HUST

ĐẠI HỌC BÁCH KHOA HÀ NỘI

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

APPLIED ALGORITHMS

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BACKTRACKING, BRANCH AND BOUND

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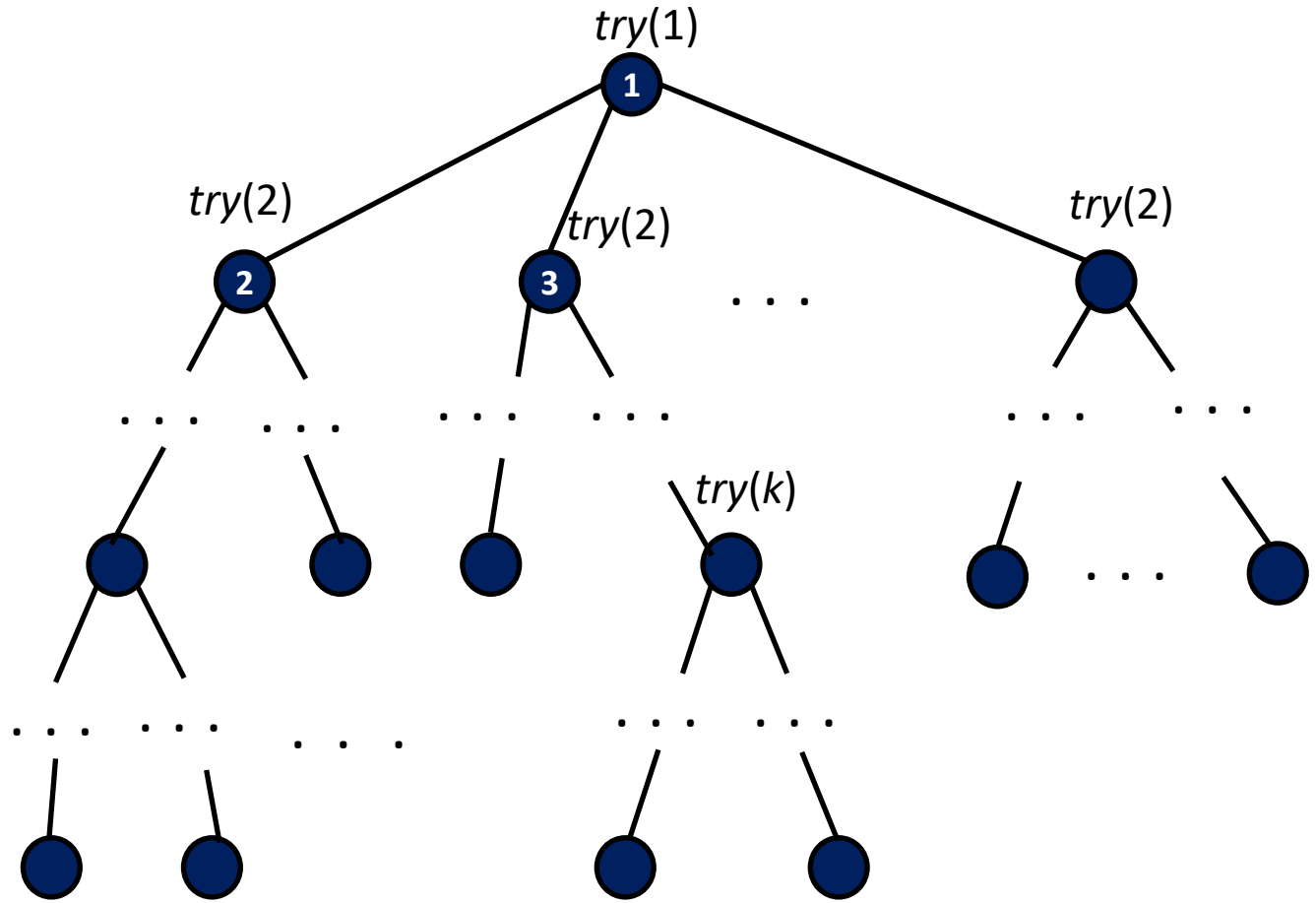
CONTENTS

- General diagram of backtracking, branch and bound
- The problem of bus routes picking up and dropping off passengers
- Delivery truck route problem

GENERAL DIAGRAM OF BACKTRACKING, BRANCHING AND BOUND

- The backtracking algorithm allows us to solve combinatorial enumeration problems and combinatorial optimization problems
- The alternative is modeled by a sequence of decision variables $X_1, X_2, \ldots, X_n$
- Need to find for each variable X_i a value selected from a given discrete set A_i such that
  - The constraints of the problem are satisfied
  - Optimize a given objective function
- Backtracking algorithm
  - Traverse through all variables (e.g. order from $X_1, X_2, \ldots, X_n$), for each variable X_k:
    - Traverse through all possible values that could be assigned to X_k, for each value v:
      - Check constraints
      - Assign $X_k = v$
      - If $k = n$ then record a solution to the problem
      - Otherwise, consider the variable X_{k+1}

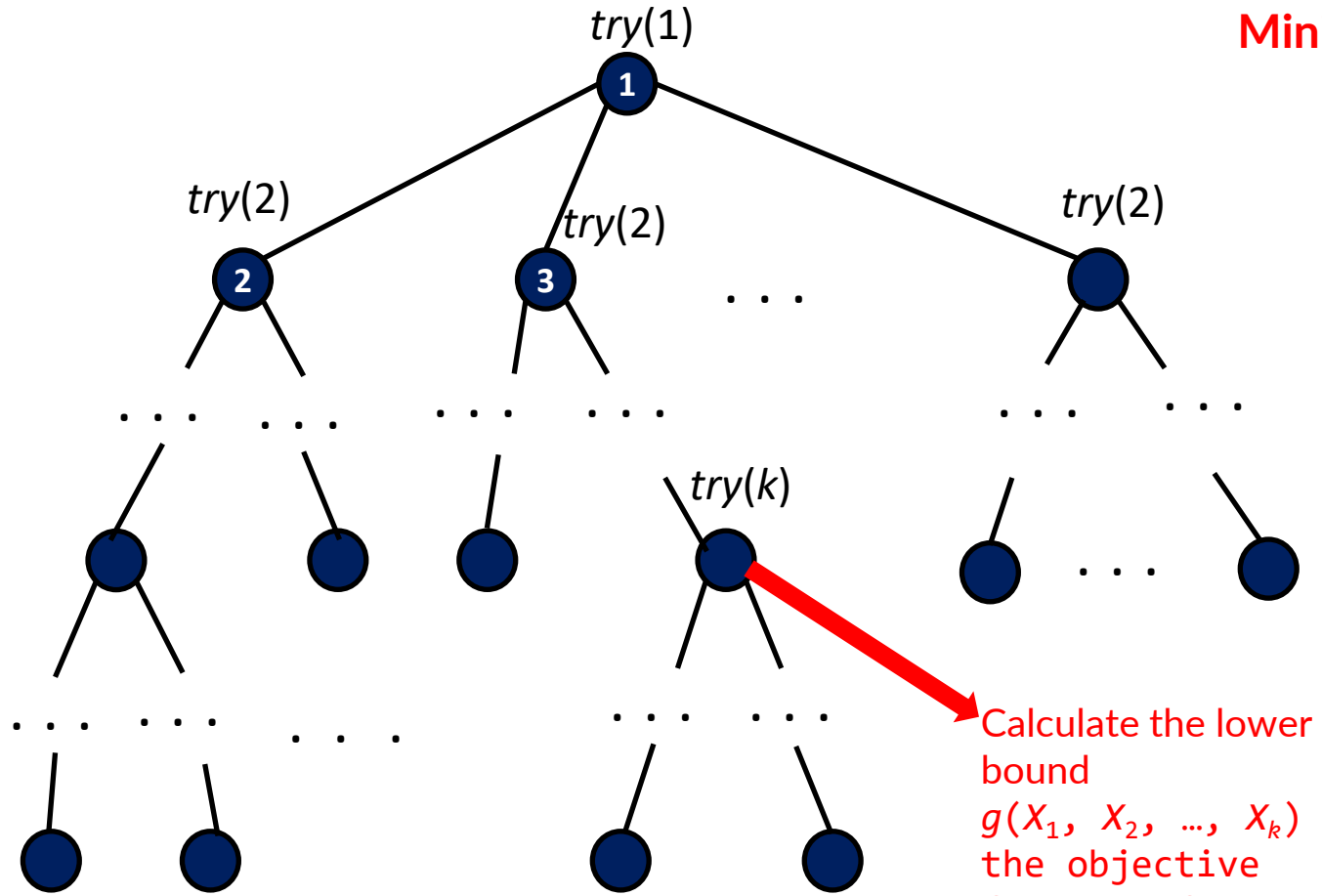
GENERAL DIAGRAM OF BACKTRACKING, BRANCHING AND BOUND



Enumeration problem

```
try(k){ //Try out the possible values assigned to X_k
  for v in A_k do {
     if check(v,k){
        X_k = v;
        [Update a data structure D]
        if k = n then solution();
        else {
            try(k+1);
        }
        [Recover the data structure D]
     }
  }
}
```

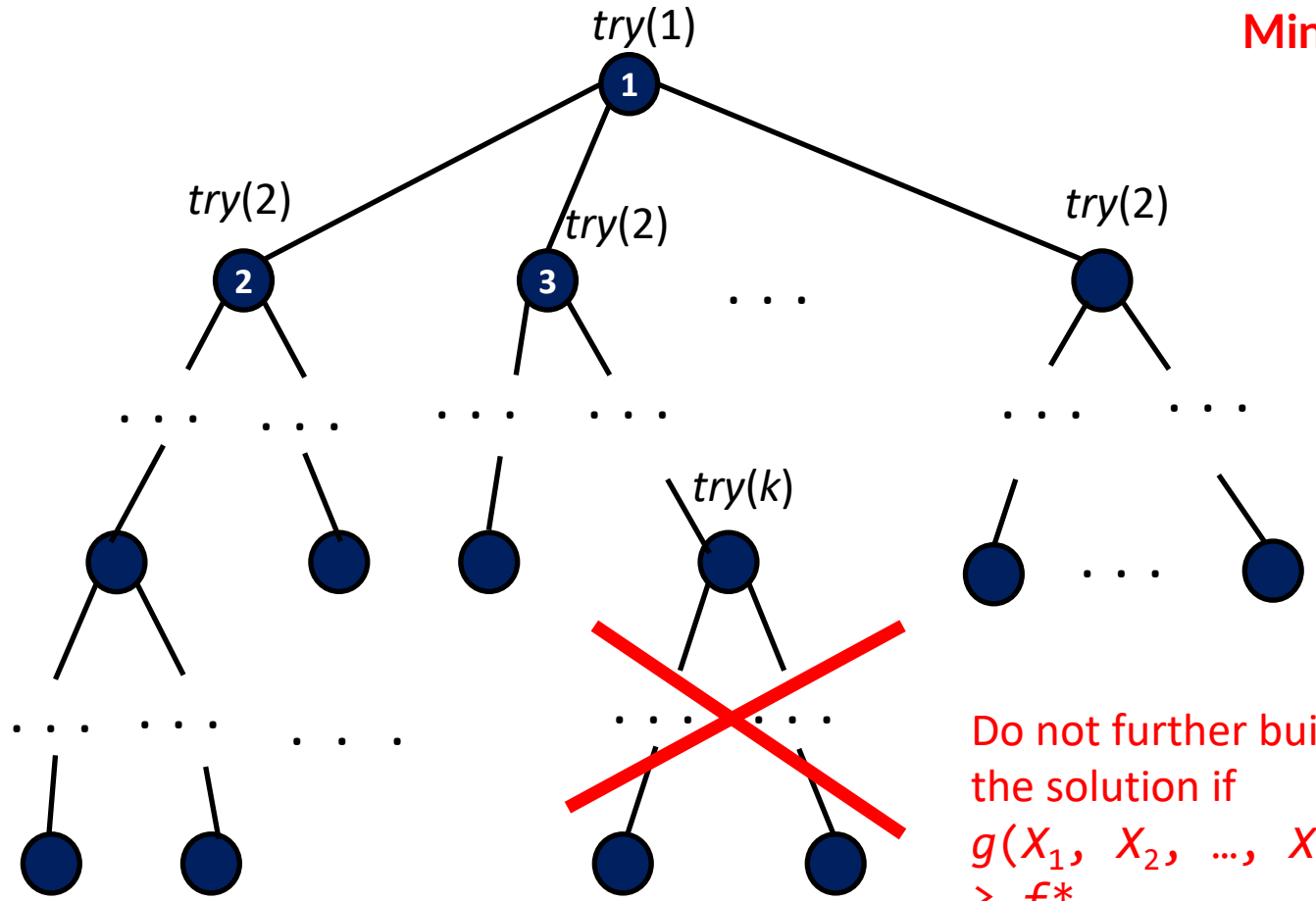
GENERAL DIAGRAM OF BACKTRACKING, BRANCHING AND BOUND



Minimize optimization problem (Denote f^* : optimal value)

```
try(k){//Try out the possible values assigned to X_k
  for v in A_k do {
     if check(v,k){
        X_k = v;
        [Update a data structure D]
        if k = n then updateBest();
        else {
            if g(X_1, X_2, …, X_k) < f* then
                try(k+1);
        }
        [Recover the data structure D]
     }
  }
}
```

GENERAL DIAGRAM OF BACKTRACKING, BRANCHING AND BOUND



Do not further build the solution if $g(X_1, X_2, \ldots, X_k) \geq f^*$

Minimize optimization problem (Denote f^* : optimal value)

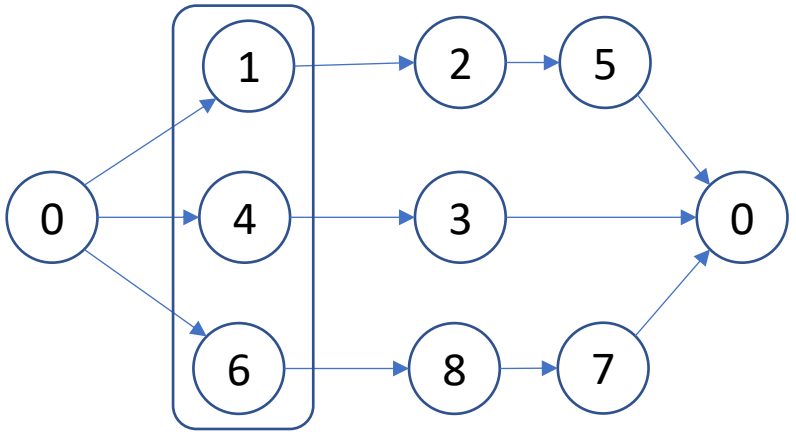
```
try(k){//Try out the possible values assigned to X_k
  for v in A_k do {
     if check(v,k){
        X_k = v;
        [Update a data structure D]
        if k = n then updateBest();
        else {
            if g(X_1, X_2, …, X_k) < f* then
                try(k+1);
        }
        [Recover the data structure D]
     }
  }
}
```

DELIVERY TRUCK ROUTE PROBLEM

- A fleet of *K* identical trucks needs to be assigned to transport pepsi boxes from the central warehouse (point 0) to delivery points 1,2,...,*N* and back to the warehouse. The travel distance from point *i* to point *j* is *c*(*i*,*j*)
- Each truck has a load capacity of Q (each trip can only transport a maximum of Q boxes)
- Each delivery point *i* has a required quantity of *d*[*i*] boxes, *i* = 1,..., *N*.
- It is necessary to develop a transportation plan so that:
  - Each delivery point can only be delivered by exactly one vehicle and only once.
  - The total amount of boxes on the vehicle must not exceed the vehicle's load.
  - The total route length of the vehicles is the smallest
- Note: do not need to use all K vehicles.

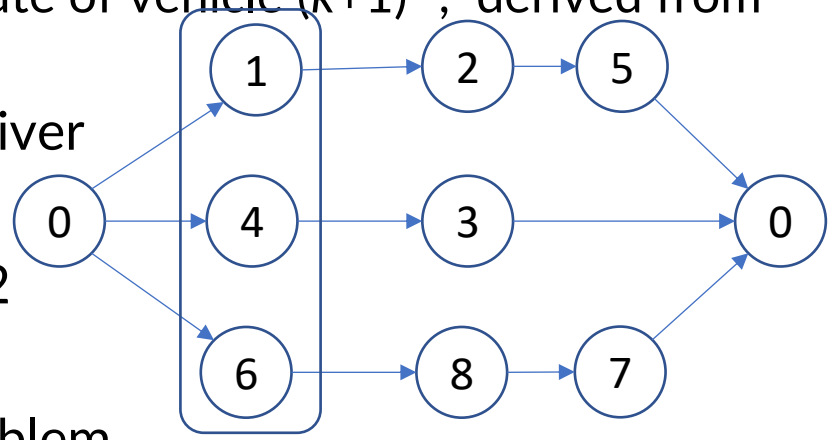
DELIVERY TRUCK ROUTE PROBLEM

- Design data
  - $y[k]$ first delivery point of the *kth* vehicle ($y[k] \in \{0, 1, 2,..., N\}$, where $k = 1, 2,..., K$)
    - $y[k] = 0$ means the vehicle k will not be used for route planning
  - $x[i]$ is the next point of the delivery point i on the route ($x[i] \in \{0,1,2,...,N\}$, với $i = 1, 2,..., N$)
  - Since the vehilces are identical, we can assume $y[k] \leq y[k+1]$, $k = 1, 2,..., K-1$
    - If $y[k] > 0$ then $y[k+1] > y[k]$
  - Variables associated with the partial solution:
    - visited[v] = true if v has been visited by a vehicle
    - load[k]: total amount of boxes on the vehicle k
    - f: total length of the current partial solution
    - f^*: best distance that has been found



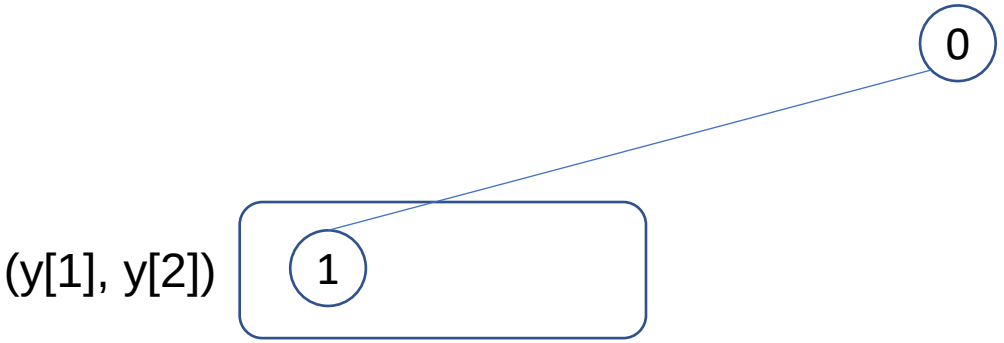
DELIVERY TRUCK ROUTE PROBLEM

- Strategy to traverse
  - Start by traversing values for the tuple ($y[1],. . ., y[K]$)
  - For each complete values of tuple ($y[1],. . ., y[K]$), start traversing values for the $x[1,...,N]$ derived f rom $x[y[1]]$
  - Each time: try to assign $x[v] = u$ for the kth vehicle:
    - If $u > 0$ (not the starting point): try continuing to browse the value for $x[u]$ still on the kth vehicle
    - If $u = 0$ (starting point) then
      - If $k = K$ (all routes for K vehicles are complete) and all delivery points are visited, then obtain a solution to the problem
      - otherwise, try continuing to assign the values for the route of vehicle $(k+1)^{th}$, derived from $x[y[k+1]]$
  - Variable nbR: the number of vehicles that has been used to deliver
  - Variable segments
    - The number of segments (segment: connection between 2 consecutive delivery points on the route)
    - when segments = N+nbR then obtain a solution to the problem



DELIVERY TRUCK ROUTE PROBLEM

- K = 2, N= 6



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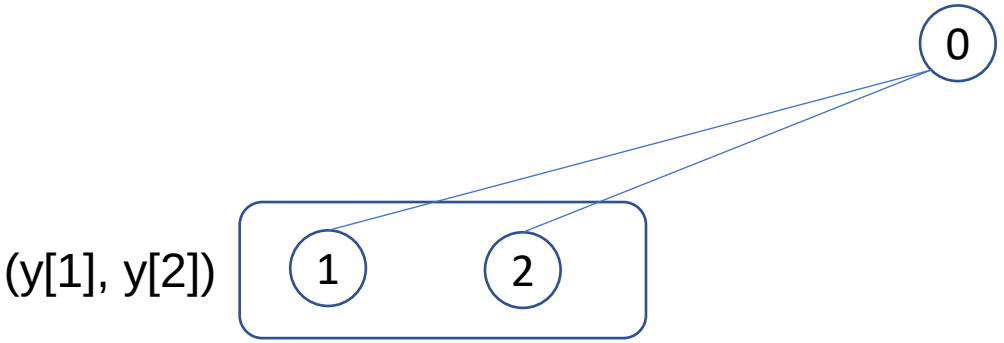

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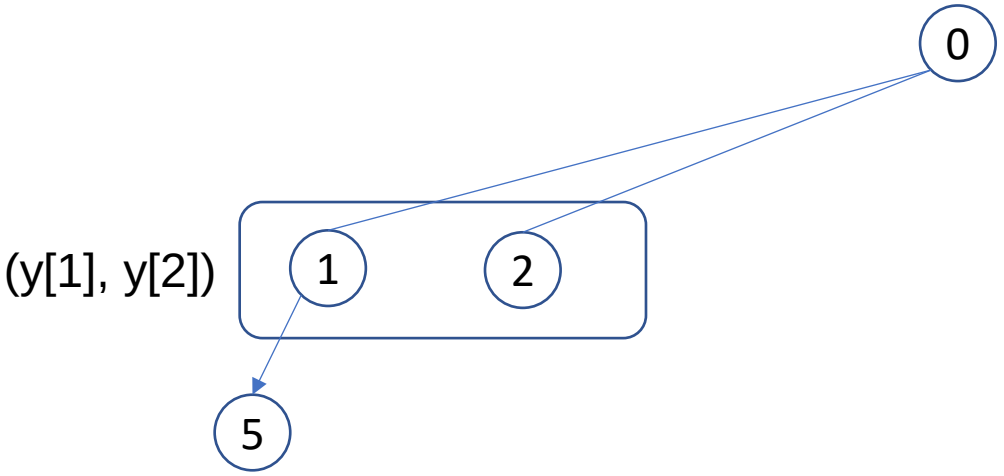

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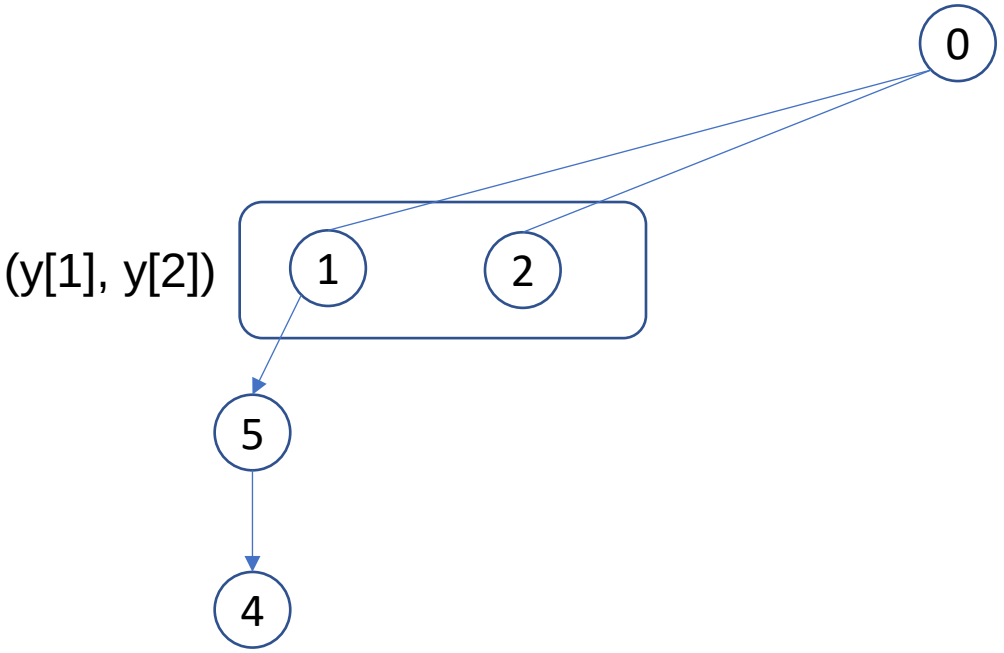

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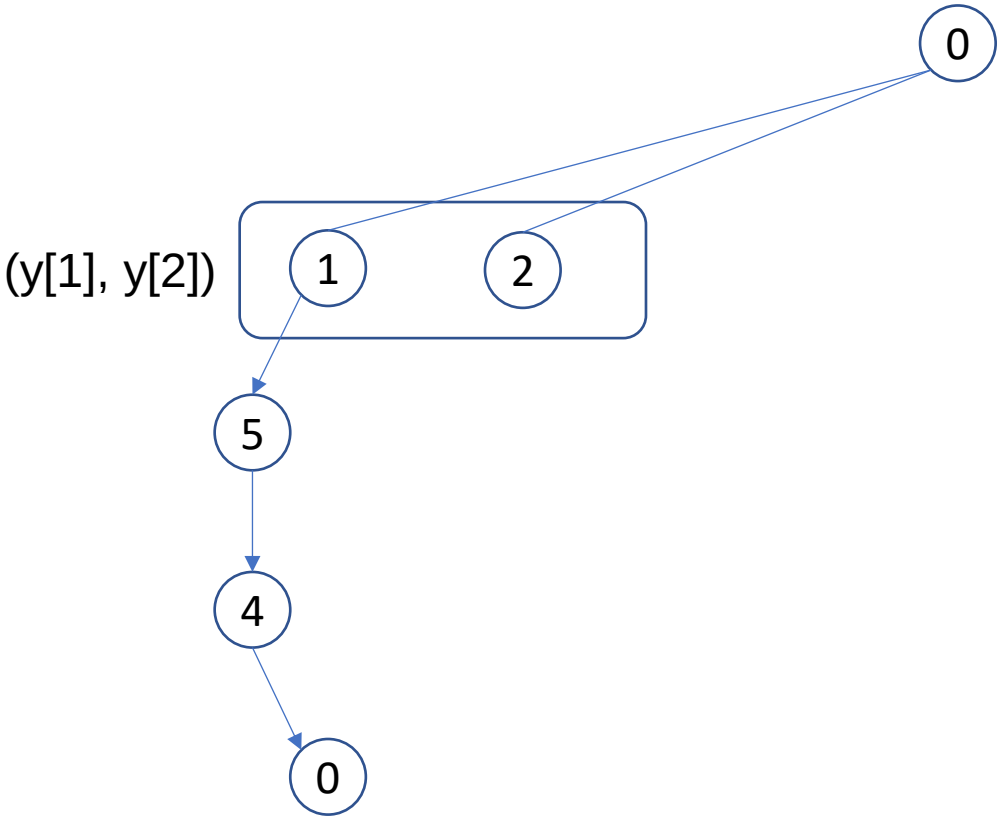

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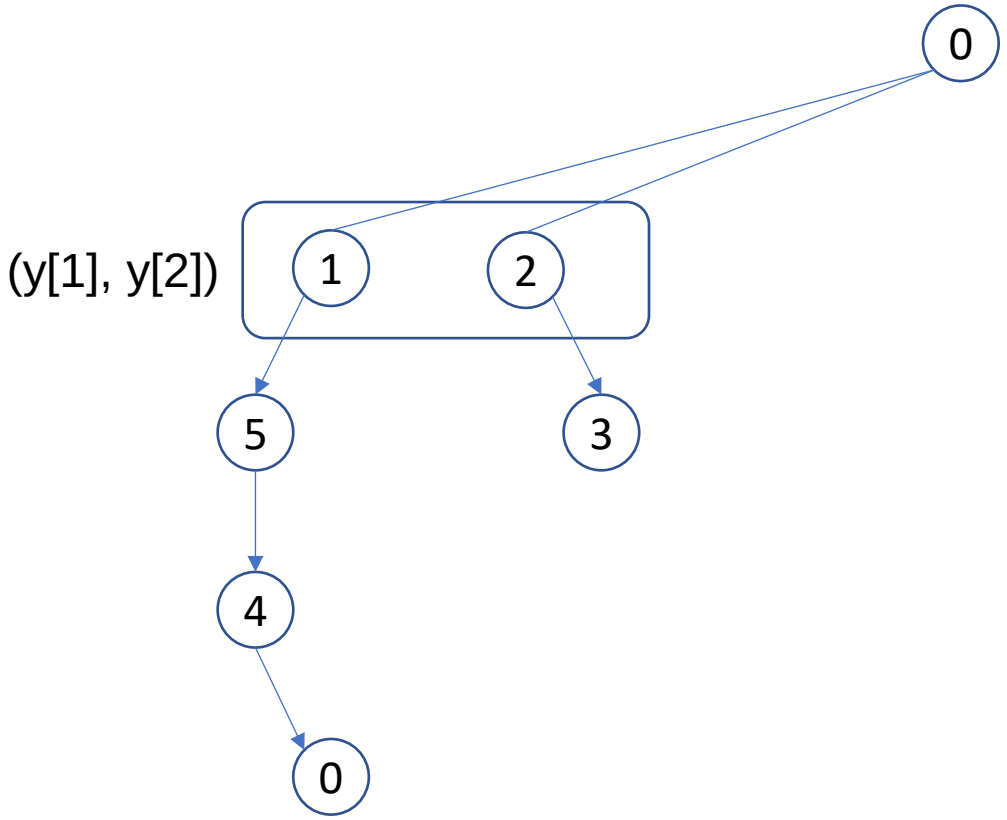

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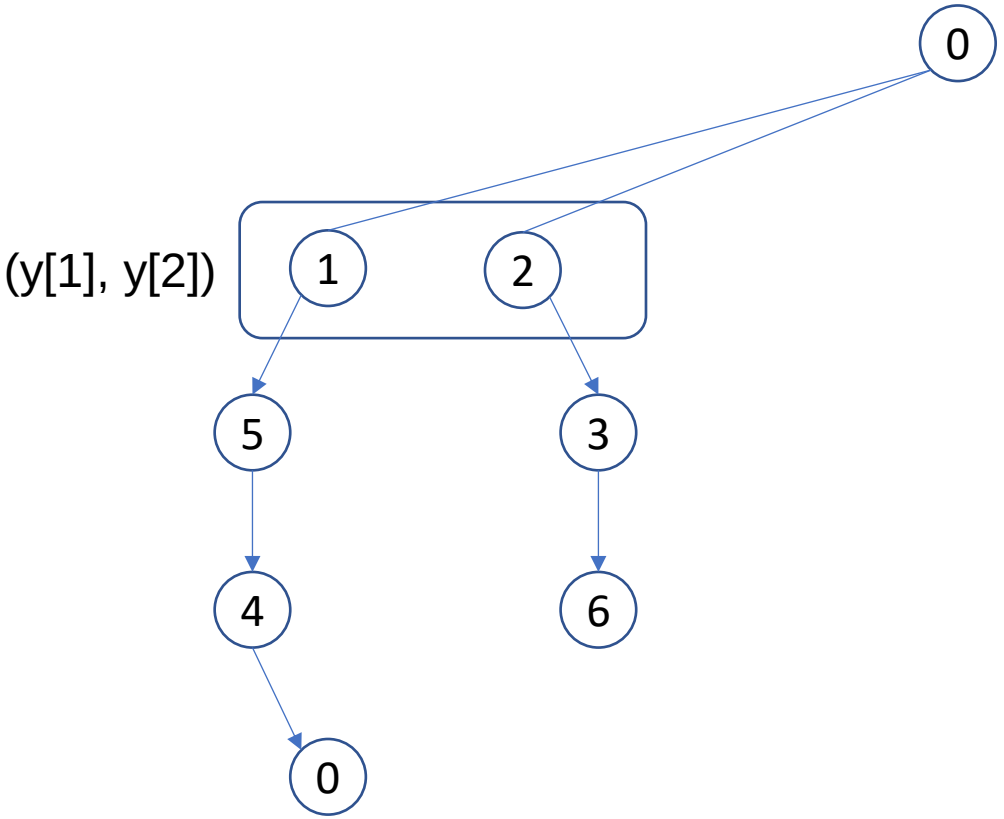

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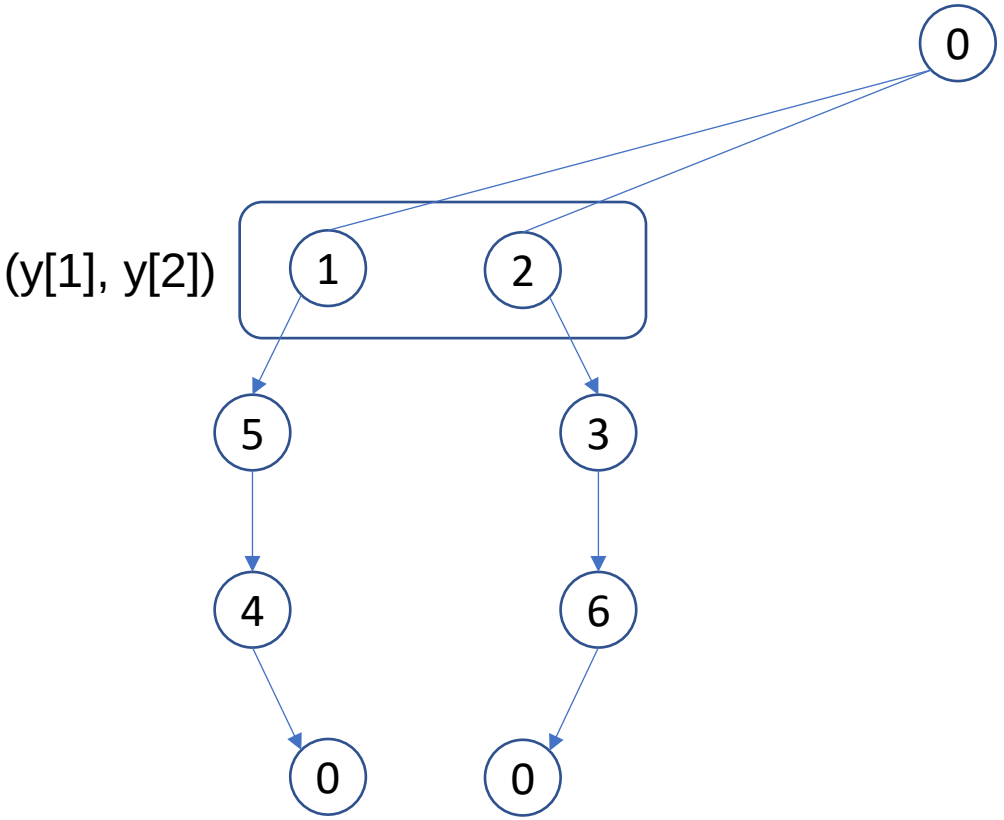

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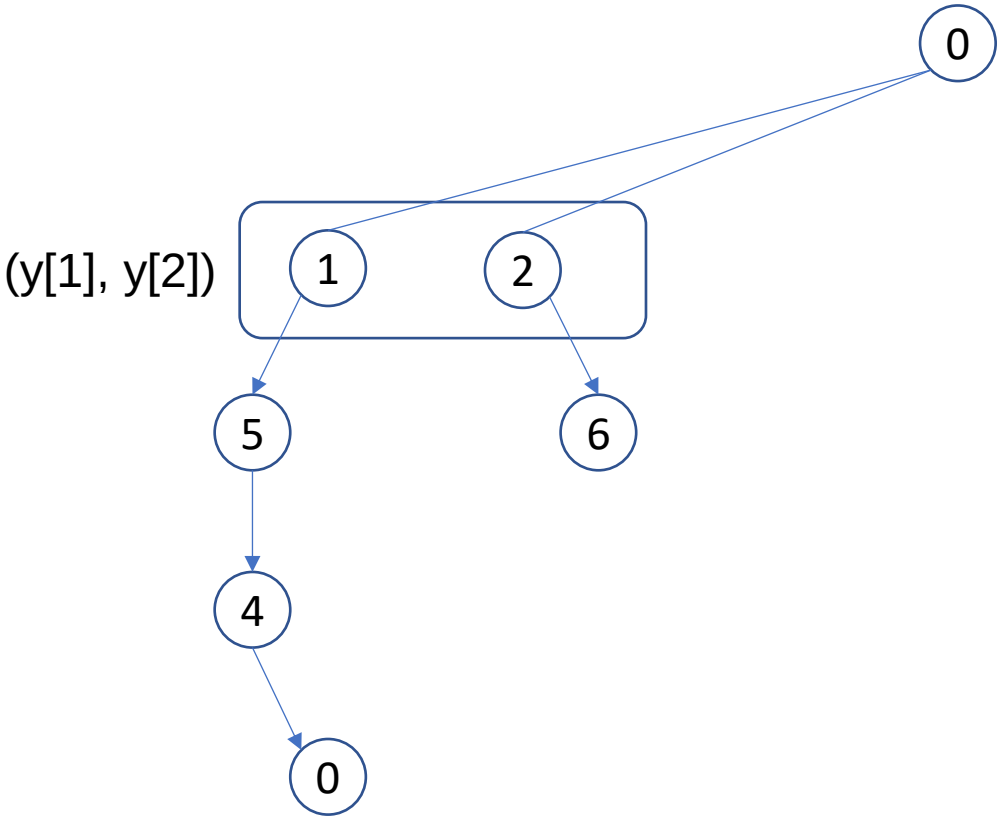

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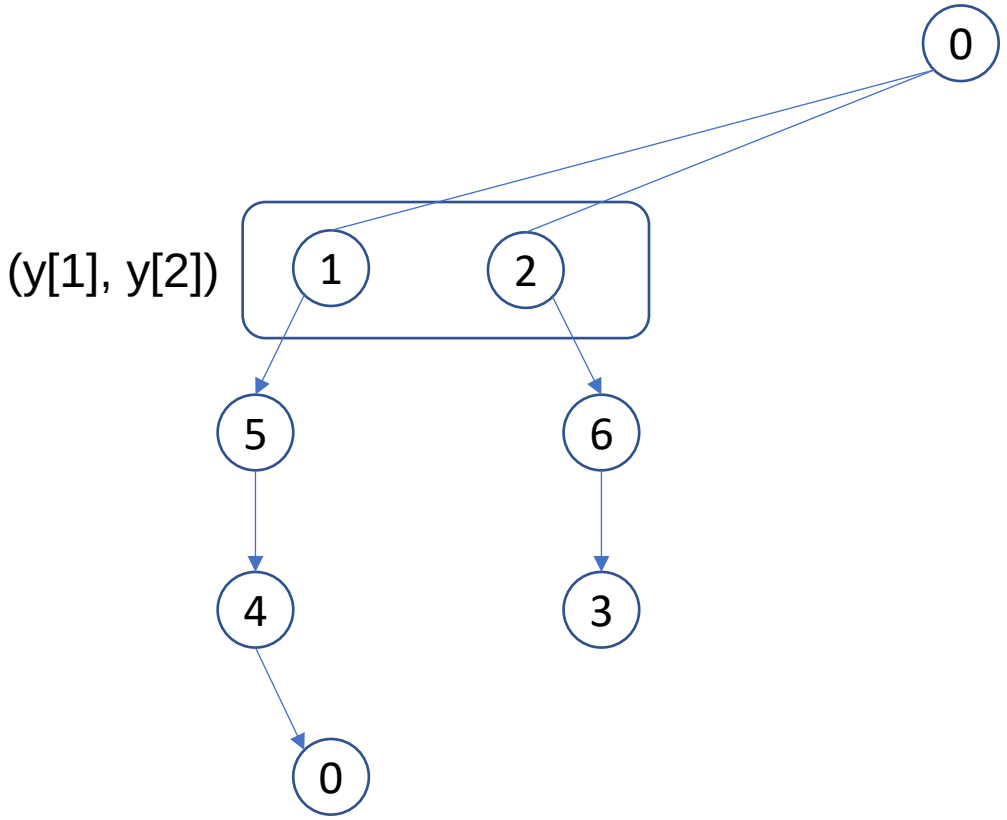

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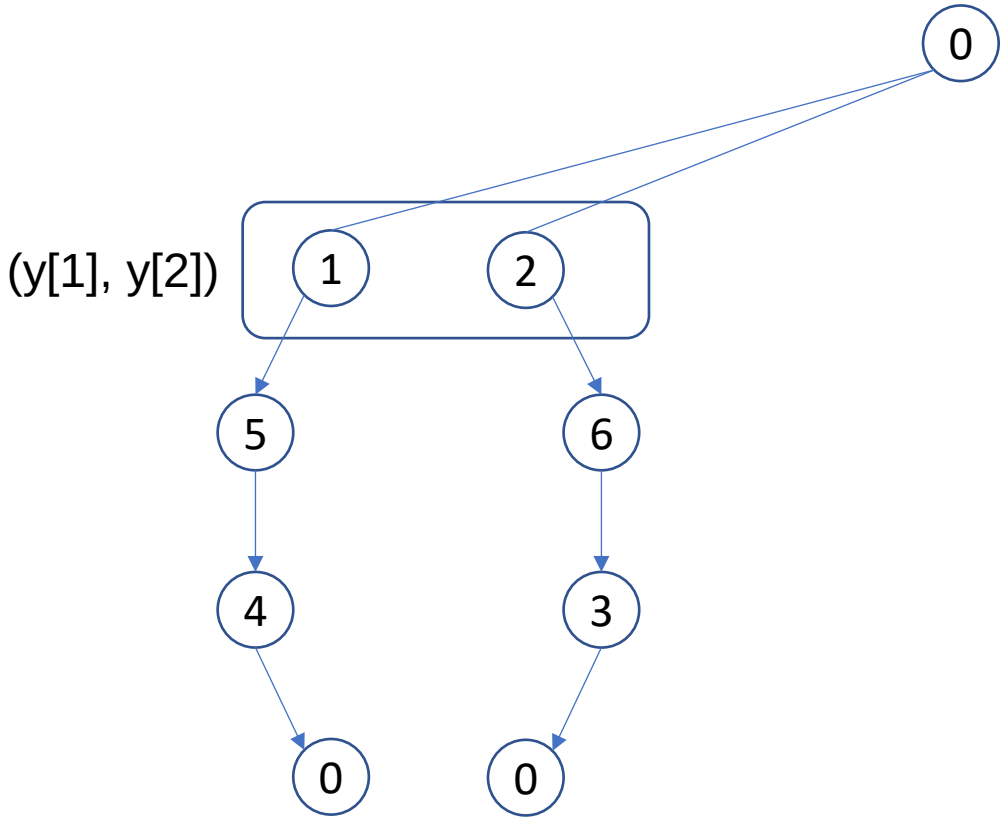

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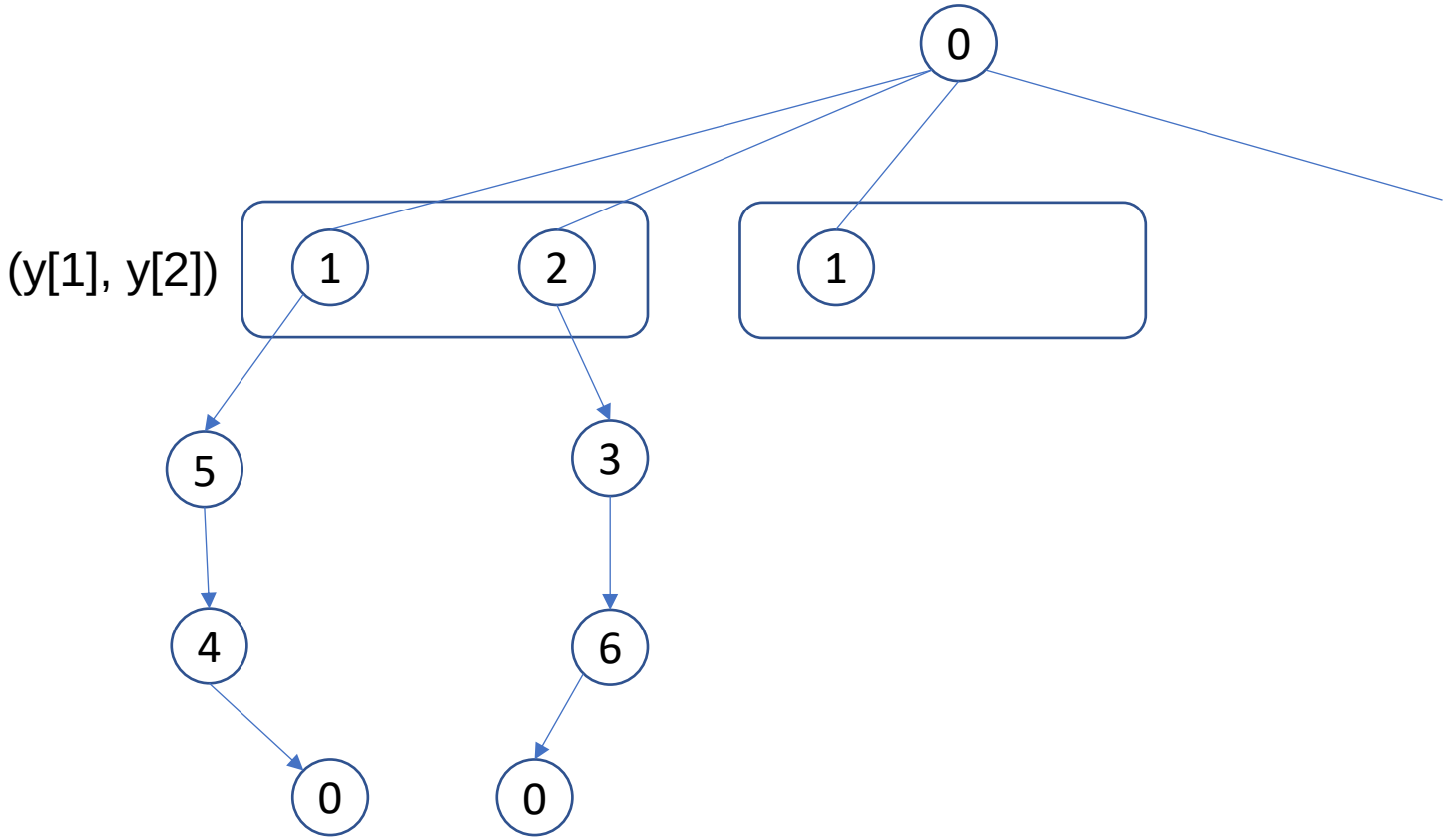

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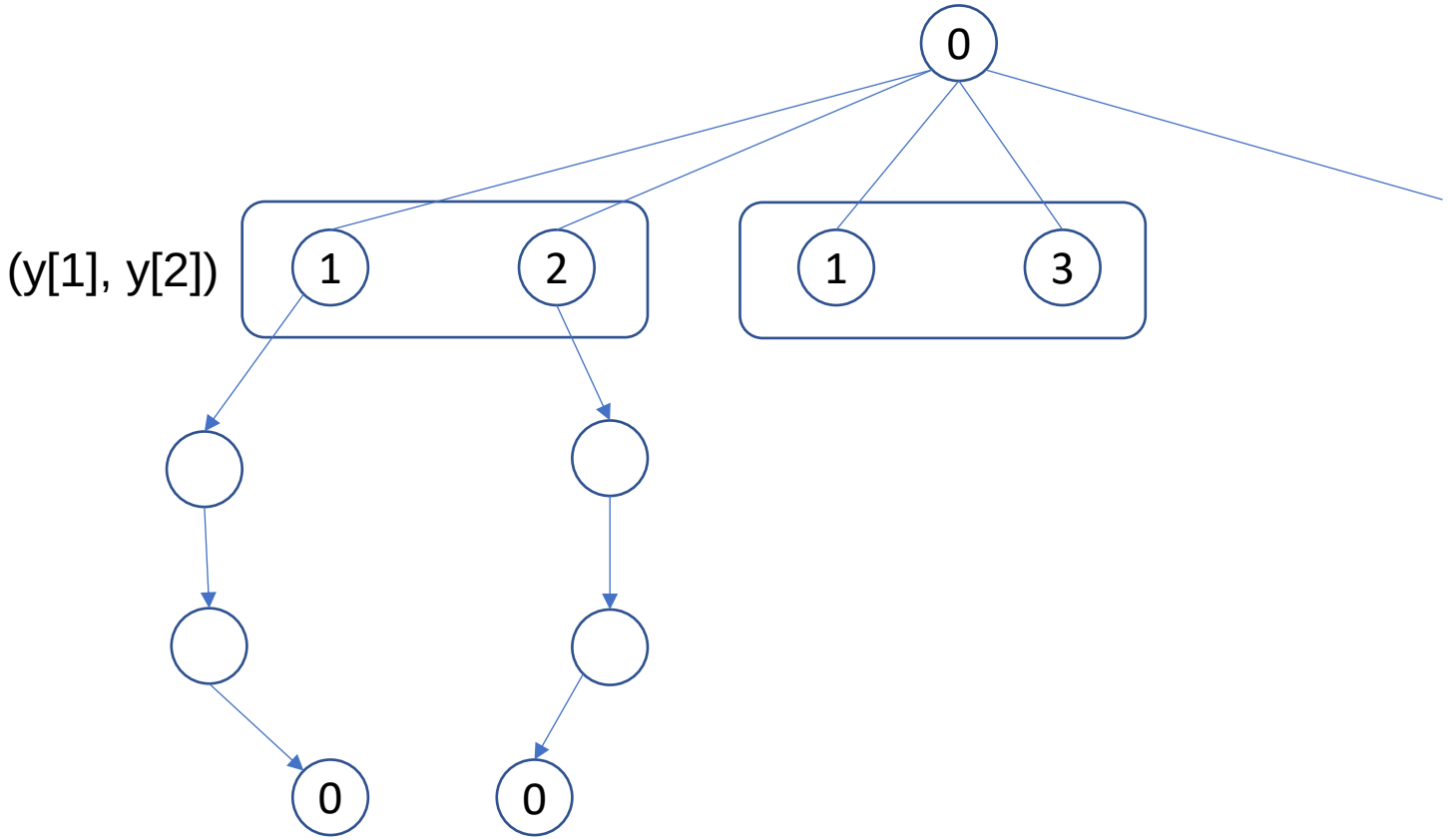

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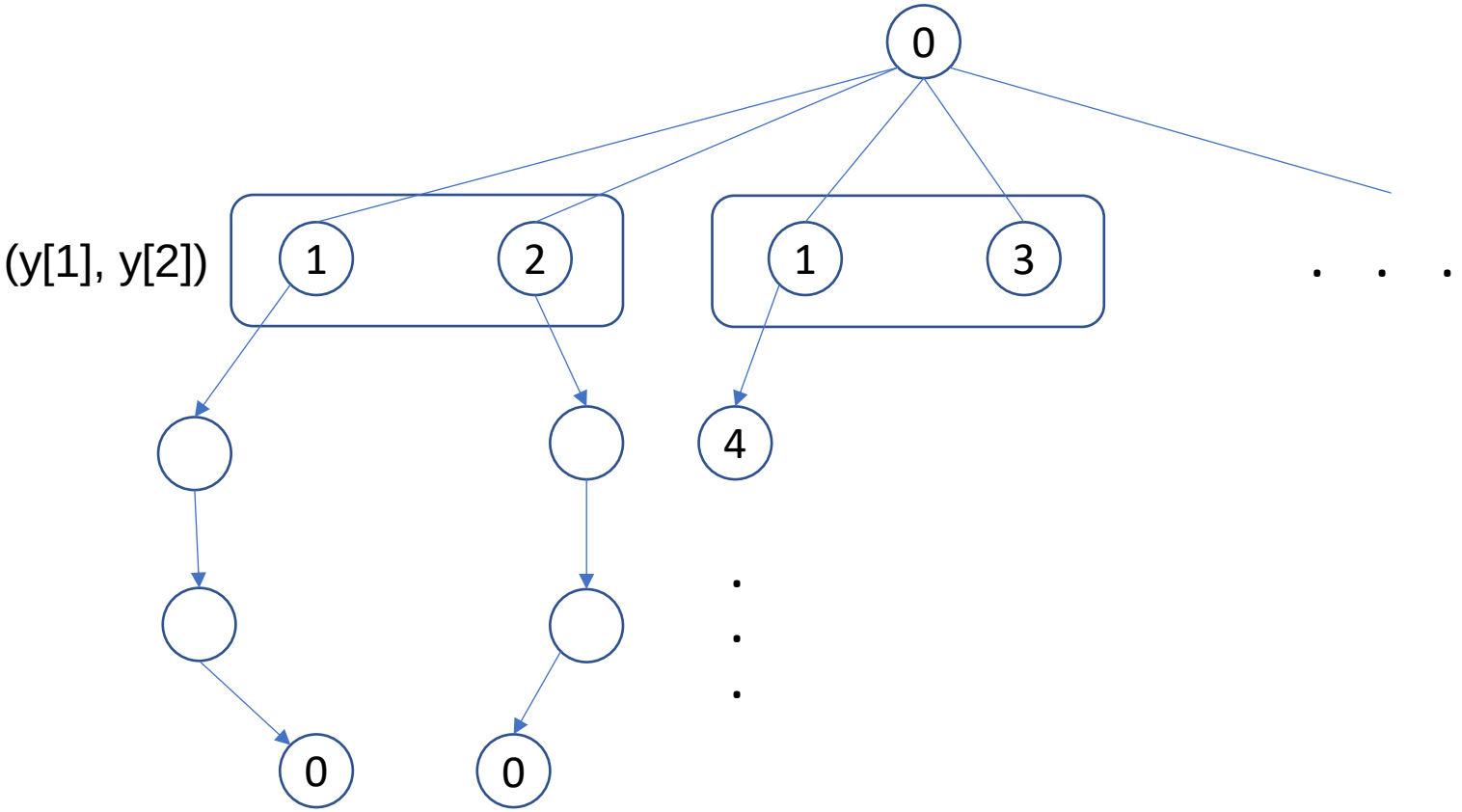

DELIVERY TRUCK ROUTE PROBLEM

```
TRY_X(s, k){// thử giá trị cho x[s]
  if(s = 0) then{
    if k < K then
       TRY_X(y[k+1],k+1);
    return
  }
  [. . .]
}
```

```
checkX(v, k){
   if v > 0 and visited[v]
        then return false;
   if load[k] + d[v] > Q
        then return false;
   return true;
}
```

```
for v = 0 to n do {
    if checkX(v,k) then {
      x[s] = v; visited[v] = true; f = f + c[s,v];
      load[k] = load[k] + d[v]; segments = segments + 1;
      if v > 0 then { if f + (n + nbR – segments)*Cmin < f* then TRY_X(v,k); }
      else{
        if k = K then {
          if segments = n + nbR then updateBest();
        }else{
          if f + (n + nbR – segments)*Cmin < f* then  TRY_X(y[k+1],k+1);
        }
      }
      visited[v] = false; f = f – c[s,v];
      load[k] = load[k] – d[v]; segments = segments – 1;
    }
}
```

DELIVERY TRUCK ROUTE PROBLEM

```
checkY(v, k){
   if v = 0 then return true;
   if load[k] + d[v] > Q then
     return false;
   if visited[v] = true then
     return false;
   return true;
}
```

```
solve(){
  f = 0; f* = +∞; y[0] = 0;
  for v = 1 to n do
    visited[v] = false;
  TRY_Y(1);
  output(f*);
}
```

```
TRY_Y(k){ // thử giá trị cho y[k]
   s = 0;
   if y[k-1] > 0 then s = y[k-1] + 1;
   for v = s to n do {
     if checkY(v,k) then  {
        y[k] = v;
        if v > 0 then segments = segments + 1;
        visited[v] = true;  f = f + c[0,v]; load[k] = load[k] + d[v];
        if k < K then TRY_Y(k+1);
        else { nbR = segments; TRY_X(y[1],1);   }
        load[k] = load[k] – d[v]; visited[v] = false; f = f – c[0,v];
        if v > 0 then segments = segments – 1;
     }
   }
}
```

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THANK YOU !