SET 1 SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS

FINAL EXAMINATION CALCULUS I
MI1016. Term 20201. Time duration: 90 mins

All materials are forbidden. Problem sheet must be submitted with your answer sheets.

Question 1. Find the domain and the range of the function

$$f(x) = \arcsin(2^x).$$

Question 2. Find the volume of the solid of revolution obtained by rotating the region $x^2 + 4y^2 \leq 4$, $x \geq 0$, about the Ox axis.

Question 3. Find an equation of the tangent plane to the surface $z(x,y) = \frac{x^3}{3} + 2xy + y^2 - 3x$ at $A(3;2)$.

Question 4. Compare the following pair of infinitesimals

$$\alpha(x) = \ln(1 + 2x \arctan x), \quad \beta(x) = \sin(x^3 - 2x^5) \text{ as } x \to 0.$$

Question 5. Find the third degree Taylor polynomial of the function $f(x) = \frac{1}{\sqrt{x}}$ centered at $x = 1$. Using this polynomial, approximate the value $f(1,2)$.

Question 6. Express the following limit as a definite integral, then evaluate it

$$\lim_{n\to\infty} \frac{1}{n^2}\left(\sin\frac{\pi}{n} + 2\sin\frac{2\pi}{n} + \ldots + (n-1)\sin\frac{(n-1)\pi}{n}\right).$$

Question 7. a) Compute the partial derivative $f'_y(x,y)$ of the function $f(x,y) = \begin{cases} \dfrac{2x^3 - 3y^3}{4x^2 + y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

b) Is the partial derivative $f'_y(x,y)$ continuous at $(0,0)$? Explain.

Question 8. Test for convergence $\displaystyle\int_0^{\infty} \frac{\arctan(2\sqrt{x})}{\ln(1+3x) + x^2}dx.$

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SET 2 SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS

FINAL EXAMINATION CALCULUS I
MI1016. Term 20201. Time duration: 90 mins

All materials are forbidden. Problem sheet must be submitted with your answer sheets.

Question 1. Find the domain and the range of the function

$$f(x) = \arccos(2^x).$$

Question 2. Find the volume of the solid of revolution obtained by rotating the region $x^2 + 4y^2 \leq 4$, $y \geq 0$, about the Oy axis.

Question 3. Find an equation of the tangent plane to the surface $z(x,y) = x^2 - 6xy - 8x + 2y^3$ at $A(1;-2)$.

Question 4. Compare the following pair of infinitesimals

$$\alpha(x) = e^{x\sin(2x)} - 1, \quad \beta(x) = \arctan(2x + 3x^2) \text{ as } x \to 0.$$

Question 5. Find the third degree Taylor polynomial of the function $f(x) = \frac{1}{\sqrt{x}}$ centered at $x = 1$. Using this polynomial, approximate the value $f(0,8)$.

Question 6. Express the following limit as a definite integral, then evaluate it

$$\lim_{n\to\infty} \frac{1}{n^2}\left(\cos\frac{\pi}{n} + 2\cos\frac{2\pi}{n} + \ldots + (n-1)\cos\frac{(n-1)\pi}{n}\right).$$

Question 7. a) Compute the partial derivative $f'_x(x,y)$ of the function $f(x,y) = \begin{cases} \dfrac{2x^3 - 3y^3}{4x^2 + y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

b) Is the partial derivative $f'_x(x,y)$ continuous at $(0,0)$? Explain.

Question 8. Test for convergence $\displaystyle\int_0^{\infty} \frac{\arctan(2\sqrt{x})}{\sin^2(\sqrt{3x}) + 2x^2}dx.$

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