Chapter 1: VECTORS AND THE GEOMETRY OF SPACE

Reference: James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition.* Thomson, Brooks/Cole Publishing Company.

Three-dimensional coordinate systems

1. Find the lengths of the sides of the triangle PQR. Is it a right triangle? Is it an isosceles triangle?

a) $P(3;-2;-3)$, $Q(7;0;1)$, $R(1;2;1)$. b) $P(2;-1;0)$, $Q(4;1;1)$, $R(4;-5;4)$.

2. Find an equation of the sphere with center $(1;-4;3)$ and radius 5. Describe its intersection with each of the coordinate planes.

3. Find an equation of the sphere that passes through the origin and whose center is $(1;2;3)$.

4. Find an equation of a sphere if one of its diameters has end points $(2;1;4)$ and $(4;3;10)$.

5. Find an equation of the largest sphere with center $(5,4,9)$ that is contained in the first octant.

6. Write inequalities to describe the following regions

a) The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$.

b) The solid upper hemisphere of the sphere of radius 2 centered at the origin.

7. Consider the points P such that the distance from P to $A(-1;5;3)$ is twice the distance from P to $B(6;2;-2)$. Show that the set of all such points is a sphere, and find its center and radius.

8. Find an equation of the set of all points equidistant from the points $A(-1;5;3)$ and $B(6;2;-2)$. Describe the set.

Vectors

9. Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2;4)$.

10. Find the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/6;1)$.

11. Find the unit vectors that are perpendicular to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/6; 1)$.

12. Let C be the point on the line segment AB that is twice as far from B as it is from A. If $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$, show that $\mathbf{c} = \mathbf{\frac{2}{3}a} + \mathbf{\frac{1}{3}b}$.

The dot product

13. Determine whether the given vectors are orthogonal, parallel, or neither

a) $a = (-5; 3; 7), \quad b = (6; -8; 2)$

b) $a = (4; 6), \quad b = (-3; 2)$

c) $a = -i + 2j + 5k, \quad b = 3i + 4j - k$

d) $u = (a, b, c), \quad v = (-b; a; 0)$

14. For what values of b are the vectors $(-6; b; 2)$ and $(b; b^2; b)$ orthogonal?

15. Find two unit vectors that make an angle of 60^o with $v = (3; 4)$.

16. If a vector has direction angles $\alpha = \pi/4$ and $\beta = \pi/3$, find the third direction angle γ.

17. Find the angle between a diagonal of a cube and one of its edges.

18. Find the angle between a diagonal of a cube and a diagonal of one of its faces.

The cross product

19. Find the area of the parallelogram with vertices $A(-2; 1)$, $B(0; 4)$, $C(4; 2)$, and $D(2; -1)$.

20. Find the area of the parallelogram with vertices $K(1; 2; 3)$, $L(1; 3; 6)$, $M(3; 8; 6)$ and $N(3; 7; 3)$.

21. Find the volume of the parallelepiped determined by the vectors a, b, and c.

a) $a = (6; 3; -1), \quad b = (0; 1; 2), \quad c = (4; -2; 5)$.

b) $a = i + j - k, \quad b = i - j + k, \quad c = -i + j + k$.

22. Let $v = 5j$ and let u be a vector with length 3 that starts at the origin and rotates in the xy-plane. Find the maximum and minimum values of the length of the vector $u \times v$. In what direction does $u \times v$ point?

Equations of lines and planes

23. Determine whether each statement is true or false.

a) Two lines parallel to a third line are parallel.

b) Two lines perpendicular to a third line are parallel.

c) Two planes parallel to a third plane are parallel.

d) Two planes perpendicular to a third plane are parallel.

e) Two lines parallel to a plane are parallel.

f) Two lines perpendicular to a plane are parallel.

g) Two planes parallel to a line are parallel.

h) Two planes perpendicular to a line are parallel.

i) Two planes either intersect or are parallel.

j) Two lines either intersect or are parallel.

k) A plane and a line either intersect or are parallel.

24. Find a vector equation and parametric equations for the line.

a) The line through the point $(6; -5; 2)$ and parallel to the vector $(1; 3; -2/3)$.

b) The line through the point $(0; 14; -10)$ and parallel to the line $x = -1+2t; y = 6-3t; z = 3+9t$.

c) The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$.

25. Find parametric equations and symmetric equations for the line of intersection of the plane $x + y + z = 1$ and $x + z = 0$.

26. Find a vector equation for the line segment from $(2; -1; 4)$ to $(4; 6; 1)$.

27. Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

a) $L_1 : x = -6t, y = 1 + 9t, z = -3t;\quad L_2 : x = 1 + 2s, y = 4 - 3s, z = s.$

b) $L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3};\quad L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}.$

28. Find an equation of the plane.

a) The plane through the point $(6; 3; 2)$ and perpendicular to the vector $(-2; 1; 5)$

b) The plane through the point $(-2; 8; 10)$ and perpendicular to the line $x = 1+t, y = 2t, z = 4-3t$.

c) The plane that contains the line $x = 3 + 2t, y = t, z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$.

29. Find the cosine of the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.

30. Find parametric equations for the line through the point $(0; 1; 2)$ that is perpendicular to the line $x = 1 + t, y = 1 - t, z = 2t$, and intersects this line.

31. Find the distance between the skew lines with parametric equations $x = 1+t, y = 1+6t, z = 2t$ and $x = 1 + 2s, y = 5 + 15s, z = -2 + 6s$.

Quadric surfaces

32. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the y-axis.

33. Find an equation for the surface consisting of all points that are equidistant from the point $(-1; 0; 0)$ and the plane $x = 1$. Identify the surface.

Chapter 2: VECTOR FUNCTIONS

Reference: James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition.* Thomson, Brooks/Cole Publishing Company.

Vector functions

34. Find the domain of the vector function.

a) $r(t) = (\sqrt{4-t^2}, e^{-3t}, \ln(t+1))$

b) $r(t) = \frac{t-2}{t+2}i + \sin tj + \ln(9-t^2)k$

35. Find the limit

a) $\lim\limits_{t\to 0}(\frac{e^t-1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{t+1})$

b) $\lim\limits_{t\to\infty}(\arctan t, e^{-2t}, \frac{\ln t}{t+1})$

36. Find a vector function that represents the curve of intersection of the two surfaces.

a) The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

b) The paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

37. Suppose u and v are vector functions that possess limits as $t \to a$ and let c be a constant. Prove the following properties of limits.

a) $\lim\limits_{t\to a}[u(t) + v(t)] = \lim\limits_{t\to a} u(t) + \lim\limits_{t\to a} v(t)$

b) $\lim\limits_{t\to a} cu(t) = c\lim\limits_{t\to a} u(t)$

c) $\lim\limits_{t\to a}[u(t).v(t)] = \lim\limits_{t\to a} u(t). \lim\limits_{t\to a} v(t)$

d) $\lim\limits_{t\to a}[u(t) \times v(t)] = \lim\limits_{t\to a} u(t) \times \lim\limits_{t\to a} v(t)$

38. Find the derivative of the vector function.

a) $r(t) = (t\sin t, t^3, t\cos 2t)$.

b) $r(t) = \arcsin ti + \sqrt{1-t^2}j + k$

c) $r(t) = e^{t^2}i - \sin^2 tj + \ln(1+3t)$

39. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. Illustrate by graphing both the curve and the tangent line on a common screen.

a) $x = t, y = e^{-t}, z = 2t - t^2; (0; 1; 0)$

b) $x = 2\cos t, y = 2\sin t, z = 4\cos 2t; (\sqrt{3}, 1, 2)$

c) $x = t\cos t, y = t, z = t\sin t; (-\pi, \pi, 0)$

40. Find the point of intersection of the tangent lines to the curve $r(t) = (\sin \pi t, 2\sin \pi t, \cos \pi t)$ at the points where $t = 0$ and $t = 0.5$

41. Evaluate the integral

a) $\int_0^{\pi/2}(3\sin^2 t \cos t\, i + 3\sin t \cos^2 t\, j + 2\sin t \cos t\, k)dt$

b) $\int_1^2(t^2\, i + t\sqrt{t-1}\, j + t\sin \pi t\, k)dt$

c) $\int(e^t\, i + 2t\, j + \ln t\, k)dt$

d) $\int(\cos \pi t\, i + \sin \pi t\, j + t^2\, k)dt$

42. If a curve has the property that the position vector $r(t)$ is always perpendicular to the tangent vector $r'(t)$, show that the curve lies on a sphere with center the origin.

Arc length and curvature

43. Find the length of the curve.

a) $r(t) = (2\sin t, 5t, 2\cos t), \quad -10 \le t \le 10$

b) $r(t) = (2t, t^2, \frac{1}{3}t^3), \quad 0 \le t \le 1$

c) $r(t) = \cos t\, i + \sin t\, j + \ln \cos t\, k, \quad 0 \le t \le \pi/4$

44. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(6; 18; 36)$.

45. Suppose you start at the point $(0; 0; 3)$ and move 5 units along the curve $x = 3\sin t, y = 4t, z = 3\cos t$ in the positive direction. Where are you now?

46. Reparametrize the curve

$$r(t) = \left(\frac{2}{t^2+1} - 1\right) i + \frac{2t}{t^2+1} j$$

with respect to arc length measured from the point $(1; 0)$ in the direction of increasing . Express the reparametrization in its simplest form. What can you conclude about the curve?

47. Find the curvature

a) $r(t) = t^2\, i + t\, k$

b) $r(t) = t\, i + t\, j + (1 + t^2)\, k$

c) $r(t) = 3t\, i + 4\sin t\, j + 4\cos t\, k$

d) $x = e^t \cos t, y = e^t \sin t$

e) $x = t^3 + 1, y = t^2 + 1$

48. Find the curvature of $r(r) = (e^t \cos t, e^t \sin t, t)$ at the point (1, 0, 0).

49. Find the curvature of $r(r) = (t, t^2, t^3)$ at the point (1, 1, 1).

50. Find the curvature

a) $y = 2x - x^2$,

b) $y = \cos x$,

c) $y = 4x^{5/2}$.

51. At what point does the curve have maximum curvature? What happens to the curvature as $x \to \infty$?

a) $y = \ln x$,

b) $y = e^x$.

52. Find an equation of a parabola that has curvature 4 at the origin.

Chapter 3: DOUBLE INTEGRALS

Reference: James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition.* Thomson, Brooks/Cole Publishing Company.

Double integrals

53. Calculate the iterated integral

a) $\displaystyle\int_1^3\int_0^1 (1+4xy)dxdy$ b) $\displaystyle\int_0^2\int_0^1 (2x+y)^8 dxdy$ c) $\displaystyle\int_0^1\int_1^2 \frac{xe^x}{y}dydx$

d) $\displaystyle\int_1^4\int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right)dxdy$ e) $\displaystyle\int_0^1\int_0^1 xy\sqrt{x^2+y^2}dxdy$ f) $\displaystyle\int_0^2\int_0^\pi r\sin^2\varphi d\varphi dr.$

54. Calculate the double integral

a) $\iint_D \frac{1+x^2}{1+y^2}dxdy, \quad D=\{(x,y)|0\le x\le 1, 0\le y\le 1\}$ c) $\iint_D \frac{x}{x^2+y^2}dxdy, \quad D=[1,2]\times[0,1]$

b) $\iint_D \frac{x}{1+xy}dxdy, \quad D=\{(x,y)|0\le x\le 1, 0\le y\le 1\}$ d) $\iint_D xye^{x^2y}dxdy, \quad D=[0,1]\times[0,2]$

55. Find the volume of the solid that lies under the hyperbolic paraboloid $z=4+x^2-y^2$ and above the square $D=[-1;1]\times[0;2]$

56. Find the volume of the solid enclosed by the surface $z=1+e^x\sin y$ and the planes $x=\pm1, y=0, y=\pi$ and $z=0$.

57. Find the volume of the solid in the first octant bounded by the cylinder $z=16-x^2$ and the plane $y=5$.

58. Evaluate the iterated integral

a) $\int_0^4\int_0^{\sqrt{y}} xy^2dxdy,$ b) $\int_0^2\int_y^{2y} xydxdy,$ c) $\int_0^1\int_0^v \sqrt{1-v^2}dudv.$

59. Evaluate the double integral

a) $\iint_D \frac{y}{1+x^5}dxdy, \quad D=\{(x,y)|0\le x\le 1, 0\le y\le x^2\}$

b) $\iint_D y^2e^{xy}dxdy, \quad D=\{(x,y)|0\le y\le 4, 0\le x\le y\}$

c) $\iint_D x\sqrt{y^2-x^2}dxdy, \quad D=\{(x,y)|0\le y\le 1, 0\le x\le y\}$

d) $\iint_D (x+y)dxdy$, D is bounded by $y=\sqrt{x}$ and $y=x^2$

e) $\iint_D y^3dxdy$, D is the triangle region with vertices $(0;2)$, $(1;1)$ and $(3;2)$

f) $\iint_D xy^2dxdy$, D is enclosed by $x=0$ and $x=\sqrt{1-y^2}$

60. Find the volume of the given solid

a) Under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

b) Enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0$, $y = 1$, $y = x$, $z = 0$

c) Enclosed by the cylinders $z = x^2$, $y = x^2$ and the planes $z = 0$, $y = 4$.

d) Bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant

e) Bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.

f) The solid enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 3y$, $z = 2 + y$.

61. Sketch the region of integration and change the order of integration.

a) $\displaystyle\int_0^4 \int_0^{\sqrt{x}} f(x,y)dydx$, b) $\displaystyle\int_0^1 \int_{4x}^4 f(x,y)dydx$, c) $\displaystyle\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x,y)dxdy$.

d) $\displaystyle\int_0^3 \int_0^{\sqrt{9-y}} f(x,y)dxdy$, e) $\displaystyle\int_1^2 \int_0^{\ln x} f(x,y)dydx$, f) $\displaystyle\int_0^1 \int_{\arctan x}^{\pi/4} f(x,y)dydx$.

62. Evaluate the integral by reversing the order of integration

a) $\displaystyle\int_0^1 \int_{3y}^3 e^{x^2} dxdy$ b) $\displaystyle\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)dxdy$ c) $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dydx$

d) $\displaystyle\int_0^1 \int_x^1 e^{x/y} dydx$ e) $\displaystyle\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}dxdy$ f) $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dxdy$.

Double integrals in polar coordinates

63. Evaluate the given integral by changing to polar coordinates.

a) $\iint_D (x + y)dxdy$ where D is the region that lies to the left of the y-axis, between the circles $x^2 + y^2 = 1$, and $x^2 + y^2 = 4$.

b) $\iint_D \cos(x^2+y^2)dxdy$ where D is the region that lies above the x-axis within the circle $x^2+y^2 = 9$.

c) $\iint_D \sqrt{4 - x^2 - y^2}dxdy$ where $D = \{(x,y)|x^2 + y^2 \le 4, x \ge 0\}$.

d) $\iint_D ye^x dxdy$ where D is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 25$.

e) $\iint_D \arctan(y/x)dxdy$ where $D = \{(x,y)|1 \le x^2 + y^2 \le 4, 0 \le y \le x\}$.

f) $\iint_D xdxdy$ where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

64. Use a double integral to find the area of the region.

a) The region enclosed by the curve $r = 4 + 3\cos\varphi$

b) The region inside the cardioid $r = 1 + \cos\varphi$ and outside the circle $r = 3\cos\varphi$.

65. Use polar coordinates to find the volume of the given solid.

a) Below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy-plane

b) Bounded by the paraboloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$ in the first octant.

c) Above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$

d) Bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

66. Evaluate the iterated integral by converting to polar coordinates

a) $\displaystyle\int_0^a \int_{-\sqrt{a^2-y^2}}^0 x^2 y\,dx\,dy$, b) $\displaystyle\int_0^1 \int_y^{\sqrt{2y-y^2}} (x+y)\,dx\,dy$, c) $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$.

Applications of double integrals

67. Find the mass and center of mass of the lamina that occupies the region D and has the given density function ρ.

a) D is the triangular region enclosed by the lines $x = 0$, $y = x$ and $2x + y = 6$, $\rho(x, y) = x^2$.

b) D is bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$, $\rho(x, y) = y$.

c) D is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 1$, $\rho(x, y) = x$.

d) D is bounded by the parabolas $y = x^2$, and $x = y^2$, $\rho(x, y) = x$.

68. A lamina occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

Chapter 4: TRIPLE INTEGRALS

Reference: James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition.* Thomson, Brooks/Cole Publishing Company.

69. Evaluate the iterated integral.

$$\text{a)} \int_0^1\int_x^{2x}\int_0^y 2xyz\,dz\,dy\,dx, \quad \text{b)} \int_0^3\int_0^1\int_0^{\sqrt{1-z^2}} ze^y\,dx\,dz\,dy, \quad \text{c)} \int_0^1\int_0^z\int_0^y ze^{-y^2}\,dx\,dy\,dz.$$

$$\text{d)} \int_0^{\pi/2}\int_0^y\int_0^x \cos(x+y+z)\,dz\,dx\,dy, \quad \text{e)} \int_0^{\sqrt{\pi}}\int_0^x\int_0^{xz} x^2\sin y\,dy\,dz\,dx.$$

70. Evaluate the triple integral

a) $\iiint\limits_E y\,dV$, where E is bounded by the planes $x=0$, $y=0$, $z=0$, and $2x+2y+z=4$

b) $\iiint\limits_E x^2e^y\,dV$, where E is bounded by the parabolic cylinder $z=1-y^2$ and the planes, $z=0$, $x=1$, and $x=-1$.

c) $\iiint\limits_E xy\,dV$, where E is bounded by the parabolic cylinder $y=x^2$ and $x=y^2$ and the planes, $z=0$ and $z=x+y$.

d) $\iiint\limits_E xyz\,dV$, where E is the solid tetrahedron with vertices $(0,0,0),(1,0,0),(0,1,0)$ and $(0,0,1)$.

e) $\iiint\limits_E x\,dV$, where E is the bounded by the paraboloid $x=4y^2+4z^2$ and the plane $x=4$.

f) $\iiint\limits_E z\,dV$, where E is the bounded by the cylinder $y^2+z^2=9$ and the planes $x=0$, $y=3x$, and $z=0$ in the first octant.

71. Find the volume of the given solid

a) The solid bounded by the cylinder $y=x^2$ and the planes $z=0$, $z=4$, and $y=9$.

b) The solid enclosed by the cylinder $x^2+y^2=9$and the planes $y+z=5$ and $z=1$.

c) The solid enclosed by the paraboloid $x=y^2+z^2$ and the plane $x=16$.

72. Evaluate $\iiint\limits_E (x^3+xy^2)\,dV$, where E is the solid in the first octant that lies beneath the paraboloid $z=1-x^2-y^2$.

73. Evaluate $\iiint\limits_E e^z dV$, where E is enclosed by the paraboloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$, and the xy-plane.

74. Evaluate $\iiint\limits_E x dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

75. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

76. Find the volume of the region E bounded by the paraboloids $z = x^2+y^2$ and $z = 36-3x^2-3y^2$.

77. Evaluate the integral by changing to cylindrical coordinates

a) $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz dz dx dy.$ b) $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} \sqrt{x^2+y^2} dz dy dx.$

78. A solid lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. Write a description of the solid in terms of inequalities involving spherical coordinates.

79. Use spherical coordinates

a) Evaluate $\iiint\limits_H (9 - x^2 - y^2) dV$, where H is the solid hemisphere $x^2 + y^2 + z^2 \leq 9$, $z \geq 0$.

b) Evaluate $\iiint\limits_E z dV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

c) Evaluate $\iiint\limits_E e^{\sqrt{x^2+y^2+z^2}} dV$, where E is enclosed by the sphere $x^2 + y^2 + z^2 = 9$ in the first octant.

d) Evaluate $\iiint\limits_E x^2 dV$, where E is bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

80. Evaluate the integral by changing to spherical coordinates.

a) $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} xy dz dy dx.$ b) $\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2z + y^2z + z^3) dz dx dy.$

81. Calculate $\iiint\limits_E y^2z^2 dV$, where E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

82. Evaluate the triple integral $\iiint\limits_V y dx dy dz$, where V is bounded by the cone $y = \sqrt{x^2+z^2}$ and the plane $y = h$, $(h > 0)$.

83. Evaluate the triple integral

$$\iiint\limits_V \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) dx dy dz, \quad \text{where } V: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1, (a, b, c > 0).$$

84. Evaluate $\iiint\limits_V \sqrt{x^2+y^2+z^2} dx dy dz$, where V is defined by $x^2 + y^2 + z^2 \leq z$.

85. Evaluate $\iiint\limits_V \sqrt{(6x - x^2 - y^2 - z^2)^3} dx dy dz$, where V is the sphere defined by $x^2 + y^2 + z^2 \leq 6x$.

86. Evaluate $\iiint\limits_V \frac{z}{1 + x^2 + y^2} dx dy dz$, where V is bounded by $z = 6 - \sqrt{x^2+y^2}$, $z = 5$.

Chapter 5: LINE INTEGRALS

Reference: James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition.* Thomson, Brooks/Cole Publishing Company.

87. Evaluate the line integral, where C is the given curve

a) $\int\limits_C x \sin y ds$, C is the line segment from $(0, 3)$ to $(4, 6)$.

b) $\int\limits_C (x^2y^3 - \sqrt{x})dy$, C is the arc of the curve $y = \sqrt{x}$ from $(1, 1)$ to $(4, 2)$.

c) $\int\limits_C xe^y dx$, C is the arc of the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$.

d) $\int\limits_C \sin x dx + \cos y dy$, C consists of the top half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ and the line segment from $(-1, 0)$ to $(-2, 3)$.

e) $\int\limits_C xyz ds$, $C : x = 2 \sin t, y = t, z = -2 \cos t$, $0 \le t \le \pi$.

f) $\int\limits_C xyz^2 ds$, C is the line segment from $(-1, 5, 0)$ to $(1, 6, 4)$.

g) $\int\limits_C x^2y\sqrt{z}dz$, $C : x = t^3, y = t, z = t^2$, $0 \le t \le 1$.

h) $\int\limits_C zdx + xdy + ydz$, $C : x = t^2, y = t^3, z = t^2$, $0 \le t \le 1$.

k) $\int\limits_C (x + yz)dx + 2xdy + xyzdz$, C consists of line segments from $(1, 0, 1)$ to $(2, 3, 1)$ and from $(2, 3, 1)$ to $(2, 5, 2)$.

l) $\int\limits_C x^2dx + y^2dy + z^2dz$, C consists of line segments from $(0, 0, 0)$ to $(1, 2, -1)$ and from $(1, 2, -1)$ to $(3, 2, 0)$.

88. Evaluate the following line integrals

a) $\int\limits_C (x - y)ds$, where C is the circle $x^2 + y^2 = 2x$.

b) $\int\limits_C (x^2 + y^2 + z^2)ds$, where C is the helix $x = a \cos t$, $y = a \sin t$, $z = bt$, $(0 \le t \le 2\pi)$.

89. Evaluate the line integral $\int_C F \cdot dr$, where $F(x, y, z) = xi - zj + yk$ and C is given by $r(t) = 2ti + 3tj - t^2k$, $-1 \le t \le 1$.

90. Find the work done by the force field $F(x, y, z) = (y + z, x + z, x + y)$ on a particle that moves along the line segment from $(1; 0; 0)$ to $(3; 4; 2)$.

91. Evaluate the line integral by two methods: (a) directly and using Green's Theorem

a) $\oint_C (x-y)dx + (x+y)dy$, C is the circle with center the origin and radius 2.

b) $\oint_C xydx + x^2dy$, C is the rectangle with vertices $(0;0)$, $(3;0)$, $(3;1)$, and $(0;1)$.

c) $\oint_C ydx + xdy$, C consists of the line segments from $(0;1)$ to $(0;0)$ and from $(0;0)$ to $(1;0)$ and the parabola $y = 1 - x^2$ from $(1;0)$ to $(0;1)$.

92. Use Green's Theorem to evaluate the line integral along given positively oriented curve

a) $\int_C (y + e^{\sqrt{x}})dx + (2x + \cos y)dy$, C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

b) $\int_C xe^{-2x}dx + (x^4 + 2x^2y^2)dy$, C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

c) $\int_C (e^x + x^2y)dx + (e^y - xy^2)dy$, C is the circle $x^2 + y^2 = 25$.

d) $\int_C (2x - x^3y^5)dx + x^3y^8dy$, C is the ellipse $4x^2 + y^2 = 4$.

93. Show that the line integral is independent of path and evaluate the integral

a) $\int_C (1 - ye^{-x})dx + e^{-x}dy$, C is any path from $(0,1)$ to $(1,2)$.

b) $\int_C 2y^{3/2}dx + 3x\sqrt{y}dy$, C is any path from $(1,1)$ to $(2,4)$.

Curl and Divergence

94. Determine whether or not F is a conservative vector field. If it is, find a function f such that $F = \nabla f$.

a) $F(x,y) = (2x - 3y)i + (-3x + 4y - 8)j$

b) $F(x,y) = e^x \cos y i + e^x \sin y j$

c) $F(x,y) = (xy\cos xy + \sin xy)i + (x^2 \cos xy)j$

d) $F(x,y) = (\ln y + 2xy^3)i + (3x^2y^2 + x/y)j$

e) $F(x,y) = (ye^x + \sin y)i + (e^x + x\cos y)j$

95. Find a function f such that $F = \nabla f$ and then evaluate $\int_C F \cdot dr$ along the given curve C.

a) $F(x,y) = xy^2i + x^2yj$, $C:\ r(t) = (t + \sin \frac{1}{2}\pi t, t + \cos \frac{1}{2}\pi t)$, $0 \le t \le 1$.

b) $F(x,y) = \dfrac{y^2}{1+x^2}i + 2y \arctan x j$, $C:\ r(t) = t^2 i + 2tj$, $0 \le t \le 1$.

c) $F(x,y) = (2xz + y^2)i + 2xyj + (x^2 + 3z^2)k$, $C:\ x = t^2, y = t+1, z = 2t - 1$, $0 \le t \le 1$.

d) $F(x,y) = e^y i + xe^y j + (z+1)e^z k$, $C:\ x = t, y = t^2, z = t^3$, $0 \le t \le 1$.

Chapter 6: SURFACE INTEGRALS

Reference: James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition.* Thomson, Brooks/Cole Publishing Company.

96. Evaluate the surface integral

a) $\iint\limits_S xy dS$, S is the triangular region with vertices $(1,0,0)$, $(0,2,0)$, and $(0,0,2)$.

b) $\iint\limits_S yz dS$, S is the part of the plane $x+y+z=1$ that lies in the first octant.

c) $\iint\limits_S yz dS$, S is the surface with parametric equations $x=u^2$, $y=u\sin v$, $z=u\cos v$, $0\le u\le 1, 0\le v\le \pi/2$.

d) $\iint\limits_S z dS$, S is the surface $x=y+2z^2$, $0\le y\le 1$, $0\le z\le 1$.

e) $\iint\limits_S y^2 dS$, S is the part of the sphere $x^2+y^2+z^2=4$ that lies inside the cylinder $x^2+y^2=1$ and above the $xy-$plane.

97. Evaluate the surface integral $\iint\limits_S F\cdot dS$ for the given vector field F and the oriented surface S. In other words, find the flux of F across S. For closed surfaces, use the positive (outward) orientation.

a) $F(x,y,z)=xze^yi-xze^yj+zk$, S is the part of the plane $x+y+z=1$ in the first octant and has downward orientation.

b) $F(x,y,z)=xi+yj+z^4k$, S is the part of the cone $z=\sqrt{x^2+y^2}$ beneath the plane $z=1$ with downward orientation.

c) $F(x,y,z)=xzi+xj+yk$, S is the hemisphere $x^2+y^2+z^2=25, y\ge 0$, oriented in the direction of the positive $y-$axis.

d) $F(x,y,z)=xyi+4x^2j+yzk$, S is the surface $z=xe^y$, $0\le x\le 1, 0\le y\le 1$, with upward orientation.

e) $F(x,y,z)=x^2i+y^2j+z^2k$, S is the boundary of the solid half-cylinder $0\le z\le\sqrt{1-y^2}$, $0\le x\le 2$.

98. a) Find the center of mass of the hemisphere $x^2+y^2+z^2=a^2$, $z\ge 0$, if it has constant density.

b) Find the mass of a thin funnel in the shape of a cone $z = \sqrt{x^2 + y^2}$, $1 \le z \le 4$, if its density function is $\rho(x, y, z) = 10 - z$.

Stokes Theorem

99. Use Stokes Theorem to evaluate $\iint\limits_S \mathrm{curl} F \cdot dS$

a) $F(x, y, z) = 2y \cos z i + e^x \sin z j + xe^y k$, S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \ge 0$, oriented upward.

b) $F(x, y, z) = x^2 z^2 i + y^2 z^2 j + xyzk$, S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$, oriented upward.

The Divergence Theorem

100. Use the Divergence Theorem to calculate the surface integral $\iint\limits_S F \cdot dS$; that is, calculate the flux of F across S

a) $F(x, y, z) = x^3 y i - x^2 y^2 j - x^2 yzk$, S is the surface of the solid bounded by the hyperboloid $x^2 + y^2 - z^2 = 1$ and the planes $z = -2$ and $z = 2$.

b) $F(x, y, z) = (\cos z + xy^2) i + xe^{-z} j + (\sin y + x^2 z) k$, S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

c) $F(x, y, z) = 4x^3 zi + 4y^3 zj + 3z^4 k$, S is the sphere with radius R and center the origin.