CSE512 Fall 2018 - Machine Learning - Homework 7

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→ MANUAL CALCULATION OF ONE ROUND of EM for a GMM.

Given,

$$R = \begin{bmatrix} 1 & 0 \\ 0.3 & 0.7 \\ 0 & 1 \end{bmatrix}$$

M-Step

1 Likelihood function to be optimized

given $R_{i,c}$ is the probability of observation x_i, belonging to cluster c (for data point)

$$= \sum_i \sum_c R_{i,c} \log \pi_c + \sum_i \sum_c R_{i,c} \log \mathcal{N}(x_i \mid \theta_c)$$

2 The new values after performing M step for mixing weights π_1 & π_2.

We have, $\pi_c = \sum_i R_{i,c}$, when $N = 3$ for first column of R.

$$\therefore \pi_1 = \frac{1}{N} \sum_i R_{i,1} = \frac{1}{3}[1 + 0.3 + 0] = \frac{1.3}{3} = \boxed{0.4333}$$

$$\& \quad \pi_2 = \frac{1}{N} \sum_i R_{i,2} = \frac{1}{3}[0 + 0.7 + 1] = \frac{1.7}{3} = \boxed{0.5666}$$

3 The new values after performing the M step for the means μ_1 & μ_2.

We have, $\mu_c = \dfrac{\sum_i R_{i,c} \cdot x_i}{R_c}$, given $x = [1, 10, 20]$

$$\therefore \mu_1 = \frac{\sum_i R_{i,1} \cdot x_i}{R_{c=1}} = \frac{(1 \cdot 1 + 0.3 \cdot 10 + 0 \cdot 20)}{1 + 0.3 + 0} = \frac{4}{1.3}$$

$$\boxed{\mu_1 = 3.0769}$$

$$\mu_2 = \frac{\sum_i R_{i,2} \cdot x_i}{R_{c=2}} = \frac{0 \cdot 1 + 0.7 \cdot 10 + 1 \cdot 20}{0.0 + 0.7 + 1.0} = \frac{27}{1.7}$$

$$\boxed{\mu_2 = 15.8823}$$

(4) After performing M steps for the standard deviation σ_1 & σ_2, the new values for standard deviation are :

We know, $$\sigma_c^2 = \frac{\sum_i R_{i,c} \cdot x_i^2}{R_c} - \mu_c^2$$

$$\therefore \sigma_1^2 = \frac{1 \cdot 1 + 0.3 \cdot 100 + 0 \cdot 400}{1.3} - 9.4673$$

$$= \frac{1 + 30}{1.3} - 9.4673 = 23.8461 - 9.4673$$

$$= 14.3788$$

$$\therefore \sigma_1 = \sqrt{14.3788} = \boxed{3.7919}$$

& $$\sigma_2^2 = \frac{0 \cdot 1 + 0.7 \cdot 100 + 1 \cdot 400}{1.7} - 252.2474$$

$$= \frac{70 + 400}{1.7} - 252.2474 = 276.4705 - 252.2474$$

$$= 24.2231$$

$$\therefore \sigma_2 = \sqrt{24.2231} = \boxed{4.9216}$$

E - Step :

1. Formula for the probability of observation x_i belonging to cluster c.

We know for E-step $R_{i,c}$ can be written as:

$$R_{i,c} = \frac{\pi_c \mathcal{N}(x_i \mid \mu_c^{t-1})}{\sum_c \left(\pi_c \left(\mathcal{N}(x_i \mid \mu_c^{t-1})\right)\right)}$$

$$= \frac{\pi_c \cdot \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left(-\frac{(x_i - \mu_c^{t-1})^2}{2\sigma_c^2}\right)}{\sum_c \left(\pi_c \cdot \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left(-\frac{(x_i - \mu_c^{t-1})^2}{2\sigma_c^2}\right)\right)}$$

$\longrightarrow$ the function is a Normal Distribution. or Gaussian Distribution.

2. New value of R after performing the E-step.

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ R_{31} & R_{32} \end{bmatrix}$$

; here we will calculate all the values of R's to compute final new R after E-step.

$$R_{1,1} = \frac{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(x_1 - \mu_1^{t-1})^2}{2\sigma_1^2}\right)}{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(x_1 - \mu_1^{t-1})^2}{2\sigma_1^2}\right) + \pi_2 \cdot \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{(x_1 - \mu_2^{t-1})^2}{2\sigma_2^2}\right)}$$

R_{11} numerator $= 0.4333 \cdot \frac{1}{9.5025} \exp\left(-\frac{(1-3.0769)^2}{28.7576}\right)$

$= 0.0455 \exp\left(-\frac{4.3135}{28.7576}\right) = 0.0455(\exp(-0.1499))$

$= (0.0455)(0.8607)$

$= 0.039$

R_{11} denominator 2nd half.

$= 0.5666 \cdot \frac{1}{12.33} \exp\left(-\frac{(1-15.8823)^2}{48.4462}\right)$

$= 0.0459 \cdot \exp\left(-\frac{221.482}{48.4462}\right) = 0.0459 \cdot (0.0103)$

$= 0.000472$

$\therefore R_{11} = \frac{0.039}{0.039 + 0.000472} = \boxed{0.988}$

$R_{12} = 1 - R_{11} = \boxed{0.012}$

R_{21} numerator $= 0.4333 \frac{1}{9.5025} \exp\left(-\frac{(10-3.0769)^2}{28.7576}\right)$

$= (0.0455)(0.1890) = \underline{0.0085}$

R_{21} denominator 2nd half.

$= 0.0459 \cdot \exp\left(-\frac{(10-15.8823)^2}{48.4462}\right) = (0.0459)(0.4916)$

$= 0.022$

$\therefore R_{21} = \frac{0.0085}{0.0085 + 0.022} = \boxed{0.273}$

$R_{22} = 1 - R_{21} = \boxed{0.727}$

$R_{31} \rightarrow$ numerator $= 0.0455 \exp\left(-\frac{(20-3.0769)^2}{28.7576}\right)$

$= 0.0455(0.0000473) = \underline{0.000002152}$

denominator 2nd half

$= 0.0459\left(\exp\left(-\frac{(20-15.8827)^2}{48.4462}\right)\right) = (0.0459)(0.70539)$

$= \underline{0.03237}$

$\therefore R_{31} = \frac{0.000002152}{0.000002152 + 0.03237} = \boxed{0.0000664 62}$

$\therefore R_{32} = 1 - R_{31} = 0.99993$

$\therefore$ new R becomes.

$$R_{new} = \begin{bmatrix} 0.988 & 0.012 \\ 0.273 & 0.727 \\ 0.000066462 & 0.99993 \end{bmatrix}$$

My final rank and score for the test model

55	new	**Astitv Nagpal**		0.75400	4	~10s

Your Best Entry ↑

Your submission scored 0.73200, which is not an improvement of your best score. Keep trying!