UNIT 3

DISCRETE SIGNALS

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Contents

1. The forms of discrete signal representations
2. The fundamental discrete signals
3. The operations with discrete signals

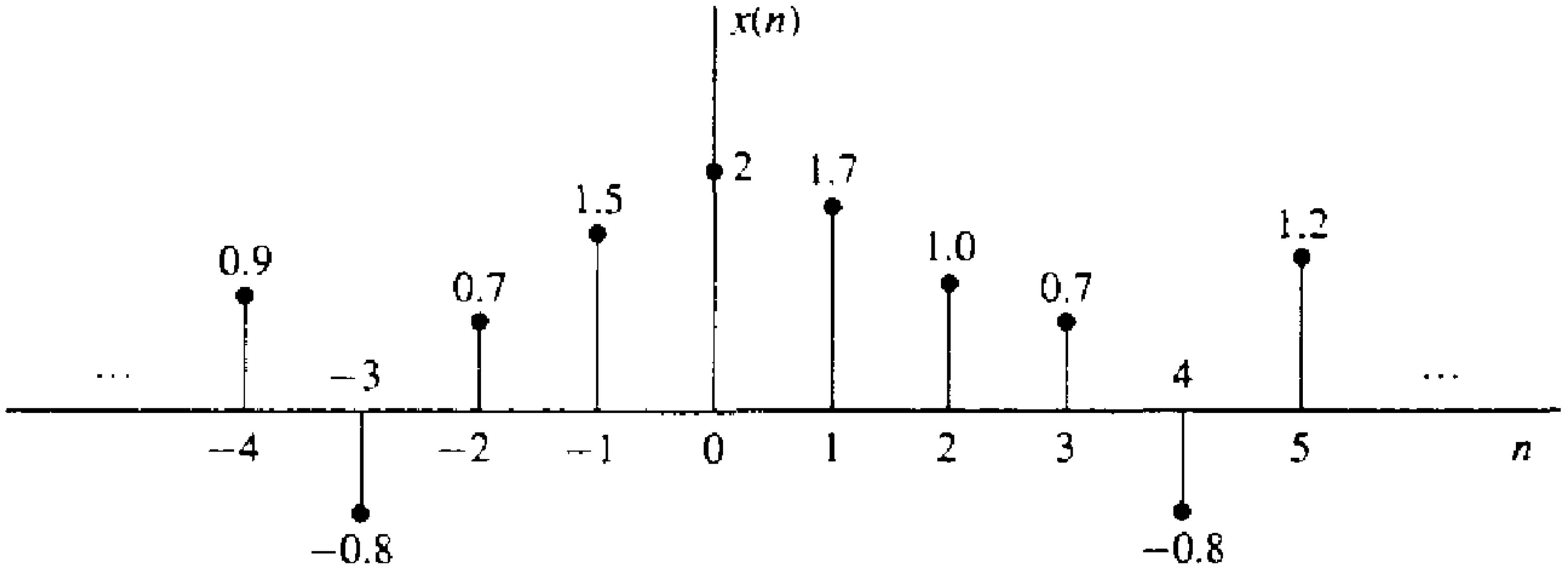
Learning Objectives

After studying this lesson, you will be able to:

- Understand the methods of representing discrete signals
- Identify some fundamental discrete signals
- Understand the concepts of energy and power of a signal
- Perform basic operations with discrete signals.

1. Representation forms of discrete signals

- Discrete signal x(n): only specify for integer values of n.
- Graphical representation



Representation forms of discrete signals

- Representation by function

$$x(n) = \begin{cases} 1, & for\ n = 1,3 \\ 4, & for\ n = 2 \\ 0, & elsewhere \end{cases}$$

- Representation by table

n	…	-2	-1	0	1	2	3	4	5	…
x(n)	…	0	0	0	1	4	1	0	0	…

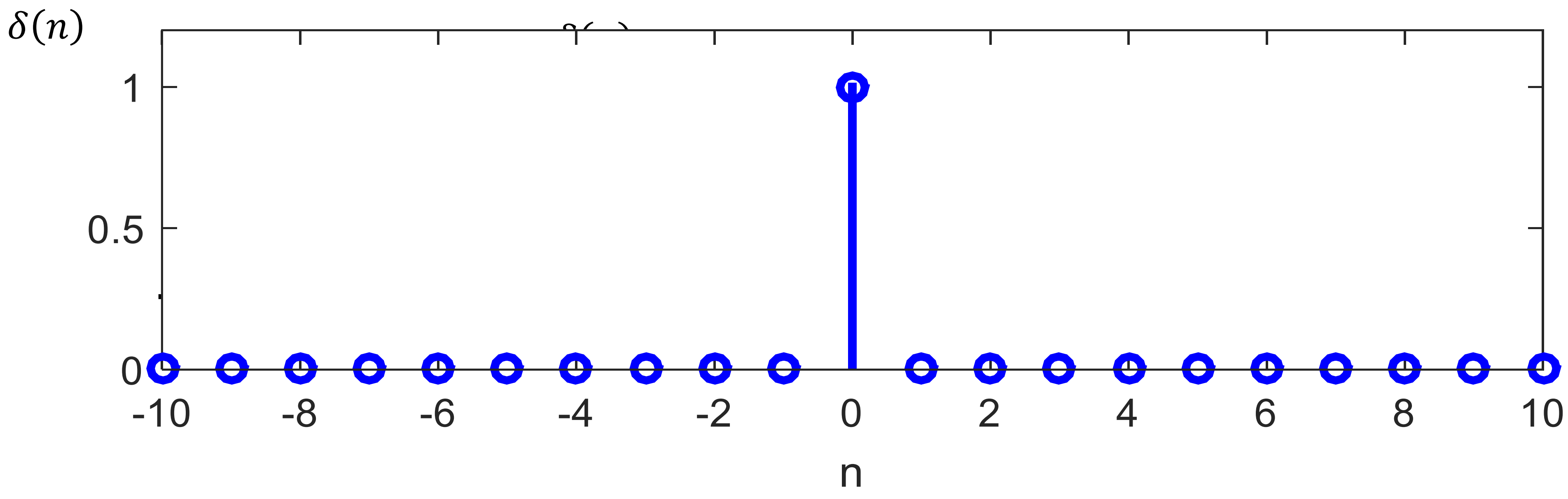
- Representation by sequence of numbers.

$$\mathrm{x(n)} = \{\ldots 0, 0, \underset{\uparrow}{1}, 4, 1, 0, 0, \ldots\}$$

2. Fundamental discrete signals

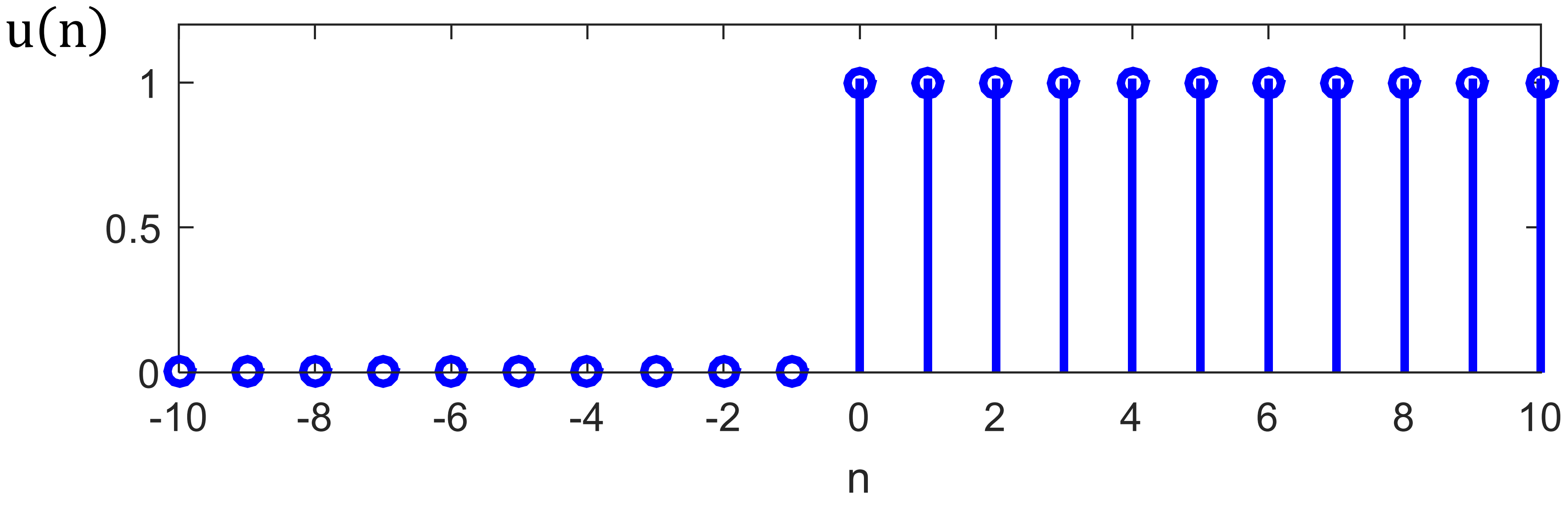
- Impulse signal

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



Unit step signal

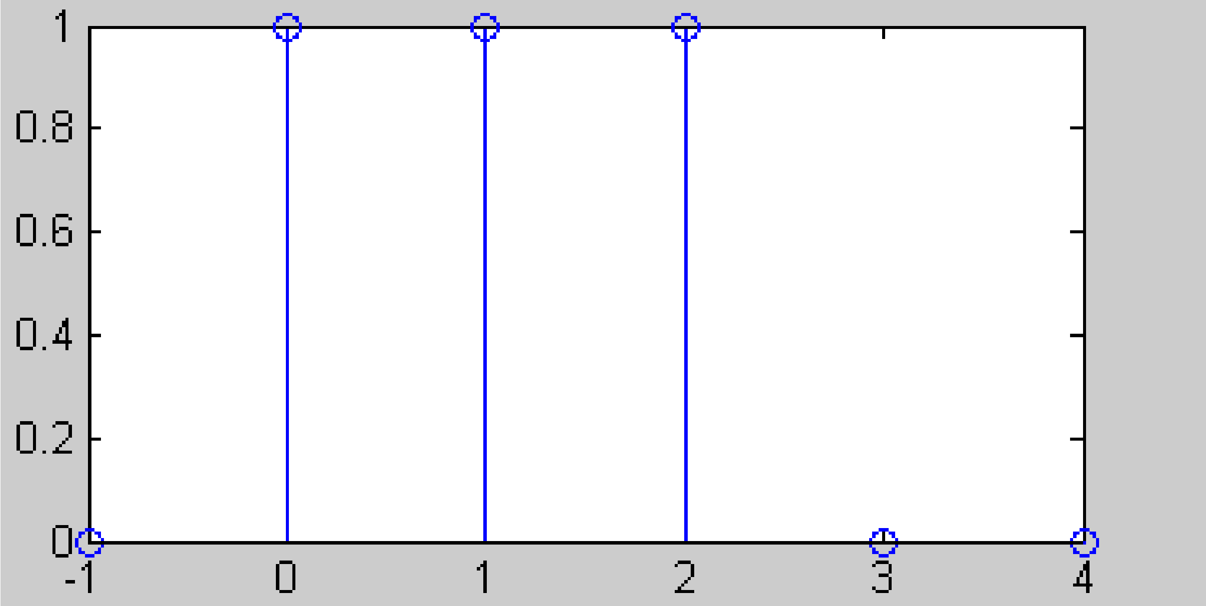
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Rectangle signal

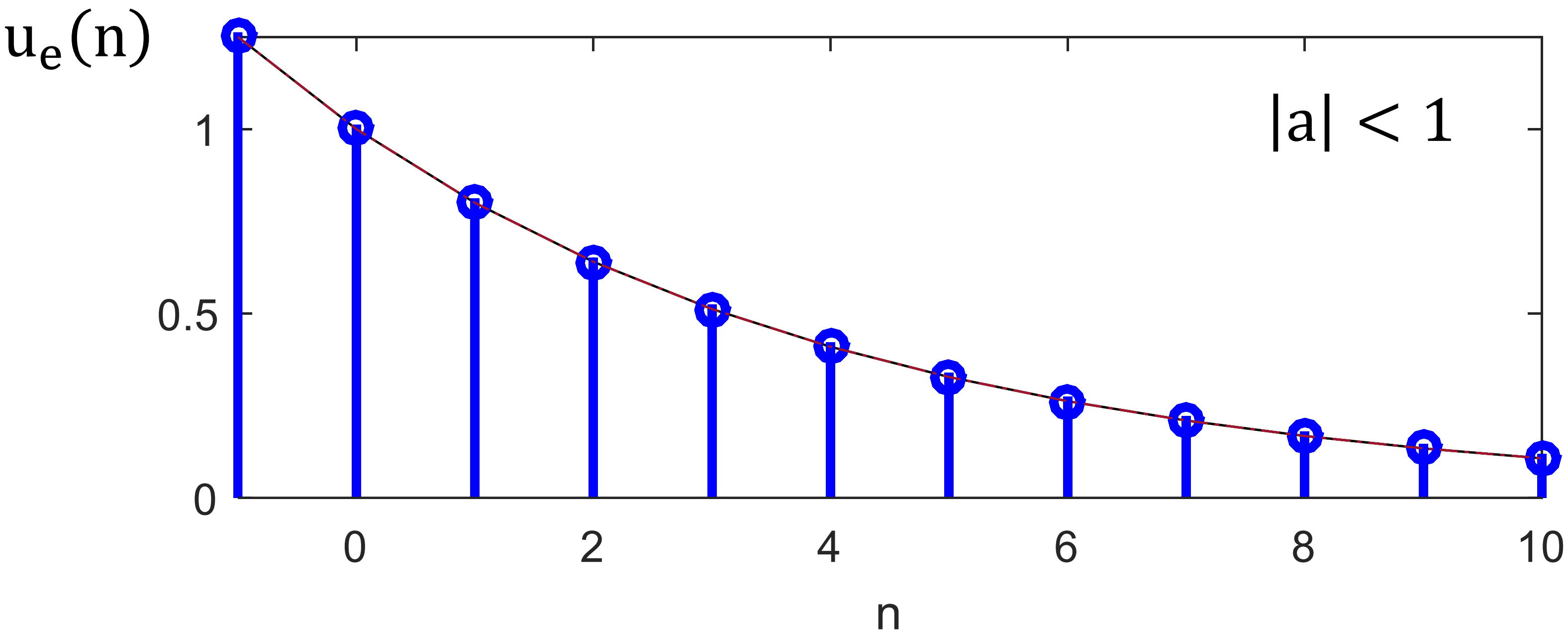
$$\mathrm{rect}_N(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & n < 0 \text{ và } n > N-1 \end{cases}$$

$$\mathrm{rect}_3(n) = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & n < 0 \end{cases}$$



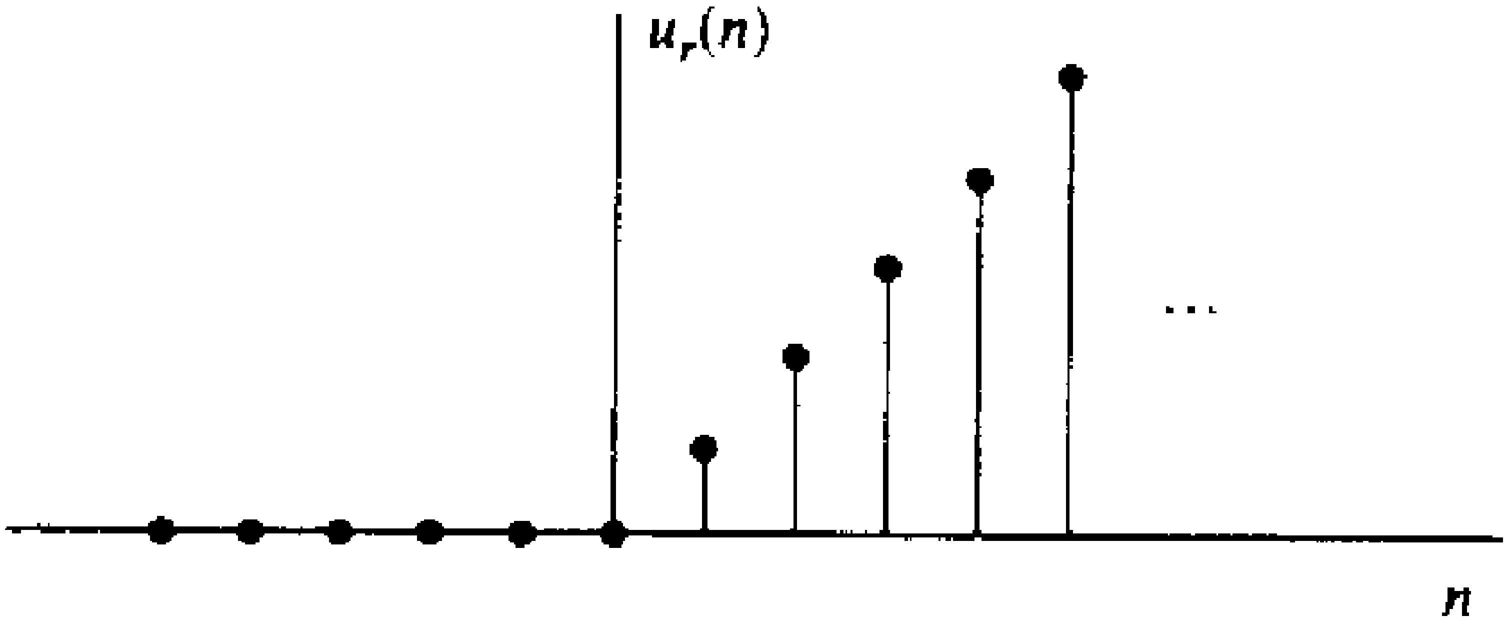
Casual exponential signal

$$u_e(n) = \begin{cases} a^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Ramp signal

$$u_r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



The energy and power of the signal

- The energy of the signal $x(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

- Energy signal is a signal whose energy is finite $0 < E < \infty$

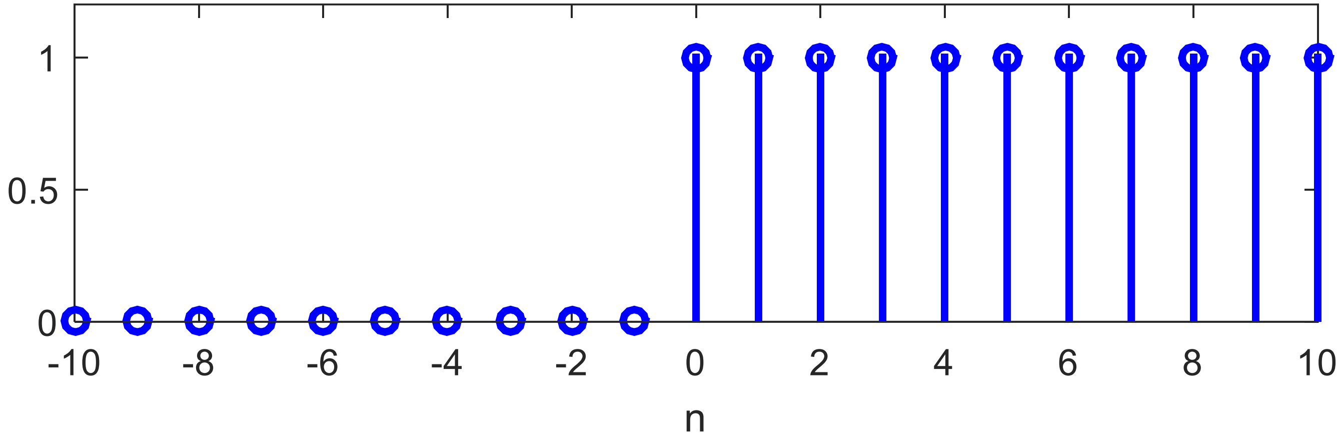
- The energy of the signal $x(n)$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

- Power signal is a signal whose power is finite $0 < P < \infty$

Example

- Compute energy and power of the signal $u(n)$



$$E = \sum_{n=0}^{\infty} u(n)^2 = \infty \qquad P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} u(n)^2 = \lim_{N\to\infty} \frac{N+1}{2N+1} = \lim_{N\to\infty} \frac{1+1/N}{2+1/N} = \frac{1}{2}$$

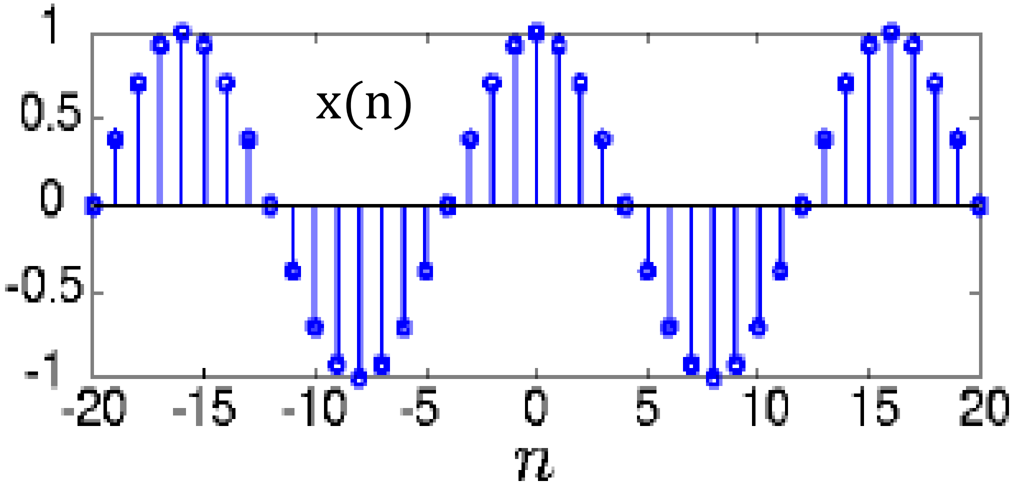
The signal $u(n)$ has a finite power and an infinite energy

Periodic signal

- The signal $x(n)$ is periodic with period N $(N > 0)$ if and only if

$$x(n+N) = x(n) \quad \forall n$$

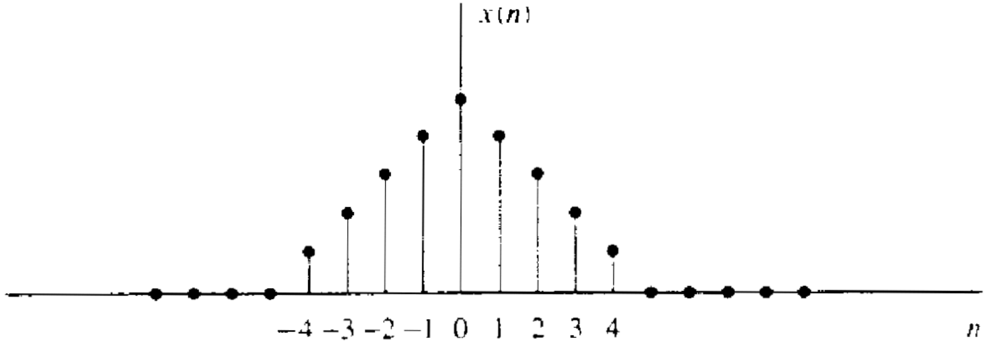
- The smallest N is the fundamental period



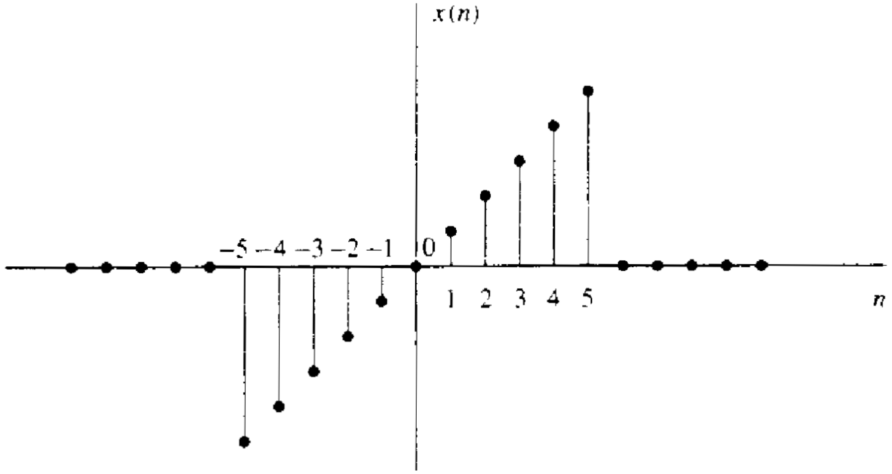
- Nonperiodic signal has an infinite fundamental period

Even and odd signals

- Even signal: $x(-n) = x(n)$



- Odd signal: $x(-n) = -x(n)$



- Signal decomposition:

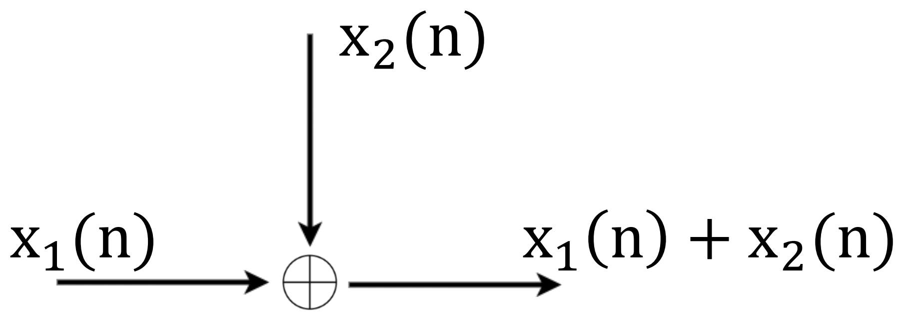
$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

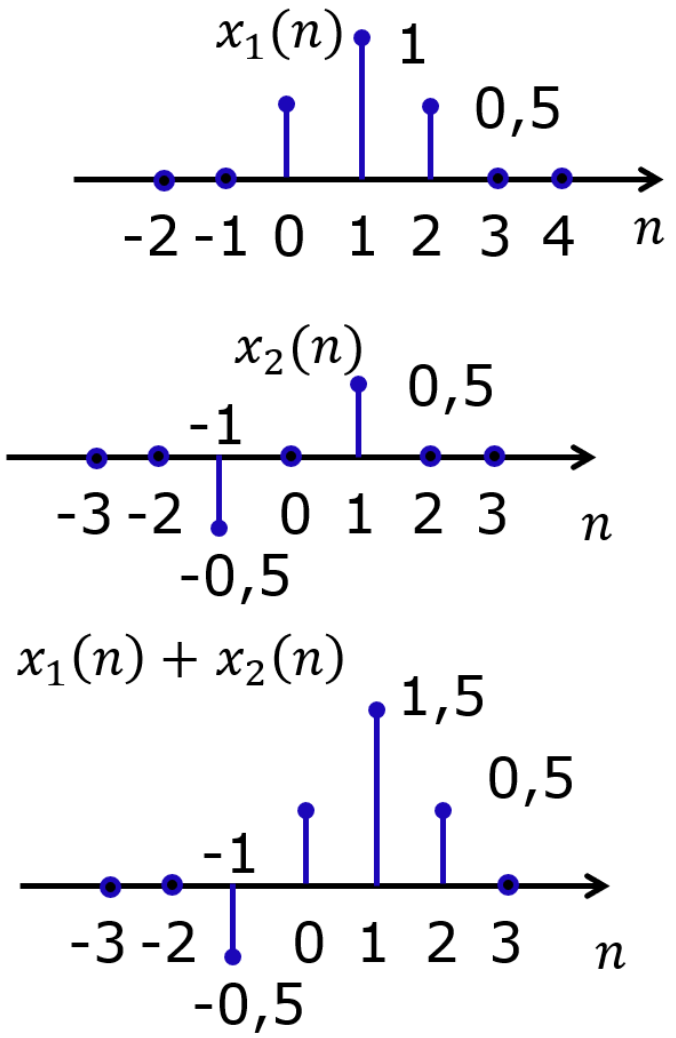
$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$x(n) = x_e(n) + x_o(n)$$

3. Basic operations with discrete signal

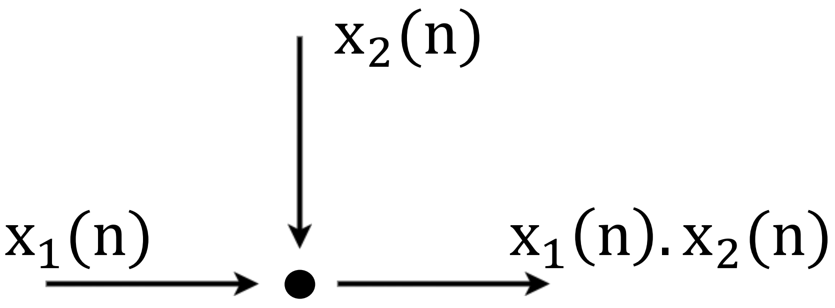
- Sum of two signals



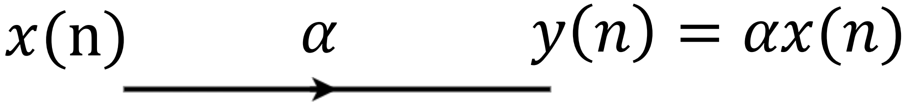


Basic operations with discrete signal

- Product of two signals



- Amplification of a signal



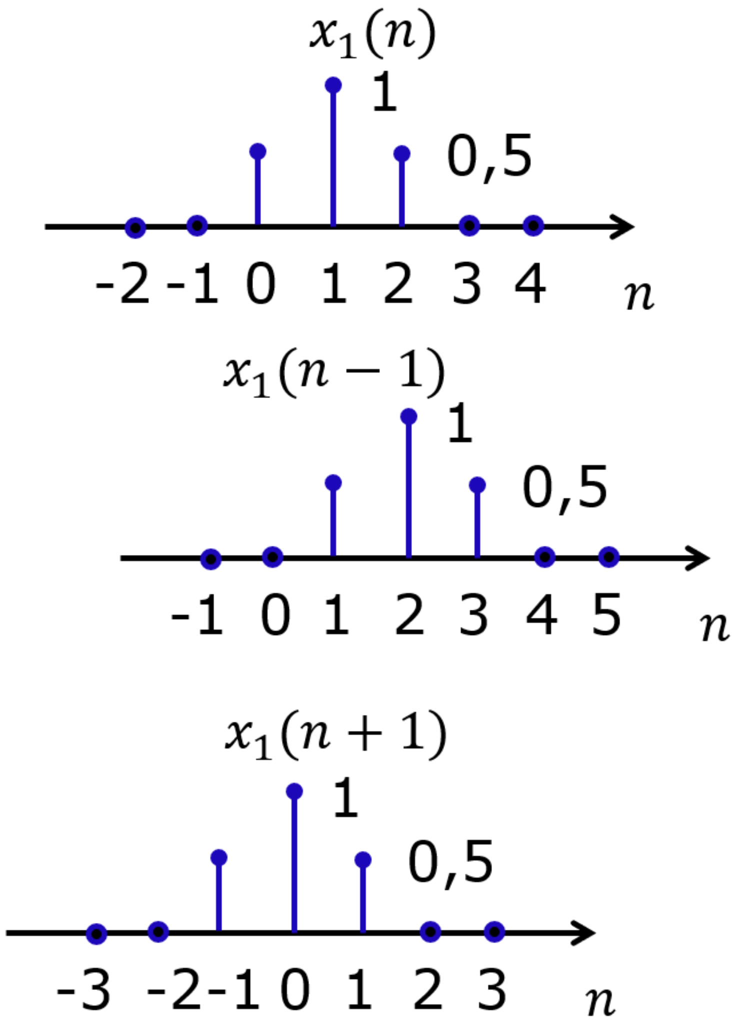
Time shifting

- Right shifting n_0 samples:

$$y(n) = x(n - n_0)$$

- Left shifting n_0 samples:

$$y(n) = x(n + n_0)$$



Signal delay and analysis

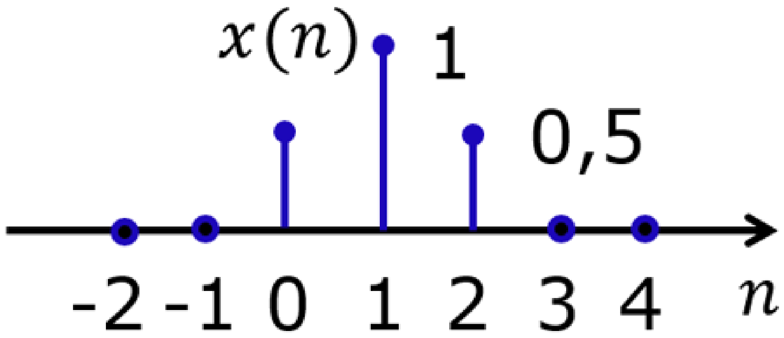
- Delay:

$$x(n) \longrightarrow \boxed{D} \longrightarrow x(n-1)$$

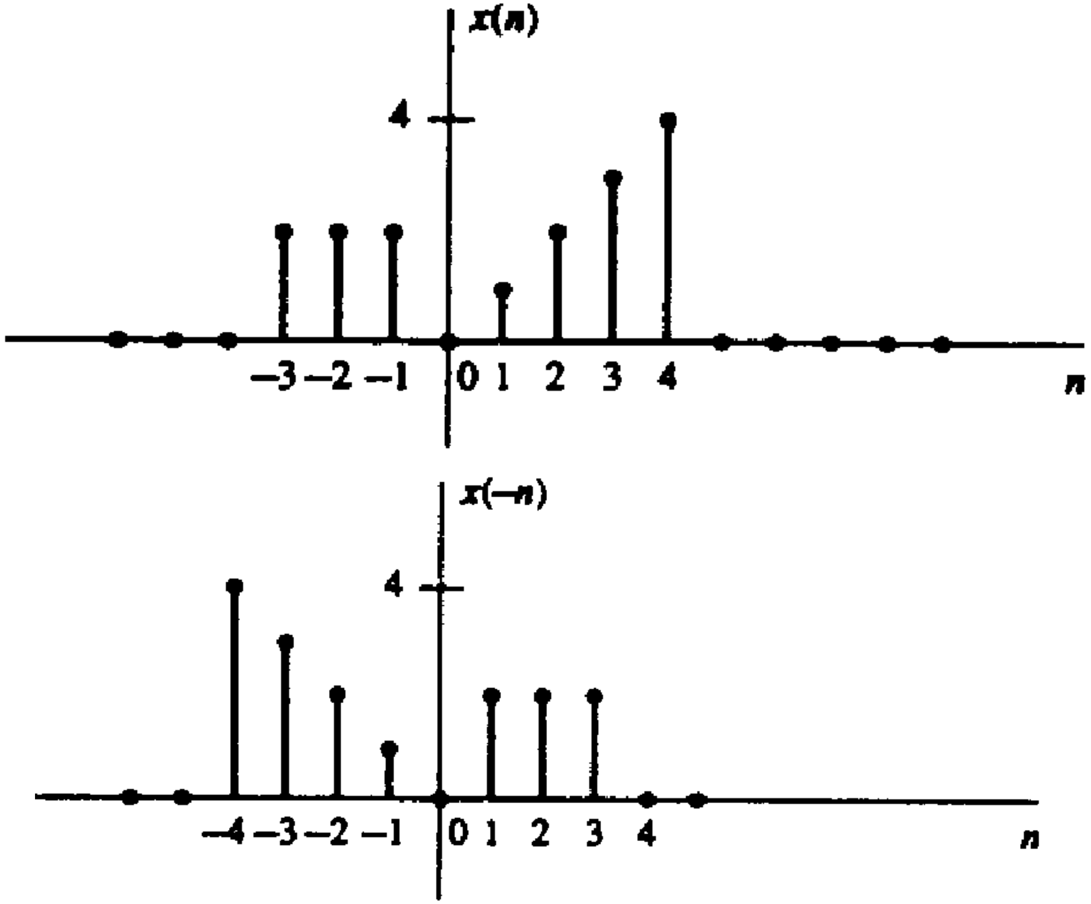
- Analysis: any discrete signal $\mathrm{x(n)}$ can be represented in terms of a sum of weighted delayed impulses

$$\mathrm{x(n)} = \sum_{\mathrm{k}=-\infty}^{\infty} \mathrm{x(k)}\delta(\mathrm{n-k})$$

- Example: $\mathrm{x(n)} = 0.5\,\delta(\mathrm{n}) + 1.\,\delta(\mathrm{n}-1) + 0.5\,\delta(\mathrm{n}-2)$



Time reversal: $y(n) = x(-n)$



Assignment

Given a discrete signal:

$$x(n) = \begin{cases} 1 + \frac{n}{2} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & n\# \end{cases}$$

a. Represent the signal $x(n)$ as a sequence of numbers and draw the signal $x(n)$.
b. Perform a time-reversal operation on the signal $x(n)$ to obtain the signal $x_1(n)$. Then, perform a 4-sample delay on this signal to obtain the signal $x_2(n)$. Represent $x_2(n)$ as a sequence of numbers and draw the signal $x_2(n)$.
c. Perform a 4-sample delay on the signal $x(n)$ to obtain the signal $x_3(n)$. Then, perform a time-reversal operation on this signal to obtain the signal $x_4(n)$. Represent $x_4(n)$ as a sequence of numbers and draw the signal $x_4(n)$.
d. Compare the signals $x_2(n)$ and $x_4(n)$. From this, derive a rule to obtain the signal $x(-n + k)$ from the signal $x(n)$.
e. Represent the signal $x(n)$ in terms of the signals $\delta(n)$ and $u(n)$.

Assignment – a

$$x(n) = 0 \ \forall \ n < -3$$

$$x(-3) = 1 - \frac{3}{2} = -\frac{1}{2}$$

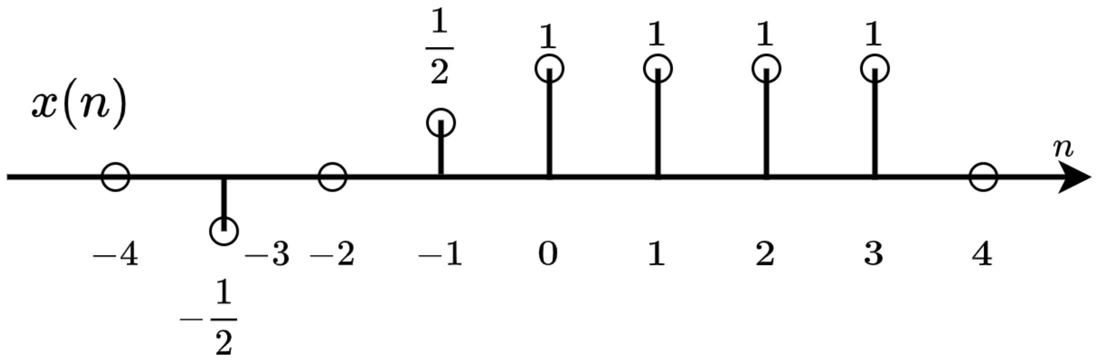
$$x(-2) = 1 - \frac{2}{2} = 0$$

$$x(-1) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$x(0) = x(1) = x(2) = x(3) = 1$$

$$x(n) = 0 \ \forall \ n > 3$$

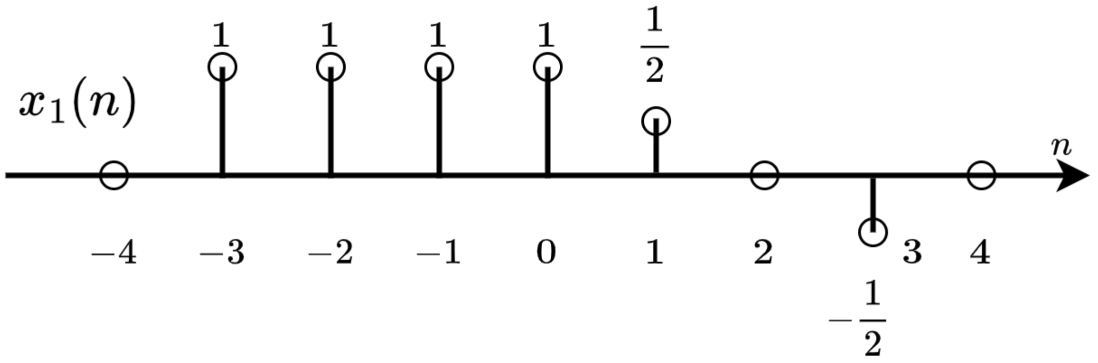
$$\Rightarrow x(n) = \left\{\cdots, 0, -\frac{1}{2}, 0, \frac{1}{2}, \underset{\uparrow}{1}, 1, 1, 1, 0, \cdots\right\}$$



Assignment – b

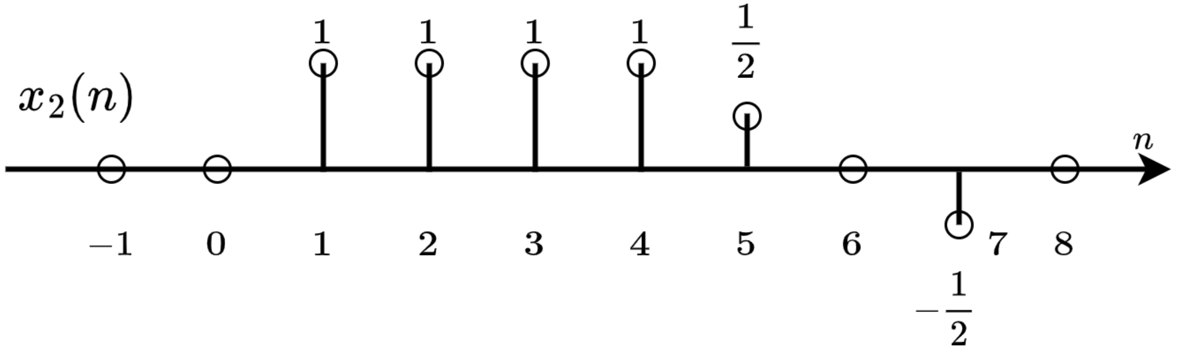
Performing time-reversal on the signal x(n) we obtain:

$$x_1(n) = \left\{\cdots, 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{1}{2}, 0, -\frac{1}{2}\, 0, \cdots\right\}$$



Continuing with a 4-sample delay on the signal $x_1(n)$ we have:

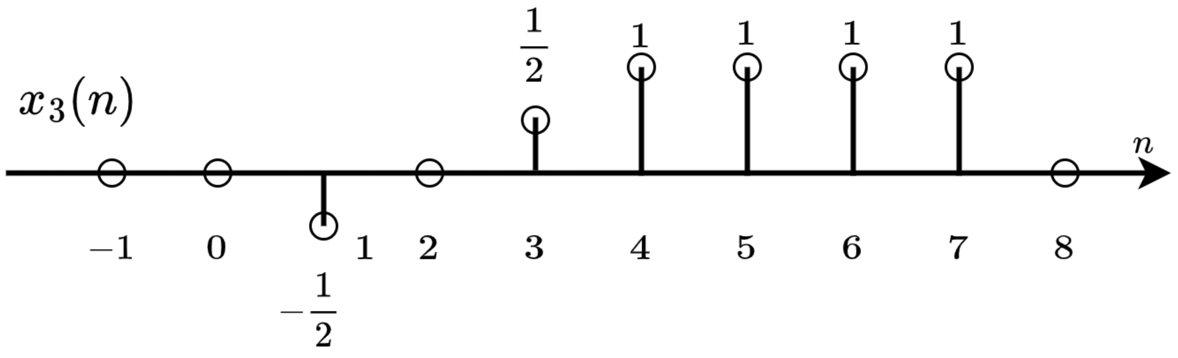
$$x_2(n) = \left\{\cdots, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{1}{2}, 0, -\frac{1}{2}\, 0, \cdots\right\}$$



Assignment – c

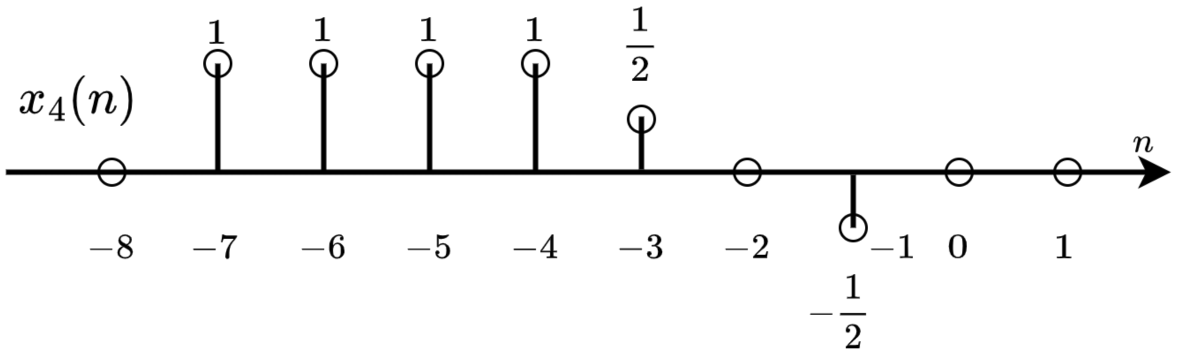
Performing a 4-sample delay on the signal x(n) we obtain:

$$x_3(n) = \left\{\cdots, 0, -\frac{1}{2}, \underset{\uparrow}{0}, \frac{1}{2}, 1, 1, 1, 1, 0, \cdots\right\}$$



Continuing with a 4-sample delay on the signal $x_3(n)$ we have:

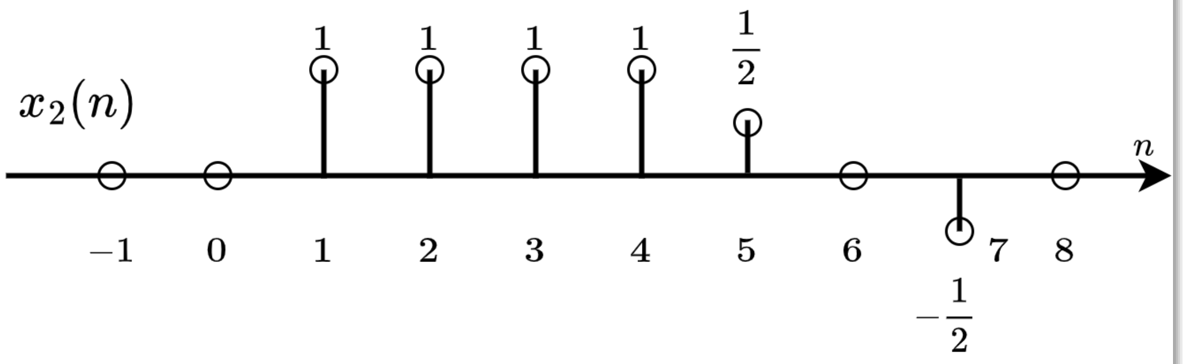
$$x_4(n) = \left\{\cdots, 0, 1, 1, 1, 1, \frac{1}{2}, 0, -\frac{1}{2}, \underset{\uparrow}{0}, \cdots\right\}$$

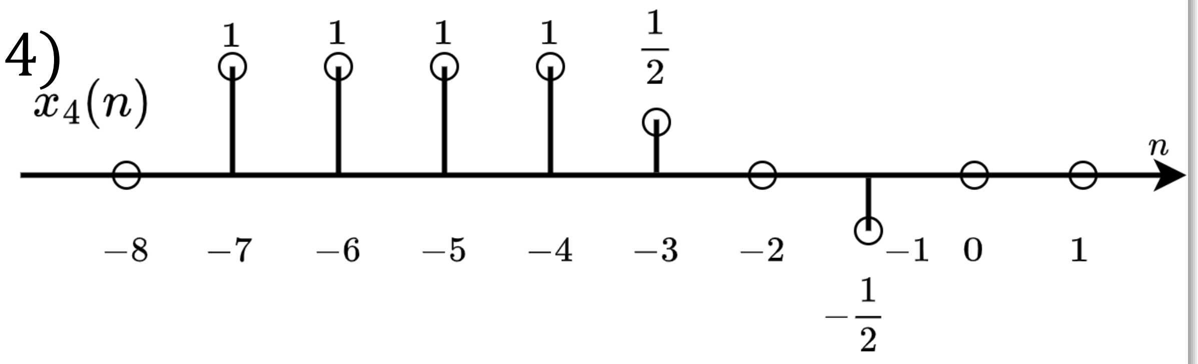


Assignment – d

Discussion:

- The signal $x_2(n)$ is delayed by 8 samples compared to the signal $x_4(n)$
- Relationship between $x_2(n)$, $x_4(n)$ and the original signal $x(n)$:
  - $x_2(n) = x(-n+4)$, since $x_1(n) = x(-n)$
  - $x_4(n) = x(-n-4)$, since $x_3(n) = x(n-4)$
- To obtain the signal $x(-n+k)$, we perform a time-reversal operation to obtain $x(-n)$. Then we shift this signal to the right by k samples if $k > 0$ or to the left by $-k$ samples if $k < 0$.





Assignment – e

Performing time reversals operation on the signal $x(n)$ we obtain:

$$x(n) = -\frac{1}{2}\delta(n+3) + \frac{1}{2}\delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$$

We observe that:

$$\begin{aligned} \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) &= rect_4(n) \\ &= u(n) - u(n-4) \end{aligned}$$

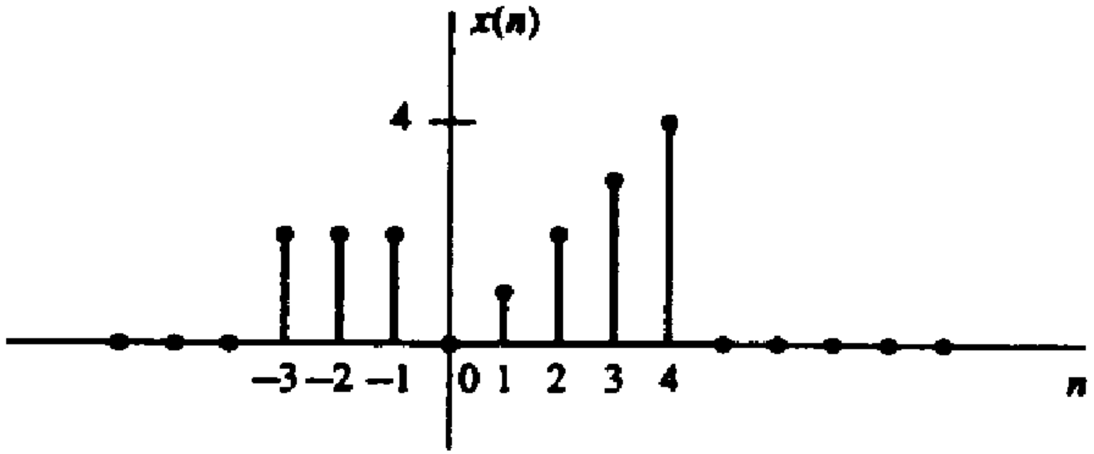
Thus:

$$x(n) = -\frac{1}{2}\delta(n+3) + \frac{1}{2}\delta(n+1) + u(n) - u(n-4)$$

4. Summary

- Discrete signals can be represented using graphs, functions, numerical tables, or sequences of numbers.
- Serveral fundamental types of discrete signals include unit impulses, unit step signal, rectangular pulses, exponential signals, and ramp signals.
- There are several basic operations that can be performed on discrete signals.

5. Assignment



- Assignment 1

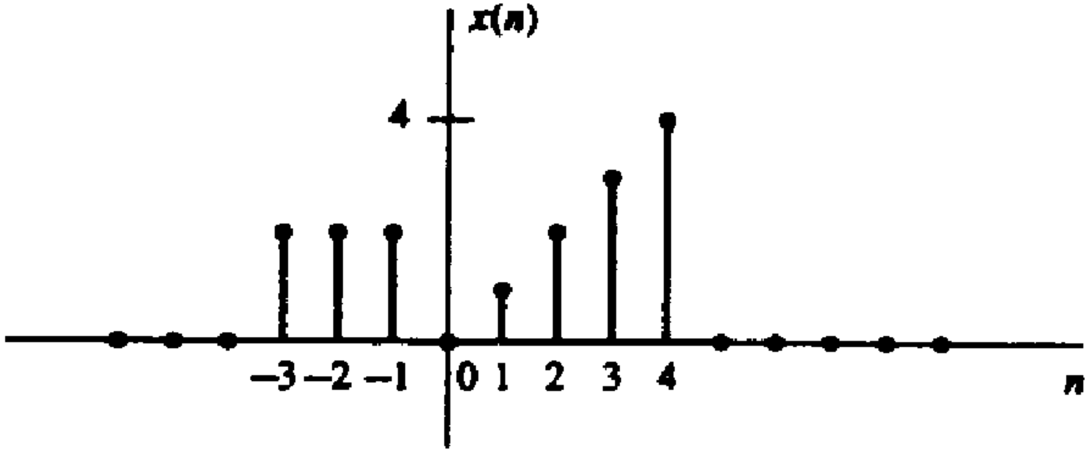
  a. Performing a 2-sample delay on the signal x(n) to yields the signal $x_1(n)$, then performing a time-reversal operation on $x_1(n)$ to produces the signal $x_2(n)$. Determine and plot the signals $x_1(n)$ and $x_2(n)$.

  b. Performing a time-reversal operation on the signal x(n) to yields the signal $x_3(n)$, then performing a 2-sample delay on $x_3(n)$ to produces the signal $x_4(n)$. Determine and plot the signals $x_3(n)$ and $x_4(n)$.

  c. Compare the signals $x_2(n)$ and $x_4(n)$.

Home work

- Assignment 2
  - Given the following signal $x(n)$:



  - Determine and plot the following signals, and comment on the role of each operation :
    - $x_1(n) = x(n-2)$
    - $x_2(n) = x(-n)$

The next unit 4

LINEAR TIME-INVARIANT DISCRETE-TIME SYSTEMS

References:

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

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Wishing you all the best in your studies!