HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

LESSON 20

CHARACTERISTICS AND APPLICATIONS OF EACH TYPE OF FIR FILTERS

PhD. Nguyen Hong Quang
Assoc. Prof. Trinh Van Loan
PhD. Doan Phong Tung
Computer Engineering Department

❑ CONTENT

1. Type 1 FIR filter.
2. Type 2 FIR filter.
3. Type 3 FIR filter.
4. Type 4 FIR filter.

❑ Lesson Objectives

After completing this lesson, you will be able to understand the following topics:

- The characteristics of each type of FIR filter.
- Application of each type of FIR filter.

Type 1 FIR Filter

- $h(n)$ symmetric, N odd
- For example:

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$h(0) = h(6), h(1) = h(5), h(2) = h(4)$$

$$H(e^{j\omega})$$

$$= h(0)\left[1 + e^{-6j\omega}\right] + h(1)\left[e^{-j\omega} + e^{-5j\omega}\right] + h(2)\left[e^{-2j\omega} + e^{-4j\omega}\right] + h(3)e^{-3j\omega}$$

$$= e^{-3j\omega}\left\{h(0)\left[e^{3j\omega} + e^{-3j\omega}\right] + h(1)\left[e^{2j\omega} + e^{-2j\omega}\right] + h(2)\left[e^{j\omega} + e^{-j\omega}\right] + h(3)\right\}$$

$$H(e^{j\omega}) = e^{-j3\omega}\{2h(0)\cos(3\omega) + 2h(1)\cos(2\omega) + 2h(2)\cos(\omega) + h(3)\}$$

$$= e^{j\theta(\omega)}[H_R(\omega)]$$

- Group delay equal to constant and equal to $\frac{N-1}{2}$

2. Type 2 FIR Filter

- h(n) symmetric, N even
- For example:

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4}$$
$$+h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7}$$
$$\mathrm{h}(0) = \mathrm{h}(6), \mathrm{h}(1) = \mathrm{h}(6), \mathrm{h}(2) = \mathrm{h}(5), \mathrm{h}(3) = \mathrm{h}(4)$$

$$\mathrm{H}\left(e^{j\omega}\right)$$
$$= \mathrm{h}(0)\left[1 + \mathrm{e}^{-7\mathrm{j}\omega}\right] + \mathrm{h}(1)\left[\mathrm{e}^{-j\omega} + \mathrm{e}^{-5\mathrm{j}\omega}\right] + \mathrm{h}(2)\left[\mathrm{e}^{-2\mathrm{j}\omega} + \mathrm{e}^{-5\mathrm{j}\omega}\right] + \mathrm{h}(3)[e^{-3\mathrm{j}\omega} + \mathrm{e}^{-4\mathrm{j}\omega}]$$

$$\mathrm{H}\left(\mathrm{e}^{-\mathrm{j}\omega}\right)$$
$$= e^{-j3.5\omega}\{2h(0)\cos(3.5\omega) + 2h(1)\cos(2.5\omega) + 2h(2)\cos(1.5\omega) + 2h(3)\cos(0.5\omega)\}$$
$$= \mathrm{e}^{\mathrm{j}\theta(\omega)}[\mathrm{H_R}(\omega)]$$

Characteristics of Type 2 FIR filter

- $h(n)\ symmetric, N\ even$
- Group delay equal to constant and equal to $\frac{\mathrm{N}-1}{2}$
- Because at $\omega = \pi$ then $H(\omega) = 0$: type 2 FIR filter is not suitable for high-pass and band-pass filter design but only for low-pass filter and band-pass filter.

3. Type 3 FIR Filter

- $h(n)$ antisymmetric, N odd

- For example:

$$H(z^{-1}) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$h(0) = -h(6), h(1) = -h(5), h(2) = -h(4), h(3) = 0$$

$$\begin{aligned} H(e^{j\omega}) &= h(0)[1 - e^{-6j\omega}] + h(1)[e^{-j\omega} - e^{-5j\omega}] + h(2)[e^{-2j\omega} - e^{-4j\omega}] \\ &= e^{-3j\omega}\{h(0)[e^{3j\omega} - e^{-3j\omega}] + h(1)[e^{2j\omega} - e^{-2j\omega}] + h(2)[e^{j\omega} - e^{-j\omega}]\} \end{aligned}$$

$$\begin{aligned} H(e^{-j\omega}) &= j.e^{-j3\omega}\{2h(0)\sin(3\omega) + 2h(1)\sin(2\omega) + 2h(2)\sin(\omega)\} \\ &= e^{j\theta(\omega)}[H_R(\omega)] \end{aligned}$$

- Group delay equal to constant and equal to $\frac{N-1}{2}$

- Since at $\omega = 0$ and $\omega = \pi$ then $H(\omega) = 0$, the class 3 FIR filter is suitable for band-pass filter design, not suitable for low-pass, high-pass and band-passing

4. Type 4 FIR Filter

- h(n) antisymmetric, N even
- For example:

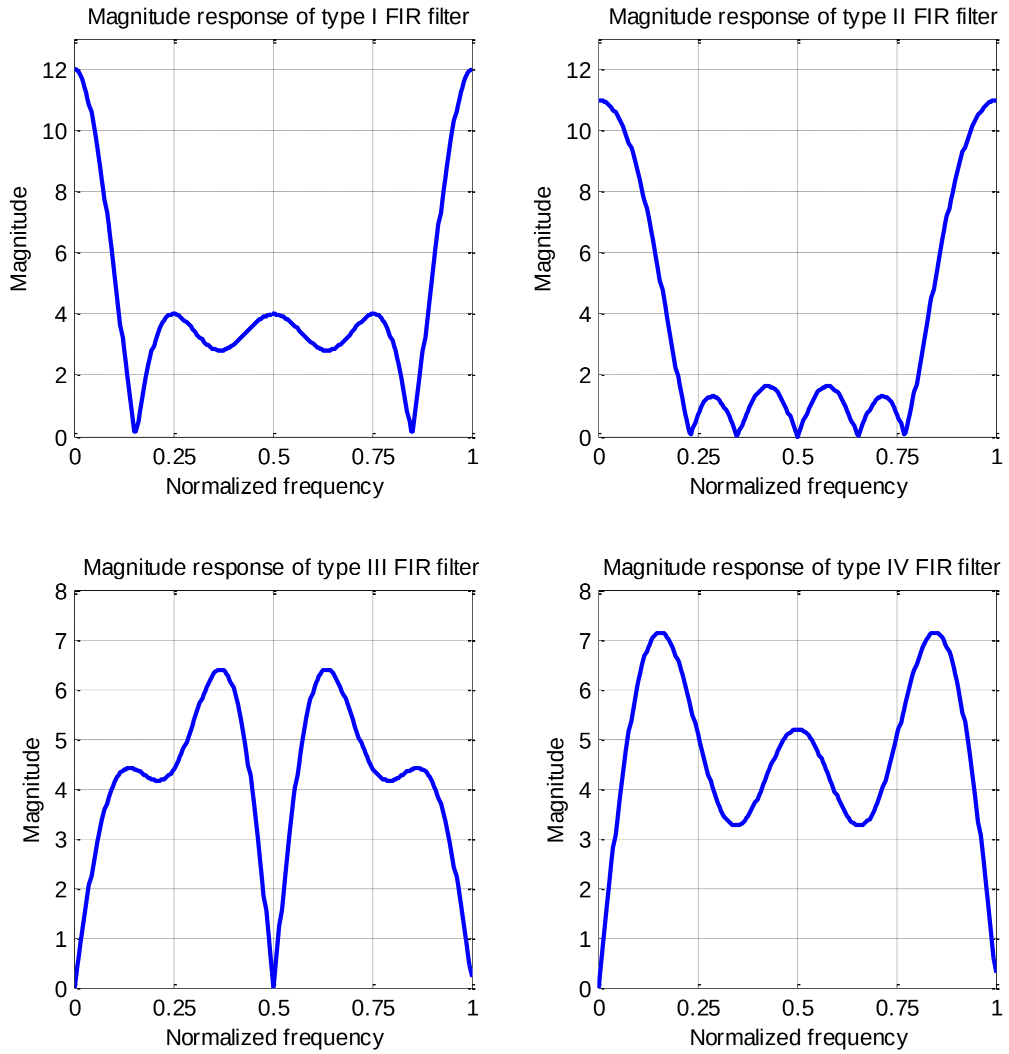
$$H(z^{-1}) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5}$$
$$h(0) = -h(5), h(1) = -h(4), h(2) = -h(3)$$

$$\begin{aligned} H(e^{j\omega}) &= h(0)\left[1 - e^{-5j\omega}\right] + h(1)\left[e^{-j\omega} - e^{-4j\omega}\right] + h(2)\left[e^{-2j\omega} - e^{-3j\omega}\right] \\ &= e^{-2.5j\omega}\left\{h(0)\left[e^{2.5j\omega} - e^{-2.5j\omega}\right] + h(1)\left[e^{1.5j\omega} - e^{-1.5j\omega}\right] + h(2)\left[e^{0.5j\omega} - e^{-0.5j\omega}\right]\right\} \end{aligned}$$

$$\begin{aligned} H(e^{-j\omega}) &= j.e^{-j2.5\omega}\{2h(0)\sin(2.5\omega) + 2h(1)\sin(1.5\omega) + 2h(2)\sin(0.5\omega)\} \\ &= e^{j\theta(\omega)}[H_R(\omega)] \end{aligned}$$

- Group delay equal to constant and equal to $\frac{N-1}{2}$
- Since at $\omega = 0$ then $H(\omega) = 0$, type 4 FIR filters are suitable for high-pass and band-pass filter design, not suitable for low-pass and band-passing.

Example



Magnitude response of type I FIR filter
Magnitude response of type II FIR filter
Magnitude response of type III FIR filter
Magnitude response of type IV FIR filter
Magnitude
Normalized frequency
0
0.25
0.5
0.75
1
0
2
4
6
8
10
12
1
3
5
7

4. Summary

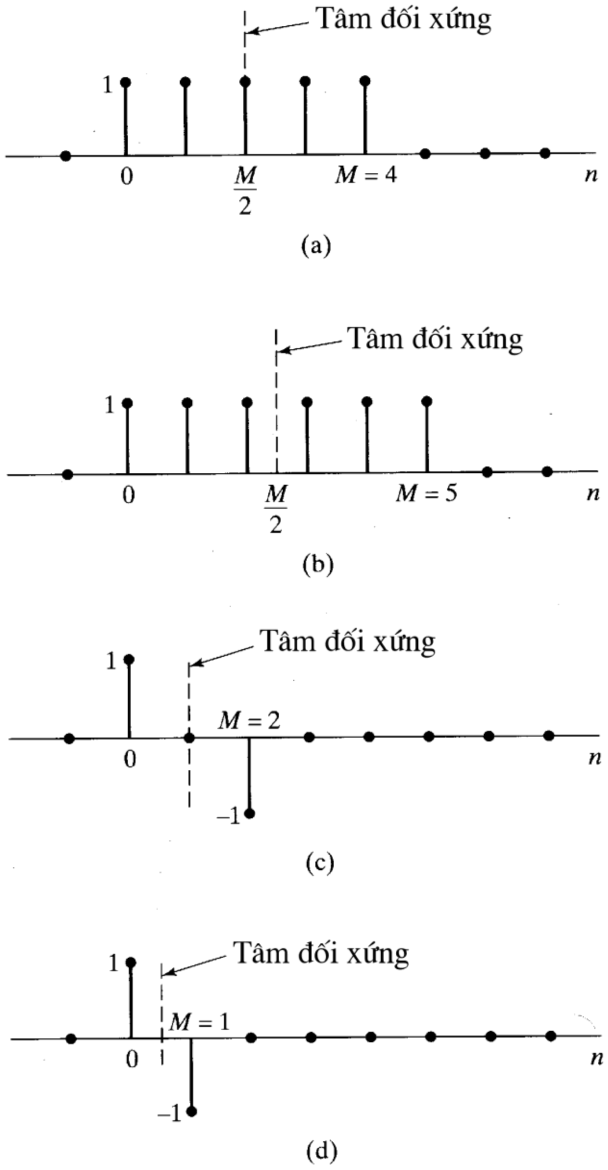
- Type 2 FIR filters are not suitable for high-pass and band-pass filter designs, but only for low-pass and band-pass filters.
- Class 3 FIR filter is suitable for bandpass filter design, not suitable for low pass, high pass and band blocking.
- Class 4 FIR filters are suitable for high-pass and band-pass filter designs, not for low-pass and band-passing.

5. Exercises

- Exercise 1
  - ❑ Show that type 4 FIR filter is suitable for bandpass and high pass, not suitable for low pass and band blocking.

Homework

- Exercise 2
  - ❑ Determine the amplitude response, phase response and comment on the characteristics of each filter type.



Next lesson. Lesson 21

SYNTHESIS OF LINEAR PHASE FIR FILTER USING WINDOW METHOD

References :

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

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Wish you all good study!