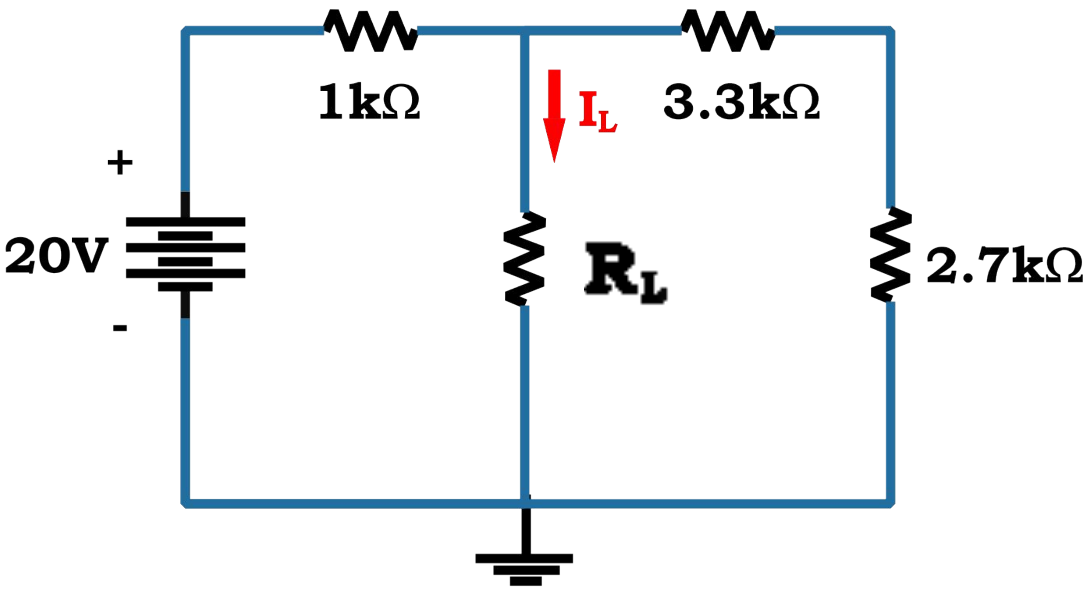
The maximum power transferred is obtained by substituting (2) in (1)

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

applies only when $R_L = R_{th}$

Problem-1.18

Find the value of R_L for maximum power transfer in the circuit shown in Fig. Find the maximum power P_{max}



Sol:

- Calculate R_{Th} and V_{Th} to form the Thevenin's equivalent circuit.
- The maximum power transfer is,

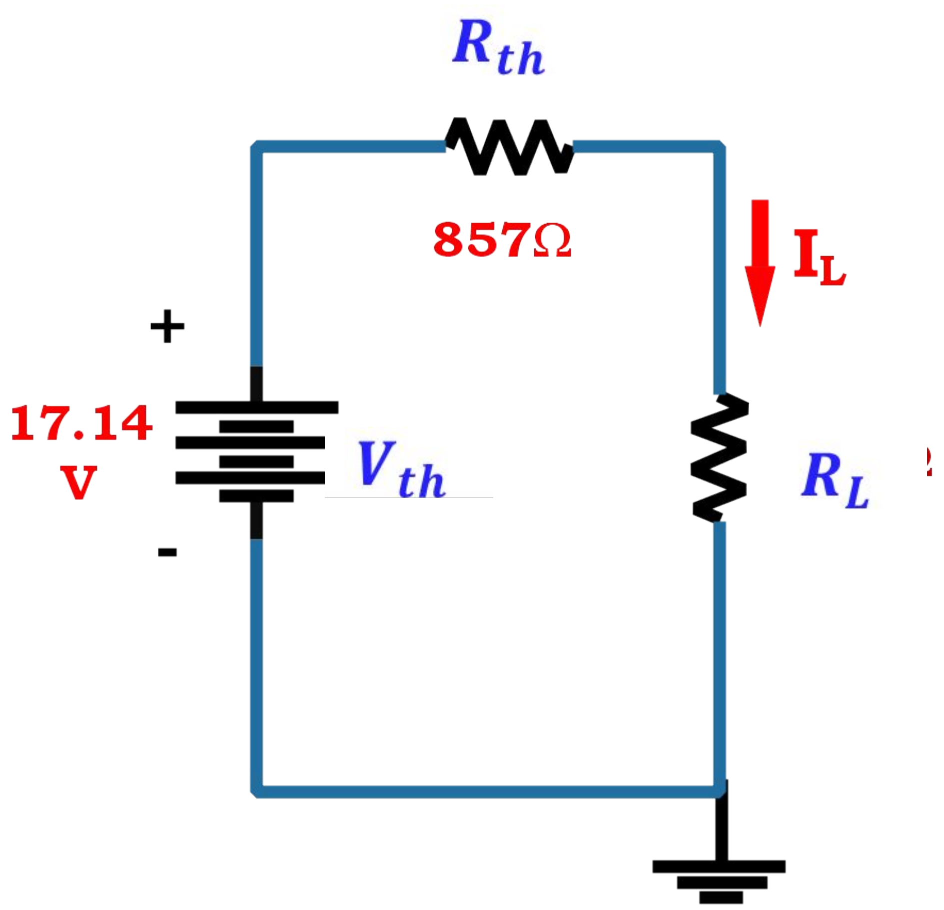
$$V_{th} = 17.14\ V$$

$$R_{th} = 857\ \Omega$$

For maximum power transfer,

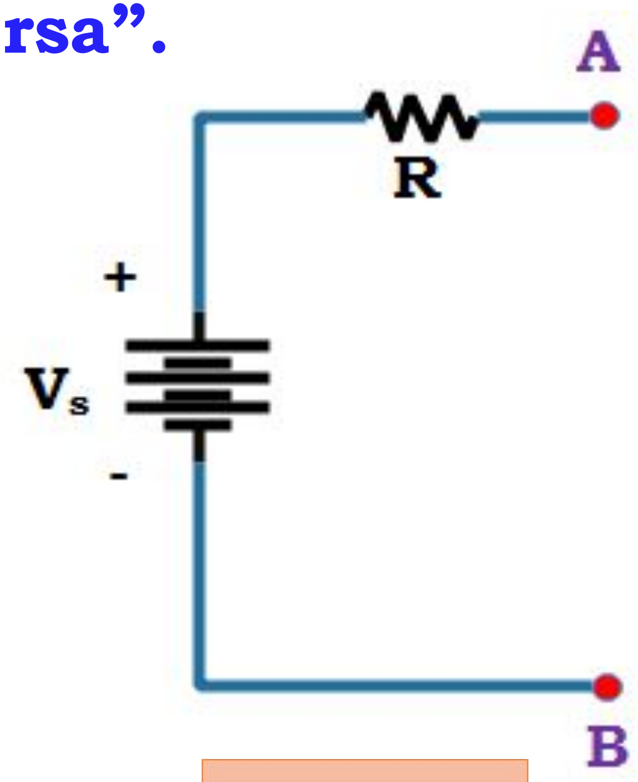
$$R_L = R_{th} = 857\ \Omega$$

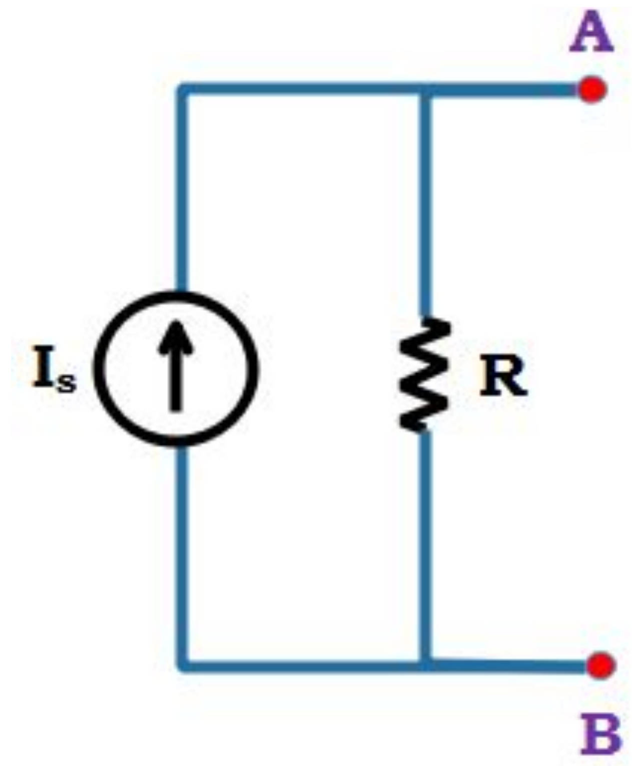
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = 85.7\ mW$$



Source Transformation:

"The process of replacing a voltage source (V_s) in series with a resistor (R) by a current source (I_s) in parallel with a resistor R, or vice versa".

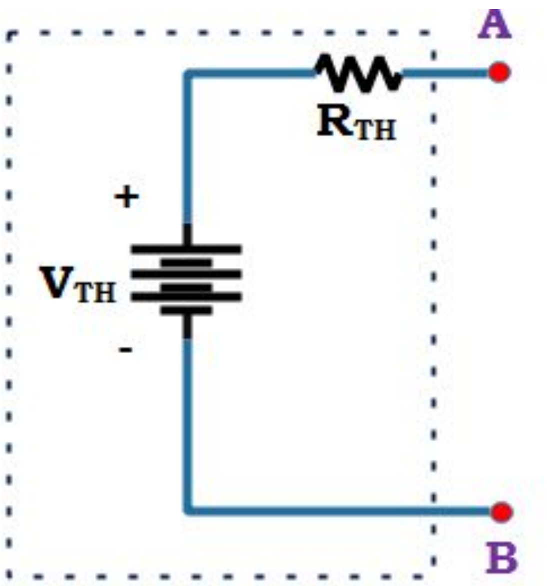




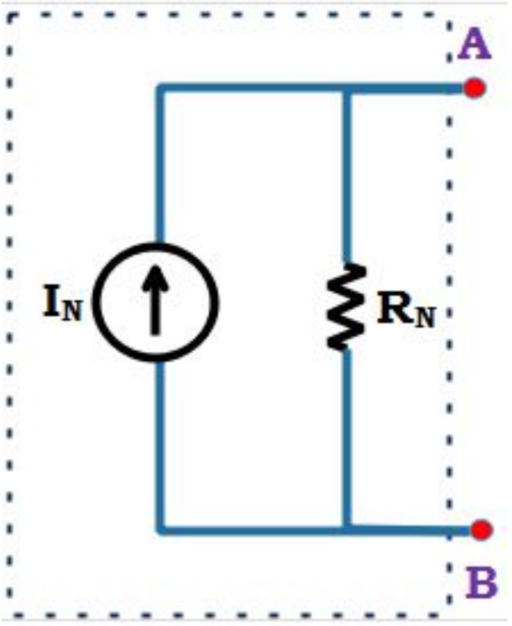
$$V_s = 36V$$
$$R = 6\Omega$$

$$I_s = 6A$$
$$R = 6\Omega$$

Relationship between Norton's and Thevenin's theorems



$$R_N = R_{Th}$$



$$V_{Th} = V_{oc}$$

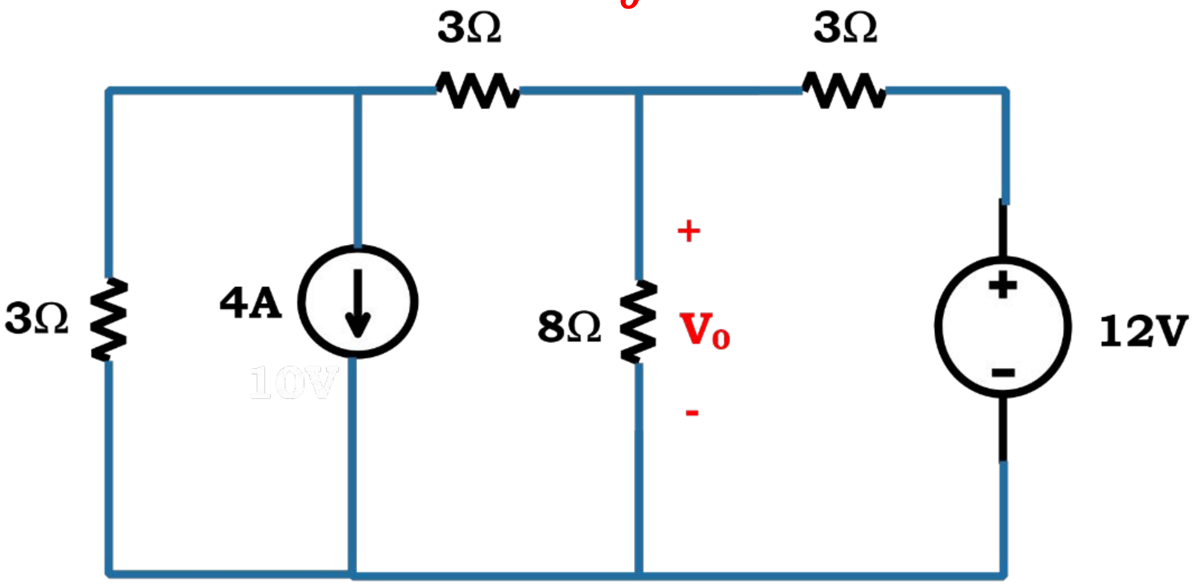
$$I_N = I_{sc}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = R_N$$

$$I_N = \frac{V_{Th}}{R_{Th}}$$

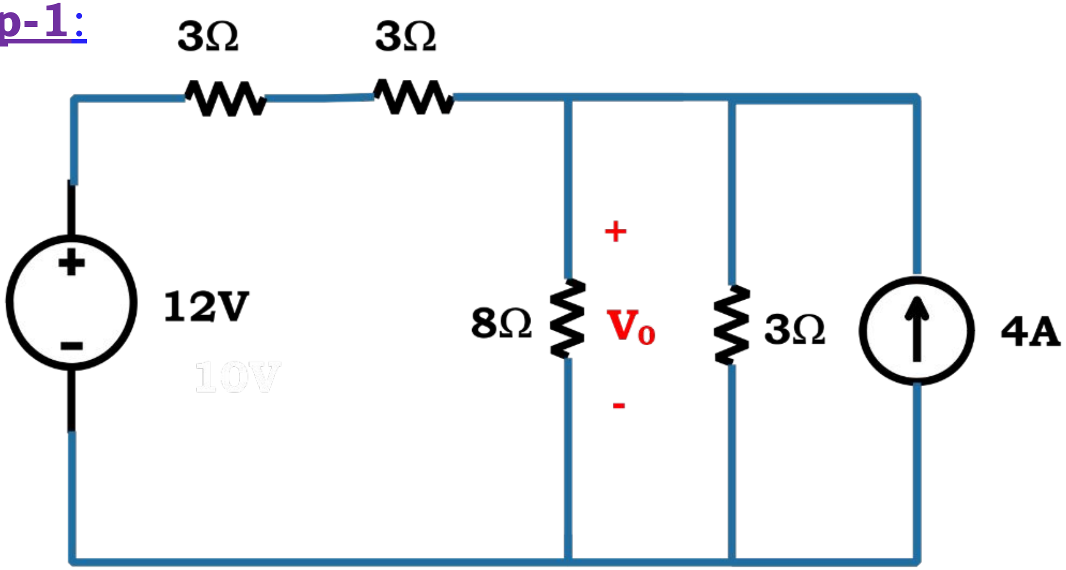
Problem-1.18

Using source transformation, determine V_o in the circuit shown in Fig.

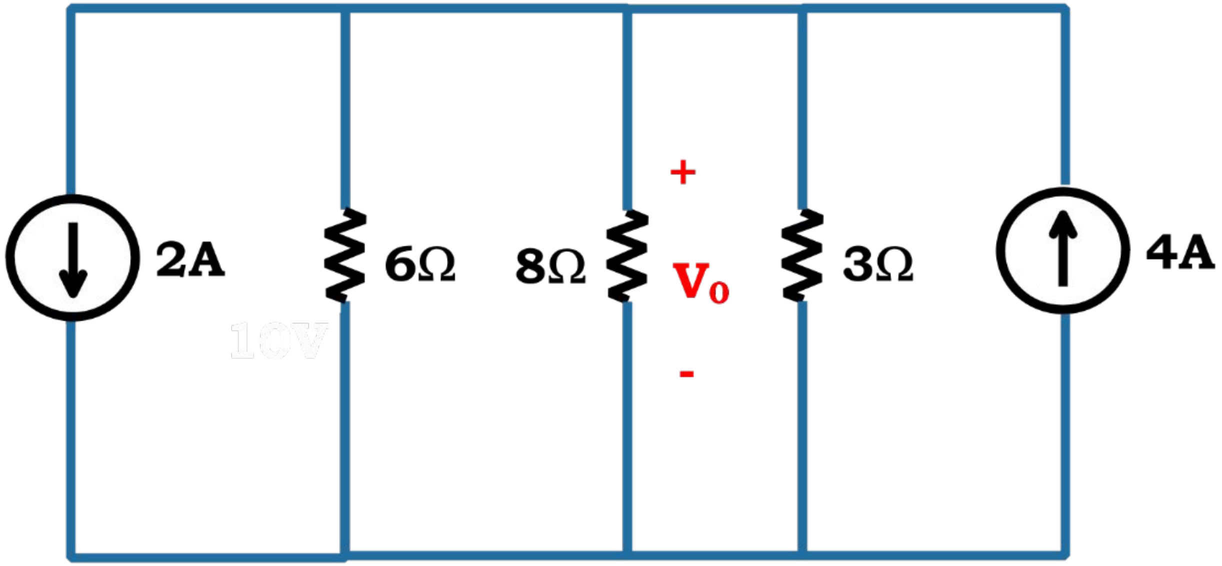


Sol:

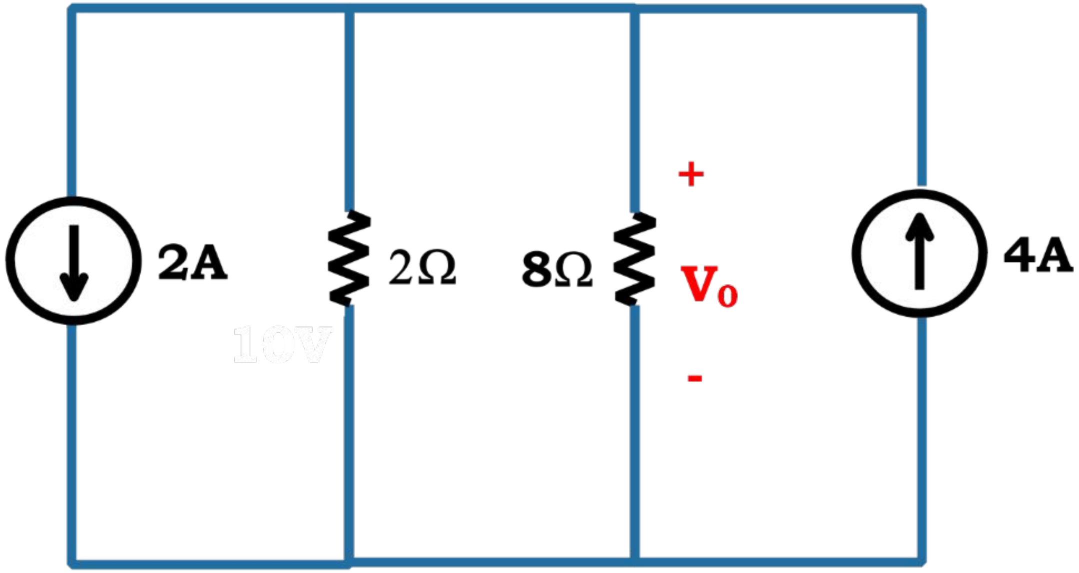
Step-1:



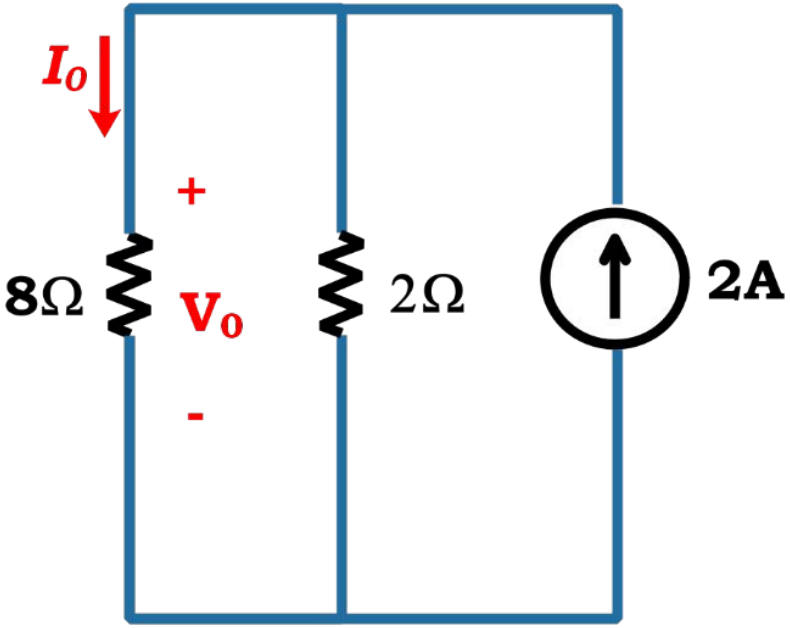
Step-2:



Step-3:



Step-4:



$$I_o = \frac{2}{2+8} \times 2 = 0.4\ A$$

$$V_o = 8I_o = 8 \times 0.4 = 3.2\ V$$