# BT tuần 4

VD2: $I = \iiint_V z\,dx\,dy\,dz$ với $V$ $\begin{cases} z^2 = 9(x^2+y^2) \\ z = 9 \end{cases}$

Đặt $\begin{cases} x = r/3 \cos\varphi \\ y = r/3 \sin\varphi \\ z = z \end{cases} \rightarrow |J| = r$

$V \rightarrow V'$ $\begin{cases} 0 \le r \le z \\ 0 \le \varphi \le 2\pi \\ 0 \le z \le 9 \end{cases}$

$$\rightarrow I = \iiint_V zr\,dr\,d\varphi\,dz$$

$$= \frac{1}{9}\int_0^{2\pi} d\varphi \int_0^9 dz \int_0^z zr\,dr$$

$$= \frac{2\pi}{9}\int_0^9 \frac{zr^2}{2}\Big|_0^z dz$$

$$= \frac{2\pi}{9}\int_0^9 \frac{z^3}{2} dz$$

$$= \frac{2\pi}{9}\cdot\frac{z^4}{8}\Big|_0^9$$

$$= \frac{729\pi}{4}$$

VD3: $I = \iiint_V [(x+y)^2 + (x-z)^3]\,dxdydz$ $\quad V: 1 \le x^2+y^2+z^2 \le 9$

$$I = \iiint_V (x^2 + 2xy + y^2 + x^3 - 3x^2z + 3xz^2 - z^3)\,dxdydz$$

$$= \iiint_V (x^2+y^2)\,dxdydz + \iiint_V (2xy + x^3 + 3xz^2)\,dxdydz$$

$$+ \iiint_V (-3x^2z - z^3)\,dxdydz = I_1 + I_2 + I_3$$

Do $\begin{cases} V \text{ đối xứng qua mặt phẳng } x=0 \\ \text{Hàm số } f(x,y,z) = 2xy + x^3 + 3xz^2 \text{ là hàm lẻ đối với } x \end{cases}$

$\to I_2 = 0$

Do $\begin{cases} V \text{ đối xứng qua mặt phẳng } z=0 \\ \text{Hàm số } g(x,y,z) = -3x^2z - z^3 \text{ là hàm lẻ theo biến } z \end{cases}$

$\to I_3 = 0$

$$\Rightarrow I = I_1 = \iiint_V (x^2+y^2)\,dxdydz$$

Đặt $\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \to |J| = r^2\sin\theta$

$V \to V' \begin{cases} 1 \le r \le 2 \\ 0 \le \varphi \le 2\pi \\ 0 \le \theta \le \pi \end{cases}$

$$\to I = \iiint_V r^2\sin^2\theta \cdot r^2\sin\theta\,drd\varphi\,d\theta$$

$$= \iiint_V r^4\sin^3\theta\,drd\varphi\,d\theta$$

$$= \int_0^{2\pi} d\varphi \int_1^2 r^4 dr \int_0^{\pi} \frac{3\sin\theta - \sin 3\theta}{4}\,d\varphi$$

$$= 2\pi \cdot \frac{r^5}{5}\Big|_1^2 \left(-\frac{3}{4}\cos\theta + \frac{\cos 3\theta}{12}\right)\Big|_0^{\pi}$$

$$= 2\pi \cdot \frac{31}{5} \cdot \frac{4}{3} = \frac{248\pi}{15}$$