$\boxed{\text{Set 1}}$ SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS

MIDTERM EXAM - CALCULUS 1 - SEMESTER 20221

Course code MI1016. Time duration: 60 mins.

Question 1. (1pt) Determine the domain of the function $y = \tan(\frac{1}{x+2})$.

Question 2. (1pt) Classify the discontinuity $x = 0$ of the function $y = \dfrac{\cos x}{1 + e^{\frac{2}{x}}}$.

Question 3. (1pt) Use the first differential to approximate $\sqrt[3]{8.02}$.

Question 4. (1pt) Find the limit $\lim\limits_{x\to 0^+} \frac{x+\sin^2 x+\arctan^3 x+\sinh^4 x}{e^x-1+\ln^2(x+1)-\sqrt{\arcsin(x^3)}}$.

Question 5. (1pt) Compare the following pair of infinitesimals as $x \to 0$

$$\alpha(x) = (e^x - 1)\sin x, \quad \beta(x) = x^2 + x^3.$$

Question 6. (1pt) Determine a such that the following function is differentiable at $x = 0$:

$$f(x) = \begin{cases} e^x + a\sin x & \text{if} \quad x \le 0, \\ \dfrac{1}{1+x} & \text{if} \quad x > 0. \end{cases}$$

Then compute $f'(0)$.

Question 7. (1pt) Given the function $f(x) = \dfrac{1}{\sqrt{1-x}}$. Compute $f^{(n)}(0)$, $n \in \mathbb{N}^*$.

Question 8. (1pt) Find the local extrema of $f(x) = \dfrac{e^x}{x^e}$ on $(0, +\infty)$. Using the variation of $f(x)$, compare e^π and π^e.

Question 9. (1pt) Find the second order Maclaurin expansion of the function $\sin(\cos 2x)$.

Question 10. (1pt) Let f be a twice differentiable function on $[0, 1]$ which satisfies $f(0) = f(\frac{1}{2}) = f(1)$. Prove that there exists $c \in (0, 1)$ such that

$$f''(c) = 2022(f'(c))^2.$$

<u>Note:</u> ***Students are not allowed to use any documents & Proctors sign to confirm the number of problem sheets.***

$\boxed{\text{Set 2}}$ SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS

MIDTERM EXAM - CALCULUS 1 - SEMESTER 20221

Course code MI1016. Time duration: 60 mins.

Question 1. (1pt) Determine the domain of the function $y = \tan(\frac{1}{2-x})$.

Question 2. (1pt) Classify the discontinuity $x = 0$ of the function $y = \dfrac{\cos 2x}{1 + e^{\frac{1}{x}}}$.

Question 3. (1pt) Use the first differential to approximate $\sqrt[3]{8.04}$.

Question 4. (1pt) Find the limit $\lim\limits_{x\to 0^+} \frac{x+\sin^3 x+\arcsin^3 x+\sinh^2 x}{e^x-1+\ln^3(x+1)-\sqrt{\arctan(x^3)}}$.

Question 5. (1pt) Compare the following pair of infinitesimals as

$$\alpha(x) = \tan x \ln(1+x), \quad \beta(x) = x^2 + x^4.$$

Question 6. (1pt) Determine a such that the following function is differentiable at $x = 0$:

$$f(x) = \begin{cases} 2e^x + a\sin x & \text{if} \quad x \le 0, \\ \dfrac{2}{1+x} & \text{if} \quad x > 0. \end{cases}$$

Then compute $f'(0)$.

Question 7. (1pt) Given the function $f(x) = \dfrac{1}{\sqrt{2-x}}$. Compute $f^{(n)}(1)$, $n \in \mathbb{N}^*$.

Question 8. (1pt) Find the local extrema of $f(x) = \dfrac{x^e}{e^x}$ on $(0, +\infty)$. Using the variation of $f(x)$, compare e^π and π^e.

Question 9. (1pt) Find the second order Maclaurin expansion of the function $\sin(\cos 3x)$.

Question 10. (1pt) Let f be a twice differentiable function on $[0, 1]$ which satisfies $f(0) = f(\frac{1}{2}) = f(1)$. Prove that there exists $c \in (0, 1)$ such that

$$f''(c) = -2022(f'(c))^2.$$

<u>Note:</u> ***Students are not allowed to use any documents & Proctors sign to confirm the number of problem sheets.***