LESSON 13

1-sided Z TRANSFORM

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❑ CONTENT

1. Definition of one-sided Z transform
2. Application of the one-sided Z transform to solve the differential equation

❑ Lesson Objectives

After completing this lesson, you will be able to understand the following topics:

- The method of applying one-sided Z transform to solve differential equations.
- Analyze components in the response of a discrete system

1. One-sided Z transform

$$X^{+}(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

- Comment :
  - The one-sided Z-transform contains no information about the signal x(n) at times $n < 0$
  - The one-sided Z-transform is uniquely determined for the causal signal (x(n)=0, when n<0)
- For example :

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\}$$

$$\overset{z^{+}}{\longleftrightarrow} \quad X_1^{+} = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

Example

$$x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\} \xleftrightarrow{z^+} X_2^+(z) = 5 + 7z^{-1} + z^{-3}$$

$$x_3(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\} \xleftrightarrow{z^+} X_3^+(z) = 5 + 7z^{-1} + z^{-3}$$

For a non-causal signal, the Z . transform only one non-corresponding side. For example: $X_2^+(z) = X_3^+(z)$ nhưng $x_2(n) \neq x_3(n)$

2. Properties of one-sided Z transform

- Almost all the properties studied for the two-way Z-transform can be applied to the one-sided Z-transform: except the translation (delay) property.

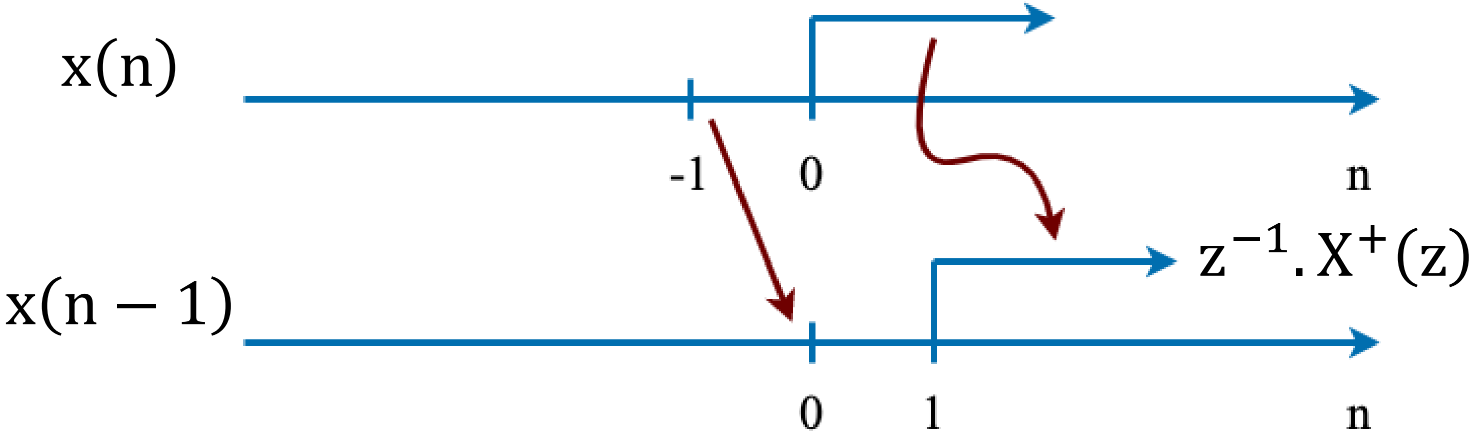
$$x(n) \xleftrightarrow{z^+} X^+(z) \qquad k > 0$$

$$x(n-k) \xleftrightarrow{z^+} z^{-k}\left[X^+(z) + \sum_{n=1}^{k} x(-n)z^{n}\right]$$

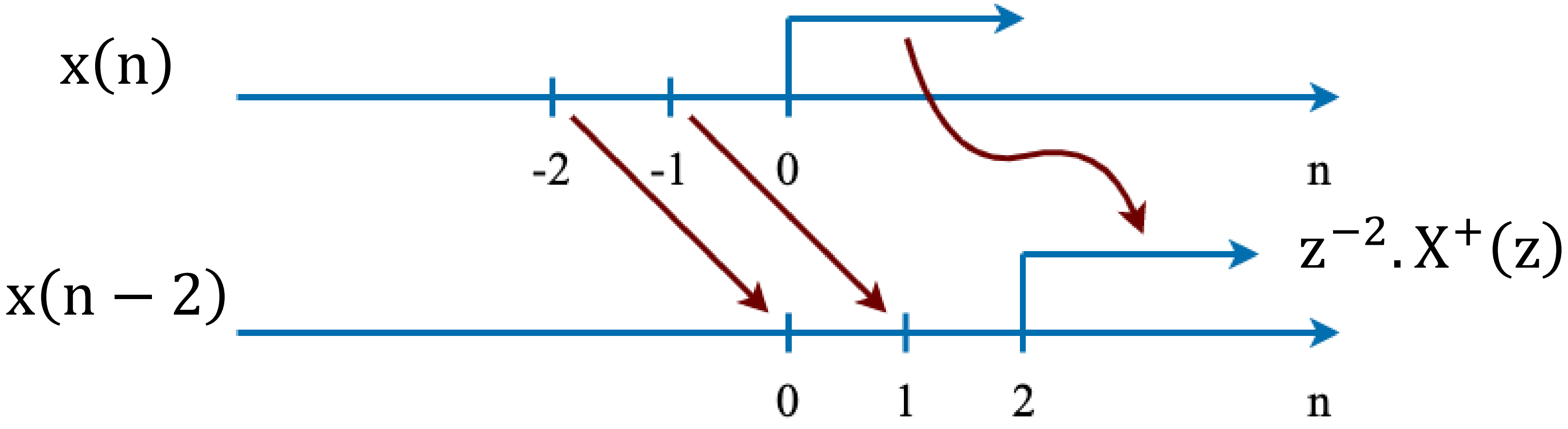
$$= \left[x(-k) + x(-k+1)z^{-1} + \cdots + x(-1)z^{-k+1}\right] + z^{-k}X^+(z) \qquad k > 0$$

Delay

$$x(n-1) \xrightarrow{ZT^+} z^{-1}.X^+(z) + x(-1)$$



$$x(n-2) \xrightarrow{ZT^+} z^{-2}.X^+(z) + x(-2) + x(-1).z^{-1}$$



Example

- Find the one-sided Z-transform of the signal:

  a. $x(n) = a^n u(n)$

  b. $x_1(n) = x(n-2)$ với $x(n) = a^n$

$$x(n) = a^n u(n) \quad \rightarrow \quad X^+(z) = \frac{1}{1 - az^{-1}}$$

$$\begin{aligned} x_1(n) = x(n-2) \rightarrow z^{-2}[X^+(z) + x(-1)z + x(-2)z^2]. \\ = z^{-2}X^+(z) + x(-1)z^{-1} + x(-2) \end{aligned}$$

$$\Rightarrow \quad X_1^+(z) = \frac{z^{-2}}{1 - az^{-1}} + a^{-1}z^{-1} + a^{-2}$$

3. Solve the differential equation

- Determine the response of the following system to the signal u(n):

$$y(n) = a.y(n-1) + x(n)\ ;\ -1 < a < 1$$

- Initial condition: $y(-1) = 1$

- Solve. Take the one-sided Z transform:

$$Y^+(z) = \alpha[z^{-1}Y^+(z) + y(-1)] + X^+(z)$$

$$Y^+(z) = \frac{\alpha}{1 - \alpha z^{-1}} + \frac{1}{(1-\alpha z^{-1})(1-z^{-1})}$$

$$\boxed{\begin{aligned} y(n) &= \alpha^{n+1}u(n) + \frac{1-\alpha^{n+1}}{1-\alpha}u(n) \\ &= \frac{1}{1-\alpha}(1-\alpha^{n+2})u(n) \end{aligned}}$$

Zero Input Response

Zero State Response

4. Summary

- The one-sided Z-transform is a Z-transform that only considers the causal part of the signal.
- The one-sided Z transform is applied to solve the differential equation representing the system.
- The system response consists of two components: zero input response and zero state response

Homework

- Exercise 1
  - Apply one-sided Z transform to solve the differential equation:

$$y(n) - \frac{1}{2}y(n-1) = x(n) - \frac{1}{2}x(n-1)$$

know $\mathrm{x(n)} = \delta(\mathrm{n}),\ \mathrm{y}(-1) = 0.$

Exercise 2

- A causal system is represented by the following differential equation:

$$y(n) = 0.5y(n-1) + x(n) + 2x(n-1)\ ;\ -1 < a < 1$$

- Initial condition: $y(-1) = 1$
- Determine the response of the system to the action u(n).
- Determine the zero-state response and zero input response of the system.

Next lesson. Lesson 14

SPECTRUM ANALYSIS OF CONTINUOUS SIGNALS

References:

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

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Wish you all good study!