## PROBABILITY LAWS

- Additive Rule
- Conditional Probability
- Multiplicative Rule
- Independence
- Law of Total Probability
- Bayes' Rule
- Bernoulli Trials

**Problem 1.31.** A manufacturer of a flu vaccine is concerned about the quality of its flu serum. Batches of serum are processed by three different departments having rejection rates of 0.15, 0.12, and 0.09, respectively. The inspections by the three departments are sequential and independent.
(a) What is the probability that a batch of serum survives the first departmental inspection but is rejected by the second department?
(b) Given that a batch of serum has been rejected, what is the probability that it is rejected by the second department?

**Problem 1.32.** An aerospace company has submitted bids on two separate federal government defense contracts. The company president believes that there is a $40\%$ probability of winning the first contract. If they win the first contract, the probability of winning the second is $70\%$. However, if they lose the first contract, the president thinks that the probability of winning the second contract decreases to $50\%$.
(a) What is the probability that they win both contracts?
(b) What is the probability that they lose both contracts?
(c) What is the probability that they win only one contract?

**Problem 1.33.** An investor believes that on a day when the Dow Jones Industrial Average (DJIA) increases, the probability that the NASDAQ also increases is $77\%$. If the investor believes that there is a $60\%$ probability that the DJIA will increase tomorrow, what is the probability that the NASDAQ will increase as well?

**Problem 1.34.** A unions executive conducted a survey of its members to determine what the membership felt were the important issues to be resolved during upcoming negotiations with management. The results indicate that $74\%$ of members felt that job security was an important issue, whereas $65\%$ identified pension benefits as an important issue. Of those who felt that pension benefits were important, $60\%$ also felt that job security was an important issue. One member is selected at random.
(a) What is the probability that he or she felt that both job security and pension benefits were important?
(b) What is the probability that the member felt that at least one of these two issues was important?

**Problem 1.35.** In a random experiment, $A, B, C$ and $D$ are events with probabilities $\mathbb{P}(A) = \frac{1}{4}$, $\mathbb{P}(B) = \frac{1}{8}$, $\mathbb{P}(C) = \frac{5}{8}$ and $\mathbb{P}(D) = \frac{3}{8}$. Furthermore, $A$ and $B$ are disjoint, while $C$ and $D$ are independent.
(a) Find $\mathbb{P}(A \cap B)$, $\mathbb{P}(A \cup B)$, $\mathbb{P}(A \cap B^c)$, $\mathbb{P}(A \cup B^c)$.
(b) Are $A$ and $B$ independent?
(c) Find $\mathbb{P}(C \cap D)$, $\mathbb{P}(C \cap D^c)$, $\mathbb{P}(C^c \cap D^c)$.
(d) Are $C^c$ and $D^c$ independent?

**Problem 1.36.** In a random experiment, $A, B, C$ and $D$ are events with probabilities $\mathbb{P}(A \cup B) = \frac{5}{8}$, $\mathbb{P}(A) = \frac{3}{8}$, $\mathbb{P}(C \cap D) = \frac{1}{3}$ and $\mathbb{P}(C) = \frac{1}{2}$. Furthermore, $A$ and $B$ are disjoint, while $C$ and $D$ are independent.

(a) Find $\mathbb{P}(A \cap B)$, $\mathbb{P}(B)$, $\mathbb{P}(A \cap B^c)$, $\mathbb{P}(A \cup B^c)$.
(b) Are $A$ and $B$ independent?
(c) Find $\mathbb{P}(D)$, $\mathbb{P}(C \cap D^c)$, $\mathbb{P}(C^c \cap D^c)$, $\mathbb{P}(C|D)$.
(d) Find $\mathbb{P}(C \cup D)$, $\mathbb{P}(C \cup D^c)$.
(e) Are $C$ and $D^c$ independent?

**Problem 1.37.** You have a six-sided die that you roll once. Let $R_i$ denote the event that the roll is $i$. Let $G_j$ denote the event that the roll is greater than $j$. Let $E$ denote the event that the roll of the die is even-numbered.
(a) What is $\mathbb{P}(R_3|G_1)$, the conditional probability that 3 is rolled given that the roll is greater than 1?
(b) What is the conditional probability that 6 is rolled given that the roll is greater than 3?
(c) What is $\mathbb{P}(G_3|E)$, the conditional probability that the roll is greater than 3 given that the roll is even?
(d) Given that the roll is greater than 3, what is the conditional probability that the roll is even?

**Problem 1.38.** Two different suppliers, A and B, provide a manufacturer with the same part. All suppliers of this part are kept in a large bin. In the past, 5 percent of the parts supplied by A and 9 percent of the parts supplied by B have been defective. A supplies four times as many parts as B. Suppose you reach into the bin and select a part and find it is non-defective. What is the probability that it was supplied by A?

**Problem 1.39.** Suppose that 30 percent of the bottles produced in a certain plant are defective. If a bottle is defective, the probability is 0.9 that an inspector will notice it and remove it from the filling line. If a bottle is not defective, the probability is 0.2 that the inspector will think that it is defective and remove it from the filling line.
(a) If a bottle is removed from the filling line, what is the probability that it is defective?
(b) If a customer buys a bottle that has not been removed from the filling line, what is the probability that it is defective?

**Problem 1.40.** A factory has three machines A, B, and C. Past records show that the machine A produced $40\%$ of the items of output, the machine B produced $35\%$ of the items of output, and machine C produced $25\%$ of the items. Furthermore, $2\%$ of the items produced by machine A were defective, $1.5\%$ produced by machine B were defective, and $1\%$ produced by machine C were defective.
(a) If an item is drawn at random, what is the probability that it is defective?
(b) An item is acceptable if it is not defective. What is the probability that an acceptable item comes from machine A?

**Problem 1.41.** A financial analyst estimates that the probability that the economy will experience a recession in the next 12 months is $25\%$. She also believes that if the economy encounters a recession, the probability that her mutual fund will increase in value is $20\%$. If there is no recession, the probability that the mutual fund will increase in value is $75\%$. Find the probability that the mutual funds value will increase.

**Problem 1.42.** The Rapid Test is used to determine whether someone has HIV (the virus that causes AIDS). The false-positive and false-negative rates are $2.7\%$ and $8\%$, respectively. A physician has just received the Rapid Test report that his patient tested positive. Before receiving the result, the physician assigned his patient to the low-risk group (defined on the basis of several variables) with only a $0.5\%$ probability of having HIV. What is the probability that the patient actually has HIV?

**Problem 1.43.** A customer-service supervisor regularly conducts a survey of customer satisfaction. The results of the latest survey indicate that $8\%$ of customers were not satisfied with the service they received at their last visit to the store. Of those who are not satisfied, only $22\%$ return to the store within a year. Of those who are satisfied, $64\%$ return within a year. A customer has just entered the store. In response to your question, he informs you that it is less than 1 year since his last visit to the store. What is the probability that he was satisfied with the service he received?

**Problem 1.44.** A telemarketer sells magazine subscriptions over the telephone. The probability of a busy signal or no answer is $65\%$. If the telemarketer does make contact, the probability of 0, 1, 2, or 3 magazine subscriptions is 0.5, 0.25, 0.2, and 0.05, respectively. Find the probability that in one call she sells no magazines.

**Problem 1.45.** There are 3 boxes of marbles: the first box contains 3 red marbles, 2 white marbles; the second box contains 2 red marbles, 2 white marbles; the third box has no marbles. Draw randomly 1 marble from the first box and 1 marble from the second box and put them in the third box. Then, from the third box, 1 marble is drawn at random. Given that the marble drawn from the third box is red, what is the probability that the marble drawn from the first box is red?

**Problem 1.46.** Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is a zero with probability 0.8, independent of any other bit.
(a) What is the probability of the code word 00111?
(b) What is the probability that a code word contains exactly three ones?

**Problem 1.47.** Suppose each day that you drive to work a traffic light that you encounter is either green with probability $\frac{7}{16}$, red with probability $\frac{7}{16}$ , or yellow with probability $\frac{1}{8}$, independent of the status of the light on any other day. If over the course of five days, $G, Y$, and $R$ denote the number of times the light is found to be green, yellow, or red, respectively, what is the probability that $\mathbb{P}(G = 2, Y = 1, R = 2)$? Also, what is the probability $\mathbb{P}(G = R)$?

**Problem 1.48.** The probability that a salesperson closed a sale in each call to a customer is $70\%$. How many calls will she have to make such that the probability that she will close at least one sale is more than 0.99?

**Problem 1.49.** Let $A$ and $B$ be independent events; show that $A^c$, $B$ are independent, and deduce that $A^c$, $B^c$ are independent.

**Problem 1.50.** Let $X$ and $Y$ be the scores on two fair dice taking values in the set $\{1, 2, \dots, 6\}$. Let $A_1 = \{X + Y = 9\}$, $A_2 = \{X \in \{1, 2, 3\}\}$, and $A_3 = \{X \in \{3, 4, 5\}\}$. Show that

$$\mathbb{P}(A_1 \cap A_2 \cap A_3) = \mathbb{P}(A_1)\,\mathbb{P}(A_2)\,\mathbb{P}(A_3).$$

Are these three events independent?

**Problem 1.51.** A biased coin is tossed repeatedly. Each time there is a probability $p$ of a head turning up. Find the probability that an even number of heads has occurred after $n$ tosses (zero is an even number).

**Problem 1.52.** Three players A, B and C take turns tossing a fair coin. Suppose that A tosses the coin first, B tosses the second and C tosses third and cycle is repeated indefinitely until someone wins by being the first player to obtain a head. Determine the probability that each of the three players will win.

**Problem 1.53.** Let $A, B$ and $C$ be three independent events with probabilities $\mathbb{P}(A) = \frac{1}{4}$, $\mathbb{P}(B) = \frac{1}{3}$, $\mathbb{P}(C) = \frac{1}{2}$.
(a) What is the probability that none of these three events will occur?
(b) Determine the probability that exactly one of these three events will occur.

**Problem 1.54.** Consider a trial of rolling three fair dice simultaneously.
(a) Repeat this trial 10 times. Find the probability that there are exactly 3 trials (out of 10 trials) each of which has a sum of 8.
(b) Repeat this trial until the sum of three fair dice is 8. Determine the probability that this trial is needed to repeat $n$ times.

**Problem 1.55.** There are two lots of items. The first lot contains 8 non-defective items and 2 defective items. The second lot contains 5 non-defective items and 3 defective items. Two items are chosen at random from the first lot and then placed in the second lot. Then, two items are chosen at random from the second lot. Find the probability that there are $k$ non-defective items out of two items chosen from the second lot, with $k \in \{0, 1, 2\}$.

**Problem 1.56.** A box contains 8 white and 5 red marbles. One marble is taken out of the box at random. If the marble taken is white, it is replaced and 3 additional white marbles are also added in the box. If the marble taken is red, it is replaced and 3 additional red marbles are also put into the box. A marble is then taken out of the box at random the second time. Find the probability that the marble taken at the second stage is red.

**Problem 1.57.** A box contains 8 white and 5 red marbles. One marble is taken out of the box at random. If the marble taken is white, it is replaced and 3 additional white marbles are also added in the box. If the marble taken is red, it is not replaced in the urn and no additional balls are added. A marble is then taken out of the box at random the second time. Find the probability that the marble taken at the second stage is red.

**Problem 1.58.** In a factory, there are ten machines that function independently. The probability failure of each machine in a given period time is 0.09. Given that there are at least two machines that fail to function in that given period time, determine the probability that there are exactly 4 failed machines.

**Problem 1.59.** A machine needs at least one of its three parts to work correctly. The probability of part 1 failing is $p_1$, the probability of part 2 failing is $p_2$, and the probability of part 3 failing is $p_3$. Each part failing is independent of any other part failing. Find the probability that the machine fails.

**Problem 1.60.** Suppose that $A$, $B$, and $C$ are three independent events in an experiment such that $\mathbb{P}(A) = 1/4$, $\mathbb{P}(B) = 1/3$, and $\mathbb{P}(C) = 1/2$. What is the probability that at least one of these three events will occur?

**Problem 1.61.** Suppose that $A$ and $B$ are two events in an experiment such that $\mathbb{P}(A) = 0.3$ and $\mathbb{P}(B|\overline{A}) = 0.6$. What is $\mathbb{P}(\overline{A} \cap \overline{B})$?

**Problem 1.62.** In a random experiment, let $A$ and $B$ be two independent events such that $\mathbb{P}(A \cup B) = 0.64$ and $\mathbb{P}(B) = 0.4$. What is $\mathbb{P}(A)$?

**Problem 1.63.** In a random experiment, $A$ and $B$ are two mutually exclusive events with $\mathbb{P}(A \cup B) = 0.625$ and $\mathbb{P}(A) = 0.375$. What are $\mathbb{P}(A \cap \overline{B})$ and $\mathbb{P}(B)$?

**Problem 1.64.** In a random experiment, let $A$ and $B$ be two events such that $\mathbb{P}(A) = 0.2$, $\mathbb{P}(B) = 0.6$, and $\mathbb{P}(B|A) = 0.5$. Find $\mathbb{P}(\overline{B}|\overline{A})$.

**Problem 1.65.** Three people, A, B and C, fire at a target simultaneously and independently. Their probabilities of hitting the target are 0.2, 0.4 and 0.3 respectively. Let $p_1$ be the probability that the target is hit exactly by one of them, and let $p_0$ be the probability that the target is not hit by any one of them. Find $p_1$ and $p_0$.

**Problem 1.66.** Suppose that traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.7 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green. Let $A$ be "the second light is green" and $B$ be "you wait for at least one light". Find $\mathbb{P}(A)$ and $\mathbb{P}(B)$.

**Problem 1.67.** Suppose that $n$ guests check their hats when they arrive at a restaurant, and that these hats are returned to them in a random order when they leave. What is the probability that no guest will receive their proper hat when $n = 3$ or $n = 4$?