EXERCISE 1

Each CSP formulation concerns three components: the variables, domains, and constraints

Solution A:

- A variable corresponding to **each tile of the n^2 positions** on the board
- Each variable's domain is the set **{'occupied', 'vacant'}**
- (1)Pairs of variables (tiles) separated by a knight's move cannot take on the value 'occupied' simultaneously and (2) Total number of 'occupied'-valued variables (tiles) should be k ~ global constraint

Solution B:

- A variable corresponding to **each of the k knights**
- Each variable's domain is the set of **all tiles on the board**
- Pairs of variables (knights) cannot have the same value (be on the same tile) or have values (tiles) separated by a knight's move

EXERCISE 2

An arc (X → Y) is consistent of constraints, if and only if for every value x of variable X there is some (i.e., at least one) value y of variable Y such that the constraints between X and Y are satisfied.

AC-3 algorithm

1. Turn each binary constraint into two arcs e.g. A = B becomes A = B and B = A
2. Add all the arcs to an queue
3. Repeat until the queue is empty - Take an arc (X_i, X_j) off the queue to check - For every value of X_i, there must be some value of X_j to satisfy the constraints - Remove any inconsistent values from X_i - If X_i's domain is changed, add all arcs with X_i on the right-hand side to the queue, unless the arc is already on the queue

Refer to the image below for the detailed solution to a). The removal of values from the domain and the added arcs are color-coded. b) and c) follows the exact same paradigm

a) $A \in \{2, 3\}$, $B \in \{1, 2\}$, $C \in \{1, 2\}$
b) $A \in \{3, 4, 6\}$, $B \in \{2, 3, 4\}$, $C \in \{1, 2, 3, 4\}$
c) $A \in \{1, 2, 3, 4, 5\}$, $B \in \{1, 2, 3, 4, 5, 6, 7\}$, $C \in \{1, 2, 3, 4, 5\}$

Vars & domains:

$A \in \{\cancel{1}, 2, 3\} = D(A)$

$B \in \{1, 2, \cancel{3}\} = D(B)$

$C \in \{1, 2, \cancel{3}\} = D(C)$

Arcs:

$A > B$

$B < A$

$B = C$

$C = B$

Queue:

$A > B \rightarrow$ remove 1 from $D(A)$

$B < A \rightarrow$ remove 3 from $D(B)$

$B = C \rightarrow$ consistent

$C = B \rightarrow$ remove 3 from $D(C)$

$A > B \rightarrow$ consistent

$B = C \rightarrow$ consistent

DONE!

EXERCISE 3

1. When choosing a variable to assign values to, why should we choose the most constrained variable (a.k.a. minimum remaining values) heuristic?

Answer: To reduce the search space more quickly. More constraints mean failures will occur sooner, and we can backtrack earlier. Think of it as tackling a “difficult task” first, before doing “easier” ones.

2. Why should we choose the least-constraining value (the value that rules out the fewest values in the remaining variables)?

Answer: By selecting the value that rules out the fewest values in the remaining variables, you leave more options open for future variable assignments, which increases the chances of finding a solution.

3. What does forward checking do?

After a variable is assigned a value, it might be the case that for unassigned variables, some values in their domains are no longer possible. Forward checking looks at all the constraints that involve the variable being assigned, then looks at the domains of unassigned variables to remove such impossible values. If a variable's domain becomes null (no value to assign), the assignment fails, and this failure is captured earlier compared to normal backtracking.

CRYPTARITHMETIC PROBLEM

The exact steps depend on certain choices you are free to make.

a. Choose the X3 variable. Its domain is {0,1}.
b. Choose the value 1 for X3. (We can't choose 0; it wouldn't survive forward checking, because it would force F to be 0, and the leading digit of the sum must be non-zero.)
c. Choose F, because it has only one remaining value.
d. Choose the value 1 for F.
e. Now X2 and X1 are tied for minimum remaining values at 2; let's choose X2.
f. Either value survives forward checking, let's choose 0 for X2.
g. Now X1 has the minimum remaining values.
h. Again, arbitrarily choose 0 for the value of X1.
i. The variable O must be an even number (because it is the sum of T + T less than 5 (because O + O = R + 10 × 0). That makes it most constrained.
j. Arbitrarily choose 4 as the value of O.
k. R now has only 1 remaining value.
l. Choose the value 8 for R.
m. T now has only 1 remaining value.48 Chapter 6. Constraint Satisfaction Problems
n. Choose the value 7 for T.
o. U must be an even number less than 9; choose U.
p. The only value for U that survives forward checking is 6.
q. The only variable left is W.
r. The only value left for W is 3.
s. This is a solution.