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ĐẠI HỌC BÁCH KHOA HÀ NỘI
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Electronics for Information Technology

(Điện tử cho Công nghệ Thông tin)

IT3420E

Đỗ Công Thuần

Department of Computer Engineering

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General Information

- Course: **Electronics for Information Technology**
- ID Number: IT3420
- Credits: 2 (2-1-0-4)
- Lecture/Exercise: 32/16 hours (48 hours, 16 weeks)
- Evaluation:
 - Midterm examination and weekly assignment: **50%**
 - Final examination: **50%**
- Learning Materials:
 - Lecture slides
 - Textbooks
 - *Introductory Circuit Analysis* (2015), 10th – 13th ed., Robert L. Boylestad
 - *Electronic Device and Circuit Theory* (2013), 11th ed., Robert L. Boylestad, Louis Nashelsky
 - *Microelectronics Circuit Analysis and Design* (2006), 4th ed., Donald A. Neamen
 - *Digital Electronics: Principles, Devices and Applications* (2007), Anil K. Maini

Contact Your Instructor

- You can reach me through office in **Room 802, B1 Building**, HUST.
 - You should make an appointment by email before coming.
 - If you have urgent things, just come and meet me!
- You can also reach me at the following **email** any time. This is the best way to reach me!
 - thuandc@soict.hust.edu.vn

Course Contents

- The Concepts of Electronics for IT
- **Chapter 1: Passive Electronic Components and Applications**
- **Chapter 2: Semiconductor Components and Applications**
- **Chapter 3: Operational Amplifiers**
- **Chapter 4: Fundamentals of Digital Circuits**
- **Chapter 5: Logic Gates**
- **Chapter 6: Combinational Logic**
- **Chapter 7: Sequential Logic**

Chapter 5:

Logic Gates

1. Introduction to Boolean Algebra
2. Variables, Literals & Terms in Boolean Expressions
3. Laws and Rules of Boolean Algebra
4. Logic Simplification
5. Basic Logic Gates

References:

***Digital electronics: Principles, Devices, and Applications, Anil Kumar Maini 2007
John Wiley & Sons***

***Fundamentals of Logic Design, Seventh Edition, Charles H. Roth, Jr. and Larry L.
Kinney***

***Digital Fundamentals, Thomas L. Floyd, Eleventh Edition, Pearson Education
Limited 2015***

Contents

1. Introduction to Boolean Algebra
2. Variables, Literals & Terms in Boolean Expressions
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1. Introduction to Boolean Algebra
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Introduction to Boolean Algebra

- Boolean algebra was introduced by George Boole in the 1850s.
- A mathematical system for formulating logic statements with symbols so that problems can be written and solved in a manner similar to ordinary algebra.
- Boolean algebra is applied in the **design and analysis of digital systems**.

Introduction to Boolean Algebra

- In a digital circuit (logic circuit), the signal has only one of two discrete levels at a time.
 - Each level is interpreted as one of two different states (on/off, true/false, 1/0, ...).
 - Ex: $0 \rightarrow 0.8V$: logic 0
 $2.5 \rightarrow 5V$: logic 1

 **Boolean algebra can be efficiently used to analyze and design digital systems.**

Introduction to Boolean Algebra

- **Binary variable** is used to represent the **input or output** of a **switching circuit**, and can take on only 2 different values, “0” and “1”.
- **Boolean function/expression** consists of binary variables, the constants “0” and “1”, and the logic operation symbols. A Boolean function also have 2 different values, “0” and “1”.
- **3 basic logic functions:**
 - “AND”
 - “OR”
 - “NOT”

Introduction to Boolean Algebra

Logic 0	Logic 1
False	True
Off	On
Low	High
No	Yes
Open switch	Closed switch

Contents

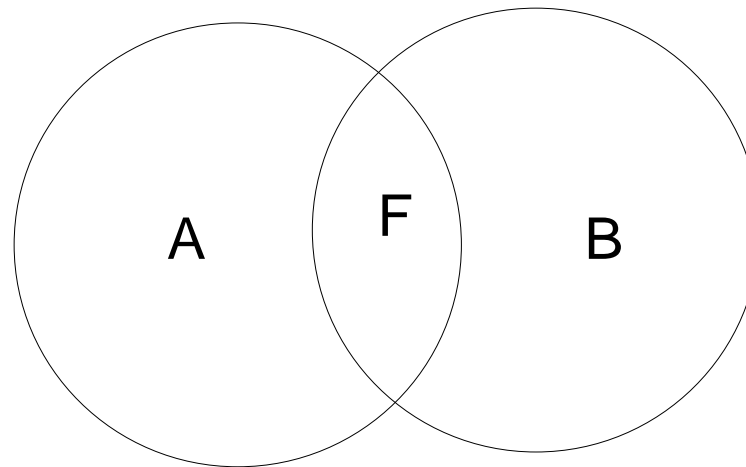
1. Introduction to Boolean Algebra
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Variables, Literals & Terms in Boolean Expressions

- Venn Diagram
- Logic Function
- Truth Table
- Karnaugh Map
- Timing Diagram

Venn Diagram

- A Venn diagram uses simple closed curves (circles or ellipses) drawn on a plane to represent sets.
- It shows all **possible logical relations** between a finite collection of different sets.
- Each logic variable is composed of 2 different sets, corresponding to *Logic 0* and *Logic 1*.
- Example: $F = A \text{ AND } B$



Logic Function

- Basic functions:
 - **AND**, \cdot
 - **OR**, $+$
 - **NOT**, $\bar{}$
- Examples:
 - $F = A \text{ AND } B$ or $F = A.B$
 - $F = A \text{ OR } B$ or $F = A+B$
 - $F = \text{NOT}(A)$ or $F = \bar{A}$

Truth Table

- A true table lists **all possible combinations** of input binary variables and the corresponding outputs of a logic system.
- If a logic circuit has n binary inputs, its truth table will have:
 - 2^n possible input combinations, or 2^n rows
 - $(n+1)$ columns

$$F = A+B$$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Karnaugh Map

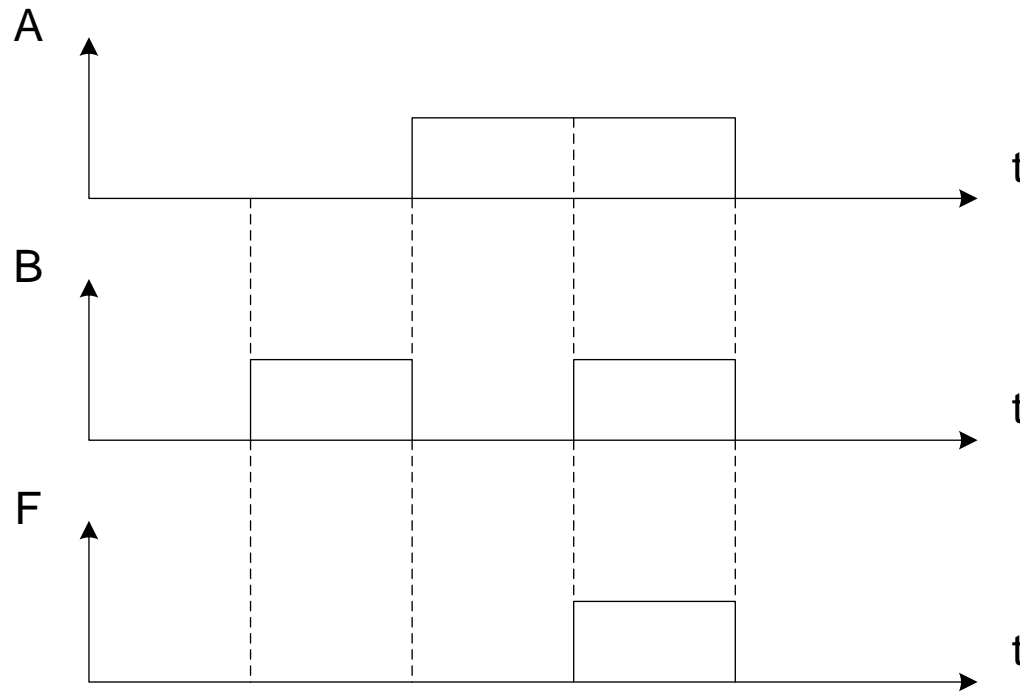
- A Karnaugh map is a graphical representation of the logic system.
- It is an array of cells in which each **cell** represents a **binary value of the input variables**.
- The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells.

$$\mathbf{F = A.B}$$

a\b	0	1
0	0	0
1	0	1

Timing Diagram

- A timing diagram is a graphical representation of a set of signals in the **time domain**.
- Example, for **$F = A.B$** :



Contents

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Laws and Rules of Boolean Algebra

- Equivalent, Complement, Dual of Boolean Expression
- 10 postulates
- 17 basic theorems (**Homework**)

Equivalent of Boolean Expression

- Two given Boolean expressions are said to be equivalent if **one of them equals '1' only when the other equals '1'** and also **one equals '0' only when the other equals '0'**.

Complement of Boolean Expression

- Two given Boolean expressions are said to be the complement of each other if one expression equals '1' only when the other equals '0', and vice versa.
- **Method:** complementing each literal, changing
- all '.' to '+' and all '+' to '.', all 0s to 1s and all 1s to 0s.

- Given expression:

$$\overline{A}.B + A.\overline{B}$$

- Corres. complement:

$$(A + \overline{B}).(\overline{A} + B)$$

- Given expression:

$$\overline{A}.\overline{B} + A.B$$

- Corres. complement:

$$(A + B).(\overline{A} + \overline{B})$$

Example 1

- Find the complement expression of:

$$[(A.\overline{B} + \overline{C}).D + \overline{E}].F$$

- ✓Corresponding complement:

$$[(\overline{A} + B).C + \overline{D}].E + \overline{F}$$

Dual of Boolean Expression

- The dual of a Boolean expression is obtained by replacing all ‘.’ operations with ‘+’ operations, all ‘+’ operations with ‘.’ operations, all 0s with 1s and all 1s with 0s and leaving all literals unchanged.

- Given expression:

$$\overline{A}.B + A.\overline{B}$$

- Corres. dual:

$$(\overline{A} + B).(A + \overline{B})$$

- Given expression:

$$(A + B).(\overline{A} + \overline{B})$$

- Corres. dual:

$$A.B + \overline{A}.\overline{B}$$

Example 2

- Find the dual of:

$$A.\overline{B} + B.\overline{C} + C.\overline{D}$$

- ✓ Corresponding dual:

$$(A + \overline{B}).(B + \overline{C}).(C + \overline{D})$$

Example 3

- Simplify:

$$(A.B + C.D).[(\bar{A} + \bar{B}).(\bar{C} + \bar{D})]$$

- ✓ Solution:

$$(A.B + C.D).[(\bar{A} + \bar{B}).(\bar{C} + \bar{D})] = 0$$

Postulates

- $1 \times 1 = 1$

- $1 \times 0 = 0$

- $0 \times 1 = 0$

- $0 \times 0 = 0$

- $\overline{0} = 1$

- $\overline{1} = 0$

- $0 + 0 = 0$

- $0 + 1 = 1$

- $1 + 0 = 1$

- $1 + 1 = 1$

Theorems of Boolean Algebra

- Theorem 1: (Operations with '0' and '1')

$$(a) 0.X = 0$$

$$(b) 1 + X = 1$$

- Proof:

$$X = 0$$

$$\text{LHS} = 0.X = 0.0 = 0 = \text{RHS}$$

$$X = 1$$

$$\text{LHS} = 0.1 = 0 = \text{RHS}$$

Theorems of Boolean Algebra

- Theorem 2 (Operations with '0' and '1')

(a) $1.X = X$

(b) $0 + X = X$

- Proof:

$$X = 0 \quad \text{LHS} = 1.0 = 0 = \text{RHS}$$

$$X = 1 \quad \text{LHS} = 1.1 = 1 = \text{RHS}$$

Theorems of Boolean Algebra

- Theorem 3 (Idempotent or Identity Laws)

$$(a) X.X.X \dots X = X$$

$$(b) X + X + X + \dots + X = X$$

- Example:

$$\begin{aligned} & (A.\bar{B}.\bar{B} + C.C).(A.\bar{B}.\bar{B} + A.\bar{B} + C.C) \\ &= (A.\bar{B} + C).(A.\bar{B} + A.\bar{B} + C) \\ &= (A.\bar{B} + C).(A.\bar{B} + C) \\ &= A.\bar{B} + C \end{aligned}$$

Theorems of Boolean Algebra

- Theorem 4 (Complementation Law)

(a) $X.\overline{X} = 0$

(b) $X + \overline{X} = 1$

- Proof:

$$X = 0, \overline{X} = 1$$

$$X.\overline{X} = 0.1 = 0$$

$$X = 1, \overline{X} = 0$$

$$X.\overline{X} = 1.0 = 0$$

- Further illustration:

$$(A + B.C)(\overline{A + B.C}) = 0$$

$$(A + B.C) + (\overline{A + B.C}) = 1$$

Example 4

- Simplify the following:

$$[1 + L.M + L.\overline{M} + \overline{L}.M].[(L + \overline{M}).(\overline{L}.M) + \overline{L}.\overline{M}.(L + M)]$$

✓ Solution:

$$1.(0 + 0) = 1.0 = 0.$$

Theorems of Boolean Algebra

- Theorem 5 (Commutative Laws)

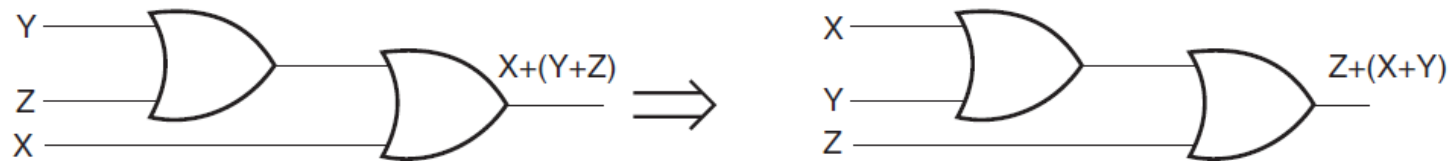
$$(a) \quad X + Y = Y + X$$

$$(b) \quad X.Y = Y.X$$

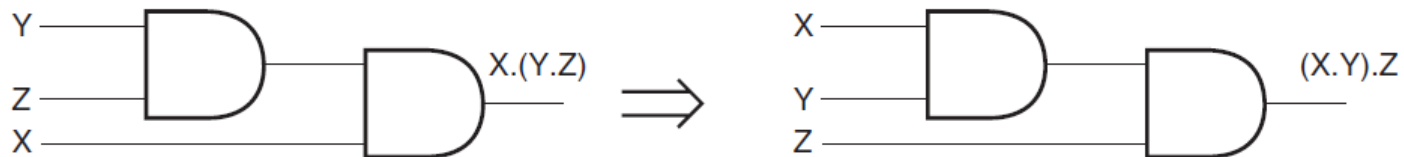
Theorems of Boolean Algebra

- Theorem 6 (Associative Laws)

$$(a) \quad X + (Y + Z) = Y + (Z + X) = Z + (X + Y)$$



$$(b) \quad X.(Y.Z) = Y.(Z.X) = Z.(X.Y)$$



Theorems of Boolean Algebra

- Theorem 7 (Distributive Laws)

(a) $X.(Y + Z) = X.Y + X.Z$

(b) $X + Y.Z = (X + Y).(X + Z)$

- Proof:

X	Y	Z	Y+Z	XY	XZ	X(Y+Z)	XY+XZ
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Example 5

- Simplify the following:

$$\begin{aligned}\overline{A}.\overline{B} + \overline{A}.B + A.\overline{B} + A.B &= \\ &= \overline{A}.(\overline{B} + B) + A.(\overline{B} + B) \\ &= \overline{A}.1 + A.1 \\ &= \overline{A} + A \\ &= 1\end{aligned}$$

Example 6

- Simplify the following:

$$\begin{aligned}(\bar{A} + \bar{B}).(\bar{A} + B).(A + \bar{B}).(A + B) &= \\&= (\bar{A} + \bar{B}.B).(A + \bar{B}.B) \\&= (\bar{A} + 0).(A + 0) \\&= \bar{A}.A \\&= 0\end{aligned}$$

Theorems of Boolean Algebra

- Theorem 8:

$$(a) \quad X.Y + X.\bar{Y} = X$$

$$(b) \quad (X + Y).(X + \bar{Y}) = X$$

- Proof:

$$X.Y + X.\bar{Y} = X.(Y + \bar{Y}) = X.1 = X$$

$$(X + Y).(X + \bar{Y}) = X + Y.\bar{Y} = X + 0 = X$$

Example 7

- Simplify the following:

$$\begin{aligned} &A.\overline{B}.\overline{C}.\overline{D} + A.\overline{B}.\overline{C}.D + A.\overline{B}.C.\overline{D} + A.\overline{B}.C.D \\ &+ A.B.\overline{C}.\overline{D} + A.B.\overline{C}.D + A.B.C.\overline{D} + A.B.C.D \end{aligned}$$

✓ Solution:

$$= A$$

Theorems of Boolean Algebra

- Theorem 9:

$$(a) (X + \overline{Y}).Y = X.Y$$

$$(b) X.\overline{Y} + Y = X + Y$$

- Theorem 9(b) is the dual of theorem 9(a) and hence stands proved.

Theorems of Boolean Algebra

- Theorem 10: (Absorption Law or Redundancy Law)

$$(a) \quad X + X.Y = X$$

$$(b) \quad X.(X + Y) = X$$

- Proof:

$$X + X.Y = X.(1 + Y) = X.1 = X$$

Example 8

- Simplify the LHS:

$$A + A.\overline{B} + A.\overline{B}.\overline{C} + A.\overline{B}.C + \overline{C}.B.A = A$$

- Simplify the LHS :

$$(\overline{A} + B + \overline{C}).(\overline{A} + B).(C + B + \overline{A}) = \overline{A} + B$$

Theorems of Boolean Algebra

- Theorem 11:

$$(a) Z.X + Z.\bar{X}.Y = Z.X + Z.Y$$

$$(b) (Z + X).(Z + \bar{X} + Y) = (Z + X).(Z + Y)$$

- Proof:

X	Y	Z	ZX	ZY	Z \bar{X}	Z $\bar{X}Y$	ZX + Z $\bar{X}Y$	ZX+ZY
0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	0	1	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	1	1
1	1	0	0	0	0	0	0	0
1	1	1	1	1	0	0	1	1

Example 9

- Simplify the following:

$$\begin{aligned} & (A + \bar{B}).(\bar{A} + \bar{B} + C).(\bar{A} + \bar{B} + D) \\ &= (A + \bar{B}).(\bar{B} + C).(\bar{A} + \bar{B} + D) \\ &= (A + \bar{B}).(\bar{B} + C).(\bar{B} + D) \end{aligned}$$

Theorems of Boolean Algebra

- Theorem 12 (Consensus Theorem)

$$(a) \quad X.Y + \overline{X}.Z + Y.Z = X.Y + \overline{X}.Z$$

$$(b) \quad (X + Y).(\overline{X} + Z).(Y + Z) = (X + Y).(\overline{X} + Z)$$

- Proof:

X	Y	Z	XY	$\overline{X}Z$	YZ	$XY + \overline{X}Z + YZ$	$XY + \overline{X}Z$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	1
0	1	0	0	0	0	0	0
0	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	1	0	1	1	1

Example 10

- Simplify the following:

$$A.B.C + \overline{A}.C.D + \overline{B}.C.D + B.C.D + A.C.D =$$

✓Solution:

$$= A.B.C + C.D$$

Example 11

- Prove that:

$$\begin{aligned} &A.B.C.D + A.B.\overline{C}.\overline{D} + A.B.C.\overline{D} + A.B.\overline{C}.D + \\ &+ A.B.C.D.E + A.B.\overline{C}.\overline{D}.\overline{E} + A.B.\overline{C}.D.E \\ &= A.B \end{aligned}$$

✓Solution:

$$\begin{aligned} &= A.B.C.D + A.B.\overline{C}.\overline{D} + A.B.C.\overline{D} + A.B.\overline{C}.D \\ &= A.B.(C.D + \overline{C}.\overline{D} + C.\overline{D} + \overline{C}.D) = A.B \end{aligned}$$

Theorems of Boolean Algebra

- Theorem 13 (DeMorgan's Theorem)

$$(a) \overline{[X_1 + X_2 + X_3 + \dots + X_n]} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3} \cdot \dots \cdot \overline{X_n}$$

$$(b) \overline{[X_1 \cdot X_2 \cdot X_3 \cdot \dots \cdot X_n]} = [\overline{X_1} + \overline{X_2} + \overline{X_3} + \dots + \overline{X_n}]$$

- Proof:

$$\text{LHS} = \overline{[X_1 + X_2 + X_3 + \dots + X_n]} = \overline{[0 + 0 + 0 + \dots + 0]} = \overline{0} = 1$$

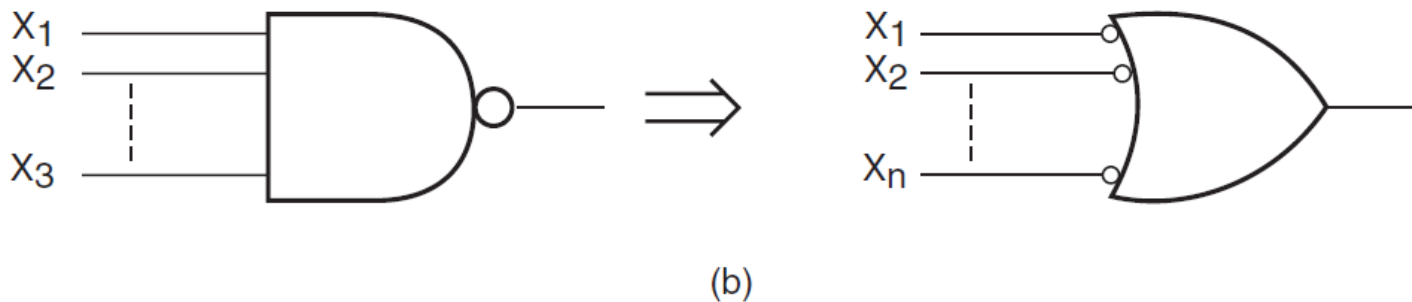
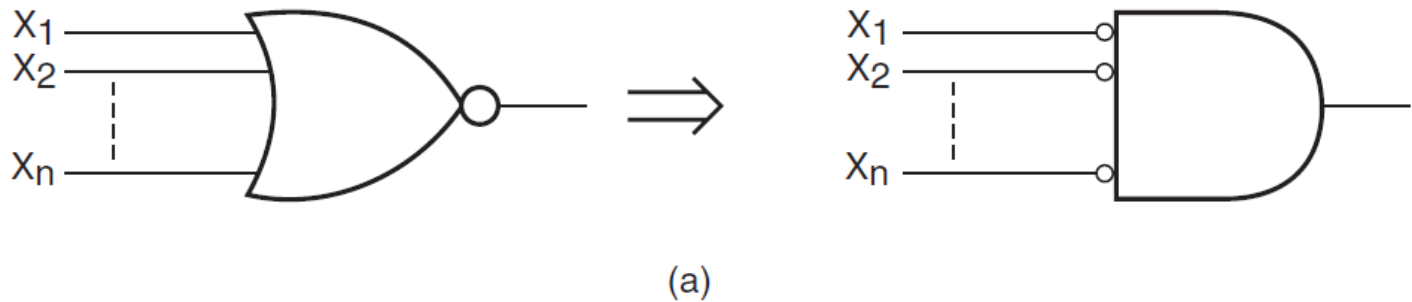
$$\text{RHS} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3} \cdot \dots \cdot \overline{X_n} = \overline{0} \cdot \overline{0} \cdot \overline{0} \cdot \dots \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot \dots \cdot 1 = 1$$

$$\text{LHS} = \overline{[X_1 + X_2 + X_3 + \dots + X_n]} = \overline{[1 + 0 + 0 + \dots + 0]} = \overline{1} = 0$$

$$\text{RHS} = \overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3} \cdot \dots \cdot \overline{X_n} = \overline{1} \cdot \overline{0} \cdot \overline{0} \cdot \dots \cdot \overline{0} = 0 \cdot 1 \cdot 1 \cdot \dots \cdot 1 = 0$$

Theorems of Boolean Algebra

- Theorem 13 (DeMorgan's Theorem)



Theorems of Boolean Algebra

- Theorem 14 (Transposition Theorem)

$$(a) \quad X.Y + \bar{X}.Z = (X + Z).(\bar{X} + Y)$$

$$(b) \quad (X + Y).(\bar{X} + Z) = X.Z + \bar{X}.Y$$

- Proof:

X	Y	Z	XY	$\bar{X}Z$	X+Z	$\bar{X} + Y$	$XY + \bar{X}Z$	$(X+Z)(\bar{X} + Y)$
0	0	0	0	0	0	1	0	0
0	0	1	0	1	1	1	1	1
0	1	0	0	0	0	1	0	0
0	1	1	0	1	1	1	1	1
1	0	0	0	0	1	0	0	0
1	0	1	0	0	1	0	0	0
1	1	0	1	0	1	1	1	1
1	1	1	1	0	1	1	1	1

Example 12

$$\overline{A}.B + A.\overline{B} = (A + B).(\overline{A} + \overline{B})$$

$$A.B + \overline{A}.\overline{B} = (A + \overline{B}).(\overline{A} + B)$$

Theorems of Boolean Algebra

- Theorem 15:

(a) $X.f(X, \overline{X}, Y, Z, \dots) = X.f(1, 0, Y, Z, \dots)$

(b) $X + f(X, \overline{X}, Y, Z, \dots) = X + f(0, 1, Y, Z, \dots)$

- Or:

(a) $\overline{X}.f(X, \overline{X}, Y, Z, \dots) = \overline{X}.f(0, 1, Y, Z, \dots)$

(b) $\overline{X} + f(X, \overline{X}, Y, Z, \dots) = \overline{X} + f(1, 0, Y, Z, \dots)$

Example 13

- Simplify the following:

$$A.[\overline{A}.B + A.\overline{C} + (\overline{A} + D).(A + \overline{E})] =$$

✓Solution:

$$= A.[0.B + 1.\overline{C} + (0 + D).(1 + \overline{E})]$$

$$= A.(\overline{C} + D)$$

Example 14

- Simplify the following:

$$\overline{A} + [\overline{A}.B + A.\overline{C} + (\overline{A} + B).(A + \overline{E})] =$$

✓Solution:

$$\begin{aligned} &= \overline{A} + [0.B + 1.\overline{C} + (0 + B).(1 + \overline{E})] \\ &= \overline{A} + \overline{C} + B \end{aligned}$$

Theorems of Boolean Algebra

- Theorem 16:

$$(a) f(X, \bar{X}, Y, \dots, Z) = X.f(1, 0, Y, \dots, Z) + \bar{X}.f(0, 1, Y, \dots, Z)$$

$$(b) f(X, \bar{X}, Y, \dots, Z) = [X + f(0, 1, Y, \dots, Z)][\bar{X} + f(1, 0, Y, \dots, Z)]$$

- Proof:

$$\begin{aligned} f(X, \bar{X}, Y, \dots, Z) &= X.f(X, \bar{X}, Y, \dots, Z) + \bar{X}.f(X, \bar{X}, Y, \dots, Z) \\ &= X.f(1, 0, Y, \dots, Z) + \bar{X}.f(0, 1, Y, \dots, Z) \end{aligned}$$

$$\begin{aligned} f(X, \bar{X}, Y, \dots, Z) &= [X + f(X, \bar{X}, Y, \dots, Z)][\bar{X} + f(X, \bar{X}, Y, \dots, Z)] \\ &= [X + f(0, 1, Y, \dots, Z)][\bar{X} + f(1, 0, Y, \dots, Z)] \end{aligned}$$

Theorems of Boolean Algebra

- Theorem 17 (Involution Law)

$$\overline{\overline{X}} = X$$

Example 15

- Prove the following:

$$L.(M + \overline{N}) + \overline{L}.\overline{P}.Q = (L + \overline{P}.Q).(\overline{L} + M + \overline{N})$$

✓ Solution:

- Assume: $L = X$, $(M + \overline{N}) = Y$ and $\overline{P}.Q = Z$

$$\begin{aligned} L.(M + \overline{N}) + \overline{L}.\overline{P}.Q &= X.Y + \overline{X}.Z \\ &= (X + Z).(\overline{X} + Y) \\ &= (L + \overline{P}.Q)(\overline{L} + M + \overline{N}) \\ &= \text{RHS} \end{aligned}$$

Example 16

- Prove the following:

$$\begin{aligned}[A.\overline{B} + \overline{C} + \overline{D}].[D + (E + \overline{F}).G] &= \\ &= D.(A.\overline{B} + \overline{C}) + \overline{D}.G.(E + \overline{F})\end{aligned}$$

✓Solution:

- Assume: $\overline{D} = X$, $A.\overline{B} + \overline{C} = Y$ and $(E + \overline{F}).G = Z$

$$\begin{aligned}[A.\overline{B} + \overline{C} + \overline{D}].[D + (E + \overline{F}).G] &= \\ &= (X + Y).(\overline{X} + Z) = X.Z + \overline{X}.Y \\ &= \overline{D}.G.(E + \overline{F}) + D.(A.\overline{B} + \overline{C}) = \text{RHS}\end{aligned}$$

Example 17

- Simplify the following:

$$A.B.C + A.B.\overline{C} + A.\overline{B}.C + A.\overline{B}.\overline{C} + \\ + \overline{A}.B.C + \overline{A}.B.\overline{C} + \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{B}.C =$$

✓ Solution:

$$= 1$$

Example 18

- Simplify the following:

$$(\bar{A} + B + \bar{C}).(\bar{A} + B + C).(C + D).(C + D + E) =$$

✓Solution:

$$= (\bar{A} + B).(C + D)$$

Example 19

- Simplify the following:

$$\begin{aligned} & \overline{B}.\overline{C}.\overline{D}.\overline{E} + B.\overline{C}.\overline{D}.E + \overline{A}.B.C.E + A.B.C.D.E + \\ & + A.\overline{B}.C.\overline{D}.\overline{E} + \overline{A}.B.\overline{C}.D.E + \overline{A}.\overline{B}.D.\overline{E} \\ & + \overline{A}.\overline{B}.C.\overline{D}.\overline{E} + A.\overline{B}.\overline{C}.D.\overline{E} = \end{aligned}$$

✓ Solution:

$$\begin{aligned} & = B.E + \overline{B}.D.\overline{E} + \overline{B}.\overline{D}.\overline{E} \\ & = B.E + \overline{B}.\overline{E} \end{aligned}$$

Summary

- Basic laws:

Commutative $X.Y = Y.X, \quad X + Y = Y + X$

Associative $X.(Y.Z) = (X.Y).Z, \quad X + (Y + Z) = (X + Y) + Z$

Distributive $X.(Y + Z) = X.Y + X.Z, \quad (X + Y).(X + Z) = X + Y.Z$

Summary

- Basic theorems:

#	Theorem	Minterm	Maxterm
1	0, 1	$\mathbf{X.1 = X}$	$\mathbf{X + 0 = X}$
2	0, 1	$\mathbf{X.0 = 0}$	$\mathbf{X + 1 = 1}$
3	Complement	$\mathbf{X.\bar{X} = 0}$	$\mathbf{X + \bar{X} = 1}$
4	Idempotent	$\mathbf{X.X = X}$	$\mathbf{X + X = X}$
5	Absorption	$\mathbf{X + X.Y = X}$	$\mathbf{X.(X + Y) = X}$
6	Involution	$\mathbf{\bar{\bar{X}} = X}$	
7	DeMorgan's	$\mathbf{\overline{(X.Y.Z...) = \bar{X} + \bar{Y} + \bar{Z} + ...}}$	$\mathbf{\overline{(X + Y + Z + ...) = \bar{X}.\bar{Y}.\bar{Z}...}}$

Contents

1. Introduction to Boolean Algebra
2. Variables, Literals & Terms in Boolean Expressions
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4. Logic Simplification
5. Basic Logic Gates

Logic Simplification

- The primary objective of all simplification procedures is to obtain an expression that has the **minimum number of terms**.
- If there is more than one possible solution with the same number of terms, the one having the minimum number of literals is the choice.
- Some simplification techniques:
 - Algebraic method
 - Karnaugh map method, or
 - Quine-McCluskey method

Algebraic Method

- This method uses Boolean theorems to simplify a logic function.

- Example: Simplify the following expression

$$f = AB + \bar{A}C + BC$$

- Apply $A + \bar{A} = 1$ and $X + XY = X$

$$\begin{aligned} f &= AB + \bar{A}C + BC(A + \bar{A}) \\ &= AB + ABC + \bar{A}C + \bar{A}BC \\ &= AB + \bar{A}C \end{aligned}$$

Example 20

- Simplify this expression:

$$f = AB + BCD + \bar{A}C + \bar{B}C$$

- Apply $A + \bar{A} = 1$ and $X + XY = X$

$$\begin{aligned} f &= AB + BCD(A + \bar{A}) + \bar{A}C + \bar{B}C \\ &= (AB + ABCD) + (\bar{A}BCD + \bar{A}C) + \bar{B}C \\ &= AB + \bar{A}C + \bar{B}C = AB + \overline{AB}.C \\ &= AB(1 + C) + \overline{AB}.C \\ &= AB + C \end{aligned}$$

Truth Table (Review)

- Getting the Boolean expression from the Truth Table:
 $f(A,B,C)$

$$= ABC = m_7$$

$$= (A+B+C)(A+B+C')(A+B'+C)(A+B'+C')(A'+B+C)(A'+B+C')(A'+B'+C)$$

m	A	B	C	f
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	0
m_4	1	0	0	0
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1

Std. Forms of Boolean Expressions

- A given Boolean function can be in either of 2 forms: **minterm** (SoP) or **maxterm** (PoS).
- A **standard form** (or **canonical form**) contain as many literals as the Boolean function has.
- In general, a Boolean function of n variables, it can be represented into the SoP form (m_i : minterm)

$$f(X_{n-1}, \dots, X_0) = \sum_{i=0}^{2^n-1} a_i m_i$$

or the PoS form (M_i : maxterm):

$$f(X_{n-1}, \dots, X_0) = \prod_{i=0}^{2^n-1} (a_i + M_i) \quad a_i = 0 \text{ or } 1$$

Minterm

- In general: $f(X_{n-1}, \dots, X_0) = \sum_{i=0}^{2^n-1} a_i m_i \quad a_i = 0 \text{ or } 1$

<i>A</i>	<i>B</i>	<i>C</i>	<i>Y</i>
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$Y = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot B \cdot C + A \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot C + A \cdot B \cdot C$$

➡ $f(A, B, C) = \sum 0, 3, 5, 6, 7$

Maxterm

- In general: $f(X_{n-1}, \dots, X_0) = \prod_{i=0}^{2^n-1} (a_i + M_i) \quad a_i = 0 \text{ or } 1$

<i>A</i>	<i>B</i>	<i>C</i>	<i>Y</i>
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$Y = (A + B + \overline{C}).(A + \overline{B} + C).(\overline{A} + B + C)$$

➡ $f(A, B, C) = \prod 1, 2, 4$

Maxterm \rightarrow Minterm

- ✓ Multiplying out the given expression
- ✓ Removing redundancy

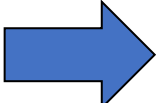
$$\begin{aligned} Y &= (A + B + \overline{C}).(A + \overline{B} + C).(\overline{A} + B + \overline{C}) \\ &= \overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot C + A \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot C \\ &\quad + A \cdot B \cdot C \end{aligned}$$

Minterm \rightarrow Maxterm

- ✓ Taking the dual of the given expression
- ✓ Multiplying out different terms to get the SoP form
- ✓ Removing redundancy
- ✓ Taking a dual to get the equivalent PoS form

$$\begin{aligned} Y &= A.B + \bar{A}.\bar{B} \\ \text{Dual} &= (A + B).(\bar{A} + \bar{B}) \\ &= A.\bar{A} + A.\bar{B} + B.\bar{A} + B.\bar{B} \\ &= 0 + A.\bar{B} + B.\bar{A} + 0 \\ &= A.\bar{B} + \bar{A}.B \end{aligned}$$

The dual of $(A.\bar{B} + \bar{A}.B) = (A + \bar{B}).(\bar{A} + B)$

 $A.B + \bar{A}.\bar{B} = (A + \bar{B}).(\bar{A} + B)$

Karnaugh Map Method

- Construction of a K-Map
 - An n-variable K-map has 2^n squares.
 - Each possible combination of inputs is allotted a square.
- In the case of a minterm K-map:
 - '1' is placed in the squares for which the output is '1'.
 - '0' is placed in the square for which the output is '0'.
 - 'X' is placed in the squares corresponding to 'don't care' conditions.
- In the case of a maxterm K-map:
 - '1' is placed in square for which the output is '0'.
 - '0' is placed in the square for which the output is '1'.
 - 'X' is placed in the square corresponding to 'don't care' conditions.
- **The designation of adjacent rows and adjacent columns should be the same except for one of the literals being complemented.**
- Also, the extreme rows and extreme columns are considered adjacent.

A \ B	0	1
0		
1		

A \ BC	00	01	11	10
0				
1				

AB \ CD	00	01	11	10
00				
01				
11				
10				

Karnaugh Map Method

- Efficient for ≤ 5 variables.
- **Guidelines:**
 - Each square containing a '1' must be considered at least once, although it can be considered as often as desired.
 - The objective should be to account for all the marked squares in the minimum number of groups.
 - The number of squares in a group must always be a power of 2, i.e. groups can have 1, 2, 4, 8, 16, ... squares.
 - Each group should be as large as possible; a group of two squares should not be made if the involved squares can be included in a group of four squares and so on.
 - 'Don't care' entries (marked 'X') can be used in accounting for all of 1-squares to make optimum groups. Such entries that can be used to advantage should be used.

CD \ AB	00	01	11	10
00			1	1
01			1	1
11	1	1	1	1
10			1	1

$f_1 = AB$ $f_2 = C$

Quine McCluskey Method

- Efficient for > 5 variables.
- Based on the complementation theorem:

$$X.Y + X.\bar{Y} = X$$

- **Step-by-step procedure:**

1. Divide all the minterms (and don't cares) of a function (F) into groups.
2. Merge minterms from adjacent groups to form a new implicant table.
3. Repeat Step 2 until no more merging is possible.
4. Put all prime implicants (PIs) in a cover table (**don't cares excluded**).
5. Identify essential minterms, and hence essential prime implicants (EPIs).
6. Add prime implicants to the minimum expression of F until all minterms of F are covered.

Quine McCluskey Method

- Example: $f(A, B, C, D) = \sum(10,11,12,13,14,15)$

Bảng a		Bảng b	
Hạng tích sắp xếp	Nhị phân (ABCD)	Rút gọn lần 1 (ABCD)	Rút gọn lần thứ 2 (ABCD)
10	1 0 1 0	1 0 1 - # (10,11)	1 1 - - (12,13,14,15)
<u>12</u>	<u>1 1 0 0</u>	1 - 1 0 # (10,14)	1 - 1 - (10,11,14,15)
11	1 0 1 1	1 1 0 - # (12,13)	
13	1 1 0 1	<u>1 1 - 0</u> # (12,14)	
<u>14</u>	<u>1 1 1 0</u>	1 - 1 1 # (11,15)	
15	1 1 1 1	1 1 - 1 # (13,15)	
		1 1 1 - # (14,15)	

Quine McCluskey Method

- Example: $f(A, B, C, D) = \sum(10,11,12,13,14,15)$

<i>A BCD</i>	10	11	12	13	14	15
1 1 - -			x	x	x	x
1 - 1 -	x	x			x	x

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3. Laws and Rules of Boolean Algebra
4. Logic Simplification
5. Basic Logic Gates

Basic Logic Gates

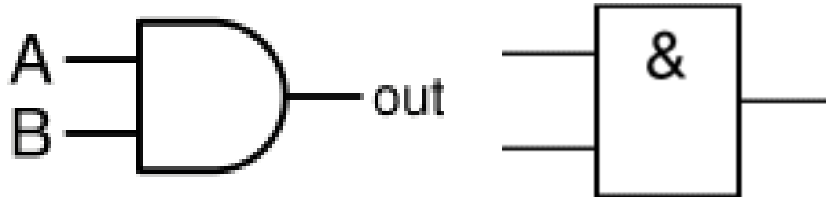
- Concepts
- Implementation of AND, OR gates using diodes
- Implementation of a NOT gate using transistors
- Integrated circuits (ICs)

Concepts

- 3 basic logic functions:
 - AND
 - OR
 - NOT
- 3 basic logic gates to implement logic functions:
 - **AND** gate
 - **OR** gate
 - **NOT** inverter
- Other logic gates: NAND, NOR, XOR, XNOR

AND Gate

- Functionality:
 - Performing an ANDing operation on two or more than two logic variables.
- 2-input AND gate:
 - Symbol:



- Truth Table:
- Expression: $out = A \cdot B$

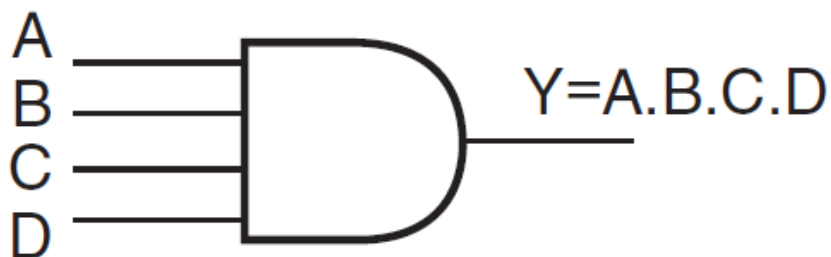
A	B	out
0	0	0
0	1	0
1	0	0
1	1	1

AND Gate

- 3-input AND gate:



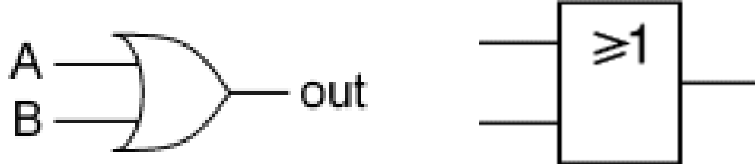
- 4-input AND gate:



A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

OR Gate

- Functionality:
 - Performing an ANDing operation on two or more than two logic variables.
- 2-input OR gate:
 - Symbol:

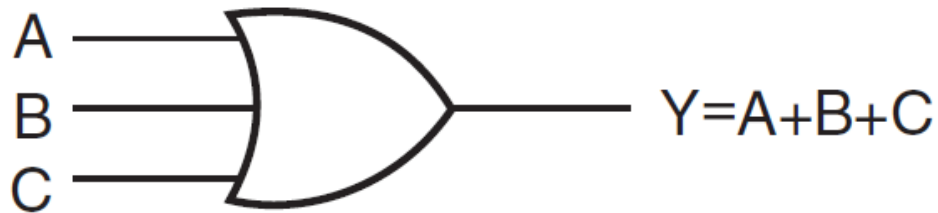


- Truth Table:
- Expression: $out = A + B$

A	B	out
0	0	0
0	1	1
1	0	1
1	1	1

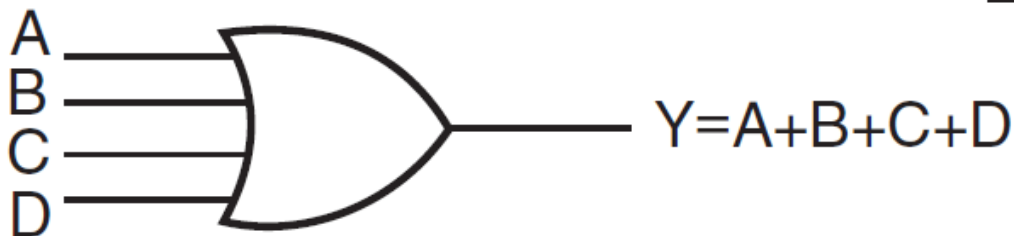
OR Gate

- 3-input OR gate:



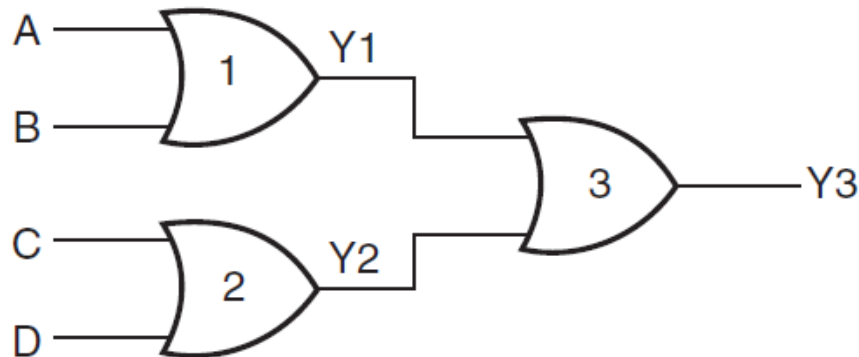
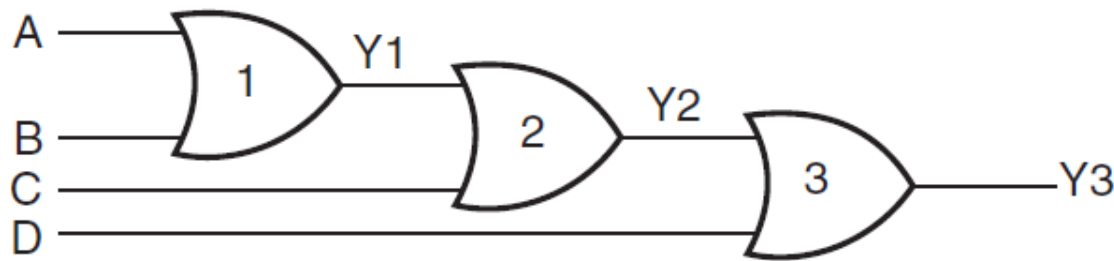
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- 4-input OR gate:



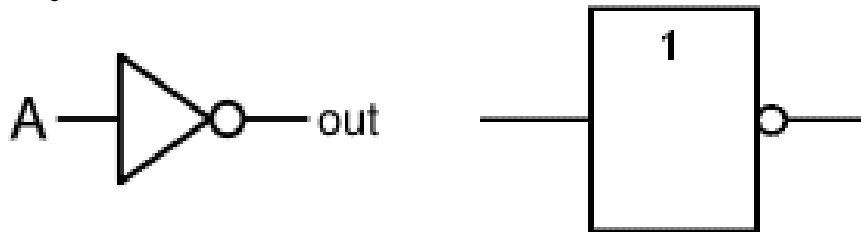
Example 21

- Show the logic arrangement for implementing a four-input OR gate using two-input OR gates only.



NOT Inverter

- Functionality:
 - The output is always the complement of the input.
- A NOT inverter has only one input.
 - Symbol:



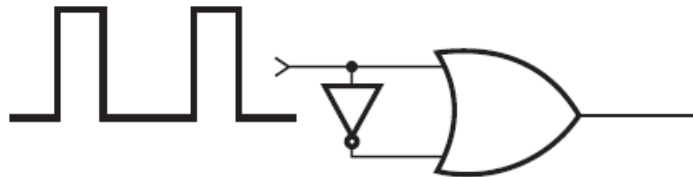
- Truth Table:
- Expression:

$$\text{out} = \overline{A}$$

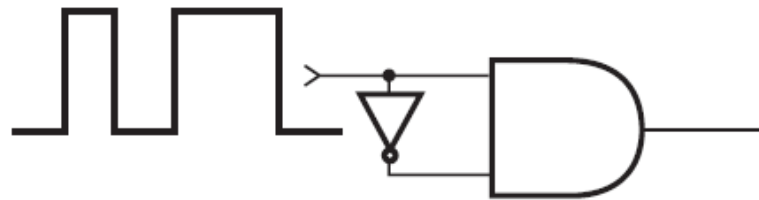
A	out
0	1
1	0

Example 22

- Draw the output waveform:



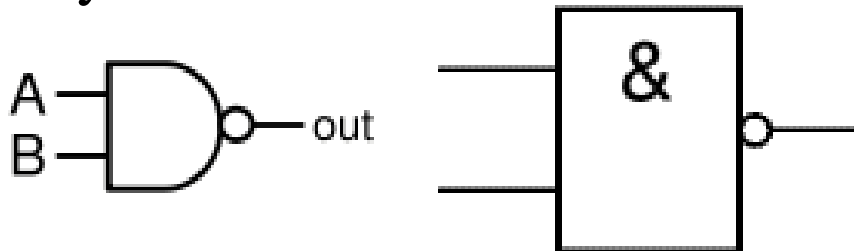
(a)



(b)

NAND Gate

- Functionality:
 - Performing an ANDing operation followed by a NOT operation on two or more than two logic variables.
- 2-input NAND gate:
 - Symbol:



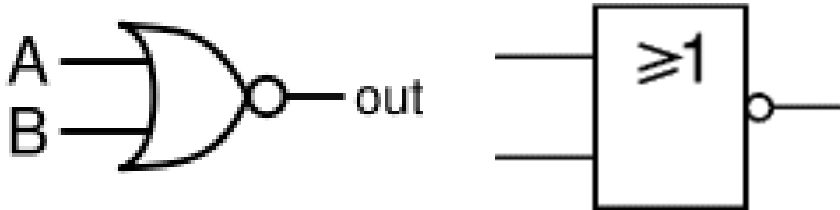
- Truth Table:
- Expression:

$$\text{out} = \overline{A \cdot B}$$

A	B	out
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate

- Functionality:
 - Performing an ORing operation followed by a NOT operation on two or more than two logic variables.
- 2-input NOR gate:
 - Symbol:



- Truth Table:
- Expression:

$$\text{out} = \overline{A + B}$$

A	B	out
0	0	1
0	1	0
1	0	0
1	1	0

Exclusive-OR Gate (XOR Gate)

- Functionality:

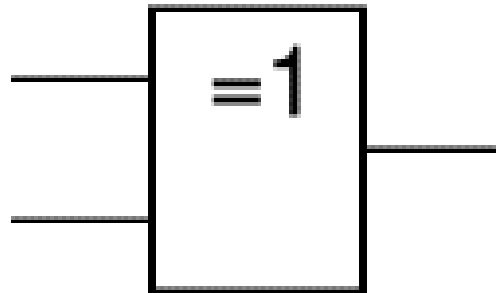
- The output of a multiple-input EX-OR logic function is a logic '1' when the number of 1s in the input sequence is odd and a logic '0' when the number of 1s in the input sequence is even, including zero.

- 2-input XOR gate:

- Symbol:



- Truth Table:
- Expression:

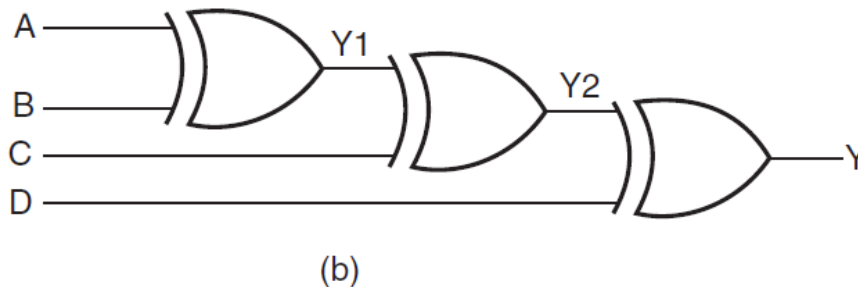
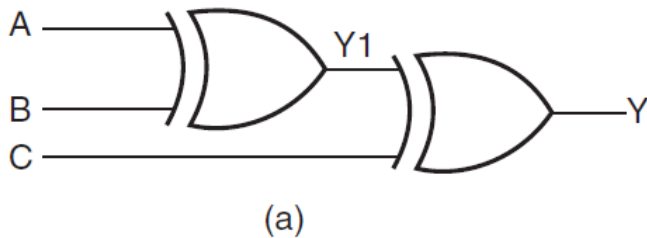


A	B	out
0	0	0
0	1	1
1	0	1
1	1	0

$$out = A \oplus B = \bar{A}.B + A.\bar{B}$$

Example 23

- How do you implement three-input and four-input EX-OR logic functions with the help of two-input EX-OR gates?



3-input XOR gate			
A	B	C	Output
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Example 24

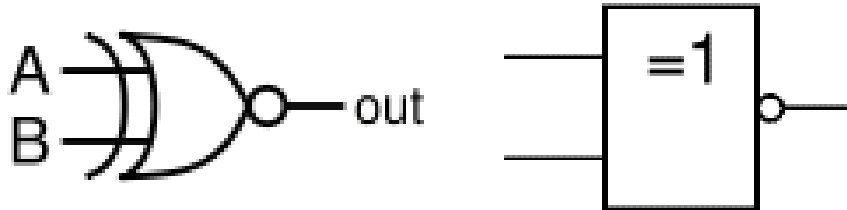
- How can you implement a NOT circuit using a two-input EX-OR gate?



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

XNOR Gate

- Functionality:
 - Complementing the output of an EX-OR gate.
- 2-input XNOR gate:
 - Symbol:



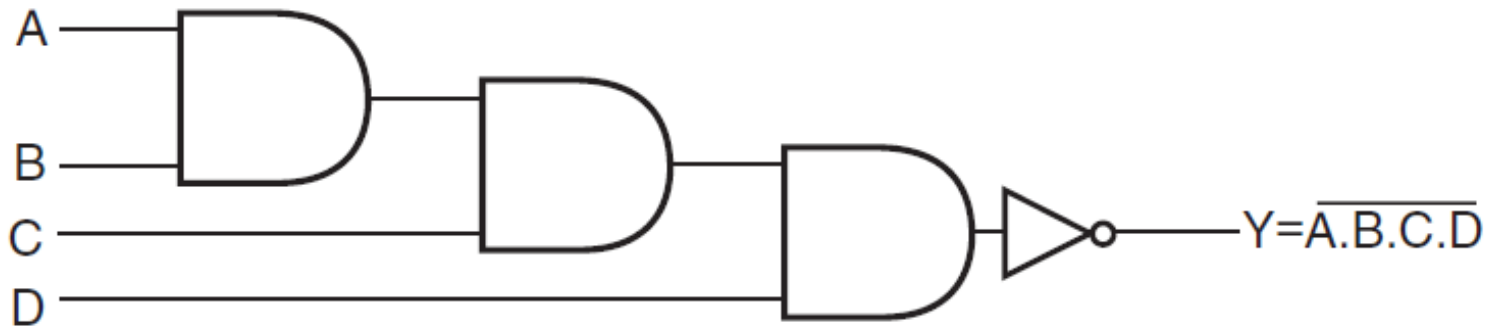
- Truth Table:
- Expression:

$$out = \overline{A \oplus B} = A.B + \overline{A}.\overline{B}$$

A	B	out
0	0	1
0	1	0
1	0	0
1	1	1

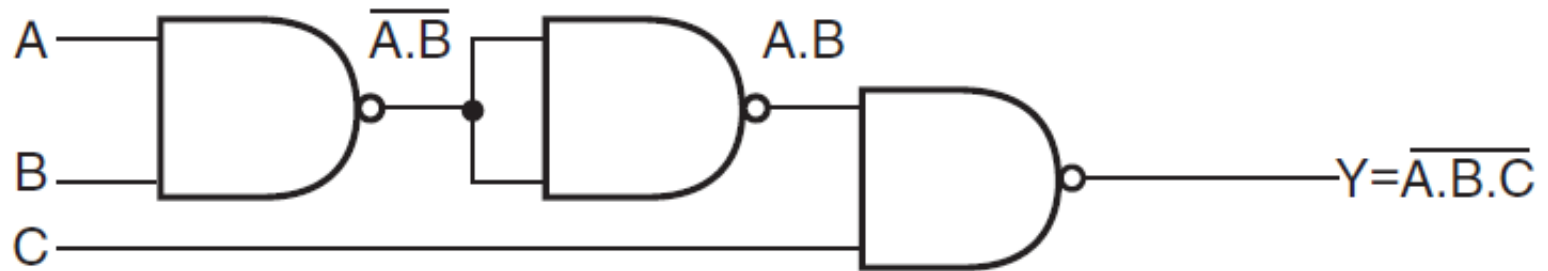
Example 25

- How do you implement a three-input EX-NOR function using only two-input EX-NOR gates?



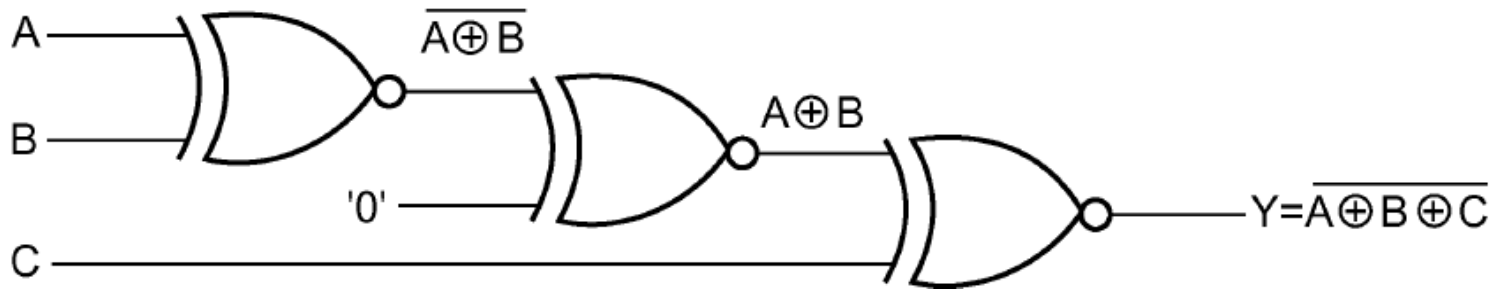
Example 26

- Show the logic arrangements for implementing a three-input NAND gate using two-input NAND gates.



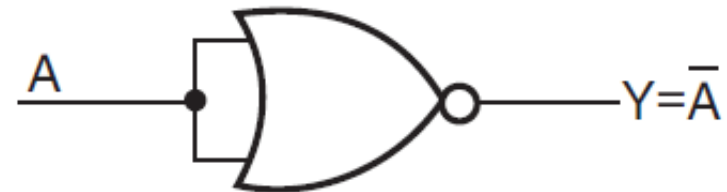
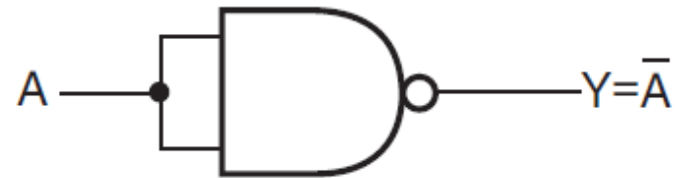
Example 27

- Show the logic arrangements for implementing a three-input XNOR gate using two-input XNOR gates.



Example 28

- Show the logic arrangements for implementing a NOT gate using:
 - 2-input NAND gates
 - 2-input NOR gates
 - 2-input XNOR gates



Exercise 1

a) Show the logic arrangements for implementing a 8-input NAND gate using 2-input AND gates and 2-input NAND gates.

b) Show the logic arrangements for implementing a 8-input XNOR gate using a minimum number of 2-input logic gates.

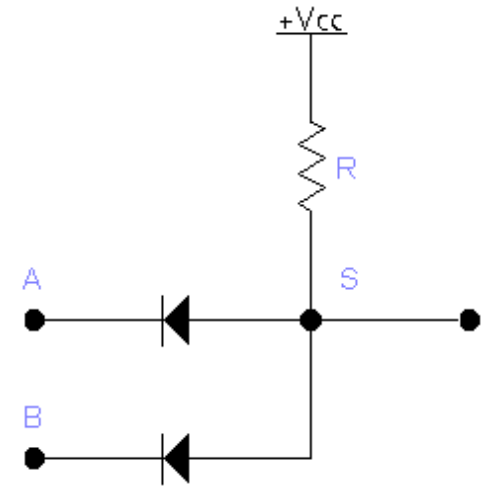
Basic Logic Gates

- Concepts
- Implementation of AND, OR gates using diodes
- Implementation of a NOT gate using transistors
- Integrated circuits (ICs)

Implementing a 2-Input AND Gate using Diodes

- Standard TTL
- Input A = 5V, Input B = 5V \rightarrow Output S \approx 5V
- Input A = 0V, Input B = 5V \rightarrow Output S \approx 0V
- Input A = 5V, Input B = 0V \rightarrow Output S \approx 0V
- Input A = 0V, Input B = 0V \rightarrow Output S \approx 0V

$$S = A.B$$



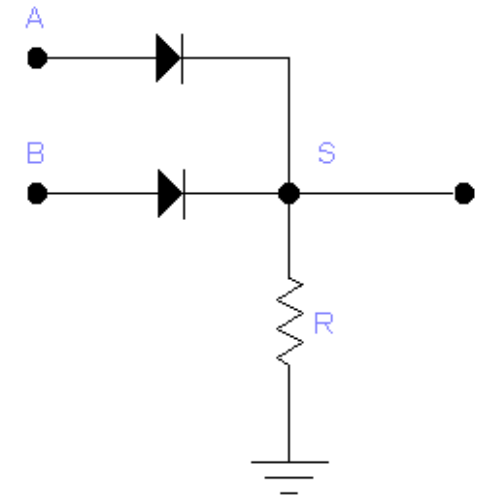
U_A	U_B	U_S	
0	0	0	D_A, D_B thông
0	5	0	D_A thông, D_B tắt
5	0	0	D_A tắt, D_B thông
5	5	5	D_A, D_B tắt

\rightarrow

A	B	S
0	0	0
0	1	0
1	0	0
1	1	1

Implementing a 2-Input OR Gate using Diodes

- Standard TTL
- Input A = 5V, Input B = 5V \rightarrow Output S \approx 4.3V
- Input A = 0V, Input B = 5V \rightarrow Output S \approx 4.3V
- Input A = 5V, Input B = 0V \rightarrow Output S \approx 4.3V
- Input A = 0V, Input B = 0V \rightarrow Output S \approx 0V



$$S = A + B$$

U_A	U_B	U_S	
0	0	0	D_A, D_B tắt
0	5	5	D_A tắt, D_B thông
5	0	5	D_A thông, D_B tắt
5	5	5	D_A, D_B thông

\rightarrow

A	B	S
0	0	0
0	1	1
1	0	1
1	1	1

Basic Logic Gates

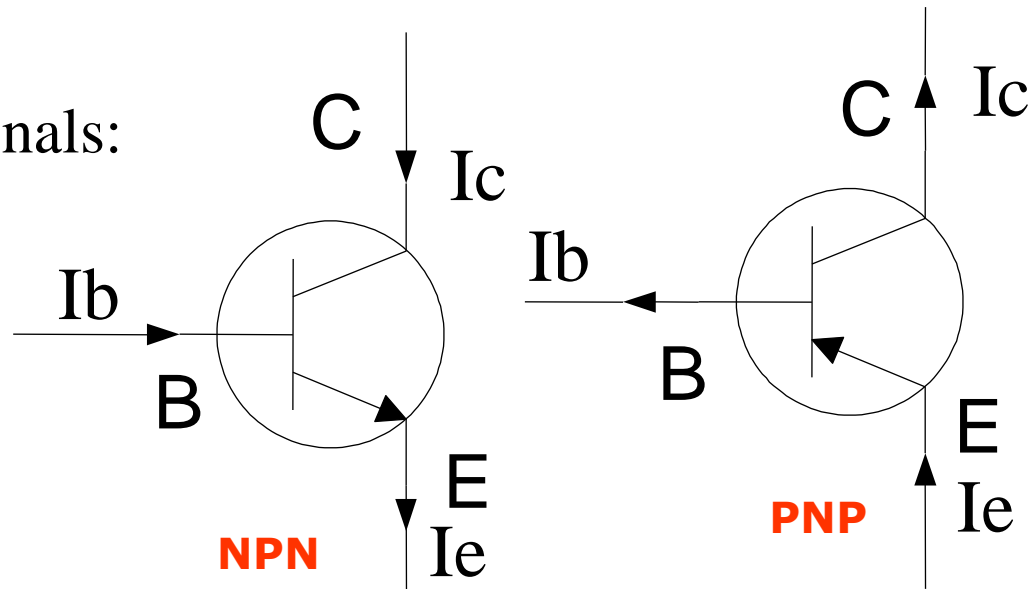
- Concepts
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Implementing a NOT Gate using Transistors

- BJT: NPN & PNP

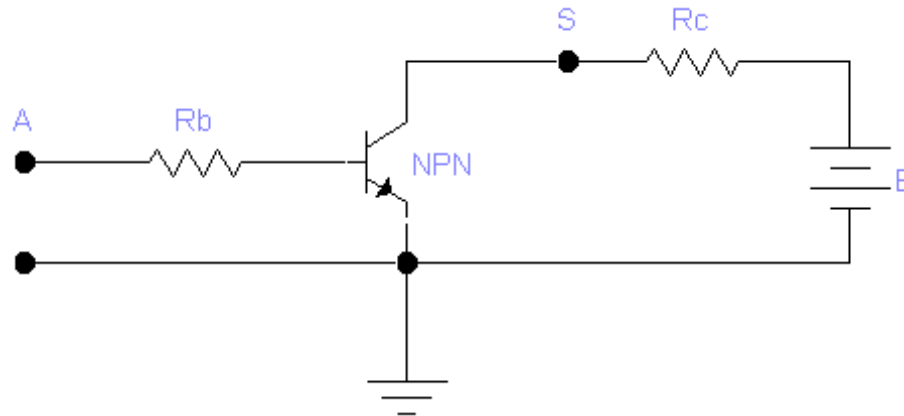
- A BJT has 3 terminals:

- B: Base
 - C: Collector
 - E: Emitter



- Functionality: controlling I_B to get amplified I_C
- Operation:
 - $I_B = 0$, the BJT is in cut-off mode, $I_C = 0$
 - $I_B > 0$, the BJT is in forward-active mode, $I_C = \beta \cdot I_B$, where β is the common-emitter current gain.

Implementing a NOT Gate using Transistors



- Standard TTL, small R_b
- Input $A = 0V \rightarrow$ the BJT off \rightarrow Output $S \approx 5V$
- Input $A = 5V \rightarrow$ the BJT on \rightarrow Output $S \approx 0V$

U_A	U_S
0	5 T tắt
5	0 T thông

\rightarrow

A	S
0	1
1	0

$$S = \overline{A}$$

Basic Logic Gates

- Concepts
- Implementation of AND, OR gates using diodes
- Implementation of a NOT gate using transistors
- Integrated circuits (ICs)

Integrated Circuits (ICs)

- An IC is a set of electronic circuits on one small flat piece (or "chip") of semiconductor material, usually silicon. [Wiki]
 - **Advantage:** cost and performance.
 - **Disadvantage:** high cost of designing ICs and fabricating the required photomasks.
- 2 types of integrated circuits:
 - Analog IC: to handle continuous signals such as audio signals
 - Digital IC: to handle discrete signals such as binary values

Classification of ICs

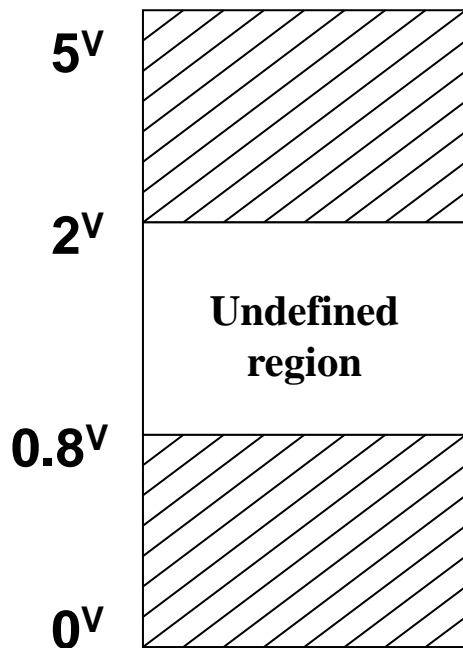
- Based on the chip size:
 - SSI - Small Scale Integration: <10 gates/chip, each gate consists of 2~10 transistors
 - MSI - Medium Scale Integration: 10~100 gates/chip
 - LSI - Large Scale Integration: 100~1000 gates/chip
 - VLSI - Very Large Scale Integration: $10^3 \sim 10^6$ gates/chip
 - ULSI - Ultra Large Scale Integration: $>10^6$ gates/chip

Classification of ICs

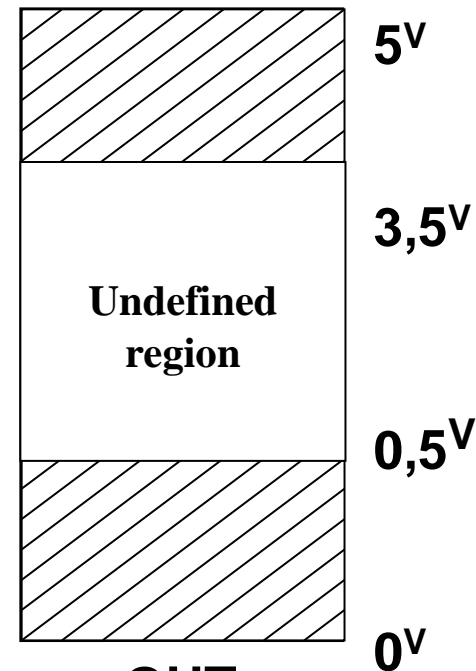
- Based on the logic family:
 - Using BJT:
 - RTL – Resistor Transistor Logic
 - DTL – Diode Transistor Logic
 - TTL – Transistor Transistor Logic
 - ECL – Emitter Coupled Logic
 - Using FET (Field Effect Transistor)
 - MOS – Metal Oxide Semiconductor
 - CMOS – Complementary MOS

Electrical Characteristics of ICs

- Logic levels are associated directly with voltage ranges.
 - **Example:** a TTL input signal is defined as “low” between 0V and 0.8V with respect to ground, and "high" when between 2V and 5V (V_{cc}).



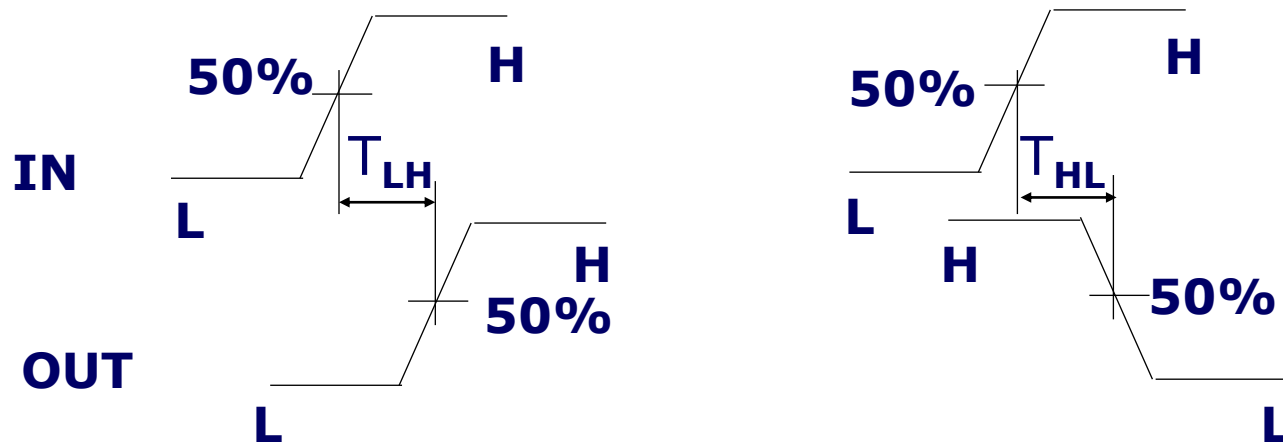
IN



OUT

Electrical Characteristics of ICs

- The **propagation delay** is the time taken to respond when there is change on its inputs.

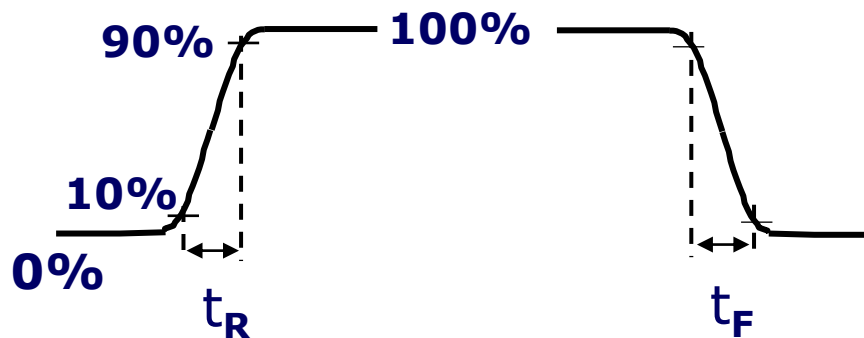


- Defined as the average of these two times:

$$T_{avg} = (T_{LH} + T_{HL})/2$$

Electrical Characteristics of ICs

- The **rise and fall times** are the time taken by a signal to change its logic states from 0 to 1 or vice versa.
 - Ideally, the rise and fall times of a signal are 0.
 - In fact, the rise and fall times of a signal are typically measured from the 10% level to the 90% level (or vice versa) of its voltage.

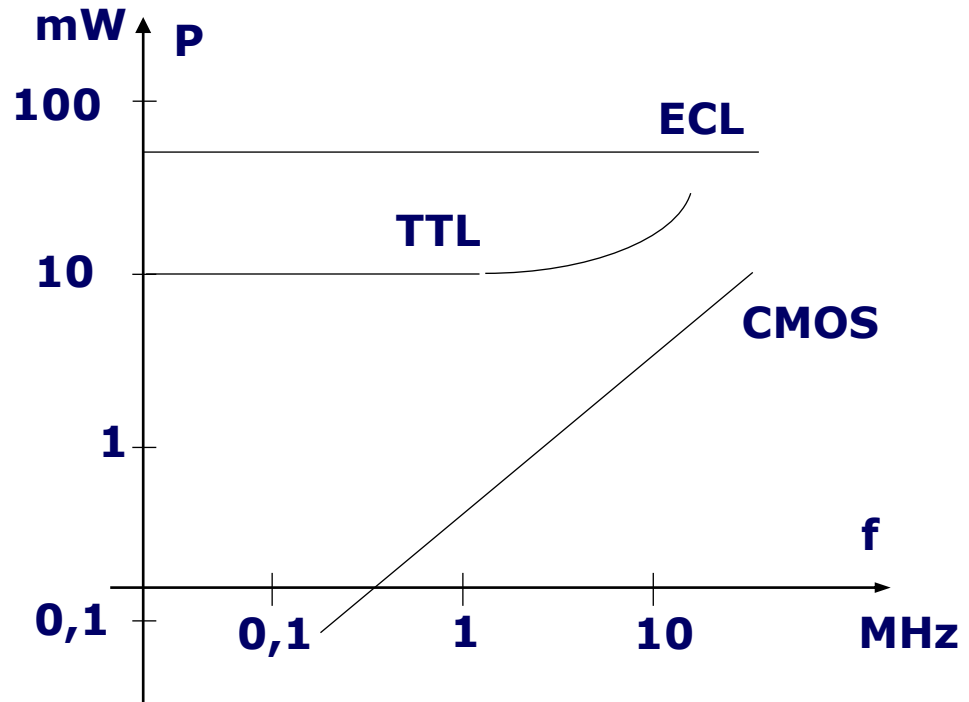


t_R : rise time
 t_F : fall time

Electrical Characteristics of ICs

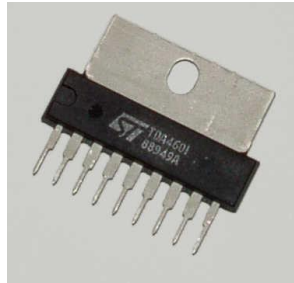
- Power dissipation: **static power dissipation** vs **dynamic power dissipation**.
- The dynamic power dissipation of an IC depends on the frequency of switching and logic families.

Dynamic power dissipation of several logic families

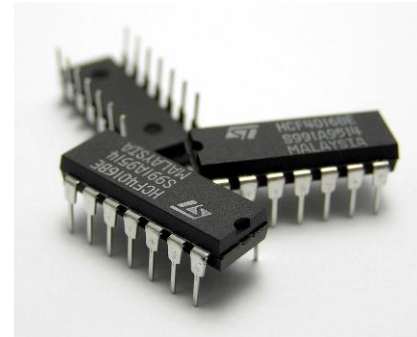


IC Packaging Types

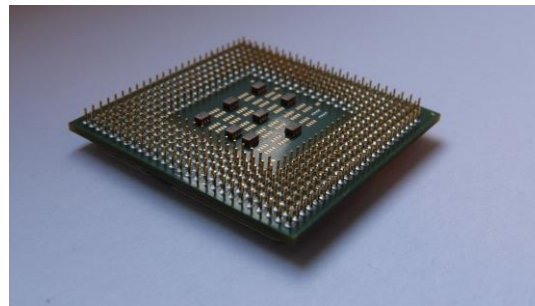
- SIP (Single Inline Package) or SIPP (Single In-line Pin Package)



- DIP (Dual In-line Package)

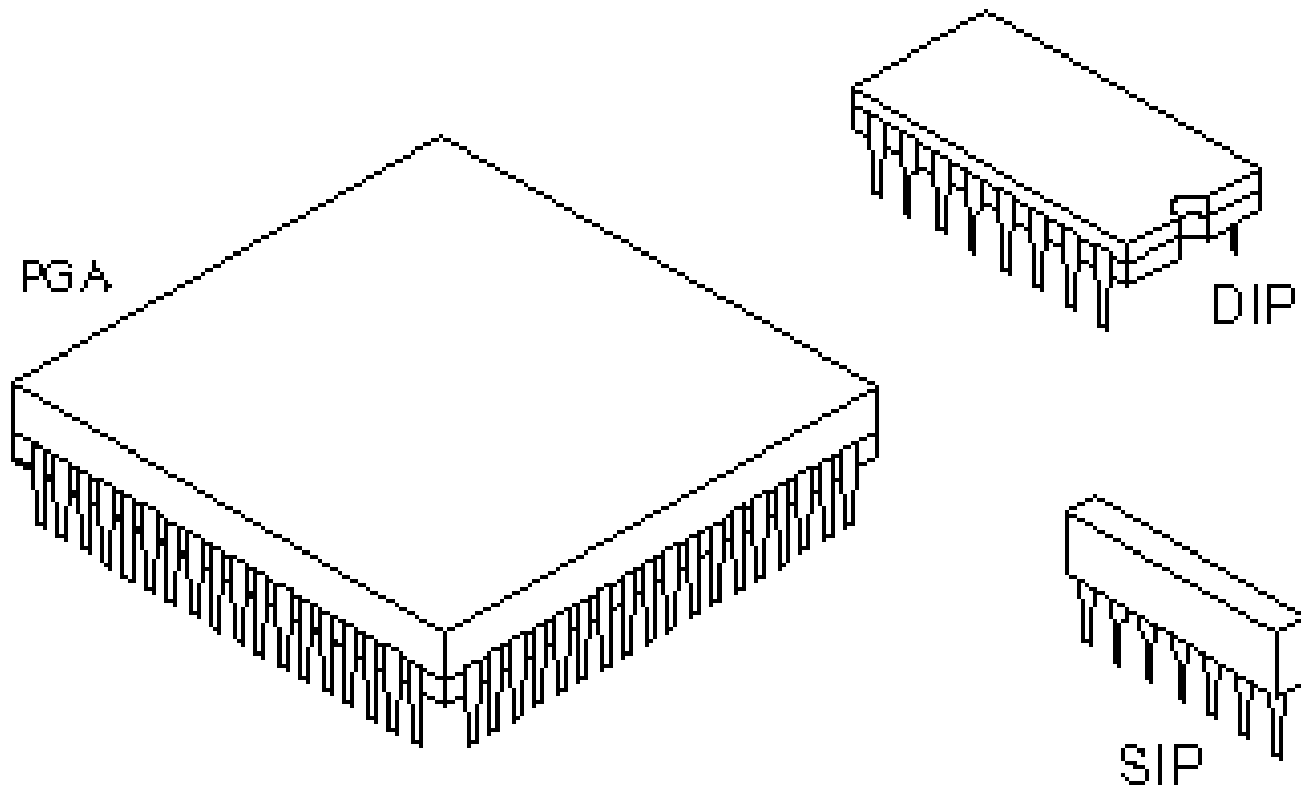


- PGA (Pin Grid Array)



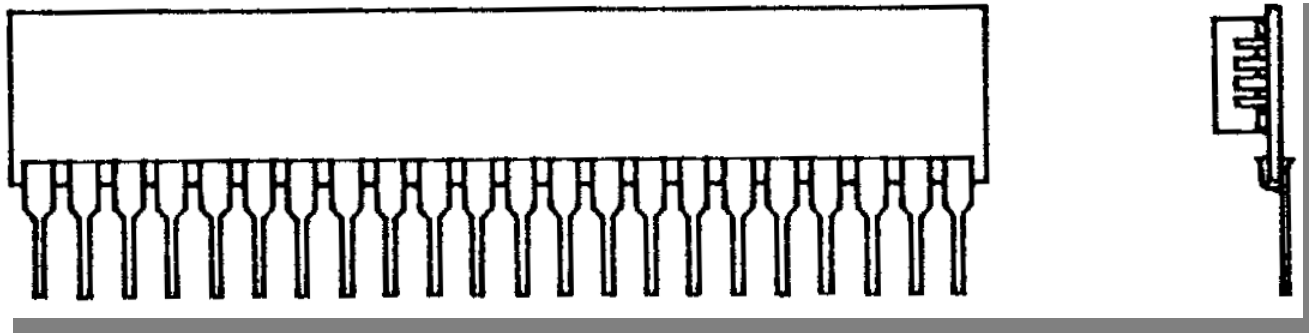
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IC Packaging Types



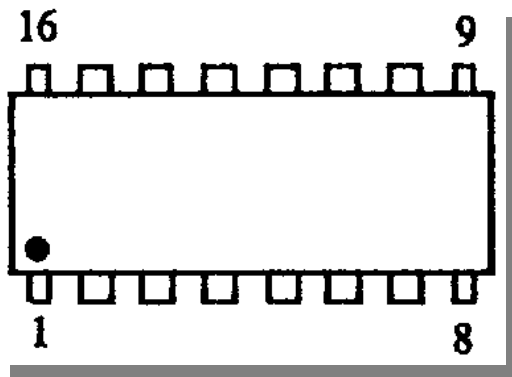
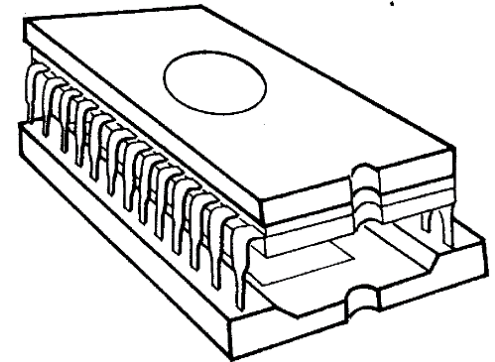
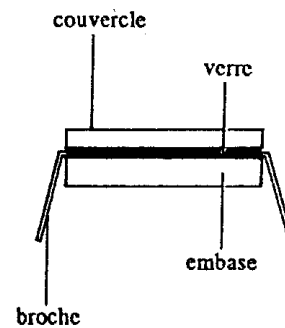
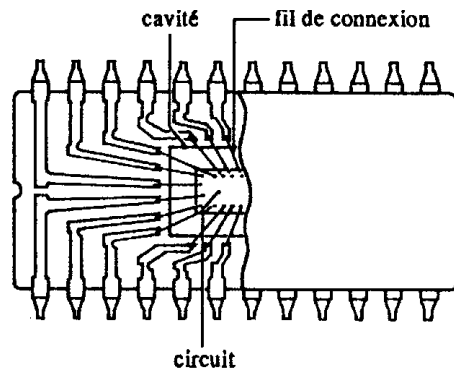
IC Packaging Types

- SIL (Single In Line)



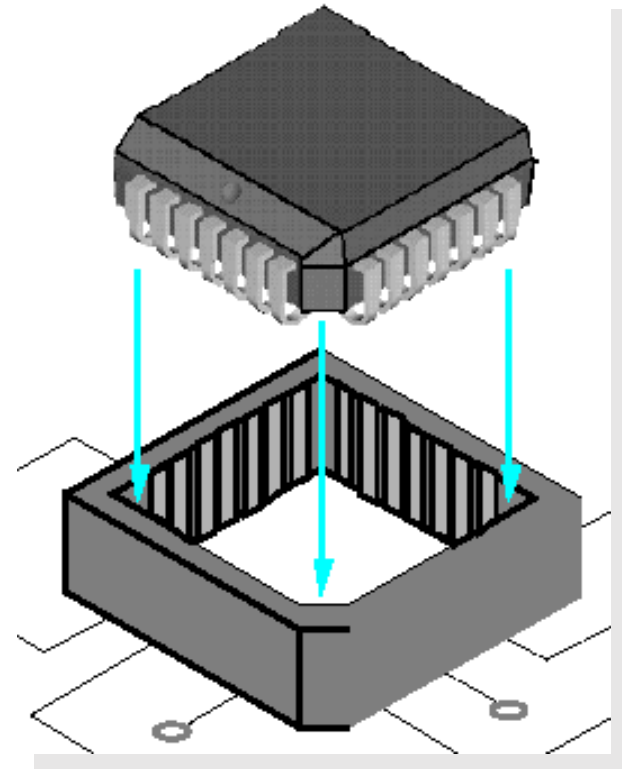
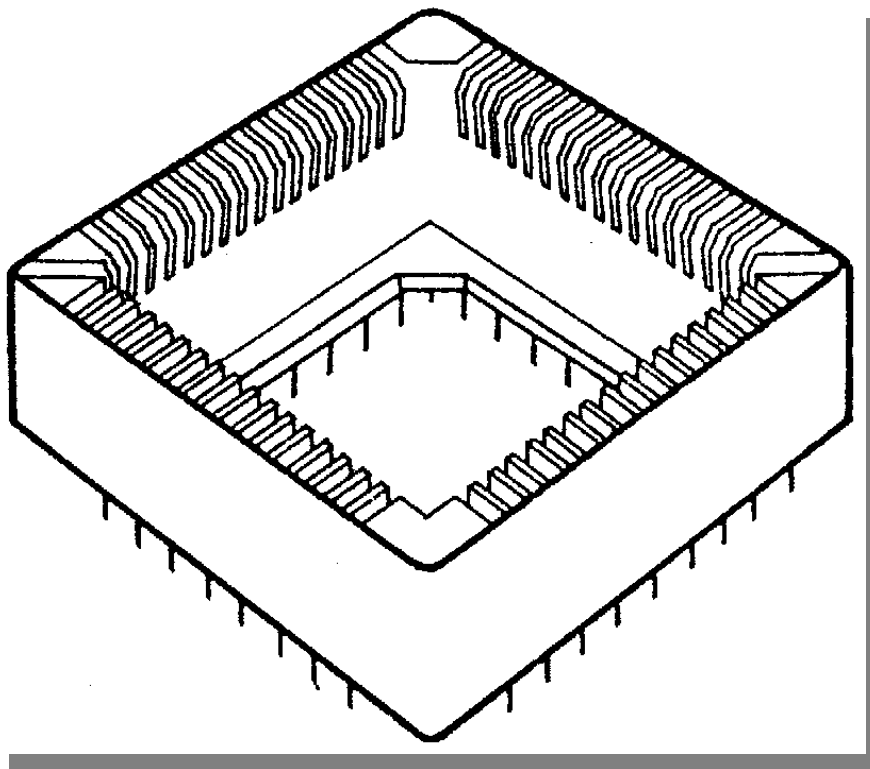
IC Packaging Types

- DIL (Dual In Line): 8 to 64 pins



IC Packaging Types

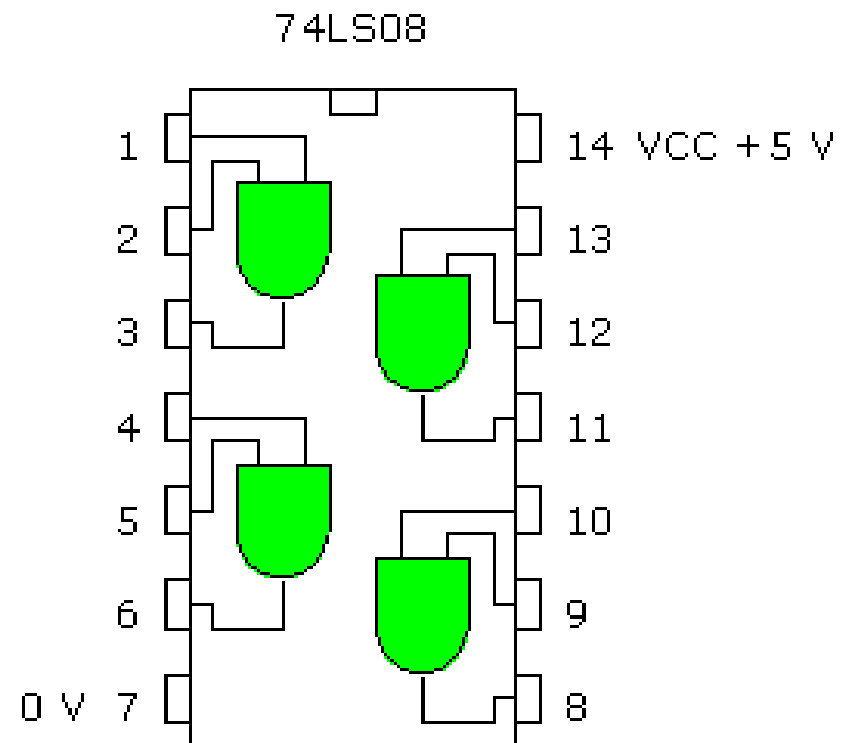
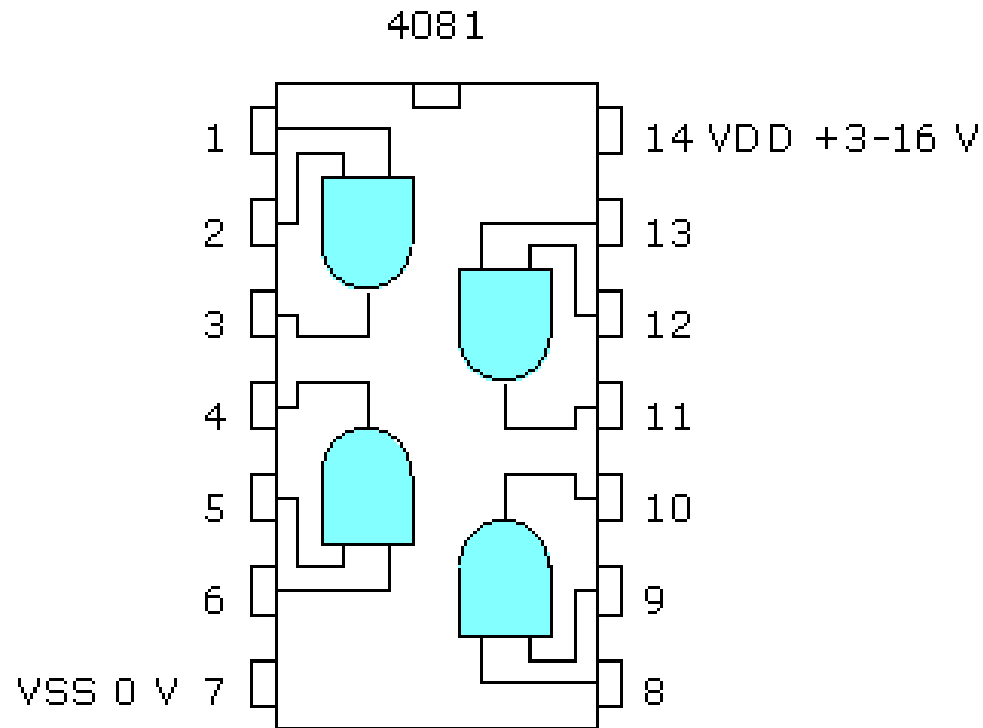
- PGA (Pin-Grid Array)



Thermal Characteristics

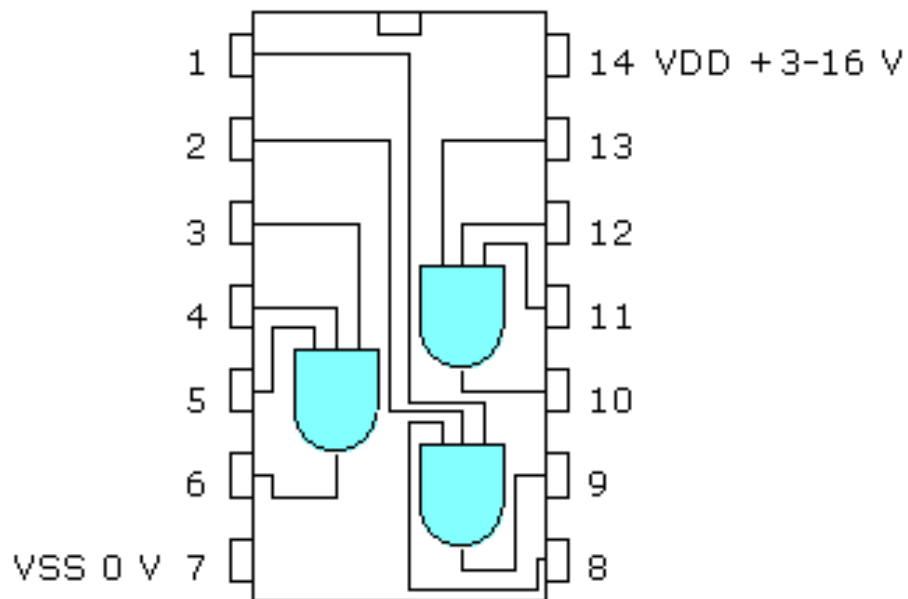
- The **thermal resistance** of an IC package is the measure of the package's ability to transfer heat generated by the IC (die) to the circuit board or the ambient.
- **Operating Temperature**: the allowable temperature range of the local ambient environment at which ICs operate.
 - Commercial: 0 °C to 70 °C
 - Industrial: -40 °C to 85 °C
 - Military: -55 °C to 125 °C

IC of AND Gates

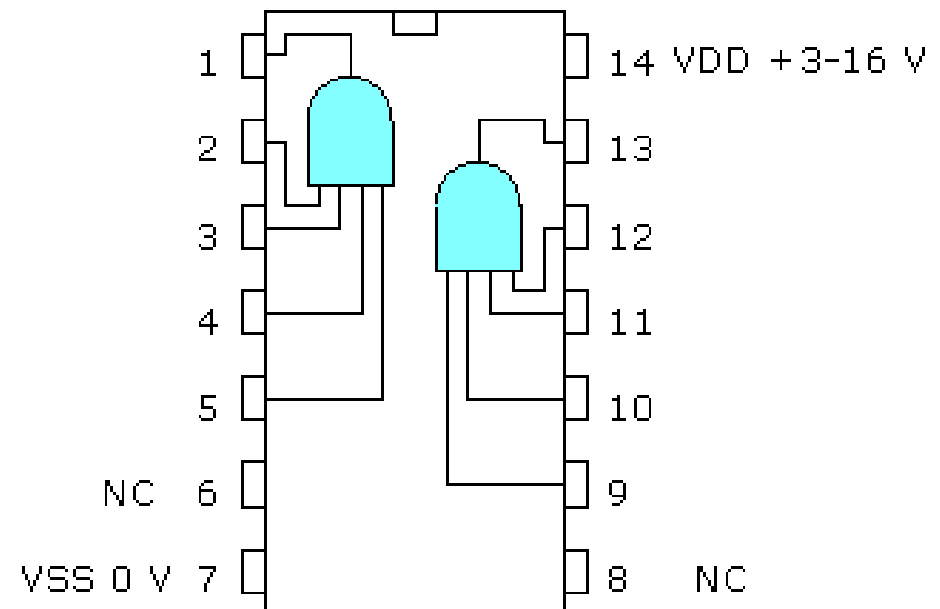


IC of AND Gates

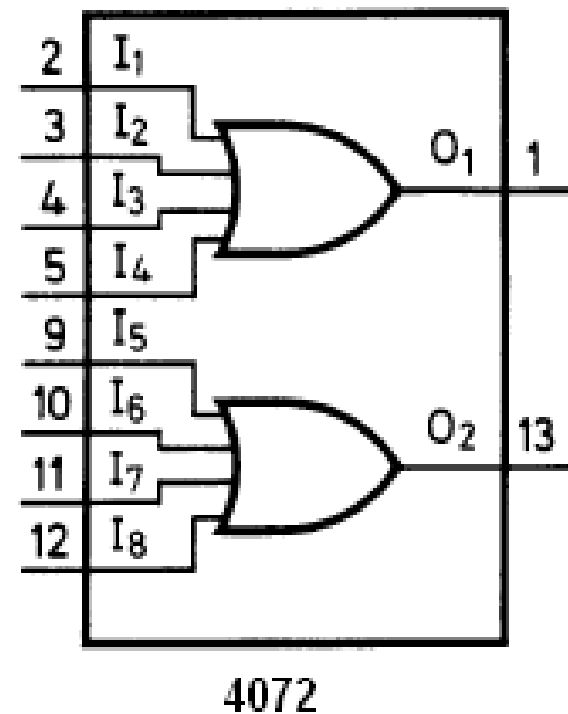
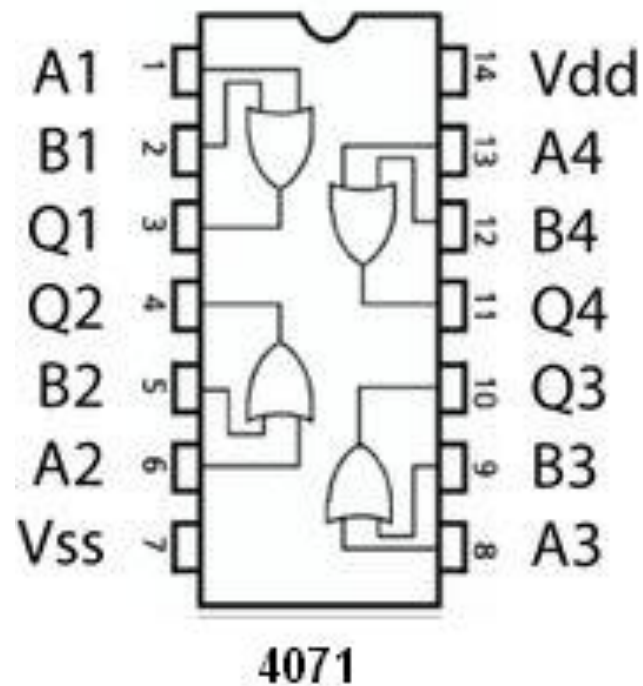
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4082

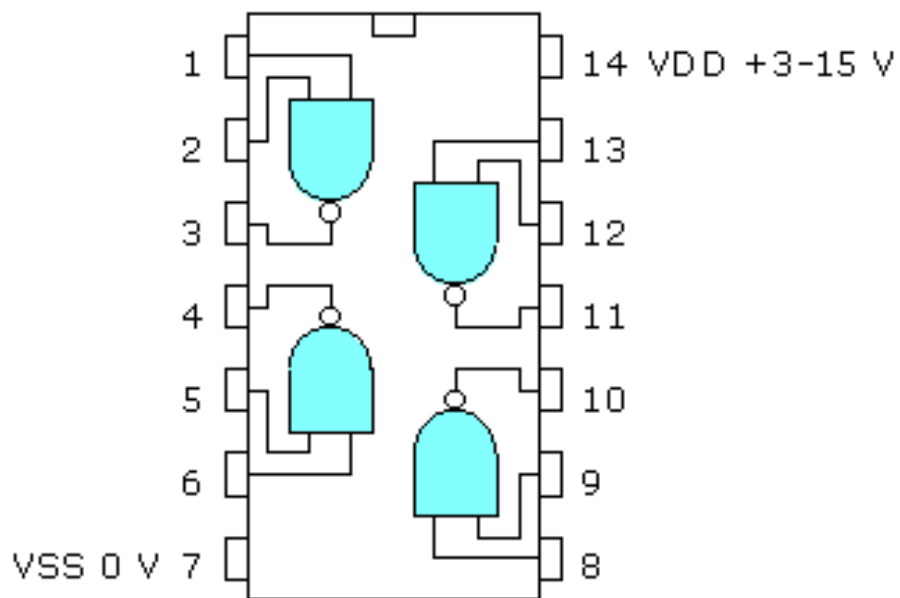


IC of OR Gates

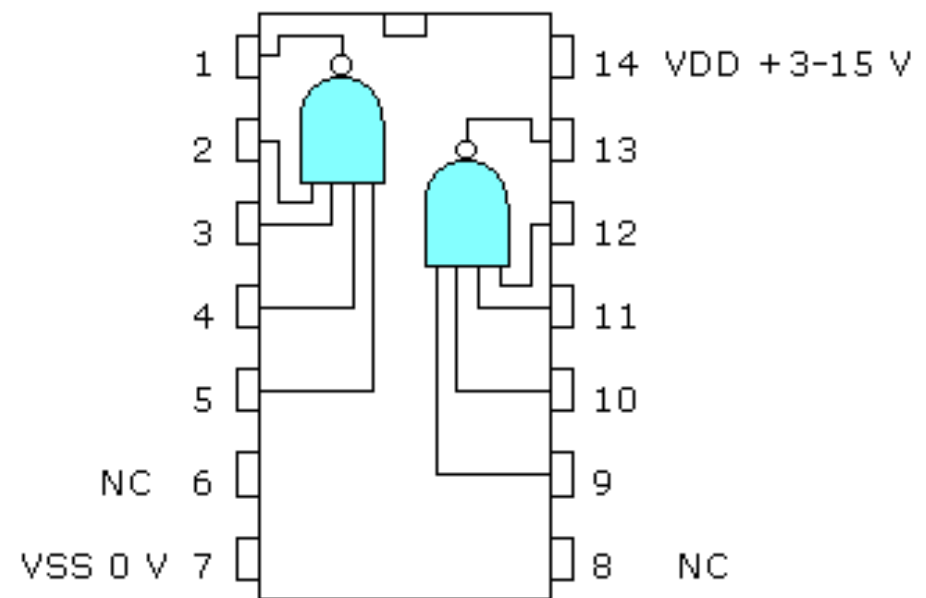


IC of NAND Gates

4011

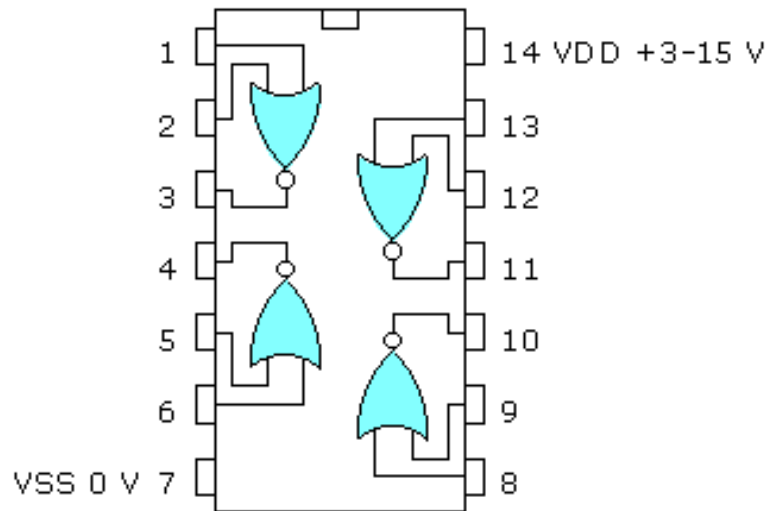


4012

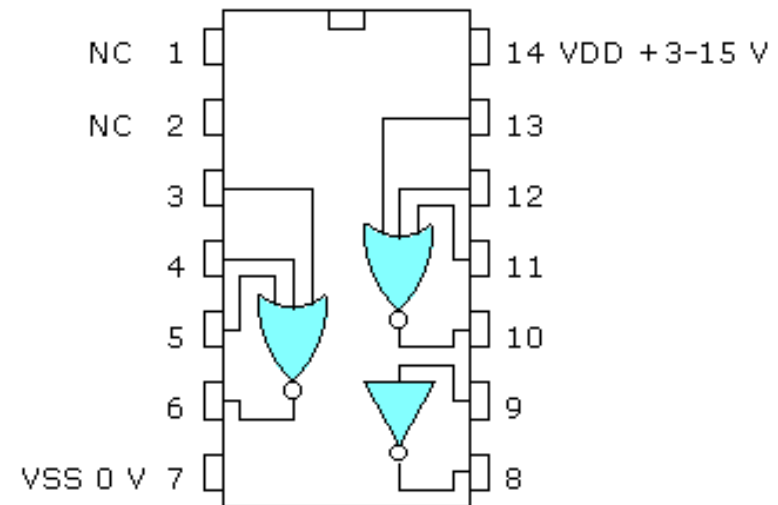


IC of NOR Gates

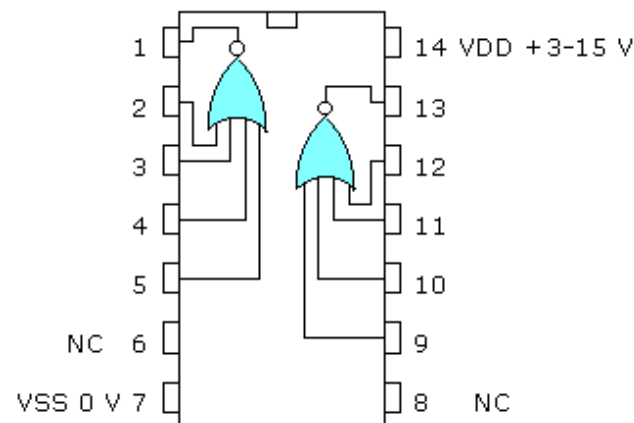
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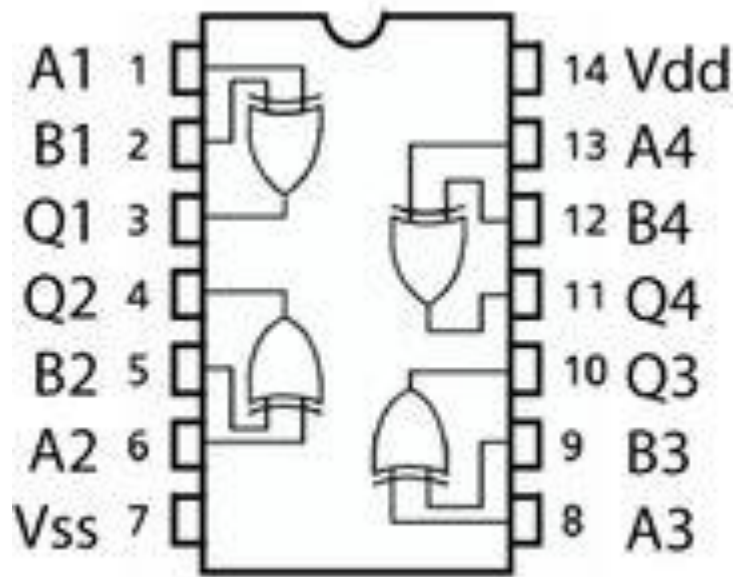
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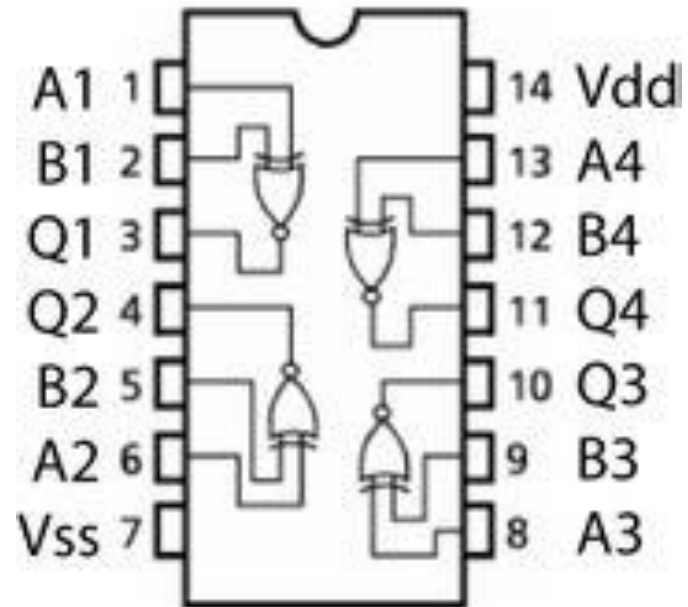
002



IC of XOR/XNOR Gates



4070/4030



4077

List of Logic Gate ICs

- AND: 74LS08
- OR: 74LS32
- NOT: 74LS04/05
- NAND: 74LS00
- NOR: 74LS02
- XOR: 74LS136
- NXOR: 74LS266

Read more: [MultiSim](#)