Lesson 3 -

Kinetics

*"The study of motion and of physical concepts such as force and mass is called **dynamics.** The part of dynamics that describes motion without regard to its causes is called **kinematics.**"*

Lecturer: **Le Tuan**, Ph. D.
Institute of Engineering Physics
Hanoi University of Science and Technology

Coordinate Systems

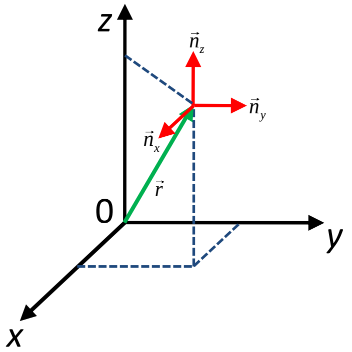
A coordinate system is a reference system consisting of a set of points, lines, and/or surfaces, and a set of rules, used to define the positions of points in space in either two or three dimensions.

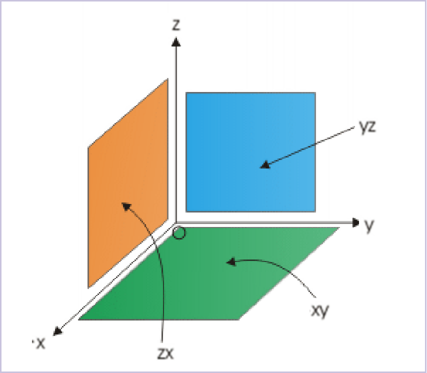
- There are many basic coordinate systems familiar to students of geometry and trigonometry.
- These systems can represent points in two-dimensional or three-dimensional space.
- René Descartes (1596-1650) introduced systems of coordinates based on orthogonal (right angle) coordinates.
- These two and three-dimensional systems used in analytic geometry are often referred to as Cartesian systems.
- When at least one coordinate is angle, the coordinate system are referred as polar. In physics, the two following polar coordinate systems are frequently used: cylindrical and spherical.

A *frame of reference* is a choice of coordinate axes that defines the starting point for measuring any quantity, an essential first step in solving virtually any problem in mechanics.

Rectangular (Cartesian) coordinate system

In Decartes rectangular coordinate systems there is a right-hand set of three perpendicular axes each to other (*x*, *y*, *z*). Each of coordinates can vary from - ∞ to + ∞.





$$\vec{r} = x \cdot \vec{n}_x + y \cdot \vec{n}_y + z \cdot \vec{n}_z$$

$\vec{n}_x, \quad \vec{n}_y, \quad \vec{n}_z$ - unit vector along x-, y-, and z-axis

Distance between two points in rectangle coordinate system

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

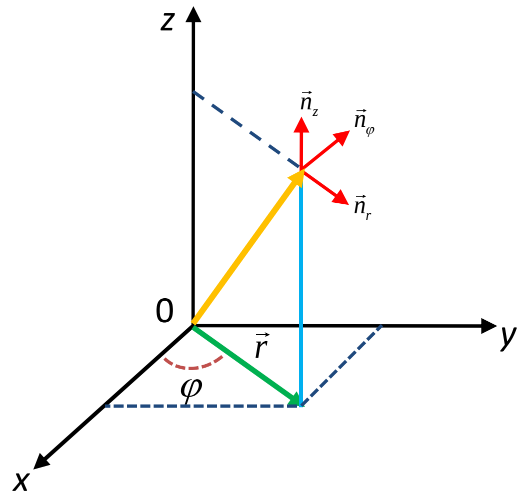
Derivatives

$$\nabla = \frac{\partial}{\partial x}\vec{n}_x + \frac{\partial}{\partial y}\vec{n}_y + \frac{\partial}{\partial z}\vec{n}_z$$

$$\nabla^2 = \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Cylindrical coordinate system

In cylindrical coordinate system the location of a point in three dimensional space may be specified by an ordered set of numbers (r, φ, z).



• The ranges for the coordinate parameters:

$$0 \le r \le \infty \;\; ; \quad 0 \le \varphi \le 2\pi \;\; ; \quad -\infty \le z \le +\infty$$

• The relationship between rectangular and cylindrical coordinates

$$x = r\cos\varphi \qquad r = +\sqrt{x^2 + y^2}$$

$$y = r\sin\varphi \qquad \theta = \arctan\left(y/x\right)$$

$$z = z$$

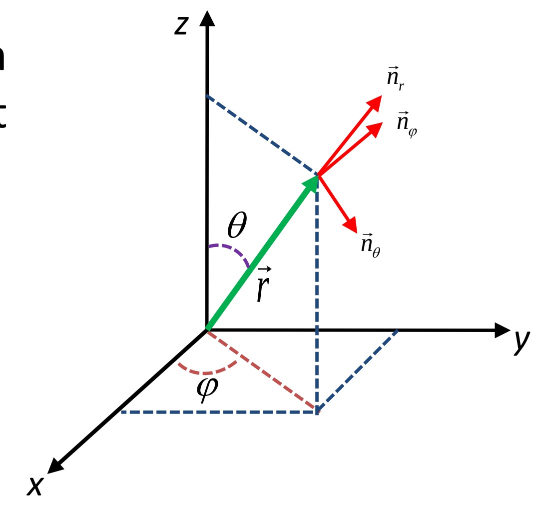
Derivatives

$$\frac{\partial}{\partial\theta}\vec{n}_r = \vec{n}_\theta \qquad \frac{\partial}{\partial\theta}\vec{n}_\theta = -\vec{n}_r$$

$$\nabla = \frac{\partial}{\partial r}\vec{n}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\vec{n}_\theta + \frac{\partial}{\partial z}\vec{n}_z$$

$$\nabla^2 = \Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Spherical coordinate system



In spherical coordinate system the location of a point in three dimensional space may be specified by an ordered set of numbers (r, φ, z).

- The ranges for the coordinate parameters:

$$0 \le r \le \infty \quad ; \quad 0 \le \theta \le \pi \quad ; \quad 0 \le \varphi \le 2\pi$$

- The relationship between rectangular and spherical coordinates

$$x = r\sin\theta\cos\varphi \qquad r = +\sqrt{x^2+y^2+z^2}$$

$$y = r\sin\theta\sin\varphi \qquad \theta = \arctan\frac{\sqrt{x^2+y^2}}{z}$$

$$z = r\cos\theta \qquad \varphi = \arctan\left(y/x\right)$$

Derivatives $$\frac{\partial}{\partial\varphi}\vec{n}_r = \vec{n}_\varphi\sin\theta \qquad \frac{\partial}{\partial\varphi} = -\vec{n}_r\sin\theta - \vec{n}_\theta\cos\theta$$

$$\nabla = \frac{\partial}{\partial r}\vec{n}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\vec{n}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\vec{n}_\varphi$$

$$\nabla^2 = \Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

Space and Time

Isaac Newton shared the popular belief that both **absolute time** and **absolute space** exist: the *grid* that defined absolute space was *undetectable*, but that there is a *universal time* that ticks away, and that universal time, which can be accurately measured by *clocks*.

• ***Absoluteness*** of Newtonian **space** exhibits in the ***invariance*** of the *distance* between two Euclidian space's fixed points in any coordinate system, for instance, in some Cartesian system:

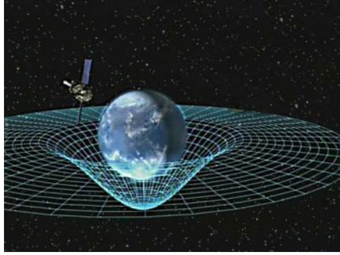
$$l_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \mathrm{const}$$

+ **Distance** may be either ***absolute*** (in relation to a frame of reference, or obtained directly) or ***relative*** (when determined through other objects, distances, *etc.*)
+ **Measuring** position of a point is *dependent* from the choice of a frame of reference, but the frame of reference, in its turn, is **arbitrary**. Hence, the coordinate of a point in the space is ***relative***. The length unit is **unchanged** for whole space.
+ We believe, therefore, that a *classical* ***motion*** *is* ***relative***.

• **Time** (classical) is ***absolute***, because it:

+ ticks *away* from the "absolute" beginning t = 0. In other words, time must be $t > 0$,
+ ticks *by the same manner* for all observers in all locations,
+ ticks *by the same manner* in the past, at the present and in the future.

Space and Time (cont.)



Albert Einstein though differently. He supposed (1905 – 1915) that both **space** and **time** are ***relative***. Now, the *invariant* is (when c – light speed)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 + c^2 (t_2 - t_1)^2} = \text{const}$$

Space and Time are all part of a single physical entity, the ***SpaceTime Continuum***. SpaceTime has four dimensions, and while we are driving in any direction, we are always driving from our past to our future – that is basically space and time are linked.

Frames of reference may be thought of as invisible "coordinate map grids“, attached to every observer so that the observer can measure the position of surrounding objects.

Space-time is essentially a "curved" geometric construct that allows for *the relativity of simultaneity*: if one observer correctly concludes that two events occur simultaneously, the same events would appear to take place at different times to an observer who was in motion relative to the first observer.

Relativity tells us that *time and distance change depending on the relative motion of the observers*: the time and distance measured by two observers in relative motion to each other is different, only the speed of light measured by all observers is the same.

Special relativity allows us to *define a distance from the origin for all the points* in the set of history of observer in SpaceTime, each of which *has physically distinguishable properties*.

(In this course, we are mainly working with the Classical Space and Time !!!)

Motion – Velocity and Acceleration

Motion of a body is an apparent change of its position with the time.

Deplacement Δs is a piece of motion in a some time interval. Generally $\Delta s \neq \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

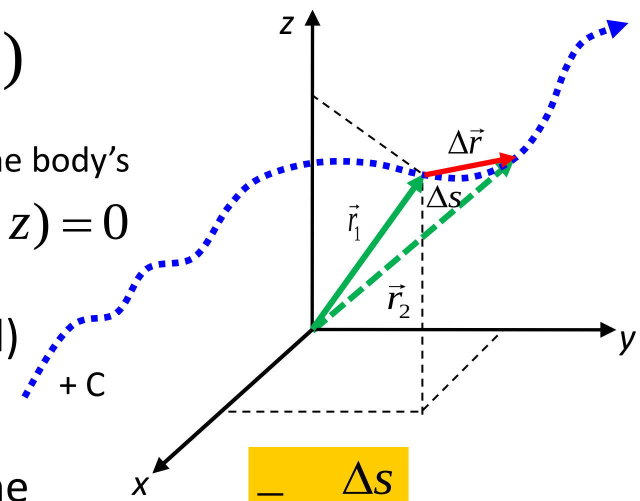
For simplicity, the object under study firstly is a body with infinitesimally small dimension, or ***material point***. Usually, a coordinate system is assigned to the frame of reference. The vector starting from the frame of reference to the actual position of the material point is a ***radius-vector***.

Trajectory is set of the end positions of the radius-vector for an object moving with the time evolution, *i.e.*, the directed set of object's positions in its whole motion.

Motion equation is expression describing the evolution of body position in space with time

$$\vec{r} = \vec{r}(t)$$

If position is known for time moment t_0 it is always possible to determine body's position at any other time moment t.



$$f(x, y, z) = 0$$

Trajectory equation describe the relation between object's coordinates in motion (time variable is excluded)

It gives an image about the form of object's trajectory.

Average velocity is the ratio of displacement to the time interval. Average value is NOT a vector!!!

$$\overline{v} = \frac{\Delta s}{\Delta t}$$

Instantaneous velocity is the limit of the average velocity as the time interval becomes infinitesimally small.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} \approx \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

Acceleration is a physical quantity representing how the velocity changes in the body's movement.

Average acceleration is a scalar value for the mean variation of velocity over a time interval.

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration by value is the limit of the average acceleration as the time interval becomes infinitesimally small.

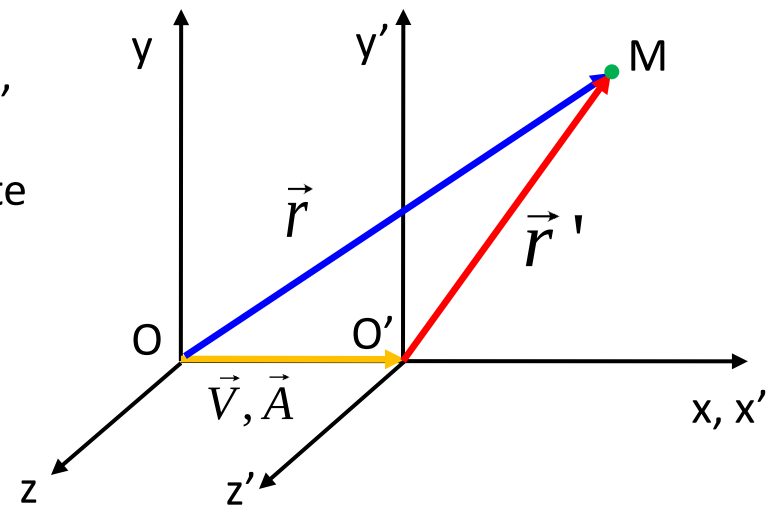
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}}$$

Relative velocity and acceleration

Measurements of velocity, and then, acceleration, depend on the **reference frame** of the observer. Reference frames are assigned with the coordinate systems.

A **stationary frame of reference** **O** usually related to the Earth. We assume the body's velocity and acceleration are $\vec{v}$ and $\vec{a}$, respectively.

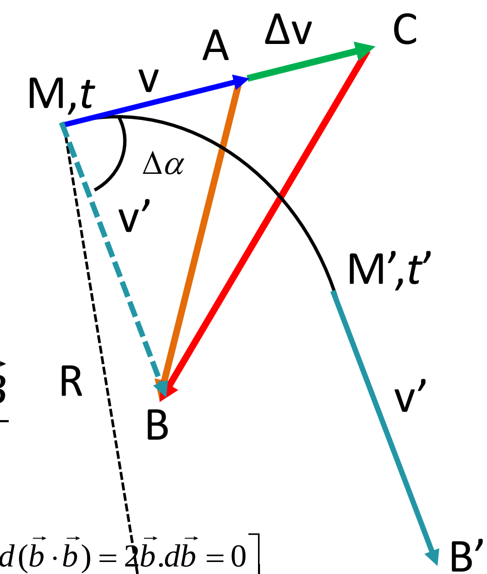
If the **moving reference frame** **O'** initially in the same place with **O**, moves with velocity $\vec{V}$ and acceleration $\vec{A}$ related to **O**, then relative body's velocity and acceleration related to **O'** are:



$$\boxed{\begin{aligned} \vec{v}\,' &= \vec{v} - \vec{V} \\ \vec{a}\,' &= \vec{a} - \vec{A} \end{aligned}}$$

Normal and tangential accelerations

Assuming a body in his non-linear motion has made a displacement MM' over time interval ($t' - t$). At the places M and M', the body's velocities are respectively $\overrightarrow{MA} = \vec{v}$ and $\overrightarrow{M'B'} = \vec{v}\,'$. *What is the body acceleration*?



By definition

$$\vec{a} = \lim_{(t'-t)\to 0} \frac{\vec{v}\,' - \vec{v}}{(t'-t)} = \lim_{\Delta t\to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t\to 0} \frac{\overrightarrow{M'B'} - \overrightarrow{MA}}{\Delta t} = \lim_{\Delta t\to 0} \frac{\overrightarrow{MB} - \overrightarrow{MA}}{\Delta t} = \lim_{\Delta t\to 0} \frac{\overrightarrow{AB}}{\Delta t}$$

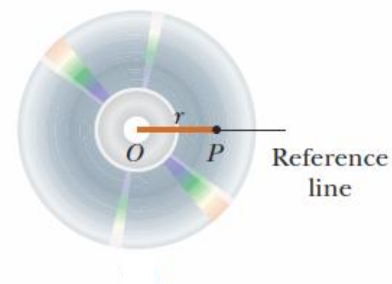
$$\Rightarrow \quad \vec{a} = \lim_{\Delta t\to 0} \frac{\overrightarrow{AB}}{\Delta t} = \lim_{\Delta t\to 0} \frac{\overrightarrow{AC}}{\Delta t} + \lim_{\Delta t\to 0} \frac{\overrightarrow{CB}}{\Delta t} = \vec{a}_t + \vec{a}_n$$

$$\left[\begin{array}{c} \text{If } \vec{b} = \text{const, then } d(\vec{b}\cdot\vec{b}) = 2\vec{b}.d\vec{b} = 0 \\ \text{and} \quad d\vec{b} \perp \vec{b} \end{array}\right]$$

Tangential acceleration	Normal acceleration
- Based at M - In the velocity direction - Value is $\lvert\vec{a}_t\rvert = a_t = \lim_{\Delta t\to 0} \frac{v' - v}{\Delta t} = \lim_{\Delta t\to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \dot{v}$	- Based at M - Perpendicular to the trajectory and directed to the center of curvature. - Value is (where R – radius of curvature) $\lvert\vec{a}_n\rvert = a_n = \lim_{\Delta t\to 0} \frac{CB}{\Delta t} = \lim_{\Delta t\to 0} \frac{v.\Delta\alpha}{\Delta t} = v\frac{d\alpha}{dt} = v\frac{ds}{Rdt} = \frac{v^2}{R}$
Expresses velocity change in **value**	Exhibits velocity change in **direction**

Angular velocity and angular acceleration

In linear motion, the important concepts are *displacement* $\Delta\vec{r}$ [m], *velocity* $\vec{v}$ [m/s], and *acceleration* $\vec{a}$ [m/s^2]. *Each of these concepts has its analog in rotational motion: angular displacement* $\Delta\vec{\varphi}$ [rad], *angular velocity* $\vec{\omega}$ [rad/s], *and angular acceleration* $\vec{\beta}$ [rad/s^2].

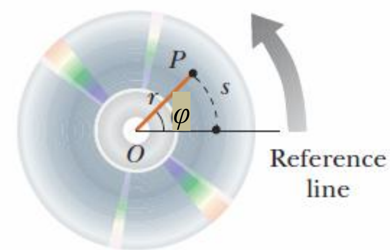


The ***average angular velocity*** of a rotating rigid object during the time interval Δt *is defined as the angular displacement* $\Delta\varphi$ *divided by* Δt*:*

$$\bar{\omega} = \frac{\varphi_2 - \varphi_1}{t_2 - t_1} = \frac{\Delta\varphi}{\Delta t}$$

Similarly, the ***average angular acceleration*** is:

$$\bar{\beta} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$



The ***instantaneous angular velocity*** (a vector value) of a rotating rigid object is defined as the limit of the average velocity *as the time interval* Δt *approaches zero:*

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\varphi}}{\Delta t} = \frac{d\vec{\varphi}}{dt} = \dot{\vec{\varphi}}$$

And, the ***instantaneous angular acceleration*** (also a vector) is defined as:

$$\vec{\beta} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt} = \dot{\vec{\omega}} = \frac{d^2\vec{\varphi}}{dt^2} = \ddot{\vec{\varphi}}$$

For a rotating rigid body with the radius ***R*** of circulation, relations

- between linear and angular velocities: $$\vec{v} = \vec{\omega} \wedge \vec{R}$$
- between tangential and angular accelerations: $$\vec{a}_t = \vec{\beta} \wedge \vec{R}$$

Examples of 1-D, 2-D motions

I. Linear motion under constant acceleration and rotational motion under constant angular acceleration

Linear motion	Rotational motion
$v(t) = v_0 + a\,t$	$\omega(t) = \omega_0 + \beta\,t$
$s(t) = v_0\,t + ½\,a\,t^2$	$\varphi(t) = \omega_0\,t + ½\,\beta\,t^2$
$v^2 - v_0^2 = 2\,a\,s$	$\omega^2 - \omega_0^2 = 2\,\beta\,\varphi$

II. Projectile motion

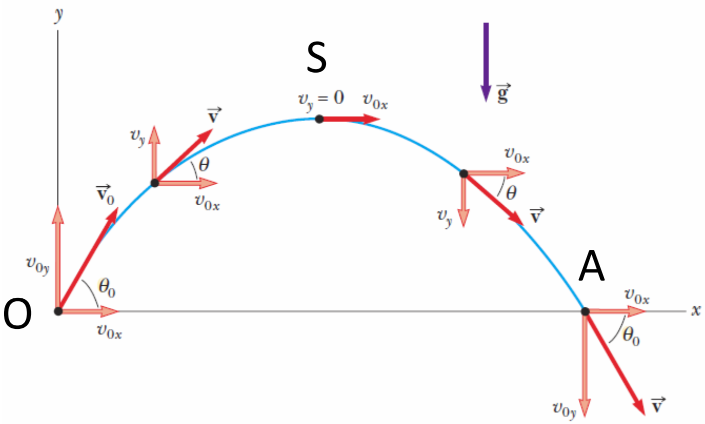
The most important experimental fact about projectile motion in two dimensions is that **the horizontal and vertical motions are completely independent of each other.**

$$\begin{cases} x = v_0 \cos\theta . t \\ y = -\dfrac{1}{2} g t^2 + v_0 \sin\theta . t \end{cases}$$

$$y = -\frac{1}{2}\frac{g}{v_0^2 \cos^2\theta} x^2 + \tan\theta . x$$

$$A \begin{cases} x_A = \dfrac{v_0^2 \sin 2\theta}{g} \\ y_A = 0 \end{cases}$$

$$S \begin{cases} x_s = \dfrac{v_0^2 \sin 2\theta}{2g} \\ y_s = \dfrac{v_0^2 \sin^2\theta}{2g} \\ t_s = \dfrac{v_0 \sin\theta}{g} \end{cases}$$



Thank you for attention!

Any suggestion or comment, please, send to:

***Le Tuan**, Ph.D.*
Department of Electronic Materials
Institute of Engineering Physics
Hanoi University of Sciences and Technology
Room 211, Building C9, № 1, Dai Co Viet Str., Hanoi, Vietnam
E-mail: letuan_2002@yahoo.com; letuan-iep@mail.hut.edu.vn
Tel.: +84(04)-3869 2223 ; Fax: +84(04)-3869 3498