Calculus 2 Final mock exam

Time: 90 minutes

Q1. Find the tangent plane and normal line of the surface $S: x^2+3y^2-z^2=3$ at the point A(1,1,1).

Q2. Find the point M on the curve (P): $y=-2x^2-4x$ such that the curvature of (P) at M reaches the maximum value.

Q3. Evaluate $I=\iint\limits_D 4xy\,dxdy$ where $D=\{(x,y)|-x\le y\le 1-x, x-2\le y\le x-1\}$.

Q4. Evaluate $I=\iiint\limits_V \frac{x^2}{\sqrt{x^2+y^2+z^2}}dxdydz$, where V is the region bounded by the sphere $x^2+y^2+z^2=4$ and the quadratic cone $z=\sqrt{\frac{x^2+y^2}{3}}$.

Q5. Evaluate $\int\limits_C y^3dx+2x^3dy$, with C is a part of the circle $x^2+y^2=1$ from $A(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$ to $B(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$.

Q6. Calculate the mass of a wire takes the shape of the curve C, where C is the curve $y=e^{\frac{x}{2}}+e^{-\frac{-x}{2}}$, $0\le x\le 2$. The mass density is described by the function $\rho(x,y)=\frac{1}{y}$.

Q7. Evaluate $\iint\limits_S (x-z)dS$, where S is the surface bounded by $x^2+y^2=4, z=x-3$ and $z=x+2$.

Q8. Vector field $\vec{F}=(x^2-y, x+2y, x+y+z)$. Find the flux of $\vec{F}$ through the surface $S: |x-y|+|x+2y|+|x+y+z|=1$ outward.

Q9. Evaluate the flux of $\vec{F} = xz^2\vec{i} + x^2y\vec{j} + y^2(z+2)\vec{k}$ across half of the sphere $S: x^2+y^2+z^2 = 1, z \leq 0$, oriented downward.

Q10. Evaluate the following line integral:

$$\int_C (x^2+y^2+z^2+yz)dx + (x^2+y^2+z^2+xz)dy + (x^2+y^2+z^2+xy)dz$$

where C is the intersection of the sphere $x^2+y^2+z^2 = 4$ and the surface $z = x^2+(y-1)^2$, oriented clockwise viewed from the origin O.