**HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY**
**ADVANCED PROGRAMS - PHYSICS HOMEWORK**

# 1. MECHANICS

## 1.5 DYNAMICS OF RIGID BODIES

Name:..... Date of birth:.....

Class:..... Student ID:.....

**Grade table**

| Question: | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
|---|---|---|---|---|---|---|---|---|
| Points: | 12 | 10 | 7 | 10 | 12 | 6 | 10 | 5 |
| Score: | | | | | | | | |

| Question: | **9** | **10** | **11** | **12** | **13** | **14** | **15** | Total |
|---|---|---|---|---|---|---|---|---|
| Points: | 2 | 9 | 4 | 5 | 12 | 9 | 6 | 119 |
| Score: | | | | | | | | |

**1.** **12 p**

(**a**) For a solid cylinder of mass $m$ and radius $R$, show that the moment of inertia $I$ about its central axis is calculated as [6]

$$I = \frac{1}{2}mR^2$$

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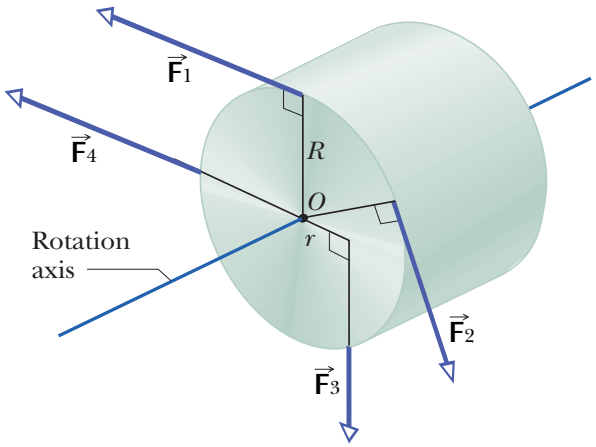


**Fig. 1.1**

(**b**) In **Fig. 1.1**, a cylinder having a mass of 2.0 kg can rotate about its central axis through point O. [6]

Forces are applied as shown: $F_1 = 6.0\,\text{N}$, $F_2 = 4.0\,\text{N}$, $F_3 = 2.0\,\text{N}$, and $F_4 = 5.0\,\text{N}$.

During the rotation, the forces maintain their same angles relative to the cylinder.

Also, $r = 5.0\,\text{cm}$ and $R = 12\,\text{cm}$.

Find the magnitude and direction of the angular acceleration of the cylinder.

**2.** In **Fig. 2.1**, block 1 has mass $m_1 = 460\,\text{g}$, block 2 has mass $m_2 = 500\,\text{g}$, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius $R = 5.00\,\text{cm}$. **10 p**

When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley.

(**a**) What is the magnitude of the acceleration of the blocks? [5]

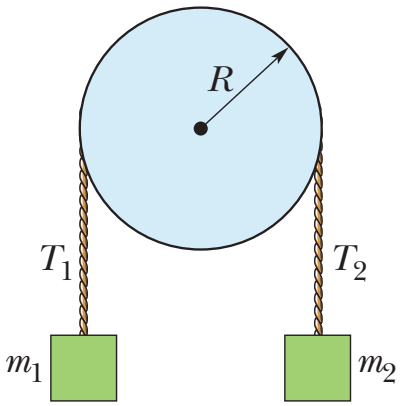


**Fig. 2.1**

(**b**) Calculate the tensions $T_1$ and $T_2$. [2]

(**c**) Determine the rotational inertia of the pulley. [3]

**3.** **Fig. 3.1** shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.00 cm and a mass of 20.0 grams and is initially at rest. [7]

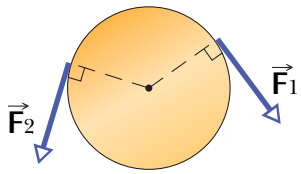


**Fig. 3.1**

Starting at time $t = 0$, two forces are to be applied tangentially to the rim as indicated, so that at time $t = 1.25\,\text{s}$ the disk has an angular velocity of $250\,\text{rad}\,\text{s}^{-1}$ counterclockwise.

The force $\vec{\mathbf{F}}_1$ has a magnitude of 0.100 N. What is magnitude of the force $\vec{\mathbf{F}}_2$? **7 p**

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**4.** **10 p**

(**a**) Prove that the moment of inertia $I$ of a system about a given axis is calculated as [5]

$$I = I_{\text{cm}} + md^2$$

where $I_{\text{cm}}$ is the moment of inertia of the system about the axis which passes through the center of mass of the system, is parallel to and is at distance $d$ from the given axis.

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(**b**) In **Fig. 4.1**, an irregularly shaped plastic plate with uniform thickness and density is to be rotated around an axle that is perpendicular to the plate face and through point **O**. The rotational inertia of the plate about that axle is measured with the following method. [5]

A circular disk of mass 0.500 kg and radius 2.00 cm is glued to the plate, with its center aligned with point **O**, as shown in **Fig. 4.2**. A string is wrapped around the edge of the disk the way a string is wrapped around a top. Then the string is pulled for 5.00 s.

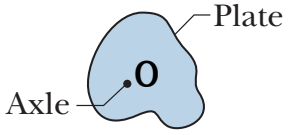


**Fig. 4.1**

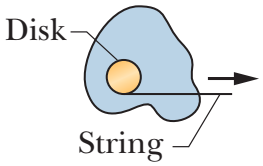


**Fig. 4.2**

As a result, the disk and plate are rotated by a constant force of 0.400 N that is applied by the string tangentially to the edge of the disk. The resulting angular speed is 114 rad s$^{-1}$.

What is the rotational inertia of the plate about the axle?

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**5.** **12 p**

(**a**) Show that the moment of inertia of a uniform sphere of mass $M$ and radius $R$ about an axis passing through its center is [5]

$$I = \frac{2}{5}MR^2$$

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$I = \frac{2}{3}MR^2$

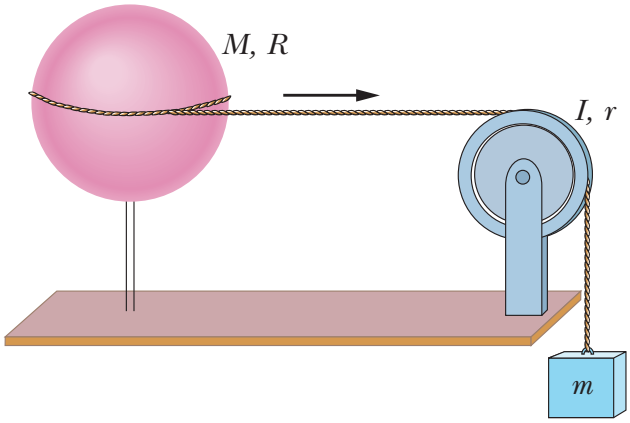


**Fig. 5.1**

(**b**) A uniform spherical shell of mass $M = 4.5\,\text{kg}$ and radius $R = 8.5\,\text{cm}$ can rotate about a vertical axis on frictionless bearings, as shown in **Fig. 5.1**. [7]

A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}\,\text{kg m}^2$ and radius $r = 5.0\,\text{cm}$, and is attached to a small object of mass $m = 0.60\,\text{kg}$. There is no friction on the pulley's axle; the cord does not slip on the pulley.

What is the speed of the object when it has fallen 82 cm after being released from rest?

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$v = 1.4(2)\ \text{m s}^{-1}$

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**6.** A pulley, with a rotational inertia of $1.0 \times 10^{-3}\,\mathrm{kg\,m^2}$ about its axle and a radius of 10 cm, is acted on by a force applied tangentially at its rim. **6 p**

The force magnitude varies in time as $F = 0.50t + 0.30t^2$, with $F$ in newtons and $t$ in seconds.

The pulley is initially at rest. Determine, at $t = 3.0\,\mathrm{s}$,

(**a**) its angular acceleration, [3]

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(**b**) its angular speed. [3]

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**7.** **10 p**

(**a**) Derive the formula for the moment of inertia of a thin rod of length $l$ and mass $m$ about a perpendicular axis passing through one of its ends. [4]

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(**b**) A thin rod of length 0.75 m and mass 0.42 kg is suspended freely from one end. The rod is pulled to one side and then allowed to swing like a pendulum, passing through its lowest position with angular speed $4.0\,\mathrm{rad\,s^{-1}}$. Neglecting friction and air resistance.

**i**. Find the rod's kinetic energy at its lowest position, [3]

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**ii**. How far above its lowest position does the rod's center of mass rise? [3]

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**8.** The bolts on the cylinder head of an engine require tightening to a torque of 95 N m. **5 p**

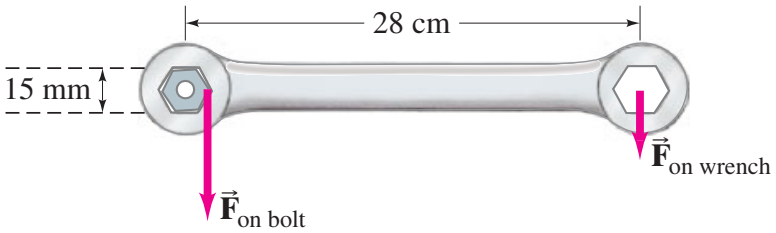


**Fig. 8.1**

(**a**) If a wrench is 28 cm long, as shown in **Fig. 8.1**, what force perpendicular to the wrench must the mechanic exert at its end? [2]

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(**b**) If the six-sided bolt head is 15 mm across, estimate the force applied near each of the six points by a wrench. [3]

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**9.** A uniform horizontal rod of mass $M$ and length $l$ rotates with angular velocity $\omega$ about a vertical axis through its center. Attached to each end of the rod is a small mass $m$. [2]

Determine the angular momentum of the system about the axis.

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**10.** An *Atwood machine* consists of two masses, $m_A = 65\,\text{kg}$ and $m_B = 75\,\text{kg}$, connected by a massless inelastic cord that passes over a pulley free to rotate, as shown in **Fig. 10.1**. **9 p**

The pulley is a solid cylinder of radius $R = 0.45\,\text{m}$ and mass $m = 6.0\,\text{kg}$.

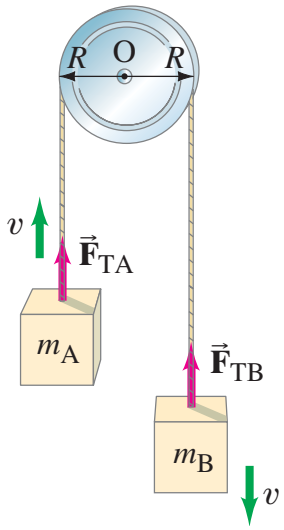


**Fig. 10.1**

(**a**) Determine the acceleration of each mass. [5]

(**b**) What percentage error would be made if the moment of inertia of the pulley is ignored? [4]

**11.** A uniform disk turns at $3.3\,\text{rev}\,\text{s}^{-1}$ around a frictionless central axis. A nonrotating rod, of the same mass as the disk and length equal to the disk's diameter, is dropped onto the freely spinning disk, as shown in **Fig. 11.1**. They then turn together around the axis with their centers superposed. [4]

What is the angular frequency in $\text{rev}\,\text{s}^{-1}$ of the combination?

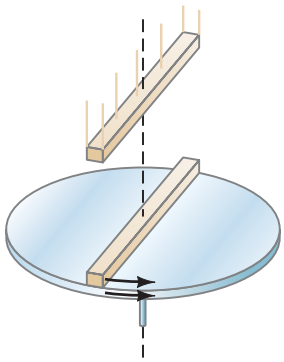

**Fig. 11.1**

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**12.** Let us treat a helicopter rotor blade as a long thin rod, as shown in **Fig. 12.1**. **5 p**

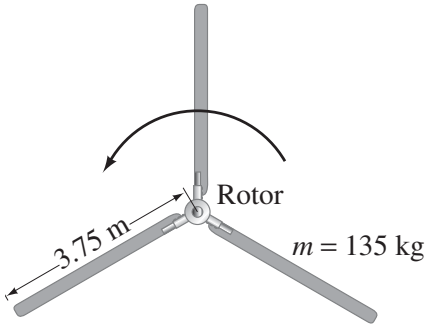


**Fig. 12.1**

(**a**) If each of the three rotor helicopter blades is 3.75 m long and has a mass of 135 kg, calculate the moment of inertia of the three rotor blades about the axis of rotation. [2]

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(**b**) How much torque must the motor apply to bring the blades from rest up to a speed of $6.0\,\text{rev}\,\text{s}^{-1}$ in 8.0 s? [3]

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**13.** An asteroid of mass $1.0 \times 10^5\,\text{kg}$, traveling at a speed of $35\,\text{km}\,\text{s}^{-1}$ relative to the Earth, hits the Earth at the equator tangentially, in the direction of Earth's rotation, and is embedded there. The Earth's radius is $R = 6400\,\text{km}$ and the Earth's mass is $M = 6.0 \times 10^{24}\,\text{kg}$. [12]

Estimate the percent change in the angular speed of the Earth as a result of the collision.

**14.** Two blocks are connected by a light string passing over a pulley of radius 0.15 m and moment of inertia $I$. The blocks move (towards the right) with an acceleration of 1.00 m s$^{-2}$ along their frictionless inclines, as shown in **Fig. 14.1**. **9 p**

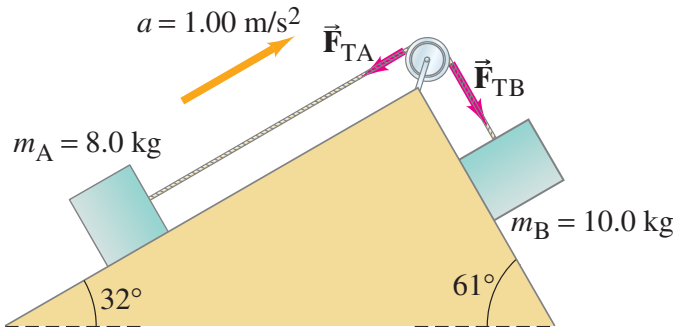


**Fig. 14.1**

(**a**) Determine the tensions $F_{TA}$ and $F_{TB}$ in the two parts of the string. [4]

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(**b**) Find the net torque acting on the pulley, and determine its moment of inertia, $I$. [5]

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**15.** A person stands, hands at his side, on a platform that is rotating at a rate of $0.90\,\text{rev}\,\text{s}^{-1}$. **6 p**

If he raises his arms to a horizontal position, **Fig. 15.1**, the speed of rotation decreases to $0.60\,\text{rev}\,\text{s}^{-1}$.

**Fig. 15.1**

(**a**) Why? [3]

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(**b**) By what factor has his moment of inertia changed? [3]

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