# ĐỀ THI GIỮA KÌ GT3 HỌC KÌ 20193 – NHÓM NGÀNH 2

Câu 1:

$$a)\ \sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}.\,Có\ u_n > 0 \forall n \geq 2 \rightarrow chuỗi\ dương$$

$$Xét\ f(x) = \frac{1}{x\sqrt[3]{\ln x}}$$

$$+)\ f'(x) = -\frac{\frac{1}{3}.\ln^{-\frac{2}{3}} x}{\left(x.\sqrt[3]{\ln x}\right)^2} < 0 \rightarrow f(x)\ đơn\ điệu\ giảm$$

$$+)\ \lim_{x\to\infty} \frac{1}{x\sqrt[3]{\ln x}} = 0$$

$$+)\ \int_2^{\infty} \frac{1}{x\sqrt[3]{\ln x}}dx = \int_{\ln 2}^{\infty} t^{-\frac{1}{3}}dt = \frac{3}{2}t^{\frac{2}{3}}\Big|_{\ln 2}^{\infty} = \infty \rightarrow Tp\ phân\ kì$$

$$\rightarrow Chuỗi\ đã\ cho\ phân\ kì\ theo\ TC\ tích\ phân$$

$$b)\ \sum_{n=1}^{\infty} \sqrt[3]{n}\left(e^{\frac{1}{n^2}} - 1\right).\,u_n > 0 \forall n \geq 1$$

$$u_n = \sqrt[3]{n}\left(e^{\frac{1}{n^2}} - 1\right) \sim \sqrt[3]{n}.\frac{1}{n^2} = \frac{1}{n^{\frac{5}{3}}}\ \ (\,do\ e^t - 1 \sim t\ khi\ t \rightarrow 0\,)$$

$$Mà\ \sum_{n=1}^{\infty} \frac{1}{n^{\frac{5}{3}}}\ hội\ tụ\ \left(\frac{5}{3} > 1\right)$$

$$\rightarrow Chuỗi\ đã\ cho\ hội\ tụ\ theo\ tiêu\ chuẩn\ so\ sánh$$

Câu 2:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}\left(\frac{2x^3}{x^6 + 4}\right)^n$$

$$u_n(x) = \frac{1}{\sqrt{n}}\left(\frac{2x^3}{x^6 + 4}\right)^n = \frac{1}{\sqrt{n}}\left(\frac{\frac{x^3}{2}}{\frac{x^6}{4} + 1}\right)^n \leq \frac{1}{\sqrt{n}}.\left(\frac{1}{2}\right)^n = \frac{1}{2^n.\sqrt{n}}$$

$$Xét\ \sum_{n=1}^{\infty} \frac{1}{2^n.\sqrt{n}}\ là\ chuỗi\ dương\ do\ \frac{1}{2^n.\sqrt{n}} > 0 \forall n \geq 1$$

$$\lim_{n\to\infty} \frac{2^n.\sqrt{n}}{2.2^n.\sqrt{n+1}} = \frac{1}{2} < 1 \rightarrow Chuỗi\ hội\ tụ\ theo\ TC\ D'Alembert$$

$$\rightarrow Chuỗi\ đã\ cho\ hội\ tụ\ đều\ trên\ R\ theo\ TC\ Weierstrass$$

Câu 3:

$$\sum_{n=0}^{\infty}\left(\frac{n-1}{2n+1}\right)^n(2x+1)^n$$

$Xét$:

$$\lim_{n\to\infty}\left|\frac{1}{\sqrt[n]{u_n(x)}}\right| = \lim_{n\to\infty}\left|\frac{2n+1}{n-1}\right|.\left|\frac{1}{2x+1}\right| = \frac{2}{|2x+1|} < 1$$

$$\to -2 < 2x+1 < 2 \to \frac{-3}{2} < x < -\frac{1}{2}$$

$Vậy\ miền\ hội\ tụ\ cần\ tìm\ l$à $\left(-\frac{3}{2};-\frac{1}{2}\right)$

Câu 4:

$$f(x) = xln(2-x) = x.\ln 2 + x.\ln\left(1-\frac{x}{2}\right)$$

$$\to f(x) = x.\ln 2 + x.\sum_{n=0}^{\infty} -\frac{\left(\frac{x}{2}\right)^{n+1}}{n+1}\ \forall\left|\frac{x}{2}\right| < 1$$

$$= x.\ln 2 + \sum_{n=0}^{\infty} -\frac{1}{2}.\frac{1}{2^n(n+1)}.x^{n+2}\ \forall|x| < 2$$

Câu 5:

$a)\ (1-x) + xy'y = 0$

$$\to (1-x) = -x\frac{dy}{dx}y \to \frac{x-1}{x}dx = ydy \to x - \ln|x| + C = \frac{y^2}{2}$$

$$\to y = \pm\sqrt{2x - 2\ln|x| + C}$$

$b)(x-2y)dx + xdy = 0$

$+)\ x = 0\ l$à $nghiệm\ k$ì $dị$

$+)\ x \neq 0.\,Có$

$$x - 2y + x.y' = 0 \to y' - \frac{2}{x}y = 1\ (PTVP\ tuyến\ tính\ )$$

$$\to y = e^{\int\frac{2}{x}dx}\left(\int 1.e^{\int -\frac{2}{x}dx}dx + C\ \right)$$

$$= x^2\left(-\frac{1}{x} + C\right) = -x + Cx^2$$

$c)(3x^2y + 2\cos y)dx + (x^3 - 2xsin\ y)dy = 0$

$P(x;y) = 3x^2y + 2\cos y \to P'_y = 3x^2 - 2\sin y$

$Q(x;y) = x^3 - 2xsin\ y \to Q'_x = 3x^2 - 2\sin y$

$\to PTVP\ to$àn $phần$

$$C=\int_0^x P(x;0)dx+\int_0^y Q(x;y)dy=\int_0^x 2dx+\int_0^y x^3-2x\sin y\, dy$$
$$=2x+x^3y+2x\cos y$$

Câu 6:

$$f(x)=\begin{cases}0 \text{ nếu } -2<x<0\\ 1 \text{ nếu } 0<x<2\end{cases} \text{tuần hoàn chu kỳ } 4$$

$$a_0=\frac{1}{2}\int_{-2}^{2}f(x)dx=\frac{1}{2}\left(\int_{-2}^{0}0dx+\int_0^2 1dx\right)=\frac{1}{2}.2=1$$

$$a_n=\frac{1}{2}\int_{-2}^{2}f(x)\cos\frac{n\pi x}{2}dx=\frac{1}{2}\left(\int_{-2}^{0}0.\cos\frac{n\pi x}{2}dx+\int_0^2\cos\frac{n\pi x}{2}dx\right)$$
$$=\frac{1}{2}.\sin\frac{n\pi x}{2}.\frac{2}{n\pi}\Big|\begin{matrix}x=2\\x=0\end{matrix}=\frac{1}{n\pi}\sin n\pi=0\ \text{ với } n=1,2,3,\dots$$

$$b_n=\frac{1}{2}\int_{-2}^{2}f(x)\sin\frac{n\pi x}{2}dx=\frac{1}{2}\left(\int_{-2}^{0}0.\sin\frac{n\pi x}{2}dx+\int_0^2\sin\frac{n\pi x}{2}dx\right)$$
$$=-\frac{1}{2}.\cos\frac{n\pi x}{2}.\frac{2}{n\pi}\Big|\begin{matrix}x=2\\x=0\end{matrix}$$
$$=-\frac{1}{n\pi}\cos n\pi+\frac{1}{n\pi}=\frac{(-1)^{n+1}+1}{n\pi}\ \text{ với } n=1,2,3,\dots$$

→ *Khai triển Fourier của hàm là*:

$$f(x)=\begin{cases}\dfrac{1}{2}+\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n\pi}.\sin\frac{n\pi x}{2}\ \text{ với } x\neq 0\\ \dfrac{f(0+0)+f(0-0)}{2}=\dfrac{1}{2}\ \text{ với } x=0\ \ (\text{Định lí Dirichlet})\end{cases}$$