Discrete Probability Distributions Cheat Sheet

1 Basics of Discrete Distributions

- **Discrete Random Variable (X)**: A variable that takes a finite/countable number of values.
- **Probability Mass Function (PMF)**:
    - Defines the probability of each discrete outcome.
    - P(X=x)=f(x)P(X = x) = f(x)P(X=x)=f(x)
    - Must satisfy:
        - 0≤P(X=x)≤10 \leq P(X=x) \leq 10≤P(X=x)≤1
        - ∑P(X=x)=1\sum P(X=x) = 1∑P(X=x)=1
- **Probability Density Function (PDF) vs PMF**:
    - PDF is for continuous variables (integrates to 1).
    - PMF is for discrete variables (sum equals 1).
- **Cumulative Distribution Function (CDF)**:
    - F(x)=P(X≤x)F(x) = P(X \leq x)F(x)=P(X≤x)
    - Summation of all probabilities up to xxx.

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2 Important Discrete Distributions

A. Binomial Distribution

- **Used when**: Fixed number of independent trials, each with two outcomes (Success/Failure).
- **PMF**:
- P(X=k)=(nk)pk(1−p)n−kP(X = k) = \binom{n}{k} p^k (1-p)^{n-k}P(X=k)=(kn)pk(1−p)n−k
    - nnn = Number of trials
    - kkk = Number of successes
    - ppp = Probability of success
    - (1−p)(1-p)(1−p) = Probability of failure
- **Mean**: E(X)=npE(X) = npE(X)=np
- **Variance**: Var(X)=np(1−p)Var(X) = np(1-p)Var(X)=np(1−p)

Example: Probability of getting 3 heads in 5 coin tosses.

B. Geometric Distribution

- **Used when**: Counting the number of trials until the first success.
- **PMF**:
- P(X=k)=(1−p)k−1pP(X = k) = (1-p)^{k-1} pP(X=k)=(1−p)k−1p
    - ppp = Probability of success
    - kkk = Trial number of first success

- **Mean**: E(X)=1pE(X) = \frac{1}{p}E(X)=p1
- **Variance**: Var(X)=1−pp2Var(X) = \frac{1-p}{p^2}Var(X)=p21−p

Example: Rolling a die until the first 6 appears.

C. Negative Binomial Distribution

- **Used when**: Counting the number of trials until a fixed number of successes occurs.
- **PMF**:
- P(X=k)=(k−1r−1)pr(1−p)k−rP(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}P(X=k)=(r−1k−1)pr(1−p)k−r
- rrr = Number of required successes
- kkk = Total trials to achieve rrr successes
- **Mean**: E(X)=rpE(X) = \frac{r}{p}E(X)=pr
- **Variance**: Var(X)=r(1−p)p2Var(X) = \frac{r(1-p)}{p^2}Var(X)=p2r(1−p)

Example: Number of dice rolls to get 3 sixes.

D. Poisson Distribution

- **Used when**: Counting the number of events occurring in a fixed time/space interval.
- **PMF**:
- P(X=k)=e−λλkk!P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}P(X=k)=k!e−λλk
- λ\lambdaλ = Expected number of occurrences in the interval
- kkk = Number of events
- **Mean & Variance**: E(X)=Var(X)=λE(X) = Var(X) = \lambdaE(X)=Var(X)=λ

Example: Number of customer arrivals at a shop in an hour.

Summary Table

Distribution	PMF	Mean	Variance
Binomial	P(X=k)=(nk)pk(1−p)n−kP(X = k) = \binom{n}{k} p^k (1-p)^{n-k}P(X=k)=(kn)pk(1−p)n−k	npnpnp	np(1−p)np(1-p)np(1−p)
Geometric	P(X=k)=(1−p)k−1pP(X = k) = (1-p)^{k-1} pP(X=k)=(1−p)k−1p	1p\frac{1}{p}p1	1−pp2\frac{1-p}{p^2}p21−p
Negative Binomial	P(X=k)=(k−1r−1)pr(1−p)k−rP(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}P(X=k)=(r−1k−1)pr(1−p)k−r	rp\frac{r}{p}pr	r(1−p)p2\frac{r(1-p)}{p^2}p2r(1−p)

Distribution	PMF	Mean	Variance
Poisson	P(X=k)=e−λλkk!P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}P(X=k)=k! e−λλk	λ\lambdaλ	λ\lambdaλ

This **cheat sheet** covers all the key concepts in **Discrete Probability Distributions** . Let me know if you need more details!