№4.

Пусть $\xi \in N(a, \sigma)$, $\theta = (a, \sigma)$ — оцениваемые параметры распределения.

$$L(a, \sigma) = \prod_{i=1}^{n} p(\xi_i, a, \sigma) =$$

$$= \frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{1}{2}\sum_{i=1}^{n}\frac{(\xi_i - a)^2}{\sigma^2}}$$

$$\Downarrow$$

$$\ln L(a, \sigma) = -n\ln\sigma - n\ln\sqrt{2\pi} -$$

$$-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(\xi_i - a)^2$$

находим экстремум:

$$\frac{\partial \ln L}{\partial a} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(\xi_i - a) = 0$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(\xi_i - a)^2 = 0$$

$$\begin{cases} \sum_{i=1}^{n} (\xi_i - a) = 0 \\ \frac{1}{\sigma^2} \sum_{i=1}^{n} (\xi_i - a)^2 = n \end{cases}$$

$$\begin{cases} \sum_{i=1}^{n} \xi_i = na \\ \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (\xi_i - a)^2 \end{cases}$$

$$\begin{cases} a = \frac{1}{n} \sum_{i=1}^{n} \xi_i \\ \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (\xi_i - a)^2 \end{cases}$$

$$\Downarrow$$

оценки: $\hat{a} = \frac{1}{n} \sum_{i=1}^{n} \xi_i = \overline{\xi}$, $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (\xi_i - \overline{\xi})^2$