Introduction to Communications Engineering

IT4593E

Đỗ Công Thuần, Ph.D.

Dept. of CE, SoICT, HUST

Email: thuandc@soict.hust.edu.vn

ONE LOVE. ONE FUTURE.

Thông tin chung

- Tên học phần: **Nhập môn kỹ thuật truyền thông**
- Mã học phần: **IT4593E**
- Khối lượng: **2 TC (2-1-0-4)**
- Lý thuyết và bài tập: **10 buổi lý thuyết, 5 buổi bài tập**
- Đánh giá học phần:

  30% QT (kiểm tra + bài tập/project + chuyên cần-quiz)

  70% CK (trắc nghiệm + tự luận)
- Tài liệu tham khảo:
  - Lecture slides
  - Lecture notes
  - Textbooks, ví dụ ***Communication Systems Engineering***, 2nd Edition, by John G. Proakis Masoud Salehi
  - Internet

Part 2: Digital Modulations

Lec 09: Pulse Amplitude Modulation (PAM)

Modulation Techniques

For each modulation technique, we will consider:

- Characteristics
- Signal Space (Constellation) (Signal set/Vector set)
- Binary Labelling (Gray coding)
- Transmitted Waveform
- Signal Spectrum
- Bandwidth and Spectral Efficiency
- Modulator Structure / Transmitter
- Receiver Structure
- Error Probability
- Practical Applications

Baseband modulation
(Power Spectral Density concentrated around DC)
e.g., PAM

Bandpass modulations
(Power Spectral Density concentrated around $f_0 \neq 0$)
e.g., PSK, QAM, FSK

$p(t)$ is the impulse response of the low-pass filter

Typically, we are interested in the following three low-pass filters:

- $p(t)$ = ideal low pass filter
- $p(t)$ = root raised cosine with roll-off factor α
- $p(t) = P_T(t)$ = Square pulse with duration T

Digital Modulation Techniques

PAM Signal Space

2-PAM Signal Space: Characteristics

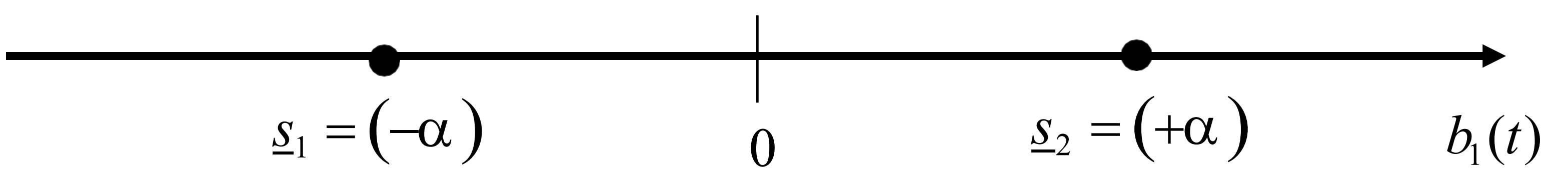
1. Base-band modulation
2. 1-dimensional signal space
3. Antipodal binary constellation
4. Information is "hidden" in the amplitude of the PAM pulse (Pulse Amplitude Modulation)

2-PAM: Không gian tín hiệu

Signal Set $$M = \{s_1(t) = -\alpha\, p(t)\,,\, s_2(t) = +\alpha\, p(t)\}$$

Vector $$b_1(t) = p(t) \qquad (d=1)$$

Vector Space $$M = \{\underline{s}_1 = (-\alpha)\,,\, \underline{s}_2 = (+\alpha)\} \subseteq R$$



$k = 1$ $\boxed{T = T_b}$ $\boxed{\boxed{R = R_b}}$

SOICT

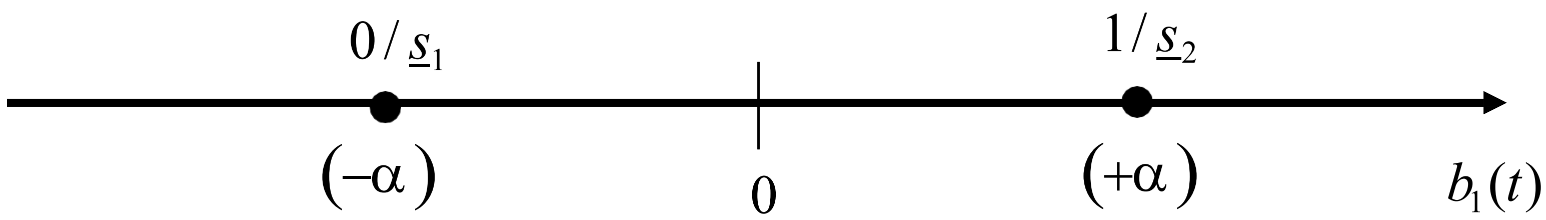
2-PAM: Binary Labelling

Example:

$$e : H_1 \leftrightarrow M$$

$$e(0) = \underline{s}_1$$

$$e(1) = \underline{s}_2$$



2-PAM: Transmitted Waveform

$$s(t) = \sum_{n=-\infty}^{+\infty} a[n]p(t - nT)$$

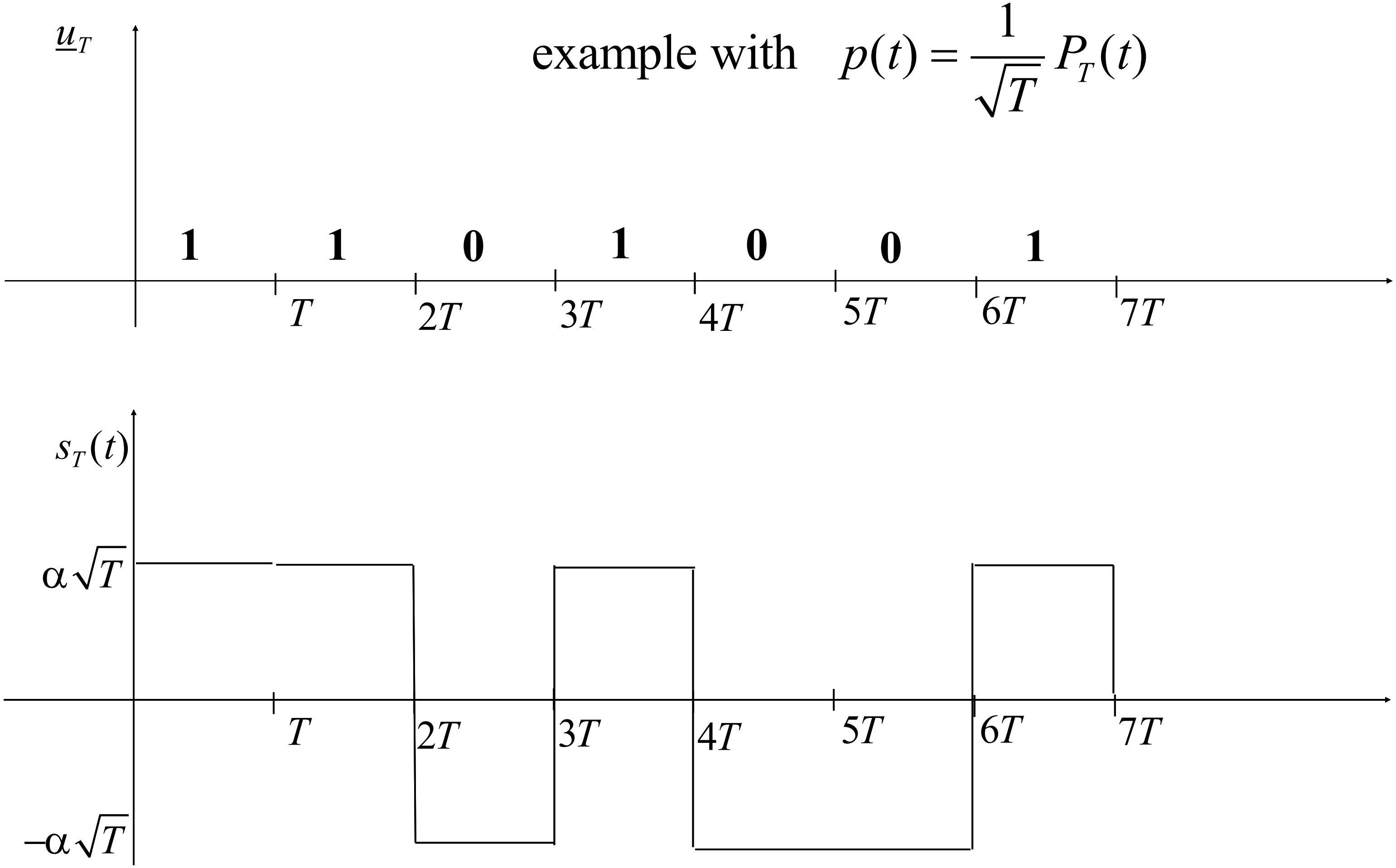
where

$$\boxed{T = T_b}$$

$$a[n] \in \{-\alpha, +\alpha\}$$

Transmitted Waveforms

example with $p(t) = \frac{1}{\sqrt{T}} P_T(t)$

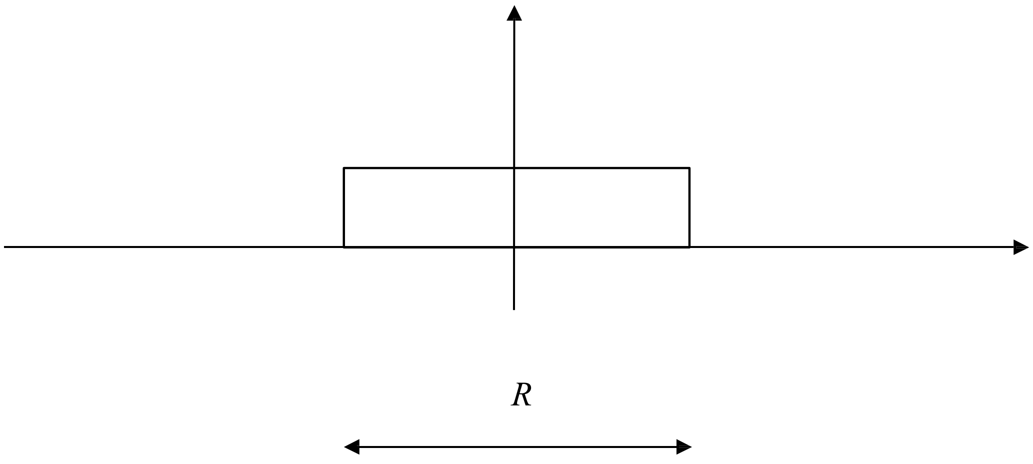


SOICT

2-PAM: Signal Spectrum

$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = x|P(f)|^2 \qquad x \in R$$

Case 1: $p(t)$ = ideal low pass filter



Bandwidth Definition

Bandwidth B [Hz] = frequency region containing the most significant part of the power spectral density $G_s(f)$

Other definitions:

1. Total Bandwidth (contains the entire spectrum)
2. Half-Power Bandwidth (from -3dB below the peak of the spectrum upwards)
3. Equivalent Noise Bandwidth (**square** (with height equal to the maximum value) containing all signal power)
4. “Null-to-null” Bandwidth (width of the main lobe)

SOICT

5. 99% Bandwidth (99.9% etc.) contains 99% of the signal power

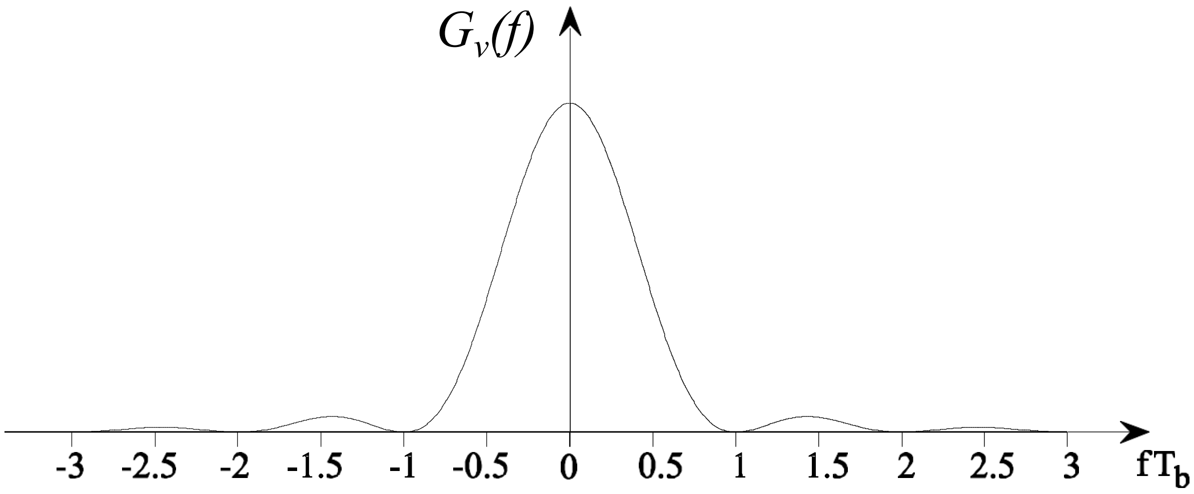
6. Power Spectral Density Bandwidth -35 dB (-50 dB) ($G_s(f)$ from -35 dB below the maximum spectral value)

Example:

Antipodal binary constellation with square pulse:

$$p(t) = b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$$

$$\boxed{G_s(f) = A^2 T \left(\frac{\sin(\pi f T)}{(\pi f T)} \right)^2}$$



Example:

Bandwidth concepts:

1. TOTAL BANDWIDTH $= \infty$
2. Half power bandwidth $\geq 0.44/T_b$
3. Equivalent noise bandwidth $= 0.5/T_b$
4. Null to null bandwidth $= 1/T_b$
5. 99% bandwidth $\geq 10.29/T_b$
6. -35 dB bandwidth $\geq 17.57/T_b$
7. -50 dB bandwidth $\geq 100.52/T_b$

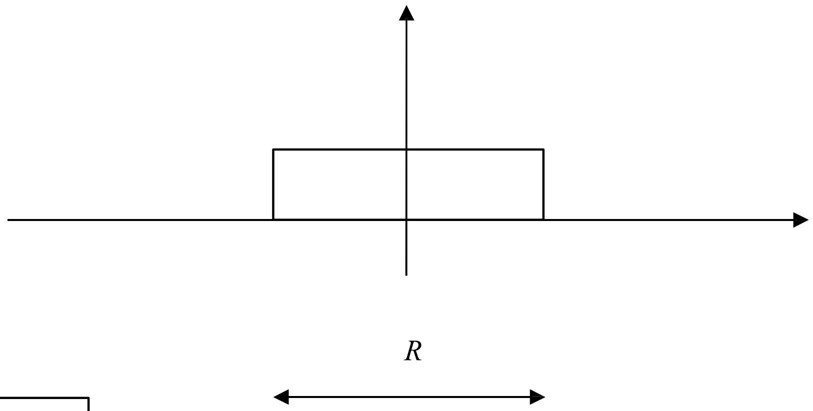
Spectral Efficiency

Spectral Efficiency [bps/Hz]

$$\boxed{\eta = \frac{R_b}{B}}$$

Bandwidth and Spectral Efficiency

Case 1: $p(t)$ = ideal low pass filter



Total Bandwidth (ideal case)

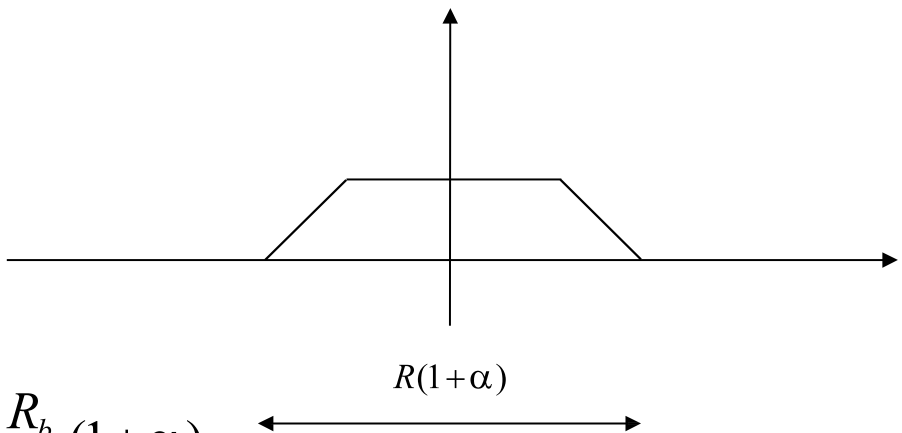
$$\boxed{B_{id} = \frac{R}{2} = \frac{R_b}{2}}$$

Spectral Efficiency (ideal case)

$$\boxed{\eta_{id} = \frac{R_b}{B_{id}} = 2\ bps/Hz}$$

SOICT

Case 2: $p(t)$ = RRC filter with roll-off factor α



Total Bandwidth $$B = \frac{R}{2}(1+\alpha) = \frac{R_b}{2}(1+\alpha)$$

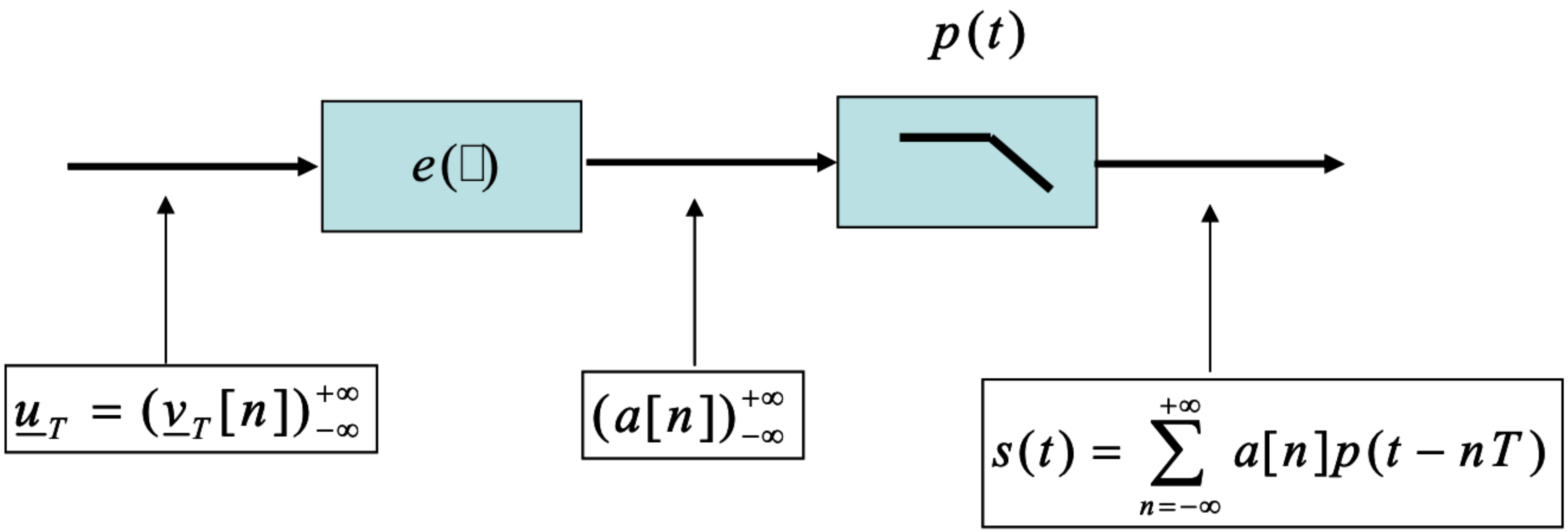
Bandwidth Efficiency $$\boxed{\eta = \frac{R_b}{B} = \frac{2}{(1+\alpha)}\; bps/Hz}$$

Exercise

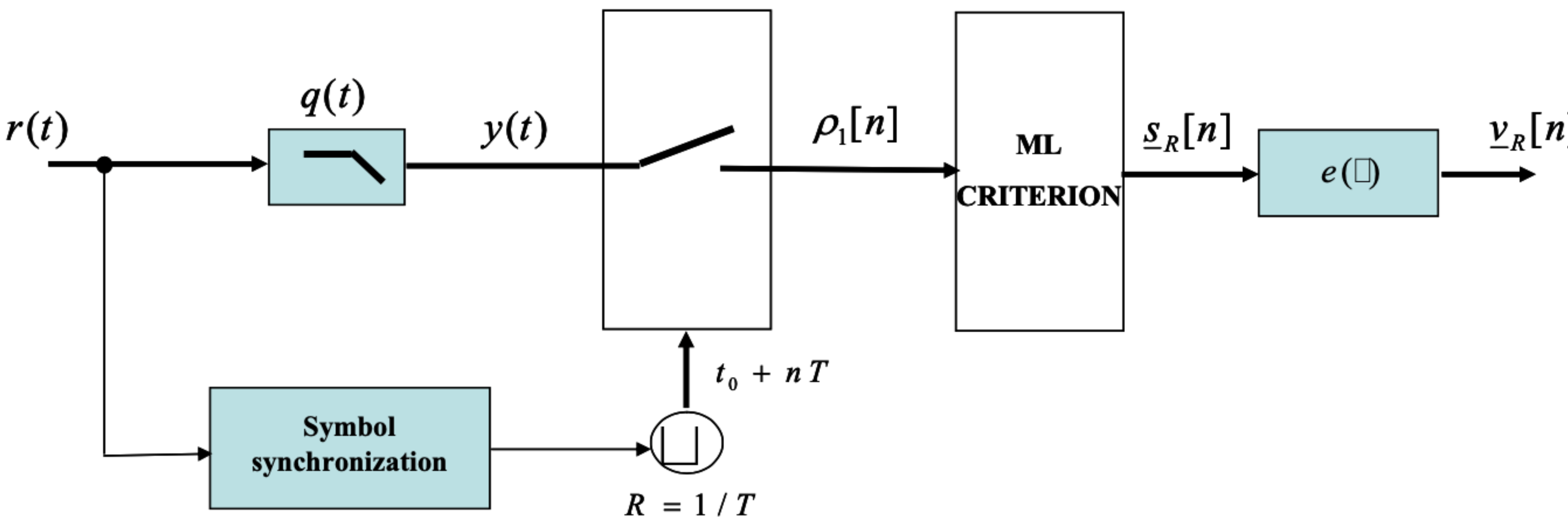
Given a baseband channel with bandwidth B up to 4000 Hz, calculate the maximum bit rate R_b we can transmit over this channel using 2-PAM constellation in two cases:

- Ideal low-pass filter
- RRC filter with alpha = 0.25

2-PAM: Modulator



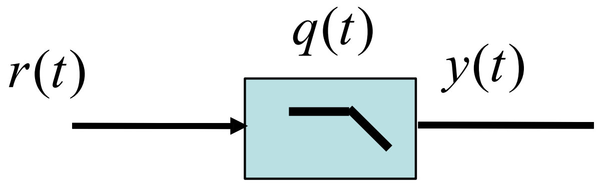
2-PAM: Demodulator



Eye Diagram

For the output of the Matched Filter (MF):

- Divide the output into segments of duration $2T$
- Overlay the segments (oscilloscope)

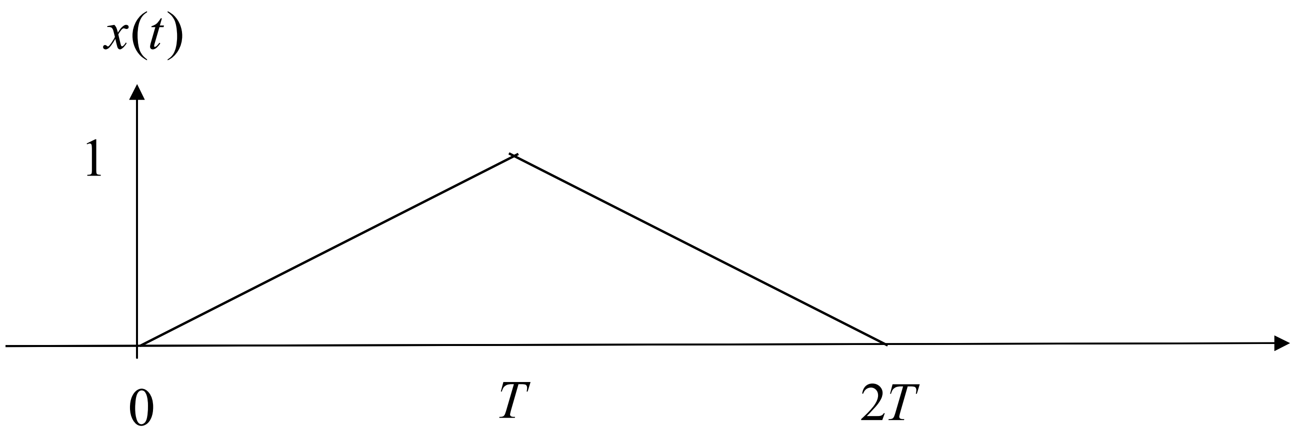


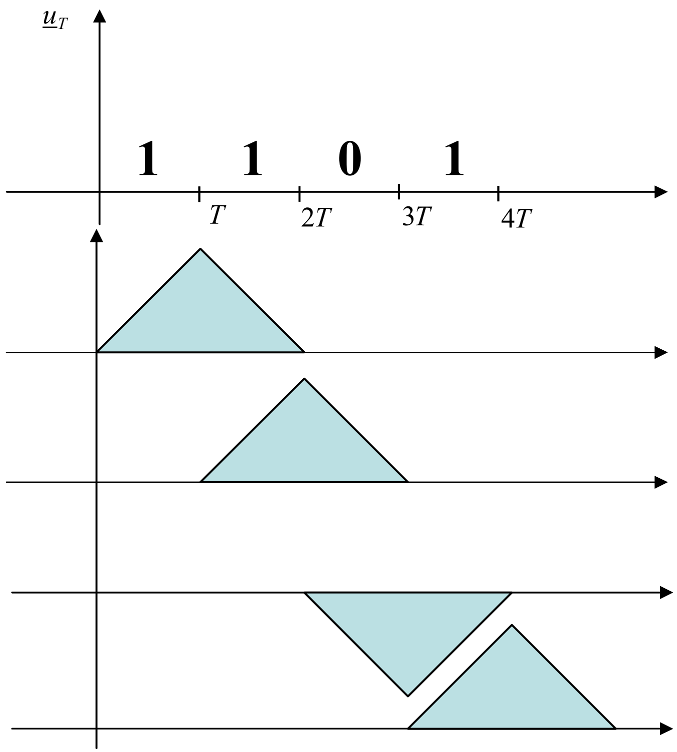
Example:

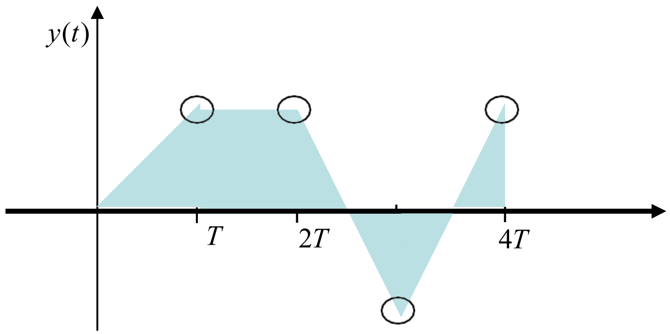
$$p(t) = b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$$

$$q(t) = p(T - t) = \frac{1}{\sqrt{T}} P_T(t)$$

$$x(t) = p(t) * q(t)$$





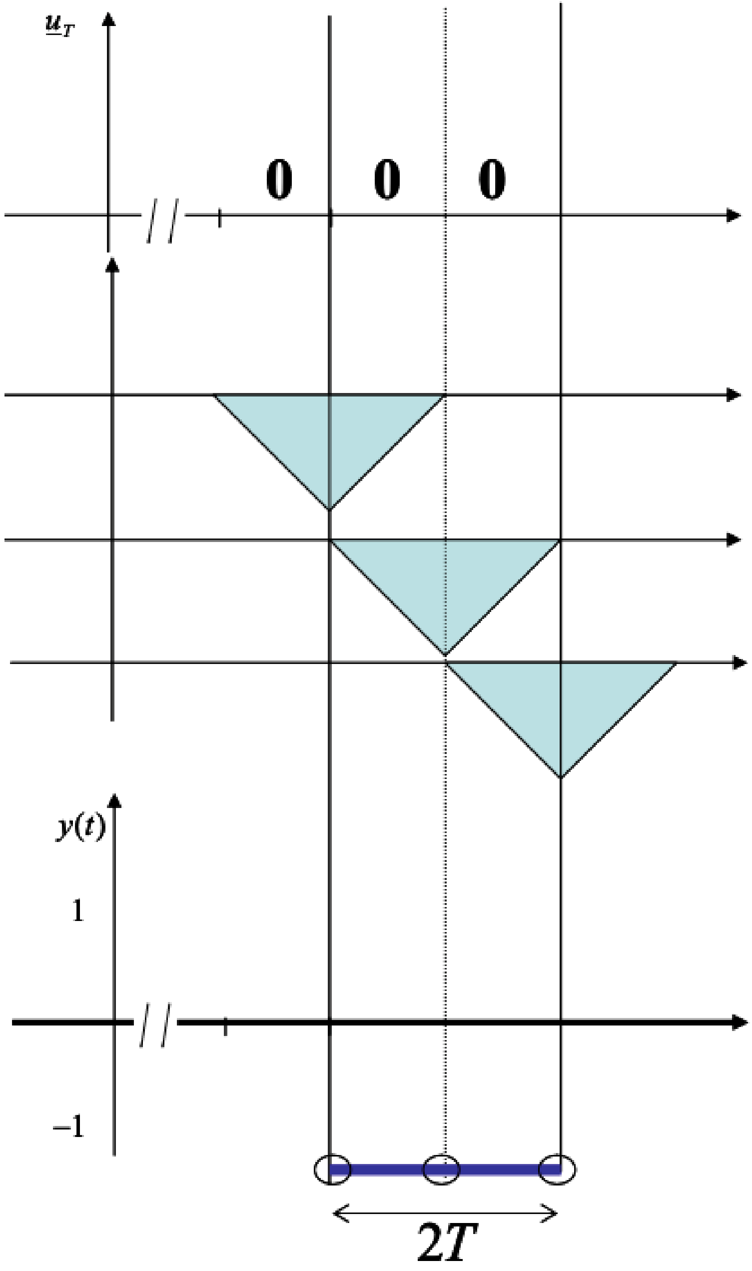


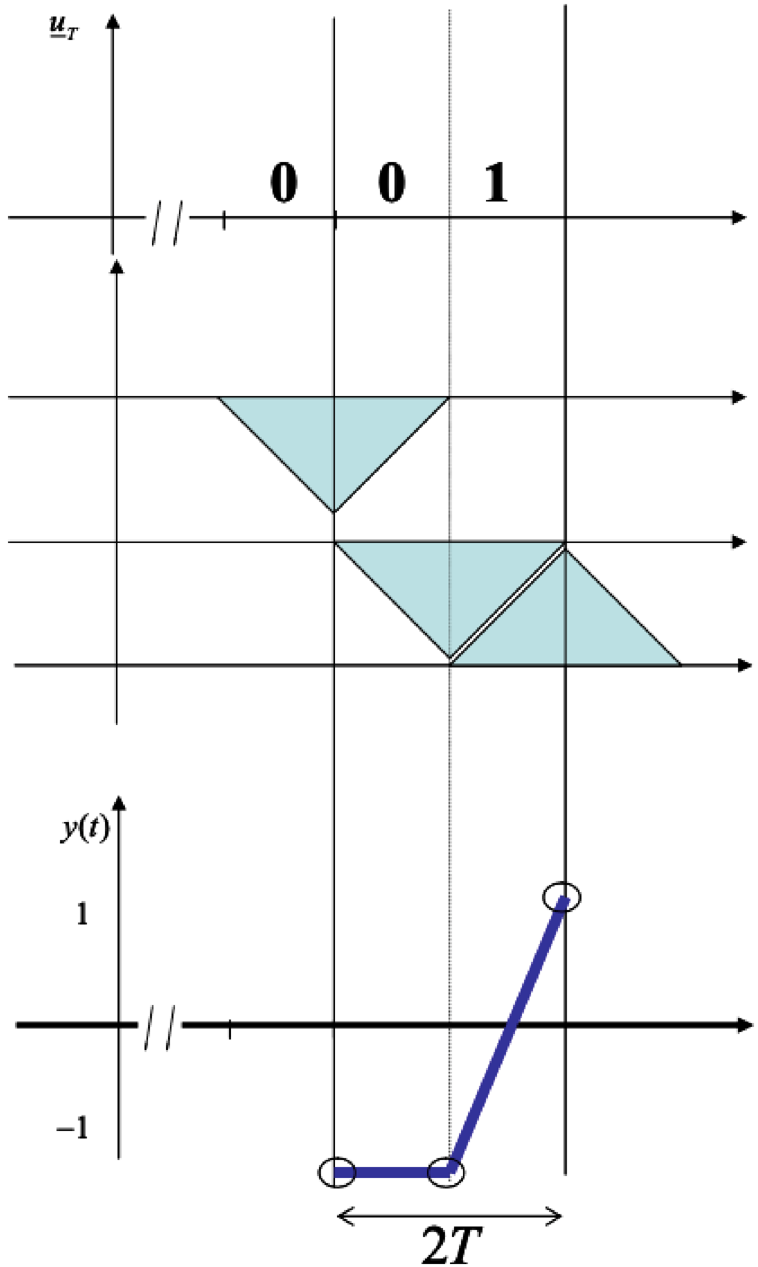
$$y(t) = \sum_n a[n]x(t - nT)$$

$$\rho[n] = y(T + nT) = a[n]$$

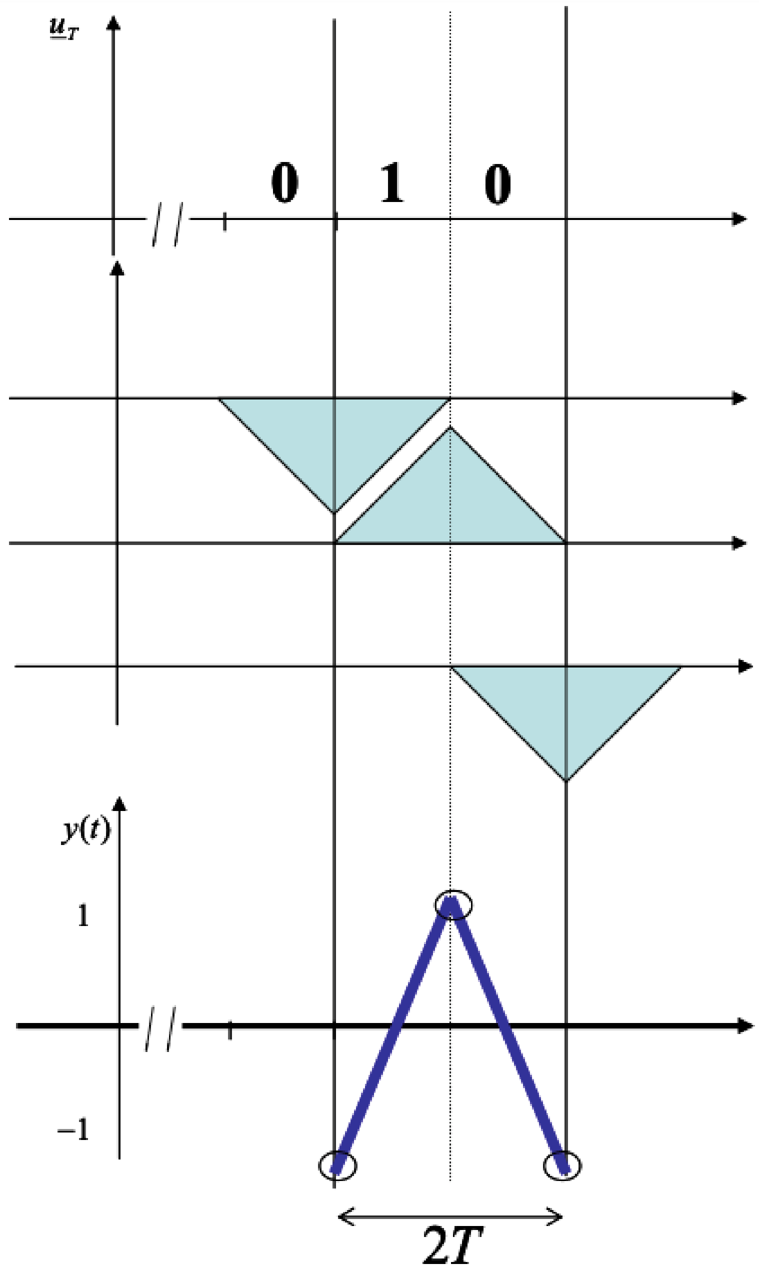
$\boxed{\textbf{NO ISI}}$

Consider all segments of duration $2T$

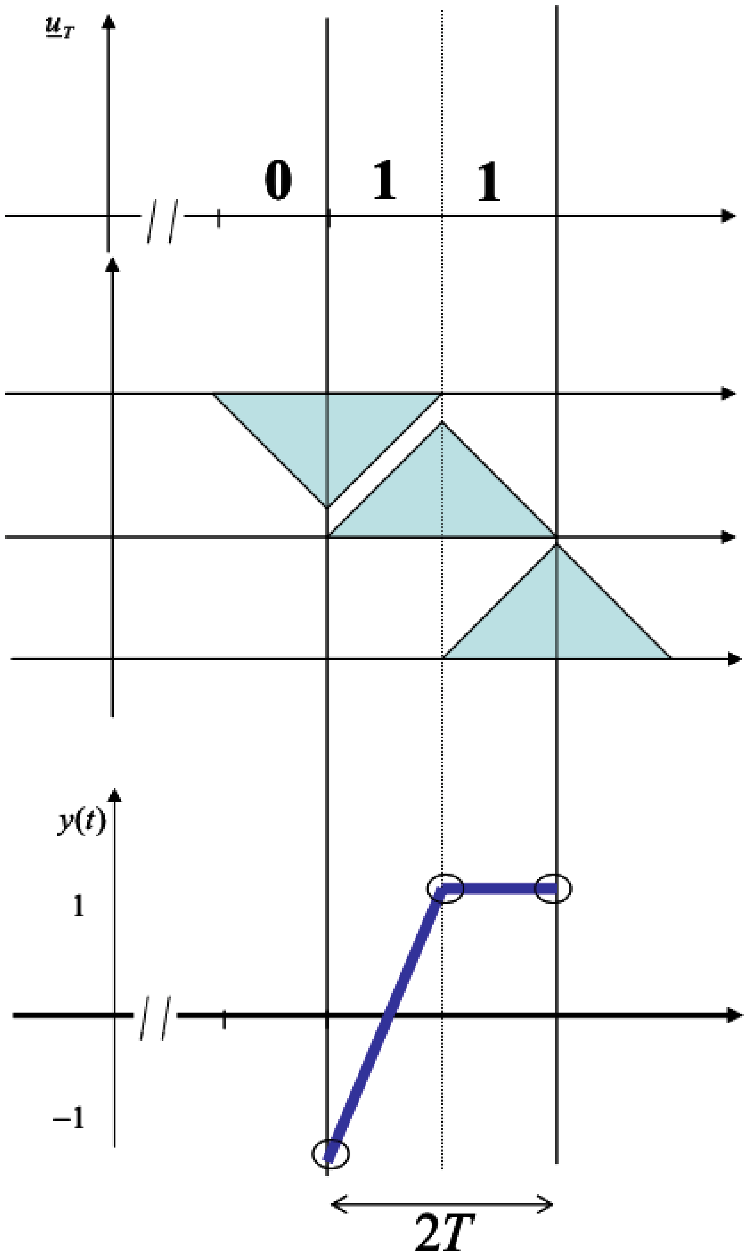




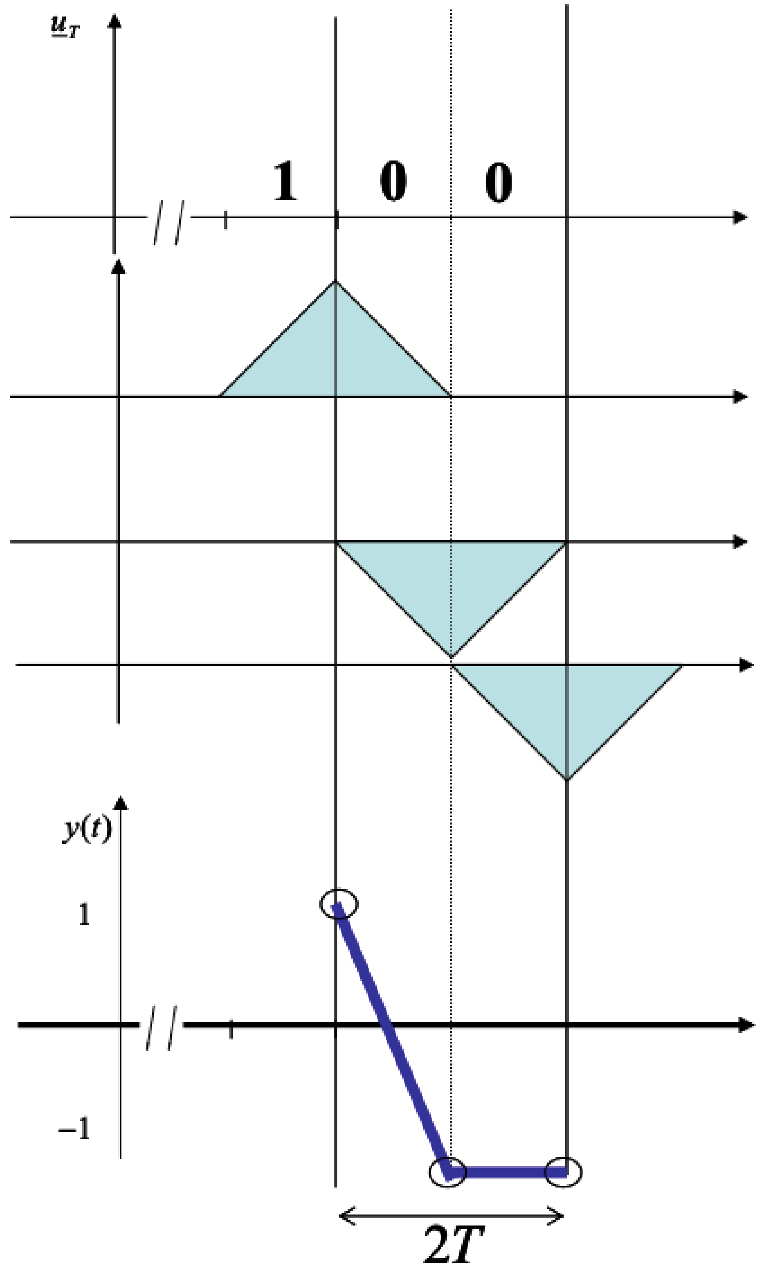
$\underline{u}_T$
0
0
1
$y(t)$
1
−1
$2T$



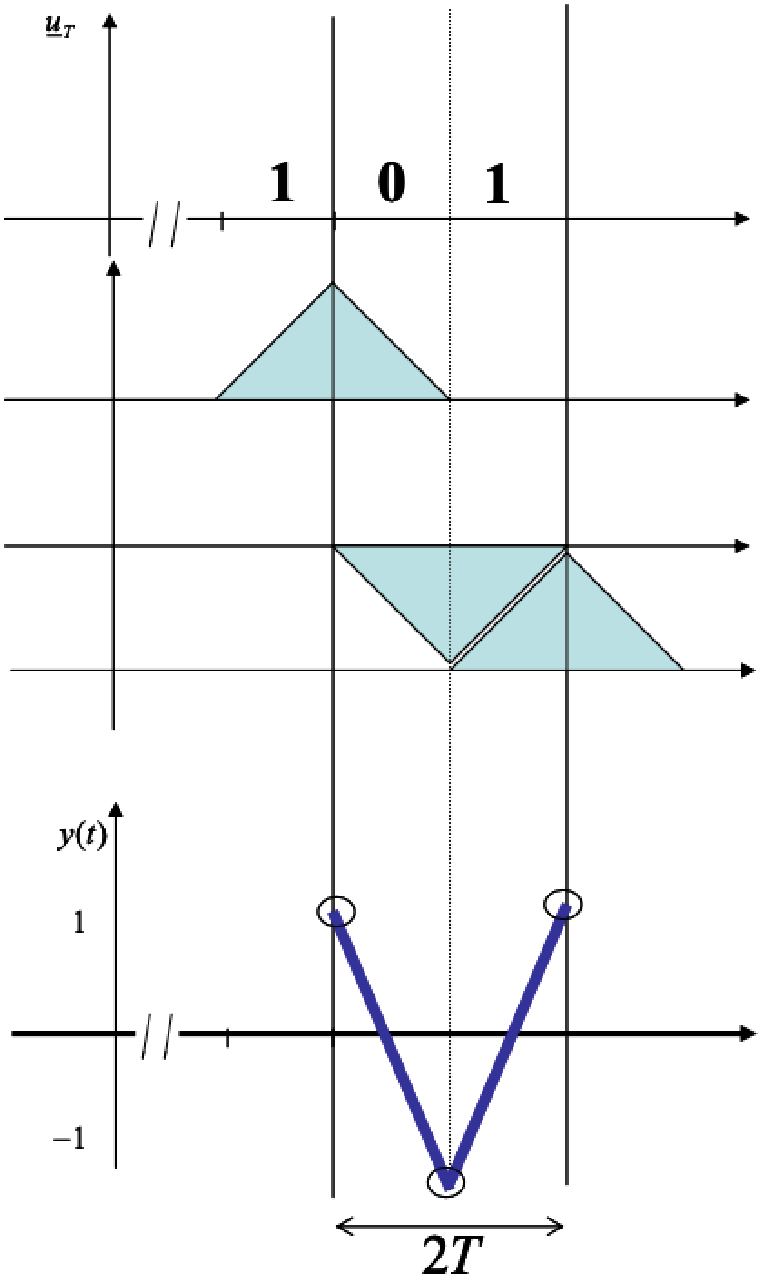
u_T
0
1
0
$y(t)$
1
–1
2T



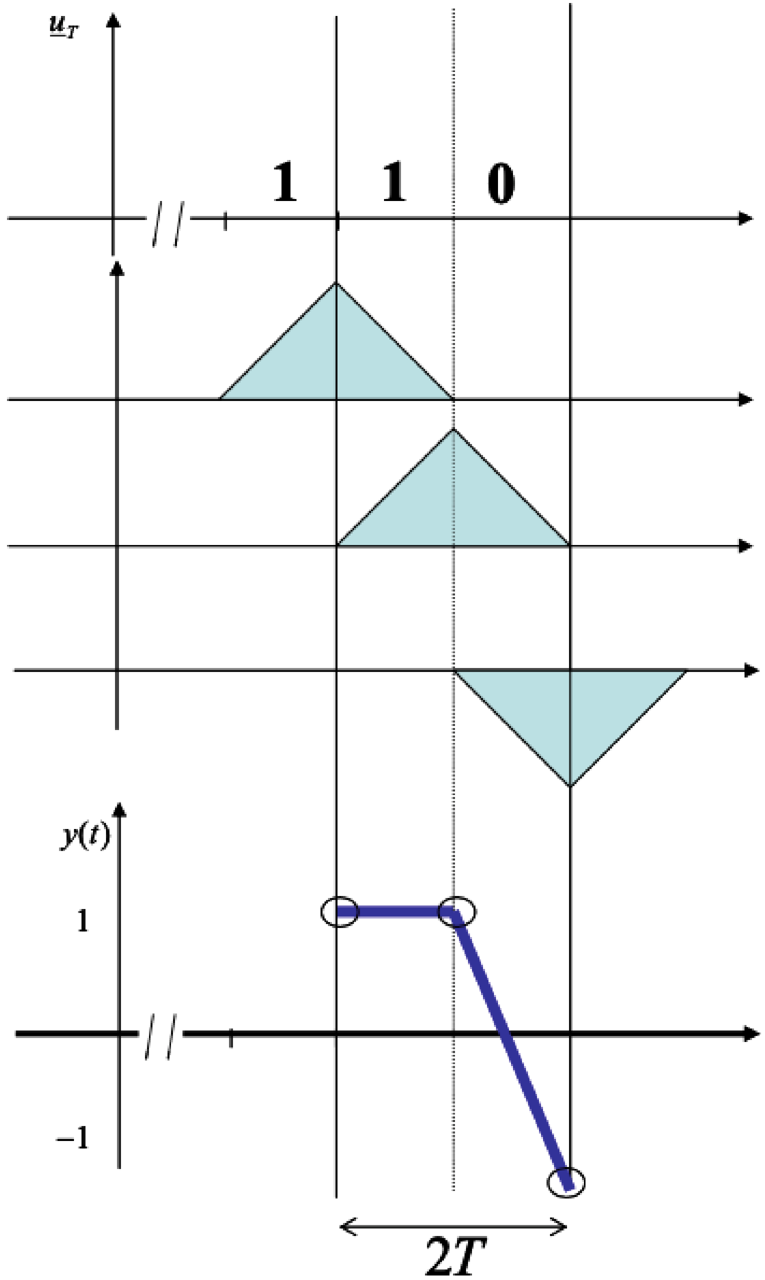
$\underline{u}_T$
0
1
1
$y(t)$
1
−1
$2T$



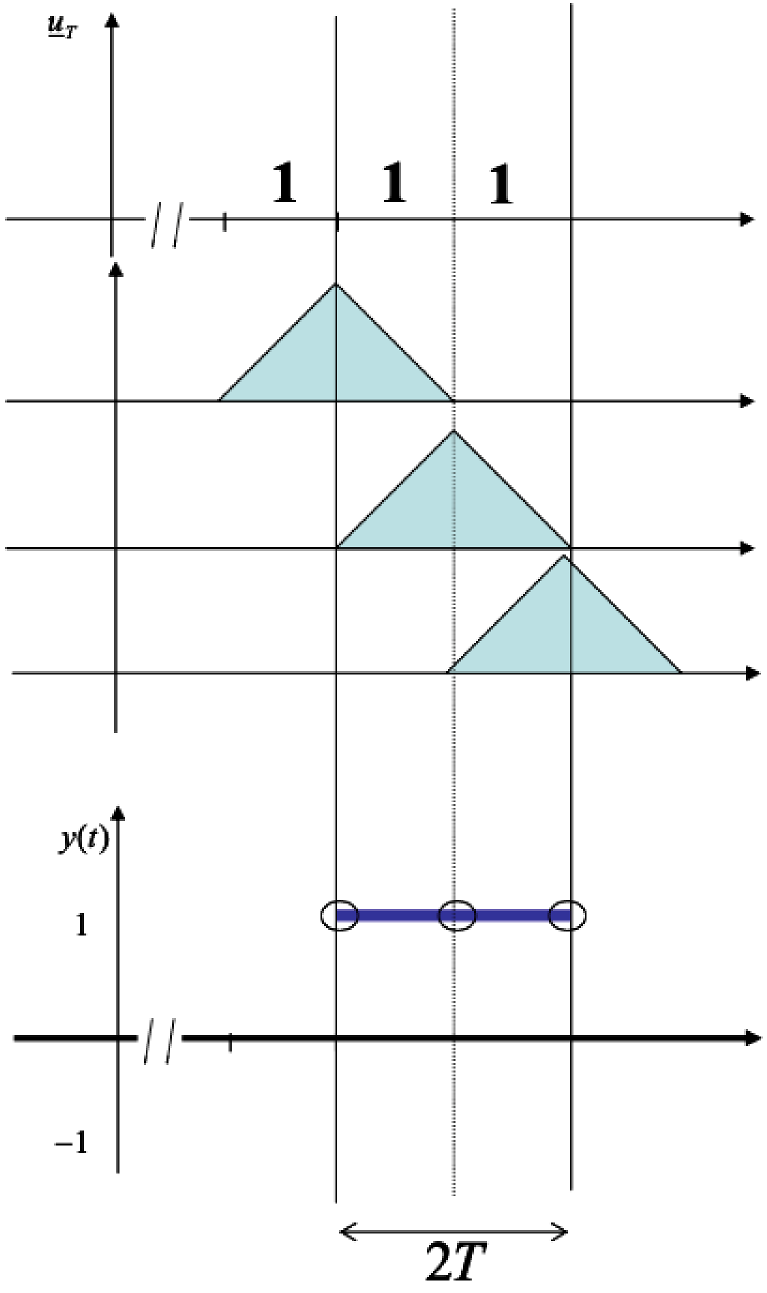
$\underline{u}_T$
1
0
0
$y(t)$
1
−1
$2T$



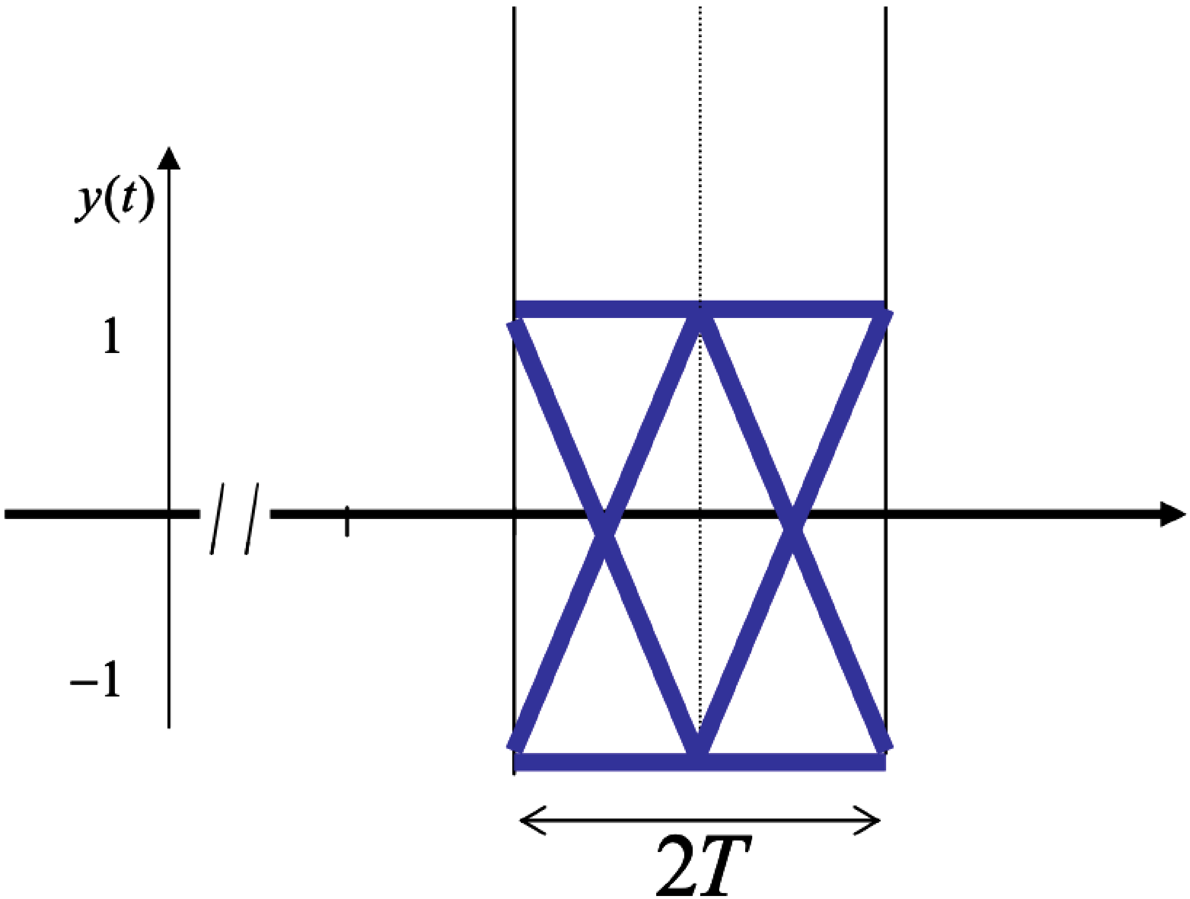
u_T
1
0
1
$y(t)$
1
−1
$2T$



$\underline{u}_T$
1
1
0
$y(t)$
1
−1
$2T$


$\underline{u}_T$
1
1
1
$y(t)$
1
–1
2T

Overlaying Segments

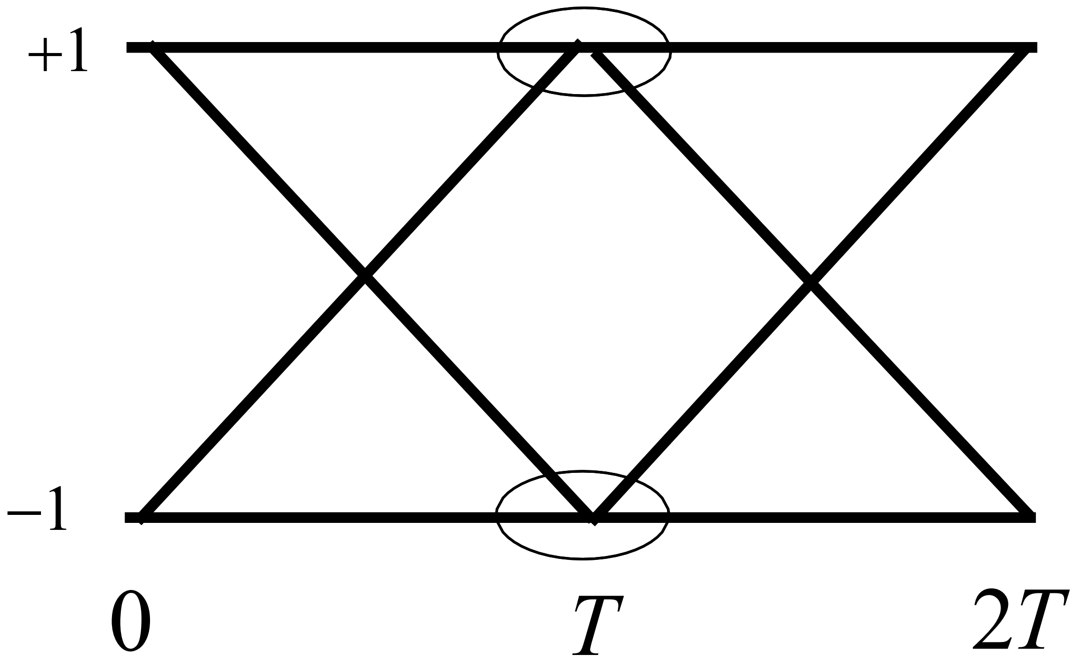


SOICT

Eye Diagram

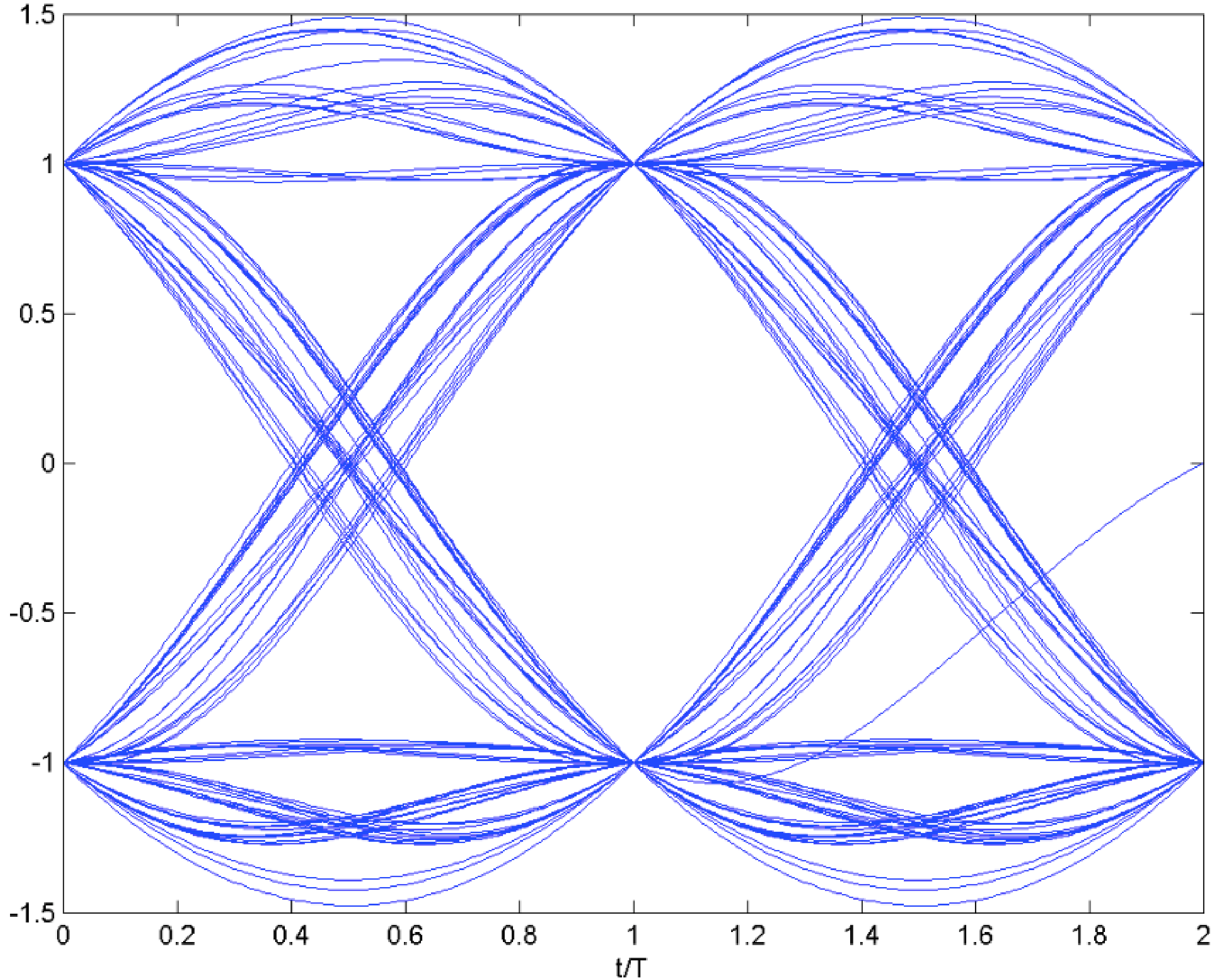
2-PAM constellation with rectangular window

$$p(t) = \frac{1}{\sqrt{T}} P_T(t)$$

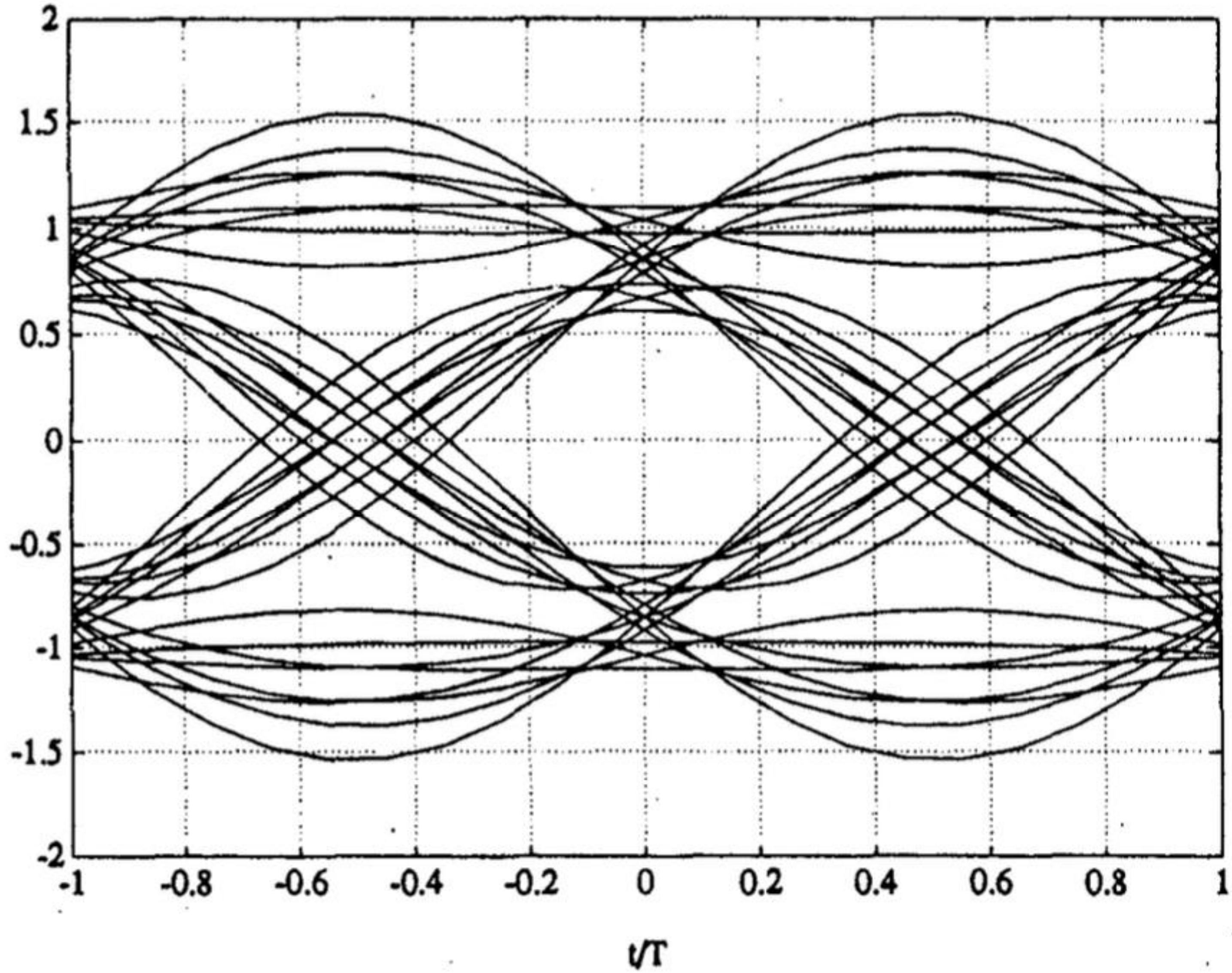


SOICT

Using RRC filter ($alpha$=0.5)



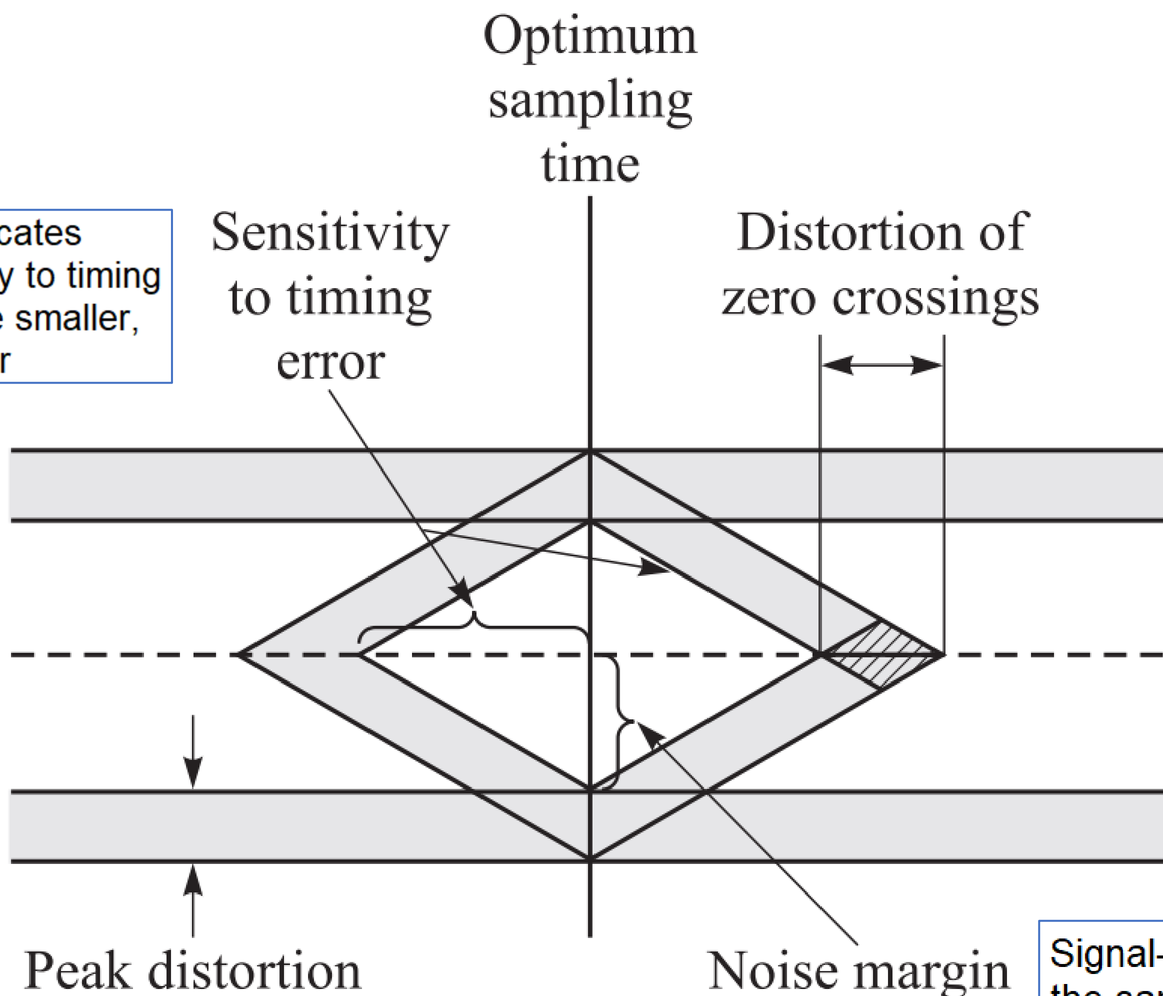
2-PAM in practice



Basic Quantities:

Best time to sample (decision point)
Most open part of eye = best signal-to-noise ratio

Slop indicates sensitivity to timing error; the smaller, the better



Signal-to-noise ratio at the sampling point

Amount of distoration (set by signal-to-noise ratio)

2-PAM: Error Probability

$$BER = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$