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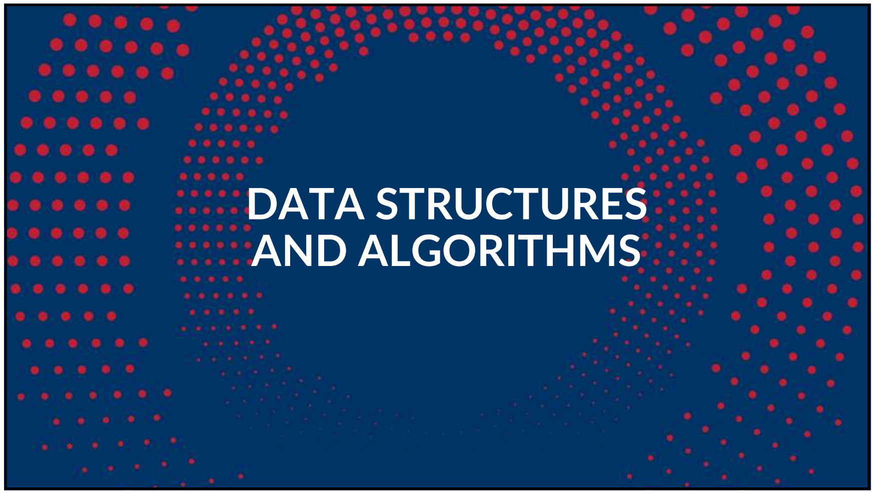
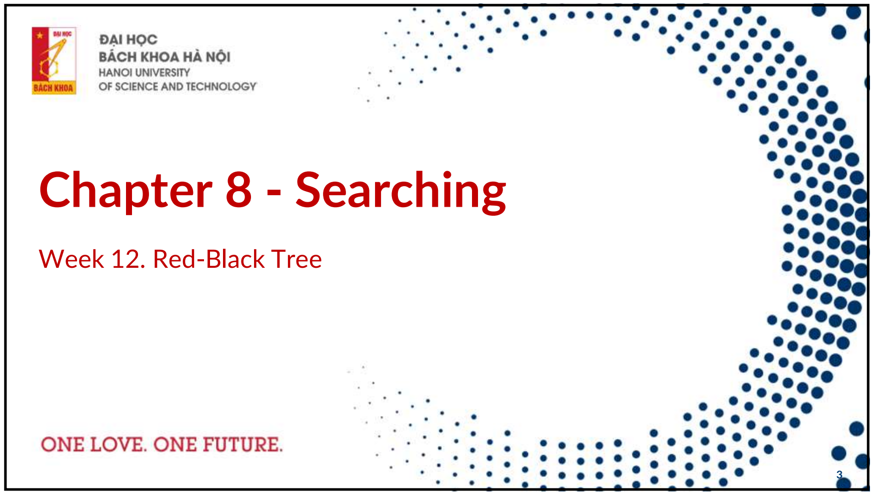
ONE LOVE. ONE FUTURE.



DATA STRUCTURES AND ALGORITHMS



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Chapter 8 - Searching

Week 12. Red-Black Tree

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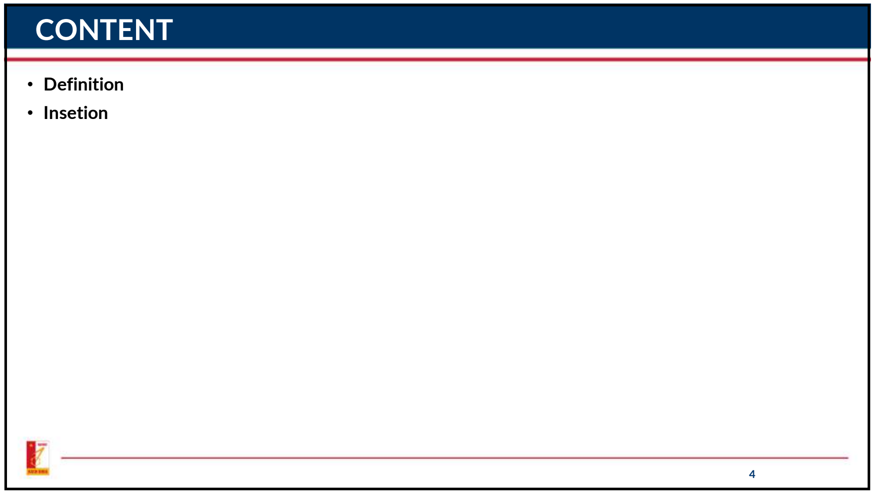
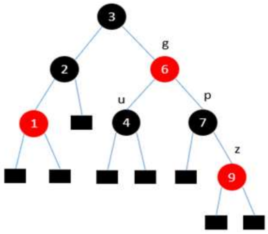


CONTENT

- Definition
- Insetion

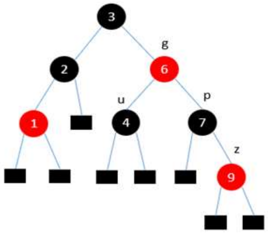
Definition

- Red-Black tree is a binary search tree with properties:
  - Each node has a color (red or black)
  - Color of root is black
  - Leaf (or NULL node) has color black (is presented by a rectangle)
  - A red node has 2 children with color black
  - Paths from a node to leaf nodes have the same number of black nodes



Definition

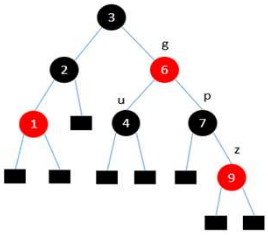
- Red-Black (RB) tree is a binary search tree with properties:
  - (1) Each node has a color (red or black)
  - (2) Color of root is black
  - (3) Leaf (or NULL node) has color black (is presented by a rectangle)
  - (4) A red node has 2 children with color black
  - (5) Paths from a node to leaf nodes have the same number of black nodes
- Notation: *bh*(*x*): number of black nodes (except *x*) on the path from *x* to a leaf
- Lemma 1. A RB tree contains at least $2^{bh(x)}$-1 internal nodes
- Lemma 2. Height of a RB tree containing *n* nodes is at most 2log(*n*+1)



Definition

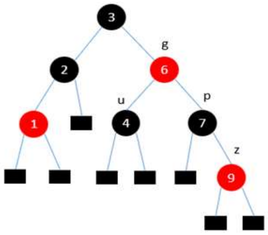
- Typical data structure of a node on a RB tree

```
Node {
   key;  // key of the node
   color; // color of the node
   p; // pointer to the parent
   left;  // pointer to the left-child
   right: // pointer to the right-child
}
```



Insertion

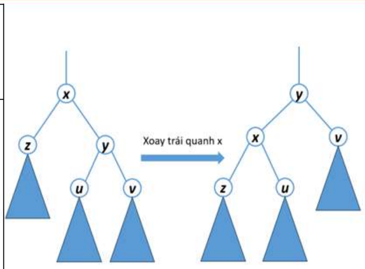
- When inserting a new node (node z) in the RB tree T
  - Insert node z into T as in binary search tree
  - Assign red color to this node z
  - If the RB property is not satisfied, then we perform rotations and change the color of some nodes to recover the RB property
- Notation
  - p: the parent node of z
  - u: sibling node of p (uncle node of z)
  - g: parent of p (grand parent of z)



Insertion: Left Rotation

```
Algorithm leftRotate(r, x)
Input: pointer r to the root of the RB tree T, pointer x to
some node of T
Output: perform left rotation on x, return the pointer to the
root of the resulting tree
```

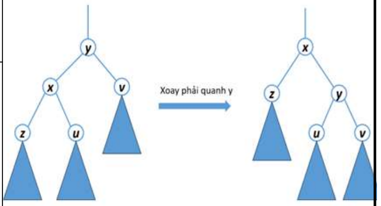
```
y = x.right;
x.right = y.left;
if y.left != NULL then {
      y.left.p = x;
}
y.p = x.p;
if x.p = NULL then {
      r = y;
} else if x = x.p.left then {
      x.p.left = y;
} else {
      x.p.right = y;
}
y.left = x;
x.p = y;
return r;
```



Insertion: Right Rotation

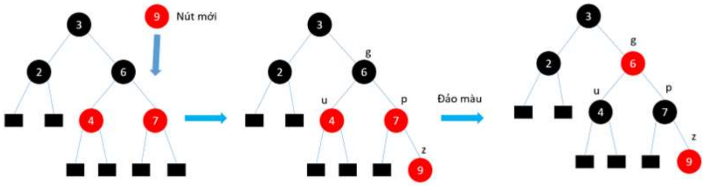
```
Thuật toán rightRotate(r, y)
Input: pointer r to the root of the RB tree T, pointer y to
some node of T
Output: perform right rotation on y, return the pointer to
the root of the resulting tree
```

```
x = y.left;
y.left = x.right;
If x.right != NULL {
     x.right.p = y;
}
x.p = y.p;
if y.p == NULL {
     r = x;
} else if y = y.p.left {
     y.p.left = x;
} else {
     y.p.right = x;
}
x.right = y;
y->p = x;
return r;
```



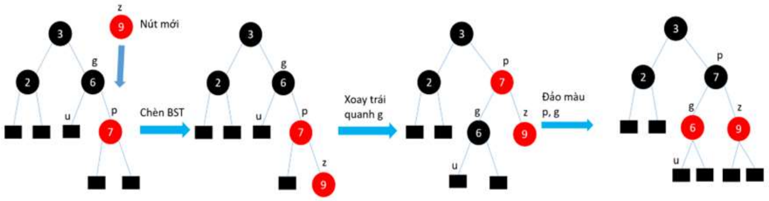
Fix RB property

- Case 1: T is empty, make z as the root, assign black color to z
- Case 2: (**node p – parent of z – with black color**) RB property is satisfied, do nothing
- Case 3: (**node p has color red**), property RB (4) is not satisfied. Node g has color black (as before insertion, T is a RB tree with RB property)
  - Case 3.1: (**node u (if exists) has color red**) → change the color of p and u from red to black and change the color of g by the color red, repeat this process with node g (Figure below).



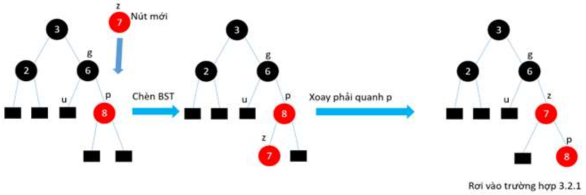
Fix RB property

- Case 3: (**node p has color red**), property RB (4) is not satisfied. Node g has color black (as before insertion, T is a RB tree with RB property)
  - Case 3.2: (node u (if exists) has color black) → perform a single-rotation or double-rotation depending on z is the left child or right child of p
    - Case 3.2.1 (**p is a right child of g and z is a right child of p**), perform left rotation on g and flip the color of p and g (Figure below)



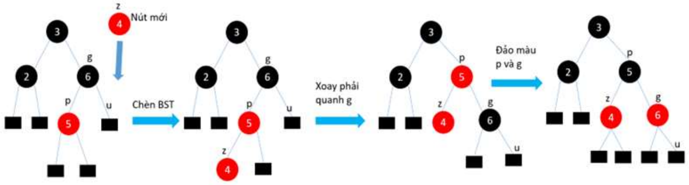
Fix RB property

- Case 3: (**node p has color red**), property RB (4) is not satisfied. Node g has color black (as before insertion, T is a RB tree with RB property)
    - Case 3.2: (node u (if exists) has color black) → perform a single-rotation or double-rotation depending on z is the left child or right child of p
        - Case 3.2.2 (**p is a right child of g and z is a left child of p**), perform right rotation on p (Figure below) return to the case 3.2.1 (process as in the case 3.2.1)



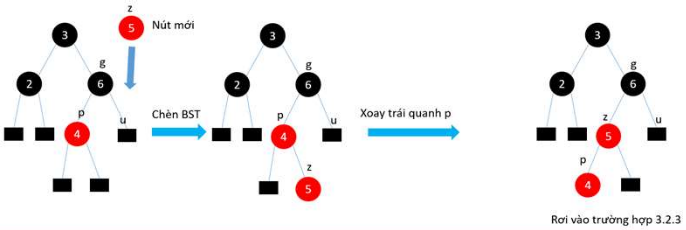
Fix RB property

- Case 3: (**node p has color red**), property RB (4) is not satisfied. Node g has color black (as before insertion, T is a RB tree with RB property)
    - Case 3.2: (node u (if exists) has color black) → perform a single-rotation or double-rotation depending on z is the left child or right child of p
        - Case 3.2.3 (**p is a left child of g and z is a left child of p**), perform right rotation on g and flip the color of p and g (Figure below)



Fix RB property

- Case 3: (**node p has color red**), property RB (4) is not satisfied. Node g has color black (as before insertion, T is a RB tree with RB property)
    - Case 3.2: (node u (if exists) has color black) → perform a single-rotation or double-rotation depending on z is the left child or right child of p
        - Case 3.2.4 (**p is a left child of g and z is a right child of p**), perform left rotation on p (Figure below) return to the case 3.2.3 (process as in the case 3.2.3)



Insertion: Fix RB property

```
Insert(r, k) {
  x = r;
  y = NULL;
  while x !=  NULL do {
    y = x;
    if  k < x.key then
      x = x->left;
    else
      x = x->right;
  }
  z = createNode(k,'RED');
  z.p = y;
  if y = NULL then {
    z.color = 'BLACK';
    return z;
  } else if z.key < y.key then
    y.left = z;
  else
    y.right = z;
  return insertFixUp(r,z);
}
```

```
insertFixUp(r,z) {
  while z.p != NULL and z.p.color = ‘RED' do {
    if z.p = z.p.p.left then {// z->p is a left-child of its parent
      u = z.p.p.right; // y is the uncle of z
      if u != NULL and u.color = 'RED'){// case 3.1
        z.p.color = 'BLACK'; u.color = 'BLACK';  z.p.p.color = 'RED';
        z = z.p.p;// repeat with the grand-parent of z
      } else {
        if z = z.p.right then { z = z.p;  r = leftRotate(r,z);  }
        z.p.color = 'BLACK'; z.p.p.color = 'R';  r = rightRotate(r, z.p.p);
      }
    } else {// z->p is the right-child of its parent
      u = z.p.p.left;
      if u != NULL and u.color = 'RED' then {// case 3.1
        z.p.color = 'BLACK';  u.color = 'BLACK'; z.p.p.color = 'RED'; z = z.p.p;
      } else {
        if z = z.p.left then {  z = z.p;   r = rightRotate(r, z); }
        z.p.color = 'BLACK';  z.p.p.color = 'RED'; r = leftRotate(r, z.p.p);
      }
    }
  }
  r.color = 'BLACK';  return r;
}
```

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THANK YOU !