Higher Order Differential Equations – Cheat Sheet

1. Linear Differential Equations (Constant Coefficients)

General Form:

any(n)+an−1y(n−1)+...+a1y′+a0y=f(x)a_n y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = f(x)any(n)+an−1 y(n−1)+...+a1y′+a0y=f(x)

Steps to Solve:

1st
2nd**Find Complementary Function (CF):**

- Solve the characteristic equation: anrn+an−1rn−1+...+a0=0a_n r^n + a_{n-1} r^{n-1} + ... + a_0 = 0anrn+an−1rn−1+...+a0=0
- Based on roots:
  - **Distinct roots** → yc=C1er1x+C2er2x+...y_c = C_1 e^{r_1 x} + C_2 e^{r_2 x} + ...yc=C1er1x+C2er2 x+...
  - **Repeated roots** → yc=(C1+C2x)er1xy_c = (C_1 + C_2x)e^{r_1 x}yc=(C1+C2x)er1x
  - **Complex roots** → yc=eax(C1cosbx+C2sinbx)y_c = e^{ax} (C_1 \cos bx + C_2 \sin bx)yc=eax(C1 cosbx+C2sinbx)

3rd**Find Particular Integral (PI):**

- If f(x)f(x)f(x) is of the form eax,xn,sinx,cosxe^{ax}, x^n, \sin x, \cos xeax,xn,sinx,cosx, use standard formulas.
- If f(x)f(x)f(x) is already a solution of CF, multiply by xxx.

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2. Euler’s Linear Equation (Variable Coefficients)

General Form:

xny(n)+an−1xn−1y(n−1)+...+a1xy′+a0y=f(x)x^n y^{(n)} + a_{n-1} x^{n-1} y^{(n-1)} + ... + a_1 x y' + a_0 y = f(x)xny(n)+an−1xn−1y(n−1)+...+a1xy′+a0y=f(x)

Steps to Solve:

1st Assume y=xry = x^ry=xr.
2ndSolve the characteristic equation for rrr.
3rdForm the general solution using roots rrr.

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3. Method of Variation of Parameters

Used when the standard method fails.

Steps to Solve:

1st Solve the homogeneous equation to get **CF**.

2ndAssume **Particular Integral (PI)** as:
yp=u1y1+u2y2y_p = u_1 y_1 + u_2 y_2yp=u1y1+u2y2
3rd Solve for u1u_1u1 and u2u_2u2 using:
u1′y1+u2′y2=0u_1' y_1 + u_2' y_2 = 0u1′y1+u2′y2=0
u1′y1′+u2′y2′=f(x)u_1' y_1' + u_2' y_2' = f(x)u1′y1′+u2′y2′=f(x)
4th Integrate to find u1u_1u1 and u2u_2u2.
5th General solution:
y=yc+ypy = y_c + y_py=yc+yp.

Quick Summary Table

Topic	Key Steps
Higher Order LDE	Solve characteristic equation, find CF & PI
Euler's Equation	Use y=xry = x^ry=xr, solve for rrr
Variation of Parameters	Use known solutions to construct PI

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