BT tuần 9

Bài 1:

$I = \iint_S y\,dS \quad : \quad S: x^2 + y^2 + z^2 = 9,\ z \geq 0.$

Đặt $\begin{cases} x = 3\sin\theta\cos\varphi \\ y = 3\sin\theta\sin\varphi \\ z = 3\cos\theta \end{cases}$ với $\begin{matrix} 0 \leq \varphi \leq 2\pi \\ 0 \leq \theta \leq \pi/2 \end{matrix}$

$\vec{r}_\theta{}' \wedge \vec{r}_\varphi{}' = \begin{vmatrix} i & j & k \\ 3\cos\theta\cos\varphi & 3\cos\theta\sin\varphi & -3\sin\theta \\ -3\sin\theta\sin\varphi & 3\sin\theta\cos\varphi & 0 \end{vmatrix}$

$= 9\sin^2\theta\cos\varphi\,\vec{i} + 9\sin^2\theta\sin\varphi\,\vec{j} + 9\sin\theta\cos\theta\,\vec{k}$

$\to |\vec{r}_\theta{}' \wedge \vec{r}_\varphi{}'| = 9\sin\theta$

$\to I = \int_0^{2\pi} d\varphi \int_0^{\pi/2} 3\sin\theta\sin\varphi \cdot 9\sin\theta\,d\theta$

$= 27\int_0^{2\pi} \sin\varphi \int_0^{\pi/2} \sin^2\theta\,d\theta$

$= 27\left(-\cos\varphi\right)\Big|_0^{2\pi} \cdot \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)\Big|_0^{\pi/2}$

$= 0$

Bài 2:

$I = \iint_S |xy|\,dS, \quad S \begin{cases} x^2 + y^2 + z^2 = 1 \\ z \geq 0 \end{cases}$

$\begin{cases} \text{Miền } S \text{ đối xứng qua } Ox, Oy \\ f(x,y) = |xy| \text{ là hàm lẻ theo biến } x, y \end{cases}$

$\Rightarrow \iint_S |xy|\,dS = 4\iint_{S_1} xy\,dS$ với $S_1 \begin{cases} x^2 + y^2 + z^2 = 1 \\ x \geq 0, y \geq 0, z \geq 0 \end{cases}$

Đặt $\begin{cases} x = \sin\theta\cos\varphi \\ y = \sin\theta\sin\varphi \\ z = \cos\theta \end{cases}$ với $\begin{cases} 0 \leq \varphi \leq \frac{\pi}{2} \\ 0 \leq \theta \leq \frac{\pi}{2} \end{cases}$

$|r'_\theta \wedge r'_\varphi| = \sin\theta$

$\Rightarrow I = 4\int_0^{\pi/2} d\theta \int_0^{\pi/2} \sin^3\theta\cos\varphi\sin\varphi\,d\varphi$

$= 2\int_0^{\pi/2} \sin^3\theta\,d\theta \int_0^{\pi/2} \sin 2\varphi\,d\varphi$

$$= 2\int_0^{\pi/2} \frac{3\sin\theta - \sin 3\theta}{4}\, d\theta \int_0^{\pi/2} \sin 2\varphi\, d\varphi$$

$$= 2\left(\frac{-3}{4}\cos\theta + \frac{\cos 3\theta}{12}\right)\Big|_0^{\pi/2} . \frac{-\cos 2\varphi}{2}\Big|_0^{\pi/2}$$

$$= 2 . \frac{2}{3} . 1 = \frac{4}{3}$$

Bài 3:

$I = \iint\limits_S x^2 y\, dS$, S là phần mặt nón $y = \sqrt{x^2+z^2}$, $1 \le y \le 2$

Hình chiếu của S trên Oxz là miền: $D: 1 \le \sqrt{x^2+z^2} \le 2$

$$dS = \sqrt{y_x'^2 + 1 + y_z'^2}\, dx dz = \sqrt{\left(\frac{x}{\sqrt{x^2+z^2}}\right)^2 + 1 + \left(\frac{z}{\sqrt{x^2+z^2}}\right)^2}\, dx dz$$

$$= \sqrt{2}\, dx dz$$

$$\to I = \iint\limits_D x^2\sqrt{x^2+z^2} . \sqrt{2}\, dx dz$$

Đặt $\begin{cases} x = r\cos\varphi \\ z = r\sin\varphi \end{cases} \to |J| = r$

$D \to D' \begin{cases} 0 \le \varphi \le 2\pi \\ 1 \le r \le 2 \end{cases}$

$$\to I = \iint\limits_{D'} r^2\cos^2\varphi\, r\sqrt{2}\, r\, dr\, d\varphi$$

$$= \sqrt{2}\int_0^{2\pi} \cos^2\varphi\, d\varphi \int_1^2 r^4 dr$$

$$= \frac{\sqrt{2}}{2}\int_0^{2\pi}(1+\cos 2\varphi)\, d\varphi . \int_1^2 r^4 dr$$

$$= \frac{\sqrt{2}}{2}\left(\varphi + \frac{\sin 2\varphi}{2}\right)\Big|_0^{2\pi} . \frac{r^5}{5}\Big|_1^2$$

$$= \frac{\sqrt{2}}{2} . 2\pi . \frac{31}{5}$$

$$= \frac{31\sqrt{2}\pi}{5}$$

Bài 4:

$I = \iint_S \frac{dS}{(2+x+y+z)^2}$ S là biên tứ diện $\begin{cases} x+y+z \le 1 \\ x \geq 0; y \geq 0; z \geq 0 \end{cases}$

Do x, y, z có vai trò như nhau trong S nên ta có:

$$I = \iint_{S'} \frac{dS}{(2+x+y+z)^2} + 3\iint_D \frac{dS}{(2+x+y)^2}$$

Với $S \begin{cases} x+y+z = 1 \\ x \geq 0, y \geq 0; z \geq 0 \end{cases}$ và $D \begin{cases} x+y = 1 \\ x \geq 0; y \geq 0 \end{cases}$

$$I = \iint_D \frac{\sqrt{3}\,dxdy}{(2+x+y+1-x-y)^2} + 3\iint_D \frac{dxdy}{(2+x+y)^2}$$

$$= \int_0^1 dx \int_0^{1-x} \frac{\sqrt{3}}{9} dy + 3\int_0^1 dx \int_0^{1-x} \frac{dy}{(2+x+y)^2}$$

$$= \frac{\sqrt{3}}{9} \int_0^1 (1-x)\,dx + 3\int_0^1 \frac{-1}{2+x+y}\Big|_0^{1-x} dx$$

$$= \frac{\sqrt{3}}{9}\left(x - \frac{x^2}{2}\right)\Big|_0^1 - 3\left(\frac{x}{3} - \ln|x+2|\right)\Big|_0^1$$

$$= \frac{\sqrt{3}}{18} - 1 + 3\ln\frac{3}{2}$$