<u>CHAPTER 3:</u> IMPORTANT PROBABILITY DISTRIBUTIONS

DISCRETE PROBABILITY DISTRIBUTIONS	CONTINUOUS PROBABILITY DISTRIBUTIONS
Discrete Uniform Distribution Bernoulli Distribution Binomial Distribution Poisson Distribution Geometric Distribution (Pascal) Negative Binomial Distribution	Continuous Uniform Distribution Exponential Distribution Normal Distribution (Gaussian Distribution) Standard Normal Distribution Chi-Squared Distribution Student's t-Distribution
For these discrete random variables, • PMF • Expectation • Variance • Mode value for Binomial Distribution r.v	For these continuous random variables, • PDF • CDF • Expectation • Variance • Memoryless property of Exponential Distribution
Approximation of Binomial Distribution by a Poisson Distribution Normal Approximation to the Binomial Distribution	

Problem 3.1. Traffic accidents at a particular intersection follow Poisson distribution with an average rate of 1.4 per week (7 days) (so that the mean number of accidents in a given period of time will be proportional to the length of time).
(a) What is the probability that the next week is accident-free?
(b) What is the probability that there will be exactly 3 accidents next week?
(c) What is the probability that there will be at most 2 accidents next week?
(d) What is the probability that there will be at least 2 accidents during the next two weeks?
(e) What is the probability that there will be exactly 5 accidents during the next four weeks?
(f) What is the probability that there will be exactly 2 accidents tomorrow?
(g) What is the probability that the next accident will not occur for three days?
(h) What is the probability that there will be exactly three accident-free weeks during the next eight weeks?
(i) What is the probability that there will be exactly five accident-free days during the next week?
(j) What is the probability that during the next week there are less than 2 days in which there are at least 2 accidents in a single day?

Problem 3.2. The lifetime of a light bulb is X hours, where X can be modeled by an exponential distribution with parameter $\lambda = 0.0125$.
(a) Find the mean and variance of the lifetime of a light bulb.
(b) Find the probability that the lifetime of a bulb is less than 100 hours.
(c) Find the probability that the lifetime of a bulb is between 50 hours and 150 hours.

Problem 3.3. The time, T seconds, between the arrival of successive vehicles at a zebra crossing on a road can be modeled by an exponential distribution with parameter $\lambda = 0.025$.
(a) Write down the mean and the variance of T.
(b) A pedestrian takes 30 seconds to cross the road using this zebra crossing. Calculate the probability that:
(b1) no vehicle arrives whilst the pedestrian is crossing;
(b2) no vehicle arrives whilst the pedestrian makes two independent crossings.
(c) A person starts crossing the road immediately after a vehicle has passed. How long should this person take to cross the road to ensure the probability of a vehicle arriving before they have crossed is less than 0.2?

Problem 3.4. The probability that the lifetime, H, of a certain type of electrical component is more than h hours is given by

$$\mathbb{P}(H > h) = e^{-\frac{h}{1000}}, \qquad h > 0.$$

(a) Calculate the probability that a randomly selected component has a lifetime of
(a1) more than 1500 hours;
(a2) between 1000 and 2000 hours;
(a3) precisely 1200 hours.
(b) Calculate the probability that three components, chosen at random, all have lifetimes of more than 1500 hours.
(c) Derive the probability density function for H, and hence state the mean and variance of the component lifetime.

Problem 3.5. The lifetime of a computer is X years, where X can be modeled by an exponential distribution with parameter λ.
(a) Know that $\mathbb{P}(X > 10) = 0,286$, determine the value of λ.
(b) Find the probability that the lifetime of the computer is less than 6 months.
(c) Know that the computer worked for 8 years, what is the probability that the lifetime of the computer is more than 10 years?

Problem 3.6. A clothing store has determined that 30% of the people who enter the store will make a purchase. Eight people enter the store during a one-hour period.
(a) Find the probability that exactly four people will make a purchase
(b) Find the probability that at least one person will make a purchase.
(c) Find the average number of people that make a purchase.
(d) Find the most likely number of people that make a purchase.

Problem 3.7. Let X be a continuous uniform random variable on $(-5, 5)$. Find
(a) the PDF and the CDF
(b) $\mathbb{E}[X]$, Var(X), $\mathbb{E}[X^5]$, $\mathbb{E}[e^X]$.

Problem 3.8. A manufacturer claims that a newly-designed computer chip has a 1% chance of failure because of overheating. To test their claim, a sample of 120 chips are tested. What is the probability that at least two chips fail on testing?

Problem 3.9. A manufacturer of electric light globes knows from past experience that 2% of globes produced are defective. What is the probability that, out of the next 200 globes, less than 2 are defective?
(i) Use the Binomial distribution
(ii) Use the Poisson distribution to approximate that probability.

Problem 3.10. The lifetime of a computer chip is T hours, where T follows an exponential distribution with $\mathbb{E}[T] = 800$ hours and the following PDF

$$f_T(x) = \begin{cases} \dfrac{1}{\lambda} e^{-\frac{x}{\lambda}} & x > 0, \\ 0 & x \leq 0. \end{cases}$$

(a) Find λ and the CDF of T.
(b) Find the probability that the lifetime of the computer chip is more than 1600 hours.