

Calculate probabilities $P(X_t = i | e_{1:t})$ for the following sequence of observations:

NS, NSW time slots 1, 2 respectively.

1	2	3	4	5
6		7		8
9	10	11	12	13

We are going to use the following matrices

$R = T^T$, $R[i][j] = P(X_t = i | X_{t-1} = j)$, pre-calculated $P(e | X)$ given discrepancy d , and $F[i] = P(X_0 = i)$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		0.5				0.5							
2	0.5		0.33										
3		0.5		0.5			0.5						
4			0.33		0.5								
5				0.5				0.5					
6	0.5								0.5				
7		0.33								0.33			
8					0.5								0.5
9						0.5				0.5			
10							0.5				0.33		
11								0.5				0.5	
12											0.33		0.5
13								0.5				0.5	

d	$P(e X) = \epsilon^d (1 - \epsilon)^{4-d}$
0	0.6561
1	0.0729
2	0.0081
3	0.0009
4	0.0001

1	1/13
2	1/13
3	1/13
4	1/13
5	1/13
6	1/13
7	1/13
8	1/13
9	1/13
10	1/13
11	1/13
12	1/13
13	1/13

1. $t = 1, e_1 = NS = 1100$

Calculate $Y = R \times F$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		0.5				0.5							
2	0.5		0.33										
3		0.5		0.5			0.5						
4			0.33		0.5								
5				0.5				0.5					
6	0.5								0.5				
7		0.33								0.33			
8					0.5								0.5
9						0.5				0.5			
10							0.5				0.33		
11								0.5				0.5	
12											0.33		0.5
13								0.5				0.5	

X

1	1/13
2	1/13
3	1/13
4	1/13
5	1/13
6	1/13
7	1/13
8	1/13
9	1/13
10	1/13
11	1/13
12	1/13
13	1/13

=

1	1/13
2	1/6
3	3/26
4	1/6
5	1/13
6	1/13
7	2/39
8	1/13
9	1/13
10	1/6
11	3/26
12	1/6
13	1/13

$O[i][i] = P(e_1 | X_1 = i)$ for $d_1 = \langle 2, 0, 1, 0, 2, 4, 4, 4, 2, 0, 1, 0, 2 \rangle$

Calculate $Z = O \times Y$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.0081												
2		0.6561											
3			0.0729										
4				0.6561									
5					0.0081								
6						0.0001							
7							0.0001						
8								0.0001					
9									0.0081				
10										0.6561			
11											0.0729		
12												0.6561	
13													0.0081

X

1	1/13
2	1/6
3	3/26
4	1/6
5	1/13
6	1/13
7	2/39
8	1/13
9	1/13
10	1/6
11	3/26
12	1/6
13	1/13

=

1	0.000623077
2	0.10935
3	0.008411538
4	0.10935
5	0.000623077
6	7.69231E-06
7	5.12821E-06
8	7.69231E-06
9	0.000623077
10	0.10935
11	0.008411538
12	0.10935
13	0.000623077

Sum up the rows of Z: $sum \approx 0.456736$

Update $F[i] = P(X_1 = i | e_1) = Z[i] / sum$

1	0.001364195
2	0.239416259
3	0.018416635
4	0.239416259
5	0.001364195
6	1.68419E-05
7	1.12279E-05
8	1.68419E-05
9	0.001364195
10	0.239416259
11	0.018416635
12	0.239416259
13	0.001364195

2. $t=2, e_1=NSW=1110$

1	2	3	4	5
6		7		8
9	10	11	12	13

Calculate $Y = R \times F$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		0.5				0.5							
2	0.5		0.33										
3		0.5		0.5			0.5						
4			0.33		0.5								
5				0.5				0.5					
6	0.5								0.5				
7			0.33								0.33		
8					0.5								0.5
9						0.5				0.5			
10								0.5			0.33		
11							0.5					0.5	
12											0.33		0.5
13								0.5					0.5

X

=

1	0.001364195
2	0.239416259
3	0.018416635
4	0.239416259
5	0.001364195
6	1.68419E-05
7	1.12279E-05
8	1.68419E-05
9	0.001364195
10	0.239416259
11	0.018416635
12	0.239416259
13	0.001364195

1	0.119716551
2	0.006820976
3	0.239421873
4	0.006820976
5	0.119716551
6	0.001364195
7	0.012277757
8	0.001364195
9	0.119716551
10	0.006820976
11	0.239421873
12	0.006820976
13	0.119716551

$O[i][i] = P(e_1 | X_1 = i)$ for $d_1 = \langle 1, 1, 2, 1, 2, 3, 3, 3, 1, 1, 2, 1, 2 \rangle$

Calculate $Z = O \times Y$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.0729												
2		0.0729											
3			0.0081										
4				0.0729									
5					0.0081								
6						0.0009							
7							0.0009						
8								0.0009					
9									0.0729				
10										0.0729			
11											0.0081		
12												0.0729	
13													0.0081

X

=

1	0.119716551
2	0.006820976
3	0.239421873
4	0.006820976
5	0.119716551
6	0.001364195
7	0.012277757
8	0.001364195
9	0.119716551
10	0.006820976
11	0.239421873
12	0.006820976
13	0.119716551

1	0.008727337
2	0.000497249
3	0.001939317
4	0.000497249
5	0.000969704
6	1.22778E-06
7	1.105E-05
8	1.22778E-06
9	0.008727337
10	0.000497249
11	0.001939317
12	0.000497249
13	0.000969704

Sum up the rows of Z: $sum \approx 0.025275$

Update $F[i] = P(X_1 = i | e_1) = Z[i] / sum$

1	0.34529224
2	0.019673388
3	0.076728011
4	0.019673388
5	0.038365804
6	4.85763E-05
7	0.000437186
8	4.85763E-05
9	0.34529224
10	0.019673388
11	0.076728011
12	0.019673388
13	0.038365804