Z-Transforms Cheat Sheet (Theory Only)

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1. Z-Transform – Definition & Importance

The **Z-Transform** converts a **discrete-time signal** into its corresponding **complex frequency domain representation**.
Used in **digital signal processing (DSP), control systems, and difference equations**.
Helps analyze **discrete-time systems, stability, and system response**.

Applications of Z-Transform

Digital Filters → Used in digital signal processing.
Control Systems → Helps in stability analysis.
Difference Equations → Used to solve recurrence relations.
Economic & Financial Models → Time-series analysis.

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2. Elementary Properties of Z-Transform

1. Linearity Property

The Z-Transform of a **sum of signals** is the sum of their individual Z-transforms.
Helps in **superposition-based solutions**.

2. Time Shifting (Delay & Advance) Property

Delaying a signal in the time domain introduces a **multiplicative factor** in the Z-domain.
Used in **control systems and digital filters**.

3. Scaling Property

Expanding or compressing a discrete-time sequence **modifies its Z-transform accordingly**.

4. Differentiation in Z-Domain

Used for analyzing **discrete-time systems with derivatives**.

5. Convolution Theorem

The Z-Transform of the **convolution of two sequences** is the **product of their individual Z-transforms**.
Used in **signal processing and system response analysis**.

6. Initial & Final Value Theorems

Determine the **first and last values** of a sequence without computing the inverse Z-transform.
Useful in **control system stability analysis**.

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3. Inverse Z-Transform Methods

Converts a function from the **Z-domain** back to the **time-domain**.
Used to obtain the original discrete-time sequence from its Z-transform.

Methods to Compute Inverse Z-Transform

1. Partial Fraction Method

Used when the given Z-transform is a **rational function (polynomial ratio)**.
Steps:

- Express in **partial fraction form**.
- Use **table lookup** for inverse transform.
  Helps solve **difference equations and digital control systems**.

2. Residue Method

Based on **contour integration and inverse transform computation**.
Steps:

- Find **poles of the function** in the Z-domain.
- Compute **residues for each pole**.
  Used in **digital signal processing and stability analysis**.

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4. Solution of Difference Equations Using Z-Transform

Difference equations are the **discrete equivalent of differential equations**.
Z-Transform helps convert them into **algebraic equations**, which are easier to solve.

Steps to Solve a Difference Equation Using Z-Transform

1st **Apply Z-Transform** to both sides of the equation.
2nd **Use Z-transform properties** (e.g., time shifting).
3rd **Convert to an algebraic equation** in Z-domain.
4th **Solve for the sequence function** in Z-domain.
5th **Find the inverse Z-transform** to get the solution in the time domain.

Applications of Difference Equations

Digital Filters → FIR & IIR filter design in DSP.
Population Growth Models → Predicting future population sizes.
Control Systems → Analyzing stability and system response.

This **Z-Transform Cheat Sheet** covers **Z-transforms, elementary properties, inverse Z-transform (partial fraction & residue methods), and solving difference equations using Z-transform**. Let me know if you need further explanations!