Life Insurance Reserves Calculator

R

November 14, 2025

1 Project Objectives

We will regress on claims in claims ladders to make projections for future claims. First we will do this manually using R's inbuilt linear regression functionality, but we will then use the ChainLadder Mack Model package to very easily apply industry standard models to the problem. We will also critique the output and whether they line up with requirements.

2 Dataset

The dataset used is taken from ChainLadder itself. ABC is a neatly formatted claims triangle for the worker's compensation portfolio of a large company. The data comes from 11 accident years and includes 11 development years, collected by (B. Zehnwirth and G. Barnett, 2000).

Table 1: ABC Data

origin	1	2	3	4	5	6	7
1977	153638	342050	476584	564040	624388	666792	698030
1978	178536	404948	563842	668528	739976	787966	823542
1979	210172	469340	657728	780802	864182	920268	958764
1980	211448	464930	648300	779340	858334	918566	964134
1981	219810	486114	680764	800862	888444	951194	1002194
1982	205654	458400	635906	765428	862214	944614	NA
1983	197716	453124	647772	790100	895700	NA	NA
1984	239784	569026	833828	1024228	NA	NA	NA
1985	326304	798048	1173448	NA	NA	NA	NA
1986	420778	1011178	NA	NA	NA	NA	NA
1987	496200	NA	NA	NA	NA	NA	NA

Or, visually (Figure 1).
For the Mack Model, and any claims triangle modelling in general, the base assumption is that there exists some vector of variables relating historic claims to future:

$$f_i = \frac{\sum_i^T C_{i,j+1}}{\sum_i^T C_{i,j}}$$

Table 2: Table 1 (contd)

origin	8	9	10	11
1977	719282	735904	750344	762544
1978	848360	871022	889022	NA
1979	992532	1019932	NA	NA
1980	1002134	NA	NA	NA
1981	NA	NA	NA	NA
1982	NA	NA	NA	NA
1983	NA	NA	NA	NA
1984	NA	NA	NA	NA
1985	NA	NA	NA	NA
1986	NA	NA	NA	NA
1987	NA	NA	NA	NA

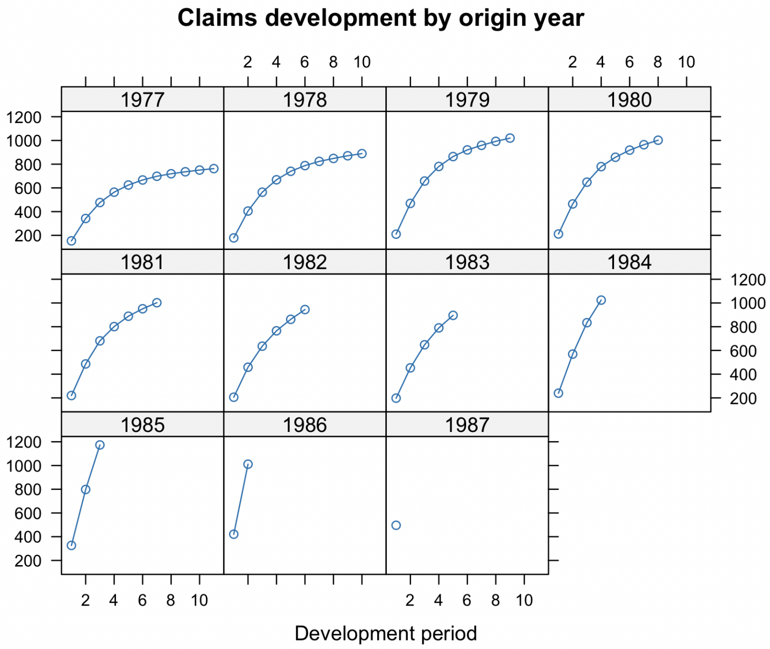

Figure 1: Claims developments over time

Where $C_{i,j}$ can be depicted according to Table 3.

Table 3: Generalised Claims Triangle

Year	1 year claims	2 year claims	3 year claims	4 year claims
2000	$C_{1,1}$	$C_{1,2}$	$C_{1,3}$	$C_{1,4}$
2001	$C_{2,1}$	$C_{2,2}$	$C_{2,3}$	
2002	$C_{2,1}$	$C_{3,2}$		
2003	$C_{4,1}$			

```
Age2AgeCalculator <- function(data = ABC) {
    size <- dim(data)
    len <- size[1] - 1
    a2a <- sapply(1:len,
              function(i){
                  sum(data[c(1:(size[1] - i)), i+1])
                  /sum(data[c(1:(size[1] - i)),i])
              }) # regressing for vector
    return(a2a)
}
devRat <- Age2AgeCalculator() # vector of development ratios
```

Figure 2: Age-to-age factor calculator

If we now log our development ratios, and regress it on time, we can extract our critical hypothesised constants linking past to future claims values. We can visualise these using ggplot2 (Figure 3). Looking at the Age-to-Age effects, it's possible a mildly curved model would be a better fit than a linear regression, but the points are roughly well fitted over the time period.
More importantly, we can now use these cumulative effects to project future claims for each oncoming year. We can again use ggplot2 to graph these out over the future (Figure 4).
We can also display this as a claims table, where the upper triangle is past claims, and the lower is projected claims (Tables 4 & 5). With this, we see that our total expected unpaid losses are £5,563,411.78. But we can push these models further.

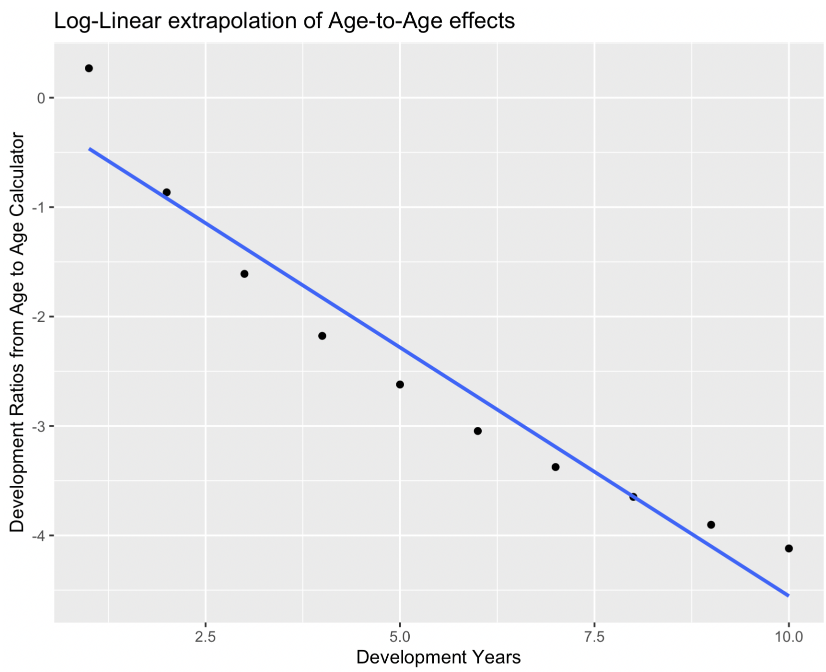

Figure 3: Estimated Age to Age effects

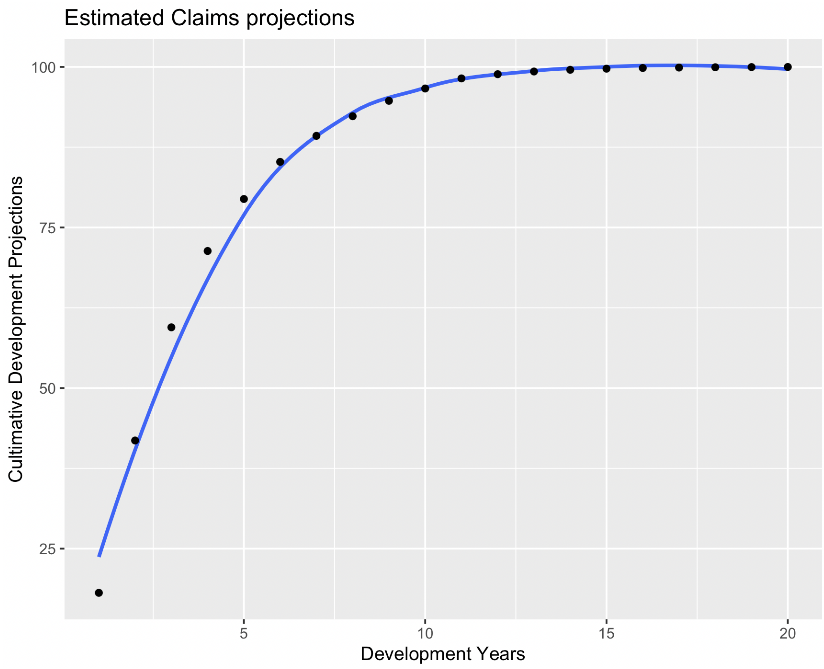

Figure 4: Projected Claims

Table 4: Full Rank Claims Dataset

Year	1	2	3	4	5	6	7
1977	153638	342050	476584	564040	624388	666792	698030
1978	178536	404948	563842	668528	739976	787966	823542
1979	210172	469340	657728	780802	864182	920268	958764
1980	211448	464930	648300	779340	858334	918566	964134
1981	219810	486114	680764	800862	888444	951194	1002194
1982	205654	458400	635906	765428	862214	944614	989539
1983	197716	453124	647772	790100	895700	960849	1006547
1984	239784	569026	833828	1024228	1140421	1223371	1281553
1985	326304	798048	1173448	1408060	1567797	1681832	1761818
1986	420778	1011178	1436983	1724284	1919895	2059540	2157490
1987	496200	1145527	1627905	1953379	2174979	2333178	2444141

Table 5: Full Rank Claims Dataset (contd)

Year	8	9	10	11	Total
1987	719282	735904	750344	762544	776598
1978	848360	871022	889022	903477	920128
1979	992532	1019932	1040522	1057440	1076929
1980	1002134	1028236	1048994	1066050	1085697
1981	1036480	1063477	1084946	1102586	1122907
1982	1023392	1050048	1071246	1088663	1108728
1983	1040981	1068095	1089658	1107375	1127784
1984	1325396	1359918	1387372	1409929	1435915
1985	1822091	1869551	1907292	1938303	1974027
1986	2231299	2289417	2335635	2373610	2417357
1987	2527757	2593597	2645956	2688977	2738536

3 Mack Model

Mack's Model generalises claims prediction down to a more precise linear regression that is unbiased under a fairly small set of assumptions:

1. There exist a vector of constants like the ones we found earlier,
2. Future shifts in claims developments happen independently of past ones (claims are i.i.d.),
3. The variance of claims in a period are directly related to claims in the past period.

In practice, the Mack model is weighted linear autoregression of claims. Luckily, ChainLadder provides a fully functional Mack implementation, so we can use the package to project claims.

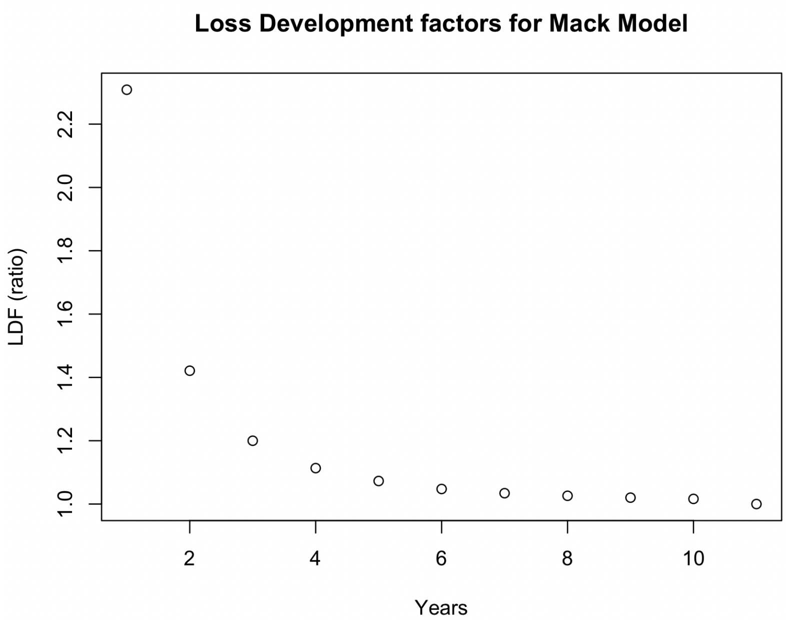

Figure 5: LDFs for the Mack Model

Looking at Figure 5, we can see that claims taper off after about 10 years, and we can very easily get full tables of projected claims over the next 10 years (Tables 6 & 7).

The ChainLadder package even provides easy summaries of key variables.
We can very easily see that the latest actual claims have been worth just over £10 million and the Estimated ultimate claims are about £15.5 million (Table 8). Beyond this, we can also evaluate the Mack Model using the visual summary defaults provided.
Looking at Figure 6, we can see a clear increasing trend in forecasting claims over time, implying that our assumptions of claims being i.i.d. distributed over the

Table 6: Mack Model Claims Projections

Year	1	2	3	4	5	6	7
1977	153638	342050	476584	564040	624388	666792.0	698030.0
1978	178536	404948	563842	668528	739976	787966.0	823542.0
1979	210172	469340	657728	780802	864182	920268.0	958764.0
1980	211448	464930	648300	779340	858334	918566.0	964134.0
1981	219810	486114	680764	800862	888444	951194.0	1002194.0
1982	205654	458400	635906	765428	862214	944614.0	989538.9
1983	197716	453124	647772	790100	895700	960849.4	1006546.5
1984	239784	569026	833828	1024228	1140421	1223370.6	1281553.0
1985	326304	798048	1173448	1408060	1567797	1681831.7	1761818.0
1986	420778	1011178	1436983	1724284	1919895	2059540.0	2157489.7
1987	496200	1145527	1627905	1953379	2174979	2333177.7	2444141.5

Table 7: Mack Model Claims Projections (contd)

Year	8	9	10	11
1977	719282	735904	750344	762544.0
1978	848360	871022	889022	903476.8
1979	992532	1019932	1040522	1057440.1
1980	1002134	1028236	1048994	1066049.7
1981	1036480	1063477	1084946	1102586.1
1982	1023392	1050048	1071246	1088663.4
1983	1040981	1068095	1089658	1107374.6
1984	1325396	1359918	1387372	1409929.1
1985	1822091	1869551	1907292	1938303.4
1986	2231299	2289417	2335635	2373610.5
1987	2527757	2593597	2645956	2688976.8

Table 8: Summary Statistics for the Mack Model

Variable	Value
Latest Actual Claims Costs:	1.022119e+07
Chain-Ladder Development (to date):	6.594764e-01
(Estimated) Ultimate Claims Loss:	1.549895e+07
Incurred but Not Reported claims (estimated):	5.277760e+06
Mack's Standard Error:	1.522831e+05
Mack's Coefficient of Variance:	2.885374e-02

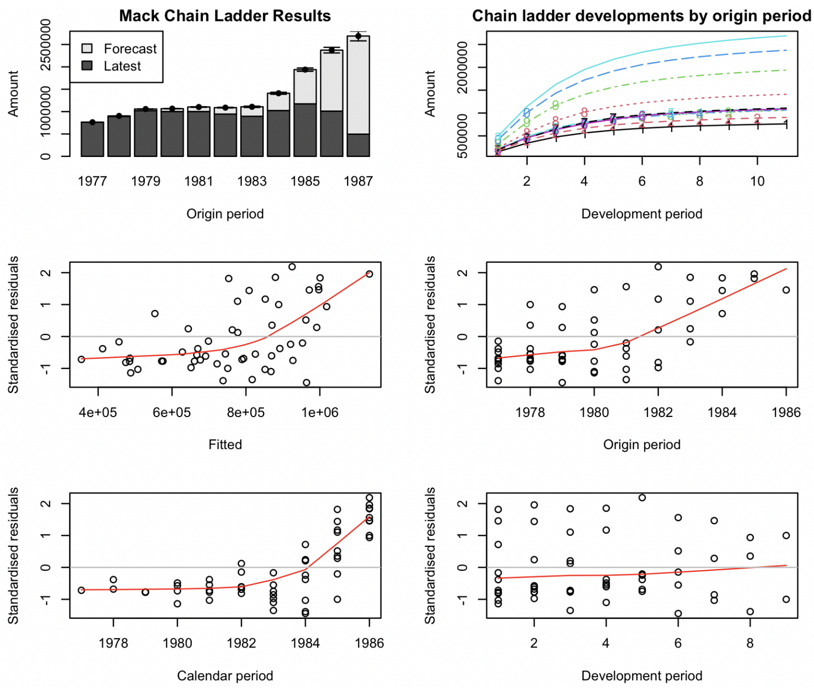

Figure 6: Mack Model Summary

observation period are invalid. We can also see that our fitted residuals are clearly trending, implying that linear regression was an improper model. We can, in response to these facts, only weight the latest 5 years in the Mack Model regression. Rerunning with these specifications:

Looking at Figure 7, this specification reduced a decent amount of residual trending, but there are still some trends in the fitted residuals that either need to be better studied or approached with alternative, more flexible models.

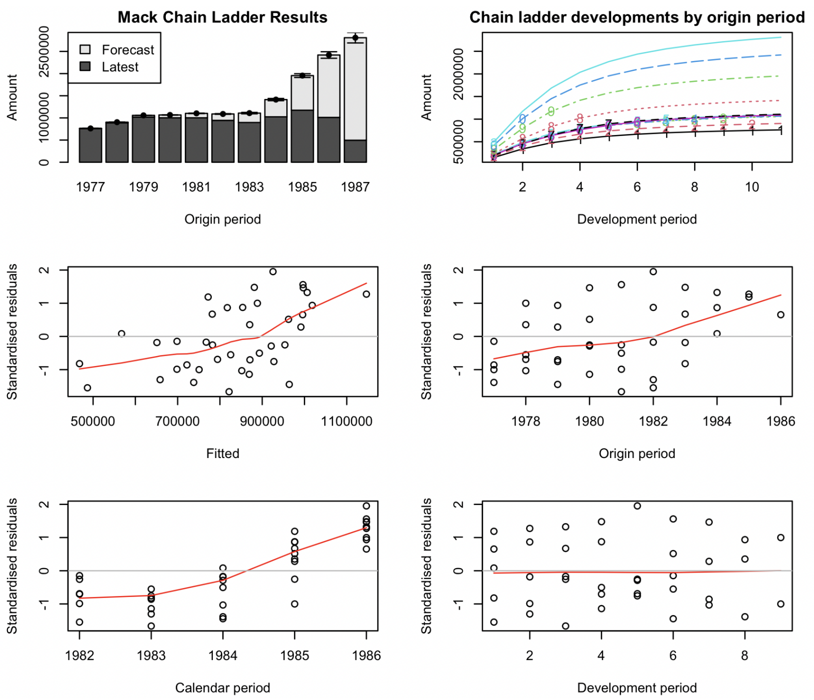

Figure 7: Mack Model Summary (Only latest 5 years used in Estimation)

4 References

B. Zehnwirth and G. Barnett (2000). *Best Estimates for Reserves*, .