HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS
DEPARTMENT OF APPLIED MATHEMATICS

MI2020E

EXERCISES OF PROBABILITY AND STATISTICS

Hanoi - 2023

Chapter 4

ESTIMATION

Problem 4.1.

The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint:

$$3.4, 2.5, 4.8, 2.9, 3.6, 2.8, 3.3, 5.6, 3.7, 2.8, 4.4, 4.0, 5.2, 3.0, 4.8$$

Assuming that the measurements represent a random sample from a normal population $N(\mu, \sigma^2)$, where σ is supposed to be 0.9.

(a) Find the point estimate of μ by the method of moments.
(b) Find the point estimate of μ by the maximum likelihood estimation (MLE) method.
(c) Prove that the estimator of μ by MLE method or method of moments is unbiased.
(d) Find a 95% confidence interval estimate of μ.
(e) What is the confidence level $1 - \alpha$ such that the error of confidence interval estimation equals to 0.4.

Problem 4.2.

The number of cars sold annually by used car salespeople is normally distributed with a standard deviation of 15. A random sample of 15 salespeople was taken, and the number of cars each sold is listed here:

$$79, 43, 58, 66, 101, 63, 79, 33, 58, 71, 60, 101, 74, 55, 88$$

(a) Find the 95% confidence interval estimate of the population mean μ. Interpret the interval estimate.
(e) Determine the sample size necessary to estimate the population mean μ to within 5 with 90% confidence level.

Problem 4.3.

The heights of students at a college follow a normal distribution $N(\mu, \sigma^2)$. The following measurements were recorded for a sample of 20 students:

$$160.3, 162.1, 165.8, 161.1, 170.9, 165.0, 168.3, 166.9, 158.8, 166.8,$$

$$159.2, 167.7, 169.8, 169.4, 168.4, 160.7, 171.6, 167.3, 158.3, 163.4$$

(a) Find the point estimate of μ and σ^2 by the method of moments.
(b) Find the point estimate of μ and σ^2 by the maximum likelihood estimation (MLE) method.
(c) Find a 95% confidence interval estimate of μ.
(d) Find a 90% confidence interval estimate of σ^2.

Problem 4.4.

A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, 1.02 and 1.03 centimeters. Suppose that the diameter of pieces from this machine follows a normal distribution $N(\mu, \sigma^2)$.

(a) Find a 90% confidence interval estimate for the mean diameter of pieces from this machine.
(b) What can we assert with 90% confidence about the possible size of our error if we estimate the mean diameter of all pieces to be 1.007 centimeters?
(c) Find a 95% confidence interval estimate of σ^2.

Problem 4.5.

A random sample of delivery times (X) for 35 deliveries to an address across town by a courier service was recorded. These data (in hours) are shown here.

X	[3.5,4.0)	[4.0,4.5)	[4.5,5.0)	[5.0, 5.5)	[5.5,5.0)	[6.0, 6.5)
Frequency	4	5	9	8	6	3

Find a 98% confidence interval estimate for the average delivery time.

Problem 4.6.

The contents (X) of similar containers of sulfuric acid have a mean of μ and a standard deviation of 0.3 (liters). The content of a random sample of 40 similar containers was recorded and the data are shown here.

X	[9.4,9.6)	[9.6,9.8)	[9.8,10.0)	[10.0, 10.2)	[10.2,10.4)	[10.4, 10.6)
Frequency	6	6	9	14	4	1

Find a 95% confidence interval estimate for μ.

Problem 4.7.

In a random sample of $n = 500$ families owning television sets in the city of Hamilton, Canada, it is found that $m = 340$ subscribe to HBO.

(a) Find a 95% confidence interval for the actual proportion p of families with television sets in this city that subscribe to HBO.
(b) What is the error of 90% confidence interval estimate of p?
(c) How large a sample is required if we want to be 95% confident that our estimate of p is within 0.02 of the true value?

Problem 4.8.

A random sample of 200 voters in a town is selected, and 114 are found to support an annexation suit.

(a) Find the 96% confidence interval for the fraction of the voting population favoring the suit.
(b) What can we assert with 96% confidence about the possible size of our error if we estimate the fraction of voters favoring the annexation suit to be 0.57?

Problem 4.9.

Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled nicely by the exponential distribution with a rate parameter of λ. A random sample of size 50 drawn from the population T are given as follows:

T	[0, 5)	[5,10)	[10,15)	[15, 20)	[20,25)
Frequency	25	10	7	5	3

(a) Find the point estimate of λ by the method of moments.
(b) Find the point estimate of λ by the maximum likelihood estimation (MLE) method.
(c) Find a 95% confidence interval estimate of λ using the central limit theorem

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{X} - 1/\lambda}{(1/\lambda)/\sqrt{n}} \to N(0;1).$$

(d) Find a 95% confidence interval estimate of λ using the following limit theorem

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{\bar{X} - 1/\lambda}{S/\sqrt{n}} \to N(0;1).$$

Problem 4.10.

Recorded the number of customers enter a store during 50 periods of one hour and obtained the following data:

T	2	3	4	6	7	8	9	10	11	12	13	14	15	‘6	17
Frequency	1	1	1	3	4	5	10	7	6	3	3	1	2	2	1

Suppose that the number of customers enter a store per hour follows a Poisson distribution a parameter of λ

(a) Find the point estimate of λ by the method of moments.
(b) Find the point estimate of λ by the maximum likelihood estimation (MLE) method.
(c) Find a 95% confidence interval estimate of λ using the central limit theorem

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{X} - \lambda}{\sqrt{\lambda}/\sqrt{n}} \to N(0;1).$$

(d) Find a 95% confidence interval estimate of λ using the following limit theorem

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{\bar{X} - \lambda}{S/\sqrt{n}} \to N(0;1).$$