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HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

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APPLIED ALGORITHMS

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BACKTRACKING, BRANCH AND BOUND

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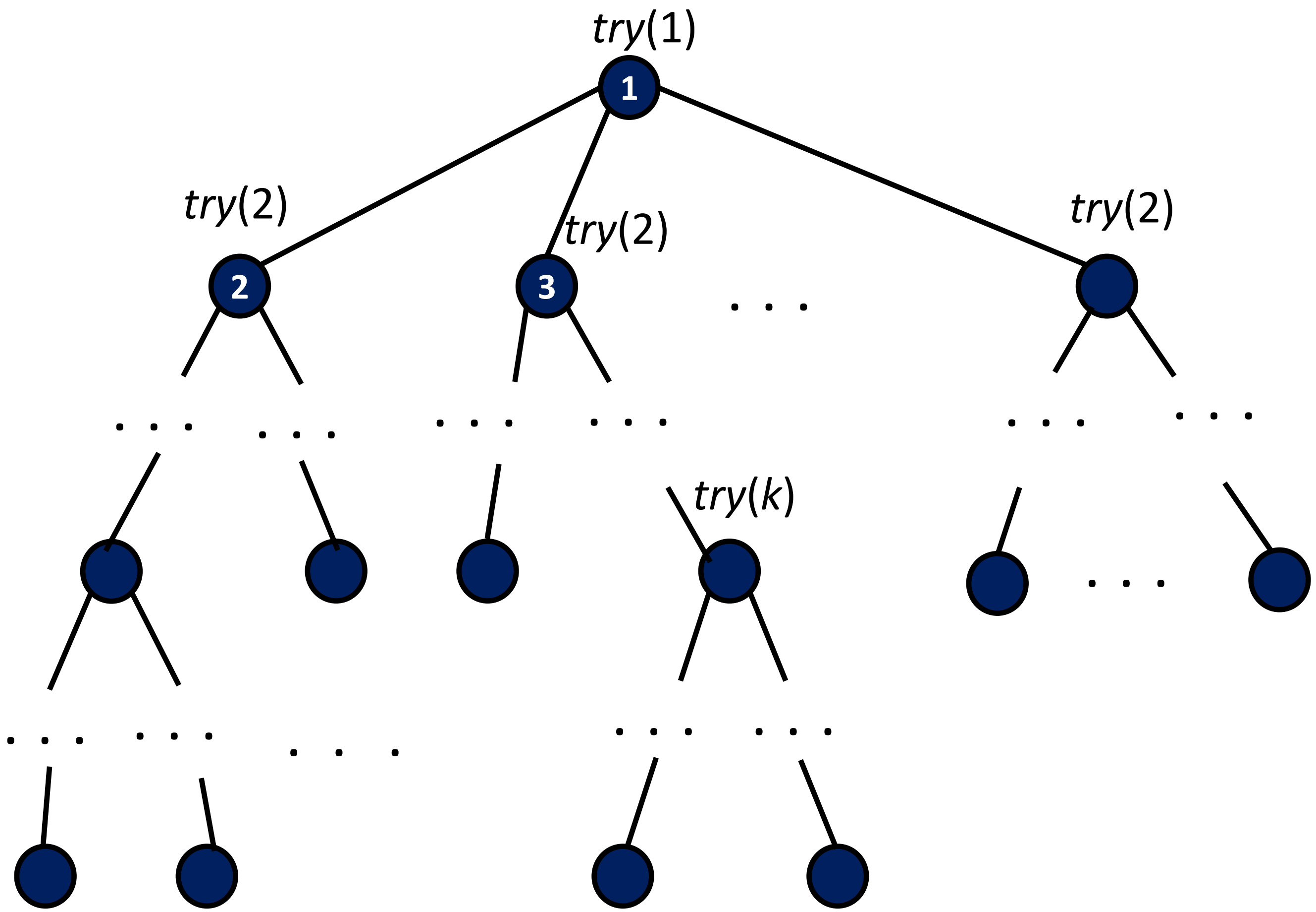
- General diagram of backtracking, branch and bound
- The problem of bus routes picking up and dropping off passengers
- Delivery truck route problem
- 2D material cutting problem

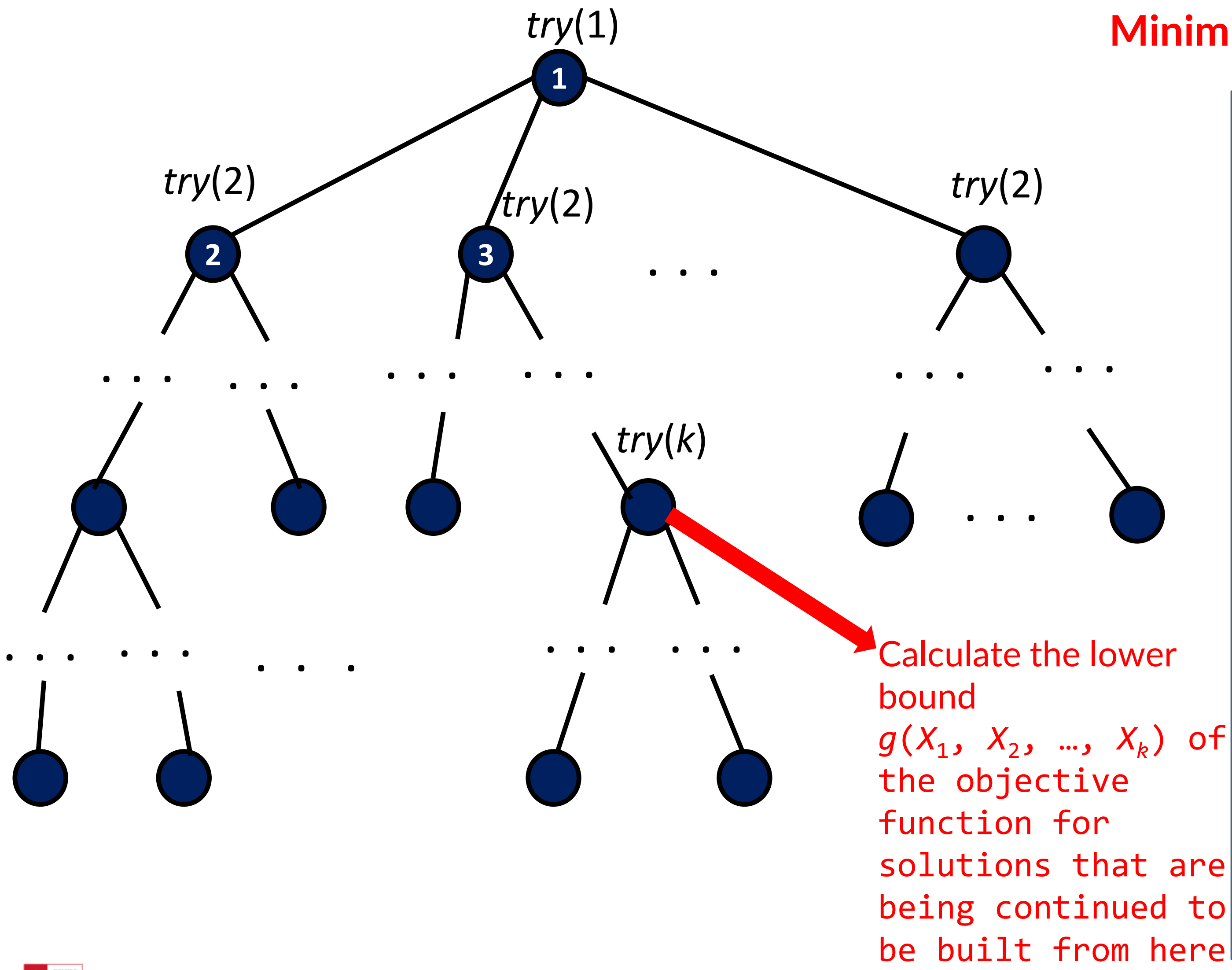
GENERAL DIAGRAM OF BACKTRACKING, BRANCHING AND BOUND

- The backtracking algorithm allows us to solve combinatorial enumeration problems and combinatorial optimization problems
- The alternative is modeled by a sequence of decision variables $X_1, X_2, \ldots, X_n$
- Need to find for each variable X_i a value selected from a given discrete set A_i such that
  - The constraints of the problem are satisfied
  - Optimize a given objective function
- Backtracking algorithm
  - Traverse through all variables (e.g. order from $X_1, X_2, \ldots, X_n$), for each variable X_k :
    - Traverse through all possible values that could be assigned to X_k, for each value v:
      - Check constraints
      - Assign $X_k = v$
      - If $k = n$ then record a solution to the problem
      - Otherwise, consider the variable X_{k+1}



Enumeration problem

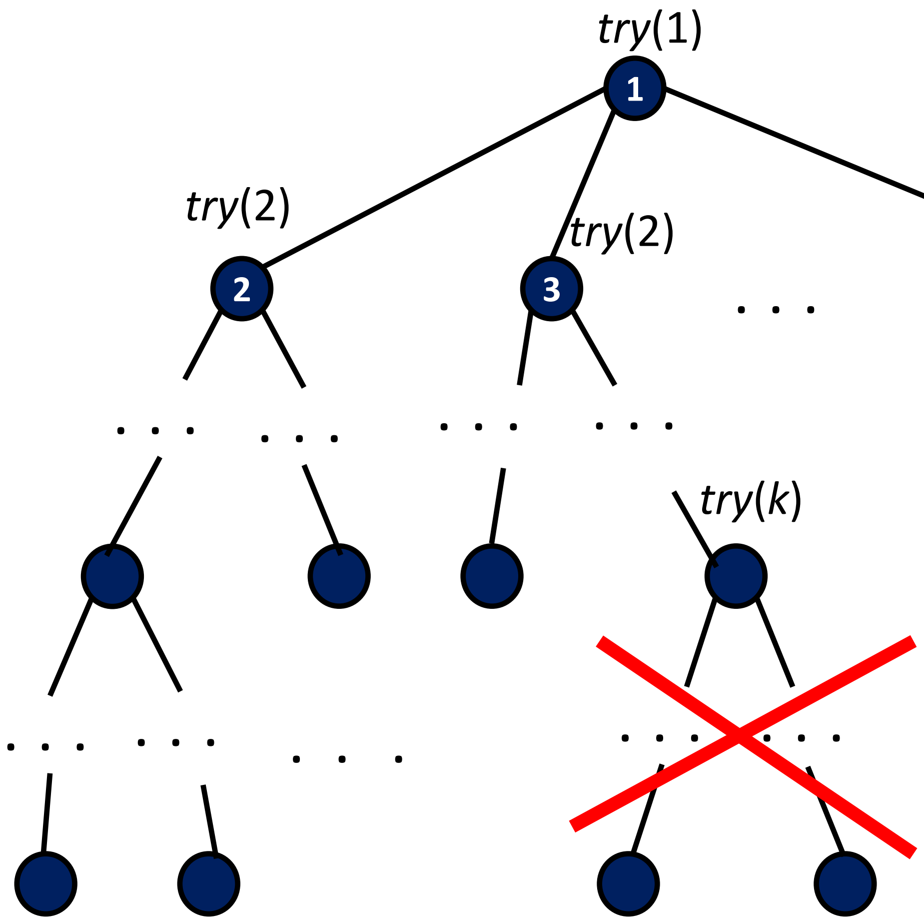
```
try(k){ //Try out the possible values assigned to X_k
  for v in A_k do {
     if check(v,k){
        X_k = v;
        [Update a data structure D]
        if k = n then solution();
        else {
            try(k+1);
        }
        [Recover the data structure D]
     }
  }
}
```



Minimize optimization problem (Denote f^* : optimal value)

```
try(k){//Try out the possible values assigned to X_k
  for v in A_k do {
     if check(v,k){
        X_k = v;
        [Update a data structure D]
        if k = n then updateBest();
        else {
            if g(X_1, X_2, …, X_k) < f* then
                try(k+1);
        }
        [Recover the data structure D]
     }
  }
}
```

GENERAL DIAGRAM OF BACKTRACKING, BRANCHING AND BOUND



Do not further build the solution if $g(X_1, X_2, \ldots, X_k) \geq f^*$

Minimize optimization problem (Denote f^* : optimal value)

```
try(k){//Try out the possible values assigned to X_k
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                try(k+1);
        }
        [Recover the data structure D]
     }
  }
}
```

The problem of bus routes picking up and dropping off passengers

- A bus departing from point 0 needs to build a route that could serve n passengers and return to point 0. Passenger i has: the pick-up point is i and the drop-off point is $i + n$ ($i = 1, 2, \ldots, n$). The bus has K seats to serve passengers. The travel distance from point i to point j is d(i, j), with $i, j = 0, 1, 2, \ldots, 2n$. Calculate the route for the bus so that the total distance traveled is minimal, and the number of passengers on the bus never exceeds K.

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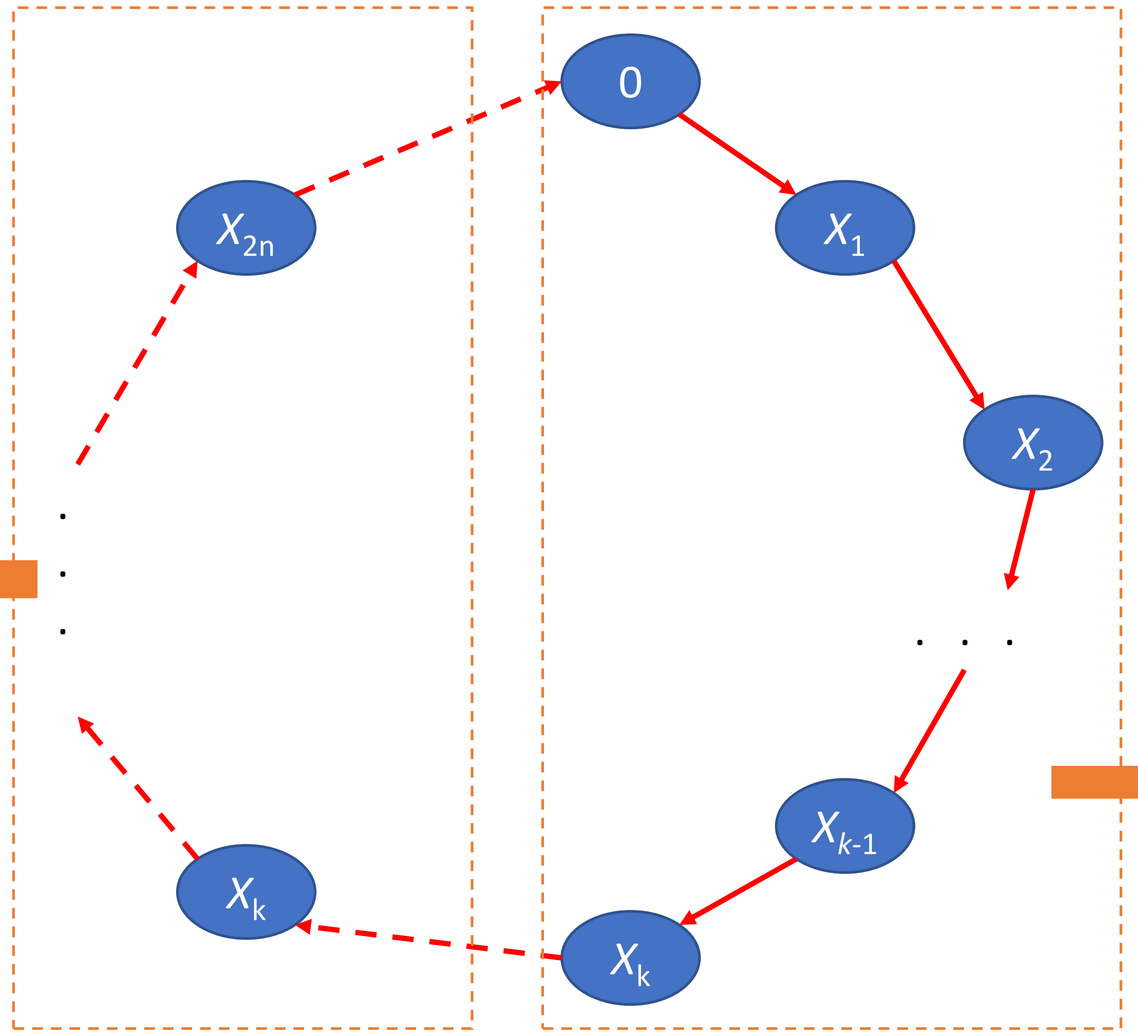
- Branch and bound algorithm
  - Modelling problem: $X_1, X_2, \ldots, X_{2n}$ is the sequence of pick-up and drop-off points on the bus route (a permutation of 1, 2, …, 2n).
  - *Cmin*: the smallest distance among the distances between 2 points
  - Marker array: visited[v] = true means point *v* has appeared on the route and visited[v] = false, otherwise
  - load: number of passengers present in the vehicle
    - When the route reaches the pick-up point, the load increases by 1, and when it reaches the drop-off point, the load decreases by 1
  - *f*: length of the partial route
  - *f**: shortest route length that has been found

The problem of bus routes picking up and dropping off passengers

- **Analyze the lower bound**

- Untraveled route, including $2n+1-k$ segments, each segment has length $\geq Cmin$
- The length of the complete route developing further from X_k will be $\geq f + Cmin*(2n+1-k)$



The partial route that has gone through:

- k segments
- Length f

The problem of bus routes picking up and dropping off passengers

```
try(k){
  for v = 1 to 2n do {
     if check(v,k){
        Xk = v;
        f = f + d(Xk-1,Xk); visited[v] = true;
        if v ≤ n then load += 1; else load -= 1;
        if k = 2n then updateBest();
        else {
            if f + Cmin*(2n+1-k) < f* then
                try(k+1);
        }
        if v ≤ n then load -= 1; else load += 1;
        f = f - d(Xk-1,Xk); visited[v] = false;
     }
  }
}
```

```
check(v,k){
  if visited[v] = true then return false;
  if v > n then {
    if visited[v-n] = false then return false;
  }else{
    if load + 1 > K then return false;
  }
  return true;
}
```

```
updateBest(){
  if f + d(X2n,0) < f* then {
    f* = f + d(X2n,0);
  }
}
```

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THANK YOU !