# Week 2 questions

1) Show that $\forall \alpha, \beta, \gamma > 0$

$$\lim_{x \to +\infty} \frac{x^\alpha}{(\ln x)^\beta} = \lim_{x \to +\infty} \frac{e^{\gamma x}}{x^\alpha} = +\infty$$

2) Show that $\forall m, n \in \mathbb{N}$

a) $o(x^m) + o(x^n) = o(x^{\min(m,n)})$

b) $x^m \cdot o(x^n) = o(x^{m+n})$

c) $o(x^m) \cdot o(x^n) = o(x^{m+n})$

d) $(o(x^m))^n = o(x^{m \cdot n})$

e) $\forall m > n \quad o(x^n) + x^m = o(x^n)$

3) Classify the discontinuities of the following functions

a) $\frac{x}{\tan x}$ b) $e^{\tan x}$ c) $\arccos(\cos x)$

4) Construct a function with a removable discontinuity at $x = 0$ and a type II discontinuity at $x = 1$

5) Show that $\ln x$ is uniformly continuous on $[1, +\infty)$