

Introduction to Communications Engineering

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ONE LOVE. ONE FUTURE.

Thông tin chung

- Tên học phần: **Nhập môn kỹ thuật truyền thông**
- Mã học phần: **IT4593E**
- Khối lượng: **2 TC (2-1-0-4)**
- Lý thuyết và bài tập: **10 buổi lý thuyết, 5 buổi bài tập**
- Đánh giá học phần:
 - 30% QT (kiểm tra + bài tập/project + chuyên cần-quiz)**
 - 70% CK (trắc nghiệm + tự luận)**
- Tài liệu tham khảo:
 - Lecture slides
 - Lecture notes
 - Textbooks, ví dụ ***Communication Systems Engineering***, 2nd Edition, by John G. Proakis Masoud Salehi
 - Internet

Part 2: Digital Modulations

Lec 10: Phase Shift Keying (PSK)

2-PSK: characteristics

1. Bandpass modulation
2. One-dimensional signal space and antipodal binary constellation (equal to 2-PAM)
3. TX filter $p(t)\cos(2\pi f_0 t)$
4. Information associated to the carrier phase = Phase Shift Keying

2-PSK: constellation

SIGNAL SET $M = \{s_1(t) = +\alpha p(t) \cos(2\pi f_0 t), s_2(t) = -\alpha p(t) \cos(2\pi f_0 t)\}$

Information associated to the impulse amplitude
BUT
we can also write

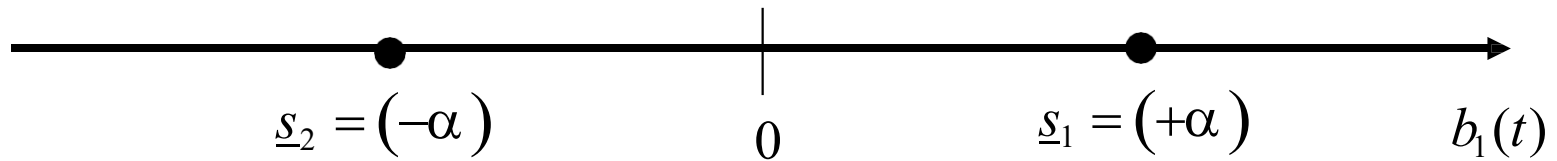
SIGNAL SET $M = \{s_1(t) = +\alpha p(t) \cos(2\pi f_0 t), s_2(t) = +\alpha p(t) \cos(2\pi f_0 t - \pi)\}$

Information associated to the carrier phase

2-PSK: constellation

Vector $b_1(t) = p(t) \cos(2\pi f_0 t) \quad (d=1)$

VECTOR SET $M = \{s_1 = (+\alpha), s_2 = (-\alpha)\} \subseteq R$



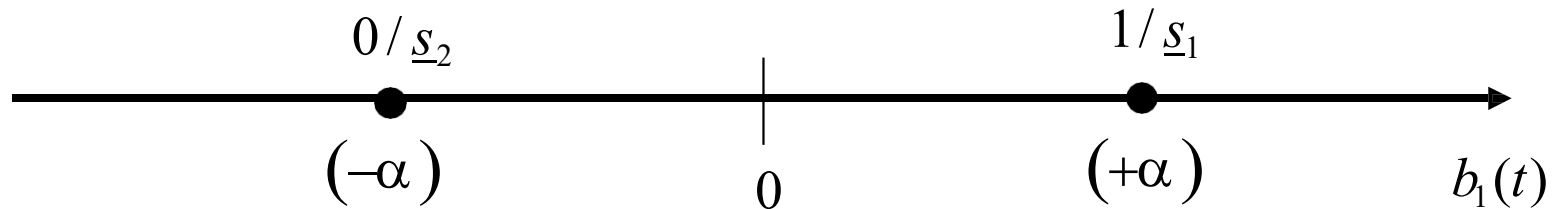
2-PSK: binary labeling

(example)

$$e: H_1 \leftrightarrow M$$

$$e(1) = \underline{s}_1$$

$$e(0) = \underline{s}_2$$



2-PSK: transmitted waveform

$$m = 2 \rightarrow k = 1$$

$$R = R_b$$

$$T = T_b$$

Transmitted waveform

$$s(t) = \sum_{n=-\infty}^{+\infty} a[n]b_1(t - nT)$$

where

$$a[n] \in \{+\alpha, -\alpha\}$$

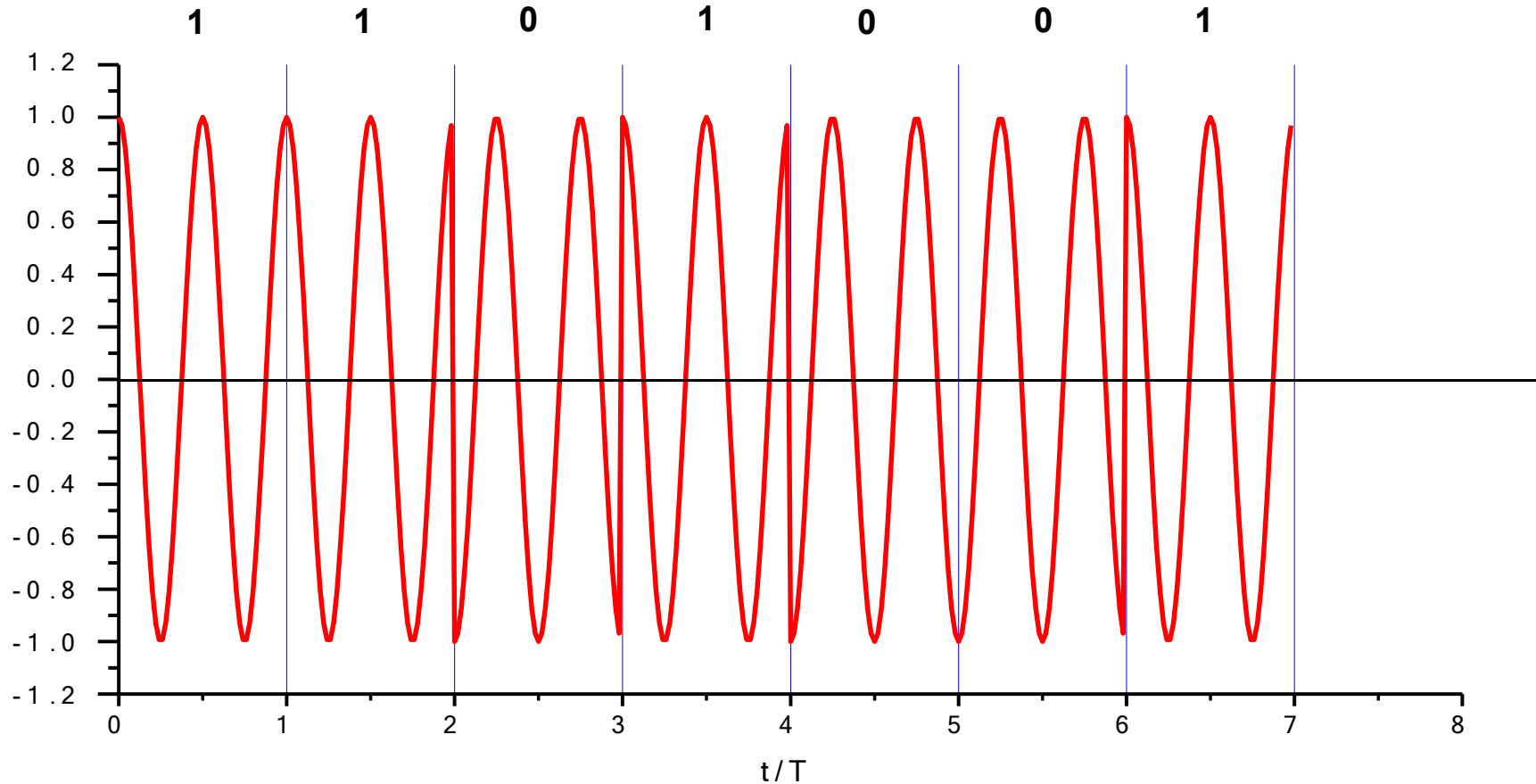
$$b_1(t) = p(t)\cos(2\pi f_0 t)$$

2-PSK: transmitted waveform

example for $p(t) = \frac{1}{\sqrt{T}} P_T(t)$

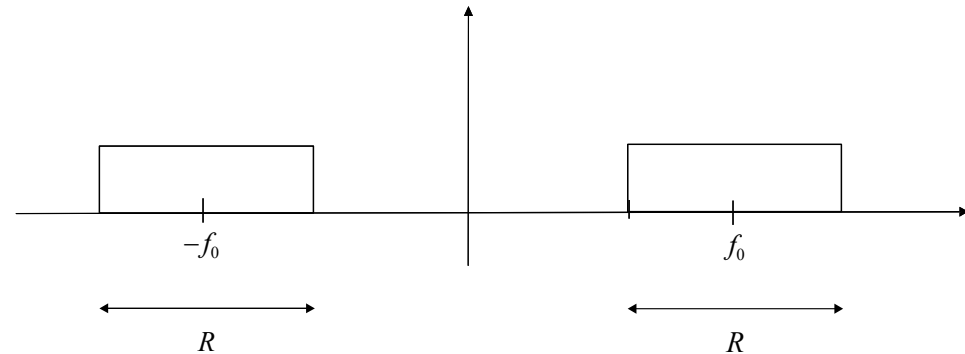
$$f_0 = 2R_b$$

$$\alpha = \sqrt{T}$$



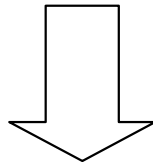
2-PSK: bandwidth and spectral efficiency

Case 1: $p(t)$ = ideal low pass filter



Total bandwidth
(ideal case)

$$B_{id} = R = R_b$$

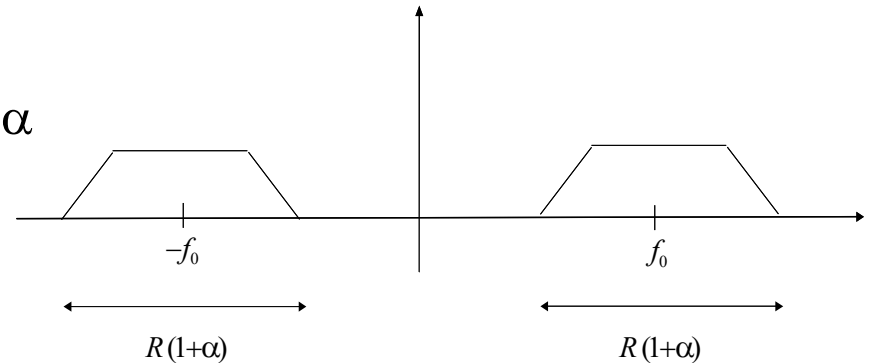


Spectral efficiency
(ideal case)

$$\eta_{id} = \frac{R_b}{B_{id}} = 1 \text{ bps / Hz}$$

2-PSK: bandwidth and spectral efficiency

Case 2: $p(t)$ = RRC filter with roll off α



Total bandwidth $B = R(1 + \alpha) = R_b(1 + \alpha)$

Spectral efficiency

$$\eta = \frac{R_b}{B} = \frac{1}{(1 + \alpha)} \text{ bps / Hz}$$

Exercise

Given a bandpass channel with bandwidth $B = 4000$ Hz, centred around $f_0 = 2$ GHz, compute the maximum bit rate R_b we can transmit over it with a 2-PSK constellation in the two cases:

- Ideal low pass filter
- RRC filter with $\alpha = 0.25$

2-PSK: modulator

The transmitted waveform is given by $s(t) = \sum_n a[n]b_1(t - nT)$

where $b_1(t) = p(t) \cos(2\pi f_0 t)$

Then we must generate $s(t) = \sum_n a[n]p(t - nT) \cos(2\pi f_0(t - nT))$

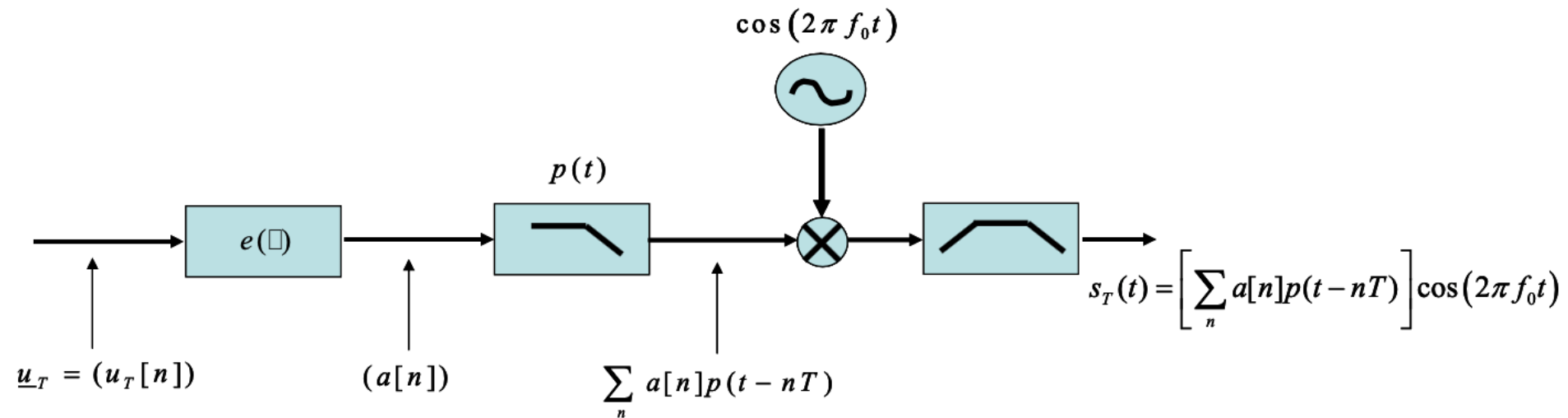
We choose f_0 multiple of $R=1/T$

It follows $\cos(2\pi f_0(t - nT)) = \cos(2\pi f_0 t - 2\pi f_0 nT) = \cos(2\pi f_0 t)$

Then we can generate

$$s(t) = \left[\sum_n a[n]p(t - nT) \right] \cos(2\pi f_0 t)$$

2-PSK: modulator



2-PSK: demodulator

Given the received signal $\rho(t)$

the received symbol is obtained by projecting it
on the vector $b_1(t) = p(t) \cos(2\pi f_0 t)$

$$\rho[0] = \int_{-\infty}^{+\infty} \rho(t) b_1(t) dt = \int_{-\infty}^{+\infty} \rho(t) p(t) \cos(2\pi f_0 t) dt$$

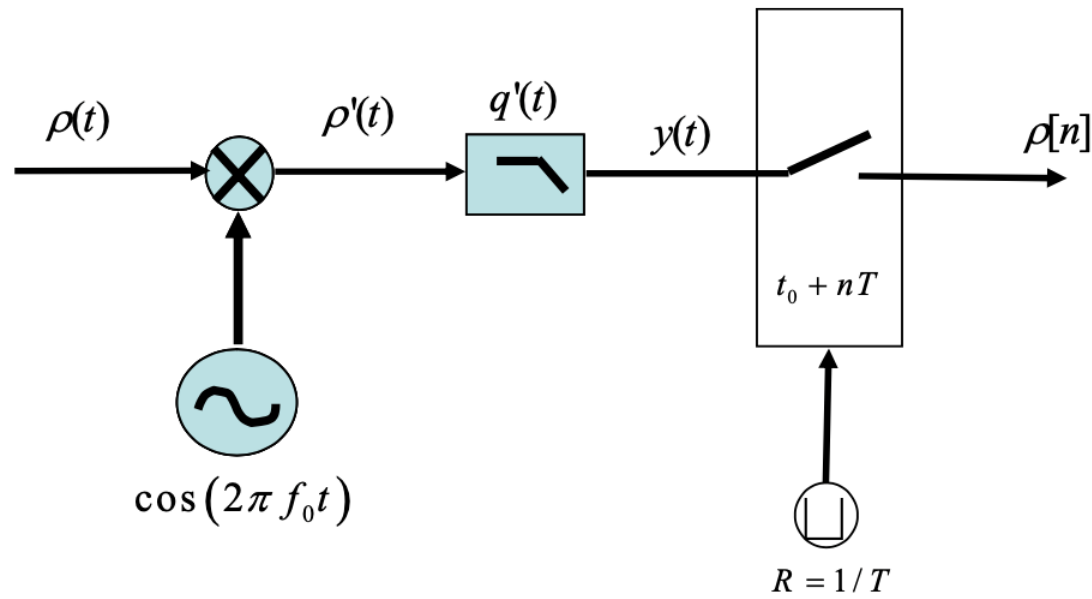
This projection could be computed by using a matched filter

$$q(t) = b_1(T - t) = p(T - t) \cos(2\pi f_0 (T - t))$$

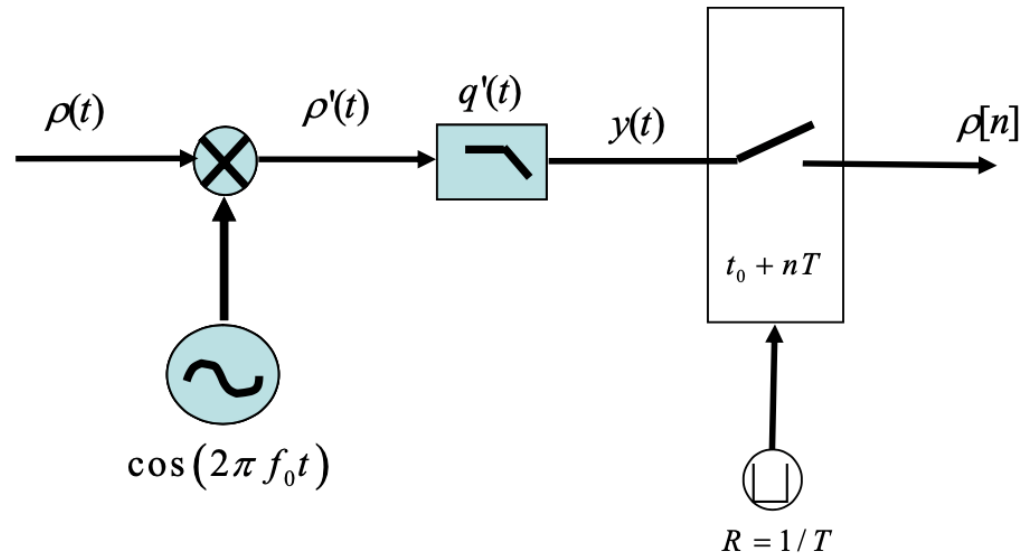
2-PSK: demodulator

As an alternative, we can work as follows:

1. Given the received signal $\rho(t)$ multiply it by $\cos(2\pi f_0 t)$
2. Use a filter matched to $p(t)$: $q'(t) = p(T - t)$



2-PSK: demodulator

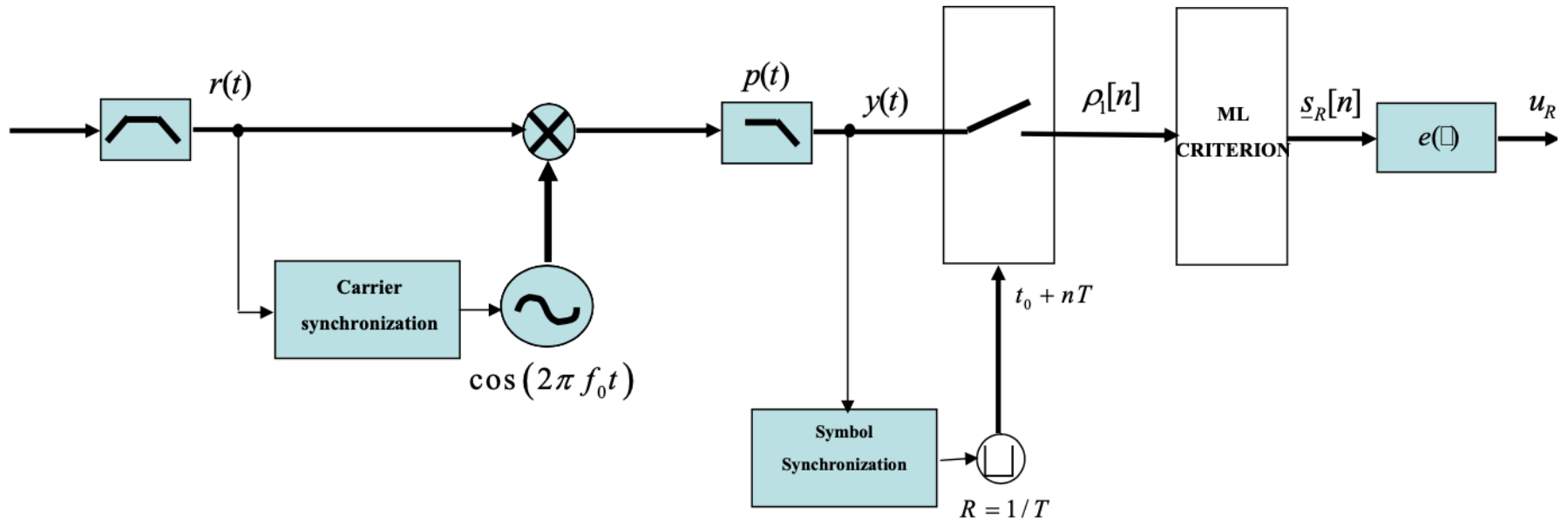


By sampling the matched filter output waveform we obtain

$$y(t) = \int_{-\infty}^{+\infty} \rho'(\tau) q'(t - \tau) d\tau = \int_{-\infty}^{+\infty} \rho(\tau) \cos(2\pi f_0 \tau) p(T - t + \tau) d\tau$$

$$y(t = T) = \int_{-\infty}^{+\infty} \rho(\tau) \cos(2\pi f_0 \tau) p(\tau) d\tau = \rho[0]$$

2-PSK: demodulator



2-PSK: interpretation

We generate a baseband signal

$$v(t) = \sum_n a[n]p(t - nT)$$

Multiplication by cosine shifts the spectrum around f_0

$$s(t) = v(t) \cos(2\pi f_0 t)$$

2-PSK: interpretation

At the receiver side, multiplication by cosine generates

$$s(t) \cos(2\pi f_0 t) = v(t) \cos(2\pi f_0 t) \cos(2\pi f_0 t) = v(t) \cos^2(2\pi f_0 t) = v(t) \left[\frac{1 + \cos(2\pi (2f_0) t)}{2} \right]$$

This signal enters the matched filter $q(t) = p(T-t)$.

It is a low pass filter: the high frequency component around $2f_0$ is eliminated.

Only the baseband component $v(t) = \sum_n a[n] p(t - nT)$ survives.

The matched filter output is then equal to $a[n]$ when sampled at $t_0 + nT$

2-PSK: analytic signal

The 2-PSK transmitted waveform

$$s(t) = \left[\sum_n a[n] p(t - nT) \right] \cos(2\pi f_0 t)$$

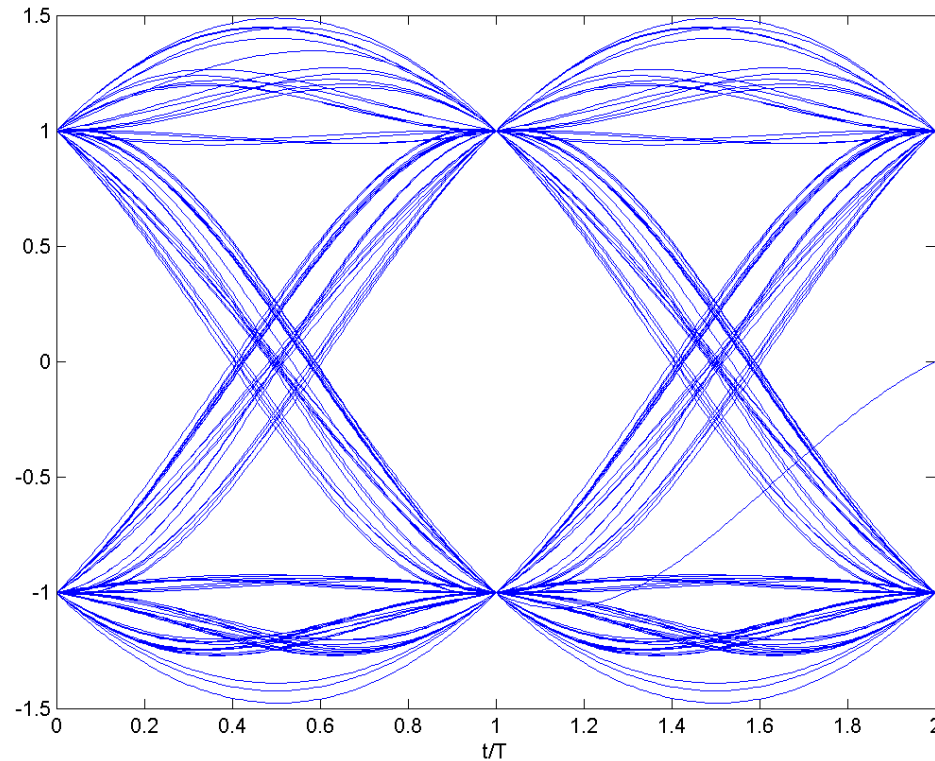
can be written as

$$s(t) = \text{Re}[\dot{s}(t)] = \text{Re} \left[\sum_n a[n] p(t - nT) e^{j2\pi f_0 t} \right]$$

where $\dot{s}(t)$ is called the **analytic signal** associated with $s(t)$

2-PSK: Eye diagram

2-PSK constellation with RRC filter ($\alpha = 0.5$)



2-PSK: error probability

$$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

ERROR PROBABILITY

