UNIT - II: Fourier Transforms

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Fourier Transform Definition

The Fourier Transform (FT) converts a time-domain function into its frequency-domain representation:
[F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx]

Inverse Fourier Transform

The Inverse Fourier Transform retrieves the original function from its frequency domain:
[f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega x} d\omega]

Properties of Fourier Transform

Key properties include:
• Linearity: FT of a sum is the sum of individual transforms.
• Time Shifting: FT of (f(x-a)) introduces a phase shift.
• Frequency Shifting: Multiplication by (e^{j\omega_0 x}) shifts frequency.
• Scaling: Scaling in time domain affects frequency domain.
• Differentiation: FT of (f'(x)) is (j\omega F(\omega)).

Fourier Sine and Cosine Transforms

Used for functions defined on semi-infinite domains:
• Fourier Sine Transform:
[Fs(\omega) = \int0^{\infty} f(x) \sin(\omega x) dx]
• Fourier Cosine Transform:
[Fc(\omega) = \int0^{\infty} f(x) \cos(\omega x) dx]

Properties of Sine and Cosine Transforms

Similar to FT properties, but applicable to sine and cosine transforms:
• Linearity
• Time shifting
• Scaling
• Differentiation in time domain corresponds to multiplication in frequency domain