LESSON 17

DIGITAL FILTERS CONCEPT

PhD. Nguyen Hong Quang
Assoc. Prof. Trinh Van Loan
PhD. Doan Phong Tung
Computer Engineering Department

❑ CONTENT

1. The basic characteristics of the digital system are represented by the constant coefficient linear difference equation.
2. Digital filter concept.

Lesson Objectives

After completing this lesson, you will be able to understand the following topics:

- Basic characteristics of the number system
- Basic concepts and principles of digital filters.

1. Digital system represented by the constant coefficient linear difference equation

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$$y(n) = h(n) * x(n) = \sum_{m=-\infty}^{\infty} h(m)x(n-m) = x(n) * h(n) = \sum_{m=-\infty}^{\infty} x(m)h(n-m)$$

- $h(n)$: impulse response of the system
- Difference equation $$\sum_{k=0}^{n} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r)$$
- Transfer function $H(z)$: $$H(Z) = ZT[h(n)] = \frac{Y(Z)}{X(Z)} = \frac{\sum_{r=0}^{M} b_r Z^{-r}}{\sum_{k=0}^{N} a_k Z^{-k}}$$
- Frequency Response: $$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{r=0}^{M} b_r e^{-J\omega r}}{\sum_{k=0}^{N} a_k e^{-J\omega k}} \qquad Y(e^{J\omega}) = H(e^{j\omega}).X(e^{j\omega})$$

FIR system

- Differential Equation : $y(n) = h(0).x(n) + h(1).x(n-1) + \dots + h(N).x(n-N)$
- Impulse response has a finite length: $h(n)$ is zero outside the range 0 to $N-1$
- Transfer function: $$H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n}$$
- Frequency response: $$H(\omega) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}$$
- Causality: the system is always causal because $h(n) = 0, \forall\, n < 0$
- Stability: the system is always stable because $h(n)$ satisfies the stability test condition

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{N-1} |h(n)| < \infty$$

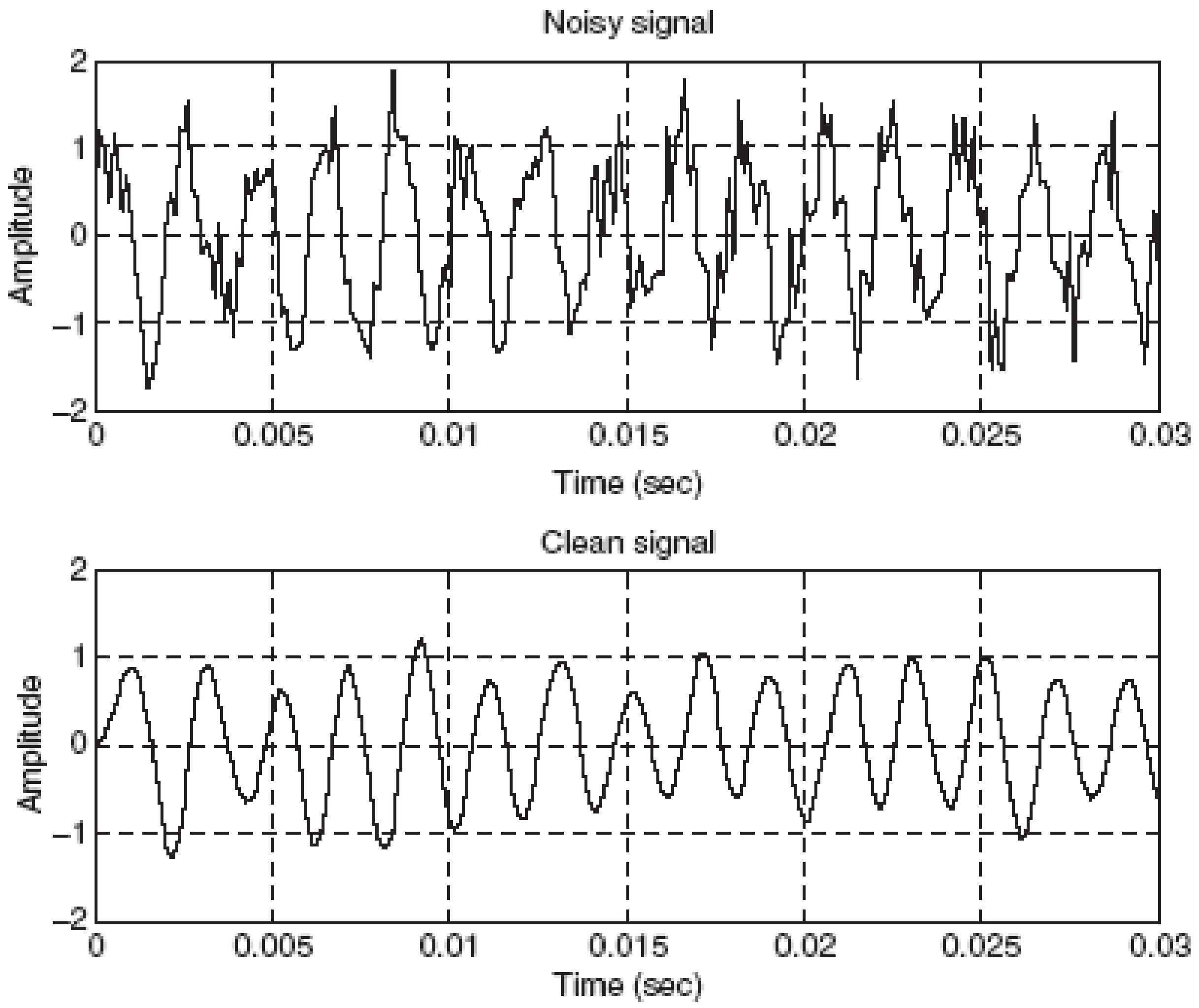
IIR system

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r)$$

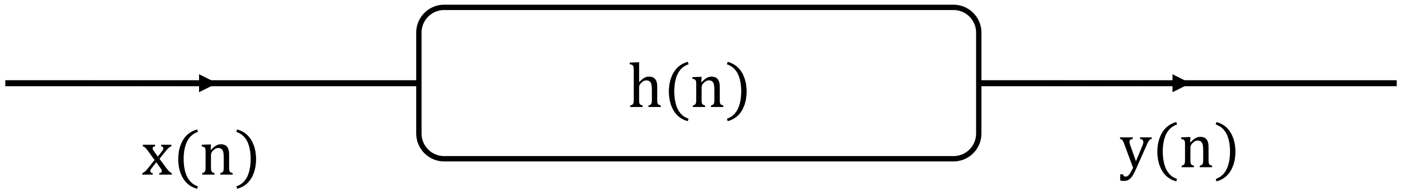
- The impulse response h(n) has an infinite length
- Causality with $h(n) = 0, \forall\, n < 0$ however stability needs to be investigated.

2. Digital filter concept

- In many different applications, it is often necessary to remove certain frequency components.
- Such systems are called digital filters.
- Example: signal noise filter.



The basic principle of the filter



$$x(n) = \sum_{i=1}^{L} A_i \cos(\omega_i n + \theta_i)$$

$$y(n) = \sum_{i=1}^{L} A_i |H(\omega_i)| \cos[\omega_i n + \varphi_i + \theta(\omega_i)]$$

- The system adjusts the amplitude of the input frequency components through the amplitude response
- Digital filter design: design for amplitude response according to filter specifications

4. Summary

- A digital filter is a digital system capable of attenuating and removing unwanted frequency components in an input signal.
- The system adjusts the amplitude and delay of the input frequency components through the amplitude response and phase response.

5. Exercise

- Let's learn and get examples of digital filters in action

Next lesson. Lesson 18

IDEAL DIGITAL FILTERS

References :

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

Wish you all good study!