PRACTICE 5

PROPOSITIONAL LOGIC

1. Which of the following are correct?

c.
$$(A \land B) \models (A \Leftrightarrow B)$$
.

$$\mathbf{f}$$
. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$.

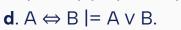
e.
$$A \Leftrightarrow B \models \neg A \lor B$$
.

g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$.

i. $(A \lor B) \land (\neg C \lor \neg D \lor E) = (A \lor B) \land (\neg D \lor E)$.

h. $(A \lor B) \land (\neg C \lor \neg D \lor E) = (A \lor B)$.

j. (A \vee B) $\wedge \neg$ (A \Rightarrow B) is satisfiable. **k**. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable.























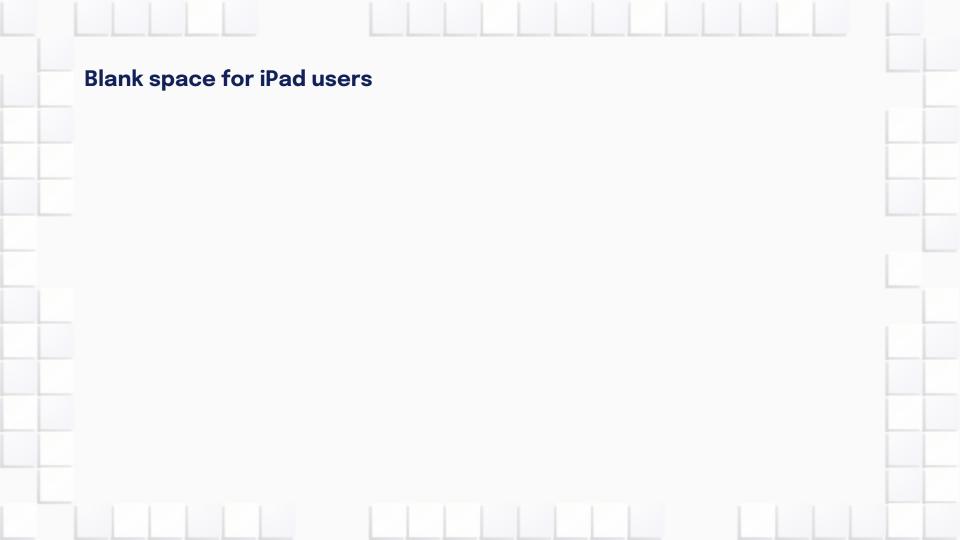












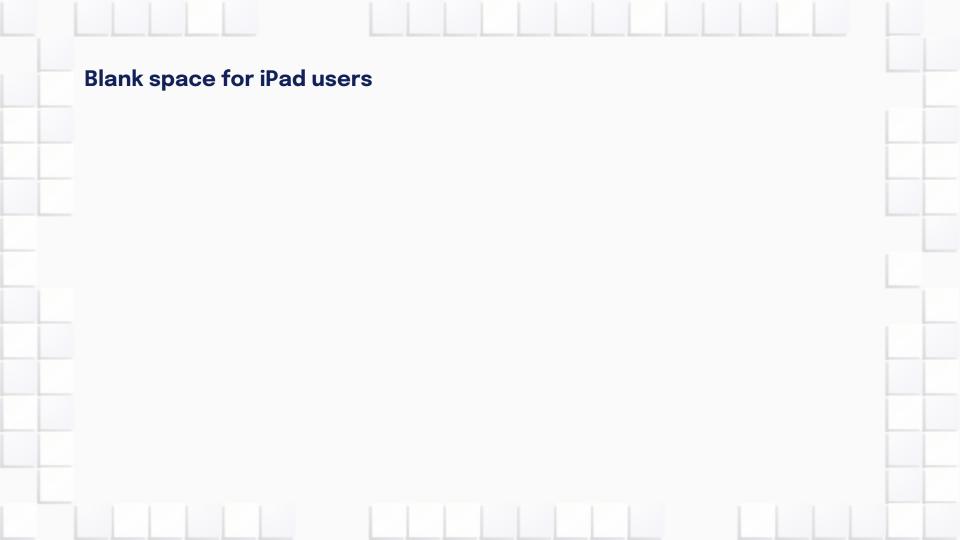
2. Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences? a. B v C **b.** ¬A ∨ ¬B ∨ ¬C ∨ ¬D **c.** $(A \Rightarrow B)$

3. Determine whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules. a. Smoke ⇒ Smoke **b.** Smoke \Rightarrow Fire **c.** (Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke $\Rightarrow \neg$ Fire) d. Smoke V Fire V ¬Fire **e.** ((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire)) **f.** (Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)

3. Determine whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules.

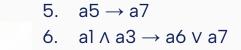
$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\ (\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\ ((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\ ((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\ \neg(\neg \alpha) \equiv \alpha \quad \text{double-negation elimination} \\ (\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha) \quad \text{contraposition} \\ (\alpha \Rightarrow \beta) \equiv (\neg \alpha \vee \beta) \quad \text{implication elimination} \\ (\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\ \neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta) \quad \text{De Morgan} \\ \neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta) \quad \text{De Morgan} \\ (\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\ (\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

Figure 7.11 Standard logical equivalences. The symbols α , β , and γ stand for arbitrary sentences of propositional logic.



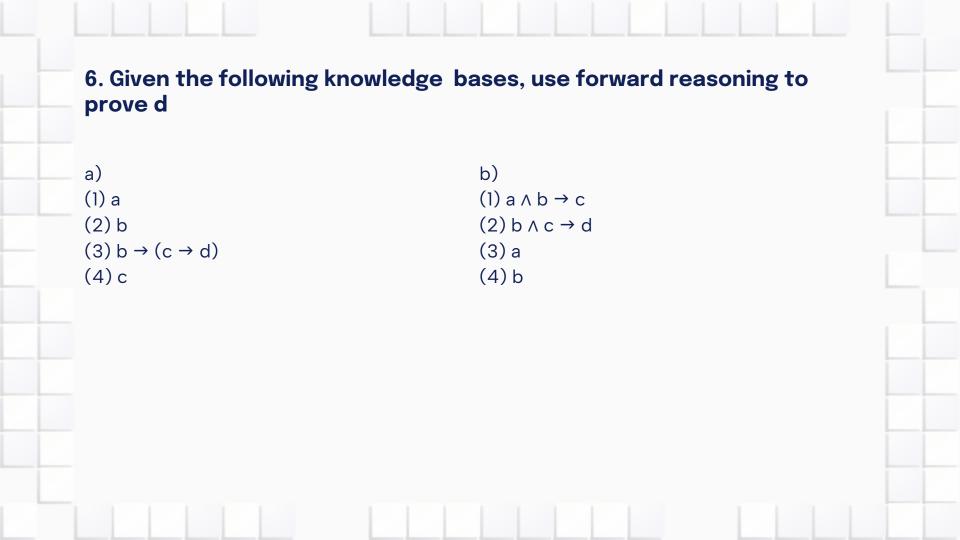
4. Given the following expressions. (a) Convert them to Conjunctive Normal Form (CNF) (b) Prove $\neg A \land \neg B$ by resolution. S1: $A \Leftrightarrow (B \lor E)$ **S2:** E ⇒ D **S3:** $C \land F \Rightarrow \neg B$ **S4:** E ⇒ B **S5:** B ⇒ F **S6:** B ⇒ C

5. Given the following expressions. (a) Convert them to Conjunctive Normal Form (CNF) (b) Prove a7 by resolution. 1. al ∨ a2 → a3 ∨ a4 2. al → a5 3. a2 ∧ a3 → a5



4. $a2 \land a4 \rightarrow a6 \land a7$

Suppose that all and a2 are true.



7. Given the following knowledge bases, use backward reasoning to prove (a) t (b) g a) b) (1) p(1) $(a \lor b) \land c \rightarrow (c \land d)$ (2) $p \rightarrow q$ (2) $a \wedge m \wedge d \rightarrow f$ (3) $q \wedge r \wedge s \rightarrow t$ (3) $m \rightarrow b \wedge c$ $(4) a \rightarrow c$ $(4) p \rightarrow u$ (5) $v \rightarrow w$ $(5) (a \wedge f) \rightarrow (\neg e \vee g)$ (6) $u \rightarrow v$ (6) $(m \land f) \rightarrow g$

(7) a

(8) m

(7) $v \rightarrow t$

(8) r

(9) s