CHAPTER 5: HYPOTHESIS TESTING

ONE SAMPLE: TEST ON A SINGLE MEAN

σ^2**: KNOWN** Population $X \sim N(\mu, \sigma^2)$	σ^2**: UNKNOWN** Population $X \sim N(\mu, \sigma^2)$
• Null and alternative hypotheses: $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{cases}$	
Standardized test statistic: $Z = \dfrac{\overline{X} - \mu_0}{\sigma}\sqrt{n} \sim N(0,1)$ when H_0 is true. Rejection region for H_0: $W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$	Test statistic: $T = \dfrac{\overline{X} - \mu_0}{S}\sqrt{n} \sim t(n-1)$ when H_0 is true. $W_\alpha = (-\infty, -t_{\frac{\alpha}{2},n-1}) \cup (t_{\frac{\alpha}{2},n-1}, +\infty)$
• Null and alternative hypotheses: $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu < \mu_0 \end{cases}$	
Rejection region for H_0: $W_\alpha = (-\infty, -z_\alpha)$	$W_\alpha = (-\infty, -t_{\alpha,n-1})$
• Null and alternative hypotheses: $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu > \mu_0 \end{cases}$	
Rejection region for H_0: $W_\alpha = (z_\alpha, +\infty)$	$W_\alpha = (t_{\alpha,n-1}, +\infty)$
σ^2**: KNOWN**; **Sample size** $n \geq 30$ **Any** population X with $\mathbb{E}[X] = \mu$, Var$(X) = \sigma^2$	σ^2**: UNKNOWN**; **Sample size** $n \geq 30$ **Any** population X, $\mathbb{E}[X] = \mu$, Var$(X) = \sigma^2$
• $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{cases}$ Test statistic: $Z = \dfrac{\overline{X} - \mu_0}{\sigma}\sqrt{n} \approx N(0,1)$ when H_0 is true. $W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$	• $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{cases}$ Test statistic: $T = \dfrac{\overline{X} - \mu_0}{S}\sqrt{n} \approx N(0,1)$ when H_0 is true. $W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$
• $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu < \mu_0 \end{cases}$ $W_\alpha = (-\infty, -z_\alpha)$	• $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu < \mu_0 \end{cases}$ $W_\alpha = (-\infty, -z_\alpha)$
• $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu > \mu_0 \end{cases}$ $W_\alpha = (z_\alpha, +\infty)$	• $\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu > \mu_0 \end{cases}$ $W_\alpha = (z_\alpha, +\infty)$

ONE SAMPLE: TEST ON A SINGLE PROPORTION

Let p be the population proportion. Let $\widehat{p}$ be the sample proportion.

• $\begin{cases} H_0 : p = p_0 \\ H_1 : p \neq p_0 \end{cases}$ (Check: $np_0 \geq 5$ and $n(1-p_0) \geq 5$)

Standardized test statistic: $Z = \dfrac{\widehat{p} - p_0}{\sqrt{p_0(1-p_0)}}\sqrt{n} \approx N(0,1)$ when H_0 is true.

$$W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$$

• $\begin{cases} H_0 : p = p_0 \\ H_1 : p < p_0 \end{cases}$ $W_\alpha = (-\infty, -z_\alpha)$ • $\begin{cases} H_0 : p = p_0 \\ H_1 : p > p_0 \end{cases}$ $W_\alpha = (z_\alpha, +\infty)$

TWO SAMPLES: TEST ON TWO MEANS

TWO POPULATIONS X, Y WITH MEANS μ_1, μ_2 and VARIANCES σ_1^2, σ_2^2

TWO VARIANCES σ_1^2, σ_2^2: **KNOWN**

$X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$	**Any** X, Y **and** $n_1 \geq 30, n_2 \geq 30$

- $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{cases}$

Test statistic: $Z = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ when H_0 is true.

(Any X, Y and $n_1 \geq 30, n_2 \geq 30$): $Z = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \approx N(0,1)$ when H_0 is true.

$$W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$$

- $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 < \mu_2 \end{cases}$ $\quad W_\alpha = (-\infty, -z_\alpha)$
- $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 > \mu_2 \end{cases}$ $\quad W_\alpha = (z_\alpha, +\infty)$

TWO VARIANCES σ_1^2, σ_2^2: **UNKNOWN**

$X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$; **EQUAL VARIANCES** $\sigma_1^2 = \sigma_2^2 = \sigma^2$	**Any** X, Y **and** $n_1 \geq 30, n_2 \geq 30$

- $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{cases}$

Test statistic: $T = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t(n_1 + n_2 - 2)$ when H_0 is true.

$W_\alpha = (-\infty, -t_{\frac{\alpha}{2}, n_1+n_2-2}) \cup (t_{\frac{\alpha}{2}, n_1+n_2-2}, +\infty)$

(Any X, Y and $n_1 \geq 30, n_2 \geq 30$): $Z = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \approx N(0,1)$ when H_0 is true.

$W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$

- $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 < \mu_2 \end{cases}$

$W_\alpha = (-\infty, -t_{\alpha, n_1+n_2-2})$ | $W_\alpha = (-\infty, -z_\alpha)$

- $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 > \mu_2 \end{cases}$

$W_\alpha = (t_{\alpha, n_1+n_2-2}, +\infty)$ | $W_\alpha = (z_\alpha, +\infty)$

TWO SAMPLES: TEST ON TWO PROPORTIONS

Let p_1, p_2 be two population proportions. Let $\widehat{p}_1, \widehat{p}_2$ be two sample proportions.

- $\begin{cases} H_0 : p_1 = p_2 \\ H_1 : p_1 \neq p_2 \end{cases}$ (Check: $n_1\widehat{p}_1 \geq 5$, $n_1(1-\widehat{p}_1) \geq 5$, and $n_2\widehat{p}_2 \geq 5$, $n_2(1-\widehat{p}_2) \geq 5$)

Standardized test statistic: $Z = \dfrac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \approx N(0,1)$ when H_0 is true.

Here, $\overline{p} = \dfrac{n_1\widehat{p}_1 + n_2\widehat{p}_2}{n_1 + n_2}$. Rejection region: $W_\alpha = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$

- $\begin{cases} H_0 : p_1 = p_2 \\ H_1 : p_1 < p_2 \end{cases}$ $\quad W_\alpha = (-\infty, -z_\alpha)$
- $\begin{cases} H_0 : p_1 = p_2 \\ H_1 : p_1 > p_2 \end{cases}$ $\quad W_\alpha = (z_\alpha, +\infty)$

Problem 5.1. A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.42 ounces. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

Problem 5.2. A business student claims that, on average, an MBA student is required to prepare more than five cases per week. To examine the claim, a statistics professor asks a random sample of 10 MBA students to report the number of cases they prepare weekly. The results are exhibited here

$$2, 7, 4, 8, 9, 5, 11, 3, 7, 4$$

Can the professor conclude at the 5% significance level that the claim is true, assuming that the number of cases is normally distributed with a standard deviation of 1.5?

Problem 5.3. A courier service advertises that its average delivery time is less than 6 hours for local deliveries. A random sample of times for 12 deliveries to an address across town was recorded. These data are shown here.

$$3.03, 6.33, 7.98, 4.82, 6.50, 5.22, 3.56, 6.76, 7.96, 4.54, 5.09, 6.46$$

Is this sufficient evidence to support the couriers advertisement, at the 5% level of significance? Assume that the delivery time is normally distributed.

Problem 5.4. An electrical firm manufactures light bulbs that have a length of life that is normally distributed. A sample of 20 bulbs were selected randomly and found to have an average of 655 hours and a standard deviation of 27 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm. Test $H_0 : \mu = 660$ against $H_1 : \mu \neq 660$? Use a 0.02 level of significance.

Problem 5.5. Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal.

Problem 5.6. To determine whether car ownership affects a students academic achievement, two random samples of 100 male students were each drawn from the students' body. The grade point average for the $n_1 = 100$ non-owners of cars had an average and variance equal to $\overline{x}_1 = 2.70$ and $s_1^2 = 0.36$, while $\overline{x}_2 = 2.54$ and $s_2^2 = 0.40$ for the $n_2 = 100$ car owners. Do the data present sufficient evidence to indicate a difference in the mean achievements between car owners and non-owners of cars? Test using $\alpha = 0.05$.

Problem 5.7. To determine whether car ownership affects a students academic achievement, two random samples of 100 male students were each drawn from the students' body. The grade point average for the $n_1 = 100$ non-owners of cars had an average equal to $\overline{x}_1 = 2.70$, while $\overline{x}_2 = 2.54$ for the $n_2 = 100$ car owners. Do the data present sufficient evidence to indicate a difference in the mean achievements between car owners and non-owners of cars? Test using $\alpha = 0.05$. Assume that the two populations have variances $\sigma^2_{non-owners} = 0.36$ and

$\sigma^2_{owners} = 0.42$.

Problem 5.8. Random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\overline{x}_1 = 81$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3.4$, has a mean $\overline{x}_2 = 76$. Test the hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative $H_1 : \mu_1 \neq \mu_2$. Use a significance level of 0.05.

Problem 5.9. A study was conducted to see if increasing the substrate concentration has an appreciable effect on the velocity of a chemical reaction. With a substrate concentration of 1.5 moles per liter, the reaction was run 15 times, with an average velocity of 7.5 micromoles per 30 minutes and a standard deviation of 1.5. With a substrate concentration of 2.0 moles per liter, 12 runs were made, yielding an average velocity of 8.8 micromoles per 30 minutes and a sample standard deviation of 1.2. Is there any reason to believe that this increase in substrate concentration causes an increase in the mean velocity of the reaction of more than 0.5 micromole per 30 minutes? Use a 0.01 level of significance and assume the populations to be approximately normally distributed with equal variances.

Problem 5.10. To find out whether a new serum will arrest leukemia, 9 mice, all with an advanced stage of the disease, are selected. Five mice receive the treatment and 4 do not. Survival times, in years, from the time the experiment commenced are as follows:

Treatment	2.1	5.3	1.4	4.6	0.9
No Treatment	1.9	0.5	2.8	3.1	

At the 0.05 level of significance, can the serum be said to be effective? Assume the two populations to be normally distributed with equal variances.

Problem 5.11. A men's softball league is experimenting with a yellow baseball that is easier to see during night games. One way to judge the effectiveness is to count the number of errors. In a preliminary experiment, the yellow baseball was used in 10 games and the traditional white baseball was used in another 10 games. The number of errors in each game was recorded and is listed here

Yellow	5	2	6	7	2	5	3	8	4	9
White	7	6	8	5	9	11	8	3	6	10

Can we infer that there are fewer errors on average when the yellow ball is used at the 0.05 level of significance? Assume the two populations to be normally distributed with equal variances.

Problem 5.12. A marketing expert for a pasta-making company believes that 40% of pasta lovers prefer lasagna. If 9 out of 20 pasta lovers choose lasagna over other pastas, what can be concluded about the experts claim? Use a 0.05 level of significance.

Problem 5.13. A fuel oil company claims that one-fifth of the homes in a certain city are heated by oil. Do we have reason to believe that fewer than one-fifth are heated by oil if, in a random sample of 1000 homes in this city, 136 are heated by oil?

Problem 5.14. A study was conducted to compare between the proportions of smokers in two universities. Two independent random samples gave the following data:

	Univ. (1)	Univ. (2)
Sample size	200	300
Number of smokers	80	111

Does this data provide sufficient statistical evidence to indicate that the percentage of students who smoke differs for these two universities? Test using a 0.01 level of significance.

Problem 5.15. A recent survey stated that male college students smoke less than female college students. In a survey of 1245 male students, 361 said they smoke at least one pack of cigarettes a day. In a survey of 1065 female students, 341 said they smoke at least one pack a day. At $\alpha = 0.01$, can you support the claim that the proportion of male college students who smoke at least one pack of cigarettes a day is lower than the proportion of female college students who smoke at least one pack a day?

Problem 5.16. In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportions of urban and suburban residents who favor the construction of the nuclear plant? Use a 0.01 level of significance.

Problem 5.17. Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the companys new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows:
Brand A: $\overline{x}_A = 37900$ kilometers, $s_A = 5100$ kilometers.
Brand B: $\overline{x}_B = 39800$ kilometers, $s_B = 5900$ kilometers.
Test the hypothesis that there is no difference in the average wear of the two brands of tires. Assume the populations to be approximately normally distributed with equal variances. Use a 0.01 level of significance.