HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS
DEPARTMENT OF APPLIED MATHEMATICS

MI2020E

EXERCISES OF PROBABILITY AND STATISTICS

Hanoi - 2023

Chapter 5

HYPOTHESIS TESTING

Problem 5.1.
The average weekly earnings for female social workers is \$670. Do men in the same positions have average weekly earnings that are higher than those for women? A random sample of $n = 40$ male social workers showed $\overline{x} = \$725$. Assuming a population standard deviation of \$102, test the appropriate hypothesis using $\alpha = 0.01$. Find the power of test if the true average weekly earning of male social workers is 720\$.

Problem 5.2.
A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.42 ounces. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

Problem 5.3.
A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 18 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$? Assume that the distribution of the length of a phone call is normal. Find the power of test if the true average length of phone calls is 7.6 minutes.

Problem 5.4.

A business student claims that, on average, an MBA student is required to prepare more than five cases per week. To examine the claim, a statistics professor asks a random sample of 10 MBA students to report the number of cases they prepare weekly. The results are exhibited here.

$$2, 7, 4, 8, 9, 5, 11, 3, 7, 4$$

Can the professor conclude at the 5% significance level that the claim is true, assuming that the number of cases is normally distributed with a standard deviation of 1.5? Find the probability of type II error when the students prepare 7 cases per week, on average.

Problem 5.5.

A courier service advertises that its average delivery time is less than 6 hours for local deliveries. A random sample of times for 12 deliveries to an address across town was recorded. These data are shown here.

$$3.03, 6.33, 7.98, 4.82, 6.50, 5.22, 3.56, 6.76, 7.96, 4.54, 5.09, 6.46$$

Is this sufficient evidence to support the courier's advertisement, at the 5% level of significance? Assume that the delivery time is normally distributed.

Problem 5.6.

According to a dietary study, high sodium intake may be related to ulcers, stomach cancer, and migraine headaches. The human requirement for salt is only 220 milligrams per day, which is surpassed in most single servings of ready-to-eat cereals. If a random sample of 20 similar servings of a certain cereal has a mean sodium content of 244 milligrams and a sample standard deviation of 24.5 milligrams, does this suggest at the 0.05 level of significance that the average sodium content for a single serving of such cereal is greater than 220 milligrams? Assume the distribution of sodium content to be normal.

Problem 5.7.

The daily yield for a local chemical plant has averaged 880 tons for the last several years. The quality control manager would like to know whether this average has changed in recent months. She randomly selects 50 days from the computer database and computes the average and sample standard deviation of the $n = 50$ yields as $\overline{x} = 871$ tons and $s = 21$ tons, respectively. Test the appropriate hypothesis using $\alpha = 0.05$.

Problem 5.8.
A college claims that more than 94% of their graduates find employment within 6 months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance? Find the probability of type II error when 96% of their graduates find employment within 6 months of graduation.

Problem 5.9.
A cigarette manufacturer claims that $1/8$ of the US adult population smokes cigarettes. In a random sample of 100 adults, 5 are cigarette smokers. Test the claim at $\alpha = 0.05$.

Problem 5.10.
It is believed that at least 60% of the residents in a certain area favor an annexation suit by a neighboring city. What conclusion would you draw if only 110 in a sample of 200 voters favored the suit? Use a 0.05 level of significance. Find the power of test if the true proportion of residents favoring the annexation suit is 70%.

Problem 5.11.
A high school math teacher claims that students in her class will score higher on the math portion of the ACT then students in a colleague's math class. The mean ACT math score for 49 students in her class is 22.1 and the sample standard deviation is 4.8. The mean ACT math score for 44 of the colleague's students is 19.8 and the sample standard deviation is 5.4. At $\alpha = 0.10$, can the teacher's claim be supported?

Problem 5.12.
To find out whether a new serum will arrest leukemia, 9 mice, all with an advanced stage of the disease, are selected. Five mice receive the treatment and 4 do not. Survival times, in years, from the time the experiment commenced are as follows:

Treatment	2.1	5.3	1.4	4.6	0.9
No Treatment	1.9	0.5	2.8	3.1	

At the 0.05 level of significance, can the serum be said to be effective? Assume the two populations to be normally distributed with equal variances.

Problem 5.13.

A men's softball league is experimenting with a yellow baseball that is easier to see during night games. One way to judge the effectiveness is to count the number of errors. In a preliminary experiment, the yellow baseball was used in 10 games and the traditional white baseball was used in another 10 games. The number of errors in each game was recorded and is listed here.

Yellow	5	2	6	7	2	5	3	8	4	9
White	7	6	8	5	9	11	8	3	6	10

Can we infer that there are fewer errors on average when the yellow ball is used at the 0.05 level of significance? Assume the two populations to be normally distributed with equal variances.

Problem 5.14.

A manufacturer claims that the average tensile strength of thread A exceeds the average tensile strength of thread B by at least 12 kilograms. To test this claim, 50 pieces of each type of thread were tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with a standard deviation of 6.28 kilograms, while type B thread had an average tensile strength of 77.8 kilograms with a standard deviation of 5.61 kilograms. Test the manufacturer's claim using a 0.05 level of significance.

Problem 5.15.

Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the company's new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows:

- Brand A: $x_A = 37,900$ kilometers, $s_A = 5100$ kilometers.
- Brand B: $x_B = 39,800$ kilometers, $s_B = 5900$ kilometers.

Test the hypothesis that there is no difference in the average wear of the two brands of tires. Assume the populations to be approximately normally distributed with equal variances. Use a 0.01 level of significance. Find the power of test if the true difference in the average wear of the two brands of tires is 3000 kilometers.

Problem 5.16.

A recent survey stated that male college students smoke less than female college students. In a survey of 1245 male students, 361 said they smoke at least one pack of cigarettes a day. In a survey of 1065 female students, 341 said they smoke at least one pack a day. At $\alpha = 0.01$, can you support the claim that the proportion of male college students who smoke at least one pack of cigarettes a day is lower then the proportion of female college students who smoke at least one pack a day?

Problem 5.17.

In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportions of urban and suburban residents who favor the construction of the nuclear plant? Use a 0.01 level of significance. Find the power of test if the true proportions of urban and suburban residents who favor the construction of the nuclear plant are 60% and 52%.