HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

LESSON 11

TRANSFER FUNCTION

PhD. Nguyen Hong Quang
Assoc. Prof. Trinh Van Loan
PhD. Doan Phong Tung
Computer Engineering Department

❑ CONTENT

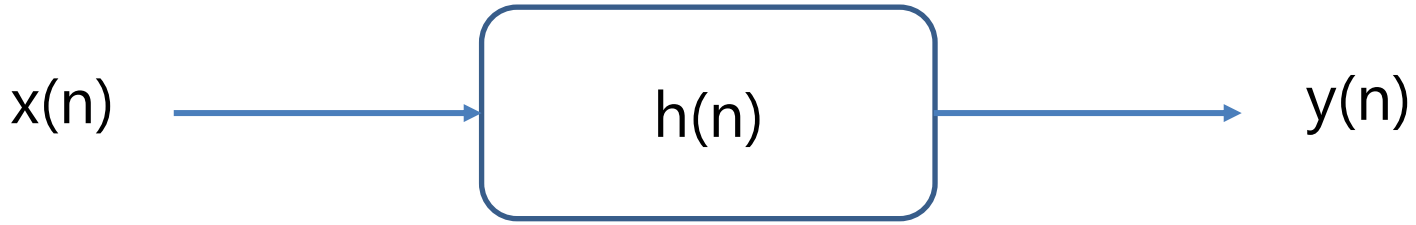
1. The transfer function of the system.
2. Determine the transfer function from the difference equation

❑ Lesson Objectives

After completing this lesson, you will be able to understand the following topics:

- The concept of the transfer function of the system
- Method to determine the transfer function of the system
- The relationship between the transfer function and the impulse response and the differential equation

1. The transfer function of the system



$$y(n) = x(n) * h(n) \rightarrow Y(z) = X(Z).H(Z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = ZT\{h(n)\} = \sum_{n=-\infty}^{\infty} h(n)z^{-n}$$

- $H(z)$: Transfer Function

2. Determine H(z) from the constant coefficient linear difference equation

- Determine H(z) from the constant coefficient linear difference equation:

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

- Take the Z transform on both sides of the differential equation:

$$\sum_{n=-\infty}^{\infty} \left[\sum_{k=0}^{N} a_k y(n-k) \right] z^{-n} = \sum_{n=-\infty}^{\infty} \left[\sum_{k=0}^{M} b_k x(n-k) \right] z^{-n}$$

The transfer function of the system

$$\sum_{k=0}^{N} a_k \left[\sum_{n=-\infty}^{\infty} y(n-k)\, z^{-n} \right] = \sum_{k=0}^{M} b_k \left[\sum_{n=-\infty}^{\infty} x(n-k)\, z^{-n} \right]$$

$$\sum_{k=0}^{N} a_k\, z^{-k}\, Y(z) = \sum_{k=0}^{M} b_k\, z^{-k}\, X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k\, z^{-k}}{\sum_{k=0}^{N} a_k\, z^{-k}}$$

- Represent H(Z) through the zeros z_r and the poles p_k:

$$H(Z) = H_0 \frac{\prod_{r=1}^{M} (z - z_r)}{\prod_{k=1}^{N} (z - z_k)}$$

4. Summary

- The transfer function H(z) of the system represents the relationship of the input and output signals on blanket Z.
- The transfer function H(z) is calculated directly from the difference equation
- The impulse response h(n) is the inverse Z transform of the transfer function H(z).

5. Exercise

- Exercise 1
  - ❑ A causal invariant linear system has the following impulse response function:

$$x(n) = 2.\,3^{n}\,u(n) + 4.5^{n}\,u(n)$$

a. Determine the differential equation of the system

b. Draw direct forms I and II implementing the system

Excercise 2

- The causal invariant linear system has the following differential equation:

$$y(n) - 3y(n-1) + 2y(n-2) = x(n-1)$$

a. Determine the transfer function, zero, and poles

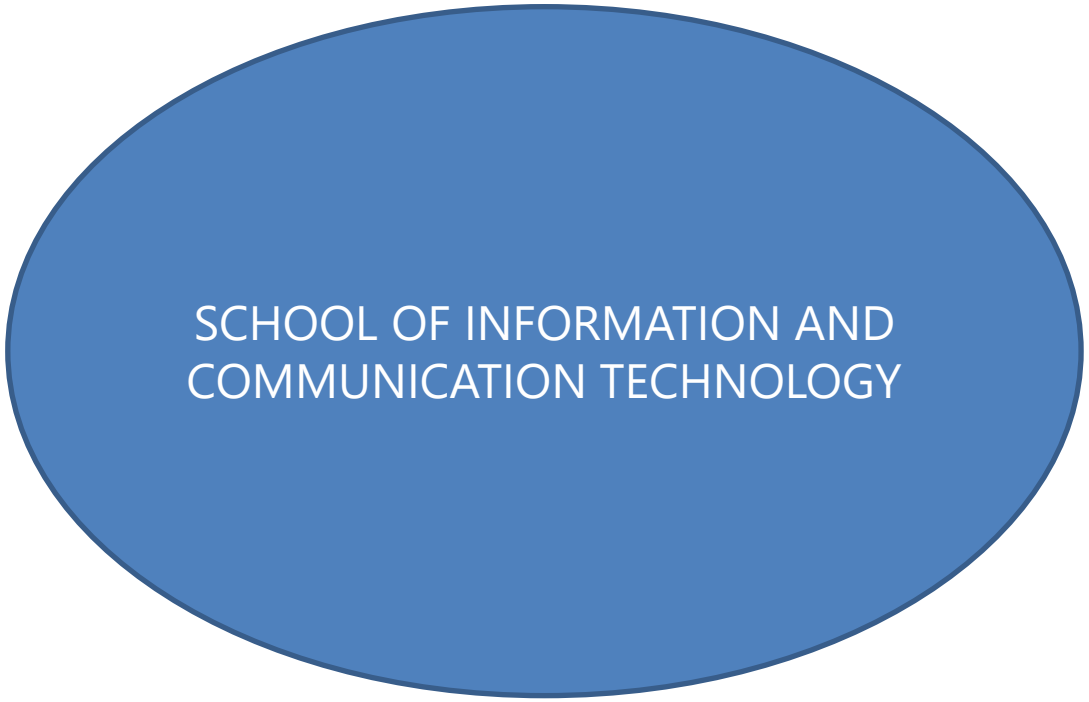
b. Determine the impulse response h(n) of the system

Next lesson. Lesson 12

CAUSALITY AND STABILITY SURVEY IN Z

References:

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***



Wish you all good study!