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# Lesson 5

## Database design: Bottom-up Approach

### Part 1: Functional Dependency

# Learning objective

• ***Upon completion of this lesson, students will be able to:***

1. Recall the concept of functional dependency, **Armstrong's axioms and secondary rules**
2. Identify closure of a **FD set**, closure of a set of attributes
3. Find a **minimal key** of a relation under a set of FDs
4. Identify the equivalence of sets of FDs and find the **minimal cover of a set of FDs**

# Outline

1. Functional Dependency
2. Armstrong's Axioms and secondary rules
3. Closure of a FD set, closure of a set of attributes
4. A minimal key
5. Equivalence of sets of FDs
6. Minimal Sets of FDs

# 1. Functional Dependency

- 1.1. Introduction
- 1.2. Definition

## 1.1. Introduction

- We have to deal with the problem of database design
  - anomalies, redundancies
- The most important concepts in relational schema design theory

## 1.2. Definition

- Suppose that  $R = \{A_1, A_2, \dots, A_n\}$ ,  $X$  and  $Y$  are non-empty subsets of  $R$ .
- A **functional dependency** (FD), denoted by  $X \rightarrow Y$ , specifies a constraint on the possible tuples that can form a relation state  $r$  of  $R$ . The constraint is that, for any two tuples  $t_1$  and  $t_2$  in  $r$  that have  $t_1[X] = t_2[X]$ , they must also have  $t_1[Y] = t_2[Y]$ .
  - $X$ : the left-hand side of the FD
  - $Y$ : the right-hand side of the FD

## 1.2. Definition

- This means that the values of the  $X$  component of a tuple uniquely  
(or **functionally**) determine the values of the  $Y$  component.
- A FD  $X \rightarrow Y$  is **trivial** if  $X \supseteq Y$
- If  $X$  is a candidate key of  $R$ , then  $X \rightarrow R$



## 1.2. Definition

- Examples

- $AB \rightarrow C$

A	B	C	D
a1	b1	c1	d1
a1	b1	c1	d2
a1	b2	c2	d1
a2	b1	c3	d1

- $\text{subject\_id} \rightarrow \text{name},$
  - $\text{subject\_id} \rightarrow \text{credit},$
  - $\text{subject\_id} \rightarrow \text{percentage\_final\_exam},$
  - $\text{subject\_id} \rightarrow \{\text{name}, \text{credit}\}$

### subject

<u>subject_id</u>	name	credit	percentage_ final_exam
IT3090	Databases	3	0.7
IT4843	Data integration	3	0.7
IT4868	Web mining	2	0.6
IT2000	Introduction to ICT	2	0.5
IT3020	Discrete Mathematics	2	0.7
IT3030	Computer Architectures	3	0.7

# 2. Armstrong's axioms

2.1. Armstrong's axioms

2.2. Secondary rules

2.3. An example

## 2.1. Armstrong's axioms

- Given
  - $R = \{A_1, A_2, \dots, A_n\}$ ,  $X, Y, Z, W$  are subsets of  $R$ .
  - $XY$  denoted for  $X \cup Y$
- Reflexivity
  - If  $Y \subseteq X$  then  $X \rightarrow Y$
- Augmentation
  - If  $X \rightarrow Y$  then  $XZ \rightarrow YZ$
- Transitivity
  - If  $X \rightarrow Y, Y \rightarrow Z$  then  $X \rightarrow Z$

## 2.2. Secondary rules

- Union
  - If  $X \rightarrow Y$ ,  $X \rightarrow Z$  then  $X \rightarrow YZ$ .
- Pseudo-transitivity
  - If  $X \rightarrow Y$ ,  $WY \rightarrow Z$  then  $XW \rightarrow Z$ .
- Decomposition
  - If  $X \rightarrow Y$ ,  $Z \subseteq Y$  then  $X \rightarrow Z$

## 2.3. An example

- Given a set of FDs:  $F = \{AB \rightarrow C, C \rightarrow A\}$
- Prove:  $BC \rightarrow ABC$ 
  - From  $C \rightarrow A$ , we have  $BC \rightarrow AB$  (Augmentation)
  - From  $AB \rightarrow C$ , we have  $AB \rightarrow ABC$  (Augmentation)
  - And we can conclude  $BC \rightarrow ABC$  (Transitivity)

# 3. Closure of a FD set, closure of a set of attributes

3.1. Closure of a FD set

3.2. Closure of a set of attributes

3.3. A problem

## 3.1. Closure of a FD set

- Suppose that  $F = \{A \rightarrow B, B \rightarrow C\}$  on  $R(A, B, C, \dots)$ . We can infer many FDs such as:

$A \rightarrow C, AC \rightarrow BC, \dots$

- Definition
  - Formally, the set of all dependencies that include  $F$  as well as all dependencies that can be inferred from  $F$  is called the **closure** of  $F$ , denoted by  $F^+$ .
- $F \models X \rightarrow Y$  to denote that the FD  $X \rightarrow Y$  is inferred from the set of FDs  $F$ .

## 3.2. Closure of a set of attributes

- Problem
  - We have  $F$ , and  $X \rightarrow Y$ , we have to check if  $F \models X \rightarrow Y$  or not
- Should we calculate  $F^+$ ?  $\Rightarrow$  Closure of a set of attributes
- Definition
  - For each such set of attributes  $X$ , we determine the set  $X^+$  of attributes that are functionally determined by  $X$  based on  $F$ ;  $X^+$  is called the **closure of  $X$  under  $F$** .



## 3.2. Closure of a set of attributes

- To find the closure of an attribute set  $X^+$  under  $F$ 
  - **Input:** A set  $F$  of FDs on a relation schema  $R$ , and a set of attributes  $X$ , which is a subset of  $R$ .  
 $X^0 := X$ ;  
**repeat**  
    for each functional dependency  $Y \rightarrow Z$  in  $F$  do  
        if  $X^{i-1} \supseteq Y$  then  $X^i := X^{i-1} \cup Z$ ;  
        else  $X^i := X^{i-1}$   
**until** ( $X^i$  unchanged);  
 $X^+ := X^i$

## 3.2. Closure of a set of attributes

- An example
  - Given  $R = \{A, B, C, D, E, F\}$  and  $F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$ .

Calculate  $(AB)^+_F$

- $X^0 = AB$
- $X^1 = ABC$  (from  $AB \rightarrow C$ )
- $X^2 = ABCD$  (from  $BC \rightarrow AD$ )
- $X^3 = ABCDE$  (from  $D \rightarrow E$ )
- $X^4 = ABCDE$
- $(AB)^+_F = ABCDE$

## 3.3. A Problem

- $X \rightarrow Y$  can be inferred from  $F$  if and only if  $Y \subseteq X^+_F$
- $F \models X \rightarrow Y \Leftrightarrow Y \subseteq X^+_F$
- An example
  - Let  $R = \{A, B, C, D, E\}$ ,  $F = \{A \rightarrow B, B \rightarrow CD, AB \rightarrow CE\}$ .

Consider whether or not  $F \models A \rightarrow C$

- $(A)^+_F = ABCDE \supseteq \{C\}$

# 4. Minimal key

4.1. Definition

4.2. An algorithm to find a minimal key

4.3. An example

## 4.1. Definition

- Minimal key
  - Given  $R = \{A_1, A_2, \dots, A_n\}$ , a set of FDs  $F$
  - $K$  is considered as a minimal key of  $R$  if:
    - $K \subseteq R$
    - $K \rightarrow R \in F^+$
    - Với  $\forall K' \subset K$ , thì  $K' \rightarrow R \notin F^+$
  - $K^+ = R$  and  $K \setminus \{A_i\} \rightarrow R \notin F^+$

## 4.2. An algorithm to find a minimal key

- To find a minimal key
  - Input:  $R = \{A_1, A_2, \dots, A_n\}$ , a set of FDs  $F$ 
    - Step<sup>0</sup>       $K^0 = R$
    - Step<sup>i</sup>      If  $(K^{i-1} \setminus \{A_i\}) \rightarrow R$  then  $K^i = K^{i-1} \setminus \{A_i\}$   
                 else  $K^i = K^{i-1}$
    - Step<sup>n+1</sup>     $K = K^n$

## 4.3. An example

- Given  $R = \{A, B, C, D, E\}$ ,  $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow DE\}$ .
- Find a minimal key
  - Step<sup>0</sup>:  $K^0 = R = ABCDE$
  - Step<sup>1</sup>: Check if or not  $(K^0 \setminus \{A\}) \rightarrow R$  (i.e,  $BCDE \rightarrow R$ ).
  - $(BCDE)^+ = BCDE \neq R$ . Vậy  $K^1 = K^0 = ABCDE$
  - Step<sup>2</sup>: Check if or not  $(K^1 \setminus \{B\}) \rightarrow R$  (i.e,  $ACDE \rightarrow R$ ).
  - $(ACDE)^+ = ABCDE = R$ . So,  $K^2 = K^1 \setminus \{B\} = ACDE$
  - Step<sup>3</sup>:  $K^3 = ACDE$
  - Step<sup>4</sup>:  $K^4 = ACE$
  - Step<sup>5</sup>:  $K^5 = AC$
- We infer that AC is a minimal key

## 4.4. An other algorithm to find a minimal key

**Input:**  $U = \{A_1, A_2, \dots, A_n\}$ ,  $F$

**Output:** a minimal key

- **Step 1:**

VT = set of all attributes on the left-side of FD in  $F$

VP = set of all attributes on the right-side of FD in  $F$

$X = U \setminus VP$ : set of attributes **that must be in K**

$Y = VP \setminus VT$ : set of attributes **that must NOT be in K**

$Z = VP \cap VT$ : set of attributes **that may be in K**

- **Step 2:** If  $(X)^+ = U$  then  $X$  is the minimal key:  $K = X$ . End!

- **Step 3:** If  $(X)^+ \neq U$  then

- $K^0 = X \cup Z$

- Repeat: check if we can remove any attribute from  $Z$  (similar to slide 22)

- $K = K^i$



## 5. Equivalence of Sets of FDs

5.1. Definition

5.2. An example

## 5.1. Definition

- Definition:
  - A set of FDs  $F$  is said to cover another set of FDs  $G$  if every FD in  $G$  is also in  $F^+$  (every dependency in  $G$  can be inferred from  $F$ ).
- Check if  $F$  and  $G$  are equivalent:
  - Two sets of FDs  $F$  and  $G$  are equivalent if  $F^+ = G^+$ .
    - Therefore, equivalence means that every FD in  $G$  can be inferred from  $F$ , and every FD in  $F$  can be inferred from  $G$ ;
    - That is,  $G$  is equivalent to  $F$  if both the conditions -  $G$  covers  $F$  and  $F$  covers  $G$  - hold.

## 5.2. An example

- Prove that  $F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$  and  $G = \{A \rightarrow CD, E \rightarrow AH\}$  are equivalent
  - For each FD of  $F$ , prove that it is in  $G^+$ 
    - $A \rightarrow C$ :  $(A)^+_G = ACD \supseteq C$ , so  $A \rightarrow C \in G^+$
    - $AC \rightarrow D$ :  $(AC)^+_G = ACD \supseteq D$ , so  $AC \rightarrow D \in G^+$
    - $E \rightarrow AD$ :  $(E)^+_G = EAHCD \supseteq AD$ , so  $E \rightarrow AD \in G^+$
    - $E \rightarrow H$ :  $(E)^+_G = EAHCD \supseteq H$ , so  $E \rightarrow H \in G^+$
    - $\Rightarrow F^+ \subseteq G^+$
  - For each FD of  $G$ , prove that it is in  $F^+$  (the same)
    - $\Rightarrow G^+ \subseteq F^+$
  - $\Rightarrow F^+ = G^+$

## 6. A minimal cover of a set of FDs

6.1. Definition

6.2. An algorithm to find a minimal cover of a set of FDs

6.3. An example

## 6.1. Definition

- Minimal Sets of FDs
  - A set of FDs  $F$  to be minimal if it satisfies:
    - Every dependency in  $F$  has a single attribute for its right-hand side.
    - We cannot replace any dependency  $X \rightarrow A$  in  $F$  with a dependency  $Y \rightarrow A$ , where  $Y$  is a proper subset of  $X$ , and still have a set of dependencies that is equivalent to  $F$ .
    - We cannot remove any dependency from  $F$  and still have a set of dependencies that is equivalent to  $F$ .
  - A set of dependencies in a standard or canonical form and with no redundancies

## 6.2. An algorithm to find a minimal cover of a set of FDs

- Finding a Minimal Cover  $F$  for a Set of FDs  $G$ 
  - Input: A set of FDs  $G$ .
    - 1. Set  $F := G$ .
    - 2. Replace each functional dependency  $X \rightarrow \{A_1, A_2, \dots, A_n\}$  in  $F$  by the  $n$  FDs  $X \rightarrow A_1, X \rightarrow A_2, \dots, X \rightarrow A_n$ .
    - 3. For each FD  $X \rightarrow A$  in  $F$ 
      - for each attribute  $B$  that is an element of  $X$
      - if  $\{F - \{X \rightarrow A\}\} \cup \{(X - \{B\}) \rightarrow A\}$  is equivalent to  $F$
      - then replace  $X \rightarrow A$  with  $(X - \{B\}) \rightarrow A$  in  $F$ .
    - 4. For each remaining functional dependency  $X \rightarrow A$  in  $F$ 
      - if  $\{F - \{X \rightarrow A\}\}$  is equivalent to  $F$ , then remove  $X \rightarrow A$  from  $F$ .

## 6.3. An example

- $G = \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$ .
- We have to find the minimal cover of  $G$ .
  - All above dependencies are in canonical form
  - In step 2, we need to determine if  $AB \rightarrow D$  has any redundant attribute
    - on the left-hand side; that is, can it be replaced by  $B \rightarrow D$  or  $A \rightarrow D$ ?
    - Since  $B \rightarrow A$  then  $AB \rightarrow D$  may be replaced by  $B \rightarrow D$ .
    - We now have a set equivalent to original  $G$ , say  $G_1: \{B \rightarrow A, D \rightarrow A, B \rightarrow D\}$ .
  - In step 3, we look for a redundant FD in  $G_1$ . Using the transitive rule on
    - $B \rightarrow D$  and  $D \rightarrow A$ , we conclude  $B \rightarrow A$  is redundant.
    - Therefore, the minimal cover of  $G$  is  $\{B \rightarrow D, D \rightarrow A\}$

## Remark

- Functional dependencies
- Armstrong axioms and their secondary rules
- Closure of a set of FDs
- Closure of a set of attributes under a set of FDs
- An algorithm to find a minimal key
- Equivalence of sets of FDs
- Finding a minimal set of a set of FDs



# Summary

## 1. Functional Dependency

- A FD  $X \rightarrow Y$ : the values of the X component of a tuple uniquely (or functionally) determine the values of the Y component

## 2. Armstrong 's Axioms and secondary rules

- Reflexivity, Augmentation, Transitivity
- Union, Pseudo-transitivity, Decomposition

## 3. Closure of a FD set, closure of a set of attributes

- All dependencies that can be inferred from F, include F, is called the closure of F, denoted by  $F^+$
- A set of attributes are functionally determined by X based on F

## 4. A minimal key

- A minimal set of attributes can determine R

## 5. Equivalence of sets of functional dependencies

- F is equivalent to G if every dependency in G can be inferred from F, and every dependency in F can be inferred from G

## 6. A minimal cover of a set of FDs

- A set of dependencies in a standard or canonical form and with no redundancies



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**Thank you for  
your attention!**

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# References

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- Elmasri and Navathe, Fundamentals of Database Systems, 6th edition, Addison-Wesley, 2011.