UNIT 2

ANALOG-TO-DIGITAL CONVERSION

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❑ Contents

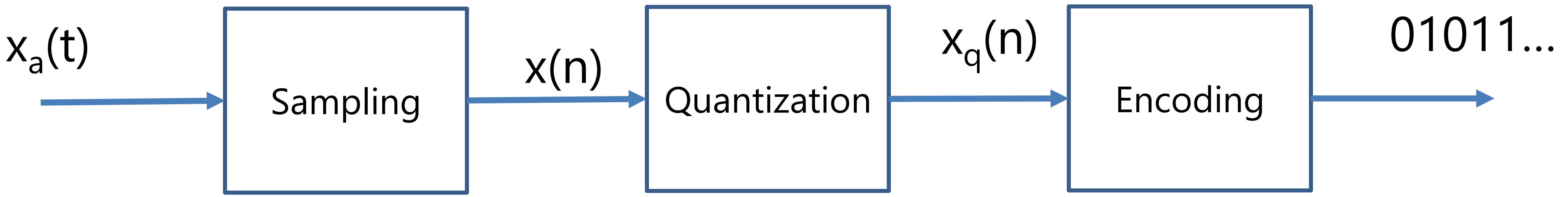
❑ Learning Objectives

After completing this lesson, you will have grasped the following topics:

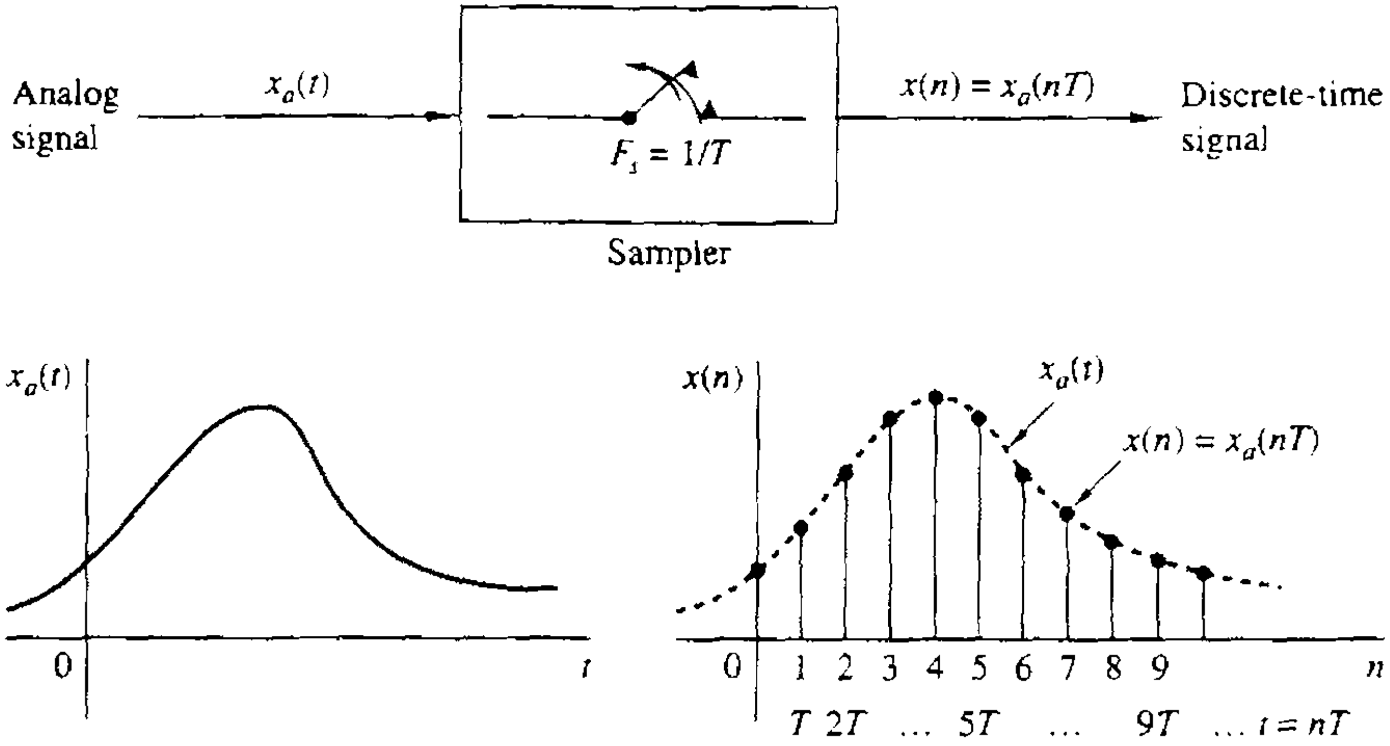
- The method of converting analog signals into digital signals.
- The sampling method of signals and the basic parameters of the sampling process.
- The quantization method of signals and the basic parameters of the quantization process.

1. Chuyển đổi tín hiệu tương tự thành tín hiệu số

- Analog-to-digital (A/D) conversion
- Devices: A/D converters (ADCs)
- The implementation steps: sampling and quantization.



2. Analog signal sampling



- Sampling rate/cycle T_s
- Sampling frequency $F_s = \frac{1}{T_s}$
- Example: audio signal sampling

$$x(n) = x(nT_s)$$

Example

- Analog singal:

$$x_a(t) = A.cos(2\pi Ft + \theta) \quad (1)$$

- Output signal of the sampling process:

$$x_a(nT_s) \equiv x(n) = A.cos(2\pi FnT_s + \theta)$$

$$= A.cos\left(\frac{2\pi Fn}{F_s} + \theta\right)$$

$$= A.cos(2\pi fn + \theta) \ (2)$$

$$\Rightarrow \quad f = \frac{F}{F_s} \quad (3)$$

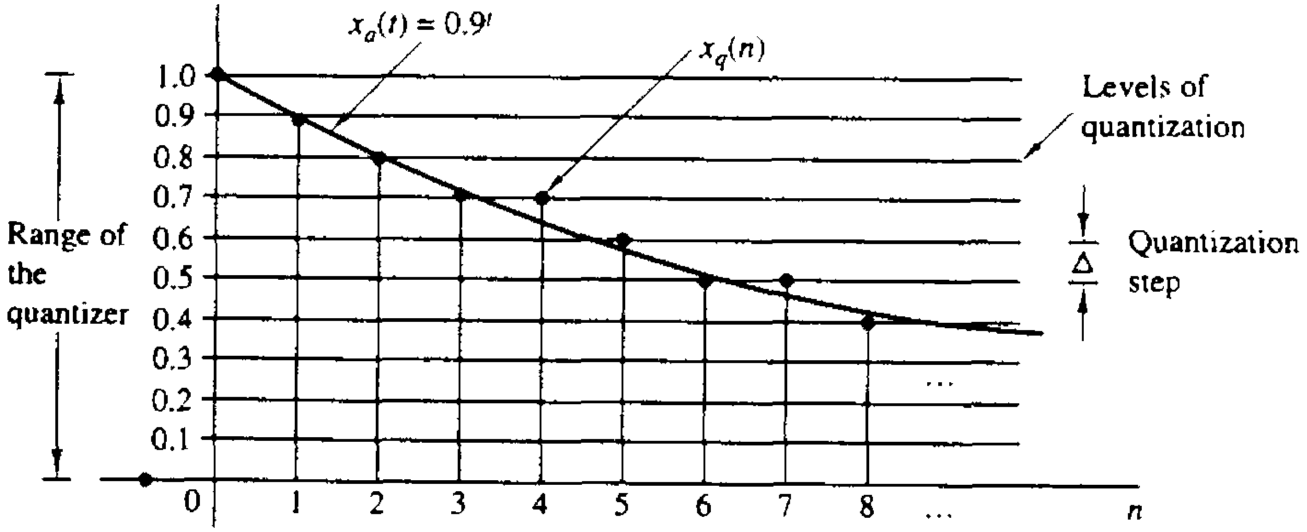
1.3. Signal quantization

$$x(n) = \begin{cases} 0.9^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

- Quantization level, quantization step
- Quantization law Rounding - rounding each sample of x(n) to the nearest quantization level.
- Q[x(n)] : Quantization function w.r.t sample x(n)

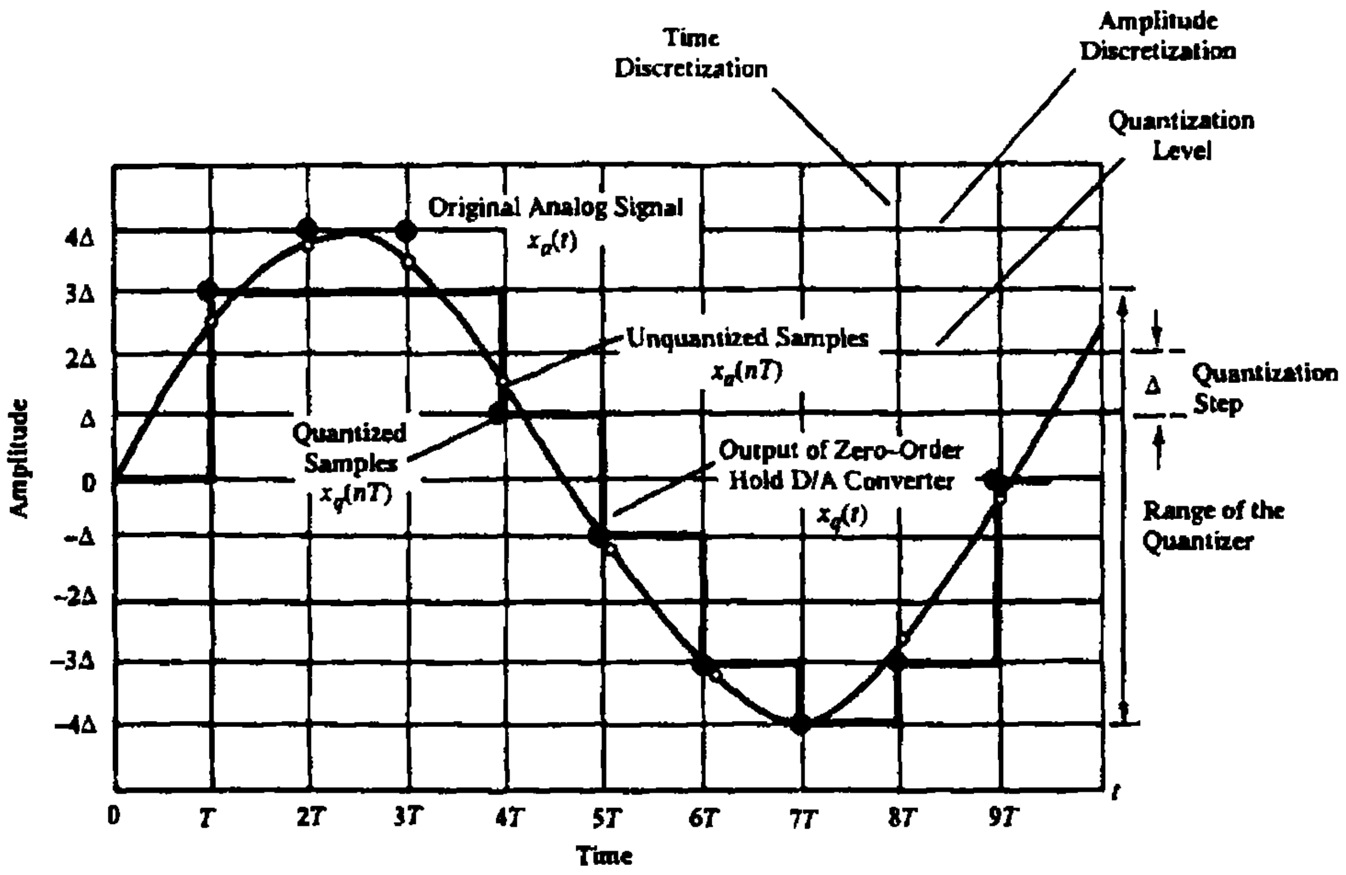
$$-\frac{\Delta}{2} \leq e_q(n) \leq \frac{\Delta}{2}$$

- $x_q(n)$: Quantizatized signal
- Quantization error : $e_q(n) = x_q(n) - x(n)$
- Increasing quantization levels:
  - Reduce quantization error
  - Increase the accuracy of the quantization system



(b)

Quantization of a sine signal

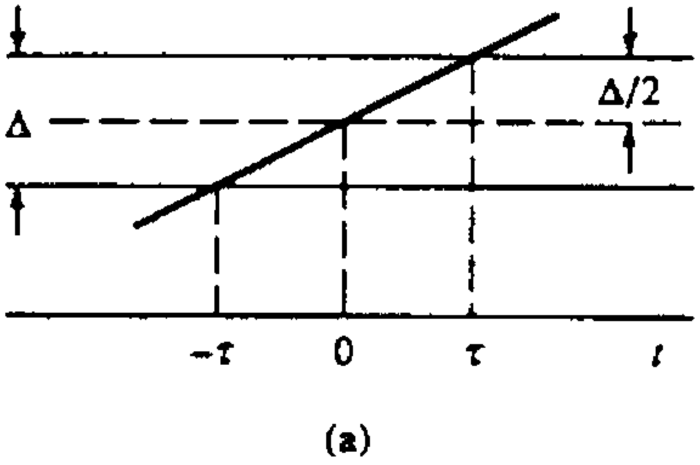


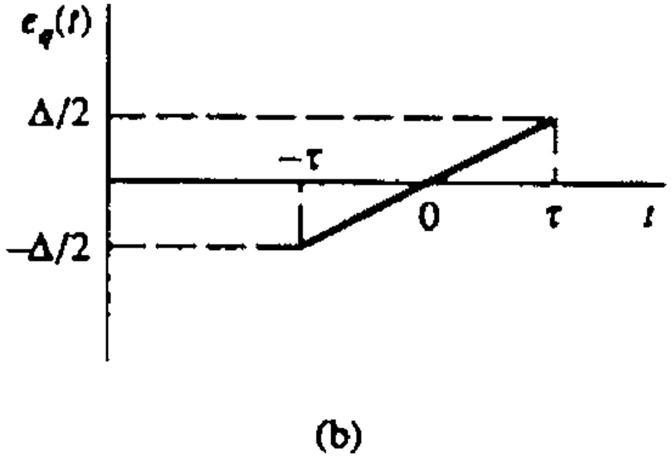
Sampling and quantization of a sine signal

$$x_a(t) = A \,.\, \sin \Omega_0 t$$

Quantization noise power P_q

- $e_q(t) = x_q(t) - x_a(t)$





$$P_q = \frac{1}{2\tau}\int_{-\tau}^{\tau} e_q^2(t)dt = \frac{1}{\tau}\int_{0}^{\tau} e_q^2(t)dt$$

$$P_q = \frac{1}{\tau}\int_{0}^{\tau} \left(\frac{\Delta}{2\tau}\right)^2 t^2 dt = \frac{\Delta^2}{12} \quad \Rightarrow \quad P_q = \frac{A^2/3}{2^{2b}}$$

Signal-to-quantization noise ratio SQNR

- Quantization noise power of a signal $x_a(t)$

$$P_x = \frac{1}{T_p}\int_0^{T_p} (A sin \Omega_0 t)^2 dt = \frac{A^2}{2}$$

- Signal-to-quantization noise ratio (SQNR)

$$SQNR = \frac{P_x}{P_q} = \frac{3}{2} \cdot 2^{2b}$$

$$SQNR(dB) = 10 log_{10} SQNR = 1.76 + 6.02b$$

4. Summary

- The process of converting analog signals into digital signals consists of two main steps: sampling and quantization.
- The sampling process is characterized by the sampling rate and the sampling frequency.
- The quantization process is represented by the quantization step and the quantization law.

5. Assignment

❑ Assignment 1

Given the following analog signal:

$$x_a(t) = 3cos100\pi t$$

(a) Determine the minimum sampling frequency to avoid aliasing.

(b) Assuming the signal is sampled with F_s = 200Hz, determine the corresponding discrete signal?

(c) Assuming the signal is sampled with F_s = 75Hz. determine the corresponding discrete signal?

(d) Determine the frequency F_a such that $0 < F_a < F_s/2$ so that the samples will match the result in part (c) ?

5. Assignment

❑ Assignment 2

- Sample the following two signals with a sampling frequency of Fs = 40 Hz

$$x_1(t) = cos2\pi(10)t$$

$$x_2(t) = cos2\pi(50)t$$

- Determine and plot the signals $x_1(n)$ và $x_2(n)$

5. Assignment

❑ Assignment 3. Determine the quantization error of a sine wave signal

- Given the following signal x(n): $x(n) = sin2\pi f_0 n$
- The quantization noise power is approximated as follows:

$$P_q = \frac{1}{N}\sum_{n=0}^{N-1} e^2(n) = \frac{1}{N}\sum_{n=0}^{N-1}\left[x_q(n) - x(n)\right]^2$$

a. For f_0 = 1/50 and N = 200, write a quantization program for the signal x(n) using the rounding quantization method with quantization levels of 64, 128, and 256. For each case, plot the signals x(n), x_q(n), e(n), and calculate the corresponding signal-to-quantization-noise ratio (SQNR)

b. Compare and comment on the SQNR results in part (a) with the theoretical formula.

The next unit 3

DISCRETE SIGNALS

References:

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

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Wishing you all the best in your studies!