UNIT - II

AC CIRCUITS.

Concepts of AC Circuits:

A dc quantity is one which has a constant magnitude irrespective of time. But, an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways

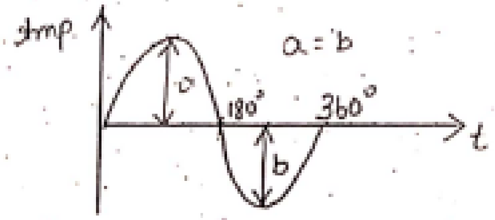
(i) by its effective (or) RMS (Root Mean Square) value.

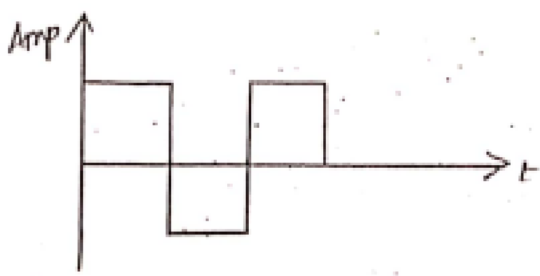
(ii) by its average value.

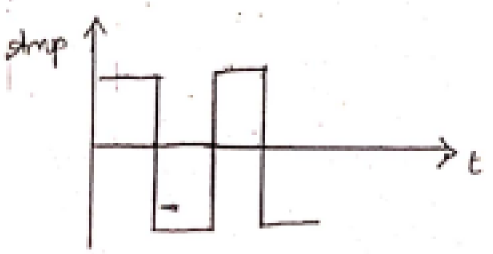
(iii) by its Peak value.

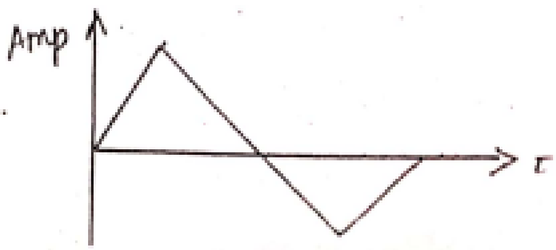
AC Circuits:

- Electric circuits fed by alternating current sources are called AC circuits.
- AC sources are which provides voltage and current periodically passed through a definite cycle of changes.
- AC circuit have any one (or) more number of passive elements namely resistors, inductors and capacitances.
- Some of the alternating voltage / current waveforms are shown below:









Terminologies of AC Signal:

(i) Waveform:

A waveform is a graph in which the instantaneous value of any quantity plotted against time

a) Alternating Waveform:

This is a wave / signal which reverses, its direction at regularly recurring intervals.

b) Periodic Waveform:

It is one which repeats itself after definite time intervals.

(ii) Cycle:

One complete set of positive and negative halves of a waveform constitute a cycle.

(iii) Amplitude:

The maximum positive (or) negative value of an alternating quantity is called the amplitude.

(iv) Frequency:

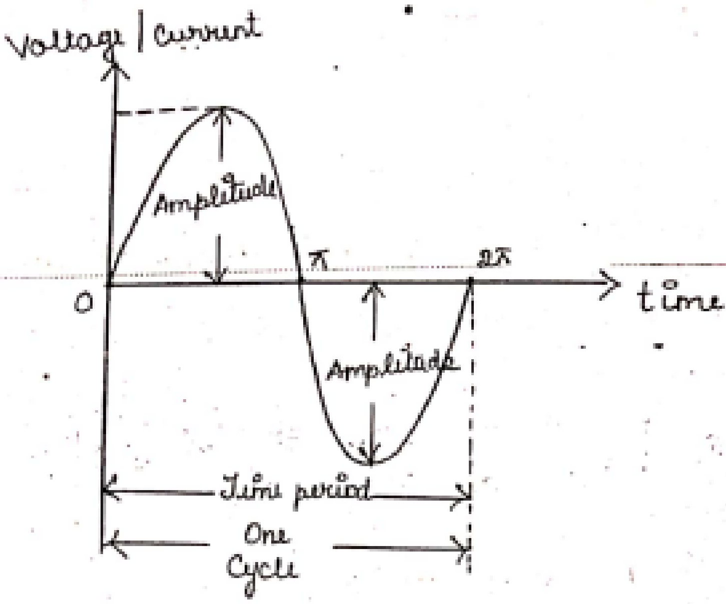
The no. of cycles per second of an alternating quantity is known as frequency.

Unit: Hertz (Hz) $f = \frac{1}{T}$ where, $f \rightarrow$ Frequency
$T \rightarrow$ Time Period.

(v) Period:

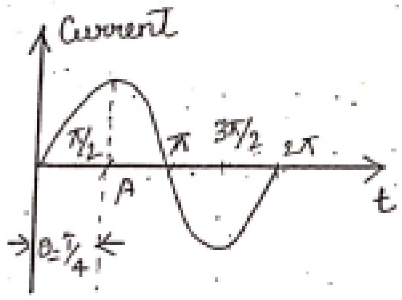
Time period of an alternating quantity is the time taken to complete one cycle. Time is expressed in 'secs'

$T = \frac{1}{f}$.



Phase :

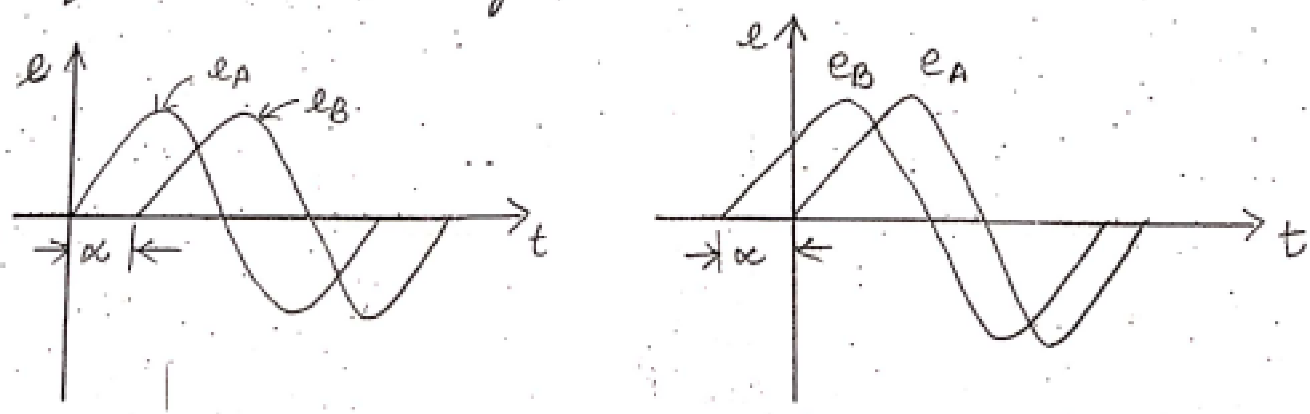
The phase at any point on a given wave is the time that has elapsed since the quantity has last passed through zero point of reference and passed periodically.



From the figure the phase of current at point A is $T/4$ sec, where T is the time period.

Phase Difference :

The term is used to compare the phase of two waveforms (or) alternating quantities.



(a) Lagging　　(b) Leading

α - phase angle

✓ In general, a lagging quantity is one which starts & progresses behind the quantity under reference.

✓ In fig (a) e_B is lagging (or) lags e_A (reference).

✓ Leading quantity is one which starts and progresses ahead of the quantity under reference.

✓ In fig (b), e_B leads e_A (Reference)

Maximum Value (or) Peak Value:

The maximum value (+ve or -ve) of an alternating (current / voltage) quantity is called amplitude. The alternating voltage represents in mathematical form,

$$V = V_m \sin\theta \rightarrow (1)$$

where, V_m - maximum amplitude

eqn (1) can be rewritten as follows,

$V = V_m \sin \omega t$ $\quad \because \theta = \omega t$

$V = V_m \sin 2\pi f t$ $\quad \because \omega = 2\pi f$

$V = V_m \sin\left(\frac{2\pi}{T}\right) . t$ $\quad \because f = \frac{1}{T}$

Representation of AC Components:

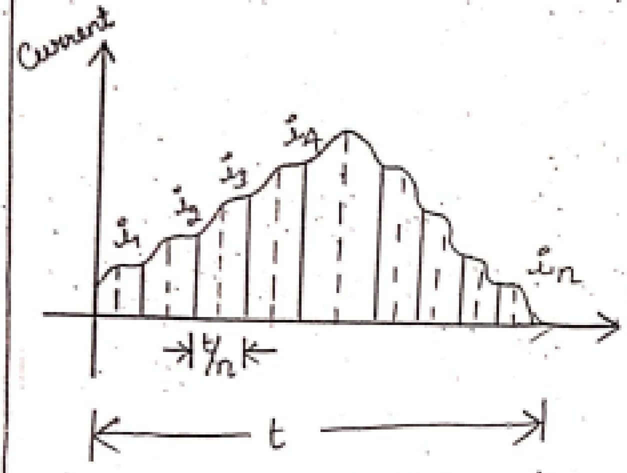
(a) Root Mean Square (RMS) or Effective Value:

RMS value of an alternating current is defined by the steady value of current (dc) which when flowing in a given circuit for a given time produces the same heat as would be produced by alternating current flowing in the same circuit for the same time.

Determination of RMS value for any Alternating Current:

For finding the r.m.s value of the symmetrical sinusoidal alternating current either "Mid-Ordinate Method" (or) "Analytical method" may be used.

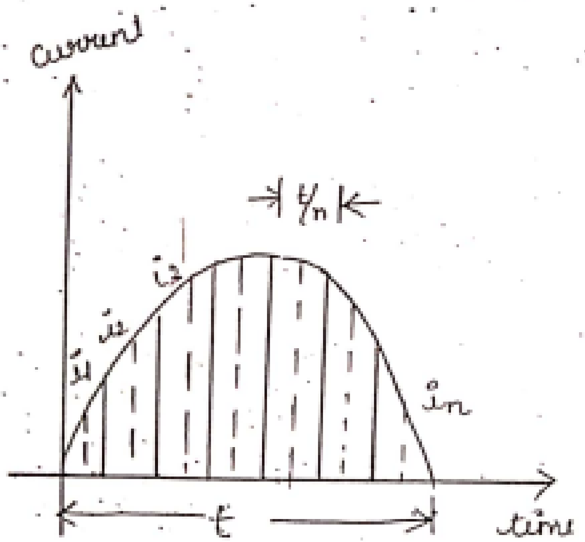
a) Mid-Ordinate Method:



Non-Sinusoidal Current

Initially divide the time base 't' into 'n' equal intervals of time each of duration t/n seconds.

Let $i_1, i_2, i_3 \ldots i_n$ be the mean instantaneous values of currents during their interval respectively.



Suppose the alternating current is passed through a circuit of resistance 'R' Ω, then.

Heat produced in 1st interval $\left.\right\} = 0.24\, i_1^2 R \cdot \frac{t}{n}$ (cal)

2nd interval $= 0.24\, i_2^2 R \cdot \frac{t}{n}$ (Cal)

⋮

nth interval $= 0.24\, i_n^2 R \cdot \frac{t}{n}$ (cal)

$\therefore$ Total heat produced in 't' seconds $\left.\right\} = 0.24\, R \cdot t \left(\dfrac{i_1^2 + i_2^2 + \cdots i_n^2}{n} \right)$

Now consider a direct current (dc) of value I producing the same heat through the same resistance during the same time 't'.

Total heat produced by Dc current in 't' secs $\left.\right\} = 0.24\, I^2 R t$ (Cal)

By definition two amount of heat produced should be equal;

$$\therefore 0.24\, I^2 \cdot Rt = 0.24\, Rt \left(\frac{i_1^2 + i_2^2 + \cdots i_n^2}{n} \right)$$

Comparing we get,

$$I^2 = \left(\frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2}{n} \right)$$

(RMS Value of alternating current) $\Rightarrow I = \sqrt{\left(\dfrac{i_1^2 + i_2^2 + \cdots \cdots i_n^2}{n} \right)}$

i.e., I = Square root of the mean of squares of the instantaneous currents.

Ill^ly, the rms value of alternating voltages is given by the expression,

$$V = \sqrt{\left(\frac{V_1^2 + V_2^2 + V_3^2 \cdots \cdots + V_n^2}{n} \right)}$$

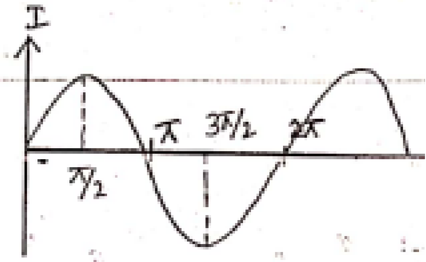
b) Analytical Method : (To obtain rms value of sinusoid... current...

Let the alternating current represented by,

$i = I_m \sin \omega t$

$= I_m \sin \theta \quad (\because \omega t = \theta)$

$\boxed{i^2 = I_m^2 \sin^2\theta}$



Mean square of AC $= \int_0^{2\pi} \frac{I_m^2 \sin^2\theta}{2\pi} \, d\theta$

$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta$ $\quad \because \sin^2\theta$ ...

$= \frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{1-\cos 2\theta}{2} \, d\theta = \frac{I_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right.$ ...

$= \frac{I_m^2}{2\pi}\left(\frac{2\pi}{2}\right)$

Mean square of AC $= \frac{I_m^2}{2}$.

RMS value of the alternating sinusoidal current is ...

$I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$

$\therefore I_{rms} = 0.707 \, I_m$.

Ill^ly for alternating voltage,

$V_{rms} = V = \sqrt{\frac{V_1^2 + V_2^2 + \cdots + V_n^2}{n}}$

for sinusoidal voltage $= V = \frac{V_m}{\sqrt{2}} = 0.707 \, V_m$.

$\boxed{V_{rms} = 0.707 \, V_m}$

General Format :

The rms value of a wave can also be ob[tained]
by the formula given below.

$$\text{RMS value} = \sqrt{\frac{\text{Area under the square wave for one cycle}}{\text{Period}}}$$

AVERAGE VALUE OF AC :

Definition :

The average value of an AC is given by th[e]
steady current which transfers across a circuit o[r]
same charge as would be transferred by the ac
across the same circuit in the same time.

Determination of Average Value :

a) Mid - Ordinate Method :

Referring the non-sinusoidal & sinusoidal
waveforms, the average value is given as,

$$I_{av} = \frac{i_1 + i_2 + \cdots i_n}{n}$$

Average value can be easily obtained by f[irst]
finding the average value for a small interval of t[ime]
and then integrating over the curve.

$$\text{i.e., } I_{avg} = \frac{1}{T}\int_0^t i\,dt$$

This is nothing but the ratio of the area u[nder]
the curve over one complete cycle to the base.

b) Analytical Method: (to obtain average value for sinus).

Let $i = I_m \sin\theta$

Since, this is a symmetrical wave it has two equal half cycles namely positive and negative halves.

Considering one half cycle. for this symmetrical wave the average value is obtained by,

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \cdot d\theta$$

$$= \frac{I_m}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

$\because \cos 0 = 1$

$-\cos\pi = 1$

$$= \frac{I_m}{\pi}(1+1)$$

$$= \frac{2I_m}{\pi}$$

$$\therefore I_{avg} = 0.637\, I_m$$

For sinusoidal voltage wave, $V_{avg} = 0.637\, V_m$

where, I_m & V_m are maximum value of current & voltage and current respectively.

FORM FACTOR & PEAK FACTOR:

The relation between average, RMS and the maximum values can be expressed by two factors namely Form Factor and Peak Factor.

(a) Form Factor : (K_f):

It is defined as the ratio of rms value to the average value.

$$\text{Form Factor } (K_f) = \frac{\text{rms value}}{\text{Average value}}$$

$$\left.\begin{array}{r}\text{Form Factor for sinusoidal} \\ \text{Current } (K_f)\end{array}\right\} = \frac{0.707\, I_m}{0.637\, I_m}$$

$$= 1.11$$

$$\left.\begin{array}{r}\text{Form Factor for sinusoidal} \\ \text{Voltage } (K_f)\end{array}\right\} = \frac{0.707\, V_m}{0.637\, V_m}$$

$$= 1.11$$

(b) Peak (or) Crest (or) Amplitude (or) Cost Factor (K_p):

It is defined as the ratio of peak value to the rms value.

$$\text{Peak Factor } (K_p) = \frac{\text{Peak Value}}{\text{Rms Value}}$$

$$\left.\begin{array}{r}K_p \text{ for sinusoidal} \\ \text{Current}\end{array}\right\} = \frac{I_m}{(I_m/\sqrt{2})} = \sqrt{2} = 1.414$$

$$K_p \text{ for sinusoidal voltage} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = 1.414$$

Power Factor:

Power Factor is defined as,

(i) cosine of the angle of lead (or) lag.

(ii) The ratio $R/Z = \dfrac{\text{Resistance}}{\text{Impedance}}$

Impedance Value:

For a series R-L circuit, Impedance $(Z) = \sqrt{R^2 + X_L^2}$.

For a series R-C circuit, Impedance $(Z) = \sqrt{R^2 + X_L^2}$.

For a series R-L-C circuit, Imp $(Z) = \sqrt{R^2 + (X_L - X_C)^2}$

where X_L or X_C is the reactance of ind (or) cap respectively

($X_{L (or) C}$ - Resistance offered by the inductance (or) Capacitance respectively in a circuit is called as Reactance)

Power $(P) = VI \cos\phi$, where $V \rightarrow$ rms voltage

$I \rightarrow$ rms current

$\cos\phi \rightarrow$ Power Factor

$P = I^2 R$ (for pure resistance circuit)

Active (or) Reactive Components of Current:

Active component is that which is inphase with the applied voltage i.e., $I\cos\phi$. It is also known as "Wattful" component.

Reactive Component:

It is that which is in quadrature V i.e., $\frac{\pi}{2}$ i.e., $I\sin\phi$. It is also known as 'Watt less' or idle component.

NOTE:

The product of voltage (Volts) and amperes in a AC circuit gives Volt-ampere (VA). Out of this, The actual power = VA $\cos\phi$ = W

Reactive Power = VA $\sin\phi$.

Expressing the values in KVA, we get two rectangular components,

(i) Active component which is obtained by multiply KVA by $\cos\phi$. It gives power in KW.

(ii) Reactive component which is obtained by multiplying KVA by $\sin\phi$. It is given box in KVAR.

The following KVA relation can be easily obta-

$$KVA = \sqrt{KW^2 + KVAR^2}$$

where, $KW - KVA \cos\phi$

$KVAR - KVA \sin\phi$.

The above equation can be easily understood by referring to KVA triangle;

NOTE : (lagging KVAR's are taken as negative)

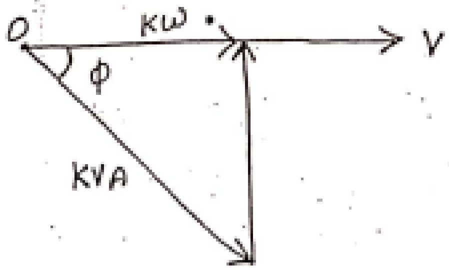


Fig: KVA Triangle.

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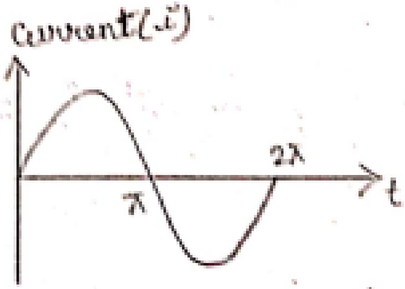
1. Derive the expression for rms value, Average value, form factor of a a) Sinusoidal alternating quantity.

The sinusoidal alternating current is shown in fig. The instantaneous value of the current is given by, $i = I_m \sin\theta$.



To find the rms Value:

$$I_{rms} = \sqrt{\int_0^{2\pi} \frac{I^2}{2\pi}\, d\theta}$$

$$= \sqrt{\int_0^{2\pi} \frac{I_m^2 \sin^2\theta}{2\pi}\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2\theta \cdot d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} (\theta)_0^{2\pi} - \frac{I_m^2}{8\pi} \left(\frac{\sin 2\theta}{2}\right)_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} [2\pi - 0] - \frac{I_m^2}{8\pi} (\sin 4\pi - \sin 0)}$$

$$= \sqrt{\frac{I_m^2}{4\pi} (2\pi - 0)}$$

$$= \sqrt{\frac{I_m^2}{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

To find the average value:

Since it is a symmetrical wave we have to consider only half cycle.

$$I_{avg} = \int_0^{\pi} \frac{i\,d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{-I_m}{\pi}(\cos\theta)_0^{\pi}$$

$$= \frac{-I_m}{\pi}(\cos\pi - \cos 0) = \frac{-I_m}{\pi}(-1-1) = \frac{2I_m}{\pi}$$

$$\boxed{I_{avg} = \frac{I_m}{\pi/2}}$$

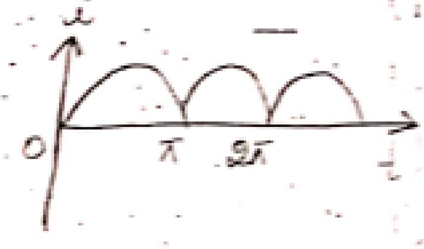
$$\text{Form Factor } (K_f) = \frac{\text{RMS Value}}{\text{Avg. Value}} = \frac{I_m/\sqrt{2}}{I_m/(\pi/2)} = 1.11$$

$$\text{Peak Factor } (K_p) = \frac{\text{Maxi. Value}}{\text{RMS Value}} = \frac{I_m}{I_m/\sqrt{2}} = 1.414.$$

b) Full Wave rectified Sine Wave:

$$\text{Mean Square Value} = \frac{\text{Area under one squared curve}}{\text{Period}}$$

$$\text{Mean square value} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta$$



$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= \frac{I_m^2}{2\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right] = \frac{I_m^2}{2\pi} \times \pi$$

$$= \frac{I_m^2}{2}.$$

$\because$ rms value $= \sqrt{\text{Mean Square Value}}$.

$\therefore$ rms. Value $= \dfrac{I_m}{\sqrt{2}}$.

$$\text{Average value} = \frac{\text{Area under the curve for one Complete cycle}}{\text{period}}$$

Since the given wave is symmetrical.

$$\text{Avg. value} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta.$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}(1+1)$$

$$= \frac{2I_m}{\pi}.$$

$$\text{Form Factor} = \frac{\text{rms value}}{\text{Average value}}$$

$$= \frac{I_m/\sqrt{2}}{(2I_m)/\pi}$$

$$= \frac{I_m \times \pi}{\sqrt{2} \times 2I_m} = 1.11$$

$$\text{Peak Factor} = \frac{\text{Peak value}}{\text{rms value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2}$$

$$\therefore \boxed{\text{Peak Factor} = \sqrt{2}.}$$

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