TEST BANK

Linear Algebra with Applications

(5th edition)

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Chapter 1: Systems of Linear Equations

1. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} x & & & & & + & w & = & 1 \\ x & & & + & z & + & w & = & 0 \\ x & + & y & + & z & & & = & -3 \\ x & + & y & & & - & 2w & = & 2 \end{array}\right.$ for z.

   a) -2 b) 2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$ e) 1 f) -1

   Answer: $\underline{-1}$.

2. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} x & + & y & - & 2z & + & 3w & = & 1 \\ x & + & 2y & - & 3z & + & w & = & 0 \\ & & y & - & 2z & + & w & = & -3 \\ -x & + & 2y & + & 5z & - & w & = & 2 \end{array}\right.$ for z.

   a) $\frac{9}{13}$ b) $\frac{11}{13}$ c) $\frac{12}{13}$ d) $\frac{15}{13}$ e) $\frac{17}{13}$ f) $\frac{19}{13}$

   Answer: $\underline{\frac{17}{13}}$.

3. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} 2x & - & 3y & + & 4z & + & w & = & 3 \\ & & & - & 2z & + & 4w & = & -5 \\ & & y & - & 2z & + & w & = & 1 \\ & & & & & & 3w & = & 2 \end{array}\right.$ and

   evaluate $6x + y + 6z + 3w$.

   a) 12 b) 44 c) 78 d) 234 e) 66 f) 158

   Answer: $\underline{66}$.

4. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} 6w & + & 5x & - & 2y & + & 4z & = & -4 \\ 9w & - & x & + & 4y & - & z & = & 13 \\ 3w & + & 4x & + & 2y & - & 2z & = & 1 \\ 3w & - & 9x & & & + & 2z & = & 11 \end{array}\right.$ for y.

   a) 1 b) $-\frac{3}{2}$ c) $\frac{3}{2}$ d) $\frac{4}{3}$ e) $\frac{5}{4}$ f) 0

   Answer: $\underline{\frac{3}{2}}$.

5. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} x & + & 2y & - & z & - & w & = & 0 \\ & & & & z & + & 2w & = & 4 \\ -x & - & 2y & + & 2z & + & 4w & = & 5 \end{array}\right.$ for y.

   a) 1 b) 2 c) 3 d) y is arbitrary e) -1 f) -2

   Answer: $\underline{y \text{ is arbitrary}}$.

6. Solve the system of equations $\left\{\begin{array}{rcrcrcr} x & + & y & + & z & = & 0 \\ -9x & - & 2y & + & 5z & = & 0 \\ -x & + & y & + & 3z & = & 0 \\ -7x & - & 2y & + & 3z & = & 0 \end{array}\right.$ for x.

a) x is arbitrary b) 1 c) -1 d) 2 e) 3 f) -2

Answer: $\underline{x \text{ is arbitrary}}$.

7. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} 2w & + & 4x & & & + & z & = & 1 \\ 2w & + & x & & & - & z & = & 1 \\ 4w & + & 5x & + & 2y & & & = & 3 \\ w & + & x & + & y & + & z & = & 0 \end{array}\right.$ for x.

a) $\frac{3}{4}$ b) $-\frac{7}{4}$ c) $\frac{4}{7}$ d) $-\frac{2}{3}$ e) $-\frac{2}{7}$ f) 1

Answer: $\underline{\frac{4}{7}}$.

8. Solve the system of equations $\left\{\begin{array}{rcrcrcrcr} x_1 & + & x_2 & & & + & 5x_4 & = & 6 \\ x_1 & + & 2x_2 & + & x_3 & & & = & 4 \\ & & 2x_2 & + & x_3 & + & x_4 & = & 6 \\ 3x_1 & & & & & - & 4x_4 & = & 2 \end{array}\right.$ for x_4.

a) -10 b) 56 c) -98 d) -8 e) -40 f) 24

Answer: $\underline{-8}$.

9. Solve the system of equations $\left\{\begin{array}{rcrcrcrcrcr} & & & - & 2x_3 & + & & + & 7x_5 & = & 12 \\ 2x_1 & + & 4x_2 & - & 10x_3 & + & 6x_4 & + & 12x_5 & = & 28 \\ 2x_1 & + & 4x_2 & - & 5x_3 & + & 6x_4 & - & 5x_5 & = & -1 \end{array}\right.$ for x_3.

a) -5 b) 3 c) -2 d) 2 e) -1 f) 1

Answer: $\underline{1}$.

10. If $C = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & -3 & 6 \\ 0 & 5 & 0 & -2 \\ 2 & 1 & 6 & 0 \end{bmatrix}$ and $C \begin{bmatrix} u \\ v \\ w \\ x \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ -2 \end{bmatrix}$, find $u + v + w + x$.

a) 208 b) 110 c) 10 d) 99 e) 363 f) -101

Answer: $\underline{99}$.

11. If the augmented matrix $[A \mid B]$ of a system $AX = B$ is row-equivalent to $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

    which of the following statements is true?

    a) The system is inconsistent.

    b) $X = (5, -2, -s, 1)$ is a solution for any value of s.

    c) $X = (5, -2, 1)$ is the unique solution of the system.

    d) $X = (5x, -2s, s)$ is a solution for any value of s.

    e) $X = (5t, -2, -s, s)$ is a solution for any value of s and t.

    f) $X = (5, -3, 1)$ is the unique solution to the system.

    Answer: $X = (5, -3, 1)$ is the unique solution to the system.

12. If the augmented matrix $[A \mid B]$ of a system $AX = B$ is row-equivalent to $\begin{bmatrix} 1 & 5 & 0 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

    which of the following statements is true?

    a) $X = (6, 1, 0)$ is a solution.

    b) $X = (6s - 5, s, 1)$ is a solution for any value of s.

    c) $X = (6 - 5s, s, 1)$ is a solution for any value of s.

    d) $X = \left(6, \frac{6}{5}, 1\right)$ is a solution.

    e) There are no solutions (the system is inconsistent).

    f) $X = (0, 0, 0)$ is a solution.

    Answer: $X = (6 - 5s, s, 1)$ is a solution for any value of s.

13. If, in a system $AX = B$, A is $n \times n$ and B is $n \times 1$, the rank of A is n and the rank of the augmented matrix $[A \mid B]$ is also n, then:

    a) the system has no solution.

    b) the system has a unique solution.

    c) the system has infinitely many solutions.

    d) the system has n solutions.

    e) the determinant of A is zero.

    f) such a system cannot exist.

    Answer: The system has a unique solution.

14. Which of the following statements is true for the system of equations

$$\left\{\begin{array}{rcrcrcr} 2x & + & y & - & z & = & 0 \\ 3x & - & y & + & 3z & = & 4 \\ x & + & y & - & 2z & = & -2 \\ 2x & - & y & + & 2z & = & 5 \end{array}\right. ?$$

a) It has no solutions.

b) It has an infinite number of solutions.

c) It has the trivial solution.

d) It has the unique solution $(3, 4, 5)$.

e) It has the solutions $\pm(4, 3, 1)$.

f) It has the unique solution $(0, -1, 2)$.

Answer: <u>It has no solutions.</u>

15. Which of the following statements is true for the system of equations

$$\left\{\begin{array}{rcrcrcr} 7x & - & 2y & + & 5z & = & 3 \\ 4x & + & 6y & - & 3z & = & -2 \\ 10x & - & 10y & + & 13z & = & 91 \end{array}\right. ?$$

a) It has infinitely many solutions with 1 free parameter.

b) It has infinitely many solutions with 2 free parameters.

c) It is inconsistent.

d) It has the trivial solution.

e) It has the unique solution $(3, -1, -1)$.

f) It has the unique solution $(-1, 3, 3)$.

Answer: <u>It is inconsistent.</u>

16. Which of the statements below is correct for the system of equations S where

$$S = \left\{\begin{array}{rcrcrcrcr} x & - & 2y & + & z & + & w & = & 2 \\ 3x & & & + & 2z & - & 2w & = & -8 \\ & & 4y & - & z & - & w & = & 2 \\ 2x & + & y & + & z & - & w & = & k \end{array}\right. ?$$

a) S has a unique solution $(0, 2, 1, 5)$ if $k = -2$.

b) S is inconsistent if $k = -2$.

c) S has an infinite number of solutions if $k = -2$.

d) S has the trivial solution of $k = -2$.

e) S has a unique solution for all k.

f) S is inconsistent for all k.

Answer: $\underline{S \text{ has an infinite number of solutions if } k = -2.}$

17. Which one of the statements below is correct for the system of linear equations

$$\left\{\begin{array}{rcrcrcr} -x & + & y & + & z & = & 4 \\ x & - & 2y & + & 3z & = & 2 \\ -2x & + & y & - & z & = & 7 \\ 3x & - & 9y & - & 2z & = & 1 \end{array}\right. ?$$

a) The system has no solutions.

b) The system has an infinite number of solutions.

c) The system has a unique solution with $x = -5$.

d) The system has a unique solution with $x \neq -5$.

e) The system has a unique solution with $z = 2$.

f) The system has a unique solution with $y = 6$.

Answer: $\underline{\text{The system has a unique solution with } x = -5.}$

18. The system of equations $\left\{\begin{array}{rcrcrcr} x & + & y & - & z & = & 3 \\ x & - & y & + & z & = & 0 \\ 2x & + & y & + & 2z & = & 3 \end{array}\right.$:

a) is inconsistent.

b) has exactly 2 solutions.

c) has exactly 1 non-trivial solution.

d) has an infinite number of solutions.

e) has exactly 3 solutions.

f) has the trivial solution only.

Answer: $\underline{\text{Has exactly 1 non-trivial solution.}}$

19. The system of linear equations $\left\{\begin{array}{rcrcrcrcrcrcr} u & + & 2v & & & - & 3x & + & y & & & = & 2 \\ u & + & 2v & + & w & - & 3x & + & y & + & 2z & = & 3 \\ u & + & 2v & & & - & 3x & + & 2y & + & z & = & 4 \\ 3u & + & 6v & + & w & - & 9x & + & 4y & + & 3z & = & 9 \end{array}\right.$:

a) is inconsistent.

b) has only one solution: $(0, 0, 1, 0, 2, 0)$.

c) has an infinite number of solutions with 1 free parameter.

d) has an infinite number of solutions with 2 free parameters.

e) has an infinite number of solutions with 3 free parameters.

f) has an infinite number of solutions with 4 free parameters.

Answer: Has an infinite number of solutions with 3 free parameters.

20. Let S be the system of equations $\left\{\begin{array}{rcrcl}(\beta-3)x & + & 2y & = & 0\\ x & + & (\beta-4)y & = & 0\end{array}\right.$.

a) S has an infinite number of solutions if $\beta = 2$ or 5.

b) S has an infinite number of solutions if $\beta = 3$ or 4.

c) S has an infinite number of solutions for all β.

d) S has one solution if $\beta = 2$ or 5.

e) S has one solution if $\beta = 3$ or 4.

f) S is inconsistent if $\beta = 2$ or 5.

Answer: S has an infinite number of solutions if $\beta = 2$ or 5.

21. Which of the following statements is correct for the system of linear equations

$$\left\{\begin{array}{rcrcrcr}3x & + & y & + & 2z & = & 1\\ x & + & y & + & z & = & 1\\ x & - & y & & & = & -1\\ & & y & - & z & = & 0\end{array}\right. ?$$

a) No solution exists.

b) There is a unique solution.

c) There are infinitely many solutions when $y = 1$.

d) There are infinitely many solutions when $y = -1$.

e) There are infinitely many solutions except when $y = 1$ or $y = -1$.

f) None of the above statements are correct.

Answer: There is a unique solution.

22. Which one of the statements below is true for the system of equations

$$\left\{\begin{array}{rcrcrcr}x & + & y & - & z & = & 0\\ 2x & + & 4y & - & z & = & 0\\ 3x & + & 11y & + & z & = & 0\end{array}\right. ?$$

a) The system has a unique solution $(x, y, z) = (0, 0, 0)$.

b) The dimension of the solution space is 2.

c) The system has solutions of the form $(1, s, s)$, s is arbitrary.

d) The system has solutions of the form $(3s, -s, 2s)$, s is arbitrary.

e) The system has a unique solution $(-3, 1, -2)$.

f) The system is inconsistent.

Answer: The system has solutions of the form $(3s, -s, 2s)$, s is arbitrary.

23. The linear system of equations $\begin{cases} x + 2y + z = 2 \\ 3x + y - 2z = 1 \\ 4x - 3y - 7z = -3 \\ 2x + 4y + 2z = 4 \end{cases}$ has:

a) infinitely many solutions.

b) no solutions.

c) infinitely many solutions with $x = 0$.

d) infinitely many solutions with $y = 2$.

e) a unique solution with $x = 0$.

f) a unique solution with $y = 2$.

Answer: Infinitely many solutions.

24. The system of equations $\begin{cases} 3x_1 + 2x_2 + x_3 = 0 \\ x_1 + x_2 + x_3 = 1 \\ x_1 - x_3 = -1 \\ x_2 - x_3 = 0 \end{cases}$:

a) has infinitely many solutions.

b) has a unique solution and $x_1 = \frac{2}{3}$.

c) has a unique solution and $x_1 = -\frac{1}{3}$.

d) has infinitely many solutions and $x_2 = \frac{2}{3}$.

e) has no solution.

f) has the unique solution $(0, 0, 0)$.

Answer: has no solution.

25. Which of the following statements is true about the system of equations S, where

$$S = \begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (a^2 - 5)z = a \end{cases} ?$$

(i) S has no solution when $a = -2$.

(ii) S has no solution when $a = 2$.

(iii) S has a unique solution when $a \neq \pm 2$.

(iv) S has infinitely many solutions when $a \neq \pm 2$.

(v) S has infinitely many solutions when $a = -2$.

a) (i) and (iii) b) (ii) and (iii) c) (i) and (iv)

d) (ii) and (iv) e) (ii) and (v) f) (i) and (ii)

Answer: (i) and (iii).

26. The system of equations $\begin{cases} x + 2y + 3z = 0 \\ 4x + ty + 6z = 0 \\ 6x + 5y + 4z = 0 \end{cases}$:

a) has infinitely many solutions for all values of t.

b) is inconsistent unless $t = 5$.

c) has a unique solution $(x, y, z) = (0, 0, 0)$ if $t \neq 5$.

d) has a unique solution $(x, y, z) = (0, 0, 0)$ if $t = 5$.

e) is inconsistent for all t.

f) has a unique solution for all values of t.

Answer: Has a unique solution $(x, y, z) = (0, 0, 0)$ if $t \neq 5$.

27. The coefficient matrix A in a homogeneous system of 12 equations in 16 unknowns is known to have rank 6. How many free parameters are there in the solution?

a) 10 b) 6 c) 4 d) none e) 12 f) 16

Answer: 10

28. For a homogeneous system S of 4 equations in 5 unknowns, which of the following statements is (are) true?

(i) S can be inconsistent.

(ii) S can have a unique solution.

(iii) S can have infinitely many solutions.

a) (i) b) (ii) c) (iii) d) (i) and (iii) e) (ii) and (iii) f) (i) and (ii)

Answer: (iii)

29. A homogeneous linear system of 6 equations in 7 unknowns must have:

a) exactly the same number of solutions as unknowns,

b) many solutions or none,

c) one solution or none,

d) many solutions or exactly one solution,

e) many solutions.

f) The system has either the trivial solution only or infinitely many solutions.

Answer: e) many solutions.

30. Consider a homogeneous system of 6 linear equations in 5 unknowns. Which of the following is true?

    a) The system can have no solution.

    b) The system has between 1 and 6 solutions.

    c) The system has between 0 and 5 solutions.

    d) The system always has infinitely many solutions.

    e) The system has only the trivial solution.

    f) The system has either the trivial solution only or infinitely many solutions.

    Answer: The system has either the trivial solution only or infinitely many solutions.

31. For a system of four equations in three unknowns, which statements are true?

    a) There is always at least one solution.

    b) There may be exactly 3 solutions.

    c) There may be exactly 4 solutions.

    d) There may be exactly 1 solutions.

    e) There may be no solution.

    f) There may be infinitely many solutions.

    g) If the system is homogeneous, then there are always infinitely many solutions.

    h) If the system is homogeneous, then there is always at least one solution.

    Answer: e), f), and h).

32. Complete the following phrase to make a true statement.

    "A system of 1100 linear equations in 550 unknowns. . ."

    a) always has a solution, which may not be unique.

    b) always has a unique solution.

    c) may be inconsistent.

    d) which is consistent always has a unique solution.

    e) which is consistent never has a unique solution.

    f) is never consistent

    Answer: May be inconsistent.

33. A homogeneous linear system of 1996 equations in 236 unknowns:

    a) can be inconsistent.

    b) is never inconsistent and will never have a unique solution.

    c) is never inconsistent and always has a unique solution.

    d) is never inconsistent and may have a unique solution.

    e) is never inconsistent and has 1730 parameters in the solution.

    f) has 1730 solutions or less.

    Answer: Is never inconsistent and may have a unique solution.

34. A linear system of 212 equations in 312 unknowns:

    a) is always consistent and has exactly 100 parameters in the general solution.

    b) is always consistent and has at most 100 parameters in the general solution.

    c) is always consistent and has at least 100 parameters in the general solution.

    d) which is consistent has exactly 100 parameters in the general solution.

    e) which is consistent has at most 100 parameters in the general solution.

    f) which is consistent has at least 100 parameters in the solution.

    Answer: Which is consistent has at least 100 parameters in the solution.

35. For a homogeneous system of 10 equations in 12 unknowns, state which combination of answers to the following questions is correct.

    - Can the system be inconsistent?
    - Can the system have infinitely many solutions?
    - Can the system have only one solution?

    a) Yes, Yes, No   b) No, No, Yes   c) Yes, No, Yes

    d) No, Yes, No   e) Yes, Yes, Yes   f) No, No, No

    Answer: No, Yes, No.

36. For a homogeneous system of 289 equations in 301 unknowns, state which combination of answers to the following questions is correct.

    - Is it possible for the system to have infinitely many solutions?
    - Is it possible for the system to have only one solution?
    - Is it possible for the system to have no solutions at all?

    (a) No, No, Yes   b) Yes, Yes, No   c) Yes, No, Yes

    d) Yes, No, No   e) Yes, Yes, Yes   f) No, No, No

    Answer: Yes, No, No.

37. For a non-homogeneous system of 17 equations in 9 unknowns, state which combination of answers to the following questions is correct.

    - Can the system have no solutions at all?
    - Can the system have one – and only one – solution?
    - Can the system have an infinite number of solutions?

    a) Yes, No, No   b) Yes, Yes, No   c) Yes, No, Yes

    d) Yes, Yes, Yes   e) No, Yes, Yes   f) No, No, No

    Answer: Yes, Yes, Yes.

38. For a non-homogeneous system of 5 equations in 14 unknowns, state which combination of answers to the following questions is correct.

- Can the system be inconsistent?
- Can the system have exactly two solutions?
- Can the system have infinitely many solutions?

a) Yes, No, Yes	b) No, Yes, Yes	c) Yes, Yes, No
d) No, No, Yes	e) Yes, Yes, Yes	f) No, Yes, No

Answer: Yes, No, Yes.

39. For a homogeneous system of 9 equations in 8 unknowns, state which combination of answers to the following questions is correct.

- Can the system be inconsistent?
- Can the system have a unique solution?
- Can the system have infinitely many solutions?

a) No, Yes, No	b) No, No, Yes	c) Yes, No, Yes
d) No, Yes, Yes	e) Yes, Yes, No	f) Yes, Yes, Yes

Answer: No, Yes, Yes.

40. For a non-homogeneous system of 12 equations with 15 unknowns, state which combination of answers to the following questions is correct?

- Is it possible for the system to be inconsistent?
- Is it possible for it to have infinitely many solutions?
- Is it possible for it to have exactly one solution?

a) No, Yes, No	b) Yes, Yes, Yes	c) Yes, Yes, No
d) No, No, No	e) Yes, No, Yes	f) No, No, Yes

Answer: Yes, Yes, No.

41. For what value(s) of λ will the system of equations $\left\{\begin{array}{ccccccccc} \lambda w & + & x & + & y & + & z & = & 1 \\ w & + & \lambda x & + & y & + & z & = & 1 \\ w & + & x & + & \lambda y & + & z & = & 1 \\ w & + & x & + & y & + & \lambda z & = & 1 \end{array}\right.$

be inconsistent?

a) -3 and 1	b) 3 and -1	c) 3 only
d) -2 only	e) -3 only	f) 1 only

Answer: -3 only.

42. For what value(s) of λ will the system of equations $\begin{cases} \lambda w + x + y + z = 1 \\ w + \lambda x + y + z = 1 \\ w + x + \lambda y + z = 1 \\ w + x + y + \lambda z = 1 \end{cases}$

have infinitely many solutions?

a) 1 only   b) -3 only   c) 1 and -3
d) 1 and -2 e) 0 only   f) 0 and -3

Answer: 1 only.

43. For what value(s) of λ will the system of equations $\begin{cases} (1+\lambda)x + y + z = 1 \\ x + (1+\lambda)y + z = 1 \\ x + y + (1+\lambda)z = 1 \end{cases}$

have infinitely many solutions?

a) -1 b) -3 c) 1   d) 0   e) 0 and 1   f) $-1, 0$ and -3

Answer: 0.

44. Find all value(s) of α so that the system of equations $\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (\alpha^2 - 5)z = \alpha \end{cases}$

has no solution.

a) -4 b) 2   c) 4   d) 2 or -2 e) anything but -2 f) -2

Answer: -2.

45. Find all values(s) of α so that the system of equations $\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (\alpha^2 - 5)z = \alpha \end{cases}$

has an infinite number of solutions.

a) 2   b) 2 or -2 c) 4 or -4 d) anything but 2 and -2 e) 0   f) anything but 0

Answer: 2.

46. Find all value(s) of α so that the system of equations $\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (\alpha^2 - 5)z = \alpha \end{cases}$

has a unique solution.

a) 2   b) 2 or -2 c) -2
d) anything but 2 and -2 e) 0   f) the system always has a unique solution.

Answer: Anything but 2 and -2.

47. Find all values of λ so that the system of equations $\left\{\begin{array}{rcrcrcl} & & (\lambda-1)y & + & z & = & 0 \\ x & + & (\lambda-2)y & + & z & = & 0 \\ & & y & + & (\lambda-1)z & = & 0 \end{array}\right.$

has non-trivial solutions.

a) $1,-1$ b) $0,1$ c) $0,2$ d) $2,1$ e) $2,-2$ f) $1,-2$

Answer: $\underline{0,2.}$

48. Find all values of λ and μ for which the system of equations $\left\{\begin{array}{rcrcrcl} x & + & y & + & 3z & = & 2 \\ x & + & 2y & + & 5z & = & 1 \\ 3x & + & 4y & + & \lambda z & = & \mu \end{array}\right.$

has a unique solution.

a) $\lambda = 11, \mu = 5$ b) $\lambda \neq 11, \mu \neq 5$ c) $\lambda \neq 11, \mu = 5$

d) $\lambda = 11, \mu \neq 5$ e) $\lambda \neq 11, \mu$ is arbitrary f) λ is arbitrary, $\mu \neq 5$

Answer: $\underline{\lambda \neq 11,\ \mu \text{ is arbitrary.}}$

49. Find all values of a and b for which the system of equations $\left\{\begin{array}{rcrcrcl} x & + & y & + & 3z & = & 2 \\ x & + & 2y & + & 5z & = & 1 \\ 4x & + & 5y & + & az & = & b \end{array}\right.$

has infinitely many solutions.

a) $a = 14, b = 7$ b) $a \neq 14, b = 7$ c) $a \neq 14, b \neq 7$

d) $a = 14, b \neq 7$ e) a is arbitrary, $b \neq 7$ f) $a = 14, b$ is arbitrary

Answer: $\underline{a = 14, b = 7.}$

50. Find all values of a and b for which the system of equations $\begin{array}{rcrcrcl} x & + & y & + & 3z & = & 2 \\ x & + & 2y & + & 5z & = & 1 \\ 2x & + & 2y & + & az & = & b \end{array}$

is inconsistent.

a) $a \neq 6, b \neq 4$ b) $a = 6, b \neq 4$ c) $a \neq 6, b = 4$

d) $a = 6, b = 4$ e) a is arbitrary, $b \neq 4$ f) $a = 6, b$ is arbitrary

Answer: $\underline{a = 6, b \neq 4.}$

51. Find all value(s) of c so that the system of equations $\left\{\begin{array}{rcrcrcl} & & cy & + & z & = & 0 \\ x & + & (c-1)y & + & z & = & 0 \\ x & + & cy & + & (c+1)z & = & 0 \end{array}\right.$

has a unique solution.

a) $0,2$ b) 1 c) -1 d) $1,2$ e) $-1,-2$ f) $1,-1$

Answer: $\underline{1,-1.}$

52. Find the value of the constant a for which the system of equation $\left\{\begin{array}{rcrcrcr} ax & + & y & + & z & = & 1 \\ 2x & + & ay & + & 3z & = & 1 \\ & & y & - & z & = & 1 \end{array}\right.$

has infinitely many solutions.

a) 2 b) 3 c) -1 d) -4 e) 1 f) -2

Answer: 1.

53. Find the value of the constant a for which the system of equations $\left\{\begin{array}{rcrcrcr} ax & + & y & + & z & = & 1 \\ 2x & + & ay & + & 3z & = & 1 \\ & & y & - & z & = & 1 \end{array}\right.$

is inconsistent.

a) 1 b) 4 c) 2 d) -1 e) -2 f) -4

Answer: -4.

54. Find all values of the constant a for which the system of equations $\left\{\begin{array}{rcrcrcr} ax & + & y & + & z & = & 1 \\ 2x & + & ay & + & 3z & = & 1 \\ & & y & - & z & = & 1 \end{array}\right.$

has a unique solution.

a) 1 or -4 b) -1 or 4 c) any $a \neq 1$ and -4
d) any $a \neq 0$ and 1 e) $0, 1, -4$ f) any $a \neq 0$ and 4

Answer: any $a \neq 1$ and -4.

55. Find the values of a for which the system of equations $\left\{\begin{array}{rcrcrcrcr} x & & & + & 3z & + & 2u & = & 0 \\ -x & + & ay & + & 2z & - & u & = & 0 \\ 2x & + & 3y & + & 5z & + & u & = & 0 \\ ax & + & 4y & - & z & - & 3u & = & 0 \end{array}\right.$

has solutions other than the trivial solution.

a) 0 or 3 b) 2 or 3 c) 1 or 3 d) 1 or 2 e) 0 or 2 f) 0 or 1

Answer: 1 or 2.

56. Find the value(s) of k for which the system of equations $\left\{\begin{array}{rcrcrcr} 2x & + & y & - & z & = & -4 \\ kx & + & y & + & 4z & = & 1 \\ 3x & & & + & kz & = & 5 \end{array}\right.$

has a unique solution.

a) -3 b) any $k \neq 5$ c) 5 or -3
d) any $k \neq 5$ and -3 e) 0 and 5 f) any $k \neq -3$

Answer: any $k \neq 5$ and -3.

57. Find all a so that the system of equations $\left\{\begin{array}{rcrcrcl} x & + & y & + & z & = & 0 \\ 2x & + & y & + & az & = & 0 \\ & - & 2y & + & (a^2-1)z & = & 0 \end{array}\right.$

has non-trivial solutions.

a) -3 or 1 b) 3 or -1 c) all $a \neq -3$ and 1

d) all a e) -3 or -1 f) all $a \neq 3$ and -1

Answer: 3 or -1.

58. Find the constraint under which the system of equations $\left\{\begin{array}{rcrcrcl} x & - & 2y & + & 7z & = & a \\ 3x & + & 5y & + & z & = & b \\ 4x & + & 3y & + & 8z & = & c \end{array}\right.$

has infinitely many solutions.

a) $a+b-c=0$ b) $a-b-c=0$ c) $a+b+c=0$

d) $a-b+c=0$ e) $a=b=c=0$ f) $a=0$ and $b-c=0$

Answer: $a+b-c=0$.

59. Find the value(s) of a for which the system of equations $\left\{\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & 4 \\ 3x & - & 4y & - & 5z & = & 4 \\ -2x & + & y & + & (a^2-a-16)z & = & a \end{array}\right.$

is inconsistent.

a) 5 b) all $a \neq 5$ c) -5

d) all $a \neq -4$ and 5 e) all a f) no a

Answer: 5.

60. Find the value(s) of a for which the system of equations $\left\{\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & 4 \\ 3x & - & 4y & - & 5z & = & 4 \\ -2x & + & y & + & (a^2-a-16)z & = & a \end{array}\right.$

has infinitely many solutions.

a) -4 only b) all a c) system cannot have infinitely many solutions

d) 5 or -4 e) 5 only f) all $a \neq -4$

Answer: -4 only.

61. Find the value(s) of a for which the system of equations $\left\{\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & 4 \\ 3x & - & 4y & - & 5z & = & 4 \\ -2x & + & y & + & (a^2-a-16)z & = & a \end{array}\right.$

has a unique solution.

a) -4 or 5 b) all a c) system cannot have only one solution
d) all $a \neq -4$ and 5 e) 4 only f) all $a \neq -3$

Answer: all $a \neq -4$ and 5.

62. Find all value(s) of k for which the system of equations $\begin{cases} x + 2y + z = 0 \\ x + 3y + 2kz = 0 \\ 2x + 3y + kz = 0 \end{cases}$

has non-trivial solutions.

a) -1 b) 1 c) $\frac{1}{2}$ d) 3 e) -3 f) 0

Answer: 1.

63. Find all value(s) of k so that the system of equations $\begin{cases} 2x + (k+2)y = 0 \\ (k+2)x + 8y = 0 \end{cases}$

has non-trivial solutions.

a) -2 or 6 b) $\frac{1}{2}$ c) 3 d) all $k \neq 2$ e) all $k \neq 6$ f) 2 or -6

Answer: 2 or -6.

64. Find the value of k for which the system of equations $\begin{cases} x + 2y + z = 0 \\ 3x + 7y + kz = 0 \\ 4x + 9y + 3z = 0 \end{cases}$

has non-trivial solutions.

a) -2 b) 0 c) 4 d) -4 e) 8 f) 2

Answer: 2.

65. Find all values of p and q for which the system of equations $\begin{cases} x + 2z = 0 \\ y + w = q \\ x + 5z + 3w = 0 \\ 2y + 3z + pw = 4 \end{cases}$

is inconsistent.

a) $p = 0, q = 0$ b) $p = 5, q$ is arbitrary c) $p \neq 5, q$ is arbitrary
d) $p = 5, q \neq 2$ e) $p = 0, q$ is arbitrary f) $p = 0, q$ is arbitrary

Answer: $p = 5, q \neq 2$.

66. Find all values of p and q for which the system of equations $\begin{cases} x + 2z = 0 \\ y + w = q \\ x + 5z + 3w = 0 \\ 2y + 3z + pw = 4 \end{cases}$

has a unique solution.

a) $p = 0, q = 0$ b) all p and q but $p = q = 0$ c) $p = 5, q$ is arbitrary
d) p is arbitrary, $q = 2$ e) $p = 5, q = 2$ f) p is arbitrary, $q = 0$

Answer: $\underline{p = 5, q = 2.}$

67. Find all values of p and q for which the system of equations $\begin{cases} x & & + & 2z & & & = & 0 \\ & y & & & + & w & = & q \\ x & & + & 5z & + & 3w & = & 0 \\ & 2y & + & 3z & + & pw & = & 4 \end{cases}$

has a unique solution.

a) $p \neq 5, q$ is arbitrary b) p is arbitrary, $q \neq 2$ c) p is arbitrary, $q = 2$
d) $p = 5, q = 2$ e) $p = 0, q = 2$ f) p is arbitrary, $q = 0$

Answer: $\underline{p \neq 5, q \text{ is arbitrary.}}$

68. For what value(s) of ω is the system of equations $\begin{cases} 2x & + & y & + & 3z & = & 4 \\ 6x & + & 5y & + & 7z & = & 8 \\ 10x & + & 9y & + & \omega z & = & 0 \end{cases}$ inconsistent?

a) no values b) all values c) -12
d) 12 c) 11 d) -11

Answer: $\underline{11.}$

69. For which value of c does the linear system of equations $\begin{cases} x & - & 3y & + & 2z & = & 4 \\ 2x & + & y & - & z & = & 1 \\ 3x & - & 2y & + & z & = & c \end{cases}$

have at least one solution?

a) 3 b) 5 c) 8 d) 12 e) 14 f) 19

Answer: $\underline{5.}$

70. Find the value of b for which the system of equation $\begin{cases} 2x & + & y & - & 3z & = & 4 \\ -x & + & 3y & + & 5z & = & 2 \\ x & + & 4y & + & (b^2 - 14)z & = & b + 2 \end{cases}$

has infinitely many solutions.

a) 0 b) 2 c) 4 d) 8 e) 12 f) 16

Answer: $\underline{4.}$

71. For what value of a does the system of equations $\begin{cases} x & + & y & + & z & = & 2 \\ 3x & + & 2y & + & 2z & = & 5 \\ 3x & + & 2y & + & (a^2 - 2)z & = & a + 7 \end{cases}$

have infinitely many solutions?

a) $\sqrt{2}$ b) $-\sqrt{2}$ c) 2 d) -2 e) $2\sqrt{2}$ f) $\frac{1}{\sqrt{2}}$

Answer: $\underline{-2.}$

72. Determine the values of k so that the system of equations $\left\{\begin{array}{rcrcrcl} x & + & y & - & z & = & 1 \\ 2x & + & 3y & + & kz & = & 3 \\ x & + & ky & + & 3z & = & 2 \end{array}\right.$

has a unique solution.

a) all $k \neq 0$ b) all $k \neq -3$ or -2 c) all $k \neq -3$ or 2

d) all $k \neq 3$ e) all $k \neq 2$ f) all $k \neq -2$ or 2

Answer: All $k \neq -3$ or 2.

73. Find the relationship between a, b and c for which the system of equations

$\left\{\begin{array}{rcrcrcl} 3x & + & 3y & - & 9z & = & a \\ x & - & y & - & 2z & = & b \\ 5x & + & y & - & 13z & = & c \end{array}\right.$ is inconsistent.

a) $a + b = 0$ b) $a + 2b - c \neq 0$ c) $a + b + c = 0$

d) $2a - b - c \neq 0$ e) $3a + 2b - 2c = 0$ f) $a + 2b - c = 0$

Answer: $\underline{a + 2b - c \neq 0.}$

74. Find all value(s) of k for which the system of equations

$\left\{\begin{array}{rcrcrcl} (5-k)x & + & 4y & + & 2z & = & 0 \\ 4x & + & (5-k)y & + & 2z & = & 0 \\ 2x & + & 2y & + & (2-k)z & = & 0 \end{array}\right.$ has infinitely many solutions.

a) 1 only b) 1 and 10 c) 10 only d) $-1, 1$ and 10 e) -1 and 1 f) all k

Answer: 1 and 10.

75. Find all values of t for which the system of equations $\left\{\begin{array}{rcrcrcl} tx & + & y & - & z & = & 1 \\ -x & + & ty & + & z & = & 1 \\ x & - & y & + & z & = & 1 \end{array}\right.$

has a unique solution.

a) $-1 \leq t \leq 1$ b) all $t \neq 1$ c) all $t \neq -1$

d) all $t \neq -1$ or 1 e) all $t \neq 0$ f) none of the above

Answer: all $t \neq -1$.

76. Find all values of p and q for which the system of equations

$\left\{\begin{array}{rcrcl} 2x & - & y & = & 3 \\ x & + & py & = & q \end{array}\right.$ has infinitely many solutions.

a) $p = \frac{1}{2}, q = 3$ b) $p = \frac{1}{2}, q = -3$ c) $p = -\frac{1}{2}, q = -3$
d) $p = 2, q = -\frac{3}{2}$ e) $p = -2, q = \frac{3}{2}$ f) $p = -\frac{1}{2}, q = \frac{3}{2}$

Answer: $\underline{p = -\frac{1}{2}, q = \frac{3}{2}.}$

77. For what value(s) of a is the system of equations $\begin{cases} 5x + 6y + 7z = 8 \\ x + 2y + 3z = 4 \\ 9x + 10y + az = 0 \end{cases}$ inconsistent?

a) no value b) all values c) 10 d) -10 e) 11 f) 13

Answer: $\underline{11.}$

78. Find the value of c for which the system of equations $\begin{cases} x - 3y = c \\ 3x + 2y = -4 \\ x + 8y = 8 \end{cases}$ is consistent.

a) -2 b) -4 c) -6 d) -8 e) -10 f) -12

Answer: $\underline{-6.}$

79. Given a linear system $\begin{cases} ax - by + 4z = 0 \\ x - 2y + 2z = 0 \\ 6z = 0 \end{cases}$, then:

a) The system is inconsistent if $2a = b$,

b) The system has infinitely many solutions if $2a = b$,

c) The system is consistent if $b = a$,

d) The system has exactly one solution if $2a \neq b$,

e) The system has exactly one solution if $a = b = 0$.

Answer: <u>b), c), and d).</u>

80. Find all solutions to the system of equations $\begin{cases} 2x + y + 2z = 6 \\ y + z = 4 \end{cases}$ for which x, y and z are non-negative integers.

a) $(1, 4, 0)$ and $(0, 2, 2)$ b) $(4, 1, 0)$ and $(2, 2, 0)$ c) $(0, 4, 0)$ and $(1, 3, 1)$
d) $(1, 0, 4)$ and $(2, 0, 2)$ e) $(1, 1, 3)$ and $(0, 2, 2)$ f) $(0, 0, 4)$ and $(0, 4, 0)$

Answer: <u>$(1, 4, 0)$ and $(0, 2, 2)$.</u>

81. Find all solutions to the system of equations $\begin{cases} x - y - z = 2 \\ 3x + y + z = 10 \end{cases}$

for which x, y and z are non-negative integers.

a) $(3,0,1)$ and $(3,1,1)$ b) $(3,0,1)$ c) $(3,0,1)$ and $(3,1,2)$
d) $(3,1,0)$ e) $(3,0,1)$ and $(3,1,0)$ f) $(3,0,0)$

Answer: $(3,0,1)$ and $(3,1,0)$.

82. Find all values of x and y so that the matrix $\begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix}$ is in reduced row-echelon form.

a) $x = 0, y$ is arbitrary b) $x = 1, y$ is arbitrary c) $x = 0, y = 0$
d) $x = 0, y = 1$ e) $x = 1, y = 1$ f) $x = 1, y = 0$

Answer: $x = 0, y = 0$.

83. Find all (x, y) so that the matrix $\begin{bmatrix} 1 & 0 & 1 \\ x & y & 0 \end{bmatrix}$ is in reduced row-echelon form.

a) $(0,0)$ b) $(0,0)$ and $(1,0)$ c) $(0,1)$
d) $(0,0)$ and $(0,1)$ e) $(1,0)$ and $(0,1)$ f) $(1,1)$

Answer: $(0,0)$ and $(0,1)$.

84. Find all (a, b, c) so that the matrix $\begin{bmatrix} a & 1 & b & b & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & c \end{bmatrix}$ is in reduced row-echelon form.

a) $(1,0,0)$ b) $(1,0,0)$ and $(0,0,0)$ c) $(0,0,0)$
d) $(1,0,0)$ and $(0,0,1)$ e) $(0,0,1)$ f) $(1,1,1)$

Answer: $(1,0,0)$ and $(0,0,0)$.

85. Find the number of leading ones in the reduced row-echelon form of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 3 \\ 1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 5 \end{bmatrix}.$$

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

Answer: 3.

86. Which of the following matrices are in reduced row-echelon form?

(1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 4 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

a) only (5) b) (1), (3) and (4) c) (3) and (5)
d) (1) and (2) e) (1), (2) and (4) f) none of the above

Answer: (3) and (5).

87. Which of the following matrices are in reduced row-echelon form?

(1) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

a) (1) and (2) b) (2) and (5) c) (1), (3) and (4)
d) (1), (2) and (5) e) (3), (4) and (5) f) only (2)

Answer: (2) and (5).

88. Examine the following matrices and select the correct statement below.

(1) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

a) (1) and (2) are in row-echelon form.
b) (3) and (4) are in reduced row-echelon form.
c) (1) and (4) are in row-echelon form.
d) (3) and (5) are in reduced row-echelon form.
e) Only (4) is in reduced row-echelon form.
f) All the matrices are in row-echelon form.

Answer: (3) and (4) are in reduced row-echelon form.

89. If a 2×3 matrix is in reduced row echelon form then (choose all correct answers):

    a) Every entry must be 0 or 1.

    b) Every column must contain at most one nonzero entry.

    c) Every row must contain at most one nonzero entry.

    d) No column can contain two ones.

    e) None of the above.

    Answer: e) None of the above.

90. The rank of $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 5 & 3 & 4 \end{bmatrix}$ is:

    a) 0 b) 2 c) 3 d) $\frac{1}{2}$ e) -1 f) -2

    Answer: 2.

91. Given that rank $\begin{bmatrix} -1 & 4 & 5 & 3 \\ 2 & 3 & -2 & 6 \\ 3 & 10 & a & 15 \end{bmatrix}$ is 2, find a :

    a) 0 b) -1 c) 4 d) $\frac{1}{2}$ e) 1 f) -2

    Answer: 1.

92. A system of three linear equations in five variables is given. If the rank of the coefficient matrix is 3, the possible values of the rank of the augmented matrix are:

    a) 2 b) 1 c) 3 d) 4 e) 0

    Answer: 3.

93. For a system of 4 linear equations in 3 variables, which statements are true:

    a) There is always at least one solution.

    b) There may be exactly three solutions.

    c) There may be infinitely many solutions.

    d) There may be exactly one solution.

    e) There may be no solution.

    f) If the system is homogeneous there are always infinitely many solutions.

    g) If the system is homogeneous there is always at least one solution.

    a) 2 b) 1 c) 3 d) 4 e) 0

    Answer: c), d), e), and g).

94. The rank of the augmented matrix for a linear system is always:

    a) the same as the rank of the coefficient matrix.

    b) less than or equal to the rank of the coefficient matrix.

c) greater than or equal to the rank of the coefficient matrix.

d) always different from the rank of the coefficient matrix.

e) none of the above.

Answer: c).

95. The difference in age between two brothers is 5. 8 years ago, the older of the two was twice as old as the younger. Find the present age of the younger brother.

a) 10   b) 11   c) 12   d) 13   e) 14

Answer: d) 13.

96. A company makes 10% butterfat coffee cream from 5% butterfat milk and 30% butterfat dairy cream. How much milk and dairy cream are needed for 100 litres of coffee cream.

a) 80 l milk, 20 l dairy   b) 85 l milk, 15 l dairy

c) 90 l milk, 10 l dairy   d) 75 l milk, 25 l dairy

Answer: 80 l milk, 20 l dairy.

97. There are nickels, dimes and pennies in a bowl, 10 coins in all, at least one of each type. If the total value of the coins is 31 cents, determine (p, n, d) where there are p pennies, n nickels and d dimes. (Note that p, n and d are integers!)

Answer: $(p, n, d) = (6, 3, 1)$.

Chapter 2: Matrix Algebra

1. The $(2,1)$-entry of the product $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ is:

   a) 12 b) 3 c) 1 d) 4 e) 0 f) 2

   Answer: 1.

2. Given two matrices $P = \begin{bmatrix} 1 & -4 & 1 \\ -2 & 3 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & 1 \\ 0 & 3 \\ 2 & 4 \end{bmatrix}$, which one of the following is the product QP?

   a) $\begin{bmatrix} 3 & -7 & 1 \\ 6 & -9 & 0 \\ 6 & -4 & -2 \end{bmatrix}$ b) $\begin{bmatrix} -3 & -7 & 3 \\ 3 & -10 & 1 \\ 0 & -12 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 3 & 0 \\ -7 & -12 & -10 \\ 1 & 3 & 4 \end{bmatrix}$

   d) $\begin{bmatrix} -3 & 7 & -1 \\ -6 & 9 & 0 \\ -6 & 4 & 2 \end{bmatrix}$ e) $\begin{bmatrix} 3 & -4 & 1 \\ -10 & -7 & -12 \\ 3 & 0 & 3 \end{bmatrix}$ f) $\begin{bmatrix} -3 & 7 & -6 \\ -6 & 9 & 4 \\ -6 & 0 & 2 \end{bmatrix}$

   Answer: $\begin{bmatrix} -3 & 7 & -1 \\ -6 & 9 & 0 \\ -6 & 4 & 2 \end{bmatrix}$.

3. If $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 10 \end{bmatrix}$, $G = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, and $H = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$, determine the $(2,2)$-entry of $HD - \left(\frac{1}{4}\right) G$.

   a) $\frac{-7}{4}$ b) $\frac{7}{8}$ c) $\frac{15}{8}$ d) $\frac{-15}{8}$ e) $\frac{7}{4}$ f) $\frac{-15}{4}$

   Answer: $\frac{-15}{8}$.

4. The $(2,1)$-entry of the product $\begin{bmatrix} 3 & 7 & -2 \\ 3 & 4 & 7 \\ 0 & -3 & 6 \end{bmatrix}\begin{bmatrix} -3 & 9 & 6 \\ -7 & -8 & 4 \\ 1 & 0 & 5 \end{bmatrix}$ is:

   a) -10 b) 10 c) -25 d) 30 e) -30 f) -45

   Answer: -30.

5. The $(2,3)$-entry of the product $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 5 & 1 \\ 4 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 5 & 1 & 0 \\ 0 & 4 & 3 \end{bmatrix}$ is:

   a) 4 b) 5 c) 6 d) 7 e) 8 f) 9

   Answer: 7.

6. If A is an $m \times n$ matrix and B is an $n \times r$ matrix where m, n and r are distinct, then the product AB is:

   a) undefined b) an $n \times n$ matrix c) an $m \times r$ matrix
   d) an $r \times n$ matrix e) an $m \times n$ matrix f) none of the above

   Answer: An $m \times r$ matrix.

7. Find all (a, b, c) so that $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

   a) $(4t, -2t, t)$, t is arbitrary b) $(0, 0, 0)$ c) $(t, -2t, 4t)$, t is arbitrary
   d) $(-2t, 4t, t)$, t is arbitrary e) $(-2, 4, 1)$ f) $(2, -4, -1)$

   Answer: $(-2t, 4t, t), t$ is arbitrary.

8. Find all (a, b, c) so that $\begin{bmatrix} a & a \\ b & c \end{bmatrix}$ commutes with $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

   a) $(2s, 3s, 5s)$, s is arbitrary b) $(3, 2, 5)$ c) $(-3s, 2s, 5s)$, s is arbitrary
   d) $(-2, 3, 5)$ e) $(0, 0, 0)$ f) there are no such (a, b, c)

   Answer: $(2s, 3s, 5s)$, s is arbitrary.

9. Find all s and t so that $B^2 = 0$ if $B = \begin{bmatrix} s & t \\ 0 & s \end{bmatrix}$.

   a) $s = 0$ or $t = 0$ b) $s = 0$ and $t = 0$ c) $s = 1$ and $t = -1$
   d) $s = 0$ and t is arbitrary e) $t = 0$ and s is arbitrary f) there are no such s and t

   Answer: $s = 0$ and t is arbitrary.

10. Find all (α, β) for which $A^2 = 0$ if $A = \begin{bmatrix} \alpha & -1 \\ 2 & \beta \end{bmatrix}$.

    a) $\pm(1, 1)$ b) $\pm(\sqrt{2}, -\sqrt{2})$ c) $\pm(\sqrt{2}, \sqrt{2})$
    d) $\pm(\sqrt{3}, -\sqrt{3})$ e) $\pm(\sqrt{3}\sqrt{3})$ f) $\pm(1, -1)$

    Answer: $\pm(\sqrt{2}, -\sqrt{2})$.

11. If $6\begin{bmatrix} 3 & 2 & -4 \\ 0 & z & \frac{x}{6} \end{bmatrix} = \begin{bmatrix} x & 4z & -y \\ 0 & y-6 & 18 \end{bmatrix}$, then:

a) $x = 3, y = 4, z = \frac{1}{2}$ b) $x = 18, y = 24, z = 3$
c) $x = 3, y = 4, z = 3$ d) $x = 6, y = 24, z = 6$
e) $x = 9, y = 4, z = 3$ f) The given matrix has no solution.

Answer: $\underline{x = 18,\ y = 24,\ z = 3.}$

12. Find all s and t so that $A^2 = I_3$, where $A = \begin{bmatrix} 1 & s & t \\ 0 & 1 & s \\ 0 & 0 & 1 \end{bmatrix}$.

a) $s = 0$ or $t = 0$ b) $s = 0$ and $t = 0$ c) $s = 0$ and t is arbitrary
d) s and t are arbitrary e) all s and t with $2t + s^3 = 0$ f) there are no such s and t

Answer: $\underline{s = 0 \text{ and } t = 0}$.

13. If three $n \times n$ matrices A, B and C satisfy $AB - BA = C$, then ABA is:

a) $A^2B - C$ b) $A^2B - CA$ c) $BA^2 + CA$
d) A^2B e) $A^2B + AC$ f) $A^2B + BC$

Answer: $\underline{BA^2 + CA}$.

14. If $A = [3 \quad 4]$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then $AB =$

a) $[3 \quad 8]$ b) $[11]$ c) $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$ e) $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ f) does not exist

Answer: $\underline{[11]}$.

15. Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -6 \\ 2 & 1 \end{bmatrix}$, and X be such that $AXB = C$. The second row of the matrix X is

a) $[\frac{-2}{3} \quad \frac{4}{3}]$ b) $[-8 \quad 4]$ c) $[\frac{-8}{3} \quad \frac{4}{3}]$
d) $[\frac{4}{3} \quad \frac{8}{3}]$ e) $[\frac{8}{3} \quad \frac{-4}{3}]$ f) $[\frac{4}{3} \quad \frac{-8}{3}]$

Answer: $\underline{[\frac{-8}{3} \quad \frac{4}{3}]}$.

16. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 15 & 23 \\ 25 & 38 \end{bmatrix}$, and suppose $AYB = C$. Compute the matrix Y.

a) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix}$ e) $\begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}}$.

17. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$. What is the $(1, 2)$-entry of the matrix $AB - BA$?

a) 3 b) 2 c) 1 d) 0 e) -1 f) -4

Answer: $\underline{-4}$.

18. Let A be a fixed 5×5 matrix and let $B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ be an elementary matrix.

Then $B^{-3}A$ is obtained from the matrix A by

a) adding 6 times row 2 to row 4
b) adding 6 times row 4 to row 2
c) adding 8 times row 2 to row 4
d) adding 8 times row 4 to row 2
e) the matrix B is not invertible
f) none of the above

Answer: <u>adding 6 times row 4 to row 2</u>.

19. Let $B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Then the second row of B^{-1} is

a) $[0 \quad 1 \quad -1]$ b) $[-1 \quad 1 \quad 0]$ c) $[0 \quad -1 \quad 1]$

d) $[1 \quad -1 \quad 0]$ e) $[1 \quad 0 \quad -1]$ f) none of the above

Answer: $\underline{[0 \quad 1 \quad -1]}$.

20. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -2 \\ 0 & 2 & 3 \end{bmatrix}$,

(i) George says that the first row of A^{-1} is $[1 \quad -3 \quad -2]$;

(ii) Ringo says that the third column of A^{-1} is $[-3 \quad 3 \quad -2]$.

a) Both George and Ringo are correct.

b) Both George and Ringo are wrong although A^{-1} exists.

c) George is correct but Ringo is wrong.

d) Ringo is correct although George is wrong.

e) Both are wrong since A^{-1} does not exist.

Answer: George is correct but Ringo is wrong.

21. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, find the main diagonal of A^{-1}.

a) $(0, 1, 0)$ b) $(-1, 0, -1)$ c) $(0, -1, 0)$

d) $(-1, 0, 0)$ e) $(0, 0, 1)$ f) $(-1, -1, 1)$

Answer: $(-1, 0, 0)$.

22. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$. What is the second row of A^{-1}?

a) $\begin{bmatrix} \frac{-15}{8} & \frac{1}{8} & \frac{3}{8} \end{bmatrix}$ b) $\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ c) $\begin{bmatrix} \frac{13}{8} & \frac{-1}{2} & \frac{-1}{8} \end{bmatrix}$

d) $\begin{bmatrix} \frac{-15}{8} & \frac{1}{2} & \frac{3}{8} \end{bmatrix}$ e) $\begin{bmatrix} \frac{-15}{4} & 1 & \frac{3}{4} \end{bmatrix}$ f) $[-15 \quad 4 \quad 3]$

Answer: $\begin{bmatrix} \frac{-15}{8} & \frac{1}{2} & \frac{3}{8} \end{bmatrix}$.

23. Find the main diagonal of the inverse of $A = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 2 & 4 \\ -3 & 0 & 2 \end{bmatrix}$.

a) $(2, \frac{-7}{2}, -1)$ b) $\left(\frac{5}{2}, \frac{7}{2}, \frac{3}{2}\right)$ c) $(2, 1, -1)$

d) $\left(-1, \frac{-7}{2}, 3\right)$ e) $\left(\frac{7}{2}, 2, -1\right)$ f) $\left(2, 1, \frac{-7}{2}\right)$

Answer: $\left(2, \frac{-7}{2}, -1\right)$.

24. What is the first row of C^{-1} if $C = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & -3 & 6 \\ 0 & 5 & 0 & -2 \\ 2 & 1 & 6 & 0 \end{bmatrix}$?

a) $[35 \quad 1 \quad 3 \quad -17]$ b) $[17 \quad 5 \quad 6 \quad 19]$ c) $[33 \quad 1 \quad 3 \quad 17]$

d) $\begin{bmatrix} 11 & \frac{1}{3} & 1 & 6 \end{bmatrix}$ e) $[0 \quad 1 \quad 3 \quad 9]$ f) C is not invertible

Answer: $[35 \quad 1 \quad 3 \quad -17]$.

25. Suppose $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}$. Which one of the following statements is true about A^{-1}?

a) It does not exist

b) The third row is $[-1 \quad -1 \quad 1]$

c) The second row is $[2 \quad 2 \quad -1]$

d) The first row is $[1 \quad 0 \quad -1]$

e) The second column is $[0 \quad 2 \quad -1]^T$

f) $A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$

Answer: The third row is $[-1 \quad -1 \quad 1]$.

26. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 0 & -3 & 1 \end{bmatrix}$ and add up the nine entries. What is the result?

a) 39 b) -18 c) 3 d) 0 e) 9 f) -33

Answer: 0.

27. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}$. Then the second column of A^{-1} is

a) $[0 \quad -1 \quad 1]^T$

b) $[2 \quad 0 \quad -1]^T$

c) $[0 \quad 1 \quad -1]^T$

d) $[1 \quad -2 \quad 1]^T$

e) $[1 \quad -1 \quad 0]^T$

f) $[0 \quad 2 \quad -1]^T$

Answer: $[2 \quad 0 \quad -1]^T$.

28. Given $A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, examine the propositions:

(i) A is in row-echelon form but not in reduced row-echelon form.

(ii) A^{-1} exists. If $A^{-1} = [b_{ij}]$ then $b_{12} = 4$.

(iii) The system $AX = 0$ admits only the trivial solution.

a) All three propositions are false.

b) Only (i) and (ii) are true.

c) Only (iii) is true.

d) Only (ii) and (iii) are true.

e) Only (i) is true.

f) All three propositions are true.

Answer: Only (ii) and (iii) are true.

29. Suppose $A = \begin{bmatrix} 1 & -2 & 0 & -1 \\ -1 & 5 & 1 & 2 \\ 0 & 6 & 1 & 3 \\ -2 & 8 & 2 & 3 \end{bmatrix}$. Which one of the following statements is correct?

    a) The third row of A^{-1} is $[0 \quad -6 \quad 5 \quad 3]$.

    b) The second column of A^{-1} is $[2 \quad 5 \quad -6 \quad -8]^T$.

    c) The fourth row of A^{-1} is $[2 \quad -8 \quad 2 \quad 3]$.

    d) A^{-1} does not exist.

    e) The first row of A^{-1} is $[1 \quad -2 \quad 0 \quad -1]$.

    f) The first column of A^{-1} is $[1 \quad 1 \quad 0 \quad 1]^T$.

    Answer: The second column of A^{-1} is $[2 \quad 5 \quad -6 \quad -8]^T$.

30. If an $n \times n$ matrix A satisfies $A^2 - 6A + 5I_n = 0$, then A^{-1}

    a) does not exist.

    b) is $\frac{1}{5}(6I_n - A)$.

    c) is $\frac{1}{5}(A - 6I_n)$.

    d) exists, but there is not enough information to determine it.

    e) exists only if $n < 6$.

    f) may exist only if $n \geq 6$.

    Answer: Is $\frac{1}{5}(6I_n - A)$.

31. If A is an $n \times n$ matrix such that $A^2 - 3A + I_n = 0$, then

    a) $A^{-1} = 3(A^2 + I)$

    b) $A^{-1} = 3I - A$

    c) $A^{-1} = A - 3I$

    d) with the present information, it is impossible to determine whether or not A is invertible.

    e) all the other responses are false.

    Answer: $A^{-1} = 3I - A$.

32. Suppose a matrix A satisfies $A^3 - 3A^2 + I = 0$, where I denotes the identity matrix. Which one of the following statements is correct?

    a) A is not invertible.

    b) $A^{-1} = 3I - A - A^2$

    c) $A^{-1} = 3A - A^2$

    d) A is the identity matrix I.

    e) A is the null matrix O.

    f) The information given is insufficient to determine whether or not A is invertible.

    Answer: $A^{-1} = 3A - A^2$.

33. The inverse of the matrix $\begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}$

a) exists if and only if $x \neq 0$ b) is $\frac{1}{1+x^2}\begin{bmatrix} 1 & -x \\ x & 1 \end{bmatrix}$ c) is $\frac{1}{1-x^2}\begin{bmatrix} 1 & -x \\ x & 1 \end{bmatrix}$

d) is $\frac{1}{1+x^2}\begin{bmatrix} x & 1 \\ 1 & -x \end{bmatrix}$ e) is $\frac{1}{1+x^2}\begin{bmatrix} 1 & x \\ -x & 1 \end{bmatrix}$ f) is $\frac{1}{1-x^2}\begin{bmatrix} x & 1 \\ 1 & -x \end{bmatrix}$

Answer: Is $\frac{1}{1+x^2}\begin{bmatrix} 1 & -x \\ x & 1 \end{bmatrix}$.

34. The inverse of the elementary matrix $E = \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} 1 & 0 & \frac{1}{x} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ if $x \neq 0$ b) $\begin{bmatrix} 1 & 0 & \frac{-1}{x} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ if $x \neq 0$ c) $\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ e) $\begin{bmatrix} x & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -x \end{bmatrix}$ f) $\begin{bmatrix} x & 0 & -1 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$

Answer: $\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

35. For what value(s) of λ is the matrix $\begin{bmatrix} \lambda - 4 & -2 \\ -3 & \lambda - 3 \end{bmatrix}$ not invertible?

a) 1 only b) 4 only c) 3 only d) 6 only e) 1 or 6 f) 3 or 4

Answer: 1 or 6.

36. Find a 2×2 matrix A such that $(A^T - 2I)^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} -3 & 1 \\ 1 & -4 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}$ e) $\begin{bmatrix} 3 & -1 \\ 1 & -4 \end{bmatrix}$ f) No such matrix A exists.

Answer: $\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}$.

37. Find the 2×2 matrix A such that $(3A^T - 2I)^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

a) $\begin{bmatrix} -1 & \frac{1}{3} \\ \frac{1}{3} & \frac{-4}{3} \end{bmatrix}$ b) $\begin{bmatrix} \frac{4}{3} & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix}$ c) $\begin{bmatrix} \frac{4}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{4}{3} \end{bmatrix}$ e) $\begin{bmatrix} \frac{1}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{1}{3} \end{bmatrix}$ f) $\begin{bmatrix} 1 & \frac{-1}{3} \\ \frac{-1}{3} & -1 \end{bmatrix}$

Answer: $\begin{bmatrix} 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{4}{3} \end{bmatrix}$.

38. If A is a 2×2 invertible matrix and $(3A)^{-1} = \begin{bmatrix} -1 & -3 \\ 4 & 5 \end{bmatrix}$, what is the $(1, 1)$-entry of A?

a) $\frac{5}{21}$ b) $\frac{-5}{21}$ c) $\frac{5}{3}$ d) $\frac{15}{7}$ e) $\frac{-25}{3}$ f) $\frac{-1}{21}$

Answer: $\frac{5}{21}$.

39. Given $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 4 & 1 \\ 3 & 4 \end{bmatrix}$, compute the matrix $(AB)^{-1}$.

a) $\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ -5 & 2 \end{bmatrix}$ c) $\begin{bmatrix} \frac{1}{2} & \frac{-5}{2} \\ 2 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 \\ \frac{-5}{2} & \frac{3}{2} \end{bmatrix}$ e) $\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$ f) $\begin{bmatrix} -1 & \frac{-3}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$

Answer: $\frac{5}{21}$.

40. Compute $\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}^{1001}$.

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix}$

d) $\begin{bmatrix} -1001 & -3003 \\ 0 & 1001 \end{bmatrix}$ e) $\begin{bmatrix} 1001 & 3003 \\ 0 & -1001 \end{bmatrix}$ f) $\begin{bmatrix} -1 & 3^{1001} \\ 0 & 1 \end{bmatrix}$

Answer: $\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$.

41. Let A be the 2×2 matrix such that $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ and $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Compute A^2.

a) $\begin{bmatrix} 25 & 4 \\ 9 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -10 & 4 \\ 6 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 29 & -13 \\ -13 & 10 \end{bmatrix}$

d) $\begin{bmatrix} 30 & -8 \\ -12 & 7 \end{bmatrix}$ e) $\begin{bmatrix} 19 & 8 \\ -12 & 5 \end{bmatrix}$ f) $\begin{bmatrix} 31 & -8 \\ -12 & 7 \end{bmatrix}$

Answer: $\begin{bmatrix} 31 & -8 \\ -12 & 7 \end{bmatrix}$.

42. Find $A^T A - (AA^T)I_3$ if $A = (1, 1, 1)$

a) $\begin{bmatrix} -2 & -2 & -2 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ e) $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ f) 0

Answer: $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$.

43. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, then $AB^T - A^{-1}B$ is:

a) $\begin{bmatrix} \frac{1}{2} & \frac{-5}{2} \\ \frac{5}{2} & \frac{-9}{2} \end{bmatrix}$ b) $\begin{bmatrix} 2 & -7 \\ 9 & -3 \end{bmatrix}$ c) $\begin{bmatrix} \frac{3}{2} & \frac{-11}{2} \\ \frac{13}{2} & \frac{-5}{2} \end{bmatrix}$

d) $\begin{bmatrix} \frac{1}{2} & \frac{-5}{2} \\ \frac{3}{2} & \frac{-3}{2} \end{bmatrix}$ e) $\begin{bmatrix} 4 & -8 \\ 3 & -9 \end{bmatrix}$ f) $\begin{bmatrix} \frac{1}{2} & \frac{-5}{2} \\ \frac{3}{2} & \frac{7}{2} \end{bmatrix}$

Answer: $\begin{bmatrix} \frac{3}{2} & \frac{-11}{2} \\ \frac{13}{2} & \frac{-5}{2} \end{bmatrix}$.

44. Every linear system can be written as a matrix equation $AX = B$ where:

a) A is the augmented matrix.

b) A is a square matrix.

c) B is the augmented matrix

d) A and B are equal

e) None of the above.

Answer: e) None of the above.

45. If A is an $n \times 2$ matrix and $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, then the second column of the matrix AB is:

a) not defined unless $n = 2$.

b) the same as the second column of A.

c) the same as the second column of B.

d) the same as the first column of A.

e) the same as the first column of B.

f) the same as the sum of the first and second columns of A.

Answer: The same as the first column of A.

46. If $C = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and D is a $3 \times m$ matrix, then the second row of the matrix CD is:

a) not defined unless $m = 2$.

b) the same as the first row of D.

c) the same as the second row of D.

d) the sum of the first and third rows of D.

e) the sum of twice the second row of D and the third row of D.

f) twice the first row of D.

Answer: The sum of the first and third rows of D.

47. Find all 2×2 matrices A for which $A^T = 2A$.

a) $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$; a, d arbitrary b) $\begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}$; b, c arbitrary c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 \\ c & d \end{bmatrix}$; c, d arbitrary e) $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$; a arbitrary f) $\begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}$; d arbitrary

Answer: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

48. Solve the following equation for A: $A^T - \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$.

a) $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ f) $\begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}}$.

49. If two $n \times n$ matrices satisfy $A^T = B$ and $B^T = -B$, then $(ABA)^T$ is:

a) $B^T B B^T$ b) $-B^3$ c) $B^T B$

d) B^3 e) $B^T B^T B$ f) not defined

Answer: $\underline{-B^3}$.

50. If two $n \times n$ matrices A and B satisfy $A^T = B^{-1}$ and $B^T = A$, then :

a) $A = I$ b) $A^2 = I$ c) $A = A^T$ d) $A = -A^T$ e) $AA^T = I$ f) $A = 2I$

Answer: $\underline{A^2 = I}$.

51. Given that the inverse of a matrix A is $A^{-1} = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}$, then $(A^T)^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is:

a) $\begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$ b) $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ c) $\begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$ e) $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ f) $\begin{bmatrix} -4 \\ 3 \\ 5 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}}$.

52. If $v = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $w = \begin{bmatrix} 3 & 2 & -1 \end{bmatrix}$, find tr $(v^T w)$.

a) 0 b) 2 c) -2 d) 4 e) -4 f) 3

Answer: $\underline{4}$.

53. Determine the row operation that will restore the matrix $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ to the identity matrix I_2.

a) Add $-\left(\frac{1}{4}\right)$ times the second row to the first row.

b) Add $-\left(\frac{1}{4}\right)$ times the first row to the second row.

c) Add -4 times the first row to the second row.

d) Add -4 times the second row to the first row.

e) Add 4 times the second row to the first row.

f) Add $\frac{1}{4}$ times the first row to the second row.

Answer: Add -4 times the second row to the first row.

54. Which of the following statements are true for all invertible 6×6 matrices?

(i) $A^3A^4 = A^4 + A^3$ (ii) $(A+B)I_6 = A+B$

(iii) $(AB)^{-1} = A^{-1}B^{-1}$ (iv) $C(A+B) = CA+CB$

(v) $AB = BA$ (vi) $(AB)C = A(BC)$

a) (i) and (iii) only b) (i), (iv) and (vi) only c) (ii), (iv) and (vi) only

d) (iv) and (v) only e) (i) and (v) only f) none of the above

Answer: (ii), (iv) and (vi) only.

55. Which of the following statements are true for invertible $n \times n$ matrices A, B, and C?

(i) $(A+B)^{-1} = A^{-1} + B^{-1}$ (ii) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(iii) $A^2B^2 = (AB)^2$ (iv) $(A+B)^2 = A^2 + 2AB + B^2$

(v) $(A+C)(A-C) = A^2 - C^2$

a) (i) and (ii) only b) (iv) and (v) only c) (ii) and (iii) only

d) (iii) and (iv) only e) (i), (iv), and (v) only f) none of the above

Answer: none of the above.

56. Only two of the following statements are true. Which two?

(i) Matrix addition and matrix multiplication are associative.

(ii) Matrix addition is associative but matrix multiplication is not.

(iii) Matrix addition and matrix multiplication are commutative.

(iv) Matrix multiplication is commutative but matrix addition is not.

(v) If the matrix product $AB = 0$, it does not follow that A or B is a zero matrix.

a) (i) and (iii) b) (iv) and (v) c) (ii) and (iii)

d) (i) and (v) e) (ii) and (iv) f) (i) and (iv)

Answer: (i) and (v).

57. Which of the following statements are true?

(i) Let A and B be arbitrary $n \times n$ matrices. Then $(A+B)^2 = A^2 + 2AB + B^2$.

(ii) The inverse of an elementary matrix E is E.

(iii) If $ad - bc \neq 0$, then there is exactly one solution to the system

$$\begin{cases} ax + by = 1 \\ cx + dy = 0 \end{cases}.$$

(iv) If X and Y are $n \times 1$ matrices, then $X^TY = Y^TX$.

a) (i) and (ii) only
b) (i) and (iii) only
c) (iii) and (iv) only
d) (ii) and (iv) only
e) none of them are true
f) all of them are true

Answer: (iii) and (iv) only.

58. Let n be a positive integer. Which of the following is always true for square matrices A, B of size n?

a) $(A+B)(A-B) = A^2 - B^2$
b) $(I_n + A)^2 = I_n + 2A + A^2$
c) $ABA = A^2B$
d) $\det(A+B) = \det(A) + \det(B)$
e) $(A+B)^2 = A^2 + 2AB + B^2$
f) All these equations are false in general

Answer: $(I_n + A)^2 = I_n + 2A + A^2$.

59. Which one of the following rules of real number arithmetic is not valid for matrix arithmetic?

(a) the commutative law for addition.

(b) the commutative law for multiplication.

(c) the associative law for addition.

(d) the associative law for multiplication.

(e) the distributive laws.

(f) all of the above laws for valid for matrix arithmetic.

Answer: The commutative law for multiplication.

60. If A and B are symmetric (that is $A^T = A$), which of the following are true:

(i) $A - B$ is symmetric.

(ii) AB is symmetric.

(iii) If A is invertible, A^{-1} is symmetric.

(iv) B^2 is symmetric.

(v) $AB^T = BA^T$.

a) (i) and (ii) only
b)
c) (iii) and (iv) only
d) (ii) and (iv) only
e) none of them are true
f) all of them are true

Answer: (i), (iii) and (iv) only.

61. If A is a 4×4 matrix and B is a 4×4 matrix, which of the following is true? List all correct answers.

(a) $A - B = B - A$.

(b) If $A = BT$, then $B = AT$.

(c) $A + B$ is the same size as BA.

(d) If $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then $B^2 = B$.

(e) If kA is the zero matrix, then either $k = 0$ or A is the zero matrix.

(f) All of the above.

Answer: b), c), d), and e).

62. If A is an $n \times n$ matrix, which of the following are true:

(i) AA^T is symmetric.

(ii) A^2 is symmetric.

(iii) $2A$ is symmetric.

(iv) $A + A^T$ is symmetric.

(v) If $A^2 = 0$ then A is symmetric.

a) (i) and (ii) only	b) (iii) and (v)	c) (i) and (iv) only
d) (ii) and (iv) only	e) none of them are true	f) all of them are true

Answer: (i) and (iv) only.

63. If A and B are $n \times n$ matrices, which of the following are true:

(i) If $A^2 = 0$ then $A = 0$.

(ii) If $AB = 0$ then either $A = 0$ or $B = 0$.

(iii) If $2A = 0$ then $A = 0$.

(iv) If $A^T = 0$ then $A = 0$.

(v) If $AR = 0$ and R is in reduced row-echelon form, then $A = 0$.

a) (i) and (iii) only	b) (iii) and (iv)	c) (i) and (iv) only
d) (ii) and (v) only	e) none of them are true	f) all of them are true

Answer: (iii) and (iv) only.

64. If A is an $n \times n$ matrix, which of the following are true:

(i) If $A^2 = A$ then A is invertible.

(ii) If $A^2 = A$ then $(I - 2A)^{-1} = I - 2A$.

(iii) If $A = A^T$ then A is invertible.

(iv) If $7A$ is invertible then A is invertible.

(v) If $7A + I$ is invertible then A is invertible.

a) (ii) and (iii) only	b) (iii) and (v)	c) (i) and (iv) only
d) (ii) and (iv) only	e) none of them are true	f) (iv) only

Answer: (ii) and (iv) only.

65. If A and AB are invertible, which of the following statements are true:

(i) B is a square matrix.

(ii) B is invertible.

(iii) BA is invertible.

(iv) rank $A =$ rank B.

(v) $AB = BA$.

Answer: All but (v) are true.

66. If A is an $m \times n$ matrix and B is an $n \times m$ matrix and the jth row of A consists entirely of zeros, and the kth column of B consists entirely of zeros then we know that: (list all correct responses)

(a) the product AB is defined,

(b) AB is the $m \times n$ zero matrix,

(c) AB has a row of zeros,

(d) BA has a row of zeros,

(e) AB has a column of zeros,

(f) BA has a column of zeros.

Answer: a), c), and e).

67. Which of the following matrices are elementary?

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -4 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \; D = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) B.

(b) Only C.

(c) D and C.

(d) Only D.

(e) None of the above.

Answer: c) D and C.

68. For the elementary matrices $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $F = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$, and $G = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, the matrix $[EF^{-1}G^{-1}F]^{-1}$ is:

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} \frac{2}{3} & 1 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 0 & 1 \\ 1 & 6 \end{bmatrix}$

Answer: e) $\begin{bmatrix} 0 & 1 \\ 1 & 6 \end{bmatrix}$.

69. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$. Find the matrix of T and compute $T\begin{bmatrix} 7 \\ 3 \end{bmatrix}$:

a) $\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}, \begin{bmatrix} -11 \\ 22 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}}, \underline{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}$.

70. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $T\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$. Find the matrix of T and compute $T\begin{bmatrix} 11 \\ -5 \end{bmatrix}$:

a) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, 16\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -16 \\ 16 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 6 \end{bmatrix}$ d) $\begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}, \begin{bmatrix} 16\sqrt{2} \\ -16\sqrt{2} \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, 16\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$.

71. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$. Find the matrix of T and compute $T\begin{bmatrix} 13 \\ -7 \end{bmatrix}$:

a) $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} 19 \\ 46 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 34 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 20 \\ 4 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -34 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -34 \end{bmatrix}}$.

72. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ and $T\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Find the matrix of T and compute $T\begin{bmatrix} 11 \\ 32 \end{bmatrix}$:

a) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 75 \\ 32 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 182 \\ 75 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 75 \\ 182 \end{bmatrix}$ d) None of these

Answer: $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 75 \\ 32 \end{bmatrix}$.

73. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$ and $T\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$. Compute $T^{-1}\begin{bmatrix} -5 \\ 8 \end{bmatrix}$:

a) $\begin{bmatrix} -182 \\ 75 \end{bmatrix}$ b) $\begin{bmatrix} 182 \\ 75 \end{bmatrix}$ c) $\begin{bmatrix} -8 \\ 5 \end{bmatrix}$ d) None of these

Answer: $\begin{bmatrix} -55 \\ 21 \end{bmatrix}$.

74. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$. Compute $T^{-1}\begin{bmatrix} 7 \\ 2 \end{bmatrix}$:

a) $\begin{bmatrix} 29 \\ -12 \end{bmatrix}$ b) $\begin{bmatrix} -12 \\ 29 \end{bmatrix}$ c) $\begin{bmatrix} 12 \\ -29 \end{bmatrix}$ d) None of these

Answer: $\begin{bmatrix} 12 \\ -29 \end{bmatrix}$.

75. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and assume that $T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 17 \end{bmatrix}$ and $T\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 23 \end{bmatrix}$. Compute $T^{-1}\begin{bmatrix} 20 \\ -14 \end{bmatrix}$:

a) $\begin{bmatrix} 75 \\ 32 \end{bmatrix}$ b) $\begin{bmatrix} 182 \\ 75 \end{bmatrix}$ c) $\begin{bmatrix} 75 \\ 182 \end{bmatrix}$ d) None of these

Answer: $\underline{\begin{bmatrix} -91 \\ 37 \end{bmatrix}}$.

76. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is rotation through $2\pi/3$, then $T \begin{bmatrix} 2 \\ -6 \end{bmatrix}$ is:

a) $\begin{bmatrix} 1 - 3\sqrt{3} \\ \sqrt{3} - 3 \end{bmatrix}$ b) $\begin{bmatrix} \sqrt{3} \\ 2 \end{bmatrix}$ c) $\begin{bmatrix} 3\sqrt{3} - 1 \\ 3 - \sqrt{3} \end{bmatrix}$ d) $\begin{bmatrix} 33 - \sqrt{3} \\ 1 + 3\sqrt{3} \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 3\sqrt{3} - 1 \\ \sqrt{3} + 3 \end{bmatrix}}$.

77. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation through $\pi/2$ followed by reflection in the line $y = x$. Then T is:

a) reflection in the X axis b) reflection in the Y axis

c) reflection about $y = x$ d) rotation through π

Answer:

78. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation through $\pi/2$ followed by reflection in the line $y = x$. Then T is: reflection in the X axis

a) reflection in the X axis b) reflection in the Y axis

c) reflection about $y = -x$ d) rotation through π

Answer: reflection in the X axis.

79. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation through π followed by reflection in the X axis. Then T is:

a) reflection in the X axis b) reflection in the Y axis

c) rotation about $y = -x$ d) rotation through π

Answer: reflection in the Y axis.

80. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be reflection in $y = x$ followed by rotation through $\pi/2$. Then T is:

a) reflection in the X axis b) reflection in the Y axis

c) rotation about $y = -x$ d) rotation through $-\pi/2$

Answer: reflection in the X axis.

81. Find $T \begin{bmatrix} x \\ y \end{bmatrix}$ if $T : \mathbb{R}^2 \to \mathbb{R}^2$ is reflection in the line $y = 2x$, followed by rotation by $\pi/2$.

a) $\frac{1}{5}\begin{bmatrix} 4x + 3y \\ -3x + 4y \end{bmatrix}$ b) $\frac{1}{5}\begin{bmatrix} -4x + 3y \\ 3x + 4y \end{bmatrix}$ c) $\frac{1}{5}\begin{bmatrix} 4x - 3y \\ -3x - 4y \end{bmatrix}$ d) $\frac{1}{5}\begin{bmatrix} -4x - 3y \\ -3x + 4y \end{bmatrix}$

Answer: $\frac{1}{5}\begin{bmatrix} -4x-3y \\ -3x+4y \end{bmatrix}$.

82. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \end{bmatrix}$. Find the matrix of T, and compute $T \circ T^2$.

a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, T$ b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, 1_{\mathbb{R}^3}$ c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, 1_{\mathbb{R}^3}$ d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, T$

Answer: $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, 1_{\mathbb{R}^3}$.

83. Three players A B and C are throwing a ball to each other. A always throws the ball to B and B always throws it to C, but C is equally likely to throw the ball to B as to A. Write down the transition matrix of this Markov chain, and answer the following questions Yes or No.

a) The Markov chain is regular.

b) The probability of going from B to C in three transitions is $\frac{1}{4}$.

c) The probability of going from C to B in three transitions is $\frac{1}{4}$.

d) The long-term probability that A has the ball is $\frac{2}{5}$.

e) The steady-state vector here is $\begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix}^T$.

Answer: $Yes, No, Yes, No, Yes.$

84. If $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix}$ has LU-factorization $A = LU$, row 3 of L is:

a) $[3 \ \ -1 \ \ 2]$ b) $[1 \ \ -2 \ \ 6]$ c) $[0 \ \ 0 \ \ 1]$ d) $[1 \ \ 2 \ \ 6]$

Answer: $[1 \ \ -2 \ \ 6]$.

85. A town has two industries, A_1 and A_2. Industry A_1 uses 40% of its own output with the rest used by A_2, while A_1 and A_2 each use 50% of the output of A_2. The first row of the input-output matrix, and the equilibrium price structure for these industries is:

a) $[0.4 \ \ 0.5], [\frac{5}{11} \ \ \frac{6}{11}]^T$ b) $[0.4 \ \ 0.6], [\frac{6}{11} \ \ \frac{5}{11}]^T$ c) $[0.5 \ \ 0.5], [\frac{4}{11} \ \ \frac{7}{11}]^T$ d) $[0.6 \ \ 0.4], [\frac{7}{11} \ \ \frac{4}{11}]^T$

Answer: $[0.4 \ \ 0.5], [\frac{5}{11} \ \ \frac{6}{11}]^T$.

Chapter 3: Determinants and Diagonalization

1. If $\begin{vmatrix} 2 & 4 \\ 3 & k \end{vmatrix} = 12$, then $k =$?

   a) 10 b) 12 c) 15 d) 18 e) 20 f) 24

   Answer: $\underline{12}$.

2. If $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 0 & -5 \\ 1 & 1 & 2 \end{bmatrix}$, find $d = \det(A)$ and $c =$ the $(1,2)$-entry of A^{-1}.

   a) $d = -30, c = \frac{-11}{3}$ b) $d = 15, c = \frac{-11}{30}$ c) $d = 30, c = \frac{1}{10}$
   d) $d = 20, c = \frac{-1}{10}$ e) $d = 15, c = \frac{-2}{3}$ f) $d = 30, c = \frac{1}{6}$

   Answer: $\underline{d = 30, c = \frac{1}{6}}$.

3. Let $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$, then the second row of A^{-1} is:

   a) $[0 \quad 1]$ b) $[1 \quad 0]$ c) $[2 \quad 3]$ d) $[3 \quad 4]$ e) $[-2 \quad -3]$ f) $[3 \quad -2]$

   Answer: $\underline{[-2 \quad -3]}$.

4. If $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, what is the $(3,3)$-entry of A^{-1}?

   a) -3 b) -2 c) -1 d) 0 e) 1 f) matrix is not invertible

   Answer: $\underline{-1}$.

5. What is the $(2,3)$-entry of A^{-1} if $A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$?

   a) $\frac{1}{2}$ b) 1 c) -2 d) $\frac{-3}{2}$ e) 2 f) $\frac{-1}{2}$

   Answer: $\underline{1}$.

6. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, what is the $(2,2)$-entry of A^{-1}?

   a) 0 b) 2 c) -2 d) 1 e) -1 f) $\frac{-1}{2}$

   Answer: $\underline{-1}$.

7. Solve for y in the system of equations $\begin{cases} x - 2y + z = 1 \\ 2x + y - z = 2 \\ x - 4y - 2z = -4 \end{cases}$.

a) 0 b) $\frac{25}{21}$ c) $\frac{-25}{21}$ d) $\frac{5}{7}$ e) $\frac{7}{5}$ f) $\frac{-1}{21}$

Answer: $\underline{\frac{5}{7}}$.

8. Solve for z in the system of equations $\begin{cases} x + 2y + 2z = 4 \\ 2x + y + 2z = 2 \\ 3x + y + 2z = 4 \end{cases}$.

a) -2 b) -3 c) 0 d) 1 e) 2 f) The system is inconsistent.

Answer: $\underline{-3}$.

9. Solve for x in the system of equations $\begin{cases} x + 2z = a \\ 2x - y + 3z = b \\ 4x + y + 8z = c \end{cases}$ if:

(i) $a = 5, b = -1, c = 4$

(ii) $a = -1, b = 2, c = 1$

a) (i) -49, (ii) 17 b) (i) -35,(ii) 21 c) i) 49, (ii) -14

d) (i) -21, (ii) -11 e) (i) 21, (ii) 17 f) (i) 49, (ii) -21

Answer: $\underline{\text{(i) } -49, \text{ (ii) } 17}$.

10. What is the value of xz in the system of equations $\begin{cases} x + 2y + 2z = -1 \\ x + 3y + z = 4 \\ x + 3y + 2z = 3 \end{cases}$?

a) 3 b) 4 c) 5 d) 6 e) 7 f) 8

Answer: $\underline{7}$.

11. Solve for z in the system of equations $\begin{cases} 3x + 4y + 2z = 3 \\ x - 2y + z = -1 \\ 2x + y - 2z = 2 \end{cases}$.

a) $\frac{1}{7}$ b) $\frac{-2}{7}$ c) $\frac{3}{7}$ d) $\frac{-4}{7}$ e) $\frac{5}{7}$ f) $\frac{-6}{7}$

Answer: $\underline{\frac{-2}{7}}$.

12. Solve for x in the system of equations $\begin{cases} x + y - z = 3 \\ x + 2y - 2z = 4 \\ 2x + y - z = 5 \end{cases}$.

a) 0 b) -1 c) 1 d) 2 e) x is arbitrary f) 3

Answer: $\underline{2}$.

13. Solve for z in the system of equations: $\begin{cases} x + 2y + z = 0 \\ -x - y + 2z = 0 \\ x + 3y + 2z = 0 \end{cases}$.

a) 0 b) 1 c) 2 d) z is arbitrary e) -1 f) 3.

Answer: $\underline{0}$.

14. Solve for z in the system of equations $\begin{cases} x + 2y - 3z = 14 \\ 2x - y + z = -3 \\ -x + 7y - 2z = 19 \end{cases}$.

a) 1 b) -1 c) 2 d) -2 e) 3 f) -3

Answer: $\underline{-3}$.

15. Solve for z in the system of equations $\begin{cases} 11x - 33y + z = 44 \\ 2x - y = -2 \\ 44x - 3z = 0 \end{cases}$.

a) $\frac{-40}{11}$ b) -40 c) 40 d) $\frac{2255}{19}$ e) $\frac{400}{11}$ f) 0

Answer: $\underline{-40}$.

16. Use Cramer's Rule to solve for y in the system of equations $\begin{cases} x + y - w = -1 \\ x + z + w = 2 \\ x - y + z = 3 \\ y - z + w = 0 \end{cases}$.

a) 6 b) -2 c) -6 d) 3 e) 4 f) 0

Answer: $\underline{-2}$.

17. The solution(s) to the system $\begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 0 & 2 & -4 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} t \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$:

a) is the trivial solution $(0, 0, 0, 0)$.

b) are all the vectors of the form $(4s, -s, 2s, s)$, s is arbitrary.

c) are all the vectors of the form $(2x, -s, 2s, s)$, s is arbitrary.

d) is the vector $(-4, 1, -2, 0)$.

e) can be found using Cramer's Rule.

f) do not exist.

Answer: Are all the vectors of the form $(4s, -s, 2s, s)$, s is arbitrary.

18. Let P and Q be $n \times n$ matrices, k a scalar, and r a positive integer. Which one of the following statements could be <u>false</u>?

a) $|PQ| = |P|\,|Q|$ b) $|kP| = k\,|P|$
c) $|P^r| = |P|^r$ d) $\left|P^t\right| = |P|$
e) $P(\operatorname{adj}(P)) = |P|\, I_n$ f) None of the above statements could be false

Answer: $\underline{|kP| = k\,|P|}$.

19. If A is an $n \times n$ matrix and $A^2 = A$, which of the following are true? List all correct answers.

a) A is invertible.

b) A is symmetric.

c) $\det A = 0$ or $\det A = 1$

d) $A = 0$ or $A = I$.

e) All of the above.

Answer: <u>c) $\det A = 0$ or $\det A = 1$.</u>

20. Let A and B be invertible $n \times n$ matrices. Which <u>two</u> of the following statements are <u>false</u>?

(i) $\det(A^{-1}BA) = \det B$

(ii) $\det(A^{-1}B^{-1}AB) = 1$

(iii) $(A^T B^T)^T = AB$

(iv) $(ABA^{-1})^{-1} = A^{-1}B^{-1}A$

(v) $\det(A^T B) = \det(B^T A)$

a) (i) and (iii) b) (ii) and (iii) c) (iii) and (iv)
d) (ii) and (iv) e) (ii) and (v) f) (i) and (v)

Answer: <u>(iii) and (iv)</u>.

21. Let A and B be $n \times n$ matrices, and k be a scalar. Which <u>two</u> of the following statements are <u>false</u>?

(i) $\det(AB) = \det A \det B$

(ii) $\det A + \det B = \det(A + B)$

(iii) $\det(kA) = k \det A$

(iv) $\det(kA) = k^n \det A$

(v) $\det(A^T) = \det A$

a) (iv) and (v) b) (i) and (v) c) (i) and (ii)
d) (ii) and (iii) e) (iii) and (iv) f) (ii) and (v)

Answer: <u>(ii) and (iii)</u>.

22. Let A and B be $n \times n$ matrices, and ζ a scalar. Which three of the following statements are false?

(i) $\det(AB) = (\det A)(\det B)$

(ii) $\det A + \det B = \det(A + B)$

(iii) $\det(\zeta A) = \zeta(\det A)$

(iv) $\det(\zeta A) = \zeta^n \det(A)$

(v) $\det(A^T) = \det(A)$

(vi) $\det(A^T) = -\det(A)$

a) (i), (iii) and (vi) b) (ii), (iii) and (vi) c) (ii), (iv) and (v)

d) (i), (iv) and (vi) e) (ii), (iii) and (v) f) (i), (iii) and (v)

Answer: (ii), (iii) and (vi).

23. If A and B are invertible 4×4 matrices, which of the following statements are always true?

(i) $\det(AB^T) = \det A \det B$

(ii) $\det(3A) = 3 \det A$

(iii) $\det(A^T) = \frac{1}{\det A}$

(iv) $\det(2A) = 16 \det A$

(v) A and B have rank 2

a) (i) and (ii) b) (i) and (iii) c) (ii) and (v)

d) (iii) and (iv) e) (i) and (v) f) (i) and (iv)

Answer: (i) and (iv).

24. Suppose A is a 3×3 matrix with determinant 5. Which two of the following statements are true?

(i) $\det(2A) = 10$

(ii) $\det(2A) = 40$

(iii) $\det(2A^{-1}) = \frac{2}{5}$

(iv) $\det(2A^{-1}) = \frac{8}{5}$

(v) $\det(2A)^{-1} = \frac{1}{10}$

(vi) $\det(2A)^{-1} = \frac{2}{5}$

a) (i) and (iii) b) (i) and (v) c) (iv) and (vi)

d) (ii) and (vi) e) (ii) and (iv) f) (iii) and (v)

Answer: (ii) and (iv).

25. Let A be a 4×4 matrix and E be the elementary matrix obtained from I_4 by adding 3 times row 2 to row 1. Which of the following statements is true?

a) $\det(2A) = 2 \det A$ and $\det(EA) = \det A$

b) $\det(2A) = 16 \det A$ and $\det(EA) = -3 \det A$

c) $\det(2A) = 16 \det A$ and $\det(EA) = \det A$

d) $\det(2A) = 2 \det A$ and $\det(EA) = -3 \det A$

e) $\det(2A) = 2 \det A$ and $\det(EA) = 3 \det A$

f) $\det(2A) = 16 \det A$ and $\det(EA) = 3 \det A$

Answer: $\underline{\det(2A) = 16 \det A \text{ and } \det(EA) = \det A}$.

26. If $\det A = 2$ and A is a 4×4 matrix, which of the following is equal to 81?

a) $\det(A \text{ adj } A)$,

b) $\det(\text{ adj } A)$

c) $\det(A^T A^2 + A^T A^2)$

d) $2 \det(A^{-1} + \text{ adj } A)$

Answer: $\underline{\text{d) } 2 \det(A^{-1} + \text{ adj } A)}$.

27. B is a 4×4 matrix and $\det B = 12$. Compute $p = 4 \det(B^{-1})$, $q = \det(5B)$, $r = \det(B^T)$, $s = \det(6B^{-2})$ and find $3p + q + r + s$.

a) 7522 b) 97 c) 937 d) 259

e) One of the four expressions cannot be computed with the given data.

f) Two of the four expressions cannot be computed with the given data.

Answer: $\underline{7522}$.

28. Let A be a triangular $n \times n$ matrix with the entry $a_{nn} = 5$ and $C_{nn}(A) = -6$. What is the product of the entries on the main diagonal of A?

a) $(-1)^n(-30)$ b) -30

c) 30 d) $(-30)^n$

e) 30^n f) not computable with the data given

Answer: $\underline{-30}$.

29. Let P and Q be $n \times n$ matrices, k a scalar, r a positive integer. State which <u>one</u> of the following is, in general, <u>false</u>.

a) $\det(PQ) = \det P \det Q$

b) $\det(kP) = k \det P$

c) $\det(P^r) = (\det(P))^r$

d) $\det(P^T) = \det(P)$, where P^T is the transpose of P.

e) $\det(PQP^{-1}) = \det Q$

Answer: $\underline{\det(kP) = k \det P}$.

30. Suppose M is a $n \times n$ matrix. If <u>one</u> of the statements below is removed, the remaining four are equivalent. Which one should be removed?

(i) M is invertible.

(ii) $MX = 0$ has a non-trivial solution.

(iii) $MX = B$ has a unique solution for every $n \times 1$ matrix B.

(iv) The determinant of M is not zero.

(v) M is row-equivalent to I_n (the identity matrix).

a) (i) b) (ii) c) (iii) d) (iv) e) (v)

Answer: (ii).

31. For a 3×3 matrix A, state which combination of answers to the following questions is correct.
    - If $\det A \neq 0$, is it true that A is invertible?
    - If $\det A = 0$, is it true that rank $A = 0$?
    - If $\det A = 0$, is it true that $AX = B$ has no solution?
    - If the system $AX = B$ has a unique solution, is it true that $\det A \neq 0$?

    a) Yes, No, Yes, No
    b) Yes, Yes, No, No
    c) Yes, No, No, Yes
    d) No, No, No, Yes
    e) Yes, Yes, No, Yes
    f) Yes, No, Yes, No

    Answer: c).

32. One way to find the equation for a general conic section is to use the 'method of determinants'. Developing this method uses certain results, some of which are given below. List all true statements.

    a) If A is $n \times n$, the system $AX = 0$ has a nontrivial solution if $\det A = 0$.
    b) If A is $n \times n$ and $AX = 0$, then $\det A = 0$.
    c) If A is $n \times n$, then $AX = 0$ has a nontrivial solution if and only if $\det A = 0$.
    d) A is invertible if and only if $\det A = 0$.
    e) If $\det A = 1$, then A is invertible.

    Answer: a), c), and e).

33. For how many distinct values of α is the determinant $\begin{vmatrix} \alpha & \sqrt{3} & 0 \\ \sqrt{3} & \alpha & \sqrt{3} \\ 0 & \sqrt{3} & \alpha \end{vmatrix}$ equal to zero?

    a) 0 b) 1 c) 2 d) 3 e) 4 f) infinitely many

    Answer: 3.

34. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & x \end{bmatrix}$. For which value(s) of x is A invertible?

    a) $x \neq -1$ b) $x \neq 1$ c) $x \neq 0$ d) -1 e) $x = 1$ f) $x \neq \pm 1$

    Answer: $x \neq -1$.

35. Find all values of x for which $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = 0.$

a) 1 b) 1 or $\frac{-1}{2}$ c) $\frac{-1}{2}$ d) 0 e) 0 or $\frac{-1}{2}$ f) 2 or 1

Answer: $\underline{1 \text{ or } -\frac{1}{2}}$.

36. Find x so that the matrix $\begin{bmatrix} 0 & x & -4 \\ 2 & 3 & -2 \\ 1 & 4 & 1 \end{bmatrix}$ is not invertible.

a) 2 b) 5 c) -5 d) -2 e) 1 f) 0

Answer: $\underline{-5}$.

37. Find all value(s) of y for which $\begin{vmatrix} 1 & y & y^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0.$

a) 2 or 3 b) 0 or 1 c) 2 or -3 d) 1 or 2 e) -1 f) there are no such y.

Answer: $\underline{2 \text{ or } 3}$.

38. Find all values of λ for which the determinant $\begin{vmatrix} \lambda-6 & 0 & 0 \\ 0 & \lambda & 3 \\ 0 & 4 & \lambda+4 \end{vmatrix}$ is zero.

a) $0, 6, 4$ b) $6, -6, 2$ c) $6, -6, -2$ d) $6, -4, 3$ e) $-6, 4, 3$ f) $6, 4, 3$

Answer: $\underline{6, -6, -2}$.

39. Find all values of λ for which $\det \begin{bmatrix} \lambda-6 & \lambda & 13 \\ 0 & \lambda & 3 \\ 0 & \lambda+4 & \lambda+7 \end{bmatrix} = 0.$

a) $0, 6, -4$ b) $6, 4, 3$ c) $-6, 4, 3$ d) $6, -6, 2$ e) $6, -6, -2$ f) $6, 2$

Answer: $\underline{6, -6, 2}$.

40. For what values of x is the matrix $\begin{bmatrix} 4-x & 2\sqrt{5} & 0 \\ 2\sqrt{5} & 4-x & \sqrt{5} \\ 0 & \sqrt{5} & 4-x \end{bmatrix}$ invertible?

a) 4 b) all $x \neq 4$ c) all $x \neq 4, 9$ and -1
d) all $x \neq 4, -9$ and 1 e) all $x \neq 9$ and -1 f) $4, 9$ or -1

Answer: $\underline{\text{All } x \neq 4, 9 \text{ and } -1}$.

41. Given $A = \begin{bmatrix} 3 & 3 & 4 \\ 1 & -1 & 2 \\ a & 0 & 5 \end{bmatrix}$, which of the following are true? List all correct answers.

a) A is invertible for all values of a.

b) A fails to be invertible for all values of a.

c) A is invertible for all values of a except zero.

d) A is invertible for all values of a except $a = 3$.

e) None of the above.

Answer: d).

42. Evaluate the determinant $\begin{vmatrix} k & -3 & 9 \\ 2 & 4 & k+1 \\ 1 & k^2 & 3 \end{vmatrix}$.

a) $k^4 + k^3 - 18k^2 - 9k + 21$

b) $-k^4 - k^3 - 18k^2 - 9k + 21$

c) $-k^4 - k^3 + 18k^2 + 9k - 21$

d) $-k^4 - k^3 + 18k^2 + 9k - 21$

e) $-k^4 - k^3 + 18k^2 + 9k - 57$

f) $k^4 + k^3 + 18k^2 + 9k - 21$

Answer: $-k^4 - k^3 + 18k^2 + 9k - 21$.

43. If B is a 3×3 matrix and $\det B = 5$, then $\det(2B^{-1})$ is:

a) $\frac{1}{10}$ b) $\frac{1}{40}$ c) $\frac{2}{5}$ d) $\frac{8}{5}$ e) $\frac{5}{8}$ f) $\frac{1}{5}$

Answer: $\frac{8}{5}$.

44. If B is a 4×4 matrix, and $\det B = 6$, then $\det(2B)^{-1}$ is:

a) $\frac{1}{6}$ b) $\frac{1}{36}$ c) $\frac{1}{16}$ d) $\frac{1}{96}$ e) $\frac{-1}{96}$ f) $\frac{1}{48}$

Answer: $\frac{1}{96}$.

45. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, find $\begin{vmatrix} 4g & a & d-2a \\ 4h & b & e-2b \\ 4i & c & f-2c \end{vmatrix}$.

a) 24 b) -24 c) 12 d) -12 e) 6 f) -6

Answer: 12.

46. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$, find $\begin{vmatrix} 3a-5g & g & d \\ 3b-5h & h & e \\ 3c-5i & i & f \end{vmatrix}$.

a) 7 b) 21 c) -21 d) 35 e) -35 f) -7

Answer: -21.

47. A is a 2×2 matrix with $\det A = 3$. Find $\det(\operatorname{adj} A)$.

a) 27 b) -1 c) 1 d) 3 e) -3 f) 9

Answer: $\underline{3}$.

48. A is a 3×3 matrix with $\det A = 2$. If $\operatorname{adj}(A)$ denotes the transpose of the matrix of cofactors of A, find $\det(\operatorname{adj}(A))$.

a) -1 b) -2 c) 2 d) 4 e) 8 f) 16

Answer: $\underline{4}$.

49. The $(1,2)$-cofactor of the matrix $\begin{bmatrix} x & y & z \\ a & b & c \\ u & v & w \end{bmatrix}$ is:

a) $aw - uc$ b) $yc - bz$ c) $xz - uw$ d) $ab - uv$ e) $uc - aw$ f) $bw - cv$

Answer: $\underline{uc - aw}$.

50. Find the first row of the adjugate of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 5 \\ 0 & 1 & -1 \end{bmatrix}$.

a) $[-5 \quad 3 \quad 3]$ b) $[-5 \quad 1 \quad -5]$ c) $[-5 \quad -1 \quad -5]$
d) $[-5 \quad -3 \quad 3]$ e) $[-5 \quad 3 \quad -5]$ f) none of the above

Answer: $\underline{[-5 \quad 1 \quad -5]}$.

51. The $(3,2)$-cofactor of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ 4 & 5 & -7 \\ 7 & 8 & 10 \end{bmatrix}$ is:

a) 1 b) -1 c) 8 d) -8 e) 4 f) -4

Answer: $\underline{-1}$.

52. The $(2,4)$-cofactor of $\begin{bmatrix} 3 & 1 & 1 & 1 \\ 0 & 3 & 2 & 5 \\ 9 & 1 & -2 & 0 \\ 0 & 7 & 0 & 3 \end{bmatrix}$ is:

a) 35 b) -35 c) -105 d) 105 e) 57 f) -57

Answer: $\underline{105}$.

53. Find the $(3,2)$-cofactor of the matrix $\begin{bmatrix} -2 & 3 & -8 & 9 \\ 3 & -6 & 2 & 0 \\ 2 & 5 & 0 & 2 \\ 1 & -2 & 4 & 3 \end{bmatrix}$.

a) 131 b) -120 c) -131 d) -150 e) 150 f) -180

Answer: $\underline{-150}$.

54. The $(3,2)$-cofactor $C_{32}(A)$ for the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$ is:

a) -22 b) 6 c) 48 d) -48 e) 22 f) -6

Answer: $\underline{6}$.

55. The $(3,2)$-cofactor of $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 3 & 0 & 6 \end{bmatrix}$ is:

a) -5 b) 5 c) 3 d) -3 e) 7 f) -7

Answer: $\underline{-3}$.

56. The $(2,2)$-cofactor of $\begin{bmatrix} -2 & 3 & -8 & 9 \\ 3 & -6 & 7 & 0 \\ 2 & 5 & 0 & 2 \\ 1 & -2 & 4 & 3 \end{bmatrix}$ is:

a) -30 b) 60 c) -120 d) 120 e) -180 f) 240

Answer: $\underline{120}$.

57. The $(3,1)$-cofactor of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 3 & 0 & 6 \end{bmatrix}$ is:

a) 2 b) 5 c) 3 d) -9 e) 7 f) -7

Answer: $\underline{2}$.

58. The adjugate of the matrix $A = \begin{bmatrix} -1 & 2 & 0 \\ -3 & -1 & 5 \\ 0 & -2 & -6 \end{bmatrix}$ is:

a) $\begin{bmatrix} 16 & -18 & 6 \\ 12 & 6 & -2 \\ 10 & 5 & 7 \end{bmatrix}$ b) $\begin{bmatrix} 16 & 18 & 6 \\ 12 & 6 & 2 \\ 10 & -5 & 7 \end{bmatrix}$ c) $\begin{bmatrix} 16 & 12 & 10 \\ 18 & 6 & -5 \\ 6 & 2 & 7 \end{bmatrix}$

d) $\begin{bmatrix} 16 & 12 & 10 \\ -18 & 6 & 5 \\ 6 & -2 & 7 \end{bmatrix}$ e) $\begin{bmatrix} -16 & -18 & 10 \\ -12 & 6 & 5 \\ 10 & 5 & 7 \end{bmatrix}$ f) $\begin{bmatrix} -16 & 18 & 6 \\ -18 & 6 & 2 \\ 10 & -5 & 7 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 16 & 12 & 10 \\ -18 & 6 & 5 \\ 6 & -2 & 7 \end{bmatrix}}$.

59. The first row of the adjugate of $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & 1 & -3 \end{bmatrix}$ is:

a) $[-3 \quad -2 \quad -2]$ b) $[3 \quad 2 \quad 2]$ c) $[-3 \quad 6 \quad -2]$
d) $[3 \quad -6 \quad -2]$ e) $[1 \quad -2 \quad 0]$ f) $[-3 \quad 2 \quad -2]$

Answer: $\underline{[-3 \quad 2 \quad -2]}$.

60. Given the matrices $A = \begin{bmatrix} 7 & 5 & 1 \\ 2 & 0 & 5 \\ 7 & 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 9 & -1 \\ 3 & 6 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 7 & 6 \\ 0 & 1 & 1 \\ 0 & 2 & -5 \end{bmatrix}$, indicate which one of the following statements is true.

a) Only A is invertible b) Only B is invertible
c) Only C is invertible d) A and B are both invertible
e) A and C are both invertible f) B and C are both invertible

Answer: Only B is invertible.

61. E_1, E_2 and E_3 are 7×7 elementary matrices;
E_1 corresponds to the interchange of rows 2 and 3;
E_2 corresponds to adding 12 times row 3 to row 7;
E_3 corresponds to the multiplication of row 5 by 8.
Find $\det(E_1 E_2 E_3)$.

a) 8 b) -96 c) 12 d) 96 e) -8 f) -12

Answer: $\underline{-8}$.

62. A and B are two matrices such that $\det A = -2$ and $\det B^T = 3$. Find $\det AB$.

a) 6 b) 1 c) 5 d) -5 e) -6 f) $-\frac{3}{2}$

Answer: $\underline{-6}$.

63. If $a = (1, 2, 3)$ and $b = (4, 5, 6)$, find $\det(a^T b)$.

a) 6 b) -6 c) 12 d) 0 e) 1 f) -3

Answer: $\underline{0}$.

64. If $A = \begin{bmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{bmatrix}$, find $\det(AA^T)$.

a) 3 b) 18 c) 9 d) 36 e) 24 f) 45

Answer: $\underline{45}$.

65. If A is a 23×23 matrix such that $A^T = -A$, then $\det A = ?$

a) -1 b) 1 c) -23 d) 0 e) 23

f) Impossible to determine from the information

Answer: $\underline{0}$.

66. If A is a 3×3 matrix with determinant -5, then which of the following statements is true?

a) $\det(4A) = -20$ and $\det(2A) = -10$ b) $\det(A^{-1}) = -0.2$ and $\det(2A) = -10$

c) $\det(A^{-1}) = -0.2$ and $\det(2A) = -40$ d) $\det(A^{-1}) = 5$ and $\det(2A) = -40$

e) $\det(A^{-1}) = -5$ and $\det(2A) = -10$ f) The inverse does not exist and $\det(4A) = -320$

Answer: $\underline{\det(A^{-1}) = -0.2 \text{ and } \det(2A) = -40}$.

67. Let A and B be 4×4 matrices with $\det A = 3$ and $\det B = -5$. Find $\det(2AB^TB^{-2})$.

a) $\frac{-48}{5}$ b) $\frac{-36}{5}$ c) $\frac{-24}{5}$ d) $\frac{24}{5}$ e) $\frac{12}{5}$ f) $\frac{48}{3}$

Answer: $\underline{\frac{-48}{5}}$.

68. Let B be a 4×4 matrix such that $\det(B^TB) = 4$. Find $\left|\det(2B^3)\right|$.

a) 64 b) 12 c) 48 d) 96 e) 128 f) 256

Answer: $\underline{128}$.

69. Q is a 3×3 matrix with $\det Q = 4$. Find $\det((3Q)^{-1})$.

a) $\frac{27}{4}$ b) $\frac{4}{27}$ c) $\frac{1}{12}$ d) $\frac{1}{108}$ e) $\frac{3}{4}$ f) $\frac{108}{27}$

Answer: $\underline{\frac{1}{108}}$.

70. B is a 4×4 matrix with $\det(2BB^T) = 64$. Find $\det(3B^2B^T)$.

a) -648 b) 128 c) 640 d) 648 e) 192 f) Not sufficient information.

Answer: Not sufficient information.

71. A and B are 4×4 matrices with $\det A = 2$ and $\det B = -1$. Find $\det(3AB^TA^{-2}BA^TB^{-1})$.

a) 81 b) -3 c) 48 d) -81 e) 12 f) -48

Answer: $\underline{-81}$.

72. A and B are 3×3 matrices with $\det A = 5$ and $\det B = 8$. Find $\det(4B^TA^2B^{-2})$.

a) $\frac{25}{2}$ b) $\frac{125}{2}$ c) 250 d) 400 e) 800 f) 200

Answer: $\underline{200}$.

73. $\begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{vmatrix}$ equals:

a) 15 b) -18 c) -36 d) -10 e) 18 f) 30

Answer: $\underline{-10}$.

74. $\begin{vmatrix} 1 & 2 & -1 & 1 \\ 3 & 0 & 1 & 4 \\ 1 & -1 & 2 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix}$ equals:

a) 6 b) -6 c) -12 d) 12 e) 18 f) -18

Answer: $\underline{-12}$.

75. $\begin{vmatrix} 2 & 6 & 1 & 6 \\ 3 & 18 & 2 & 0 \\ 4 & 24 & 3 & 0 \\ 5 & 30 & 4 & 2 \end{vmatrix}$ equals:

a) 22 b) -22 c) 12 d) 11 e) -11 f) 121

Answer: $\underline{12}$.

76. $\begin{vmatrix} 0 & 0 & 0 & 7 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 5 & 0 & 0 & 0 \end{vmatrix}$ equals:

a) -135 b) -105 c) 165 d) -205 e) -175 f) 225

Answer: $\underline{-175}$.

77. $\begin{vmatrix} 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix}$ equals:

a) 2 b) -2 c) 1 d) 0 e) -1 f) -3

Answer: $\underline{0}$.

78. $\begin{vmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{vmatrix}$ equals:

a) -6 b) -9 c) -12 d) -15 e) -18 f) -24

Answer: $\underline{-18}$.

79. $\begin{vmatrix} 2 & -1 & 1 & 0 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 1 \end{vmatrix}$ equals:

a) -39 b) -21 c) 39 d) 21 e) -15 f) 15

Answer: -15

80. $\begin{vmatrix} 2 & -5 & -10 & 1 & 6 \\ 0 & 6 & 12 & -2 & 0 \\ 0 & 0 & 5 & -1 & 3 \\ -1 & 5 & 10 & 1 & 0 \\ 1 & -3 & -6 & 0 & -2 \end{vmatrix}$ equals:

a) 180 b) -180 c) 380 d) -380 e) 580 f) -580

Answer: $\underline{-580}$.

81. $\begin{vmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{vmatrix}$ equals:

a) 12 b) 96 c) 36 d) 72 e) 48 f) 60

Answer: $\underline{48}$.

82. Find the determinant of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix} \begin{bmatrix} 13 & 14 \\ 15 & 16 \end{bmatrix}$.

a) 256 b) 4890 c) 2108 d) 1104 e) 16 f) 3636

Answer: $\underline{16}$.

83. $\begin{vmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & 3 & -3 \\ -1 & 1 & 2 & -1 \\ 2 & -1 & -3 & 4 \end{vmatrix}$ equals:

a) 36 b) -36 c) -24 d) 24 e) 18 f) -18

Answer: $\underline{36}$.

84. $\begin{vmatrix} a+b & 1 & c \\ b+c & 1 & a \\ c+a & 1 & b \end{vmatrix}$ equals:

a) $a+b+c$ b) $-a-b-c$ c) $ab+bc+ca$

d) 0 e) 1 f) $a+ab+b+bc+c+ca$

Answer: $\underline{0}$.

85. $\begin{vmatrix} 2 & -1 & 3 & 4 & -5 \\ 4 & -2 & 7 & 8 & -7 \\ -6 & 4 & -9 & -2 & 3 \\ 3 & -2 & 4 & 1 & -2 \\ -2 & 6 & 5 & 4 & -3 \end{vmatrix}$ equals:

a) -42 b) -84 c) 63 d) 21 e) 10.5 f) 168

Answer: $\underline{-84}$.

86. $\begin{vmatrix} (a+1)^2 & a^2+1 & a \\ (b+1)^2 & b^2+1 & b \\ (c+1)^2 & c^2+1 & c \end{vmatrix}$ equals:

a) 1 b) 0 c) abc

d) $a^2b^2c^2$ e) $a^2+b^2+c^2+1$ f) $-(a+b+c)+3$

Answer: $\underline{0}$.

87. $\begin{vmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{vmatrix}$ equals:

a) $abcde$ b) $acd-bce$ c) $bcd-ace$ d) $ace-bcd$ e) $abc-cde$ f) 0

Answer: $\underline{ace-bcd}$.

88. $\begin{vmatrix} 2 & -3 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ 2 & 1 & -1 & 2 \\ -1 & 2 & 1 & -1 \end{vmatrix}$ equals:

a) -1 b) 2 c) -4 d) 8 e) -16 f) 32

Answer: $\underline{-16}$.

89. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ -3 & 4 & 1 & -2 \\ -4 & -3 & 2 & 1 \end{vmatrix}$ equals:

a) 100 b) 200 c) 300 d) 400 e) 600 f) 900

Answer: $\underline{900}$.

90. $\begin{vmatrix} 4 & -1 & 2 & 3 \\ 3 & 1 & 0 & 1 \\ 3 & -4 & 1 & 4 \\ 5 & 3 & 0 & 1 \end{vmatrix}$ equals:

a) 1 b) 2 c) 4 d) -1 e) -2 f) -4

Answer: $\underline{-2}$.

91. Compute $\det \begin{bmatrix} 2 & -2 & 0 & -2 \\ 4 & 3 & 2 & 8 \\ -4 & 5 & 3 & 4 \\ 8 & 2 & 3 & 0 \end{bmatrix}$.

a) 344 b) -231 c) -344 d) -384 e) -475 f) -673

Answer: $\underline{-344}$.

92. For the matrix $A = \begin{bmatrix} 1 & 4 & 6 & 8 \\ 0 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, find $p = \det A$ and $q = \det(A^{-1})^T$.

a) $p = 12$, $q = -12$ b) $p = 9$, $q = \frac{-1}{9}$ c) $p = 6$, $q = \frac{1}{6}$
d) $p = 12$, $q = \frac{1}{12}$ e) $p = 9$, $q = -9$ f) $p = 12$, $q = \frac{-1}{12}$

Answer: $\underline{p = 12,\ q = \frac{1}{12}}$.

93. Find the value of α such that $\begin{vmatrix} 16 & 16 & 16 & 16 \\ 16 & -8 & 8 & -16 \\ 16 & 4 & 4 & 16 \\ 16 & -2 & 2 & -16 \end{vmatrix} = \alpha \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & -2 \\ 4 & 1 & 1 & 4 \\ 8 & -1 & 1 & -8 \end{vmatrix}$.

a) 16^5 b) -16 c) 0 d) 1024 e) -1024 f) 16

Answer: $\underline{1024}$.

94. The interpolating quadratic through the points $(0,0)$, $(1,1)$, $(2,3)$, is:

a) $\frac{1}{2}(x-x^2)$ b) $\frac{1}{2}(x+x^2)$ c) $x+x^2$ d) $\frac{1}{2}(2x+x^2)$.

Answer: $\underline{\text{b)}}$.

95. The interpolating quadratic through the points $(0,1)$, $(1,2)$, $(2,5)$, is:

a) $\frac{1}{2}(2+x+x^2)$ b) $\frac{1}{2}(2-x+x^2)$ c) $\frac{1}{2}(2+x-x^2)$ d) $\frac{1}{2}(2+2x+x^2)$.

Answer: $\underline{\text{a)}}$.

96. The interpolating cubic through the points $(0,1)$, $(1,2)$, $(2,3)$, $(3,2)$ is:

a) $\frac{1}{11}(11+5x-9x^2-3x^3)$ b) $\frac{1}{11}(11-5x+9x^2+3x^3)$

c) $11+5x+9x^2-3x^3$ d) $\frac{1}{11}(11+5x+9x^2-3x^3)$.

Answer: $\underline{\text{d)}}$.

97. The interpolating cubic through the points $(-1,0)$, $(0,-2)$, $(1,3)$, $(2,5)$ is:

a) $\frac{1}{2}(-4+3x+12x^2-5x^3)$ b) $\frac{1}{2}(-4-3x+12x^2-5x^3)$

c) $\frac{1}{2}(-4+3x-12x^2+5x^3)$ d) $-4+3x+12x^2-5x^3$.

Answer: $\underline{\text{a)}}$.

98. Find the eigenvalues of $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

a) $1,2,3$ b) $-1,0,1$ c) $2,3$

d) $1,3$ e) $-1,0$ f) $1,2$

Answer: $\underline{1,2}$.

99. Find the eigenvalues of $\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$.

a) $1,2,3$ b) $-1,2,3$ c) $-1,2$

d) $-2,1,3$ e) $-2,2,3$ f) $-2,1$

Answer: $\underline{-2,1,3}$.

100. Find the eigenvalues of $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 2 & -3 \end{bmatrix}$.

a) $2,3,4$ b) $-3,3,4$ c) $0,2,3$

d) $-3,0,4$ e) $-1,-2,1$ f) $0,0,2$

Answer: $\underline{-1,-2,1}$.

101. An eigenvalue of $\begin{bmatrix} 23 & 0 & -36 \\ 0 & 50 & 0 \\ -36 & 0 & 2 \end{bmatrix}$ is 50. Find another, if possible.

a) 23 b) 2 c) -36
d) -25 e) -50 f) 50 is the only eigenvalue

Answer: $\underline{-25}$.

102. The eigenvalue 48 of $\begin{bmatrix} 21 & 0 & -36 \\ 0 & 48 & 0 \\ -36 & 0 & 0 \end{bmatrix}$ has multiplicity 2. Find the other eigenvalue.

a) 21 b) -36 c) 0 d) 96 e) 23 f) -27

Answer: $\underline{-27}$.

103. One of the eigenvalues of the diagonalizable matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ -1 & 2 & -2 \\ -6 & 6 & -4 \end{bmatrix}$ is 1. The other two eigenvalues of A are:

a) 2 and -1 b) 1 and 1 c) 0 and 1
d) 2 and 2 e) -2 and 0 f) -1 and 3

Answer: $\underline{2 \text{ and } -1}$.

104. If $(1,1,1)^T$ is an eigenvector of A corresponding to the eigenvalue λ, what is the sum of the columns of A?

a) $(1,1,1)^T$ b) $(0,0,0)^T$ c) $(\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{1}{\lambda})^T$ d) $(\lambda, \lambda, \lambda)^T$ e) $(\lambda, 2\lambda, 3\lambda)^T$
f) The sum of the columns of A cannot be calculated with this information.

Answer: $\underline{(\lambda, \lambda, \lambda)^T}$.

105. If λ is an eigenvalue of the $n \times n$ invertible matrix A, select the correct statements:

a) $\lambda \neq 0$.
b) $\lambda = \pm 1$.
c) $\det(A) = \lambda^n$.
d) $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.
e) A has no eigenvalues, being invertible.
f) λ^2 is an eigenvalue of A^2.

Answer: <u>a), d), f)</u>.

106. The characteristic polynomial of $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ is

a) $(x-2)(x+1)$ b) x^2-3x+2 c) $(x+2)(x+1)$ d) $3x^2$ e) 0 f) $(x+2)(x-1)$

Answer: $\underline{x^2-3x+2}$.

107. If $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ then $P^{-1}AP$ is diagonal if P is (choose the correct answers):

a) $\begin{bmatrix} 2 & -3 \\ 1 & -3 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$

d) $\begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ f) $\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$

Answer: a) and e).

108. The eigenvalues and eigenvectors for $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ are:

a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix} \leftrightarrow 1$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix} \leftrightarrow 2$ b) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \leftrightarrow 2$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \leftrightarrow 1$

c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \leftrightarrow 1$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \leftrightarrow 2$ d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \leftrightarrow 1$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix} \leftrightarrow 2$

Answer: c).

109. The characteristic polynomial of $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ is (choose the correct answers):

a) $(x-1)(x+5)$ b) x^2+4x-5 c) $(x+1)(x+5)$

d) x^2-4x-5 e) x^2 f) $(x+1)(x-5)$

Answer: d) and f).

110. If $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ then $P^{-1}AP$ is diagonal if P is (choose the correct answers):

a) $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$ e) $\begin{bmatrix} -2 & 3 \\ 4 & 3 \end{bmatrix}$ f) $\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

Answer: b), c) and e).

111. Eigenvalues and eigenvectors for $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ are:

a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} \leftrightarrow -1$ and $\begin{bmatrix} -2 \\ -2 \end{bmatrix} \leftrightarrow 5$ b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \leftrightarrow -1$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \leftrightarrow 5$

c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \leftrightarrow -1$ and $\begin{bmatrix} -2 \\ -2 \end{bmatrix} \leftrightarrow 5$ d) $\begin{bmatrix} 1 \\ -2 \end{bmatrix} \leftrightarrow -1$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \leftrightarrow 5$

Answer: a) and d).

112. The characteristic polynomial of $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$ is (choose the correct answers):

a) $x^2 + x - 6$ b) $(x+2)(x-3)$ c) $x(x+5)$

d) $x^2 + x + 6$ e) x^2 f) $(x-2)(x+3)$

Answer: a) and f).

113. If $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$ then $P^{-1}AP$ is diagonal if P is (choose the correct answers):

a) $\begin{bmatrix} 1 & -1 \\ 1 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$ e) $\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$

Answer: a), b) and f).

114. Eigenvalues and eigenvectors for $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$ are:

a) $\begin{bmatrix} 1 \\ -4 \end{bmatrix} \leftrightarrow -3$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \leftrightarrow 2$ b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} \leftrightarrow -3$ and $\begin{bmatrix} -1 \\ -1 \end{bmatrix} \leftrightarrow 2$

c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} \leftrightarrow -3$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \leftrightarrow 2$ d) $\begin{bmatrix} -1 \\ -4 \end{bmatrix} \leftrightarrow -3$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix} \leftrightarrow 2$

Answer: a) and b).

115. Consider the matrix $A = \begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix}$. Choose the correct statements:

a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector of A. b) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is an eigenvector of A.

c) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is an eigenvector of A. d) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ is an eigenvector of A.

Answer: a) and d).

116. Consider the matrix $A = \begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}$. Which of the following are eigenvectors of A :

a) $[1\ \ 0\ \ 1]^T$ b) $[1\ \ -1\ \ 1]^T$ c) $[1\ \ 1\ \ 1]^T$ d) $[1\ \ 1\ \ -1]^T$ e) $[0\ \ 0\ \ 1]^T$ f) $[1\ \ 2\ \ 0]^T$

Answer: a), c) and f).

117. In each case answer T if the statement is true, and F if it is false.

(a) If the eigenvalues of A are real and distinct then A is diagonalizable.

(b) Every diagonalizable matrix is symmetric.

(c) If A has no inverse then $\lambda = 0$ is an eigenvalue of A.

(d) If $\lambda = 0$ is an eigenvalue of A then A has no inverse.

(e) If A has real eigenvalues then it is invertible.

(f) Every diagonal matrix is diagonalizable.

Answer: T, F, T, T, F, T.

118. In each case answer T if the statement is true, and F if it is false.

(a) Every invertible matrix is diagonalizable.

(b) Every diagonalizable matrix is invertible.

(c) If A is diagonalizable, so also is its transpose A^T.

(d) If A^T is diagonalizable, so also is A.

(e) If A is diagonalizable, so also is A^2.

(f) If A^2 is diagonalizable, so also is A.

Answer: F, F, T, T, T, F.

119. Suppose a matrix A has two eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 5$, with corresponding eigenvectors $X_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, respectively. The matrix A is (list the correct answers):

a) $\begin{bmatrix} -7 & 18 \\ -6 & 14 \end{bmatrix}$ b) $\begin{bmatrix} 7 & -18 \\ 6 & -14 \end{bmatrix}$ c) $\begin{bmatrix} 7 & -18 \\ 6 & -14 \end{bmatrix}$ d) $\begin{bmatrix} 14 & -18 \\ 6 & -7 \end{bmatrix}$

Answer: a).

120. Suppose a matrix A has two eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$, with corresponding eigenvectors $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, respectively. The matrix A is (list the correct answers):

a) $\begin{bmatrix} 2 & -1 \\ -1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 4 \\ 8 & 5 \end{bmatrix}$ c) $\frac{1}{3}\begin{bmatrix} 1 & 4 \\ 8 & 5 \end{bmatrix}$ d) $\begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

Answer: c).

121. If a 2×2, invertible matrix A has eigenvalues 2 and 5, answer the following True or False:

a) A is invertible b) A is diagonalizable c) A is symmetric d) $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

e) $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ for some invertible P f) $A^2 = 10I$.

Answer: T, T, F, F, T, F.

122. Let A be a diagonalizable matrix. If $\lambda^2 = 1$ for each eigenvalue λ of A, then (select the correct answers):

a) A is invertible b) $A^2 = I$ c) $A^T = -A$ d) $A^T = A$ e) $A^2 = A$ f) $A^3 = A$.

Answer: a), b), f).

123. Let λ be an eigenvalue of the matrix A. Select the correct statements:

a) λ^2 is an eigenvalue of A^2 b) $A^2 = \lambda I$ c) λA is invertible

d) $\lambda - 3$ is an eigenvalue of $A = 3I$ e) $AX = \lambda X$ for every column $X \neq 0$

Answer: a), d).

124. Let A be a diagonalizable matrix. If $\lambda^2 = \lambda$ for each eigenvalue λ of A, then (select the correct answers):

a) A is invertible b) $A = I$ or 0 c) $A^T = -A$ d) $A^T = A$ e) $A^2 = A$ f) $A^3 = A$.

Answer: d), e), f).

125. If $\lambda = 0$ is an eigenvalue of A, then (select the correct answers):

a) $A = 0$ b) $det(A) = 0$ c) A is invertible d) A is not invertible e) $A^2 = 0$.

Answer: b), d).

126. If all the eigenvalues of A are nonzero, then (select the correct answers):

a) $A = I$ b) $A^2 = I$ c) A is invertible d) A is not invertible e) $det(A) \neq 0$.

Answer: c), e).

127. Let A be a diagonalizable matrix. If $\lambda^3 = \lambda$ for each eigenvalue λ of A, then (select the correct answers):

a) A is not invertible b) $A = I$, $-I$ or 0 c) $A^T = -A$ d) $A = -I$ e) $A^2 = A$ f) $A^3 = A$.

Answer: f).

128. Suppose that $B = QAQ^{-1}$. Select the correct statements:

a) If A is diagonalizable, so is B b) $A^2 = Q$ c) A and B are diagonalizable

d) A and B have the same eigenvalues e) If B is diagonalizable, so is A

f) If X is an eigenvector of A corresponding to λ then QX is an eigenvector of A corresponding to λ

Answer: a), d), e), f).

129. Suppose A is 3×3 and has 2 and 3 as its only eigenvalues. Then (select the correct answers):

a) A is not diagonalizable b) $A^2 = 0$ c) A is invertible

d) A is not invertible e) A has an eigenvalue of multiplicity 2.

Answer: c), e).

130. Suppose A is $n \times n$ and has 3 as its only eigenvalue. Then (select the correct answers):

a) A is not diagonalizable b) $A^2 = 0$ c) $A = 3I$

d) A is diagonalizable e) A has an eigenvalue of multiplicity 2. f) there is no such matrix

Answer: c), d).

131. For $A = \begin{bmatrix} -9 & -8 & -4 \\ 18 & 17 & 9 \\ -14 & -14 & -8 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $2, -1, -1$; not diagonalizable b) ; diagonalizable

c) $2, 2, -1$; not diagonalizable d) $2, 2, -2$; diagonalizable

e) $-2, 1, 1$; diagonalizable f) $2, -1, -1$; diagonalizable

Answer: $2, -1, -1$; diagonalizable.

132. For $A = \begin{bmatrix} -7 & -5 & 0 \\ 10 & 8 & 0 \\ 5 & 5 & -2 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $3, -2, -2$; not diagonalizable b) $3, -2, -2$; diagonalizable

c) $2, 2, 3$; not diagonalizable d) $-2, 2, 3$; diagonalizable

e) $-2, 3, 3$; diagonalizable f) $2, -3, -3$; diagonalizable

Answer: $3 - 2, -2$; diagonalizable.

133. For $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ -12 & 11 & 4 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $3, -1, -1$; not diagonalizable b) $-3, 1, 1$; diagonalizable

c) $3, 3, -1$; not diagonalizable d) $-3, -3, 1$; diagonalizable

e) $3, 3, 1$; not diagonalizable f) $3, 3, 1$; diagonalizable

Answer: $3, 3, 1$; not diagonalizable.

134. For $A = \begin{bmatrix} -1 & 1 & -3 \\ 2 & 1 & -6 \\ 2 & 1 & -6 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $3, 3, 0$; not diagonalizable b) $-3, -3, 0$; diagonalizable

c) $3, 3, 0$; diagonalizable d) $-3, -3, 0$; not diagonalizable

e) $3, 3, 1$; diagonalizable f) $3, 0, 0$; diagonalizable

Answer: $-3, -3, 0$; not diagonalizable.

135. For $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $3, -1, -1$; not diagonalizable b) $1, 1, 1$; not diagonalizable

c) $3, 1, 1$; not diagonalizable d) $3, 1, 1$; diagonalizable

e) $-3, 1, 1$; diagonalizable f) $3, -1, -1$; diagonalizable

Answer: $3, 1, 1$; not diagonalizable.

136. For $A = \begin{bmatrix} -7 & -8 & -8 \\ 4 & 5 & 4 \\ 2 & 2 & 3 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $-1, 1, 1$; not diagonalizable b) ; $1, 1, -1$; diagonalizable

c) $1, 1, -1$; not diagonalizable d) $-1, 1, 1$; diagonalizable

e) $1, 1, 1$; diagonalizable f) $1, -1, -1$; diagonalizable

Answer: $-1, 1, 1$; diagonalizable.

137. For $A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonalizable.

a) $3, 1, 1$; not diagonalizable b) $3, -1, -1$; diagonalizable

c) $-3, 1, 1$; not diagonalizable d) $3, -1, -1$; not diagonalizable

e) $-3, 1, 1$; diagonalizable f) $3, 1, 1$; diagonalizable

Answer: $3, 1, 1$; not diagonalizable.

138. For $A = \begin{bmatrix} 5 & 4 & 4 \\ -2 & -1 & -2 \\ -1 & -1 & 0 \end{bmatrix}$, find the eigenvalues of A and determine whether A is diagonal-

izable.

a) $2, 1, 1;$ not diagonalizable b) $-2, 1, 1;$ diagonalizable

c) $2, 2, -1;$ not diagonalizable d) $2, -1, -1;$ not diagonalizable

e) $2, 1, 1;$ diagonalizable f) $2, -1, -1;$ diagonalizable

Answer: $\underline{2, 1, 1;\ \text{diagonalizable}}$.

139. Find a formula for x_k if $x_0 = 1$, $x_1 = 1$ and $x_{k+2} = 10x_k - 3x_{k+1}$. The formula is:

a) $x_k = \frac{3}{7}5^k + \frac{4}{7}(-2)^k$ b) $x_k = \frac{4}{3}2^k - \frac{1}{3}5^k$ c) $x_k = \frac{1}{7}(-5)^k + \frac{6}{7}2^k$.

Answer: $\underline{\text{c)}}$.

140. Find a formula for x_k if $x_0 = 0$, $x_1 = 1$ and $x_{k+2} = 12x_k + x_{k+1}$. The formula is:

a) $x_k = \frac{1}{7}4^k - \frac{1}{7}(-3)^k$ b) $x_k = \frac{1}{7}3^k - \frac{1}{7}(-4)^k$ c) $x_k = 4^k - 3^k$.

Answer: $\underline{\text{a)}}$.

141. Find a formula for x_k if $x_0 = 1$, $x_1 = 1$ and $x_{k+2} = 28x_k + 3x_{k+1}$. The formula is:

a) $x_k = 2(4^k) - 7^k$ b) $x_k = \frac{5}{11}7^k + \frac{6}{11}(-4)^k$ c) $x_k = \frac{8}{11}4^{k+1} + \frac{3}{11}(-7)^k$.

Answer: $\underline{\text{b)}}$.

Chapter 4: Vector Geometry

1. Given the two lines $x+1=t$, $y-6=-t$, $z+4=3t$ and $x+3=-4t$, $y-6=2t$, $z-7=5t$, an equation of the plane which contains them is:

a) $7x-11y+2z=47$ b) $11x-2y+9z=23$ c) $16x-12y+z=11$

d) $11x+17y+2z=83$ e) $8x+9y-13z=46$ f) $22x+19y+4z=42$

Answer: $\underline{11x+17y+2z=83}$.

2. An equation of the plane parallel to the vector $(1,1,-2)$ and which passes through the points $(1,5,18)$ and $(4,2,-6)$ is:

a) $5x+9y+18z=0$ b) $7x-9y+18z=0$ c) $5x-3y+z=8$

d) $5x-7y-z=8$ e) $x+18y+9z=0$ f) $9x-6y+5z=0$

Answer: $\underline{5x-3y+z=8}$.

3. An equation of the plane containing $(3,-1,4)$, $(-1,5,1)$ and $(0,2,-2)$ is:

a) $4x-9y+36z=18$ b) $9x+5y-2z=14$ c) $7x-8y+5z=6$

d) $8x-11y+18z=24$ e) $3x-2y+z=0$ f) $3x+2y-z=0$

Answer: $\underline{9x+5y-2z=14}$.

4. The point $P(1,-1,-1)$ lies in the plane π. If OP is perpendicular to π, then an equation of π is:

a) $x-y-z=3$ b) $x-y-2z=4$ c) $2x-y+z=5$

d) $y+2x=-3$ e) $x-y+2z=0$ f) $x+y+z=-1$

Answer: $\underline{x-y-z=3}$.

5. An equation for the plane passing through the points $P(2,1,-1)$, $Q(3,2,1)$, and parallel to the X-axis is:

a) $-3x+7y-2z=3$ b) $2x-z=5$ c) $x+y-z=4$

d) $x-y=1$ e) $2y-z=3$ f) $x+y+z=2$

Answer: $\underline{2y-z=3}$.

6. The point $P(2,-1,1)$ lies in the plane π. If OP is perpendicular to π, then an equation of π is:

a) $x-y-z=2$ b) $x-y-2z=1$ c) $2x-y+z=6$

d) $y+2x=3$ e) $x-y+2z=5$ f) $-y-z=0$

Answer: $\underline{2x-y+z=6}$.

7. An equation of the plane which contains the point $P(-1,0,2)$ and the line of intersection of the two planes $3x+2y-z=5$ and $2x+y+2z=1$ is:

a) $20x+12y+9z=14$ b) $23x+12y+19z=15$ c) $26x+12y+29z=16$

d) $29x+12y+39z=17$ e) $32x+12y+49z=18$ f) $35x+12y+59z=19$

Answer: $\underline{23x + 12y + 19z = 15}$.

8. Find the equation of the plane π passing through the points $P(2,3,4)$ and $Q(-1,2,3)$ and parallel to the vector $\mathbf{w} = (3,4,5)$.

a) $x + 2y - z = 9$ b) $3x + 4y - 5z + 10 = 0$ c) $3x + y + z = 8$

d) $x - 2y - z + 8 = 0$ e) $2x - 3y - 5z = 10$ f) $x - 12y + 9z = 2$

Answer: $\underline{x - 12y + 9z = 2}$.

9. An equation of the plane containing the point $P(1,-1,2)$ and the line $x = 4$, $y = -1 + 2t$, $z = 2 + t$ is:

a) $x + y - 2z + 5 = 0$ b) $y - 2z + 5 = 0$ c) $y + 2z + 5 = 0$

d) $y - 2z - 5 = 0$ e) $x + y + 2z - 5 = 0$ f) $y + 2z - 5 = 0$

Answer: $\underline{y - 2z + 5 = 0}$.

10. An equation of the plane spanned by the two vectors $\mathbf{u} = (1,1,-1)$ and $\mathbf{v} = (2,3,5)$ and which passes through the origin is:

a) $7x - 8y + 3z = 0$ b) $x - 7y + 8z = 0$ c) $7x - 3y + z = 0$

d) $8x - 7y + z = 0$ e) $8x + y - 7z = 0$ f) $7x + 8y + z = 0$

Answer: $\underline{8x - 7y + z = 0}$.

11. An equation of the plane passing through the points $P(2,1,-1)$ and $Q(3,2,1)$ and parallel to the Y-axis is:

a) $2x - z - 5 = 0$ b) $2y - z - 5 = 0$ c) $2x - y - 5 = 0$

d) $x + y - z = 4$ e) $2x + z = 5$ f) $2y - z = 5$

Answer: $\underline{2x - z - 5 = 0}$.

12. Find an equation of the plane which passes through the point $Q(1,-7,8)$ and which is perpendicular to the line whose parametric equations are:

(i) $x = 2 + 2t$; t is arbitrary

(ii) $y = 7 - 4t$; t is arbitrary

(iii) $z = -3 + t$; t is arbitrary

a) $2x - 4y + z = -38$ b) $2x - 4y + z - 38 = 0$ c) $2x + 7y - 3z = -71$

d) $2x - 4y + z = -28$ e) $-4x + 2y + z + 10 = 0$ f) $-4x + 2y + z = 10$

Answer: $\underline{2x - 4y + z - 38 = 0}$.

13. An equation of the plane determined by the points $(6,-1,5)$, $(7,2,-4)$ and $(1,1,5)$ is:

a) $12x + 15y + 8z - 46 = 0$ b) $12x + 15y - 8z + 23 = 0$ c) $18x - 45y + 17z + 148 = 0$

d) $18x + 45y + 17z = 148$ e) $6x + 15y + 5z - 46 = 0$ f) $6x + 15y + 5z = -46$

Answer: $\underline{18x + 45y + 17z = 148}$.

14. An equation for the plane passing through the points $(1, 2, 3)$, $(1, 0, -1)$ and $(4, -2, 0)$ is:

a) $x = 1$ b) $5x + 6y - 3z + 8 = 0$ c) $5x + 6y - 3z = 8$

d) $6x - 5y - 3z + 8 = 0$ e) $2x + 2y - z = 3$ f) $3x - 2y - 3z = 3$

Answer: $\underline{5x + 6y - 3z = 8}$.

15. Find an equation of the plane which contains the point $(2, 4, 3)$ and which is perpendicular to the planes $x + 2y - z = 1$ and $3x - 4y = 2$.

a) $4x - 3y + 10z = -50$ b) $4x + 3y - 10z - 50 = 0$ c) $4x - 3y + 10z - 50 = 0$

d) $-4x + 3y + 10z - 50 = 0$ e) $4x + 3y + 10z = -50$ f) $4x + 3y + 10z = 50$

Answer: $\underline{4x + 3y + 10z = 50}$.

16. A linear equation for the set of all points $P = (x, y, z)$ that are at equal distance from the fixed points $A = (2, -1, 4)$ and $B = (1, 5, 2)$ is:

a) $-2x - 5y + 4z = 4$ b) $2x - 5y - 4z = 3$ c) $-3x - 2y - z = \frac{4}{5}$

d) $-x + 6y - 2z = \frac{9}{2}$ e) $-x + 6y - 2z = \frac{-9}{2}$ f) $3x - 2y + z = \frac{4}{5}$

Answer: $\underline{-x + 6y - 2z = \frac{9}{2}}$.

17. The equation $5x - y + 6z + 3 = 0$ is the equation of a:

a) line in space with direction vector $(5, -1, 6)$.

b) plane passing through the points $(9, 0, -8)$, $(1, 1, 1)$ and $(0, 3, 0)$.

c) plane with normal vector $(5, -1, 6)$ and passing through the point $(0, 3, 0)$.

d) plane with normal vector $(-1, 6, 3)$ and passing through the point $(9, 0, -8)$.

e) line in space passing through $(0, 3, 0)$ and $(9, 0, -8)$.

f) two intersecting planes, one of which passes through the point $(0, 3, 0)$ and the other one through $(9, 0, -8)$.

Answer: <u>plane with normal vector $(5, -1, 6)$ and passing through the point $(0, 3, 0)$</u>.

18. Parametric equations for the line containing $(3, -1, 4)$ and $(-1, 5, 1)$ are:

a) such a line does not exist b) $x = 3 + 4t,\ y = 3 + 6t,\ z = 4 + 3t$

c) $x = 1 - t,\ y = -1 - 6t,\ z = 4 - 3t$ d) $x = 3 + 4t,\ y = -1 - 6t,\ z = 6 + t$

e) $x = 3 + 4t,\ y = -1 - 6t,\ z = 4 + 3t$ f) $x = 1 - t,\ y = 3 + 6t,\ z = 6 + t$

Answer: $\underline{x = 3 + 4t,\ y = -1 - 6t,\ z = 4 + 3t}$.

19. Parametric equations of the line containing $(-5, 0, 1)$ and which is parallel to the two planes $2x - 4y + z = 0$ and $x - 3y - 2z = 1$ are:

a) $x = 5 + 11t,\ y = 3t,\ z = 1 + 2t$ b) $x = -5 + 5t,\ y = -5t,\ z = 1 - 10t$

c) $x = -5t,\ y = 0,\ z = t$ d) $x = -5 + 11t,\ y = -3t,\ z = 1 + 2t$

e) $x = 5t,\ y = 0,\ z = t$ f) $x = -5 + 11t,\ y = 5t,\ z = 1 - 2t$

Answer: $\underline{x = -5 + 11t,\ y = 5t,\ z = 1 - 2t}$.

20. Parametric equations for the line passing through $(1, 1, -1)$ and which is perpendicular to the plane $2x - y + 3z = 4$ are:

    a) $x = 1 - 2t$, $y = 1 + t$, $z = -1 + 3t$; t is arbitrary

    b) $x = 1 + 2t$, $y = 1 - t$, $z = -1 + 3t$; t is arbitrary

    c) $x = 1 - t$, $y = 1 + t$, $z = -1 - 6t$; t is arbitrary

    d) $x = 1 - 2t$, $y = 1 - t$, $z = -1 - 3t$; t is arbitrary

    e) $x = 1 + t$, $y = 1 + t$, $z = -1 - 3t$; t is arbitrary

    f) $x = 1 - 4t$, $y = 1 - t$, $z = -1 - 3t$; t is arbitrary

    Answer: $\underline{x = 1 + 2t,\ y = 1 - t,\ z = -1 + 3t,\ t \text{ is arbitrary}}$.

21. Find a vector parallel to the line of intersection of the planes OAB and OCD for the points $A(2, 1, -2)$, $B(0, 1, -1)$, $C(2, -2, 1)$ and $D(8, -4, 3)$.

    a) $(3, 2, 1)$ b) $(0, 1, -1)$ c) $(3, 3, 2)$

    d) $(0, -2, -3)$ e) $(2, -2, 1)$ f) $(1, 3, -2)$

    Answer: $\underline{(2, -2, 1)}$.

22. If $\mathbf{u} = (3, -1, 4)$ and $\mathbf{v} = (-1, 6, -5)$, what is $\mathbf{u} \times \mathbf{v}$?

    a) $(17, -10, 11)$ b) $(-19, 11, 17)$ c) $(-3, -6, -20)$

    d) $(-19, -11, 17)$ e) $(-17, -10, 11)$ f) $(3, -6, 20)$

    Answer: $\underline{(-19, 11, 17)}$.

23. If $\mathbf{u} = (1, 1, -1)$, $\mathbf{v} = (0, 2, -1)$ and $\mathbf{w} = (1, -3, 3)$, find $\|\mathbf{v} \times \mathbf{w}\| (\mathbf{u} \times \mathbf{w})$.

    a) $\sqrt{14}(0, 4, 4)$ b) $\sqrt{14}(0, 4, -4)$ c) $-\sqrt{14}(0, 4, -4)$

    d) $-\sqrt{14}(0, 4, 4)$ e) $2\sqrt{7}(0, 4, 4)$ f) $-2\sqrt{7}(0, 4, -4)$

    Answer: $\underline{-\sqrt{14}(0, 4, 4)}$.

24. Let $\mathbf{u} = (4, -1, 7)$, $\mathbf{v} = (2, 1, 2)$, $\mathbf{w} = (-1, -2, 3)$. Then $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ is:

    a) $(-6, 21, 12)$ b) $(-6, -21, 12)$ c) $(30, -21, 24)$

    d) $(30, 21, 24)$ e) $(-30, -21, 24)$ f) $(-30, -21, -24)$

    Answer: $\underline{(30, 21, 24)}$.

25. Let $\mathbf{u} = (-4, 2, 7)$, $\mathbf{v} = (2, 1, 2)$, $\mathbf{w} = (1, 2, 3)$. Then $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ equals:

    a) 15 b) -15 c) 16 d) -16 e) 17 f) -17

    Answer: $\underline{17}$.

26. If $\mathbf{u} = (1, 1, -1)$, $\mathbf{v} = (0, 2, 1)$ and $\mathbf{w} = (1, -3, 3)$, then $\|\mathbf{v} \times \mathbf{w}\| (\mathbf{u} \times \mathbf{w})$ is:

    a) $\sqrt{14}(0, 4, 4)$ b) $\sqrt{86}(0, 4, -4)$ c) $-\sqrt{14}(0, 6, 6)$

    d) $-\sqrt{86}(0, 4, 4)$ e) $\sqrt{14}(0, 6, 6)$ f) $-\sqrt{86}(0, 4, -4)$

    Answer: $\underline{-\sqrt{86}(0, 4, 4)}$.

27. Given vectors $\mathbf{u} = (-1, 2, -2)$, $\mathbf{v} = (4, 8, -1)$ and $\mathbf{w} = (-1, 0, 6)$, find $\beta = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$, $\eta = \mathbf{v} \times (\mathbf{w} \times \mathbf{u})$ and $\xi = \|\mathbf{u}\|\, \mathbf{v} + \|\mathbf{v}\|\, \mathbf{u}$.

   a) $\beta = 110$, $\eta = (-12, 10, 32)$, $\xi = (3, -21, 21)$.

   b) $\beta = 9$, $\eta = (-24, 20, -64)$, $\xi = (3, -42, -21)$.

   c) $\beta = -110$, $\eta = (-24, 20, 64)$, $\xi = (3, 42, -21)$.

   d) $\beta = -9$, $\eta = (24, 20, 64)$, $\xi = (-3, 42, 21)$.

   e) $\beta = 110$, $\eta = (-24, -20, -64)$, $\xi = (-3, -42, -21)$.

   f) $\beta = -18$, $\eta = (12, 10, -42)$, $\xi = (3, -21, 21)$.

   Answer: $\underline{\beta = -110,\ \eta = (-24, 20, 64),\ \xi = (3, 42, -21)}$.

28. Given $\mathbf{u} = (3, 0, 3)$, $\mathbf{v} = (-5, 1, -8)$, $\mathbf{w} = (0, 3, 4)$, then $\|(2\mathbf{u} + \mathbf{v}) \times \mathbf{w}\|$ equals:

   a) $2\sqrt{5}$ b) $5\sqrt{5}$ c) $5\sqrt{2}$ d) $10\sqrt{5}$ e) 25 f) $2\sqrt{10}$

   Answer: $\underline{5\sqrt{5}}$.

29. Let $\mathbf{u} = (3, 1, 2)$, $\mathbf{v} = (-5, 2, -7)$ and $\mathbf{w} = (0, 3, 4)$. Then $\|(2\mathbf{u} + \mathbf{v}) \times \mathbf{w}\|$ equals:

   a) 0 b) $2\sqrt{322}$ c) 322 d) 644 e) $\sqrt{645}$ f) $5\sqrt{26}$

   Answer: $\underline{5\sqrt{26}}$.

30. Find all vectors in $\mathbb{R}^3$ which are orthogonal to both $(-1, 1, 5)$ and $(2, 1, 2)$.

   a) $(2, -8, 2)$ only b) $(t + 1, -8, t + 1)$, where t is arbitrary

   c) $(t, -4t, t)$, where t is arbitrary d) $(-t, 0, t)$, where t is arbitrary

   e) $(0, 0, 0)$ only f) $(3, -12, 3)$ only

   Answer: $\underline{(t, -4, t)\text{, where } t \text{ is arbitrary}}$.

31. Which of the vectors below is orthogonal to both $(2, 1, -1)$ and $(-3, -2, 4)$?

   a) $(1, 0, 1)$ b) $(2, -5, -1)$ c) $(1, -2, 0)$

   d) $(-4, 0, 3)$ e) $(3, 0, 2)$ f) none of the above

   Answer: $\underline{(2, -5, -1)}$.

32. Construct a vector $\mathbf{w}$ which is orthogonal to $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and such that $\|\mathbf{w}\| = \sqrt{3}$.

   a) $\mathbf{w} = \left(\frac{-7}{5}, \frac{7}{5}, -1\right)$ b) $\mathbf{w} = \left(\frac{7}{5}, \frac{1}{5}, 1\right)$ c) $\mathbf{w} = \left(\frac{-1}{5}, \frac{7}{5}, \frac{7}{5}\right)$

   d) $\mathbf{w} = \left(\frac{7}{5}, \frac{-1}{5}, 1\right)$ e) $\mathbf{w} = \left(\frac{7}{5}, \frac{-1}{5}, -1\right)$ f) $\mathbf{w} = (1, 5, 7)$

   Answer: $\underline{\mathbf{w} = \left(\frac{7}{5}, \frac{1}{5}, 1\right)}$.

33. Let $\mathbf{u} = (3, 4)$ and suppose $\mathbf{v} = (a, b)$ is a unit vector orthogonal to $\mathbf{u}$. Then $|b|$ equals:

   a) 3 b) 4 c) $\frac{3}{5}$ d) $\frac{4}{5}$ e) $\frac{3}{4}$ f) $\frac{4}{3}$

   Answer: $\underline{\frac{3}{5}}$.

34. For what values of k are $(k, k, 1)$ and $(k, 5, 6)$ orthogonal?

   a) 1 b) 3 or -2 c) -1 or -3 d) -3 or -2 e) 3 f) 2 or 3

   Answer: $\underline{-3 \text{ or } -2}$.

35. For what value of k are the two planes $3kx + y - 5kz + 10 = 0$ and $2x - 3y + z + 12 = 0$ orthogonal?

    a) 1 b) -1 c) 2 d) -2 e) 3 f) -3

    Answer: $\underline{3}$.

36. Find all values of a such that $(a, 1, 1) \times (2, a, 1) = (a^2 - 1, 3a + 4, -1)$.

    a) 1 or -1 b) -1
    c) 1 d) all $a \neq -1$ and 1
    e) No value of a satisfies the equation f) All values of a satisfy the equation

    Answer: No value of a satisfies the equation.

37. Find a normal to the plane which is parallel to the vectors $(3, 2, 7)$ and $(4, 1, 8)$ and which contains the origin.

    a) $(4, 4, -6)$ b) $(5, 4, -1)$ c) $(6, 4, -2)$
    d) $(7, 4, -3)$ e) $(8, 4, -4)$ f) $(9, 4, -5)$

    Answer: $\underline{(9, 4, -5)}$.

38. A normal vector to the plane containing the points $(-7, 10, 0)$, $(4, 1, 6)$ and $(2, -1, 3)$ is:

    a) $(39, 21, -40)$ b) $(-39, 21, 40)$ c) $(12, 33, 22)$
    d) $(-12, 33, 22)$ e) $(39, -21, 40)$ f) $(12, 21, 22)$

    Answer: $\underline{(39, 21, -40)}$.

39. Which of the following is a normal vector to the plane containing the points $(-7, 1, 0)$, $(2, -1, 3)$ and $(4, 1, 6)$?

    a) $(12, 87, 22)$ b) $(-12, -21, 22)$ c) $(8, 1, 4)$
    d) $(1, 2, 3)$ e) $(0, 5, 8)$ f) $(0, 1, 0)$

    Answer: $\underline{(-12, -21, 22)}$.

40. If $\mathbf{u} = (1, 1, -1)$, $\mathbf{v} = (0, 2, -1)$, $\mathbf{w} = (1, -3, 3)$, then the cosine of the angle between $(\mathbf{v} \times \mathbf{w})$ and $(\mathbf{u} \times \mathbf{v})$ is:

    a) $\frac{2}{21}$ b) $\frac{-1}{21}$ c) $\frac{\sqrt{2}}{\sqrt{21}}$ d) $\frac{-1}{\sqrt{7}}$ e) $\frac{-1}{\sqrt{21}}$ f) $\frac{2}{\sqrt{7}}$

    Answer: $\underline{\frac{-1}{\sqrt{21}}}$.

41. The angle between $(-2, 4, 1)$ and $(1, 2, 3)$ is:

    a) $\arcsin\left(\frac{9}{7\sqrt{6}}\right)$ b) $\arccos\left(\frac{4}{126}\right)$ c) $\arccos\left(\frac{9}{21\sqrt{14}}\right)$
    d) $\arccos\left(\frac{9}{7\sqrt{6}}\right)$ e) $\frac{\pi}{2}$ f) $\arctan\left(\frac{9}{21\sqrt{14}}\right)$

    Answer: $\underline{\arccos\left(\frac{9}{7\sqrt{6}}\right)}$.

42. A triangle has vertices $A(1,1,1)$, $B(2,3,1)$ and $C(1,2,3)$. Find the cosine of the interior angle at A.

a) 0 b) $\frac{1}{5}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$ e) $\frac{4}{5}$ f) 1

Answer: $\underline{\frac{2}{5}}$.

43. Given the points $A(2,4,1)$, $B(3,0,9)$ and $C(1,4,0)$, find the angle BAC.

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$ e) $\frac{3\pi}{4}$ f) $\frac{4\pi}{3}$

Answer: $\underline{\frac{3\pi}{4}}$.

44. Given the points $A(2,4,1)$, $B(3,0,9)$, $C(1,4,0)$ and $D(2,6,2)$, find the cosine of the angle between the planes ABC and ABD.

a) $\frac{1}{\sqrt{5}}$ b) $\frac{-2}{\sqrt{5}}$ c) $\frac{9}{\sqrt{5}}$ d) $\frac{-1}{3\sqrt{5}}$ e) $\frac{-81}{\sqrt{5}}$ f) $\frac{-1}{\sqrt{5}}$

Answer: $\underline{\frac{1}{\sqrt{5}}}$.

45. Given the points $A(-1,5,0)$, $B(1,0,4)$, $C(1,4,0)$, find the cosine of the angle BAC.

a) $\frac{1}{5}$ b) $\frac{4}{5}$ c) $\frac{2}{5}$ d) $\frac{6}{5}$ e) $\frac{3}{5}$ f) $\frac{8}{5}$

Answer: $\underline{\frac{3}{5}}$.

46. Given the points $A(-1,5,0)$, $B(1,0,4)$, $C(1,4,0)$ and $D(0,6,0)$, find the angle between the line AD and the plane ABC.

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{5}$ e) $\frac{\pi}{6}$ f) $\frac{\pi}{8}$

Answer: $\underline{\frac{\pi}{4}}$.

47. Given the points $A(-1,5,0)$, $B(1,0,4)$, $C(1,4,0)$ and $D(0,6,0)$, find the cosine of the angle between the planes ABC and ABD.

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{3}{5}$ e) $\frac{4}{5}$ f) $\frac{1}{6}$

Answer: $\underline{\frac{2}{3}}$.

48. A vector which is parallel to the plane $x+4y+z=5$ and whose angles with $(4,1,1)$ and $(2,2,8)$ are equal, is:

a) $(3,2,3)$ b) $4,2,4)$ c) $2,-1,2)$

d) $(1,-2,1)$ e) $(0,2,0)$ f) $(1,3,1)$

Answer: $\underline{(2,-1,2)}$.

49. Given the points $A(2,4,1)$, $B(3,0,9)$, $C(1,4,0)$ and $D(2,6,2)$, find the cosine of the angle between the line AD and the plane ABC.

a) $\frac{1}{\sqrt{5}}$ b) $\frac{8}{\sqrt{5}}$ c) $\frac{6}{\sqrt{5}}$ d) $\frac{3}{\sqrt{5}}$ e) $\frac{4}{\sqrt{5}}$ f) $\frac{2}{\sqrt{5}}$

Answer: $\underline{\frac{2}{\sqrt{5}}}$.

50. Find the angle between the vectors $\mathbf{u} = (0,3,4)$ and $\mathbf{v} = (5\sqrt{2}, -7, -1)$.

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{4}$ e) $\frac{5\pi}{6}$ f) π

Answer: $\underline{\frac{2\pi}{3}}$.

51. The cosine of the angle between the planes $x + y + z = 1$ and $2x - y - z = 3$ is:

a) 1 b) $\frac{\sqrt{2}}{2}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) $\frac{\pi}{2}$ f) 0

Answer: $\underline{0}$.

52. The angle between $(0, 3, -3)$ and $(-2, 2, -1)$ is:

a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) $\frac{\pi}{5}$ f) $\frac{\pi}{7}$

Answer: $\underline{\frac{\pi}{4}}$.

53. Find the angle between $(2, -1, 1)$ and $(1, -17, 2)$.

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π e) $\frac{\pi}{2}$ f) $\frac{\pi}{5}$

Answer: $\underline{\frac{\pi}{3}}$.

54. Find the cosine of the angle between $(8, 3, 1)$ and $(2, 5, -1)$.

a) $\frac{15}{74}$ b) $\frac{30}{37}$ c) $\frac{30}{74}$ d) $\sqrt{\frac{30}{74}}$ e) $\frac{30}{\sqrt{74}}$ f) $\frac{\sqrt{30}}{74}$

Answer: $\underline{\sqrt{\frac{30}{74}}}$.

55. Find the cosine of the angle between $(4, -3, -1)$ and $(0, 3, -2)$.

a) $\frac{-7}{13\sqrt{2}}$ b) $\frac{-11}{13\sqrt{2}}$ c) $\frac{-7}{\sqrt{26}}$ d) $\frac{-11}{\sqrt{26}}$ e) $\frac{-7}{26\sqrt{2}}$ f) $\frac{-11}{26\sqrt{2}}$

Answer: $\underline{\frac{-7}{13\sqrt{2}}}$.

56. The angle between the vectors $(-3, 2\sqrt{3}, -\sqrt{3})$ and $(-3, 0, -\sqrt{3})$ is:

a) $\frac{\pi}{12}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) $\frac{5\pi}{12}$ f) $\frac{\pi}{2}$

Answer: $\underline{\frac{\pi}{4}}$.

57. Find the cosine of the angle between a diagonal of a cube and one of its faces.

a) $\frac{-1}{2}$ b) $\frac{\sqrt{2}}{2}$ c) $\sqrt{\frac{2}{3}}$ d) 0 e) $\frac{\sqrt{3}}{3}$ f) $\frac{1}{2}$

Answer: $\underline{\frac{\sqrt{3}}{3}}$.

58. The angle between the planes $x - z = 7$ and $y - z = 234$ is:

a) 0 b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) $\frac{\pi}{2}$ f) none of the above

Answer: $\underline{\frac{\pi}{3}}$.

59. Find the cosine of the angle between the planes $x - y + z = 2$ and $2x - 3y + 4z = 7$.

a) $\frac{3}{29}$ b) $\frac{3}{\sqrt{29}}$ c) $\frac{9}{\sqrt{87}}$ d) $\frac{9}{\sqrt{29}}$ e) $9\sqrt{3}$ f) $\frac{3}{\sqrt{87}}$

Answer: $\underline{\frac{9}{\sqrt{87}}}$.

60. The cosine of the angle between the planes $-x + y + z = 3$ and $-4x + 4y - 7z = 5$ is:

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{-1}{\sqrt{3}}$ d) $\frac{-1}{3}$ e) $\frac{1}{9\sqrt{3}}$ f) $\frac{1}{3\sqrt{3}}$

Answer: $\underline{\frac{1}{9\sqrt{3}}}$.

61. If $\mathbf{u} = (-2, 1, 1)$ and $\mathbf{v} = (1, 0, 1)$, then $\|\text{proj}_{\mathbf{v}}\mathbf{u}\|$ is:

    a) $\frac{\sqrt{6}}{6}$ b) 1 c) $\frac{\sqrt{2}}{2}$ d) 0 e) $\frac{1}{2}$ f) $\frac{1}{6}$

    Answer: $\underline{\frac{\sqrt{2}}{2}}$.

62. If $\mathbf{u} = (3, 3, 3)$ and $\mathbf{v} = (2, 1, 3)$, then $\text{proj}_{\mathbf{v}}\mathbf{u} = ?$

    a) $\frac{-9}{14}(2, 1, 3)$ b) $\frac{17}{14}(2, 1, 3)$ c) $\frac{6}{7}(2, 1, 3)$ d) $\frac{9}{7}(2, 1, 3)$ e) $\frac{11}{7}(2, 1, 3)$ f) $\frac{24}{14}(2, 1, 3)$

    Answer: $\underline{\frac{9}{7}(2, 1, 3)}$.

63. If $\mathbf{u} = (1, -3)$ and $\mathbf{v} = (4, 1)$, the projection of $\mathbf{u}$ on $\mathbf{v}$ is:

    a) $(\frac{4}{17}, \frac{1}{17})$

    b) $(\frac{1}{5}, -\frac{3}{5})$

    c) $(\frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}})$

    d) $(\frac{1}{17}, \frac{-3}{17})$

    e) $(\frac{4}{5}, \frac{1}{5})$

    f) none of the above

    Answer: $\underline{(\frac{4}{17}, \frac{1}{17})}$.

64. If $\mathbf{u} = (3, 3, 6)$ and $\mathbf{v} = (2, -1, 1)$, then the length of the projection of $\mathbf{u}$ along $\mathbf{v}$ is:

    a) $\frac{3\sqrt{6}}{2}$ b) $\frac{3\sqrt{2}}{2}$ c) 0 d) $\frac{\sqrt{6}}{2}$ e) $\frac{2\sqrt{6}}{3}$ f) $\frac{2\sqrt{2}}{3}$

    Answer: $\underline{\frac{3\sqrt{6}}{2}}$.

65. If $\mathbf{u} = (2, -1, 1)$ and $\mathbf{v} = (3, 3, 6)$, then the length of the projection of $\mathbf{u}$ along $\mathbf{v}$ is:

    a) $\frac{\sqrt{6}}{2}$ b) $\frac{3\sqrt{2}}{2}$ c) 0 d) $\frac{3\sqrt{6}}{2}$ e) $\frac{2\sqrt{6}}{3}$ f) 3

    Answer: $\underline{\frac{\sqrt{6}}{2}}$.

66. Given $\mathbf{u} = (3, 0, 3)$ and $\mathbf{v} = (-5, 1, 8)$, the orthogonal projection of $\mathbf{v}$ along $\mathbf{u}$ is:

    a) $\left(1, \frac{-1}{5}, \frac{-8}{5}\right)$ b) $\left(-1, \frac{1}{5}, \frac{8}{5}\right)$ c) $\left(\frac{-3}{2}, 0, \frac{-3}{2}\right)$

    d) $\left(\frac{3}{2}, 0, \frac{3}{2}\right)$ e) $(5, -1, -8)$ f) $(-5, 1 - 8)$

    Answer: $\underline{\left(\frac{3}{2}, 0, \frac{3}{2}\right)}$.

67. The volume of the parallelepiped with edges given by the vectors $\mathbf{u} = (1, 1, 1)$, $\mathbf{v} = (0, 2, 1)$ and $\mathbf{w} = (1, 1, 5)$ is:

    a) 4 b) $\sqrt{2}$ c) $\frac{1}{\sqrt{2}}$ d) $2\sqrt{2}$ e) 8 f) $8\sqrt{2}$

    Answer: $\underline{8}$.

68. The volume of the parallelepiped with a vertex at the origin and edges given by the vectors $\mathbf{u} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = 2\mathbf{j} + 5\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{k}$ is:

    a) 3 b) 7 c) 9 d) 10 e) 11 f) 14

    Answer: $\underline{3}$.

69. Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (1, 1, -1)$, $\mathbf{v} = (2, 0, 1)$ and $\mathbf{w} = (1, -1, 3)$.

    a) -2 b) 4 c) 6 d) 8 e) 16 f) 2

    Answer: $\underline{2}$.

70. The volume of a parallelepiped with a vertex at the origin and edges $\mathbf{u} = (1, 1, 0)$, $\mathbf{v} = (1, 0, -1)$, and $\mathbf{w} = (1, 1, 1)$

    a) -1 b) 1 c) -2 d) 2 e) 3 f) $\sqrt{3}$

    Answer: $\underline{1}$.

71. The volume of the parallelepiped with a vertex at the origin and edges $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{w} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ is:

    a) 5 b) 10 c) 15 d) 20 e) 25 f) 30

    Answer: $\underline{10}$.

72. The volume of the parallelepiped determined by vectors $\mathbf{u} = \mathbf{i} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{w} = 2\mathbf{j} - 5\mathbf{k}$ is:

    a) 2 b) 3 c) 4 d) 5 e) 6 f) 7

    Answer: $\underline{3}$.

73. Given the points $A(-1, 5, 0)$, $B(1, 0, 4)$, $C(1, 4, 0)$ and $D(0, 6, 0)$, find the volume of the tetrahedron $ABCD$.

    a) $\frac{1}{2}$ b) $\frac{8}{3}$ c) $\frac{3}{2}$ d) 2 e) $\frac{5}{2}$ f) $\frac{4}{3}$

    Answer: $\underline{2}$.

74. Consider the points $A(1, 2, 3)$, $B(1, 3, 2)$ and $C(2, 1, 3)$. Find a point D on the Z-axis so that the volume of the tetrahedron $ABCD$ is 3.

    a) $(0, 0, 2)$ b) $(0, 0, 18)$ c) $(0, 0, 6)$
    d) $(0, 0, 12)$ e) $(0, 0, -12)$ f) $(0, 0, -18)$

    Answer: $\underline{(0, 0, -12)}$.

75. The volume of the pyramid with vertices $(0, 0, 0)$, $(-2, 8, 14)$, $(-6, 7, -3)$ and $(4, 0, 2)$ is:

    a) 35 b) 45 c) 60 d) 70 e) 75 f) 85

    Answer: $\underline{70}$.

76. Find the area of the triangle with vertices $A(-1, 5, 0)$, $B(1, 0, 4)$ and $C(1, 4, 0)$.

    a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

    Answer: $\underline{6}$.

77. If we project the triangle with vertices $A(-1, 5, 0)$, $B(1, 0, 4)$ and $C(1, 4, 0)$ onto the xy-plane, find the resulting area.

    a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

    Answer: $\underline{4}$.

78. Find the area of the triangle whose vertices are $P(1,1,-1)$, $Q(2,0,1)$ and $R(1,-1,3)$.

a) 10 b) $2\sqrt{5}$ c) $\sqrt{5}$ d) 0 e) $4\sqrt{5}$ f) 5

Answer: $\underline{\sqrt{5}}$.

79. Find the area of the triangle whose vertices are the points $P(3,-1,2)$, $Q(1,1,0)$ and $R(1,2,-1)$.

a) 4 b) $2\sqrt{2}$ c) $\sqrt{2}$ d) 0 e) $4\sqrt{2}$ f) 2

Answer: $\underline{\sqrt{2}}$.

80. Find the area of the triangle whose vertices are the points $P(-3,1,2)$, $Q(1,-1,0)$ and $R(1,-2,1)$.

a) $4\sqrt{3}$ b) $2\sqrt{3}$ c) 6 d) 12 e) 24 f) $8\sqrt{3}$

Answer: $\underline{2\sqrt{3}}$.

81. The area of the triangle PQR where the points P, Q, R are $P=(1,1,1)$, $Q=(2,3,-3)$ and $R=(2,1,5)$ is:

a) $4\sqrt{2}$ b) $\sqrt{33}$ c) $3\sqrt{7}$ d) 8 e) 9 f) 10

Answer: $\underline{\sqrt{33}}$.

82. What is area of the triangle with vertices $(3,0,-1)$, $(5,2,-1)$ and $(5,9,0)$?

a) $\sqrt{17}$ b) 17 c) $\sqrt{51}$ d) $\frac{13}{2}$ e) $\frac{51}{2}$ f) $\frac{17}{2}$

Answer: $\underline{\sqrt{51}}$.

83. The area of the triangle with vertices $(-5,3,1)$, $(2,3,1)$ and $(1,-2,1)$ is:

a) 35 b) $\frac{17}{2}$ c) $\frac{35}{2}$ d) $\frac{25}{2}$ e) $\frac{7}{2}$ f) 15

Answer: $\underline{\frac{35}{2}}$.

84. The area of a parallelogram determined by the vectors $\mathbf{u}=(1,-1,0)$ and $\mathbf{v}=(2,-3,1)$ is:

a) $\sqrt{3}$ b) 3 c) -3 d) $3\sqrt{3}$ e) 27 f) 9

Answer: $\underline{\sqrt{3}}$.

85. The area of a parallelogram whose vertices are at points $A(2,1,-2)$, $B(1,1,0)$ and $C(-5,7,11)$ is:

a) $\frac{\sqrt{181}}{2}$ b) $\sqrt{181}$ c) 181 d) $\frac{181}{2}$ e) $\frac{181}{4}$ f) $\frac{\sqrt{181}}{4}$

Answer: $\underline{\sqrt{181}}$.

86. If $\mathbf{u}=2\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{v}=-\mathbf{i}+3\mathbf{j}$, $\mathbf{w}=\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{x}=\mathbf{j}-2\mathbf{k}$, which vectors are orthogonal to each other?

a) **u** and **v** b) **u** and **w** c) **u** and **x**

d) **v** and **w** e) **v** and **x** f) **w** and **x**

Answer: <u>**w** and **x**</u>.

87. Let $\mathbf{u}=(1,1,1)$, $\mathbf{v}=(0,1,1)$ and $\mathbf{w}=(1,0,1)$. Find the length of $\mathbf{x}=(3\mathbf{u}+\mathbf{v})\times\mathbf{w}$.

a) $\sqrt{13}$ b) $\sqrt{23}$ c) $\sqrt{33}$ d) $\sqrt{43}$ e) $\sqrt{53}$ f) $3\sqrt{7}$

Answer: $\underline{\sqrt{33}}$.

88. Let $\mathbf{u} = (1, -3, 2)$, $\mathbf{v} = (1, 1, 0)$, and $\mathbf{w} = (2, 2, -6)$. Compute $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$.

a) 0 b) 8 c) 12 d) $12\sqrt{2}$ e) 16 f) 24

Answer: $\underline{12}$.

89. Let $\mathbf{u} = (1, -3, 2)$, $\mathbf{v} = (1, 1, 0)$ and $\mathbf{w} = (2, 2, 4)$. Find $\|3\mathbf{u} - 4\mathbf{v} + \mathbf{w}\|$.

a) $\sqrt{142}$ b) $9\sqrt{2}$ c) $\sqrt{182}$ d) $\sqrt{202}$ e) $\sqrt{222}$ f) $11\sqrt{2}$

Answer: $\underline{\sqrt{222}}$.

90. If $\|\mathbf{u} - \mathbf{v}\| = 6$, $\|\mathbf{u} + \mathbf{v}\| = 5$, then $\mathbf{u} \cdot \mathbf{v}$ is:

a) $\frac{9}{4}$ b) $\frac{-15}{4}$ c) $\frac{3}{4}$ d) $\frac{-9}{4}$ e) $\frac{13}{4}$ f) $\frac{-11}{4}$

Answer: $\underline{\frac{-11}{4}}$.

91. Let $\|\mathbf{u}\| = \|\mathbf{v}\| = \sqrt{2}$ and $\mathbf{u} \cdot \mathbf{v} = 1$. Then $\|\mathbf{u} \times \mathbf{v}\|^2$ is:

a) 0 b) 1 c) 2 d) 3 e) $\sqrt{2}$ f) $\frac{1}{2}$

Answer: $\underline{3}$.

92. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$ such that $\|\mathbf{u}\| = 5$, $\|\mathbf{v}\| = 2$ and $\mathbf{u} \cdot \mathbf{v} = -8$. Then $\|\mathbf{u} \times \mathbf{v}\|$ is:

a) 5 b) 60 c) 4 d) 6 e) 12 f) 16

Answer: $\underline{6}$.

93. Let $\mathbf{u}$, $\mathbf{v}$ be vectors in $\mathbb{R}^3$ such that $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 4$, and $\mathbf{u} \cdot \mathbf{v} = -2$. Then $\|\mathbf{u} \times \mathbf{v}\|$ is:

a) 2 b) 4 c) $2\sqrt{15}$ d) $4\sqrt{15}$ e) $8\sqrt{15}$ f) 60

Answer: $\underline{2\sqrt{15}}$.

94. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in $\mathbb{R}^3$ such that $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 1$ and $\mathbf{u} \cdot \mathbf{v} = \sqrt{3}$. Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

a) $\frac{\pi}{12}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$ e) $\frac{\pi}{2}$ f) $\frac{11\pi}{12}$

Answer: $\underline{\frac{\pi}{6}}$.

95. If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$ such that $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 5$, $\|\mathbf{u} \times \mathbf{v}\| = \sqrt{51}$, and $\mathbf{u} \cdot \mathbf{v} = 7$, what is $\|\mathbf{u} + \mathbf{v}\|$?

a) 29 b) $\sqrt{29}$ c) 43 d) $\sqrt{43}$ e) 55 f) $\sqrt{55}$

Answer: $\underline{\sqrt{43}}$.

96. Let $\mathbf{u} = (-1, 1, -1)$, $\mathbf{v} = (1, 7, 5)$, $\mathbf{w} = (3, 1, 2)$. Which of the following are equal to 5 (there may be more than one)?

a) $\|\mathbf{u} + \mathbf{w}\|$ b) $\|\mathbf{u}\| + \|\mathbf{v}\|$ c) $\frac{\|\mathbf{v}\|}{\|\mathbf{u}\|}0$

d) $\left\|\frac{\mathbf{w}}{\|\mathbf{w}\|}\right\|$ e) $\frac{\|\mathbf{u}+\mathbf{w}\|}{\|\mathbf{v}\|}$

Answer: $\underline{\text{a), c)}}$.

97. If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal unit vectors, then $(2\mathbf{u} - 3\mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$ is:

a) -5 b) -1 c) 0

d) 1 e) 5 f) not computable with the given data

Answer: $\underline{-1}$.

98. Compute $\mathbf{i} \cdot (\mathbf{k} \times \mathbf{j})$.

a) -1 b) 0 c) $\mathbf{0}$ d) 1 e) $\mathbf{i}$ f) $-\mathbf{i}$

Answer: $\underline{-1}$.

99. Evaluate $\mathbf{j} \times (\mathbf{i} \times \mathbf{k})$.

a) $-\mathbf{i}$ b) $\mathbf{i}$ c) $\mathbf{0}$ d) $\mathbf{j}$ e) $\mathbf{k}$ f) $-\mathbf{k}$

Answer: $\underline{\mathbf{0}}$.

100. Find $(\mathbf{k} \times \mathbf{i}) \times \mathbf{j}$.

a) $\mathbf{i}$ b) $\mathbf{j}$ c) $\mathbf{k}$ d) 0 e) $-\mathbf{i}$ f) $-\mathbf{j}$

Answer: $\underline{\mathbf{0}}$.

101. Evaluate $(\mathbf{i} \times \mathbf{k}) \times (\mathbf{k} \times \mathbf{j})$.

a) $\mathbf{i}$ b) $-\mathbf{i}$ c) $\mathbf{j}$ d) $-\mathbf{j}$ e) $\mathbf{k}$ f) $-\mathbf{k}$

Answer: $\underline{-\mathbf{k}}$.

102. Find $\mathbf{j} \times (\mathbf{i} \times \mathbf{k})$.

a) $\mathbf{i}$ b) $\mathbf{j}$ c) $\mathbf{k}$ d) $-\mathbf{i}$ e) $-\mathbf{j}$ f) $\mathbf{0}$

Answer: $\underline{\mathbf{0}}$.

103. Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in $\mathbb{R}^3$. Which two of the following statements are <u>false</u>?

(i) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

(ii) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$

(iii) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{v} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{u}$

(iv) $(\mathbf{u} + 2\mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$

(v) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

a) (ii) and (iii) b) (i) and (iv) c) (iv) and (v)

d) (ii) and (v) e) (iii) and (iv) f) (i) and (iii)

Answer: <u>(ii) and (v)</u>.

104. Let $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3)$ and $\mathbf{w} = (w_1, w_2, w_3)$. How many of the following statements are <u>false</u>?

(i) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$

(ii) $(\mathbf{v} \times \mathbf{u}) \cdot \mathbf{v} = -1$

(iii) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$

(iv) $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \, \|\mathbf{v}\| \cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$

(v) $\|\mathbf{u} \cdot \mathbf{v}\| = \|\mathbf{u}\| \, \|\mathbf{v}\| \cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{3}$.

105. For vectors in $\mathbb{R}^3$, is $(\mathbf{u}+\mathbf{v}) \times \mathbf{u}$ perpendicular to $\mathbf{u}-\mathbf{v}$?

a) Yes, always.

b) No, never.

c) Sometimes yes, sometimes, no.

d) Cannot be determined.

e) Question makes no sense.

Answer: Sometimes yes, sometimes, no.

106. For vectors $\mathbf{u}$ and $\mathbf{v}$, which of the following are true? (select all correct responses).

a) If $\mathbf{u}$ is not perpendicular (orthogonal) to $\mathbf{v}$, and the projection of $\mathbf{u}$ on $\mathbf{v}$ is the same as the projection of $\mathbf{v}$ on $\mathbf{u}$, then $\mathbf{u}$ and $\mathbf{v}$ are parallel.

b) If the projection of $\mathbf{u}$ on $\mathbf{v}$ is zero then $\mathbf{u}$ is orthogonal to $\mathbf{v}$.

c) $\mathbf{u}-$ proj$_{\mathbf{v}}\mathbf{u}$ is orthogonal to $\mathbf{v}$.

d) $\mathbf{u}$ is the sum of the projection of $\mathbf{u}$ on $\mathbf{v}$ and a vector orthogonal to $\mathbf{v}$.

e) If $\mathbf{v}$ is a unit vector then the projection of $\mathbf{u}$ on $\mathbf{v}$ is $(\mathbf{u}\cdot\mathbf{v})\mathbf{v}$.

f) All of the above.

Answer: f).

107. The vector $\mathbf{v}$ whose initial point is $(-1,-2,-3)$ and whose terminal point is $(3,4,5)$ is:

a) $(-4,-6,-8)$ b) $(2,2,2)$ c) $(-2,-2,-2)$

d) $(4,6,8)$ e) $(0,0,0)$ f) none of the above

Answer: $(4,6,8)$.

108. Let $P(2,3,-2)$ and $Q(8,-3,-8)$ be two points. Then the point on the line segment connecting P and Q that is $\frac{2}{3}$ of the way from P to Q is:

a) $(6,-1,-6)$ b) $(4,-4,-4)$ c) $(-4,4,4)$

d) $(-6,1,6)$ e) $(4,1,-4)$ f) $(-4,-1,4)$

Answer: $(6,-1,-6)$.

109. If $A=(1,2,3)$, $B=(-5,-2,5)$, $C=(-2,8,-10)$ and D is the midpoint of $\overline{AB}$, find the coordinates of the midpoint of $\overline{CD}$.

a) $\left(\frac{-1}{2},1,\frac{-3}{2}\right)$ b) $(-1,-4,-5)$ c) $(-1,0,2)$

d) $(-1,-4,-1)$ e) $(-2,0,4)$ f) $(-2,4,-3)$

Answer: $(-2,4,-3)$.

110. Use vector geometry to locate the point (x,y,z) which is one third of the way from the point $(2,1,3)$ to the point $(-4,4,3)$. The point (x,y,z) is:

a) not uniquely determined

b) $(-2,5,3)$

c) $(4,0,3)$

d) $(\frac{1}{3}, \frac{8}{3}, 2)$

e) none of the above

Answer: $\underline{(4, 0, 3)}$.

111. If $P = (1, 0, 2)$ and $Q = (2, 1, 4)$ are points in $\mathbb{R}^3$, find the point R between P and Q such that $\|PR\| = \frac{3}{4}\|PQ\|$.

a) $\left(\frac{5}{4}, \frac{1}{4}, \frac{5}{2}\right)$ b) $(3, 1, 6)$ c) $\left(\frac{7}{4}, \frac{3}{4}, \frac{7}{2}\right)$

d) $(5, 3, 7)$ e) $(5, 2, 10)$ f) $\left(\frac{5}{2}, 1, \frac{5}{2}\right)$

Answer: $\underline{\left(\frac{7}{4}, \frac{3}{4}, \frac{7}{2}\right)}$.

112. A line is parallel to the vector $\mathbf{u} = (-1, 2, -3)$ and passes through the point $P(0, -5, 2)$. Find the point where this line intersects the XZ-plane.

a) $\left(\frac{-5}{2}, 0, \frac{11}{2}\right)$ b) $\left(\frac{5}{2}, 0, \frac{19}{2}\right)$ c) $\left(\frac{-5}{2}, 0, \frac{-11}{2}\right)$

d) $\left(\frac{5}{2}, 0, \frac{-19}{2}\right)$ e) $\left(\frac{-5}{2}, 0, \frac{19}{2}\right)$ f) $\left(\frac{5}{2}, 0, \frac{11}{2}\right)$

Answer: $\underline{\left(\frac{-5}{2}, 0, \frac{-11}{2}\right)}$.

113. Find the point of intersection of the plane $2x + 2y - z = 5$ and the line with parametric equations $x = 4 - t$, $y = 13 - 6t$, $z = -7 + 4t$.

a) $(-1, 2, 2)$ b) $(2, 1, 0)$ c) $(0, 1, 1)$

d) $(-1, -1, 2)$ e) $(2, 1, 1)$ f) $(-1, 2, 0)$

Answer: $\underline{(2, 1, 1)}$.

114. For what values of p will the line $x = 4 + 2t$, $y = 5t$, $z = 1 + pt$ and the plane $x + 2y + 3z = 8$ have no point in common?

a) 3 b) -12 c) -4 d) 6 e) 4 f) -8

Answer: $\underline{-4}$.

115. The line L passes through $(1, 1, 0)$ and $(2, 3, 1)$. The point of intersection of L with the plane $x + y - z = 1$ is:

a) $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ b) $\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$ c) $(1, 0, 0)$

d) $\left(0, \frac{1}{2}, \frac{-1}{2}\right)$ e) $(0, 1, 0)$ f) $(-1, 0, -1)$

Answer: $\underline{\left(\frac{1}{2}, 0, \frac{-1}{2}\right)}$.

116. The line L through the point $P(2, 1, 0)$ parallel to the vector $\mathbf{u} = (1, -1, 2)$ meets the plane $x + y + 2z = 23$ at the point:

a) $(11, 4, 4)$ b) $(2, 1, 10)$ c) $(7, -4, 10)$

d) $(7, 4, 6)$ e) $(11, 1, 4)$ f) $(10, -4, 7)$

Answer: $\underline{(7, -4, 10)}$.

117. Find the point where the lines $x - 1 = -3t$, $y + 2 = 4t$, $z - 3 = t$ and $x - 5 = 4t$, $y + 3 = -t$, $z - 5 = 2t$ intersect.

a) $(1,-2,3)$ b) $(4,5,6)$ c) $\left(\frac{-4}{7},-1,-2\right)$
d) $(1,2,3)$ e) $(0,6,-3)$ f) the lines do not intersect

Answer: $\underline{(1,-2,3)}$.

118. Where do the lines $x+1=t$, $y-6=-t$, $z+4=3t$ and $x+3=-4t$, $y-6=2t$, $z-7=5t$ intersect?

a) they do not intersect b) $(1,5,8)$ c) $(1,4,2)$
d) $(2,8,2)$ e) $(3,6,1)$ f) $(1,2,6)$

Answer: $\underline{(1,4,2)}$.

119. Consider the line which passes through the point $(5,1,-2)$ and which is perpendicular to the plane $12x-y+3z=4$. The point where this line meets the plane $2x+y+z=19$ is:

a) $\left(\frac{125}{13},\frac{-8}{13},\frac{-11}{13}\right)$ b) $\left(\frac{-125}{13},\frac{8}{13},\frac{11}{13}\right)$ c) $\left(\frac{60}{13},\frac{-8}{13},-2\right)$
d) $\left(\frac{125}{13},\frac{8}{13},\frac{-11}{13}\right)$ e) $\left(\frac{-60}{13},\frac{-8}{13},\frac{-24}{13}\right)$ f) $\left(\frac{-60}{13},\frac{8}{13},\frac{11}{13}\right)$

Answer: $\underline{\left(\frac{125}{13},\frac{8}{13},\frac{-11}{13}\right)}$.

120. The intersection of the planes $2x-3y+4z=6$ and $4x-6y+8z=11$ is described by:

a) the planes are identical

b) $\left.\begin{aligned} x &= 1+2t \\ z &= 2-3t \\ y &= 10+4t \end{aligned}\right\}$, t is arbitrary

c) $\frac{x-2}{1}=\frac{y+3}{2}=\frac{z-4}{10}$

d) $\left.\begin{aligned} x &= 1+4t \\ y &= 2+5t \\ z &= -3-7t \end{aligned}\right\}$, t is arbitrary

e) $\frac{x-4}{1}=\frac{y-5}{2}=\frac{z+7}{-3}$

f) the planes do not intersect

Answer: The planes do not intersect.

121. Find the intersection of the lines $x=1+2s$, $y=2-s$, $z=3-2s$ and $x=3+5t$, $y=3-t$, $z=5-2t$.

a) $(5,7,-3)$ b) $(-5,8,4)$ c) $(3,4,5)$
d) $(1,0,-1)$ e) $\frac{1}{3}(11,-23,17)$ f) $\frac{1}{3}(-11,13,23)$

Answer: $\underline{\frac{1}{3}(-11,13,23)}$.

122. If L_1 is a line passing through the points $(2,1,-3)$ and $(5,4,-1)$ and L_2 is a line passing through the points $(0,2,3)$ and $(-1,-2,-5)$, then find their point of intersection.

a) $(2,-1,5)$ b) $(2,-1,-5)$ c) $(2,1,5)$
d) $(-1,-2,5)$ e) $(-1,-2,-5)$ f) $(1,2,-5)$

Answer: $\underline{(-1,-2,-5)}$.

123. Let L_1 be the line containing the points $(1,-3,-2)$ and $(3,1,4)$. Let L_2 be the line containing the points $(17,4,-8)$ and $(20,5,-9)$. Which of the following points belong to both of L_1 and L_2?

a) $(0,0,0)$ b) $(2,-1,-3)$ c) $(5,5,-6)$

d) $(1,-3,2)$ e) $(1,7,3)$ f) the lines do not intersect.

Answer: The lines do not intersect.

124. What is the point of intersection of the lines

$$\begin{aligned} x+1-4t \\ y-3=t \\ z-1=0 \end{aligned} \quad \text{and} \quad \begin{aligned} x+13=12t \\ y-1=6t \\ z-2=3t \end{aligned} \quad ?$$

a) $(-2,3,1)$ b) $(4,-12,-1)$ c) $(-17,-1,1)$

d) $(13,-5,2)$ e) $(1,-3,-1)$ f) $(13,-1,-2)$

Answer: $\underline{(-17,-1,1)}$.

125. The line passing through the points $(2,5,-2)$ and $(11,-3,22)$ intersects the YZ-plane at the point:

a) $(0,42,-91)$ b) $(0,-41,-91)$ c) $\frac{1}{9}(0,61,-66)$

d) $\frac{1}{9}(0,-66,-92)$ e) $(0,42,92)$ f) $(0,61,66)$

Answer: $\underline{\frac{1}{9}(0,61,-66)}$.

126. Find the x-coordinate of the point of intersection of the line

$$L: \begin{aligned} x &= -1+2t \\ y &= 3+t \\ z &= 2 \end{aligned} \qquad \text{and the plane } 2x+y-3z=2.$$

a) $\frac{7}{5}$ b) $\frac{22}{5}$ c) -2 d) $\frac{9}{5}$ e) $\frac{19}{5}$ f) -3

Answer: $\underline{\frac{9}{5}}$.

127. The planes $5x+7y-4z-8=0$ and $x-y+8=0$ have a line L in common. An equation for L is $(x,y,z)=\,?$

a) $(-4,4,0)+t(1,3,1)$ b) $(4,4,0)+t(1,1,8)$

c) $(-4,4,0)+t(1,1,3)$ d) $(4,-4,0)+t(-1,1,-3)$

e) $(0,4,-4)+t(1,3,-1)$ f) $(4,0,4)+t(-1,3,1)$

Answer: $\underline{(-4,4,0)+t(1,1,3)}$.

128. A beam of light is emitted from the point $(0,7,1)$, is reflected by the plane $x-2y+z-5=0$, and finally arrives at the point $(-8,2,-7)$. Find the point where the beam strikes the plane.

a) $(0,2,1)$ b) $(0,2,-1)$ c) $(0,-2,1)$

d) $(0,0,0)$ e) $(2,2,2)$ f) $(-1,-1,2)$

Answer: $\underline{(0,-2,1)}$.

129. The line $(x,y,z)=(1+11t,2+5t,13t)$ intersects the plane $x-2y-2z=22$ at the point P. If the distance from the origin to P is d, then:

a) $d<9$ b) $9\le d<11$ c) $11\le d\le 13$
d) $13<d\le 16$ e) $16<d\le 19$ f) $19<d$

Answer: $\underline{16<d\le 19}$.

130. Find the distance of the point $P(0,-5,2)$ from the plane $2x+3y+5z=2$.

a) $\frac{7}{\sqrt{38}}$ b) $\frac{-7}{6}$ c) $\frac{7}{\sqrt{34}}$ d) $\frac{-7}{4\sqrt{2}}$ e) $\frac{7}{\sqrt{30}}$ f) $\frac{-\sqrt{7}}{2}$

Answer: $\underline{\frac{7}{\sqrt{38}}}$.

131. Find the distance from the point $(2,6,2)$ to the plane containing the points $(2,4,1)$, $(3,0,9)$ and $(1,4,0)$.

a) 1 b) 3 c) 6 d) 8 e) 2 f) 5

Answer: $\underline{2}$.

132. Find the distance from the point $(8,6,11)$ to the line containing the two points $(0,1,3)$ and $(3,5,4)$.

a) 1 b) 3 c) 5 d) 7 e) 9 f) 11

Answer: $\underline{7}$.

133. Find the distance from the point $(5,4,7)$ to the line containing the two points $(3,-1,2)$ and $(3,1,1)$.

a) 11 b) 9 c) 7 d) 5 e) 3 f) 1

Answer: $\underline{7}$.

134. Which of the points $(1,0,1)$ and $(1,1,1)$ is closest to the point $(-1,2,-2)$, and what is this minimal distance?

a) $(1,0,1)$; $\sqrt{17}$ b) $(1,0,1)$; $\sqrt{14}$ c) $(1,1,1)$; $\sqrt{14}$
d) $(1,1,1)$; $\sqrt{17}$ e) $(1,0,1)$; 17 f) $(1,1,1)$; 14

Answer: $\underline{(1,1,1);\ \sqrt{14}}$.

135. How far is the point $P(-5,0,2)$ from the plane $x-y=5$?

a) $5\sqrt{2}$ b) $10\sqrt{2}$ c) 10 d) 2 e) 5 f) $2\sqrt{5}$

Answer: $\underline{5\sqrt{2}}$.

136. How far is the point $P(5,-2,0)$ from the plane $y-z=5$?

a) 7 b) $7\sqrt{2}$ c) $\sqrt{7}$ d) 14 e) $\frac{7\sqrt{2}}{2}$ f) $\frac{\sqrt{7}}{2}$

Answer: $\underline{\frac{7\sqrt{2}}{2}}$.

137. The distance between the two planes $3x-2y+z=1$ and $6x-4y+2z=7$ is:

a) 0 b) $\frac{5}{2\sqrt{14}}$ c) $\frac{7}{2\sqrt{14}}$ d) $\frac{9}{2\sqrt{14}}$ e) 1 f) $2\sqrt{14}$

Answer: $\underline{\frac{5}{2\sqrt{14}}}$.

138. The distance of the line $x = -1 - 2t$, $y = 2 + 5t$, $z = -3 + 3t$ from the plane $3x + 2z = 5$ is:

a) $\frac{14}{\sqrt{38}}$ b) $\frac{9}{\sqrt{38}}$ c) $\frac{9}{\sqrt{13}}$ d) $\frac{4}{\sqrt{13}}$ e) $\frac{14}{\sqrt{13}}$ f) $\frac{4}{\sqrt{38}}$

Answer: $\underline{\frac{14}{\sqrt{13}}}$.

139. A line is parallel to the planes $3x - 2y + z = 5$ and $6x + z + 1 = 0$, and passes through the point $P(-3, 1, -1)$. How far is this line from the plane $3x - 2y + z = 5$?

a) $\frac{17}{14}$ b) $\frac{17}{7\sqrt{2}}$ c) $\frac{17}{7}$ d) $\frac{7}{\sqrt{14}}$ e) $\frac{17}{\sqrt{14}}$ f) $\frac{7}{7\sqrt{2}}$

Answer: $\underline{\frac{17}{\sqrt{14}}}$.

140. The distance d from the point $P(-2, 5, 9)$ to the plane $6x + 2y - 3z + 8 = 0$ is:

a) 2 b) 3 c) 4 d) 6 e) 9 f) 10

Answer: $\underline{3}$.

141. The distance from the point $P(4, 0, 1)$ to the plane $2x - y + 8z = 3$ is:

a) $\frac{13}{\sqrt{69}}$ b) $\frac{19}{\sqrt{69}}$ c) $\frac{15}{\sqrt{69}}$ d) 0 e) $\frac{21}{\sqrt{69}}$ f) $\frac{17}{\sqrt{69}}$

Answer: $\underline{\frac{13}{\sqrt{69}}}$.

142. The distance from the point $(2, 2, 3)$ to the plane $3x - y + 2z = 10$ is:

a) $\frac{10}{\sqrt{14}}$ b) $\frac{20}{\sqrt{14}}$ c) 0
d) $\sqrt{14}$ e) $5\sqrt{14}$ f) $15\sqrt{14}$

Answer: $\underline{0}$.

143. Find the distance between the points $(3, -1, 6)$ and $(-2, 3, 5)$.

a) $3\sqrt{2}$ b) $\sqrt{6}$ c) $\sqrt{42}$ d) $\sqrt{10}$ e) $\sqrt{30}$ f) $\sqrt{39}$

Answer: $\underline{\sqrt{42}}$.

144. A wandering point wishes to travel from the line A, defined by $x = 20 + 4t$, $y = 3 + 2t$, $z = t$, to the line B, defined by $x = t$, $y = 27 - t$, $z = 3 + t$, along the shortest distance. But the course it takes must be parallel to the plane $2y - z = 6$. Along which vector should it travel?

a) $(3, -2, -1)$ b) $(-4, 5, 6)$ c) $(1, -1, -2)$
d) $(4, 8, 12)$ e) $(0, 0, 1)$ f) $(6, 9, -3)$

Answer: $\underline{(1, -1, -2)}$.

145. A tetrahedron sits with its base ABC on the XY-plane. The three edges of the top vertex D meet to form three right angles. If $A = (0, 3, 0)$, $B = (10, 13, 0)$ and $C = (10, -5, 0)$, find D.

a) $(1, 2, 3)$ b) $(1, 2, 4)$ c) $(2, 3, 4)$
d) $(5, 10, 15)$ e) $(0, 1, 2)$ f) $(4, 8, 12)$

Answer: $\underline{(2, 3, 4)}$.

146. Find the center of the sphere containing the three points $A(10, 14, 0)$, $B(11, 0, 7)$ and $C(0, 12, 10)$, if a line tangent to the sphere at C has direction vector $\mathbf{v} = (0, -1, 1)$.

a) $(2, 3, 1)$ b) $(3, 4, 2)$ c) $(4, 5, 3)$

d) $(5, 6, 4))$ e) $(6, 7, 5)$ f) $(7, 8, 6)$

Answer: $\underline{(7, 8, 6)}$.

147. Find the center of the circle passing through the points $A(0, 3, -1)$, $B(-1, 2, 3)$ and $C(3, 3, 2)$.

a) $\left(\frac{1}{3}, \frac{4}{3}, \frac{7}{3}\right)$ b) $\left(\frac{1}{3}, \frac{2}{3}, \frac{4}{3}\right)$ c) $\left(\frac{8}{3}, \frac{4}{3}, \frac{8}{3}\right)$

d) $\left(\frac{2}{3}, \frac{8}{3}, \frac{4}{3}\right)$ e) $\left(\frac{5}{3}, \frac{4}{3}, \frac{5}{3}\right)$ f) $\left(\frac{7}{3}, \frac{1}{3}, \frac{2}{3}\right)$

Answer: $\underline{\left(\frac{2}{3}, \frac{8}{3}, \frac{4}{3}\right)}$.

148. If we write the vector $\mathbf{u} = (2, 9, 8)$ as a linear combination $a\mathbf{d} + \mathbf{v}$, where $\mathbf{v}$ is perpendicular to $\mathbf{d} = (1, 2, 5)$, what is the value of a?

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

Answer: $\underline{2}$.

149. Find the reflection of the point $P(1, -1, 3)$ in the plane OAB where $A = (-1, 0, 1)$ and $B = (4, 3, 2)$.

a) $(2, -3, 4)$ b) $(1, 0, -1)$ c) $(-1, 0, 1)$

d) $(-1, 3, 1)$ e) $(2, 0, -1)$ f) $(1, 3, -4)$

Answer: $\underline{(-1, 3, 1)}$.

150. Find the orthogonal projection of the point $P(1, -1, 3)$ onto the plane OAB where $A = (-1, 0, 1)$ and $B = (4, 3, 2)$.

a) $(0, 1, 2)$ b) $(0, 2, 1)$ c) $(1, 0, 2)$

d) $(1, 2, 0)$ e) $(2, 0, 1)$ f) $(2, 1, 0)$

Answer: $\underline{(0, 1, 2)}$.

151. P and Q are points such that:

(i) P is on the line joining $(0, 7, 3)$ and $(3, 1, 12)$;

(ii) Q is on the line joining $(-3, -2, 1)$ and $(-5, -4, 3)$;

(iii) $\overline{PQ}$ is parallel to $(1, 2, 2)$.

Find Q.

a) $(-3, -2, -1)$ b) $(-2, -1, 0)$ c) $(-1, 0, 1)$

d) $(0, 1, 2)$ e) $(1, 2, 3)$ f) $(2, 3, 4)$

Answer: $\underline{(-2, -1, 0)}$.

152. Find a vector which is orthogonal to the Z-axis and which is parallel to the plane containing $(3, 2, 7)$, $(4, 1, 8)$ and the origin.

a) $(4,-5,4)$ b) $(4,-6,3)$ c) $(4,-7,2)$
d) $(4,-8,1)$ e) $(4,-9,0)$ f) $(4,-4,5)$

Answer: $\underline{(4,-9,0)}$.

153. The point $A(2,-3,1)$ is collinear with which two of the following points?

(i) $(3,-1,0)$ (ii) $(3,1,-1)$ (iii) $(-3,1,0)$
(iv) $(0,-7,3)$ (v) $(1,-5,1)$ (vi) $(9,1,-6)$

a) (i) and (iii) b) (i) and (iv) c) (ii) and (vi)
d) (ii) and (v) e) (iii) and (vi) f) (iv) and (v)

Answer: <u>(i) and (iv)</u>.

154. A line passing through the origin intersects the plane $2x+4y-3z=\alpha$ at a point P, at right angles. If $Q=(9,18,1)$ and $R=(7,13,-7)$ lie in the plane, then:

a) $\|RQ\| \le \|PQ\| \le \|PR\|$ b) $\|PQ\| \le \|PR\| \le \|RQ\|$
c) $\|RQ\| \le \|PR\| \le \|PQ\|$ d) $\|PR\| \le \|RQ\| \le \|PQ\|$
e) $\|RQ\| \le \|PQ\| \le \|PR\|$ f) none of the above

Answer: $\underline{\|PR\| \le \|RQ\| \le \|PQ\|}$.

155. If $T:\mathbb{R}^2 \to \mathbb{R}^2$ is reflection in the line $y=-3x$, then $T\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ is:

a) $\frac{1}{5}\begin{bmatrix} 1 \\ -18 \end{bmatrix}$ b) $\frac{1}{5}\begin{bmatrix} 18 \\ 6 \end{bmatrix}$ c) $5\begin{bmatrix} -1 \\ 6 \end{bmatrix}$ d) $\frac{1}{5}\begin{bmatrix} -1 \\ 6 \end{bmatrix}$

Answer: $\underline{\frac{1}{5}\begin{bmatrix} 1 \\ -18 \end{bmatrix}}$.

156. If $T:\mathbb{R}^2 \to \mathbb{R}^2$ is projection on $y=-2x$, then $T\begin{bmatrix} 6 \\ 3 \end{bmatrix}$ is:

a) $\frac{12}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ c) $\frac{1}{5}\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ d) $\frac{1}{5}\begin{bmatrix} 2 \\ -4 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}$.

Chapter 5: The Vector Space $\mathbb{R}^n$

1. Which one of the following vectors is a linear combination of $\mathbf{u} = (2, 1, 4)$ and $\mathbf{v} = (1, -1, 3)$?

   a) $(2, 2, 3)$ b) $(5, 9, 5)$ c) $(2, 0, 6)$
   d) $(1, 2, 1)$ e) $(-3, 1, 0)$ f) $(-1, 3, 5)$

   Answer: $\underline{(1, 2, 1)}$.

2. Find the value(s) of t for which $(2, 6, 5, 2t)$ lies in the subspace spanned by $(1, 2, 2, 2)$, $(3, 7, 6, 6)$ and $(1, 2, 1, 2)$.

   a) 2 or 4 b) 2 only c) -4 only
   d) -2 or -4 e) 0 or 2 f) -2, 0 or 4

   Answer: $\underline{\text{2 only}}$.

3. For what value of a is the set of vectors $S = \{(2, 2, 2), (2, 0, 4), (2, a, 2)\}$ linearly dependent?

   a) -4 b) -2 c) 0 d) 2 e) 4 f) 6

   Answer: $\underline{2}$.

4. Find all $x \in \mathbb{R}$ such that $\{(1, 1, 2), (-2, x, 1), (2, -1, 1)\}$ is a linearly independent set.

   a) all $x \neq 0$ b) all $x \neq 0$ and 1 c) all $x \neq 2$
   d) $0, 1, 2$ e) all $x \neq 3$ f) 3

   Answer: $\underline{\text{All } x \neq 3}$.

5. If $\mathbb{R}^4$ has basis $\{X_1, X_2, X_3, X_4\}$, which of the following are bases of $\mathbb{R}^4$?

   (a) $\{X_1 - X_2,\ X_2 - X_3,\ X_3 - X_4,\ X_4 - X_1\}$.
   (b) $\{X_1 + X_2,\ X_2 + X_3,\ X_3 + X_4,\ X_4 + X_1\}$.
   (c) $\{X_1,\ X_1 + X_2,\ X_1 + X_2 + X_3,\ X_1 + X_2 + X_3 + X_4\}$.
   (d) $\{X_1,\ X_1 + X_2,\ X_1 + X_3,\ X_1 + X_4\}$.

   Answer: $\underline{\text{(b), (c), (d)}}$.

6. For what value(s) of λ are the vectors $(1, -1, 2)$, $(1, \lambda, -4)$ and $(-1, 0, \lambda)$ linearly dependent?

   a) 4 or -1 b) $\lambda \neq -4$ and 1 c) -4 or 1
   d) all λ e) no λ f) 1 only

   Answer: $\underline{-4 \text{ or } 1}$.

7. For what values of a are the vectors $(a, 1, 1)$, $(1, a, a)$ and $(8, 7, a)$ linearly dependent?

   a) $0, 1, 7$ b) $-7, 1, -1$ c) $7, -7, 1$
   d) $7, -7, -1$ e) $7, 1, -1$ f) $0, -1, -7$

   Answer: $\underline{7, 1, -1}$.

8. Find the value of t for which $(4,6,3,t)$ is a linear combination of $(1,3,-4,1)$, $(2,8,-5,-1)$ and $(-1,-5,0,2)$.

   a) 0 b) 4 c) 7 d) 11 e) 13 f) 15

   Answer: 13.

9. Let $U = \text{span}\{(1,-2,3,4),(-3,6,-5,-16),(-1,2,-5,-2)\}$. Find all t such that $(1,t,3,4) \in U$.

   a) $t=0$ b) $t \neq 0$ c) $t=-1$
   d) $t \neq -1$ e) $t=-2$ f) $t \neq -2$

   Answer: $t=-2$.

10. For what value of α does the vector $(5,3,\alpha)$ belong to the space spanned by $(3,2,0)$ and $(1,0,3)$?

    a) $\frac{1}{2}$ b) 1 c) $\frac{3}{2}$ d) 2 e) $\frac{5}{2}$ f) 3

    Answer: $\frac{3}{2}$.

11. For what value of α is the set of vectors $\{(1,1,1),(1,0,2),(1,\alpha,1)\}$ linearly dependent?

    a) -1 b) 2 c) 0 d) 1 e) $\frac{-1}{2}$ f) -2

    Answer: 1.

12. Find all values of λ so that $\{(2,-1,3),(0,\lambda,2),(8,-1,8)\}$ spans $\mathbb{R}^3$.

    a) $\lambda = \frac{3}{2}$ b) $\lambda \neq \pm\frac{3}{2}$ c) $\lambda > 0$
    d) $\lambda < 0$ e) $\lambda = 0$ f) $\lambda \neq \frac{-3}{2}$

    Answer: $\lambda \neq \frac{-3}{2}$.

13. Which of the following are not subspaces of $\mathbb{R}^3$?

    (i) $V = \{(x,y,z) \in \mathbb{R}^3 \mid 2x-3y+z=0\}$, together with the usual vector operations.

    (ii) $W = \{(x,y,z) \in \mathbb{R}^3 \mid xyz=0\}$, together with the usual vector operations.

    (iii) the set of all real 3×2 matrices whose entry in the first row and second column is zero, together with the usual vector operations on 3×2 matrices.

    a) (i) and (ii) b) (i) and (iii) c) (ii) and (iii)
    d) (i) only e) (ii) only f) (iii) only

    Answer: (ii) only.

14. Which of the following are subspaces of $\mathbb{R}^3$?

    (i) $\{(x,y,z) \mid 2x-y+3z=0\}$

    (ii) $\{(x,y,z) \mid xy=0\}$

    (iii) $\{(x,y,z) \mid 2x=5z\}$

    (iv) $\left\{(x,y,z) \mid \frac{x}{2} = \frac{y+3}{5} = 7z\right\}$

    (v) $\mathbb{R}^2$

a) (i) and (ii) b) (i) and (iii) c) (ii) and (iv)
d) (ii) and (iii) e) (i), (iii) and (iv) f) (iii) and (iv)

Answer: (i) and (iii).

15. Which of the following are subspaces of $\mathbb{R}^3$? (List all correct answers.)

(i) $\{(a, a, 0) \mid a \in \mathbb{R}\}$

(ii) $\{(5a, a, -a) \mid a \in \mathbb{R}\}$

(iii) $\{(a + b - 3, a, b) \mid a, b \in \mathbb{R}\}$

(iv) $\{(b, |a|, a) \mid a, b \in \mathbb{R}\}$

a) (i) and (ii) only b) (i) and (iv) only c) (i), (ii) and (iii) only
d) (iii) and (iv) only e) (i), (ii) and (iv) only f) none of the above

Answer: (i) and (ii) only.

16. Which of the following are bases for $\mathbb{R}^3$?

(i) $\{(4, 2, 0), (1, 4, 1), (1, 3, -1)\}$

(ii) $\{(-1, 2, 3), (3, 3, 2)\}$

(iii) $\{(-1, 3, -5), (1, -2, 4), (2, 0, 4), (5, 1, 9)\}$

a) all three b) (i) only c) (ii) only
d) (i) and (ii) e) (ii) and (iii) f) none of the three

Answer: (i) only.

17. Which of these sets: $U = \{(x, y, x - y) \mid x, y \in \mathbb{R}\}$, $V = \{(x, y, z + y) \mid x, y \in \mathbb{R}\}$ and $W = \{(x, y, xy) \mid x, y \in \mathbb{R}\}$ are subspaces of $\mathbb{R}^3$?

a) U and V only b) U and W only c) V and W only
d) U only e) V only f) W only

Answer: U and V only.

18. Examine the following statements:

(i) The set of all solutions of a homogeneous system of linear equations $Ax = 0$ is a subspace of $\mathbb{R}^n$ for some n.

(ii) The set of all solutions of a non-homogeneous system of linear equations $Ax = B$ is a subspace of $\mathbb{R}^n$ for some n.

Which of the following is correct?

a) (i) and (ii) are both true b) (i) and (ii) are both false
c) (i) is true but (ii) is false d) (i) is false but (ii) is true
e) (i) is false but (ii) depends on whether A is invertible f) none of the above

Answer: (i) is true but (ii) is false.

19. Consider the following three sets in $\mathbb{R}^3$?

    (i) $\{(1,0,1),(6,4,5),(-4,-4,7)\}$

    (ii) $\{(3,-1,2),(5,1,1),(1,1,1)\}$

    (iii) $\{(2,1,3),(3,1,-3),(1,1,9)\}$

    Which of the following is true?

    a) (ii) and (iii) are bases
    b) (i) and (iii) are bases
    c) only (i) is a basis
    d) Only (iii) is a basis
    e) (i) and (ii) are bases
    f) none of these sets forms a basis

    Answer: (i) and (ii) are bases.

20. Which of the following is a subspace of $\mathbb{R}^4$?

    a) $\{(a,b,c,d) : a = b = 0\}$
    b) $\{(a,b,c,d) : a = 1, b = 0 \text{ and } c + d = 1\}$
    c) $\{(a,b,c,d) : a > 0 \text{ and } b < 0\}$
    d) $\{(a,b,c,d) : a > 0 \text{ and } b > 0\}$
    e) $\{(a,b,c,d) : a + b + c + d = 1\}$
    f) none of the above

    Answer: $\{(a,b,c,d) : a = b = 0\}$.

21. Which of the following is a subspace of $\mathbb{R}^4$?

    a) the subset $\{(a,b,c,d)\}$, where $a + d = 0$.
    b) the subset $\{(a,b,c,d)\}$, where $ad = 1$.
    c) the subset $\{(a,b,c,d)\}$, where a, b, c, d are integers.
    d) the subset $\{(a,b,c,d)\}$, where $ad - bc = 0$.
    e) the subset $\{(a,b,c,d)\}$, where $a = 1$
    f) none of the above.

    Answer: The subset $\{(a,b,c,d\}$, where $a + d = 0$.

22. Let A be an $n \times n$ invertible matrix. Which of the following statements is true?

    a) $\det A = 0$
    b) The homogeneous system $AX = 0$ has infinitely many solutions.
    c) The rank of A is not equal to n.
    d) The column vectors of A are linearly independent.
    e) The row vectors of A are linearly dependent.
    f) Each of the above statements is false.

    Answer: The column vectors of A are linearly independent.

23. Let U be a subspace of $\mathbb{R}^6$. Which one of the following statements is true?

    a) $\dim U < 6$
    b) any six vectors will span U.
    c) there is a basis for U containing five vectors.
    d) the zero vector of $\mathbb{R}^6$ is in U.
    e) all vectors having length ≤ 6 are in U.
    f) none of the above

    Answer: The zero vector of $\mathbb{R}^6$ is in U.

24. Let A be an $n \times n$ matrix. Which one of the following statements is not equivalent to the other four?

(i) A is not invertible.

(ii) The equation $AX = \mathbf{b}$ has a unique solution X for any n-vector $\mathbf{b}$.

(iii) The rows of A are linearly independent.

(iv) A can be row-reduced to the identity matrix I_n.

(v) The column rank of A is n.

a) (i) b) (ii) c) (iii)

d) (iv) e) (v) f) they are all equivalent

Answer: <u>(i)</u>.

25. Which two of the following subsets are <u>not</u> subspaces of $\mathbb{R}^4$?

$$\begin{aligned} R &= \{(a,b,c,d) : c = a + 2b, d = a - 3b\} \\ S &= \{(a,b,c,d) : a = 0, b = 0\} \\ T &= \{(a,b,c,d) : a - b = 2, c = d\} \\ U &= \{(a,b,c,d) : a \geq 0, b \geq 0\} \end{aligned}$$

a) R and T b) T and U c) S and T

d) T and U e) S and U f) R and U

Answer: <u>T and U</u>.

26. A basis for the solution space of the system $\begin{aligned} u - 2x + 3y + 4z &= 0 \\ -2u + 4x - 5y - 6z &= 0 \end{aligned}$ is:

a) $\{(0,0,0,0)\}$ b) $\{(2,1,0,0)\}$

c) $\{(1,2,0,0)\}$ d) $\{(2,1,0,0),(1,-3,-4,1)\}$

e) $\{(2,1,0,0),(2,0,-2,1)\}$ f) $\{(2,0,-2,1)\}$

Answer: <u>$\{(2,1,0,0),(2,0,-2,1)\}$</u>.

27. If we denote by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ the columns of the matrix $A = \begin{bmatrix} 1 & 3 & -1 & 4 & 1 \\ 2 & 1 & 8 & 3 & 0 \\ -1 & 2 & -9 & 1 & 1 \\ 1 & 4 & -3 & 5 & 0 \end{bmatrix}$,

which of the following is a basis for the column space of A?

a) $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$ b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ c) $\{\mathbf{v}_2, \mathbf{v}_5\}$

d) $\{\mathbf{v}_1, \mathbf{v}_2\}$ e) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5\}$ f) $\{\mathbf{v}_1, \mathbf{v}_5\}$

Answer: <u>$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5\}$</u>.

28. If we denote the rows of $A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 1 & 5 \\ -1 & 0 & -1 \\ -2 & 5 & 3 \end{bmatrix}$ by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ then which of the following sets is a basis for the row space of A?

    a) $\{\mathbf{v}_1\}$ b) $\{\mathbf{v}_1, \mathbf{v}_3\}$ c) $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

    d) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ e) $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$ f) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

    Answer: $\underline{\{\mathbf{v}_1, \mathbf{v}_3\}}$.

29. Suppose $B = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 2 & 1 & 0 & -2 & 0 \\ -1 & -1 & -2 & 2 & -1 \\ -1 & 1 & -1 & -2 & -2 \end{bmatrix}$, and let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ denote, respectively, the four rows of B. A basis for the row space of B is:

    a) $\{\mathbf{v}_1, \mathbf{v}_2\}$ b) $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$ c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

    d) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ e) $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ f) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

    Answer: $\underline{\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}}$.

30. Suppose $B = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & 1 \\ 3 & 6 & -2 & -4 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & -1 & -2 \end{bmatrix}$, and let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ denote, respectively, the four columns of B. A basis for the column space of B is:

    a) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ b) $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$ c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$

    d) $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ e) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ f) Any 3 of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$

    Answer: $\underline{\text{Any 3 of } \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4}$.

31. Which one of the following is a basis for the subspace of $\mathbb{R}^3$ defined by $G = \{(x, y, z) : 2x - y + 3z = 0\}$?

    a) $(1, 2, 0)$ and $(0, 3, 1)$ b) $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$

    c) $(1, 2, 0)$ d) $(1, 0, 0)$ and $(1, 2, 0)$

    e) $(3, 0, -2)$ f) $(-3, 0, 2)$ and $(1, 0, 0)$

    Answer: $\underline{(1, 2, 0) \text{ and } (0, 3, 1)}$.

32. If $\mathbf{u} = (1, 1, 2)$, $\mathbf{v} = (-2, 3, 1)$ and $\mathbf{w} = (2, -1, 1)$, then $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$ for some constants a and b, one of them is equal to:

    a) $\frac{1}{5}$ b) $\frac{-1}{5}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$ e) $\frac{4}{5}$ f) 1

Answer: $\frac{4}{5}$.

33. Let $\mathbf{u} = (1, 1, 1)$, $\mathbf{v} = (1, 2, 3)$, $\mathbf{w} = (1, 3, 7)$ and $\mathbf{x} = (0, -3, -10)$. Which of the following statements is true?

(i) $\mathbf{x}$ is a linear combination of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

(ii) $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is linearly independent.

(iii) $\mathbf{u}$ belongs to the span of $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$.

(iv) the matrix with $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}$ as columns has rank 3.

(v) $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is linearly dependent.

(vi) $\dim(\text{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}\}) = 3$.

a) (i) only b) (ii) only c) (i) and (iii) only

d) (iv), (v) and (vi) only e) (i), (iii), (iv), (v) and (vi) f) all of them

Answer: (i), (iii), (iv), (v) and (vi).

34. Which of the vectors below is a linear combination of $\mathbf{u} = (1, 1, 2)$, $\mathbf{v} = (-2, 3, 1)$ and $\mathbf{w} = (2, -1, 1)$?

a) $(1, 0, 0)$ b) $(0, 1, 0)$ c) $(0, 0, 1)$

d) $(0, 1, 1)$ e) $(1, 1, 0)$ f) $(1, 1, 1)$

Answer: $(0, 1, 1)$.

35. If (x, y, z, t) is expressed as a linear combination of the vectors $\mathbf{v}_1 = (1, 1, 1, 1)$, $\mathbf{v}_2 = (1, 1, -1, -1)$, $\mathbf{v}_3 = (1, -1, 1, -1)$, and $\mathbf{v}_4 = (1, -1, -1, 1)$, the coefficient of $\mathbf{v}_3$ is:

a) $\frac{1}{4}(x - y + z - t)$ b) $\frac{1}{2}(x + y - z - t)$ c) $\frac{1}{2}(x - y - z + t)$ d) $\frac{1}{2}(x - y + z - t)$

e) $\frac{1}{\sqrt{2}}(x - y + z - t)$

Answer: $\frac{1}{2}(x - y + z - t)$.

36. If (x, y, z, t) is expressed as a linear combination of the vectors $\mathbf{v}_1 = (2, -1, 4, 5)$, $\mathbf{v}_2 = (0, 1, -1, 1)$ $\mathbf{v}_3 = (0, 3, 2, -1)$, and $\mathbf{v}_4 = (21, 1, -4, -5)$, the coefficient of $\mathbf{v}_4$ is:

a) $\frac{1}{\sqrt{3}}(y - z + t)$ b) $\frac{1}{\sqrt{483}}(21x + y - 4z - 5t)$ c) $\frac{1}{483}(21x + y - 4z - 5t)$

d) $\frac{1}{\sqrt{46}}(21x + y - 4z - 5t)$ e) $(2x - y + 4z - t)$

Answer: $\frac{1}{\sqrt{483}}(21x + y - 4z - 5t)$.

37. Express the vector $\mathbf{x} = \begin{bmatrix} 2 & 9 & -10 & 6 \end{bmatrix}$ as a linear combination of $\mathbf{u} = \begin{bmatrix} 1 & 2 & -3 & 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 4 & 5 & -4 & 2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -2 & 3 & 6 & 4 \end{bmatrix}$:

a) $\mathbf{x} = 2\mathbf{u} - 2\mathbf{v} - 2\mathbf{w}$ b) $\mathbf{x} = 5\mathbf{u} - \frac{1}{2}\mathbf{v} + \frac{1}{2}\mathbf{w}$

c) $\mathbf{x} = 3\mathbf{u} - \frac{1}{2}\mathbf{v} - \frac{1}{2}\mathbf{w}$ d) $\mathbf{x} = 2\mathbf{u} + 3\mathbf{v} + \mathbf{3w}$

e) $\mathbf{x} = 3\mathbf{u} - \mathbf{v} + 2\mathbf{w}$ f) $\mathbf{x} = -3\mathbf{u} - 2\mathbf{v} - \mathbf{w}$

Answer: $\mathbf{x} = 5\mathbf{u} - \frac{1}{2}\mathbf{v} + \frac{1}{2}\mathbf{w}$.

38. Let $X = (4, -w, -u, 3)$, $Y = (3, -2, -4, 1)$, $Z = (u, -3, -6, v)$. Find u, v and w so that $Y = 2X - Z$.

a) $u = 5,\ v = 5,\ w = \frac{5}{2}$
b) $u = \frac{5}{2},\ v = \frac{5}{2},\ w = \frac{5}{2}$
c) $u = 5,\ v = 5,\ w = \frac{5}{2}$
d) $u = 5,\ v = \frac{5}{2}, w = 5$
e) $u = \frac{5}{2},\ v = 5,\ w = 5$
f) $u = \frac{5}{2},\ v = \frac{5}{2},\ w = 5$

Answer: $\underline{u = 5,\ v = 5,\ w = \frac{5}{2}}$.

39. Which of the following vectors belong to the column space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & -3 \end{bmatrix}?$$

(i) $\mathbf{u} = (-1, -2, 1)$
(ii) $\mathbf{v} = (1, 2, 3)$
(iii) $\mathbf{w} = (-1, -2, 3)$
(iv) $\mathbf{x} = (1, 0, 0)$
(v) $\mathbf{y} = (2, 4, -2)$
(vi) $\mathbf{z} = (0, 1, 0)$

a) (i) only
b) (i) and (ii)
c) (i) and (iii)
d) (i) and (v)
e) (ii), (iii) and (iv)
f) (ii), (iii) and (vi)

Answer: <u>(i) and (v)</u>.

40. If A is a $4{\times}5$ matrix, let null $A = \left\{\mathbf{x} \in \mathbb{R}^5 \mid A\mathbf{x} = \mathbf{0}\right\}$ and im $A = \left\{\mathbf{b} \in \mathbb{R}^4 \mid \mathbf{b} = A\mathbf{x} \text{ for some } \mathbf{x} \text{ in } \mathbb{R}^5\right\}$. Then if rank $A = 0$:

a) null $A = \{0\}$, im $A = \mathbb{R}^5$
b) null $A = \{0\}$, Im $A = \mathbb{R}^4$
c) null $A = \mathbb{R}^5$, Im $A = \mathbb{R}^4$
d) null $A = \mathbb{R}^5$, Im $A = \{0\}$
e) null $A = \{0\}$, Im $A = \{0\}$
f) null $A = \{0\}$, Im A has dimension 1

Answer: <u>null $A = \mathbb{R}^5$, Im $A = \{0\}$</u>.

41. A is a $4{\times}5$ matrix and null $A = \left\{\mathbf{x} \in \mathbb{R}^5 \mid A\mathbf{x} = \mathbf{0}\right\}$ and Im $A = \left\{\mathbf{b} \in \mathbb{R}^4 \mid \mathbf{b} = A\mathbf{x} \text{ for some } \mathbf{x} \in \mathbb{R}^5\right\}$. Then if rank $A = 4$:

a) Im $A = \mathbb{R}^4$, dim null $A = 1$
b) Im $A = \mathbb{R}^4$, dim null $A = 4$
c) dim Im $A = 1$, dim null $A = 1$
d) dim Im $= 1$, null $A = \mathbb{R}^5$
e) dim Im $A = 4$, null $A = \mathbb{R}^5$
f) Im $A = \{0\}$, dim null $A = 4$

Answer: <u>Im $A = \mathbb{R}^4$, dim null $A = 1$</u>.

42. Let A be an 8×6 matrix such that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

- What is the rank of A?
- Is $A\mathbf{x} = B$ consistent for all $B \in \mathbb{R}^8$?

a) 0, Yes
b) 6, Yes
c) 6, No
d) 8, Yes
e) 8, No
f) 2, Yes

Answer: <u>6, No</u>.

43. If A is a 7×12 matrix, what is the largest possible dimension of the row space of A?

a) 7 b) 8 c) 9 d) 10 e) 11 f) 12

Answer: $\underline{7}$.

44. If A is a 9×13 matrix, what is the largest possible dimension of the column space of A?

a) 9 b) 10 c) 11 d) 12 e) 13 f) none of the above

Answer: $\underline{13}$.

45. If A is a 6×9 matrix, then the column vectors of A :

a) span a space of dimension 6.
b) span a space of dimension 9 or less.
c) are always linearly independent.
d) are sometimes linearly dependent.
e) are always linearly dependent.
f) span a space of dimension 54.

Answer: <u>Are always linearly dependent</u>.

46. Let $A = \begin{bmatrix} 0 & 1 & 0 & -3 \\ 1 & 1 & 3 & 0 \\ 2 & 1 & 3 & 2 \\ 1 & 0 & 0 & 2 \end{bmatrix}$. If $\mathbf{x} \in \mathbb{R}^4$, the dimension of the solution-space of $A\mathbf{x} = \mathbf{0}$ is:

a) 4 b) 3 c) 2 d) 1 e) 0 f) Infinite

Answer: $\underline{1}$.

47. What is the dimension of the solution space of the following system:

$$\left\{\begin{array}{rcrcrcrcl} x & + & 2y & + & 2z & - & w & = & 0 \\ x & + & 3y & + & 2z & + & w & = & 0 \\ 3x & + & 8y & + & 6z & + & w & = & 0 \\ 5x & + & 14y & + & 10z & + & 5w & = & 0 \end{array}\right. ?$$

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{1}$.

48. A basis for the solution space of $A\mathbf{x} = \mathbf{0}$ if $A = \begin{bmatrix} 1 & -2 & 3 & 4 \\ 3 & -5 & 7 & 8 \end{bmatrix}$ is:

a) $\{(1,0),(0,1)\}$
b) $\{(1,0,-1,-4),(0,1,-2,-4)\}$
c) $\{(1,2,1,0)\}$
d) $\{(0,1,-2,-4)\}$
e) $\{(4,4,0,1)\}$
f) $\{(1,2,1,0),(4,4,0,1)\}$

Answer: $\underline{\{(1,2,1,0),(4,4,0,1)\}}$.

49. The distance between the points $(1,-3,-5,2,-1)$ and $(3,1,-1,4,2)$ of $\mathbb{R}^5$ is:

a) $\sqrt{41}$ b) 8 c) $\sqrt{30}$ d) 7 e) 6 f) $\sqrt{35}$

Answer: $\underline{7}$.

50. If $\mathbf{u} = (6,0,0,3,0)$ and $\mathbf{w} = (-1,4,2,1,3)$, find $\|\mathbf{u} - 3\mathbf{w}\|$ in $\mathbb{R}^5$.

a) $\sqrt{271}$ b) $3\sqrt{38}$ c) $3\sqrt{39}$ d) $\sqrt{306}$ e) $\sqrt{398}$ f) $2\sqrt{61}$

Answer: $\underline{3\sqrt{38}}$.

51. If $\mathbf{u} = (2,3,-1,3)$ and $\mathbf{v} = (-2,4,3,1)$, then in $\mathbb{R}^4$, $\|\mathbf{u} + \mathbf{v}\|$ is:

a) $\sqrt{53}$ b) 53 c) $\sqrt{85}$ d) 85 e) $\sqrt{69}$ f) 69

Answer: $\underline{\sqrt{69}}$.

52. For which real values of x is the triangle with vertices $A(2,1,-1)$, $B(4,2,0)$ and $C(3,-1,x)$ a right angled triangle?

a) -1 and -5 b) 1 and -5 c) -1 and 5 d) -1, 5 and $\frac{1}{2}\left(1 \pm \sqrt{21}\right)$

e) None of the above f) There is no such value of x.

Answer: $\underline{-1 \text{ and } 5}$.

53. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, which of the following are true:

a) $\mathbf{v} \bullet \mathbf{w} = \frac{1}{4}[\|\mathbf{v} + \mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2]$

b) $\mathbf{v} \bullet \mathbf{w} = 0$ if and only if $\|\mathbf{v} + \mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2$

c) $\|\mathbf{u} + \mathbf{v} - \mathbf{w}\|^2 + \|\mathbf{u} - \mathbf{v} + \mathbf{w}\|^2 + \|-\mathbf{u} + \mathbf{v} + \mathbf{w}\|^2 = 3[\mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w} + \mathbf{v} \bullet \mathbf{w}]$

d) If $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| = 0$ then $\mathbf{u} = \mathbf{v} = \mathbf{w} = \mathbf{0}$.

e) $\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 = \frac{1}{2}[\|\mathbf{v} + \mathbf{w}\|^2 + \|\mathbf{v} - \mathbf{w}\|^2]$

f) If $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{w} \neq \mathbf{0}$, and if $\|\mathbf{v} + \mathbf{w}\| = \|\mathbf{v} - \mathbf{w}\|$, then $\{\mathbf{v}, \mathbf{w}\}$ is orthogonal.

Answer: $\underline{\text{a), b), e), f)}}$.

54. Compute the rank of $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{3}$.

55. The rank of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 7 \\ 1 & 2 & 5 \\ 3 & 5 & 14 \\ -5 & -2 & -4 \end{bmatrix}$ is:

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

Answer: $\underline{3}$.

56. Compute the rank of $\begin{bmatrix} 3 & 2 & -1 & 2 & 0 & 1 \\ 4 & 1 & 0 & -3 & 0 & 2 \\ 2 & -1 & -2 & 1 & 1 & -3 \\ 3 & 1 & 3 & -9 & -1 & 6 \\ 3 & -1 & -5 & 7 & 2 & -7 \end{bmatrix}$.

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

Answer: $\underline{3}$.

57. For what value(s) of x does the matrix $\begin{bmatrix} 1 & x & -1 & 2 \\ 2 & -1 & x & 5 \\ 1 & 10 & -6 & 1 \end{bmatrix}$ have the least rank?

a) 3 or 12 b) 4 or 8 c) 3 only
d) all $x \neq -3$ e) 12 only f) for all x the rank is 3

Answer: $\underline{\text{3 only}}$.

58. Compute the rank of $\begin{bmatrix} 1 & -2 & 3 & -1 & -1 & -2 \\ 2 & -1 & 1 & 0 & -2 & -2 \\ -2 & -5 & 8 & -4 & 3 & -1 \\ 6 & 0 & -1 & 2 & -7 & -5 \\ -1 & -1 & 1 & -1 & 2 & 1 \end{bmatrix}$.

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

Answer: $\underline{3}$.

59. Compute the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 3 \\ 1 & 0 & 1 & 0 & 1 & 6 \\ 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 1 & -1 & 1 & -1 & 1 & 9 \end{bmatrix}$.

a) 1 b) 2 c) 3 d) 4 e) 5 f) 6

Answer: $\underline{3}$.

60. For what value(s) of x does the matrix $\begin{bmatrix} 3 & 1 & 1 & 4 \\ x & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix}$ have the least rank?

a) 1 and -2

b) 0 and -1

c) 0 only

d) -1, 1 and -2

e) for all x, the matrix has rank 3

f) for all x, the matrix has rank 2

Answer: 0 only.

61. Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$, which of the following statements is true?

a) The rank of the matrix is 1

b) The rank of the matrix is 5

c) The rank of the matrix is 4

d) The rank of the matrix is 3

e) The determinant of the matrix is 0

f) A is invertible

Answer: The rank of the matrix is 3.

62. Find the dimension of the column space of $\begin{bmatrix} -1 & 7 & 0 & 3 & 1 \\ 1 & -1 & 0 & -1 & -1 \\ 0 & -3 & 0 & -1 & -1 \\ 0 & 5 & 3 & 4 & -3 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 4.

63. Find the rank of $\begin{bmatrix} 1 & 0 & 2 & 1 & 1 \\ 1 & -1 & 6 & -1 & -2 \\ 2 & 1 & 0 & 4 & 5 \\ 5 & 1 & 6 & 7 & 8 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 2.

64. Find the rank of $\begin{bmatrix} 1 & 0 & 2 & 1 & 1 \\ 1 & -1 & 6 & -1 & -2 \\ 2 & 1 & 2 & 4 & 5 \\ 5 & 1 & 6 & 7 & 8 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 3.

65. Find the rank of $\begin{bmatrix} 0 & 1 & 0 & -3 \\ 1 & 1 & 3 & 0 \\ 1 & 0 & 0 & 2 \\ 2 & 1 & 3 & 2 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 3.

66. Find the rank of $\begin{bmatrix} 0 & 2 & 6 & -4 \\ 1 & 1 & 3 & 0 \\ 1 & 0 & 0 & 2 \\ 2 & 1 & 3 & 2 \end{bmatrix}$.

a) 5 b) 4 c) 3 d) 2 e) 1 f) 0

Answer: 2.

67. Find the rank of $\begin{bmatrix} 0 & 5 & 16 & 10 & 0 \\ 2 & -5 & -3 & -2 & 6 \\ 1 & -2 & 0 & 0 & 3 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 4.

68. Find the rank of $\begin{bmatrix} 1 & 0 & 3 & 1 & 1 \\ 1 & -1 & 7 & -1 & 0 \\ 2 & 1 & 2 & 4 & 3 \\ 5 & 1 & 11 & 7 & 6 \end{bmatrix}$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 2.

69. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & k+1 & 2k+2 \\ -1 & -1 & k^2-2 \end{bmatrix}$, find all value(s) of k for which rank $A = 2$.

a) all $k \neq 0$ b) 0 c) 1 or 2

d) 0 or 2 e) 2 f) no values of k

Answer: No values of k.

70. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & k+1 & 2k+2 \\ -1 & -1 & k^2-2 \end{bmatrix}$, find all value(s) of k for which rank $A = 1$.

a) all $k \neq 0$ b) 0 c) 1 or 2
d) 1 only e) 2 f) no values of k

Answer: $\underline{0}$.

71. The rank of $\begin{bmatrix} 1 & 3 \\ -1 & 6 \\ 4 & 5 \\ -2 & 3 \end{bmatrix}$ is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{2}$.

72. What is the dimension of the subspace spanned by $S = \{(1,1,1), (-1,1,-1), (1,1,3), (0,2,1)\}$?

a) 0 b) 1
c) 2 d) 3
e) 4 f) these vectors do not span a subspace

Answer: $\underline{3}$.

73. What is the dimension of the space spanned by the vectors in the set
$S = \{(1,1,1,1), (3,2,2,2), (2,2,1,3), (5,6,3,9), (1,0,0,0), (0,1,0,0), (0,1,1,2)\}$?

a) 4 b) 2 c) 7 d) 6 e) 1 f) 3

Answer: $\underline{4}$.

74. What is the dimension of the subspace spanned by $\mathbf{w} = (1,-1,4,-5)$, $\mathbf{x} = (2,1,5,-1)$, $\mathbf{y} = (0,1,-1,3)$ and $\mathbf{z} = (3,4,5,6)$?

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{2}$.

75. The dimension of the subspace spanned by the vectors $(1,0,3,1,1)$, $(1,-1,7,-1,0)$, $(2,1,2,4,3)$, and $(5,1,11,7,6)$ is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{2}$.

76. The dimension of the subspace spanned by the vectors $(1,1,0,9)$, $(1,1,1,16)$, $(1,1,-1,2)$ and $(-1,-1,-2,5)$ is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{3}$.

77. What is the dimension of the subspace of $\mathbb{R}^3$ spanned by $(1,2,-1)$, $(1,-2,1)$, $(-3,2,-1)$ and $(2,0,0)$?

a) 0 b) 1 c) 2 d) 3 e) 4 f) other

Answer: $\underline{2}$.

78. The dimension of the subspace of $\mathbb{R}^4$ spanned by $(1,1,0,9)$, $(1,1,0,-1)$, $(0,0,1,7)$ and $(0,0,1,0)$ is:

    a) 5 b) 4 c) 3 d) 2 e()1 f) 0

    Answer: $\underline{3}$.

79. Find the dimensions of the subspaces spanned by the given sets of vectors:

    (i) $\{(-4,3,0,6),(6,3,-2,-5),(0,3,-1,2)\}$

    (ii) $\{(6,8,1),(0,2,2),(1,-1,0),(3,4,0)\}$

    (iii) $\{(\alpha,-\alpha),(\beta,-\beta),(\alpha\beta,-\alpha\beta)\}$, $(\alpha\beta \neq 0)$

    a) (i) 3, (ii) 3, (iii) 1 b) (i) 3, (ii) 4, (iii) 2 c) (i) 4, (ii) 4, (iii) 3

    d) (i) 3, (ii) 2, (iii) 2 e) (i) 3, (ii) 3, (iii) 3 f) (i) 2, (ii) 3, (iii) 2

    Answer: $\underline{\text{(i) 3, (ii), 3, (iii) 1}}$.

80. Find the dimension of the subspace of $\mathbb{R}^5$ spanned by the vectors $(1,-1,7,-1,0)$, $(1,0,3,1,1)$, $(3,0,10,3,3)$ and $(3,1,5,5,4)$.

    a) 5 b) 4 c) 3 d) 2 e) 1 f) 0

    Answer: $\underline{3}$.

81. Find the dimension of the subspace of $\mathbb{R}^5$ spanned by the vectors $(1,3,-2,5,4)$, $(1,4,1,3,5)$, $(1,4,2,4,3)$ and $(2,7,-3,6,13)$.

    a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

    Answer: $\underline{3}$.

82. The matrix $\begin{bmatrix} 1 & -6 & -6 \\ -1 & 0 & 2 \\ 3 & -6 & 0 \end{bmatrix}$ has -2 as an eigenvalue. A basis for the corresponding eigenspace is:

    a) $\{(2,1,0)\}$ b) $\{(2,0,1)\}$ c) $\{(2,0,1),(2,0,1)\}$

    d) $\{(3,-1,3)\}$ e) $\{(2,1,0),(3,-1,3)\}$ f) $\{(2,1,0),(2,0,1),(3,-1,3)\}$

    Answer: $\underline{\{(2,1,0)\}}$.

83. A basis for the eigenspace corresponding to the eigenvalue 1 of the matrix $\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ is:

    a) $\{(1,-1,-1)\}$ b) $\{(-1,4,1\}$ c) $\{(-1,4,1),(1,-1,-1)\}$

    d) $\{(-2,1,3)\}$ e) $\{(1,2,1)\}$ f) $\{(1,2,1),(-1,4,1)\}$

    Answer: $\underline{\{(-1,4,1)\}}$.

84. The dimension of the eigenspace corresponding to the eigenvalue 3 of the matrix $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{3}$.

85. Which of the following vectors is an eigenvector for the matrix $\begin{bmatrix} 4 & -3 & 1 \\ -1 & 2 & -2 \\ -6 & 6 & -4 \end{bmatrix}$?

a) $(-1, 2, 1)$ b) $(1, -1, 2)$ c) $(-2, -1, 1)$
d) $(2, 1, 1)$ e) $(1, 1, 1)$ f) $(0, 0, 0)$

Answer: $\underline{(-2, -1, 1)}$.

86. The matrix $A = \begin{bmatrix} 8 & -3 \\ 18 & -7 \end{bmatrix}$ is similar to the diagonal matrix $D =$

a) $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e) $\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$ f) $\begin{bmatrix} 4 & 0 \\ 0 & -5 \end{bmatrix}$

Answer: $\underline{\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}}$.

87. A matrix which is **not** similar to $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ is:

a) $\begin{bmatrix} 3 & 2 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 9 & 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 \\ 6 & 2 & 0 \\ 8 & 2 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ f) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

Answer: $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

88. If $A = \begin{bmatrix} 4 & -3 & 1 \\ -1 & 2 & -2 \\ -6 & 6 & -4 \end{bmatrix}$ is similar to $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find a nonsingular matrix P such that $P^{-1}AP = D$.

a) $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ -1 & 4 & 1 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & 4 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Answer: $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & 4 & 0 \end{bmatrix}$.

89. If $A = \begin{bmatrix} 7 & 10 \\ -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$, answer the following questions:

(i) Is A diagonalizable?

(ii) Is B diagonalizable?

(iii) Are A and B similar?

a) (i) Yes, (ii) No, (iii) No b) (i) Yes, (ii) Yes, (iii) No
c) (i) Yes, (ii) Yes, (iii) Yes d) (i) No, (ii) Yes, (iii) No
e) (i) No, (ii) No, (iii) No f) (i) Yes, (ii) No, (iv) No

Answer: (i) Yes, (ii) Yes, (iii) Yes.

90. Let $A = \begin{bmatrix} -1 & 1 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$. Answer the following questions:

(i) Is A diagonalizable?

(ii) Is B diagonalizable?

(iii) Are A and B similar?

a) (i) Yes, (ii) Yes, (iii) Yes b) (i) Yes, (ii) No, (iii) Yes

c) (i) No, (ii) No, (iii) Yes d) (i) No, (ii) Yes, (iii) No

e) (i) Yes, (ii) Yes, (iii) No f) (i) No, (ii) No, (iii) No

Answer: (i) Yes, (ii) Yes, (iii) No.

91. Assume A is similar to B. Select the correct statements:

a) A^T is similar to B^T

b) A^{-1} is similar to B^{-1}

c) $\det(A)$ is similar to $\det(B)$

d) $\det(A) = \det(B)$

e) $B = A^k$ for some integer $k \geq 1$.

f) $A^{-1} = B^T$

Answer: a), b), d).

92. If A is similar to B, select the false statement(s):

a) If A is diagonalizable, so also is B.

b) If A has only one eigenvalue, the same is true of B.

c) A and B have the same trace.

d) If A is symmetric, so also is B.

e) $A + B$ is similar to B.

f) AB is similar to B.

Answer: d), e), f).

93. Given the matrices $A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 2 \end{bmatrix}$, decide which of the following questions are False:

a) A and B have the same trace for all a, b and c.

b) A and B have the same determinant for all a, b and c.

c) A and B have the same rank for all a, b and c.

d) A and B have the same characteristic polynomial for all a, b and c.

e) A and B have the same eigenvalues for all a, b and c.

f) A and B are similar for all a, b and c.

g) A and B are both diagonalizable for all a, b and c.

Answer: f), g).

94. Which of the following are true? (select all correct responses).

a) A set of n independent vectors in $\mathbb{R}^n$ forms a spanning set.

b) The column space of a 3×5 matrix A with 3 independent rows has dimension $5 - 3 = 2$.

c) Every spanning set for a subspace U of $\mathbb{R}^n$ can be enlarged to a basis of $\mathbb{R}^n$.

d) If A is an $m \times n$ matrix and $\dim(\text{col } A) = n$, then the rows of A span $\mathbb{R}^n$.

e) Every subspace of $\mathbb{R}^n$ is spanned by n vectors.

Answer: a), d), e).

95. Which of the following are true? (select all correct responses).

a) A set of n orthogonal vectors in $\mathbb{R}^n$ forms a basis of $\mathbb{R}^n$.

b) The dimension of the eigenspace corresponding to an eigenvalue of a matrix A equals the multiplicity of that eigenvalue.

c) null $(A^T) = \text{im } A$ for any matrix A.

d) The columns of any 3×4 matrix are linearly dependent .

e) Eigenvectors corresponding to different eigenvalues of an arbitrary $n \times n$ matrix are orthogonal.

Answer: a), d).

96. The least squares line fit to the three points $(0,1)$, $(1,2)$ and $(2,4)$ is:

a) $\frac{5}{6} + \frac{3}{2}x$ b) $\frac{3}{2} + \frac{5}{6}x$ c) $\frac{5}{6} - \frac{3}{2}x$ d) $-\frac{5}{6} + \frac{3}{2}x$

Answer: $\frac{5}{6} + \frac{3}{2}x$.

97. The least squares quadratic fit to the four points $(0,0)$, $(1,2)$, $(2,3)$ and $(3,1)$ is:

a) $-\frac{1}{10}\left(10 + 34x + x^2\right)$ b) $\frac{1}{10}\left(1 + 34x - 10x^2\right)$

c) $-\frac{1}{10}\left(1 + 34x + 10x^2\right)$ d) $\frac{1}{10}\left(1 - 34x + 10x^2\right)$

Answer: $-\frac{1}{10}\left(1 + 34x + 10x^2\right)$

98. The best least squares linear approximation to the data points $(1,2)$, $(3,4)$, and $(6,6)$ will (select all correct responses):

a) pass through all three points.

b) will have the form $y = ax + b$.

c) can be determined from $M^T M \, [a,b]^T = M^T Y$.

d) is always determined by inspection.

e) does not exist.

Answer: b), c).

99. If five data points are given with distinct x-values, and if the best least squares polynomial approximation of degree four is calculated, then (list all correct responses):

a) There is no such polynomial.

b) There is one polynomial that exactly fits all the data with no error.

c) there is one best polynomial, but it does not exactly fit all the data.

d) The polynomial will agree with the best least squares polynomial of degree five.

e) The polynomial will have zero constant term.

Answer: b).

Chapter 6: Vector Spaces

1. Determine when V is a vector space with the given operations:

   (a) V is the set $\mathbb{Q}$ of all rational numbers (fractions); usual operations.

   (b) $V = \{r \in \mathbb{R} \mid r \geq 0\}$ with the usual operations.

   (c) $V = \{\mathbf{0}\}$ where $\mathbf{0} \oplus \mathbf{0} = \mathbf{0}$ and $a \odot \mathbf{0} = \mathbf{0}$ for each a.

   (d) $V = \mathbb{R}^4$, usual addition and $a \odot (x, y, z) = (x, y, z)$.

   (e) V is the set of all continuous functions $[0, 1] \to \mathbb{R}$.

   (f) $V =$ the set of all 2×2 matrices with $A \oplus B = A^T + B^Y$ and $a \odot A = aA^T$.

   (g) $V = \{x \in \mathbb{R} \mid x > 0\}$; with $x \oplus y = xy$ and $a \odot x = x^a$.

   (h) $V = \mathbb{R}^4$, usual addition and $a \odot (x, y, z) = (0, 0, 0)$.

   Answer: (c), (e) and (g).

2. Which of the following are vector spaces?

   (i) all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(3) = 0$, together with the usual vector operations on real-valued functions.

   (ii) all polynomials whose degree is exactly 3, together with the usual vector operations on polynomials.

   (iii) all 2×4 real matrices A whose entries are positive, together with the usual vector operations on matrices.

   a) (i) and (ii) b) (i) and (iii) c) (ii) and (iii)

   d) (i) only e) (ii) only f) (iii) only

   Answer: (i) only.

3. Consider the following sets with operations:

   (i) $V = \{(x, y) \in \mathbb{R}^2 \mid x - y = 1\}$, equipped with the standard vector operations on $\mathbb{R}^2$.

   (ii) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x, y, z \text{ are rational}\}$, equipped with the standard vector operations on $\mathbb{R}^3$.

   (iii) $\mathbb{R}^2$ with addition defined by $(x, y) \oplus (x', y') = (x - x', y - y')$ and the standard multiplication by scalars.

   Choose the correct statement below:

   a) (i), (ii) and (iii) are vector spaces over $\mathbb{R}$.

   b) Only (i) and (ii) are vector spaces over $\mathbb{R}$.

   c) Only (ii) and (iii) are vector spaces over $\mathbb{R}$.

   d) Only (i) and (iii) are vector spaces over $\mathbb{R}$.

   e) Only (i) is a vector space over $\mathbb{R}$.

   f) None of (i), (ii) and (iii) are vector spaces over $\mathbb{R}$.

   Answer: None of (i), (ii) and (iii) are vector spaces over $\mathbb{R}$.

4. If we give $L = \mathbb{R}^2$ the non-standard vector operations

$$\begin{aligned} (x,y) \oplus (x',y') &= (x + x' - 9, y + y') && \text{(Vector addition)} \\ \text{and} \quad k \oplus (x,y) &= (kx - 9k + 9, ky) && \text{(Multiplication by scalar } k\text{)}, \end{aligned}$$

then L is a vector space over $\mathbb{R}$. What is the "zero" vector of L?

a) $(0,0)$ b) $(-9,0)$ c) $(9,0)$
d) $(18,0)$ e) $(9,9)$ f) $(0,-9)$

Answer: $\underline{(9,0)}$.

5. If we give $W = \mathbb{R}^2$ the non-standard vector operations

$$\begin{aligned} (x,y) \oplus (x',y') &= \big(x + x' + 11, y + y'\big) && \text{(Vector addition)} \\ \text{and} \quad k \oplus (x,y) &= (kx + 11k - 11, ky) && \text{(Multiplication by } k\text{)}, \end{aligned}$$

then W is a vector space over $\mathbb{R}$. If $\mathbf{v} = (x,y)$, then what is $-\mathbf{v}$?

a) $(-x,-y)$ b) $(-x+11,-y)$ c) $(x-11,-y)$
d) $(-x-11,-y)$ e) $(-x-22,-y)$ f) $(-x+22,-y)$

Answer: $\underline{(-x-22,-y)}$.

6. Express the matrix $X = \begin{bmatrix} 10 & 5 \\ -4 & 4 \end{bmatrix}$ as a linear combination of the following matrices:

$$A = \begin{bmatrix} 4 & 2 \\ -3 & 3 \end{bmatrix}; \ B = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}; \ C = \begin{bmatrix} -2 & -1 \\ 0 & -1 \end{bmatrix}.$$

a) $X = A + 7B - 7C$ b) $X = -A - 7B - 7C$
c) $X = -A - \left(\frac{7}{2}\right) B - \left(\frac{7}{2}\right) C$ d) $X = -A + \left(\frac{7}{2}\right) B - \left(\frac{7}{2}\right) C$
e) $X = A - \left(\frac{7}{2}\right) B + \left(\frac{7}{2}\right) C$ f) X is not a linear combination of A, B and C.

Answer: $\underline{X = -A + \left(\frac{7}{2}\right) B - \left(\frac{7}{2}\right) C}$.

7. The vector operations:

vector addition $a \oplus b = ab$ (ordinary multiplication of real numbers)
multiplication by the scalar k $k \otimes a = a^k$

make $\mathbb{R}^+ = \{a \in \mathbb{R} \mid a > 0\}$ a real vector space. The zero vector of $\mathbb{R}^+$ and the negative vector "$-a$" are, respectively:

a) $0, -a$ b) $0, \frac{1}{a}$ c) $1, \frac{1}{a}$
d) $-1, \frac{-1}{a}$ e) $1, -a$ f) $-1, -a$

Answer: $\underline{1, \frac{1}{a}}$.

8. Let V be the vector space of all real 2×3 matrices. Which of the following are subspaces of V?

   a) $\{a \in V \mid a_{22} = 0\}$

   b) $\{a \in V \mid a_{21} + a_{13} = 0\}$

   c) $\{a \in V \mid |a_{12}| + |a_{31}| = 0\}$

   d) $\{a \in V \mid a_{23} \neq 0\}$

   e) $\{a \in V \mid \text{each entry } a_{ij} \text{ is an integer}\}$

   Answer: <u>a), b), and c)</u>.

9. Which of the following are subspaces of $\mathbf{M}_{nn}$?

   (i) the set of all symmetric matrices A, that is $A = A^T$

   (ii) the set of all skew-symmetric matrices A, that is $A = -A^T$

   (iii) the set of all invertible matrices A, that is A^{-1} exists

   (iv) the set of all matrices with trace 0

   (v) the set of all idempotent matrices A, that is $A^2 = A$

   a) (i) and (ii) b) (i) and (iii) c) (i), (ii) and (iii)

   d) (ii), (iii) and (iv) e) (iii) and (iv) f) (i), (ii) and (iv)

   Answer: <u>(i), (ii) and (iv)</u>.

10. Find all $x \in \mathbb{R}$ so that the matrices $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & -9 \\ x & -3 \end{bmatrix}$ form a basis for $\mathbf{M}_{22}$.

    a) $x \neq 0$ b) $x \neq 2$ c) $x \neq 4$

    d) $x \neq 6$ e) $x \neq 8$ f) $x \neq 10$

    Answer: <u>$x \neq 8$</u>.

11. Determine which of the following are true, where V is a vector space:

    (a) $\{\mathbf{v}\}$ is independent for each $\mathbf{v} \in V$.

    (b) A basis of V is a spanning set that is as large as possible.

    (c) A basis of V is an independent set that is as large as possible.

    (d) A basis of V is a spanning set that is as small as possible.

    (e) A basis of V is an independent set that is as small as possible.

    (f) Any independent set in a subspace U of V is a basis of U.

    (g) Any spanning set for a subspace U of V is a basis of U.

    Answer: <u>(c), (d)</u>.

12. Determine which of the following are true, where V is a vector space:

    (a) If U is a subspace of V the zero of U is the same as that for V.

    (b) If U is a subspace of V the negative of $\mathbf{u} \in U$ is the same as its negative in V.

(c) If $a\mathbf{v} = \mathbf{0}$ where $a \in \mathbb{R}$ and $\mathbf{v} \in V$, then $a = 0$.

(d) If $a\mathbf{v} = \mathbf{0}$ where $a \in \mathbb{R}$ and $\mathbf{v} \in V$, then $\mathbf{v} = \mathbf{0}$.

(e) $-(-\mathbf{v}) = -\mathbf{v}$ for every $\mathbf{v} \in V$.

(f) If $\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w}$ where $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are in V, then $\mathbf{v} = \mathbf{w}$.

(g) If $\{\mathbf{u}, \mathbf{v}\}$ is dependent if and only if one is a scalar multiple of the other.

Answer: (a), (b), (f), (g).

13. Determine which of the following are true, where V is a vector space:

(a) Every subspace of V is a vector space in its own right.

(b) The intersection of two subspaces of V is again a subspace.

(c) The union of two subspaces of V is again a subspace.

(d) If U and W are subspaces of V then $U + W = \{\mathbf{u} + \mathbf{w} \mid \mathbf{u} \in U,\ \mathbf{w} \in W\}$ is a subspace.

(e) Every basis of a subspace of V is part of a basis of V.

(f) Every basis of V contains a basis of every subspace of V.

(g) Every spanning set for V contains a spanning set for every subspace of V.

Answer: (a), (b), (d), (e).

14. Which of the following are true for $U = \{A \in \mathbf{M}_{nn} \mid A^T = 3A\}$?

(a) U is a subspace of $\mathbf{M}_{nn}$.

(b) U is not a subspace of $\mathbf{M}_{nn}$.

(c) $U = \{0\}$.

(d) U has dimension 1.

Answer: (a), (c).

15. Which of the following sets are linearly independent?

(i) $\{(2, -3, 3), (1, 1, 0), (-1, 4, -3)\}$

(ii) $\{2x^2 - 3x + 3, x^2 + x, -x^2 + 4x - 3\}$

(iii) $\{(1, -1, 2, 1), (0, 1, 3, 4), (3, 2, 5, -1), (2, -1, 1, -5)\}$

a) none b) (i) only c) (ii) only

d) (iii) only e) (i) and (ii) f) (ii) and (iii)

Answer: (iii) only.

16. Which of the following sets in $\mathbf{P}_2$ are linearly independent?

a) $\{1 + t, 2t, -t^2\}$

b) $\{5 - t^2, t^2, 1 + t^2\}$

c) $\{1 + t, 2 + t^2, t - t^2\}$

d) $\{1 + t, 2 - t, 1 - t\}$

e) $\{t + t^2, 2t + 1, 3 + t^2\}$

Answer: a), c), e).

17. Which additional vectors in $\mathbf{P}_3$ will extend the linearly independent set $\{3x^3 - x^2, 2x^2 + 1\}$ to a basis of $\mathbf{P}_3$?

    a) $6x^3 + 1$, 1

    b) $3x^3 - x^2 + 1$, $x^2 + x$

    c) $x^2 + x$, $x - 1$

    d) $x^2 + 4$, x^3

    e) $4x + 3$, x^3

    Answer: b), c), e).

18. Let $U = \{(2,1,0), (0,2,-1), (1,0,0)\}$ and $W = \left\{ \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 5 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$.

    Which one of the following statements is correct?

    a) U and W are both linearly independent.

    b) U and W are both linearly dependent and span subspaces of dimensions 2 and 3, respectively.

    c) U is linearly independent; W is linearly dependent and spans a subspace of dimension 2.

    d) W is linearly independent; U is linearly dependent and spans a subspace of dimension 2.

    e) U is linearly independent; W is linearly dependent and spans a subspace of dimension 3.

    f) W is linearly independent; U is linearly dependent and spans a subspace of dimension 1.

    Answer: U is linearly independent; W is linearly dependent and spans a subspace of dimension 3.

19. Which of the following statements is false?

    (a) The polynomials 1, x, x^2 are linearly independent in $\mathbf{P}_2$.

    (b) Any five matrices in $\mathbf{M}_{32}$ are linearly dependent.

    (c) Any four vectors in $\mathbb{R}^3$ are linearly dependent.

    (d) Every set of n linearly independent vectors in $\mathbb{R}^n$ is a basis in $\mathbb{R}^n$.

    (e) Any $n+2$ polynomials in $\mathbf{P}_n$ are linearly dependent.

    (f) The polynomials 1, $1+x$ form a basis of $\mathbf{P}_1$.

    Answer: Any five matrices in $\mathbf{M}_{32}$ are linearly dependent.

20. Let $U = \text{span}\{\mathbf{u}_1, \cdots, \mathbf{u}_k\}$ where the $\mathbf{u}_i$ are independent, and let $W = \text{span}\ \{\mathbf{u}_1, \cdots, \mathbf{u}_k, \mathbf{v}\}$. Which of the following are true:

    a) $\dim(W) = 1 + \dim(U)$.

    b) $\dim(W) = 1 + \dim(U)$ even if the $\mathbf{u}_i$ are not independent.

    c) $\dim(W) = \dim(U)$ or $\dim(W) = 1 + \dim(U)$.

    d) $\dim(W) \geq \dim(U)$.

    e) $\dim(W) = \dim(U)$.

    Answer: c), d).

21. Let $\mathbf{F} = \{f \mid f : \mathbb{R} \to \mathbb{R},$ the second derivative f'' exists$\}$. Find all real values of a and b so that $W = \{f \mid f'' - 2f' + af = b\}$ is a subspace of $\mathbf{F}$.

    a) $a \in \mathbb{R}, b = 1$ b) $a \in \mathbb{R}, b = 0$ c) $a = 0, b = 0$

    d) $a = 0, b \in \mathbb{R}$ e) $a = 1, b = 0$ f) $a \in \mathbb{R}, b \in \mathbb{R}$

    Answer: $\underline{a \in \mathbb{R}, b = 0.}$

22. Find a basis of the vector space $U = \{A \in \mathbf{M}_{22} \mid AB = BA\}$ where $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

    (a) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

    (c) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ & 1 \end{bmatrix} \right\}$

    Answer: $\underline{\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}.}$

23. Find a basis of the vector space $U = \{A \in \mathbf{M}_{22} \mid AB = 2A\}$ where $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.

    (a) $\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

    (c) $\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \right\}$

    Answer: $\underline{\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.}$

24. Find a basis of the vector space $U = \{A \in \mathbf{M}_{22} \mid AB = B^T A\}$ where $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

    (a) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$

    (c) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

    Answer: $\underline{\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.}$

25. Which of the following is a basis of $U = \{p(x) \in P_2 \mid p(1) = 0\}$?

    (a) $B_1 = \{1 - x^2, x - x^2\}$ (b) $B_2 = \{1 - x, 1 - x^2\}$

    (c) $B_3 = \{1 - x, 2x - 2\}$ (d) $B_4 = \{x - x^2, x + 1\}$

    Answer: $\underline{B_1 \text{ and } B_2}$.

26. Find the dimension of U if $U = \{X \in \mathbf{M}_{33} \mid X^T = 2X\}$.

    (a) 2 (b) 1 (c) 0 (d) 3

    Answer: (c).

27. Find the dimension of U if $U = \{A \in \mathbf{M}_{22} \mid A^T B = BA\}$ where $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

    (a) 0 (b) 1 (c) 2 (d) 3

    Answer: (2).

28. Find the dimension of U if $U = \{A \in \mathbf{M}_{33} \mid A^T = -A\}$.

    (a) 2 (b) 3 (c) 4 (d) 1

    Answer: (3).

29. Find the dimension of U if $U = \{A \in \mathbf{M}_{22} \mid det(A) = 0\}$.

    (a) 2 (b) 3 (c) 1 (d) None of these

    Answer: (d).

30. Find the dimension of U if $U = \{p(x) \in \mathbf{P}_2 \mid p(0) = p(1)\}$.

    (a) 0 (b) 1 (c) 2 (d) None of these

    Answer: (c).

31. Find the dimension of U if $U = \{p(x) \in \mathbf{P}_3 \mid p(1) = p(-1)\}$.

    (a) 1 (b) 2 (c) 3 (d) None of these

    Answer: (b).

32. Find the dimension of U if $U = \{p(x) \in \mathbf{P}_4 \mid p(x) = -p(-x)\}$.

    (a) 2 (b) 3 (c) 4 (d) None of these

    Answer: (a).

33. Find the dimension of U if $U = \{p(x) \in \mathbf{P}_5 \mid p(x - 1) = 1\}$.

    (a) 4 (b) 2 (c) U is not a vector space (d) None of these

    Answer: (c).

34. Find the dimension of U if $U = \{A \in \mathbf{M}_{33} \mid A^T =\}$

    (a) 4 (b) 5 (c) 6 (d) None of these

    Answer: (c).

35. Find the dimension of U if $U = \{A \in \mathbf{M}_{23} \mid \text{Columns 2 and 3 in } A \text{ are equal}\}$.

    (a) 4 (b) 6 (c) 3 (d) None of these

    Answer: <u>(a)</u>.

36. Find the dimension of U if $U = \{A \in \mathbf{M}_{23} \mid \text{The sum of the columns of } A \text{ is zero}\}$.

    (a) 2 (b) 3 (c) 4 (d) None of these

    Answer: <u>(c)</u>.

37. Find the dimension of U if $U = \{A \in \mathbf{M}_{23} \mid A^T B = B^T A\}$ where $B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

    (a) 6 (b) 5 (c) 3 (d) None of these

    Answer: <u>(c)</u>.

38. Find the dimension of U if $U = \operatorname{span}\{1, \sin^2(x), \cos^2(x)\}$, a subspace of $F[-\pi, \pi]$. where

    (a) 1 (b) 2 (c) 0 (d) non of these

    Answer: <u>(2)</u>.

39. If V is the vector space spanned by the four polynomials

$$P(t) = 1 + 3t - t^2 + 4t^3, \quad Q(t) = -2 + 5t - 2t^2 + 3t^3,$$
$$R(t) = 1 + 4t + t^2 + 5t^3, \quad S(t) = 2 + t + t^2 + 3t^3,$$

    then a basis for V is:

    a) $\{P(t)\}$ b) $\{P(t), Q(t)\}$ c) $\{Q(t), R(t)\}$
    d) $\{R(t), S(t)\}$ e) $\{P(t), Q(t), S(t)\}$ f) $\{P(t), Q(t), R(t), S(t)\}$

    Answer: <u>$\{P(t), Q(t), S(t)\}$</u>.

40. Let $U = \{p(x) \in \mathbf{P}_3 \mid p(0) = p(1) = 0\}$. A basis for U is:

    a) $\{x - 1, x^2 - 1, x^2 - x\}$ b) $\{1, x, x^2, x^3\}$ c) $\{0, x - 1, x^2 - 1\}$
    d) $\{x - 1, x^2 - 1\}$ e) $\{x, x^2 - x\}$ f) $\{x^2 - x, x^3 - x^2\}$

    Answer: <u>$\{x^2 - x, x^3 - x^2\}$</u>.

41. A linear map $T : \mathbf{P}_3 \to \mathbf{P}_3$ is defined by $T(p(x) = x\,p'(x) - p(x)$, where $p'(x)$ denotes the derivative of p with respect to x. A basis for $\ker(T)$ is:

    a) $\{x^3, x^2, 1\}$ b) $\{x, 1\}$ c) $\{x^2, x, 1\}$
    d) $\{x\}$ e) $\{x^2\}$ f) $\{1\}$

    Answer: <u>$\{x\}$</u>.

42. Let $\mathbf{P}_2$ be the vector space of all polynomials of degree less than or equal to 2. Which of the following is a basis for $\mathbf{P}_2$?

a) $\{1-3x+2x^2, 1+x+4x^2, 1-7x\}$ b) $\{(4+6x+x^2, -1+4x+2x^2, 5+2x-x^2\}$
c) $\{1+x+x^2, x+x^2, x^2\}$ d) $\{1+x-x^2, 1+x+x^2\}$
e) $\{1-4x, 2x-x^2\}$ f) $\{5-2x^2, x^2\}$

Answer: $\underline{\{1+x+x^2, x+x^2, x^2\}}$.

43. Which of the following is a basis of the subspace $U = \{p(x) \mid p(-1) = 0\}$ of $\mathbf{P}_3$?

a) $\{1+x, 1-x^2, 1+x^3\}$ b) $\{1+x^3, x-x^3, x^2+x^3\}$
c) $\{1+x, x+x^2, x^2+x^3\}$ d) $\{1+x+x^2, x+x^2, x^2\}$
e) $\{1+x+x^2+x^3, 1+x^3\}$ f) $\{\{1+x+x^2+x^3, 1+x, 1+x^2, 1+x^3\}\}$

Answer: $\underline{\text{(a), (c)}}$.

44. Let $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The dimension of $C = \{A \in \mathbf{M}_{22} \mid JA = AJ\}$ is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{2}$.

45. The dimension of $V = \{p(x) \in \mathbf{P}_3 \mid p(x) = p(-x)\}$ is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{2}$.

46. $S = \{A \in \mathbf{M}_{22} \mid A = A^T\}$ is the subspace of symmetric matrices. The dimension of S is:

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: $\underline{3}$.

47. $K = \{A \in \mathbf{M}_{33} \mid A = -A^T\}$ is the subspace of skew-symmetric matrices. The dimension of K is:

a) 0 b) 2 c) 3 d) 4 e) 6 f) 9

Answer: $\underline{3}$.

48. What is the dimension of the subspace of $\mathbf{M}_{22}$ spanned by the matrices $A = \begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} -5 & 10 \\ 3 & 18 \end{bmatrix}$, $D = \begin{bmatrix} 2 & -4 \\ -1 & -7 \end{bmatrix}$?

a) 5 b) 4 c) 3 d) 2 e) 1 f) 0

Answer: $\underline{d)}$.

49. Find $\dim(U)$ if $U = \operatorname{span}\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$:

a) 2 b) 3 c) 4 d) 1

Answer: $\underline{b}$).

50. Find $\dim(U)$ if $U = \text{span}\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$:

a) 3 b) 2 c) 1 d) 4

Answer: a).

51. Find $\dim(U)$ if $U = \text{span}\{1+x, x+x^2, x^2+x^3, 1+x^3\}$:

a) 2 b) 3 c) 1 d) 4

Answer: b).

52. Find $\dim(U)$ if $U = \text{span}\{1+x+2x^2, 3+2x+3x^2, 2+x+x^2, 1\}$:

a) 1 b) 2 c) 3 d)4

Answer: c).

53. Find $\dim(U)$ if $U = \text{span}\{1-x+x^2+x^3, 1+x+x^2-x^3, 1+x^2, x-x^3\}$:

a) 3 b) 4 c) 2 d) 1

Answer: c).

54. Find $\dim(U)$ if $U = \text{span}\left\{ \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}, \right\}$:

a) 4 b) 1 c) 2 d) 3

Answer: d).

55. Find $\dim(U)$ if $U = \text{span}\left\{ \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \right\}$:

a) 1 b) 4 c) 3 d) 2

Answer: c).

56. Given the following subspace U of $\mathbf{M}_{22}$,

$$U = \left\{ A \mid A \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} A \right\},$$

what is the dimension of U?

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 2.

57. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a basis of a vector space V. The dimension of $U = \text{span}\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_3 + \mathbf{v}_4, \mathbf{v}_4 + \mathbf{v}_1\}$ is:

a) 0 b) 2 c) 3 d) 4 e) 5

Answer: c).

58. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a basis of a vector space V. The dimension of $U = \text{span}\{\mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3, \mathbf{v}_3 - \mathbf{v}_4, \mathbf{v}_4 - \mathbf{v}_1\}$ is:

    a) 0 b) 2 c) 3 d) 4 e) 5

    Answer: c).

59. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis of a vector space V. The dimension of $U = \text{span}\{\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_2 + 2\mathbf{v}_3, \mathbf{v}_3 - \mathbf{v}_1\}$ is:

    a) 2 b) 0 c) 3 d) 4 e) 5

    Answer: a).

60. Let $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{z}\}$ be a basis of a vector space V. The dimension of

    $$U = \text{ span}\{\mathbf{u} + \mathbf{v} + \mathbf{w}, \mathbf{u} + \mathbf{v} + \mathbf{z}, \mathbf{u} + \mathbf{w} + \mathbf{z}, \mathbf{v} + \mathbf{w} + \mathbf{z}\}$$

    is:

    a) 0 b) 1 c) 2 d) 3 e) 4

    Answer: e).

Chapter 7: Linear Transformations

1. Answer the following questions "Yes" or "No".

   (i) $F : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $F(x, y) = (2x, xy, 2x - 3y)$ is a linear map.

   (ii) $\det : \mathbf{M}_{22} \to \mathbb{R}$ defined by $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$ is a linear map.

   (iii) tr: $\mathbf{M}_{22} \to \mathbb{R}$ defined by $\operatorname{tr} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + d$ is a linear map.

   a) (i) Yes, (ii) Yes, (iii) Yes b) (i) Yes, (ii) Yes, (iii) No c) (i) Yes, (ii) No, (iii) Yes

   d) (i) Yes, (ii) No, (iii) No e) (i) No, (ii) Yes, (iii) Yes f) (i) No, (ii) No, (iii) Yes

   Answer: (i) No, (ii) No, (iii) Yes.

2. Let $T : V \to W$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ be vectors in V. Select the True statements:

   a) If $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ is independent in W then $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ is independent in V.

   b) If $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ is independent in V then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ is independent in W.

   c) If $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ spans W then $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ spans V.

   d) If $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ spans V then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ spans W.

   e) If $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ spans W then T is onto.

   f) If T is onto then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ spans W.

   g) If $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ is independent and T is one-to-one then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$ is independent.

   h) If $V = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ and T is onto then $W = \operatorname{span}\{T(\mathbf{v}_1), T(\mathbf{v}_2), \cdots, T(\mathbf{v}_k)\}$.

   Answer: a), e) g) h).

3. Let V and W be a vector spaces. Select the True statements:

   a) If there exists an onto linear transformation $T : V \to W$ then $\dim(V) \geq \dim(W)$.

   b) If there exists a one-to-one linear transformation $T : V \to W$ then $\dim(V) \leq \dim(W)$.

   c) No linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^5$ can be onto.

   d) No linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^5$ can be one-to-one.

   e) No linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ can be onto.

   f) No linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ can be one-to-one.

   Answer: a), b), c), f).

4. If $T : \mathbf{P}_2 \to \mathbb{R}^2$ is linear, $T(x + 1) = (2, -1)$ and $T(x + x^2) = (1, 0)$, then $T(2 - x + x^2)$ is:

   a) $(3, 2)$ b) $(3, -2)$ c) $(-2, 3)$ d) None of these

   Answer: b).

5. If $T : \mathbf{M}_{22} \to \mathbf{P}_2$ is linear, $T\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = x + 2$ and $T\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = x^2 - 1$, then $T\begin{bmatrix} 2 & 6 \\ 1 & 2 \end{bmatrix}$ is:

   a) $7 - 3x + x^2$ b) 0 c) $7 + 3x - x^2$ d) None of these

   Answer: c).

6. Suppose $T : \mathbf{F}[0, 2] \to \mathbb{R}$ is linear and $T(f) = 5$ and $T(g) = -3$ where $f(x) = x + 1$ and $g(x) = \dfrac{x+4}{x+1}$. If $h(x) = \dfrac{(x+2)(2x-1)}{x+1}$, then $T(h)$ is:

   a) 13 b) 7 c) -1 d) None of these

   Answer: a).

7. Let V be the vector space of all real 2×2 matrices of the form $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Which of the following are linear transformations $T : V \to \mathbb{R}$?

   a) $T(A) = \det A = ad - bc$ b) $T(A) = \text{tr } A = a + d$ c) $T(A) = a - 2b + 3c - 4d$

   d) $T(A) = (a - d)(b - c)$ e) $T(A) = abcd/4$

   Answer: b), c).

8. Let $C[a, b]$ be the vector space of all continuous functions on the interval $[a, b]$, and $C_1[a, b]$ be the vector space of all continuous, once differentiable functions on the interval $[a, b]$. Which of the following are linear transformations?

   a) $T : C_1[-1, 1] \to \mathbb{R}$ defined by $T(f) = f'(0)$.

   b) $T : C[0, 1] \to C[0, 1]$ defined by $T(f) = g$ where $g(x) = e^{2xf(x)}$.

   c) $T : C[0, 1] \to C[0, 1]$ defined by $T(f) = f + f^2$.

   d) $T : \mathbf{P}_2 \to \mathbf{P}_2$ defined by $T(a_0 + a_1x + a_2x^2) = (a_0 - 1) + (a_1 - 1)x + (a_2 - 1)x^2$.

   e) $T : \mathbf{P}_2 \to \mathbf{P}_2$ defined by $T(p(x)) = p(1) + xp'(x)$.

   Answer: a), b), and e).

9. If B is a fixed, invertible $n \times n$ matrix, which of the following transformations T from $\mathbf{M}_{nn}$ to $\mathbf{M}_{nn}$ are not linear?

   a) $T(A) = AB - BA$ b) $T(A) = AB + BA$ c) $T(A) = AB - BA^2$

   d) $T(A) = BAB^{-1}$ e) $T(A) = AB - B^{-1}A$ f) none of the above.

   Answer: f).

10. Let $T : \mathbf{P}_2 \to \mathbf{P}_4$ be the linear transformation defined by $T(p(x)) = x^3p'(x)$. Which of the following are true? (List all correct responses.)

    a) $2 + x + x^2$ is in $\ker T$. b) 3 is in $\ker T$. c) $x^3 + 2x^4$ is in im T.

    d) $x^4 + x^5$ is in im T. e) $\{1, x^3\}$ is a basis of $\ker T$. f) $\{x^3, x^4\}$ is a basis of im T.

    Answer: b), c), f).

11. Let $T : \mathbf{P}_2 \to \mathbb{R}^2$ be the linear map defined by $T(p(x)) = (p(2), p(-2))$. A basis for $\ker T$ is:

    a) $\{1, x, x^2\}$ b) $\{(1,0),(0,1)\}$ c) $\{(-4,0,1)\}$
    d) $\{x^2 - 4\}$ e) $\{(1,0),(0,1),(4,0)\}$ f) $\{x+2, x-2\}$

    Answer: $\underline{\{x^2 - 4\}}$.

12. Let W be the line of intersection of the planes $x + y + z = 0$ and $2x - y - z = 0$, and let $T : \mathbb{R}^3 \to W$ be the projection of $\mathbb{R}^3$ onto W. A basis for the range of T is:

    a) $\{(1,0,0),(0,1,1)\}$ b) $\{(1,0),(0,1),(1,1)\}$
    c) $\{(0,-1,1\}$ d) $\{(0,-1,1),(1,0,0),(0,1,1)\}$
    e) $\{0\}$ f) any basis of $\mathbb{R}^3$

    Answer: $\underline{\{(0,-1,1)\}}$.

13. Let U be the line of intersection of the planes $x + y + z = 0$ and $4x + y + z = 0$, and let $T : \mathbb{R}^3 \to U$ be the projection of $\mathbb{R}^3$ onto U. The formula for $T(x, y, z)$ is:

    $(0, y-z, z-y)$ b) $\frac{1}{2}(0, y-z, z-y)$ c) $(0, -y, z)$
    d) $\frac{1}{2}(x, -y, z)$ e) $(0,0,0)$ f) (x, y, z)

    Answer: $\underline{\frac{1}{2}(0, y-z, z-y)}$.

14. A linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$ is defined by $T(x, y, z, w) = (x - y + z, 2x - z + w)$. A basis for $\operatorname{Im} T$, the image of T, is:

    a) any basis of $\mathbb{R}^4$ will do b) any basis of $\mathbb{R}^2$ will do
    c) $\{(1,-1,1,0),(2,0,-1,1)\}$ d) $\{(1,2)\}$
    e) $\{(1,2),(-1,0),(1,-1)\}$ f) $\{(1,2),(-1,0),(1,-1),(0,-1)\}$

    Answer: Any basis of $\mathbb{R}^2$ will do.

15. A linear map $T : \mathbf{P}_2 \to \mathbf{P}_2$ is defined by $T(p(x)) = p(x) + p(-x)$. A basis for $\ker T$ is:

    a) $\{1, x, x^2\}$ b) $\{1, x\}$ c) $\{1 - x, x^2 - 1\}$
    d) $\{x\}$ e) $\{1, x^2\}$ f) $\{x, x^2\}$

    Answer: $\underline{\{x\}}$.

16. A linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$ is defined by $T(x, y, z, w) = (x - y + z, 2x - z + w)$. A basis for $\ker T$ is:

    a) $\{(1,-1,1,0),(2,0,-1,1)\}$ b) $\left\{\left(1,0,\frac{-1}{2},\frac{1}{2}\right),\left(0,1,\frac{-3}{2},\frac{1}{2}\right)\right\}$
    c) $\left\{\left(\frac{1}{2},\frac{3}{2}\right),(1,0)\right\}$ d) $\left\{\left(\frac{-1}{2},\frac{-1}{2}\right),(0,1)\right\}$
    e) $\left\{\left(\frac{1}{2},\frac{3}{2},1,0\right),\left(\frac{-1}{2},\frac{-1}{2},0,1\right)\right\}$ f) $\{e_1, e_2, e_3, e_4\}$, the standard basis of $\mathbb{R}^4$

    Answer: $\underline{\left\{\left(\frac{1}{2},\frac{3}{2},1,0\right),\left(\frac{-1}{2},\frac{-1}{2},0,1\right)\right\}}$.

17. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the transformation defined by $T(x, y, z) = (z, y - x, x - y, 0)$, and let $A = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Which of the following statements are true?

(i) T is linear.

(ii) $T(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3) = (1, 1, 2, 0)$ where $\mathbf{e}_1 = (1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0)$, and $\mathbf{e}_3 = (0, 0, 1)$.

(iii) A is the standard matrix of T.

a) (ii) only b) (i) and (iii) only c) (i) and (iii)

d) (i) and (ii) only e) (i) only f) all are false

Answer: (i) and (iii).

18. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ has standard matrix $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \\ 1 & -11 & -4 \end{bmatrix}$. What is $\dim \ker T$?

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 1.

19. Let a linear transformation $T : \mathbf{P}_n - > \mathbb{R}$ be defined by $T(a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n) = a_0 + a_1 + a_2 + \cdots a_n$. Then what is the dimension of $\ker T$?

a) 0 b) 1 c) $n - 1$ d) n e) $n + 1$

Answer: n.

20. A linear map $S : \mathbf{M}_{22} \to \mathbf{M}_{22}$ is defined by $S(A) = AJ - JA$, where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Find $\dim \ker S$.

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 2.

21. The trace map $T : \mathbf{M}_{nn} \to \mathbb{R}$ is defined by $T(A) = \mathrm{tr}(A)$ for $A \in \mathbf{M}_{nn}$. The dimension of $\ker(T)$ is:

a) n^2 b) $n^2 - 1$ c) n d) $n + 1$ e) 0 f) 2

Answer: b).

22. If $T : \mathbf{P}_2 \to \mathbb{R}^2$ is defined by $T[p(x)] = (p(1), p(-1))$ for $p(x) \in \mathbf{P}_2$, a basis for $\ker(T)$ is:

a) $\{1 - x\}$ b) $\{(1, 0), (0, 1)\}$ c) $\{x^4 - 1\}$ d) $\{1 - x^2\}$ e) $\{x - x^3\}$ f) $\{(1, 0), (1, 1)\}$

Answer: d).

23. Let $V = \mathbb{R}^2$ be given the following (non-standard) vector operations:

$$\begin{aligned}(x, y) \oplus (x', y') &= (x + x', y + y' + 2)\\ k \odot (x, y) &= (kx, ky + 2k - 2)\end{aligned}$$

These make V into a vector space, and $L \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a, 2a - 2)$ defines a linear map $L : \mathbf{M}_{22} \to V$. Find $\dim \ker L$.

a) 0 b) 1 c) 2

d) 3 e) 4 f) L is not linear.

Answer: 3.

24. A linear transformation $T : \mathbf{M}_{22} \to \mathbb{R}^2$ is defined by $T(A) = A \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (matrix multiplication).

If $a = \dim \ker T$ and $b = \dim \operatorname{Im} T$, then $(a, b) =$?

a) $(1, 1)$ b) $(0, 4)$ c) $(4, 0)$ d) $(3, 1)$ e) $(1, 3)$ f) $(2, 2)$

Answer: $(2, 2)$.

25. A linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$ has as standard matrix $\begin{bmatrix} 1 & -2 & 2 & -1 & 0 \\ 1 & 3 & -1 & 4 & 3 \\ 2 & 1 & 1 & 3 & 3 \end{bmatrix}$. What is $\dim \operatorname{Im} T$?

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

Answer: 2.

26. A linear map $T : \mathbf{P}_2 \to \mathbf{P}_2$ is defined by $T(p(x)) = p(x + 1)$. If $a =$ rank T and $b =$ rank $(T - I)$, where I denotes the identity transformation, then $(a, b) =$?

a) $(2, 3)$ b) $(3, 2)$ c) $(3, 3)$ d) $(1, 1)$ e) $(2, 1)$ f) $(0, 1)$

Answer: $(3, 2)$.

27. $E_a : \mathbf{P}_n \to \mathbb{R}$, $n \geq 1$, where $E_a[p(x)] = p(a)$. Select the True statements:

a) E_a is linear

b) $ker(E_a) = \{(x - a)q(x) \mid q(x) \in \mathbf{P}_{n-1}\}$

c) E_a is onto

d) E_a is one-to-one

e) $\dim(\ker(E_a)) = n + 1$

f) $\operatorname{rank}(E_a) = 1$

Answer: a), b), c), f).

28. If $\dim(V) = n$ and $T : V \to \mathbb{C}$ is an onto linear transformation, the dimension of $\ker(T)$ is:

a) n b) $n - 1$ c) $n - 3$ d) $n - 2$

Answer: $n - 2$.

29. Let $U = \{p(x) \in \mathbf{P}_n \mid p'(x) = 0\}$ where $p'(x)$ denotes the derivative of the polynomial $p(x)$. The dimension of U is:

a) $n+1$ b) $n-1$ c) n d) $n-2$

Answer: $\underline{n}$.

30. Let $U = \{X \in \mathbb{R}^n \mid X \bullet Y = 0\}$ where $Y \neq 0$ is a fixed column in $\mathbb{R}^n$. The dimension of U is:

a) n b) $n-1$ c) $n-2$ d) n^2

Answer: $\underline{n-1}$.

31. Let $U = \{X \in \mathbf{M}_{nn} \mid AXB = 0\}$ and $W = \{AXB \mid X \in \mathbf{M}_{nn}\}$, where A is $k \times n$ and B is $n \times m$. If $\dim(U) = d$ then $\dim(W)$ is:

a) n^2 b) $n^2 - d$ c) $n^2 - km$ d) km

Answer: $\underline{n^2 - d}$.

32. Let $U = \{p(x) \in \mathbf{P}_n \mid p(a) = 0\}$, where $a \in \mathbb{R}$. Then $\dim(U)$ is:

a) n b) $n-1$ c) $n+1$ d) 1

Answer: $\underline{n}$.

33. Let $U = \{p(x) \in \mathbf{P}_n \mid p(-x) = p(x)\}$ and $W = \{p(x) \in \mathbf{P}_n \mid p(-x) = -p(x)\}$. If $\dim(W) = d$ then $\dim(U)$ is:

a) $n-d$ b) $n-1-d$ c) $n+1-d$ d) d

Answer: $\underline{n+1-d}$.

34. Let $U = \{X \in \mathbf{M}_{nn} \mid X^T = X\}$ and $W = \{AXB \mid X^T = -X\}$. If $\dim(U) = d$ then $\dim(W)$ is:

a) $n^2 - d$ b) n^2 c) $n^2 - nd$ d) d^2

Answer: $\underline{n^2 - d}$.

35. If Z is a fixed matrix in $\mathbf{M}_{nk}$, let $U = \{A \in \mathbf{M}_{mn} \mid AZ = 0\}$ and $W = \{AZ \mid A \in \mathbf{M}_{mn}\}$. If $\dim(U) = d$ then $\dim(W)$ is:

a) $n^2 - d$ b) mn c) $mn - 1$ d) $mn - d$

Answer: $\underline{mn - d}$.

36. Which of the following are True:

a) $T : \mathbf{P}_4 \to \mathbb{R}^4$ is an isomorphism if $T[p(x)] = (p(0), p(1), p(-1), p(2))$.

b) $T : \mathbf{P}_3 \to \mathbb{R}^4$ is an isomorphism if $T(a + bx + cx^2 + dx^3) = (a, b+c, d, 0)$.

c) $T : \mathbf{M}_{22} \to \mathbf{M}_{22}$ is an isomorphism if $T(A) = A + NA$ where $N \in \mathbf{M}_{22}$ satisfies $N^2 = 0$.

d) $T : \mathbf{M}_{22} \to \mathbf{P}_3$ is an isomorphism if $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + (b+c)x^2 + dx^3$.

e) $T : \mathbf{M}_{mn} \to \mathbf{M}_{nm}$ is an isomorphism if $T(A) = A^T$.

f) $T : \mathbb{R}^3 \to \mathbf{P}_2$ is an isomorphism if $T(a, b, c) = (a-b) + (b-c)x + (c-a)x^2$.

g) $T : F[0,1] \to \mathbb{R}$ is an isomorphism if $T(f) = f(0)$.

h) $T : \mathbf{M}_{23} \to \mathbf{M}_{32}$ is an isomorphism if $T\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$.

Answer: $\underline{\text{c), e), h)}}$.

37. Let $T : V \to W$ and $S : W \to U$ be linear transformations. Select the True statements:

a) If $\dim(V) = m$ and $\dim(W) = n$ the matrices of T are $m \times n$.

b) If $\dim(V) = m$ and $\dim(W) = n$ the matrices of T are $n \times m$.

c) If S and T are one-to-one so also is the composite ST.

d) If S and T are onto so also is the composite ST.

e) If $\dim(V) = \dim(W)$ then T is invertible.

f) If T is invertible and $U = V$, then $S = T^{-1}$.

g) If $U = V$ then $ST = TS$.

h) If U, V and W are finite-dimensional and $ST = 1_V$, then $TS = 1_U$.

i) If U, V and W are finite-dimensional and ST is onto, $S = T^{-1}$.

Answer: $\underline{\text{b), c), d), h), i)}}$.

Chapter 8: Orthogonality

1. Suppose $\mathbf{v}$ and $\mathbf{w}$ are nonzero vectors in $\mathbb{R}^n$ such that $\|\mathbf{v}+\mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2$. Which are true:

   a) $\{\mathbf{v}, \mathbf{w}\}$ is independent.

   b) $\{\mathbf{v}, \mathbf{w}\}$ is orthonormal.

   c) $\{\mathbf{v}, \mathbf{w}\}$ is orthogonal.

   d) None of the above.

   e) All of a), b) and c).

   Answer: a) and c).

2. Determine which of the following are true for vectors in $\mathbb{R}^n$:

   a) Every orthogonal set of vectors is independent.

   b) Every orthogonal set of nonzero vectors is orthonormal.

   c) Every orthonormal set of vectors is orthogonal.

   d) Every orthonormal set of vectors is a basis of $\mathbb{R}^n$.

   e) Every orthogonal set of n vectors is a basis of $\mathbb{R}^n$.

   Answer: a), c), e).

3. Let U denote a subspace of $\mathbb{R}^n$, and let $\mathbf{v} \in \mathbb{R}^n$. Which of the following are true:

   a) If $\mathbf{v} = \mathrm{proj}_U(\mathbf{v})$ then $\mathbf{v} \in U$.

   b) If $\mathbf{v} = \mathrm{proj}_U(\mathbf{v})$ then $\mathbf{v} \in U^\perp$.

   c) $\mathbf{v} - \mathrm{proj}_U(\mathbf{v})$ is in $U^\perp$.

   d) If $\mathbf{v} \in U^\perp$ then $\mathbf{v} = \mathrm{proj}_U(\mathbf{v})$.

   e) If $\mathbf{0} \neq \mathbf{v} \in U^\perp$ and $\mathbf{v} \bullet \mathbf{w} = 0$ then $\mathbf{w} \in U$.

   f) If $\mathrm{proj}_U(\mathbf{v}) = \mathbf{0}$ then $\mathbf{v} \in U^\perp$.

   Answer: a), c), f).

4. Let U denote a subspace of $\mathbb{R}^n$, and let $\mathbf{v} \in \mathbb{R}^n$. Which of the following are true:

   a) Every vector $\mathbf{v}$ is the sum of a vector in U and one in $U^\perp$.

   b) If $\mathbf{v} \in U$ then $\mathbf{v} = \mathrm{proj}_U(\mathbf{v})$.

   c) $\mathbf{v} = \mathrm{proj}_U(\mathbf{v}) + \mathrm{proj}_{U^\perp}(\mathbf{v})$.

   d) $\mathrm{proj}_U(\mathbf{0}) = \mathbf{0}$.

   e) If $\dim(U^\perp) = n$ then $U = 0$.

   f) If $\dim(U^\perp) = 0$ then $U = \mathbb{R}^n$.

   Answer: a), c), d), e), f).

5. Let $U = \mathrm{span}\{(1,-2,3,4), (-3,6,-5,-16), (-1,2,-5,-2)\}$. Then $\dim U^\perp$ is:

   a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

   Answer: 2.

6. Given $\mathbb{R}^4$ with the dot product, let U be the row space of the matrix $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$. A basis for $U^\perp$ is:

   a) $\{(1,0,0,-1),(0,1,-1,0)\}$
   b) $\{(1,0,0,0),(0,0,1,1)\}$
   c) $\{(1,0,0,1),(0,1,1,0)\}$
   d) $\{(1,0,0,1),(0,1,1,0),(0,1,0,1)\}$
   e) $\{(1,0),(0,1)\}$
   f) $\{(0,-1),(-1,0)\}$

   Answer: $\underline{\{(1,0,0,1),(0,1,1,0)\}}$.

7. Let $U = \text{span}\{(1,2,1,5),(1,3,-4,7)\}$. Then $U^\perp$ is:

   a) $\text{span}\{(-11,-5,1,0),(5,-11,0,1)\}$
   b) $\text{span}\{(-1,-2,0,1),(0,1,0,0)\}$
   c) $\text{span}\{(-11,-5,1,0),(-1,-2,0,1)\}$
   d) $\text{span}\{(1,2,1,5),(2,-1,5,-1)\}$

   Answer: $\underline{\text{span}\{(-11,-5,1,0),(-1,-2,0,1)\}}$.

8. Let $U = \text{span}\{(2,3,-4,0),(3,5,2,7)\}$. Then $U^\perp$ is:

   a) $\text{span}\{(26,-16,1,0),(21,-14,0,1)\}$
   b) $\text{span}\{(26,-16,1,0),(1,0,0,0)\}$
   c) $\text{span}\{(1,2,-1,3),(21,-14,0,1)\}$
   d) $\text{span}\{(-2,-3,4,0),(11,-21,10,-28)\}$

   Answer: $\underline{\text{span}\{(26,-16,1,0),(21,-14,0,1)\}}$.

9. Let $U = \text{span}\{(1,0,-1,5),(2,-1,3,1),(3,-1,2,6)\}$. Then $U^\perp$ is:

   a) $\text{span}\{(1,5,1,0),(-5,1,0,1)\}$
   b) $\text{span}\{(5,9,0,-1),(1,1,3,2)\}$
   c) $\text{span}((1,0,-1,5),(0,1,5,1)\}$
   d) $\text{span}\{(1,5,1,0),(5,9,0,-1)\}$

   Answer: $\underline{\text{span}\{(1,5,1,0),(5,9,0,-1)\}}$.

10. Let $U = \text{span}\{(1,1,2,3),(1,2,0,2)\}$. A basis of $U^\perp$ is:

    a) $\text{span}\{(-4,2,1,0),(-4,1,0,1)\}$
    b) $\text{span}\{(1,0,-1,0),(0,0,1,0)\}$
    c) $\text{span}\{(-4,2,1,0),(2,4,0,0)\}$
    d) $\text{span}\{(-4,1,0,1),(1,4,0,0)\}$

    Answer: $\underline{\text{span}\{(-4,2,1,0),(-4,1,0,1)\}}$.

11. Let $U = \text{span}\{(1,1,1,-1),(1,-1,-2,2),(2,1,1,3)\}$. A basis of $U^\perp$ is:

    a) $\text{span}\{(1,3,2,0)\}$
    b) $\text{span}\{(1,-3,0,-2)\}$
    c) $\text{span}\{(1,-1,1,1)\}$
    d) $\text{span}\{(4,-12,7,-1)\}$

    Answer: $\underline{\text{span}\{(4,-12,7,-1)\}}$.

12. Let $U = \text{span}\{(1,0,2,-1),(2,3,9,5),(-1,1,0,2)\}$. A basis of $U^\perp$ is:

    a) $\text{span}\{(7,9,-4,-1)\}$
    b) $\text{span}\{(6,5,-3,0)\}$
    c) $\text{span}\{(9,9,-5,0)\}$
    d) $\text{span}\{(3,-7,0,3)\}$

    Answer: $\underline{\text{span}\{(7,9,-4,-1)\}}$

13. Which vector $\mathbf{v}$ makes $\{\mathbf{v},\, (1,1,1,1), (1,1,-1,-1), (1,-1,1,-1)\}$ into an orthogonal basis of $\mathbb{R}^4$:

a) $\mathbf{v} = (\,-a, a, a, -a),\ a \neq 0$ b) $\mathbf{v} = (\,-1, 1, 0, 0)$ c) $\mathbf{v} = (0, 0, 1, -1)$

d) $\mathbf{v} = (\,-1, 1, 1, -1)$

Answer: a), d).

14. If we apply the Gram-Schmidt process to the vectors $(3, 0, 4, 0)$, $(-1, 0, 7, 0)$, $(6, 3, -5, 4)$, in this order, to obtain an orthonormal set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, then $\mathbf{v}$ is:

a) $\frac{1}{5}\,(3, 0, 4, 0)$ b) $\frac{1}{5}(-1, 0, 7, 0)$ c) $\frac{1}{\sqrt{50}}(-1, 0, 7, 0)$

d) $\frac{1}{5}\,(0, 3, 4, 0)$ e) $(-4, 0, 3, 0)$ f) $\frac{1}{5}\,(-4, 0, 3, 0)$

Answer: $\frac{1}{5}\,(-4, 0, 3, 0)$.

15. If we apply the Gram-Schmidt process to the three vectors $(3, 0, 4, 0)$, $(-1, 0, 7, 0)$, $(6, 3, -5, 4)$, in this order, to obtain an orthonormal set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, then $\mathbf{w}$ is:

a) $(0, 3, 0, 4)$ b) $\frac{1}{5}\,(-1, 0, 7, 0)$ c) $\frac{1}{\sqrt{50}}\,(-1, 0, 7, 0)$

d) $\frac{1}{5}\,(0, 3, 4, 0)$ e) $(-4, 0, 3, 0)$ f) $\frac{1}{5}\,(-4, 0, 3, 0)$

Answer: $\frac{1}{5}(0, 3, 0, 4)$.

16. Given $\mathbf{v}_1 = (-1, 2, -1)$, $\mathbf{v}_2 = (1, 7, 1)$ and $\mathbf{v}_3 = (2, 4, 0)$, the Gram-Schmidt process allows us to construct an orthonormal basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ of $\mathbb{R}^3$ such that $\mathbf{u}_1$ is a multiple of $\mathbf{v}_1$, $\mathbf{u}_2$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, and $\mathbf{u}_3$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$. Then $\mathbf{u}_2$ is equal to:

a) $\frac{1}{\sqrt{6}}\,(-1, 2, -1)$ b) $\frac{1}{\sqrt{51}}\,(1, 7, 1)$ c) $\frac{1}{\sqrt{3}}\,(1, 1, 1)$

d) $\frac{1}{\sqrt{5}}\,(1, 2, 0)$ e) $\frac{1}{\sqrt{2}}\,(1, 0, 1)$ f) $(0, 1, 0)$

Answer: $\frac{1}{\sqrt{3}}\,(1, 1, 1)$.

17. Using the Gram-Schmidt process to obtain an orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for the span of $\mathbf{v}_1 = (1, 2, -1, 0)$, $\mathbf{v}_2 = (1, 0, -2, 1)$, and $\mathbf{v}_3 = (2, 1, 0, -1)$ in $\mathbb{R}^4$, we get $\mathbf{u}_2 =$?

a) $(1, -1, 1, 1)$ b) $\left(\frac{1}{2}, -1, \frac{-3}{2}, 1\right)$ c) $\left(\frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, -1\right)$

d) $\left(\frac{3}{2}, \frac{-1}{2}, \frac{1}{2}, -1\right)$ e) $\left(\frac{-1}{2}, -2, 3, \frac{3}{2}\right)$ f) $(1, 0, 2, 1)$

Answer: $\left(\frac{1}{2}, -1, \frac{-3}{2}, 1\right)$.

18. Let $U = \operatorname{span}\{(1, 0, 0, -1)\, , (0, 1, -1, 0)\}$. Compute $\operatorname{proj}_U(u, x, y, z)$.

a) $(u - z, x - y, y - x, z - u)$ b) $(z - u, y - x, x - y, u - z)$

c) $(y - x, z - u, u - z, x - y)$ d) $\frac{1}{2}\,(u - z, x - y, y - x, z - u)$

e) $\frac{1}{2}\,(z - u, y - x, x - y, u - z)$ f) $\frac{1}{2}\,(x - y, u - z, z - u, y - x)$

Answer: $\frac{1}{2}\,(u - z, x - y, y - x, z - u)$.

19. Let $U = \operatorname{span}\{(1, 1, 1, 1), (1, 1, -1, -1), (1, -1, 1, -1)\}$. Then $\operatorname{proj}_U(2, -1, 3, 7)$ is:

a) $\frac{1}{4}(1, 3, 19, 21)$ b) $\frac{1}{4}(1, 3, 18, 20)$ c) $(1, 3, 19, 21)$ d) $(3, 1, 1, -1)$

Answer: $\frac{1}{4}(1, 3, 19, 21)$.

20. The shortest distance from $P(2,-1,3,7)$ to $U = \text{span}\{(1,1,1,1),(1,1,-1,-1),(1,-1,1,-1)\}$ is:

    a) $\frac{7}{4}$ b) $\frac{5}{4}$ c) $\frac{1}{2}$ d) $\frac{7}{2}$

    Answer: $\underline{\frac{7}{2}}$.

21. Let $U = \text{span}\{(2,-1,4,5),(0,1,-1,1),(0,3,2,-1)\}$. Then $\text{proj}_U(21,1,-4,-5)$ is:

    a) $(21,1,-4,-5)$ b) $\frac{1}{483}(21,1,-4,-5)$ c) $(0,0,0,0)$ d) $(2,-1,4,5)$

    Answer: $\underline{(0,0,0,0)}$.

22. The shortest distance from $P(21,1,-4,-5)$ to $U = \text{span}\{(2,-1,4,5),(0,1,-1,1),(0,3,2,-1)\}$ is:

    a) 1 b) $\sqrt{483}$ c) 0 d) $\frac{1}{4}\sqrt{483}$

    Answer: $\underline{\sqrt{483}}$.

23. Let $U = \text{span}\{(1,-1,1,0),\ (1,0,2,1)\}$. Then $\text{proj}_U(1,5,3,4)$ is:

    a) $\frac{1}{\sqrt{15}}(2,-1,3,1)$ b) $(-1,13,11,12)$ c) $(2,-1,3,1)$ d) $\frac{1}{3}(-1,13,11,12)$

    Answer: $\underline{\frac{1}{3}(-1,13,11,12)}$.

24. The shortest distance from $P(1,5,3,4)$ to $U = \text{span}\{(1,-1,1,0),(1,0,2,1)\}$ is:

    a) $\frac{2}{3}\sqrt{6}$ b) 1 c) $\frac{4}{3}\sqrt{6}$ d) $\sqrt{6}$

    Answer: $\underline{\frac{2}{3}\sqrt{6}}$.

25. Let $U = \text{span}\{(1,0,0,-1),(0,1,-1,0)\}$. Find the shortest distance from $(2,0,2,0)$ to U.

    a) 0 b) 1 c) -1 d) 2 e) -2 f) 3

    Answer: $\underline{2}$.

26. Consider $U = \text{span}\{(3,0,4,0),(-4,0,3,0),(0,3,0,4)\}$. Find $\text{proj}_U(7,2,3,4)$.

    a) $(25,66,75,88)$ b) $(125,330,375,440)$ c) $\frac{1}{5}(175,66,75,88)$
    d) $\frac{1}{25}(175,66,75,88)$ e) $\frac{1}{125}(175,66,75,88)$ d) $\frac{1}{5}(1,2,3,4)$

    Answer: $\underline{\frac{1}{25}(175,66,75,88)}$.

27. Let $U = \text{span}\{(1,1,1),(1,2,1)\}$. If $\mathbf{v} = (2,0,-1)$ is written as $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$ where $\mathbf{v}_1 \in U$ and $\mathbf{v}_2 \in U^{\perp}$, then $\mathbf{v}_2$ is:

    a) $\frac{\sqrt{3}}{2}(1,0,-1)$ b) $\frac{1}{2}(1,0,1)$ c) $\frac{\sqrt{2}}{2}(1,0,1)$ d) $\frac{3}{2}(1,0,-1)$

    Answer: $\underline{\frac{3}{2}(1,0,-1)}$.

28. Let $U = \text{span}\{(1,-1,1,0),(2,0,1,1)\}$. If $\mathbf{v} = (2,-1,1,2)$ is written as $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$ where $\mathbf{v}_1 \in U$ and $\mathbf{v}_2 \in U^{\perp}$, then $\mathbf{v}_2$ is:

    a) $(-1,-2,-1,3)$ b) $\frac{1}{3}(7,-1,4,3)$ c) $(7,-1,4,3)$ d) $\frac{1}{3}$

    Answer: $\underline{\frac{1}{3}(-1,-2,-1,3)}$.

29. Let $U = \text{span}\{(1,0,0,1),(1,1,0,1)\}$. If $\mathbf{v} = (2,-1,1,0)$ is written as $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$ where $\mathbf{v}_1 \in U$ and $\mathbf{v}_2 \in U^\perp$, then $\mathbf{v}_2$ is:

    a) $(1,0,1,-1)$ b) $\frac{1}{3}(2,-1,0,2)$ c) $(1,-1,0,1)$ d) $\frac{1}{3}$

    Answer: $\underline{(1,0,1,-1)}$.

30. Let $U = \text{span}\{(1,-1,2,1),(1,0,2,2)\}$. If $\mathbf{v} = (5,-2,1,5)$ is written as $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$ where $\mathbf{v}_1 \in U$ and $\mathbf{v}_2 \in U^\perp$, then $\mathbf{v}_2$ is:

    a) $\frac{3}{2}(2,-1,-2,1)$ b) $\frac{1}{2}(2,-1,-2,1)$ c) $\frac{1}{2}(4,-1,8,7)$ d) $\frac{3}{2}(4,-1,8,7)$

    Answer: $\underline{\frac{3}{2}(2,-1,-2,1)}$.

31. Let $U = \text{span}\{(1,1,-1,-1),(3,1,1,-1)\}$. If $\mathbf{v} = (2,1,0,1)$ is written as $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$ where $\mathbf{v}_1 \in U$ and $\mathbf{v}_2 \in U^\perp$, then $\mathbf{v}_2$ is:

    a) $\frac{1}{2}(3,1,1,-1)$ b) $\frac{1}{2}(1,1,-1,3)$ c) $\frac{3}{2}(3,1,1,-1)$ d) $\frac{3}{2}(1,1,-1,3)$

    Answer: $\frac{1}{2}(1,1,-1,3)$.

32. Let $U = \text{span}\{(1,-1,1,1),(1,2,1,0),(1,0,1,2)\}$. If $\mathbf{v} = (2,0,1,0)$ is written as $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$ where $\mathbf{v}_1 \in U$ and $\mathbf{v}_2 \in U^\perp$, then $\mathbf{v}_2$ is:

    a) $(1,0,-1,0)$ b) $\frac{1}{2}(1,0,-1,0)$ c) $\frac{3}{2}(1,0,1,0)$ d) $\frac{1}{2}(-2,1,0,3)$

    Answer: $\frac{1}{2}(1,0,-1,0)$.

33. Suppose $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Then the dimension of the subspace of $\mathbb{R}^5$ which consists of vectors perpendicular to all the rows of A is:

    a) 0 b) 1 c) 2 d) 3 e) 4 f) 5

    Answer: $\underline{1}$.

34. If P and Q denote $n \times n$ matrices, list which of the following are true:

    a) If P is orthogonal then P^T is orthogonal.

    b) If P is orthogonal then $\det(P) = 1$.

    c) If P is orthogonal then P is invertible and P^{-1} is orthogonal.

    d) If P and Q are orthogonal then PQ is orthogonal.

    e) If PQ is orthogonal then P and Q are both orthogonal.

    f) If P is orthogonal then $\det(P) = \pm 1$.

    Answer: <u>a), c), d), f)</u>.

35. If $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, the orthogonal matrix P formed by normalizing the columns of A is:

a) $\begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ b) $\frac{1}{\sqrt{6}}\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ c) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{3} & -\sqrt{2} & -\sqrt{2} \\ \sqrt{3} & \sqrt{2} & 0 \\ \sqrt{3} & 0 & \sqrt{2} \end{bmatrix}$

d) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & -\sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{3} & 0 \\ \sqrt{2} & 0 & \sqrt{3} \end{bmatrix}$ e) P does not exist

Answer: d) .

36. If $A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{bmatrix}$, the orthogonal matrix P formed by normalizing the columns of A is:

a) $\frac{1}{3}\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{bmatrix}$ b) $\frac{1}{\sqrt{3}}\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{bmatrix}$ c) $\frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ e) P does not exist

Answer: a).

37. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, the orthogonal matrix P formed by normalizing the columns of A is:

a) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & 1 & -2 \\ \sqrt{2} & -2 & 1 \\ \sqrt{2} & 1 & 1 \end{bmatrix}$ b) $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ c) $\frac{1}{3}\begin{bmatrix} \sqrt{2} & 1 & -2 \\ \sqrt{2} & -2 & 1 \\ \sqrt{2} & 1 & 1 \end{bmatrix}$

d) $\frac{1}{\sqrt{6}}\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ e) P does not exist

Answer: e).

38. If $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$, the orthogonal matrix P formed by normalizing the columns

of A is:

a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ b) $\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ c) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

d) $\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ e) P does not exist

Answer: d).

39. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$, the orthogonal matrix P formed by normalizing the columns of A is:

a) $\frac{1}{\sqrt{3}}\begin{bmatrix} \frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & 1 \\ \frac{1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} & 1 \\ \frac{2}{\sqrt{2}} & 0 & -1 \end{bmatrix}$ b) $\frac{1}{\sqrt{6}}\begin{bmatrix} 1 & \sqrt{3} & \sqrt{2} \\ 1 & -\sqrt{3} & \sqrt{2} \\ 2 & 0 & -\sqrt{2} \end{bmatrix}$ c) $\frac{1}{6}\begin{bmatrix} 1 & \sqrt{3} & \sqrt{2} \\ 1 & -\sqrt{3} & \sqrt{2} \\ 2 & 0 & -\sqrt{2} \end{bmatrix}$

d) $\frac{1}{6}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$ e) P does not exist

Answer: b).

40. Find all values of a for which the matrix $\begin{bmatrix} 0 & a & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ is orthogonal.

a) all values of a b) 1 or -1 c) 1 only
d) -1 only e) $\frac{\sqrt{2}}{2}$ or $\frac{-\sqrt{2}}{2}$ f) no values of a

Answer: f).

41. Find all values of a and b for which the matrix $\frac{1}{b}\begin{bmatrix} 0 & a & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ is orthogonal.

a) $a = b \neq 0$ b) $a = 2$ and $b = \sqrt{2}$. c) $a = b = \sqrt{2}$
d) $a = b = -1$ e) $a = \frac{\sqrt{2}}{2}$ and $b = \frac{-\sqrt{2}}{2}$ f) no such values of a and b

Answer: c).

42. If $A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$, an orthogonal matrix P such that P^TAP is diagonal is:

a) $\frac{1}{\sqrt{5}}\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$ b) $\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ c) I_2 d) No such P

Answer: b).

43. If $A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$, an orthogonal matrix P such that P^TAP is diagonal is:

a) $\frac{1}{\sqrt{5}}\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$ b) $\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ c) I_2 d) No such P

Answer: b).

44. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, an orthogonal matrix P such that P^TAP is diagonal is:

a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ b) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ c) $\frac{1}{\sqrt{10}}\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$ d) No such P

Answer: a).

45. If $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$, an orthogonal matrix P such that P^TAP is diagonal is:

a) $\frac{1}{\sqrt{34}}\begin{bmatrix} 5 & -3 \\ 3 & 5 \end{bmatrix}$ b) $\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ d) No such P

Answer: c).

46. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$, an orthogonal matrix P such that P^TAP is diagonal is:

a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ b) I_3 c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{bmatrix}$ d) No such P

Answer: c)

47. If $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 3 & 0 \\ -2 & 0 & 3 \end{bmatrix}$, an orthogonal matrix P such that $P^T AP$ is diagonal is:

a) $\frac{1}{6}\begin{bmatrix} \sqrt{2} & 2 & 0 \\ \sqrt{2} & -1 & \sqrt{3} \\ -\sqrt{2} & 1 & \sqrt{3} \end{bmatrix}$ b) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & 2 & 0 \\ \sqrt{2} & -1 & \sqrt{3} \\ -\sqrt{2} & 1 & \sqrt{3} \end{bmatrix}$ c) $\frac{1}{3\sqrt{14}}\begin{bmatrix} \sqrt{14} & 6 & -6 \\ 2\sqrt{14} & 9 & 0 \\ -2\sqrt{14} & 0 & 9 \end{bmatrix}$

d) No such P

Answer: b)

48. If $A = \begin{bmatrix} 5 & -2 & -4 \\ -2 & 8 & -2 \\ -4 & -2 & 5 \end{bmatrix}$, an orthogonal matrix P such that $P^T AP$ is diagonal is:

a) $\frac{1}{3\sqrt{5}}\begin{bmatrix} 2\sqrt{5} & 3 & 4 \\ \sqrt{5} & -6 & 2 \\ 2\sqrt{5} & 0 & -5 \end{bmatrix}$ b) $\frac{1}{\sqrt{15}}\begin{bmatrix} 2\sqrt{15} & \sqrt{5} & 4 \\ \sqrt{15} & -2\sqrt{5} & 2 \\ 2\sqrt{15} & 0 & -5 \end{bmatrix}$ c) $\frac{1}{3\sqrt{5}}\begin{bmatrix} 2\sqrt{5} & 3 & 0 \\ \sqrt{5} & -6 & -6 \\ 2\sqrt{5} & 0 & 3 \end{bmatrix}$

d) No such P

Answer: a)

49. If $A = \begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$, an orthogonal matrix P such that $P^T AP$ is diagonal is:

a) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & 0 & \sqrt{3} \\ \sqrt{2} & -\sqrt{3} & 0 \end{bmatrix}$ b) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & \sqrt{3} & 1 \\ \sqrt{2} & 0 & -2 \\ \sqrt{2} & -\sqrt{3} & 1 \end{bmatrix}$ c) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{3} & 0 & -\sqrt{3} \\ -\sqrt{3} & \sqrt{3} & 0 \end{bmatrix}$

d) No such P

Answer: b).

50. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, an orthogonal matrix P such that $P^T AP$ is diagonal is:

a) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & -\sqrt{3} & \sqrt{3} \\ -\sqrt{2} & 0 & \sqrt{3} \\ \sqrt{2} & \sqrt{3} & 0 \end{bmatrix}$ b) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & -\sqrt{2} & \sqrt{2} \\ -\sqrt{3} & 0 & \sqrt{3} \\ 1 & 2 & 1 \end{bmatrix}$ c) $\frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & -\sqrt{3} & 1 \\ -\sqrt{2} & 0 & 2 \\ \sqrt{2} & \sqrt{3} & 1 \end{bmatrix}$

d) No such P

Answer: c).

51. If $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & a \end{bmatrix}$ is positive definite then a is:

a) $a > 0$ b) $a = 5$ c) $a > 5$ d) No such a

Answer: $\underline{a > 5}$.

52. The Cholesky factorization of $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ is $U^T U$ where U is:

a) $\frac{\sqrt{3}}{3}\begin{bmatrix} 3 & -2 \\ 0 & \sqrt{5} \end{bmatrix}$ b) $\begin{bmatrix} 3 & 2 \\ 0 & \sqrt{5} \end{bmatrix}$ c) $\frac{1}{3}\begin{bmatrix} \sqrt{3} & 2 \\ 0 & \sqrt{5} \end{bmatrix}$ d) $\frac{\sqrt{3}}{3}\begin{bmatrix} 3 & 2 \\ 0 & \sqrt{5} \end{bmatrix}$

Answer: d).

53. The Cholesky factorization of $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ is $U^T U$ where U is:

a) $\frac{\sqrt{5}}{5}\begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix}$ b) $\frac{\sqrt{5}}{5}\begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$ c) $\frac{\sqrt{5}}{5}\begin{bmatrix} 5 & -3 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 5 & 3 \\ 0 & 1/5 \end{bmatrix}$

Answer: a).

54. The Cholesky factorization of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is $U^T U$ where U is:

a) $\frac{1}{6}\begin{bmatrix} 6\sqrt{2} & -3\sqrt{2} & 3\sqrt{2} \\ 0 & 3\sqrt{6} & \sqrt{6} \\ 0 & 0 & 4\sqrt{3} \end{bmatrix}$ b) $\frac{1}{6}\begin{bmatrix} 6\sqrt{2} & 3\sqrt{2} & 3\sqrt{2} \\ 0 & 3\sqrt{6} & -\sqrt{6} \\ 0 & 0 & 4\sqrt{3} \end{bmatrix}$ c) $\frac{1}{6}\begin{bmatrix} 6\sqrt{2} & -3\sqrt{2} & 3\sqrt{2} \\ 0 & 3\sqrt{6} & -\sqrt{6} \\ 0 & 0 & 4\sqrt{3} \end{bmatrix}$ d) $\frac{1}{6}\begin{bmatrix} 6\sqrt{2} & 3\sqrt{2} & -3\sqrt{2} \\ 0 & 3\sqrt{6} & \sqrt{6} \\ 0 & 0 & 4\sqrt{3} \end{bmatrix}$

Answer: b).

55. The Cholesky factorization of $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 5 & 1 \\ -1 & 1 & 15 \end{bmatrix}$ is $U^T U$ where U is:

a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ b) $\frac{1}{\sqrt{2}}\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 2 & -3 & 1 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{bmatrix}$ d) $\frac{1}{\sqrt{2}}\begin{bmatrix} 2 & -3 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{bmatrix}$

Answer: c).

56. The QR-factorization of $\begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 1 & 0 \end{bmatrix}$ is QR where:

a) $Q = \begin{bmatrix} \sqrt{5} & 0 \\ 2\sqrt{5} & \sqrt{6} \\ \sqrt{5} & -2\sqrt{6} \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{6} & 2\sqrt{6} \\ 0 & \sqrt{5} \end{bmatrix}$

b) $Q = \frac{1}{\sqrt{30}} \begin{bmatrix} \sqrt{5} & 0 \\ 2\sqrt{5} & \sqrt{6} \\ \sqrt{5} & -2\sqrt{6} \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{6} & 2\sqrt{6} \\ 0 & \sqrt{5} \end{bmatrix}$

c) $Q = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & -2 \end{bmatrix}$, $R = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

d) None of the above

Answer: b).

57. The QR-factorization of $\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & 2 \end{bmatrix}$ is QR where:

a) $Q = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & \sqrt{3} \\ -1 & \sqrt{3} \\ 2 & 0 \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{6} & \sqrt{6} \\ 0 & \sqrt{2} \end{bmatrix}$

b) $Q = \begin{bmatrix} 1 & \sqrt{3} \\ -1 & \sqrt{3} \\ 2 & 0 \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{6} & \sqrt{6} \\ 0 & \sqrt{2} \end{bmatrix}$

c) $Q = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$, $R = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$

d) None of the above

58. Determine which of the following are true of the complex matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$:

a) Unitary

b) Hermitian

c) Normal

e) None of the above

Answer: a), b), c) .

59. Determine which of the following are true of the complex matrix $\begin{bmatrix} 1+i & i \\ -i & -1+i \end{bmatrix}$:

a) Unitary

b) Hermitian

c) Normal

d) None of the above

Answer: c) .

60. Determine which of the following are true of the complex matrix $\frac{1}{\sqrt{6}}\begin{bmatrix} 1+i & 2i \\ -2i & -1+i \end{bmatrix}$:

a) Unitary

b) Hermitian

c) Normal

d) None of the above

Answer: a), c) .

61. Determine which of the following are true of the complex matrix $\frac{1}{\sqrt{6}}\begin{bmatrix} 1 & 2i \\ -2i & -1 \end{bmatrix}$:

a) Unitary

b) Hermitian

c) Normal

d) None of the above

Answer: b), c) .

62. Determine which of the following are True:

a) Every unitary matrix has real eigenvalues.

b) Every matrix with real eigenvalues is unitary.

c) If U is unitary then $\det(U) = 1$.

d) Every real orthogonal matrix is unitary

e) If U is unitary then $\|UX\| = \|X\|$ for every column X in $\mathbb{C}^n$.

Answer: a), d) and e).

63. Determine which of the following are True:

a) Every Hermitian matrix has real eigenvalues.

b) Every Hermitian matrix is unitary.

c) Every real matrix is Hermitian.

d) If H is Hermitian then H^T is Hermitian.

Answer: a), and d) .

64. If $Z = \begin{bmatrix} 2 & 1+i \\ 1-i & 3 \end{bmatrix}$, a unitary matrix U such that U^*ZU is diagonal is:

a) $\frac{1}{\sqrt{3}} \begin{bmatrix} 1-i & 1 \\ -1 & 1+i \end{bmatrix}$ b) $\frac{1}{\sqrt{3}} \begin{bmatrix} 1+i & 1 \\ -1 & 1-i \end{bmatrix}$ c) $\frac{1}{\sqrt{3}} \begin{bmatrix} 1+i & -1 \\ 1 & 1-i \end{bmatrix}$ d) No such matrix

Answer: b).

65. If $Z = \begin{bmatrix} 4 & 3-i \\ 3+i & 1 \end{bmatrix}$, a unitary matrix U such that U^*ZU is diagonal is:

a) $\frac{1}{\sqrt{14}} \begin{bmatrix} 3-i & -2 \\ 2 & 3+i \end{bmatrix}$ b) $\frac{1}{\sqrt{14}} \begin{bmatrix} 3+i & -2 \\ 2 & 3-i \end{bmatrix}$ c) $\frac{1}{\sqrt{3}}$ [] d) No such matrix

Answer: a).

66. If $Z = \begin{bmatrix} 3 & 1-i \\ 1+i & 2 \end{bmatrix}$, a unitary matrix U such that U^*ZU is diagonal is:

a) $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ b) $\frac{1}{\sqrt{3}} \begin{bmatrix} 1-i & -1 \\ 1 & 1+i \end{bmatrix}$ c) $\frac{1}{\sqrt{3}} \begin{bmatrix} 1-i & 1 \\ -1 & 1+i \end{bmatrix}$ d) No such matrix

Answer: b).

67. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$, a best approximation Z to a solution to $AX = B$ is:

a) $\frac{1}{83} \begin{bmatrix} 35 \\ 44 \end{bmatrix}$ b) $\frac{1}{83} \begin{bmatrix} 35 \\ -44 \end{bmatrix}$ c) $\frac{1}{83} \begin{bmatrix} -35 \\ 44 \end{bmatrix}$ d) $\frac{1}{12} \begin{bmatrix} -5 \\ 6 \end{bmatrix}$

Answer: c).

68. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, a best approximation Z to a solution to $AX = B$ is:

a) $\frac{1}{101}\begin{bmatrix} 83 \\ -39 \end{bmatrix}$ b) $\frac{1}{101}\begin{bmatrix} -83 \\ 39 \end{bmatrix}$ c) $\frac{1}{5}\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ d) $\frac{1}{101}\begin{bmatrix} 83 \\ 39 \end{bmatrix}$

Answer: a).

69. If $A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 2 & 1 & 7 \\ 3 & 0 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix}$, a best approximation Z to a solution to $AX = B$ is:

a) $\frac{1}{201}\begin{bmatrix} 139 \\ 42 \\ 0 \end{bmatrix}$ b) $\frac{1}{201}\begin{bmatrix} 139 \\ -42 \\ 0 \end{bmatrix}$ c) $\frac{1}{201}\begin{bmatrix} -464 \\ -243 \\ 201 \end{bmatrix}$ d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Answer: b), c).

70. Given a Universal Product Code (UPC) with first eleven digits 8,3,4,2,6,9,0,4,5,8,4, the value of the check-digit must be:

a) 1 b) 2 c) 3 d) 4 e) 5

Answer: c).

71. Given the code C = {0000000,0010101,0101010,0111111,1001111,1011010,1100101,1110000}, the number of errors that can be detected,corrected is:

a) $1,1$ b) $2,1$ c) $2,2$ d) $3,1$ e) $3,2$

Answer: b).

72. Given the code C = {0000000000,0010100110,0101101011,0111001101,1001011010,1011111100, 1100110001,1110010111}, the number of errors that can be detected,corrected is:

a) $1,1$ b) $2,1$ c) $2,2$ d) $3,1$ e) $3,2$

Answer: d).

73. New variables that diagonalize the quadratic form $q = x_1^2 + x_2^2 + x_3^2 - 4(x_1x_2 + x_1x_3 + x_2x_3)$ are:

a) $y_1 = \frac{-3}{\sqrt{3}}(x_1 + x_2 + x_3), \quad y_2 = \frac{3}{\sqrt{2}}(x_1 - x_2), \quad y_3 = 3(x_1 + x_2 - 2x_3)$

b) $y_1 = \frac{1}{\sqrt{3}}(x_1 + x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{2}}(x_1 - x_2), \quad y_3 = x_1 + x_2 - 2x_3$

c) $y_1 = \frac{1}{\sqrt{6}}(\sqrt{2}x_1 + \sqrt{3}x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{6}}(\sqrt{2}x_1 - \sqrt{3}x_2 + x_3), \quad y_3 = \frac{1}{\sqrt{6}}(\sqrt{2}x_1 - 2x_3)$

d) $y_1 = \frac{1}{\sqrt{6}}(x_1 - 2x_2 - 2x_3), \quad y_2 = (-2x_1 + x_2 - 2x_3), \quad y_3 = (-2x_1 - 2x_2 + x_3)$

Answer: b).

74. New variables that diagonalize the quadratic form $q = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 + x_1x_3 - x_2x_3)$ are:

a) $y_1 = \frac{1}{\sqrt{3}}(x_1 + x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{2}}(x_1 + x_3), \quad y_3 = \frac{1}{\sqrt{6}}(x_1 - 2x_2 + x_3)$

b) $y_1 = \frac{1}{\sqrt{3}}(x_1 + x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{2}}(x_1 - x_3), \quad y_3 = \frac{1}{\sqrt{6}}(x_1 - 2x_2 + x_3)$

c) $y_1 = \frac{1}{\sqrt{3}}(x_1 - x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{2}}(x_1 - x_3), \quad y_3 = \frac{1}{\sqrt{6}}(x_1 + 2x_2 + x_3)$

d) None of the above

Answer: c).

75. New variables that diagonalize the quadratic form $q = x_1^2 + x_2^2 + x_3^2 - 2(x_1x_2 + x_1x_3 + x_2x_3)$ are:

a) $y_1 = \frac{1}{\sqrt{3}}(x_1 + x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{2}}(x_1 - x_3), \quad y_3 = \frac{1}{\sqrt{6}}(x_1 - 2x_2 + x_3)$

b) $y_1 = \frac{1}{\sqrt{6}}(\sqrt{2}x_1 + \sqrt{3}x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{6}}(\sqrt{3}x_1 - \sqrt{3}x_3), \quad y_3 = \frac{1}{\sqrt{6}}(\sqrt{2}x_1 - \sqrt{3}x_2 + x_3)$

c) $y_1 = \frac{1}{\sqrt{6}}(x_1 + x_2 + x_3), \quad y_2 = \frac{1}{\sqrt{6}}(x_1 - x_3), \quad y_3 = \frac{1}{\sqrt{6}}(x_1 - 2x_2 + x_3)$

d) None of the above.

Answer: a).

76. New variables that diagonalize the quadratic form $q = x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3$ are:

a) $y_1 = \frac{1}{3}(2x_1 + 2x_2 + x_3), \quad y_2 = \frac{1}{3}(x_1 - 2x_2 + 2x_3), \quad y_3 = \frac{1}{3}(-2x_1 + x_2 + 2x_3)$

b) $y_1 = \frac{1}{9}(2x_1 + x_2 - 2x_3), \quad y_2 = \frac{1}{9}(2x_1 - 2x_2 + x_3), \quad y_3 = \frac{1}{9}(x_1 + 2x_2 + 2x_3)$

c) $y_1 = \frac{1}{3}(2x_1 + x_2 - 2x_3), \quad y_2 = \frac{1}{3}(2x_1 - 2x_2 + x_3), \quad y_3 = \frac{1}{3}(x_1 + 2x_2 + 2x_3)$

d) None of the above.

Answer: c).

77. Consider the following homogeneous system of differential equations: $\begin{bmatrix} f_1' \\ f_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$.

The solution satisfying the initial condition $f_1(0) = -1$, $f_2(0) = 1$ is:

a) $\begin{bmatrix} f_1(x) & = & e^{-x} - e^{2x} \\ f_2(x) & = & e^{-x} - 4e^{2x} \end{bmatrix}$

b) $\begin{bmatrix} f_1(x) & = & -e^{x} - e^{2x} \\ f_2(x) & = & -e^{x} - 4e^{2x} \end{bmatrix}$

c) $\begin{bmatrix} f_1(x) & = & -e^{-x} + 4e^{2x} \\ f_2(x) & = & -e^{-x} + e^{2x} \end{bmatrix}$

d) $\begin{bmatrix} f_1(x) & = & -e^{-x} + e^{2x} \\ f_2(x) & = & -e^{-x} + 4e^{2x} \end{bmatrix}$

e) $\begin{bmatrix} f_1(x) & = & -e^{x} + 4e^{-2x} \\ f_2(x) & = & -e^{x} + e^{-2x} \end{bmatrix}$

Answer: c).

78. Consider the following homogeneous system of differential equations: $\begin{bmatrix} f_1' \\ f_2' \\ f_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 5 & 2 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$.

The solution satisfying the initial condition $f_1(0) = -2$, $f_2(0) = 3$, and $f_3(0) = 3$ is:

a) $\begin{bmatrix} f_1(x) & = & e^x + e^{-2x} - e^{3x} \\ f_2(x) & = & e^x - 2e^{-2x} - 3e^{3x} \\ f_3(x) & = & e^x + 4e^{-2x} + 9e^{3x} \end{bmatrix}$

b) $\begin{bmatrix} f_1(x) & = & -2e^x - e^{-2x} + e^{3x} \\ f_2(x) & = & -2e^x + 2e^{-2x} + 3e^{3x} \\ f_3(x) & = & -2e^x - 4e^{-2x} + 9e^{3x} \end{bmatrix}$

c) $\begin{bmatrix} f_1(x) & = & -e^x + e^{-2x} - e^{3x} \\ f_2(x) & = & -2e^x - 3e^{-2x} - 3e^{3x} \\ f_3(x) & = & 4e^x + 9e^{-2x} + 9e^{3x} \end{bmatrix}$

d) $\begin{bmatrix} f_1(x) & = & -e^x + e^{2x} - e^{3x} \\ f_2(x) & = & -e^x - 2e^{2x} - 3e^{3x} \\ f_3(x) & = & -e^x + 4e^{2x} + 9e^{3x} \end{bmatrix}$

e) $\begin{bmatrix} f_1(x) & = & e^x - e^{-2x} - e^{3x} \\ f_2(x) & = & -2e^x - e^{-2x} - 3e^{3x} \\ f_3(x) & = & 4e^x - e^{-2x} + 9e^{3x} \end{bmatrix}$

Answer: b).

79. Consider the system $\begin{array}{ccccccc} f_1' & = & f_1 & - & f_2 & + & 4f_3 \\ f_2' & = & 3f_1 & + & 2f_2 & - & f_3 \\ f_3' & = & 2f_1 & + & f_2 & - & f_3 \end{array}$ of differential equations. The solution satisfying $f_1(0) = f_2(0) = f_3(0) = 1$ is:

a) $\begin{bmatrix} f_1(x) & = & \frac{1}{3}(e^x + 3e^{3x} - e^{-2x}) \\ f_2(x) & = & \frac{1}{3}(-4e^x + 6e^{3x} + e^{-2x}) \\ f_3(x) & = & \frac{1}{3}(-e^x + 3e^{3x} + e^{-2x}) \end{bmatrix}$

b) $\begin{bmatrix} f_1(x) & = & e^x + 3e^{3x} - e^{-2x} \\ f_2(x) & = & -4e^x + 6e^{3x} + e^{-2x} \\ f_3(x) & = & -e^x + 3e^{3x} + e^{-2x} \end{bmatrix}$

c) $\begin{bmatrix} f_1(x) & = & \frac{1}{3}(-e^x + 3e^{3x} - e^{-2x}) \\ f_2(x) & = & \frac{1}{3}(4e^x + 6e^{3x} + e^{-2x}) \\ f_3(x) & = & \frac{1}{3}(e^x + 3e^{3x} + e^{-2x}) \end{bmatrix}$

d) $\begin{bmatrix} f_1(x) & = & \frac{1}{3}(e^x + 3e^{3x} + e^{-2x}) \\ f_2(x) & = & \frac{1}{3}(-4e^x + 6e^{3x} - e^{-2x}) \\ f_3(x) & = & \frac{1}{3}(-e^x + 3e^{3x} - e^{-2x}) \end{bmatrix}$

Answer: a).

Chapter 9: Change of Basis

1. Let $T : V \to W$ and $S : W \to U$ be linear transformations. Select the True statements:

   a) If $\dim(V) = m$ and $\dim(W) = n$ the matrices of T are $m \times n$.

2. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - 2y \\ 4x + 7y \end{bmatrix}$. The matrix of T with respect to the standard basis of $\mathbb{R}^2$ is:

   a) $\begin{bmatrix} 7 & -2 \\ 3 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 7 & -2 \\ 4 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -2 \\ 4 & 7 \end{bmatrix}$

   d) $\begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}$ e) $\begin{bmatrix} 4 & -2 \\ 7 & 3 \end{bmatrix}$ f) $\begin{bmatrix} 4 & -2 \\ 3 & 7 \end{bmatrix}$

   Answer: $\underline{\begin{bmatrix} 3 & -2 \\ 4 & 7 \end{bmatrix}}$.

3. A linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$ is defined by $T(x, y, z, w) = (x - y + z, 2x - z + w)$. Find the standard matrix of T.

   a) $\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & 0 & -1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$ e) $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

   Answer: $\underline{\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & 0 & -1 & 1 \end{bmatrix}}$.

4. Let $T : \mathbf{M}_{33} \to \mathbf{M}_{33}$ be given by $T(A) = A - A^T$. The matrix of T with respect to the standard basis of $\mathbf{M}_{33}$ is:

   a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   Answer: <u>a)</u> .

5. If $T : P_3 \to P_3$ is given by $T[p(x)] = \frac{1}{2}[p(x) + p(-x)]$, the matrix of T with respect to the standard basis of $\mathbf{P}_2$ is:

a) $\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Answer: c).

6. If $T : \mathbf{M}_{22} \to \mathbf{M}_{22}$ is given by $T(A) = A^T$, the matrix of T with respect to the standard basis of $\mathbf{M}_{22}$ is:

a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Answer: b).

7. Define $T : \mathbf{P}_2 \to \mathbb{R}^2$ by $T[p(x)] = (p(1), p(-1))$ for $p(x) \in \mathbf{P}_2$. If $B = \{x^2, x+1, x-1\}$ and $D = \{(1,1), (-1,1)\}$, then $M_{DB}(T)$ is:

a) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Answer: a).

8. Let B be the standard basis in the plane and let $B' = \{(7,11), (5,8)\}$ be another basis. Find the change matrix $P_{B' \leftarrow B}$ from B to B'.

a) $\begin{bmatrix} 7 & 5 \\ 11 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 7 & 11 \\ 5 & 8 \end{bmatrix}$ c) $\begin{bmatrix} 8 & -5 \\ -11 & 7 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -11 \\ -5 & 7 \end{bmatrix}$

Answer: c).

9. Let B be the standard basis for $\mathbb{R}^3$ and $B' = \{(2,0,5), (0,1,0), (1,0,3)\}$ be another. Find the change matrix $P_{B' \leftarrow B}$ from B to B'.

a) $\begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 0 & -5 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ -5 & 0 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 3 & -1 \\ 1 & 0 & 0 \\ 0 & -5 & 2 \end{bmatrix}$

Answer: c).

10. $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ and $B' = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ are ordered bases for $S = \{A \in \mathbf{M}_{22} \mid A = A^T\}$. Find the change matrix $P_{B \leftarrow B'}$ from B' to B.

a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Answer: a).

11. $B = \{1-x+x^2, 1-x^2, 1+x+x^2\}$ and $B' = \{1-x^2, 1+x^2, x\}$ are bases for $\mathbf{P}_2$. Find the change matrix $P_{B' \leftarrow B}$ from B to B'.

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Answer: d).

12. If $\mathbf{P}_2$ denotes the vector space of polynomials of degree at most 2, then $B = \{1+x, x+x^2, 1\}$ and $B' = \{2x+3, x, 2x^2+3\}$ are bases for $\mathbf{P}_2$. The change matrix from B to B' is:

a) $\begin{bmatrix} \frac{1}{3} & -\frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & 2 & -\frac{2}{3} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ -\frac{1}{3} & 2 & \frac{2}{3} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 3 & -2 & 3 \\ 3 & 2 & 3 \\ 0 & 2 & 0 \end{bmatrix}$ e) $\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} & \frac{2}{3} \\ -\frac{2}{3} & 2 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 \end{bmatrix}$

Answer: a).

13. $B' = \left\{\frac{1}{5}(3,0,4,0), \frac{1}{5}(-4,0,3,0)\right\}, \frac{1}{5}\left(0,-3,0,4), \frac{1}{5}(0,4,0,3)\right)$ is an orthonormal basis of $\mathbb{R}^4$. If B is the standard basis for $\mathbb{R}^4$, find the change matrix $P_{B' \leftarrow B}$ from B to B'.

a) $\frac{1}{5}\begin{bmatrix} 3 & 0 & 4 & 0 \\ -4 & 0 & 3 & 0 \\ 0 & -3 & 0 & 4 \\ 0 & 4 & 0 & 3 \end{bmatrix}$ b) $\frac{1}{5}\begin{bmatrix} 3 & -4 & 0 & 0 \\ 0 & 0 & -3 & 4 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 4 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 0 & 4 & 0 \\ -4 & 0 & 3 & 0 \\ 0 & -3 & 0 & 4 \\ 0 & 4 & 0 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -4 & 0 & 0 \\ 0 & 0 & -3 & 4 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 4 & 3 \end{bmatrix}$ e) $\frac{1}{5}\begin{bmatrix} 3 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ f) $\frac{1}{5}\begin{bmatrix} 0 & 0 & 3 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}$

Answer: a).

14. $B = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\}$ is an ordered basis for the space of 2×2 symmetric

matrices. Find the coordinates of $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ relative to B.

a) $(-2,3,-1)$ b) $(2,2,1)$ c) $(1,2,2,2)$ d) $(2,2,-1)$ e) $(-1,3,-2)$

Answer: $\underline{(2,2,-1)}$.

15. $B = \{1-x+x^2, 1-x^2, 1+x+x^2\}$ and $B' = \{1-x^2, 1+x^2, x\}$ are bases for $\mathbf{P}_2$. The coordinates of $4+4x$ relative to B and B' are, respectively:

a) $(4,4,0)$ and $(4,4,0)$ b) $(0,4,4)$ and $(4,4,0)$ c) $(-1,2,3)$ and $(2,2,4)$

d) $(2,2,4)$ and $(-1,2,3)$ e) $(0,1,-1)$ and $(1,0,0)$ f) $(1,0,0)$ and $(0,1,-1)$

Answer: $\underline{(-1,2,3) \text{ and } (2,2,4)}$.

16. Find the matrix of $T(x,y) = (x+y, -2x+4y)$ relative to the basis $\{(2,2),(4,-1)\}$.

a) $\begin{bmatrix} 4 & 3 \\ 4 & -12 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 4 \\ 2 & -1 \end{bmatrix}$ c) $\frac{1}{10}\begin{bmatrix} 1 & 4 \\ 2 & -2 \end{bmatrix}$

d) $\frac{1}{2}\begin{bmatrix} 4 & -9 \\ 0 & 6 \end{bmatrix}$ e) $\begin{bmatrix} 4 & 4 \\ 3 & -12 \end{bmatrix}$ f) $\frac{1}{2}\begin{bmatrix} 4 & 0 \\ -9 & 6 \end{bmatrix}$

Answer: $\underline{\frac{1}{2}\begin{bmatrix} 4 & -9 \\ 0 & 6 \end{bmatrix}}$.

17. A linear transformation $T : \mathbf{M}_{22} \to \mathbb{R}^2$ is defined by $T(A) = A\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (matrix multiplication). If B is the standard basis for $\mathbf{M}_{22}$, find the matrix of T relative to B.

a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

Answer: $\underline{\text{c)}}$.

18. Let $T : \mathbf{P}_2 \to \mathbb{R}^2$ be the linear map defined by $T(p(x)) = (p(2), p(-2))$. The matrix of T relative to the standard ordered bases in $\mathbf{P}_2$ and $\mathbb{R}^2$ is:

c) $\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 & 4 \\ 1 & -2 & 4 \end{bmatrix}$ f) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Answer: $\underline{\text{b)}}$.

19. A linear map $S : \mathbf{M}_{22} \to \mathbf{M}_{22}$ is defined by $S(A) = AJ - JA$, where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. If B is the standard ordered basis of $\mathbf{M}_{22}$, find the matrix of S relative to B.

a) $\begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix}$

Answer: $\underline{\text{a)}}$.

20. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined so that $T\begin{bmatrix} x \\ y \end{bmatrix}$ is obtained by rotating $\begin{bmatrix} x \\ y \end{bmatrix}$ anticlockwise through an angle of $\frac{\pi}{2}$. The matrix of T^6 with respect to the standard basis of $\mathbb{R}^2$ is:

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ e) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ f) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Answer: $\underline{\text{e)}}$.

21. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has matrix $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ relative to the ordered basis $B = \{(1,2),(3,4)\}$ of $\mathbb{R}^2$. Find $T(11,16)$.

a) $(33,55)$ b) $(77,122)$ c) $(1,1)$
d) $(18,28)$ e) $(102,148)$ f) the data is insufficient

Answer: $\underline{(102,148)}$.

22. A linear transformation $T : \mathbf{M}_{22} \to \mathbb{R}^2$ has matrix $M_{DB}(T) = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 3 & -2 & 1 & 2 \end{bmatrix}$ relative to the standard bases B and D of $\mathbf{M}_{22}$ and $\mathbb{R}^2$, respectively. Find $T\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

a) $\begin{bmatrix} a-c+3d \\ 3a-2b+c+2d \end{bmatrix}$ b) $\begin{bmatrix} a-d+3c \\ 3a-2c+b+2d \end{bmatrix}$ c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
d) $\begin{bmatrix} a+b \\ c+d \end{bmatrix}$ e) $\begin{bmatrix} a+b \\ c+d \end{bmatrix}$ f) the data is insufficient

Answer: $\underline{\text{a)}}$.

23. $B = \{(3,0,2),(4,0,3),(5,2,4)\}$ and B' $\{(1,-1,0),(1,0,1),(1,1,1)\}$ are ordered bases for $\mathbb{R}^3$. The change matrix from B to B' is:

a) symmetric b) orthogonal c) diagonal
d) upper triangular e) lower triangular f) not invertible

Answer: Symmetric.

24. $B = \{(19, 3), (9, 4)\}$ and $B' = \{(1, 2), (3, -1)\}$ are bases for $\mathbb{R}^2$. The coordinates of $(28, 7)$ relative to B and B' are, respectively:

a) $(4, 5)$ and $(3, 2)$ b) $(4, 3)$ and $(5, 2)$ c) $(1, 1)$ and $(7, 7)$

d) $(7, 7)$ and $(1, 1)$ e) $(3, 2)$ and $(4, 5)$ f) $(5, 2)$ and $(4, 3)$

Answer: $(1, 1)$ and $(7, 7)$.

25. If $\mathbf{P}_1$ denotes the vector space of polynomials of degree at most 1, then $B = \{2x + 1, 7x + 4\}$ and $B' = \{2 - x, 2x - 3\}$ are bases for $\mathbf{P}_1$. The change matrix from B to B' is:

a) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ c) $\begin{bmatrix} 8 & 5 \\ 29 & 18 \end{bmatrix}$

d) $\begin{bmatrix} -18 & 29 \\ 5 & -8 \end{bmatrix}$ e) $\begin{bmatrix} -18 & 5 \\ 29 & -8 \end{bmatrix}$ f) $\begin{bmatrix} 8 & 29 \\ 5 & 18 \end{bmatrix}$

Answer: $\begin{bmatrix} 8 & 29 \\ 5 & 18 \end{bmatrix}$.

26. A vector space V has bases B and B' and $C_B[\mathbf{v}]$ and $C_{B'}[\mathbf{v}]$ denote, respectively, the coordinates of a vector $\mathbf{v} \in V$ with respect to the bases B and B'. If $P = P_{B' \leftarrow B}$ is the change matrix from B' to B, choose the correct statement below.

a) $C_B[\mathbf{v}]P = C_{B'}[\mathbf{v}]$ b) $PC_{B'}[\mathbf{v}] = C_B[\mathbf{v}]$ c) $PC_B[\mathbf{v}] = C_{B'}[\mathbf{v}]$

d) $C_B[\mathbf{v}]C^T_{B'}[\mathbf{v}] = P$ e) $C^T_{B'}[\mathbf{v}]C_B[\mathbf{v}] = P$ f) $C_B[\mathbf{v}]C_{B'}[\mathbf{v}] = P^{-1}$

Answer: $PC_{B'}[\mathbf{v}] = C_B[\mathbf{v}]$.

27. $B = \left\{ \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \right\}$ is a basis for $\mathbf{M}_{22}$ and $B' = \{(1, 2, 3), (0, 1, -1), (2, 2, 2)\}$ is a basis for $\mathbb{R}^3$. A linear transformation $S : \mathbf{M}_{22} \to \mathbb{R}^3$ has $C_{B'B} = \begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 2 & 3 & 2 & 0 \end{bmatrix}$ as matrix relative to B and B'. Find $S\left(\begin{bmatrix} 2 & 5 \\ 3 & 5 \end{bmatrix}\right)$.

a) $(1, 1, 1, 1)$ b) $(2, 4, 7)$ c) $(16, 22, 16)$

d) $(2, 5, 3, 5)$ e) $(1, 0, 2)$ f) $(-2, 1, 3)$

Answer: $(16, 22, 16)$.

28. The characteristic polynomial $c_T(x)$ of the operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(x, y, z) = (x + y - 2, x - y + z, 2y + z)$ is:

a) $x^3 - x^2 - 5x + 2$ b) $x^3 - x^2 - x + 4$ c) $x^3 - x^2 - x + 2$

d) $x^3 - x^2 - 5x + 4$ e) $2x^3 - 2x^2 - 3x + 2$ f) $2x^3 - 2x^2 + 3x - 2$

Answer: a).

29. The characteristic polynomial $c_T(x)$ of the operator $T : \mathbf{P}_1 \to \mathbf{P}_1$ given by $T(ax+b) = (a+3b)+(3a-2b)x$ is:

a) $x^2 - x + 7$ b) $x^2 + x - 11$ c) $x^2 - x + 2$
d) $x^2 + x + 7$ e) $x^2 - 2x + 4$ f) $x^2 + 3x - 2$

Answer: b).

30. Let $T : \mathbf{P}_3 \to \mathbf{P}_3$ be defined by $T(p(x)) = x^2p''(x) - xp'(x) + p(x)$. The eigenvalues of T are:

a) 0 and 1 b) 0, 1 and 4 c) 0 and 4
d) 0 only e) 4 only f) 1 only

Answer: 0, 1 and 4.

31. A linear map $T : \mathbf{P}_2 \to \mathbf{P}_2$ is defined by $T(p(x)) = p(x+2)$. The eigenvalues of T are:

a) 0 and 2 b) 0, 1 and 2 c) 0 only
d) 1 only e) 2 only f) 0 and -1

Answer: 1 only.

32. A linear map $T : \mathbf{M}_{22} \to \mathbf{M}_{22}$ is defined by $T(A) = KA$ (matrix multiplication) where $K = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$. A basis for the eigenspace corresponding to $\lambda = 6$ is:

a) $\left\{ \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \right\}$ b) $\left\{ \begin{bmatrix} 9 & 3 \\ 3 & 9 \end{bmatrix} \right\}$ c) $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$

d) $\{(1,0,1,0),(0,1,0,1)\}$ e) $\{(1,-1),(1,1)\}$ f) $\left\{ \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \right\}$

Answer: $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$.

33. The linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y-x \\ z-y \\ x-z \end{bmatrix}$ has a unique real eigenvalue.

An eigenvector corresponding to this real eigenvalue is:

a) $(1,2,3)$ b) $(-1,1,2)$ c) $(1,0,0)$
d) $(1,1,1)$ e) $(0,0,1)$ f) $(-1,0,1)$

Answer: $(1,1,1)$.

34. Define $T : \mathbf{P}_1 \to \mathbf{P}_1$ by $T(a+bx) = (a+4b)+(3a+2b)x$. A basis of $\mathbf{P}_1$ that diagonalizes T is:

a) $\{4-3x,\ 1+x\}$ b) $\{4-3x\}$ c) $\{4-3x,\ 1-x\}$ d) There is no such basis

Answer: a) .

35. Define $T : \mathbf{P}_1 \to \mathbf{P}_1$ by $T(a+bx) = a + (a+b)x$. A basis of $\mathbf{P}_1$ that diagonalizes T is:

a) $\{x,\ 1-2x\}$ b) $\{x\}$ c) $\{x,\ 2x\}$ d) There is no such basis

Answer: d) .

36. Define $T : \mathbf{P}_1 \to \mathbf{P}_1$ by $T(a+bx) = (2a+b) + (2a+3b)x$. A basis of $\mathbf{P}_1$ that diagonalizes T is:

a) $\{1+x,\ 1-2x\}$ b) $\{x,\ 1+2x\}$ c) $\{1-x,\ 1+2x\}$ d) There is no such basis

Answer: c) .

37. Define $T : \mathbf{P}_2 \to \mathbf{P}_2$ by $T(a+bx+cx^2) = (2a-c) + 2bx - 3(2a-b+c)x^2$. A basis of $\mathbf{P}_2$ that diagonalizes T is:

a) $\{1+6x^2,\ 1-x^2,\ 1+2x\}$ b) $\{1-6x^2,\ 1-x^2,\ 1+2x\}$

c) $\{1+6x^2,\ 1+x^2,\ 1-2x\}$ d) There is no such basis

Answer: a) .

38. Define $T : \mathbf{P}_2 \to \mathbf{P}_2$ by $T(a+bx+cx^2) = a + (a+2b-3c)x + (a-b)x^2$. A basis of $\mathbf{P}_2$ that diagonalizes T is:

a) $\{2+x+x^2,\ x+x^2\}$ b) $\{3x-x^2,\ 2+x+x^2,\ x+x^2\}$

c) $\{3x-x^2,\ 2+x+x^2,\ x-x^2\}$ d) There is no such basis

Answer: b) .

39. Define $T : \mathbf{P}_2 \to \mathbf{P}_2$ by $T(a+bx+cx^2) = (a+c) + (b-2c)x + cx^2$. A basis of $\mathbf{P}_2$ that diagonalizes T is:

a) $\{1,\ x\}$ b) $\{1,\ x,\ x^2\}$ c) $\{1,\ x,\ 1-x\}$ d) There is no such basis

Answer: d) .

40. Let $V = \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} \mid a,b,c \in \mathbb{R} \right\}$ denote the space of 2×2 symmetric matrices, and define $T : V \to V$ by $T \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} -7a-5b & 10a+8b \\ 10a+8b & 5a+5b-2c \end{bmatrix}$. A basis of V that diagonalizes T is:

a) $\left\{ \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

d) There is no such basis

Answer: c) .

41. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(a, b, c) = (4a - b + c, 3a + 2b, 5c - b)$ and write $U = \text{span}\{(1, 1, 1), (1, 0, 1)\}$ and $W = \mathbb{R}(2, 3, 3)$. Determine which of the following are True:

a) U is T-invariant

b) W is T-invariant

c) $\mathbb{R}^3 = U \oplus W$

d) $M_B(T) = \begin{bmatrix} 5 & 3 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ where $B = \{(1, 1, 1), (1, 0, 1), (2, 3, 3)\}$

e) T is diagonalizable

f) The matrix of the restriction $T : U \to U$ is $\begin{bmatrix} 5 & 3 \\ -1 & 2 \end{bmatrix}$

Answer: a), b), c), f) .

42. Determine the cases in which $V = U \oplus W$:

a) $V = \mathbb{R}^5$, $U = \{(x, 0, 0, y, 0) \mid x, y \in \mathbb{R}\}\}$, $W = \{(0, u, v, 0, w) \mid u, v, w \in \mathbb{R}\}$.

b) $V = \mathbb{R}^5$, $U = \{(x, 0, 0, y, 0) \mid x, y \in \mathbb{R}\}\}$, $W = \{(0, u, v, z, w) \mid u, v, w, z \in \mathbb{R}\}$.

c) $V = \mathbb{R}^3$, $U = \{(x, y, 0) \mid x, y \in \mathbb{R}\}\}$, $W = \{(0, y, z) \mid y, z \in \mathbb{R}\}$.

d) $U = \ker(T)$, $W = \text{im}(T)$; where $T : V \to V$ satisfies $T^2 = T$.

e) $V = \mathbf{P}_n$, $U = \{p(x) \mid p(-x) = p(x)\}$, $W = \{p(x) \mid p(-x) = -p(x)\}$.

f) $V = \mathbf{M}_{nn}$, $U = \{A \mid A^T = A\}$, $W = \{A \mid A^T = -A\}$.

Answer: a), b), d).

43. Given a matrix $A = \begin{bmatrix} 3 & 1 & 2 & -1 \\ -8 & -3 & -4 & 0 \\ 0 & -1 & -1 & 1 \\ 8 & 4 & 4 & 1 \end{bmatrix}$, the matrix P such that $P^{-1}AP$ is block triangular is:

a) $\begin{bmatrix} 0 & -2 & -\frac{1}{2} & \frac{1}{4} \\ -1 & 2 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -2 & -2 & \frac{1}{5} \\ -1 & 2 & 0 & -3 \\ 1 & 4 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 3 & 2 & 2 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ -4 & -\frac{7}{2} & -1 & -\frac{5}{2} \\ -4 & -3 & -2 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 4 & 3 & 2 & 2 \\ 0 & 2 & 0 & \frac{1}{3} \\ -4 & -5 & -1 & -\frac{4}{3} \\ -4 & -3 & -2 & -1 \end{bmatrix}$

Answer: a).

44. Given a matrix $A = \begin{bmatrix} 2 & 0 & -3 & 6 \\ 0 & -1 & 0 & 0 \\ -3 & 0 & 2 & -6 \\ -3 & 0 & 3 & -7 \end{bmatrix}$, the matrix P such that $P^{-1}AP$ is block triangular is:

a) $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 1 & 2 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & -2 & 5 \\ -1 & 0 & 0 & -3 \\ 1 & 4 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 1 & 0 & 2 \\ -1 & 7 & -1 & 2 \\ -1 & -3 & -2 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & -2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Answer: d).

Chapter 10: Inner Product Spaces

1. Which of the following are inner products on V :

   a) $V = \mathbb{C}$ and $\langle a+bi, a'+b'i\rangle = aa' + ba' + ab' + 2bb'$.

   b) $V = \mathbf{P}_2$ and $\langle a+bx, a'+b'x\rangle = aa' - ba' - ab' + 2bb'$.

   c) $V = \mathbb{R}^2$ and $\langle (a,b),(a',b')\rangle = aa' + ba' + ab' + bb'$.

   d) $V = \mathbf{M}_{22}$ and $\langle A, B\rangle = \mathrm{tr}(A^T B)$.

   e) $V = \mathbf{F}[0,1]$ and $\langle f, g\rangle = f(0)g(0)$.

   f) $V = \mathbb{R}^3$ and $\langle (a,b,c),(a',b',c')\rangle = aa' + bb' - (cb' + bc')$.

   g) $V = \mathbf{P}_3$ and $\langle p(x), q(x)\rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$.

   Answer: a), b), d).

2. A real inner product $\langle\,,\,\rangle$ must satisfy the following properties for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ and scalars a, b.

   (i) Linearity: $\langle a\mathbf{u} + b\mathbf{v}, \mathbf{w}\rangle = a\langle \mathbf{u}, \mathbf{w}\rangle + b\langle \mathbf{v}, \mathbf{w}\rangle$

   (ii) Symmetry: $\langle \mathbf{u}, \mathbf{v}\rangle = \langle \mathbf{v}, \mathbf{u}\rangle$

   (iii) Positivity: $\langle \mathbf{u}, \mathbf{u}\rangle \geq 0$

   (iv) Nondegeneracy: $\langle \mathbf{u}, \mathbf{u}\rangle = 0 \Leftrightarrow \mathbf{u} = 0$

   Define $\langle\,,\,\rangle$ on $\mathbb{R}^2$ by $\langle (x,y),(x',y')\rangle = xx' + xy'$. Which of the above properties are satisfied?

   a) all of them: $\langle\,,\,\rangle$ is an inner product
   b) (i) and (ii) only
   c) (i), (ii) and (iii) only
   d) (i) only
   e) (ii) only
   f) (iii) and (iv) only

   Answer: (i) only.

3. Let $\langle \_, \_\rangle_1$ and $\langle \_, \_\rangle_2$ be inner products on a vector space V. Which of the following defined an inner product on V :

   a) $\langle \mathbf{v}, \mathbf{w}\rangle = \langle \mathbf{v}, \mathbf{w}\rangle_1 + \langle \mathbf{v}, \mathbf{w}\rangle_2$.

   b) $\langle \mathbf{v}, \mathbf{w}\rangle = (\langle \mathbf{v}, \mathbf{w}\rangle_1)^2 + (\langle \mathbf{v}, \mathbf{w}\rangle_2)^2$

   c) $\langle \mathbf{v}, \mathbf{w}\rangle = 2\,\langle \mathbf{v}, \mathbf{w}\rangle_1$

   d) $\langle \mathbf{v}, \mathbf{w}\rangle = 2\,\langle \mathbf{v}, \mathbf{w}\rangle_1 - \langle \mathbf{v}, \mathbf{w}\rangle_2$.

   Answer: a), c).

4. Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in an inner product space such that $\langle \mathbf{u}, \mathbf{v}\rangle = \langle \mathbf{u}, \mathbf{w}\rangle = \langle \mathbf{v}, \mathbf{w}\rangle = 1$ and $\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{w}\| = 1$. Which of the following expressions is equal to 0?

   a) $\|\mathbf{v} + 3\mathbf{w}\|$
   b) $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\|$
   c) $\|3\mathbf{u} - 2\mathbf{v} - \mathbf{w}\|$
   d) $\langle \mathbf{v} + \mathbf{w}, \mathbf{v} - \mathbf{w}\rangle$
   e) $\langle 2\mathbf{u} + \mathbf{v} - \mathbf{w}, 3\mathbf{v} + \mathbf{w}\rangle$
   f) $\langle \mathbf{v} - 2\mathbf{w}, \mathbf{u} - \mathbf{w}\rangle$

   Answer: c), d), and f).

5. Let $\langle \mathbf{u}, \mathbf{v} \rangle$ be an inner product for any vectors $\mathbf{u}$ and $\mathbf{v}$ in a vector space V. Which of the following are **not** identities?

   a) $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + 2\langle \mathbf{u}, \mathbf{v} \rangle + \|\mathbf{v}\|^2$

   b) $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \frac{1}{2}\|\mathbf{u}+\mathbf{v}\|^2 + \frac{1}{2}\|\mathbf{u}-\mathbf{v}\|^2$

   c) $\|\mathbf{u}-\mathbf{v}\| = \sqrt{\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2}$

   d) $\langle \mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v} \rangle = \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2$

   e) $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{4}\|\mathbf{u}+\mathbf{v}\|^2 + \frac{1}{4}\|\mathbf{u}-\mathbf{v}\|^2$

   f) $\|\mathbf{u}+\mathbf{v}-\mathbf{w}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - 2\langle \mathbf{u}, \mathbf{v} \rangle - 2\langle \mathbf{u}, \mathbf{w} \rangle - 2\langle \mathbf{v}, \mathbf{w} \rangle$

   Answer: <u>c), e), and f)</u>.

6. An inner product on $\mathbb{R}^3$ is defined by $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T A\mathbf{y}$, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. A unit vector that is orthogonal to both $\mathbf{e}_1$ and $\mathbf{e}_3$ is:

   a) $(0, 1, 0)$ b) $\frac{\sqrt{5}}{5}(1, 2, 0)$ c) $\frac{\sqrt{2}}{2}(1, -1, 0)$

   d) $(0, 0, 1)$ e) $(0, 0, 0)$ f) $(1, -1, 0)$

   Answer: $\underline{(1, -1, 0)}$.

7. Which of the following sets are orthogonal?

   a) $\{(-1, 1, 0), (0, 1, 1), (0, 1, 0)\}$

   b) $\left\{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (0, 0, 1)\right\}$

   c) $\left\{(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (0, -1, 1)\right\}$

   d) $\{(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}), (-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})\}$

   e) $\{(0, -1, 1), (4, 0, 0), (0, 1, 1)\}$

   f) $\{(1, 0, 0), (0, 1, 1), (0, 1, 0)\}$

   Answer: <u>b), d), and e)</u>.

8. If we define an inner product on $\mathbb{R}^2$ by $\langle (\mathbf{x}, \mathbf{y}), (\mathbf{x}', \mathbf{y}') \rangle = 4\mathbf{x}\mathbf{x}' - 3\mathbf{x}\mathbf{y}' - 3\mathbf{y}\mathbf{x}' + 5\mathbf{y}\mathbf{y}'$, the angle between $(1, 1)$ and $(1, 3)$ is:

   a) 0 b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) $\frac{\pi}{2}$ f) $\frac{3\pi}{4}$

   Answer: $\underline{\frac{\pi}{4}}$.

9. Given an inner product on $\mathbf{P}_2$ defined by $\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)dx$, the distance between $1 + x + x^2$ and $2x^2 - 3$ is:

   a) 10 b) $\frac{23}{6}$ c) $\frac{4}{5}$ d) $\frac{2}{3}$ e) $-\frac{5}{6}$ f) $\frac{25}{6}$

   Answer: $\underline{-\frac{5}{6}}$.

10. If U is a subspace of an inner product space V, let $U^{\perp} = \{\mathbf{v} \in V \mid \langle \mathbf{u}, \mathbf{v} \rangle = 0 \text{ for all } \mathbf{u} \in U\}$. Answer the following questions:

    (i) Is $U^{\perp}$ a subspace of V?

    (ii) Is the equation $\dim(U) + \dim(U^{\perp}) < \dim(V)$ true?

    (iii) Is $\left(U^{\perp}\right)^{\perp} = U$?

    a) (i) Yes, (ii) Yes, (iii) Yes b) (i) Yes, (ii) Yes, (iii) No c) (i) Yes, (ii) No, (iii) Yes

    d) (i) Yes, (ii) No, (iii) No e) (i) No, (ii) Yes, (iii) Yes f) (i) No, (ii) Yes, (iii) No

    Answer: (i) Yes, (ii) No, (iii) Yes.

11. If $\mathbf{M}_{22}$ is given, the inner product $\left\langle \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \right\rangle = aa' + bb + cc' + dd'$. An orthonormal basis for $W = \operatorname{span}\left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 5 & -4 \end{bmatrix} \right\}$ is:

    a) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 5 & -4 \end{bmatrix} \right\}$ b) $\left\{ \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \right\}$

    c) $\left\{ \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 5 & -4 \end{bmatrix} \right\}$ d) $\left\{ \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \frac{1}{\sqrt{66}}\begin{bmatrix} 4 & 3 \\ 5 & -4 \end{bmatrix} \right\}$

    e) $\left\{ \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$ f) $\left\{ \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \frac{\sqrt{2}}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$

    Answer: $\left\{ \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \frac{\sqrt{2}}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$.

12. An inner product on $\mathbf{P}_2$ is defined by $\langle p(x), q(x) \rangle = 3[p(-1)q(-1) + p(0)q(0) + p(1)q(1)]$. If we apply the Gram-Schmidt process to the basis $\{1, x\}$ of the subspace P_1, the orthonormal basis we obtain is:

    a) $\left\{\frac{1}{3}, x\right\}$ b) $\left\{\frac{1}{3}, \frac{x}{\sqrt{6}}\right\}$ c) $\left\{1, \frac{x}{\sqrt{6}}\right\}$

    d) $\{1, 1 - x\}$ e) $\left\{\frac{2}{3}, 2x\right\}$ f) $\left\{-1, \frac{x}{2} + 1\right\}$

    Answer: $\left\{\frac{1}{3}, \frac{x}{\sqrt{6}}\right\}$.

13. A non-euclidean inner product on $\mathbb{R}^3$ is defined by

    $$\langle (x, y, z), (x', y', z') \rangle = xx' + xy' + yx' + 2yy' + zz'.$$

    A unit vector which is orthogonal to $(1, 0, 0)$ and $(0, 0, 1)$ is:

    a) $(0, 1, 0)$ b) $\frac{1}{\sqrt{2}}(0, 1, 0)$ c) $(1, -1, 0)$

    d) $\frac{1}{\sqrt{2}}(1, -1, 0)$ e) $(0, 1, 1)$ f) $\frac{1}{\sqrt{2}}(0, 1, 1)$

    Answer: $(1, -1, 0)$.

14. An inner product on $\mathbf{P}_2$ is defined by $\langle p(x), q(x)\rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$. A polynomial of unit length which is orthogonal to x and x^2 is:

    a) 1 b) $\frac{\sqrt{2}}{2}$ c) $1-x$
    d) $\frac{\sqrt{2}}{2}(1-x)$ e) $1-x^2$ f) $\frac{\sqrt{2}}{2}(1-x^2)$

    Answer: $\underline{1-x^2}$.

15. An inner product on $\mathbf{P}_2$ is defined by $\langle p(x), q(x)\rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$. Let $U = \operatorname{span}\{1, x\}$. If $p(x) = 1 + x + x^2$, we can write $p(x) = p_1(x) + p_2(x)$ where $p_1 \in U$ and $p_2 \in U^{\perp}$. Then $(p_1(x), p_2(x))$ is:

    a) $(-\frac{2}{3}+x, \frac{5}{3}+x^2)$ b) $(\frac{5}{3}+x, -\frac{2}{3}+x^2)$
    c) $(2+x, -1+x^2)$ d) $(-\frac{1}{3}+x, \frac{4}{3}+x^2)$

    Answer: b).

16. An inner product on $\mathbf{P}_2$ is defined by $\langle p(x), q(x)\rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$. If the Gram-Schmidt algorithm is applied to the vectors $1, x, x^2$ to produce an orthonormal set $\{p_1, p_2, p_3\}$, then $p_3(x)$ is:

    a) $\frac{\sqrt{6}}{3}(x^2-\frac{2}{3})$ b) $\frac{\sqrt{6}}{2}(x^2-\frac{2}{3})$
    c) $\frac{\sqrt{3}}{\sqrt{2}}(x^2-\frac{2}{3})$ d) $x^2-\frac{2}{3}$

    Answer: b).

17. An inner product on $\mathbf{M}_{22}$ is defined by $\langle A, B\rangle = \operatorname{tr}(A^T B)$. If the Gram-Schmidt algorithm is applied to the vectors $\begin{bmatrix} 3 & 0 \\ 4 & 0 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$ and $\begin{bmatrix} 6 & 3 \\ -5 & 4 \end{bmatrix}$ to product an orthonormal set $\{A, B, C\}$, then C is:

    a) $\begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$ b) $\frac{1}{5}\begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$ c) $\frac{1}{5}\begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$ d) $\frac{1}{5}\begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$ e) $\begin{bmatrix} -4 & 0 \\ 3 & 0 \end{bmatrix}$ f) $\frac{1}{5}\begin{bmatrix} -4 & 0 \\ 3 & 0 \end{bmatrix}$

    Answer: c).

18. In P_2, let $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$ and define an inner product on P_2 by $\langle p(x), q(x)\rangle = a_0b_0 + a_1b_1 + a_2b_2$. If the Gram-Schmidt algorithm is applied to the basis $\left\{1+x, x+x^2, 1+x^2\right\}$, the resulting orthogonal basis is:

    a) $\{x+1, x^2+\frac{1}{2}x-\frac{1}{2}, \frac{4}{3}x^2-\frac{1}{3}x+\frac{2}{3}\}$
    b) $\{x+1, x^2+\frac{3}{2}x+\frac{1}{2}, \frac{4}{3}x^2-\frac{1}{6}x+\frac{7}{6}\}$
    c) $\{x+1, x^2+\frac{3}{2}x-\frac{1}{2}, \frac{2}{3}x^2-\frac{2}{3}x+\frac{2}{3}\}$
    d) $\{x+1, x^2+\frac{1}{2}x-\frac{1}{2}, \frac{2}{3}x^2+\frac{1}{3}x+\frac{5}{3}\}$
    e) $\{x+1, x^2-\frac{1}{2}x-\frac{1}{2}, \frac{2}{3}x^2+\frac{1}{3}x-\frac{2}{3}\}$
    f) $\{x+1, x^2+\frac{1}{2}x-\frac{1}{2}, \frac{2}{3}x^2-\frac{2}{3}x+\frac{2}{3}\}$

    Answer: f).

19. An inner product on $\mathbf{M}_{22}$ is defined by $\langle A, B\rangle = \mathrm{tr}(A^T B)$. Let $A = \begin{bmatrix} 7 & 2 \\ 3 & 4 \end{bmatrix}$ and define $U = \mathrm{span}\left\{ \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix} \right\}$. If $A = A_1 + A_2$ where $A_1 \in U$ and $A_2 \in U^\perp$, then (A_1, A_2) is:

a) $\left(\frac{1}{25}\begin{bmatrix} 0 & -4 \\ 0 & 3 \end{bmatrix}, \frac{1}{25}\begin{bmatrix} 175 & 66 \\ 75 & 88 \end{bmatrix} \right)$

b) $\left(\frac{1}{5}\begin{bmatrix} 175 & 66 \\ 75 & 88 \end{bmatrix}, \frac{-1}{5}\begin{bmatrix} 140 & 56 \\ 60 & 68 \end{bmatrix} \right)$

c) $\left(\frac{1}{25}\begin{bmatrix} 175 & 66 \\ 75 & 88 \end{bmatrix}, \frac{4}{25}\begin{bmatrix} 0 & -4 \\ 0 & 3 \end{bmatrix} \right)$

d) $\left(\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 2 \\ 3 & 0 \end{bmatrix} \right)$

Answer: c) .

20. An inner product on $\mathbf{M}_{22}$ is defined by $\langle A, B\rangle = \mathrm{tr}(A^T B)$. Let $A = \begin{bmatrix} 5 & -2 \\ 1 & 5 \end{bmatrix}$ and define $U = \mathrm{span}\left\{ \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \right\}$. If $A = A_1 + A_2$ where $A_1 \in U$ and $A_2 \in U^\perp$, then (A_1, A_2) is:

a) $\left(\frac{1}{2}\begin{bmatrix} 4 & -1 \\ 8 & 7 \end{bmatrix}, \frac{3}{2}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix} \right)$

b) $\left(\frac{3}{2}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 4 & -1 \\ 8 & 7 \end{bmatrix} \right)$

c) $\left(\begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -3 & 3 \end{bmatrix} \right)$

d) $\left(\frac{3}{2}\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} -4 & 1 \\ -8 & -7 \end{bmatrix} \right)$

Answer: a) .

21. An inner product on $\mathbf{M}_{22}$ is defined by $\langle A, B\rangle = \mathrm{tr}(A^T B)$. Let $A = \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix}$ and define $U = \mathrm{span}\left\{ \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \right\}$. If $A = A_1 + A_2$ where $A_1 \in U$ and $A_2 \in U^\perp$, then (A_1, A_2) is:

a) $\left(\left(\frac{1}{2}\begin{bmatrix} 3 & -3 \\ 4 & 8 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} -1 & 13 \\ 11 & 12 \end{bmatrix} \right) \right)$

b) $\left(\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 6 \\ 0 & 3 \end{bmatrix} \right)$

c) $\left(\frac{1}{2}\begin{bmatrix} -1 & 13 \\ 11 & 12 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} -3 & 3 \\ -4 & -8 \end{bmatrix} \right)$

d) $\left(\frac{1}{2}\begin{bmatrix} -1 & 13 \\ 11 & 12 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 3 & -3 \\ 4 & 8 \end{bmatrix} \right)$

Answer: d) .

22. An inner product on $\mathbf{M}_{22}$ is defined by $\langle A, B\rangle = \mathrm{tr}(A^T B)$. The shortest distance from $\begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}$ to $\mathrm{span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$ is:

a) 0 b) 1 c) -1 d) 2 e) -2 f) 3

Answer: d) .

23. An inner product on $\mathbf{M}_{22}$ is defined by $\langle A, B\rangle = \mathrm{tr}(A^T B)$. The shortest distance from $\begin{bmatrix} 2 & -1 \\ 3 & 7 \end{bmatrix}$ to span$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \right\}$ is:

a) $\frac{7}{4}$ b) $\frac{5}{4}$ c) $\frac{1}{2}$ d) $\frac{7}{2}$

Answer: d) .

24. Define $T : \mathbf{P}_2 \to \mathbf{P}_2$ by $T(a+bx+cx^2) = (a+2c) - bx + (2a+c)x^2$. Using the inner product $\left\langle a+bx+cx^2, a'+b'x+c'x^2 \right\rangle = aa'+bb'+cc'$, an orthonormal basis of eigenvectors of T is:

a) $\left\{ \frac{\sqrt{2}}{2}(1-x^2), x, \frac{\sqrt{2}}{2}(1+x^2) \right\}$ b) $\left\{ \frac{\sqrt{2}}{2}(1-x), x^2, \frac{\sqrt{2}}{2}(1+x) \right\}$

c) $\left\{ (1-x^2), x, (1+x^2) \right\}$ d) $\left\{ 1, x, x^2 \right\}$

Answer: a) .

25. Define $T : \mathbf{P}_3 \to \mathbf{P}_3$ by $T(a+bx+cx^2+dx^3) = (3a+5b)+(5a+3b)x+(d-c)x^2+(c-d)x^3$. Using the inner product $\left\langle a+bx+cx^2+dx^3, a'+b'x+c'x^2a+d'x^3 \right\rangle = aa'+bb'+cc'+dd'$, an orthonormal basis of eigenvectors of T is:

a) $\left\{ \frac{1}{2}(1-x),\ \frac{1}{2}(-x^2+x^3),\ \frac{1}{2}(x^2+x^3),\ \frac{1}{2}(1+x) \right\}$

b) $\left\{ \frac{\sqrt{2}}{2}(1-x),\ \frac{\sqrt{2}}{2}(-x^2+x^3),\ \frac{\sqrt{2}}{2}(x^2+x^3),\ \frac{\sqrt{2}}{2}(1+x) \right\}$

c) $\left\{ 1,\ x,\ x^2,\ x^3 \right\}$

d) $\left\{ 1-x,\ -x^2+x^3,\ x^2+x^3,\ 1+x \right\}$

Answer: b) .

APPENDIX A: Complex Numbers

1. Write $3[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)] + \cos\pi + i\sin\pi$ in the form $a + ib$.

   a) $-3 + i$ b) $3 - i$ c) $-1 + 3i$

   d) $-1 - 3i$ d) $3 - 3i$ f) $3 + i$

   Answer: $\underline{-1 - 3i}$.

2. Write $4[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)] + \cos\pi + i\sin\pi$ in the form $a + ib$.

   a) $-1 + 4i$ b) $1 + 4i$ c) $4 + i$

   d) $4 - i$ e) $-1 - 4i$ f) $3i$

   Answer: $\underline{-1 - 4i}$.

3. Write $4\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right] + 4\left[\cos(\pi) + \sin(\pi)\right]$ in the form $a + ib$.

   a) $-1 - 4i$ b) $-2 - i$ c) $-4 + i$

   d) $-4 - 4i$ e) $-2 + i$ f) $-2 - 2i$

   Answer: $\underline{-4 - 4i}$.

4. Evaluate the real part of z if $z = \frac{8+3i}{5-3i}$.

   a) $\frac{-31}{34}$ b) $\frac{31}{34}$ c) $\frac{31}{17}$ d) $\frac{41}{34}$ e) $\frac{-41}{34}$ f) $\frac{41}{17}$

   Answer: $\underline{\frac{31}{34}}$.

5. Evaluate the imaginary part of z if $z = \dfrac{1}{(1-i)(3-2i)}$.

   a) $\frac{1}{2}$ b) $\frac{-1}{2}$ c) $\frac{-1}{5}$ d) $\frac{5}{26}$ e) $\frac{-5}{26}$ f) $\frac{5}{13}$

   Answer: $\underline{\frac{5}{26}}$.

6. The real part of $\dfrac{(2+3i)\,(4-i)}{(6+2i)\,(1-i)}$ is:

   a) $\frac{3}{4}$ b) $\frac{33}{20}$ c) $\frac{3}{5}$ d) $\frac{33}{21}$ e) $\frac{20}{33}$ f) $\frac{4}{5}$

   Answer: $\underline{\frac{3}{5}}$.

7. If $z = \dfrac{1-i}{2-i} + \dfrac{2+i}{1-i}$, then the imaginary part of z is:

   a) $\frac{21}{10}$ b) $\frac{-21}{10}$ c) $\frac{7}{6}$ d) $\frac{-7}{6}$ e) $\frac{13}{10}$ f) $\frac{-13}{10}$

   Answer: $\underline{\frac{13}{10}}$.

8. Express $\dfrac{5+6i}{2+4i} - \dfrac{4-2i}{11i}$ in the form $a + bi$.

   a) $\frac{207}{55} + \frac{31}{110}i$ b) $\frac{-207}{55} + \frac{31}{110}i$ c) $\frac{207}{55} - \frac{31}{110}i$

   d) $\frac{207}{110} - \frac{2}{55}i$ e) $\frac{207}{110} + \frac{2}{55}i$ f) $\frac{31}{110}i - \frac{207}{55}$

   Answer: $\underline{\frac{207}{110} - \frac{2}{55}i}$.

9. Express $\dfrac{1}{1+i}$ in the form $a+bi$.

   a) $1 = i$ b) $-1+i$ c) $\frac{1}{2}+i\frac{1}{2}$

   d) $\frac{1}{2}-i\frac{1}{2}$ e) $2-i\frac{1}{4}$ f) $1-i$

   Answer: $\underline{\frac{1}{2}-\frac{1}{2}i}$.

10. Express $(2+i)(2+2i)$ in the form $a+bi$.

    a) $4+4i$ b) $2+4i$ c) $2-6i$

    d) $2+6i$ e) $6-2i$ f) 4

    Answer: $\underline{2+6i}$.

11. Express $\dfrac{(16+13i)(1+2i)}{10+5i}$ in the form $a+bi$.

    a) 3 b) $1+4i$ c) $1-4i$

    d) $\frac{1}{5}+\frac{4}{5}i$ e) $\frac{-1}{5}+\frac{4}{5}i$ f) $4i$

    Answer: $\underline{1+4i}$.

12. Compute $\dfrac{(1+2i)(2+5i)}{3+4i}$.

    a) $\frac{12+59i}{25}$ b) $\frac{12+59i}{5}$ c) $\frac{11i}{25}$

    d) $\frac{-11i}{5}$ e) $\frac{11i}{5}$ f) $\frac{-11i}{25}$

    Answer: $\underline{\frac{12+59i}{25}}$.

13. The complex number $1+2i+\dfrac{3+4i}{2+5i}$, written in the form $c+id$, is:

    a) $\frac{8+5i}{29}$ b) $\frac{21+53i}{29}$ c) $\frac{55+51i}{29}$

    d) $\frac{-8-5i}{29}$ e) $\frac{23+27i}{29}$ f) $\frac{1+29i}{29}$

    Answer: $\underline{\frac{55+51i}{29}}$.

14. Write $\dfrac{3\sqrt{3}-3i}{\sqrt{2}+i\sqrt{2}}$ in polar form.

    a) $6[\cos\left(\frac{-\pi}{12}\right)+i\sin\left(\frac{-\pi}{12}\right)]$ b) $3[\cos\left(\frac{-\pi}{12}\right)+i\sin\left(\frac{-\pi}{12}\right)]$

    c) $3[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)]$ d) $3[\cos\left(\frac{-5\pi}{12}\right)+i\sin\left(\frac{-5\pi}{12}\right)]$

    e) $2[\cos\left(\frac{-5\pi}{12}\right)+i\sin\left(\frac{-5\pi}{12}\right)]$ f) $2[\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)]$

    Answer: $\underline{3\left(\cos\left(\frac{-5\pi}{12}\right)+i\sin\left(\frac{-5\pi}{12}\right)\right)}$.

15. Write $3\sqrt{3}-3i$ in polar form.

    a) $36[\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{6}\right)]$ b) $6[\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)]$

    c) $36[\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)]$ d) $6[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)]$

    e) $36[\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)]$ f) $6[\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)]$

Answer: $\underline{6\left(\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)\right)}$.

16. Write $\dfrac{1-\sqrt{3}i}{-1+i}$ in polar form.

a) $\sqrt{2}[\cos\left(\frac{-5\pi}{12}\right)+i\sin\left(\frac{-5\pi}{12}\right)]$
b) $\sqrt{2}[\cos\left(\frac{-13\pi}{12}\right)+i\sin\left(\frac{-13\pi}{12}\right)]$
c) $\sqrt{2}[\cos\left(\frac{-7\pi}{12}\right)+i\sin\left(\frac{-7\pi}{12}\right)]$
d) $\sqrt{2}[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)]$
e) $\sqrt{2}[\cos\left(\frac{-\pi}{12}\right)+i\sin\left(\frac{-\pi}{12}\right)]$
f) $\sqrt{2}[\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)]$

Answer: $\underline{\sqrt{2}[\cos\left(\frac{-13\pi}{12}\right)+i\sin\left(\frac{-13\pi}{12}\right)]}$.

17. Write $\dfrac{5+5\sqrt{3}i}{\sqrt{2}-\sqrt{2}i}$ in polar form.

a) $5\left[\cos\left(\frac{5\pi}{12}\right)-i\sin\left(\frac{5\pi}{12}\right)\right]$
b) $5\left[\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)\right]$
c) $5\left[\cos\left(\frac{7\pi}{12}\right)-i\sin\left(\frac{7\pi}{12}\right)\right]$
d) $5\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$
e) $5\left[\cos\left(\frac{11\pi}{12}\right)-i\sin\left(\frac{11\pi}{12}\right)\right]$
f) $5\left[\cos\left(\frac{7\pi}{12}\right)+i\sin\left(\frac{7\pi}{12}\right)\right]$

Answer: $\underline{5\left[\cos\left(\frac{7\pi}{12}\right)+i\sin\left(\frac{7\pi}{12}\right)\right]}$.

18. Write $\dfrac{9+\left(3\sqrt{3}\right)i}{\sqrt{6}+\left(\sqrt{6}\right)i}$ in polar form.

a) $3\left[\cos\left(\frac{13\pi}{12}\right)+i\sin\left(\frac{13\pi}{12}\right)\right]$
b) $3\sqrt{2}\left[\cos\left(\frac{13\pi}{12}\right)+i\sin\left(\frac{13\pi}{12}\right)\right]$
c) $3\sqrt{2}\left[\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)\right]$
d) $3\left[\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)\right]$
e) $3\sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$
f) $3\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$

Answer: $\underline{3\left[\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)\right]}$.

19. Write $\dfrac{-9+\left(3\sqrt{3}\right)i}{\sqrt{6}-\left(\sqrt{6}\right)i}$ in polar form.

a) $3\left[\cos\left(\frac{13\pi}{12}\right)+i\sin\left(\frac{13\pi}{12}\right)\right]$
b) $3\sqrt{2}\left[\cos\left(\frac{13\pi}{12}\right)+i\sin\left(\frac{13\pi}{12}\right)\right]$
c) $3\sqrt{2}\left[\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)\right]$
d) $3\left[\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)\right]$
e) $3\sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$
f) $3\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$

Answer: $\underline{3\left[\cos\left(\frac{13\pi}{12}\right)+i\sin\left(\frac{13\pi}{12}\right)\right]}$.

20. Write $\dfrac{3i(1-i)^2}{1+i\sqrt{3}}$ in polar form.

a) $3[\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)]$
b) $3[\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)]$
c) $\frac{3}{\sqrt{2}}[\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)]$
d) $3[\cos\left(\frac{\pi}{6}\right)-i\sin\left(\frac{\pi}{6}\right)]$
e) $\frac{3}{\sqrt{2}}[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)]$
f) $3[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)]$

Answer: $\underline{3[\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)]}$.

21. Write $\dfrac{3+\left(3\sqrt{3}\right)i}{-2+2i}$ in polar form.

a) $\frac{3}{\sqrt{2}}\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$
b) $\frac{3}{\sqrt{2}}\left[\cos\left(\frac{5\pi}{12}\right)-i\sin\left(\frac{5\pi}{12}\right)\right]$
c) $3\left[\cos\left(\frac{5\pi}{12}\right)-i\sin\left(\frac{5\pi}{12}\right)\right]$
d) $\frac{3}{\sqrt{2}}\left[\cos\left(\frac{11\pi}{12}\right)-i\sin\left(\frac{11\pi}{12}\right)\right]$
e) $3\left[\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)\right]$
f) $3\left[\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right]$

Answer: $\underline{\frac{3}{\sqrt{2}}\left[\cos\left(\frac{5\pi}{12}\right)-i\sin\left(\frac{5\pi}{12}\right)\right]}$.

22. If z is a complex number, answer the following questions:

(i) Is it possible that $\bar{z}=z$?

(ii) Is it possible that $|\bar{z}|>|z|$?

(iii) Is it possible that $\bar{z}=2z$?

a) (i) Yes, (ii) No, (iii) Yes
b) (i) Yes, (ii) No, (iii) No
c) (i) Yes, (ii) Yes, (iii) Yes
d) (i) No, (ii) No, (iii) Yes
e) (i) No, (ii) Yes, (iii) No
f) (i) Yes, (ii) Yes, (iii) No

Answer: (i) Yes, (ii) No, (iii) Yes.

23. Let z_1 be a complex number and let $a>0$ be real. What geometric figure is $S=\{z:|z-z_1|=a\}$?

a) S is a circle with center at z_1 and radius a.

b) S is a straight line passing through z_1.

c) S consists of all the points whose distance from z_1 is smaller than a.

d) S consists of all the points whose distance from z_1 is greater than a.

e) S is a circle centered at the origin with radius $|z_1|$.

f) S is a pair of parallel lines.

Answer: S is a circle with center at z_1 and radius a.

24. Find all complex solutions of $x^3=-1$.

a) $x^3=1$ doesn't have any complex solutions
b) -1 only
c) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$, $\frac{1}{2}-i\frac{\sqrt{3}}{2}$, -1
d) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{1}{2}-i\frac{\sqrt{3}}{2}$ only
e) $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$, $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$, -1
f) $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$ only

Answer: $\frac{1}{2}+i\frac{\sqrt{3}}{2}$, $\frac{1}{2}-i\frac{\sqrt{3}}{2}$, -1.

25. Which one of the following is the imaginary part of a solution to the complex equation $z^2=-i$?

a) $\frac{\sqrt{3}}{2}$
b) $\frac{1}{2}$
c) $\frac{-\sqrt{3}}{2}$
d) $\frac{-1}{2}$
e) $\frac{\sqrt{2}}{3}$
f) $\frac{-\sqrt{2}}{2}$

Answer: $\frac{-\sqrt{2}}{2}$.

26. The equation $z^{101}-1=0$ has:

a) exactly 1 real root
b) 101 real roots.
c) exactly 50 complex roots
d) an odd number of complex roots
e) exactly 2 real roots
f) an infinite number of roots.

Answer: Exactly 1 real root.

27. The sum of the roots of the polynomial $z^2 - (4+i)z + 5(1+i)$ is:

a) $4-3i$ b) $2-i$ c) $2-3i$ d) $4+i$ e) $4-3i$ f) $1-2i$

Answer: $4+i$.

28. The "cube roots of 27" are complex numbers z which satisfy (select all correct responses):

a) $z^3 = 27$ b) $\sqrt[3]{z} = 27$ c) $|z| = 3$ d) $\sqrt[3]{z} = 3$ e) z is a power of $3e^{\frac{2\pi i}{3}}$ f) The product of any two cube roots of 27 is a cube root of 81.

Answer: a), c), e).

29. Compute $i^5 + i^6 + \cdots + i^{2006}$.

a) 1 b) -1 c) i d) $-i$ e) $i-1$ f) $i+1$

Answer: $i-1$.

30. Compute $\sum_{n=1}^{2003} i^n$.

a) -1 b) i c) $-i$ d) 1 e) $i-1$ f) $1001-1002i$

Answer: -1.

31. Compute $\left|\dfrac{3-i}{2-4i}\right|$.

a) $\frac{1}{2}$ b) $\sqrt{\frac{1}{2}}$ c) $\sqrt{\frac{8}{29}}$ d) $\frac{8}{29}$ e) $\frac{3}{2}$ f) $\sqrt{\frac{14}{29}}$

Answer: $\sqrt{\frac{1}{2}}$.

32. Compute $(1-i)^{13}$.

a) $64\,(1-i)$
b) $-64\,(1+i)$
c) $64\,(-1+i)$
d) $64\sqrt{2}\,(1-i)$
e) $64\sqrt{2}\,(-1-i)$
f) $64\,(i-1)$

Answer: $64\,(-1+i)$.

33. Compute $\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)^{14}$.

a) 1 b) -1 c) i d) $-i$ e) $i-1$ f) $1-i$

Answer: i.

34. Compute $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{-12}$.

a) -1 b) 0 c) $1+i$ d) 1 e) $-1+i$ f) $1-i$

Answer: 1.

35. Compute $\left(1 - i\sqrt{3}\right)^6$.

a) -64 b) $1 - 27i$ c) $1 + 27i$ d) 28 e) 64 f) $-28i$

Answer: $\underline{64}$.

36. Compute $(1 + i)^{11}$.

a) $32(1 - i)$ b) $32\sqrt{2}(1 - i)$ c) $-32(1 - i)$
d) $32(1 + i)$ e) $-32(1 + i)$ f) $32\sqrt{2}(1 + i)$

Answer: $\underline{-32(1 - i)}$.

37. Compute $\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)^{12}$.

a) 1 b) $1 - i$ c) -1
d) $\frac{1-i}{64}$ e) $64 - 64i$ f) $-i$

Answer: $\underline{-1}$.

38. Given $z = \sqrt{3} - i$, find the value of z^7 in the form $a + ib$.

a) $64\sqrt{3} - 64i$ b) $64\sqrt{3} + 64i$ c) $-64\sqrt{3} + 64i$
d) $64 - i64\sqrt{3}$ e) $64 + i64\sqrt{3}$ f) $-64 - i64\sqrt{3}$

Answer: $\underline{-64\sqrt{3} + 64i}$.

39. The square roots of $z = -3 + \left(3\sqrt{3}\right)i$ are:

a) $\pm\sqrt{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ b) $\pm\sqrt{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$
c) $\pm\sqrt{6}\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)$ d) $\sqrt{6}\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)$
e) $\pm\sqrt{3}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ f) $\sqrt{3}\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)$

Answer: $\underline{\pm\sqrt{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)}$.

40. The square roots of $z = -2 - i2\sqrt{3}$ are:

a) $\pm(1 - i\sqrt{3})$ b) $\pm\sqrt{2}\left(1 - i\sqrt{3}\right)$ c) $\pm\left(1 + i\sqrt{3}\right)$
d) $\pm\left(-\sqrt{3} + i\right)$ e) $\pm\left(\sqrt{3} - i\right)$ f) $\pm(1 + i)$

Answer: $\underline{\pm\left(1 - i\sqrt{3}\right)}$.

41. The square roots of $z = \frac{-3}{2} + \frac{3\sqrt{3}}{2}i$ are:

a) $\pm 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ b) $\pm\sqrt{3}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ c) $\pm\sqrt{3}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$
d) $\pm 3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ e) $\pm 3\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$ f) $\pm\sqrt{3}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

Answer: $\underline{\pm\sqrt{3}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)}$.

42. Let $z_1 = 3 + 4i$, $z_2 = 1 - 2i$. Find real numbers a and b such that $z_1 z_2 = a + ib$.

a) $a = 1$, $b = 2$ b) $a = -11$, $b = \frac{1}{2}$ c) $a = 11$, $b = -2$

d) $a = -11$, $b = -2$ e) $a = \frac{11}{2}$, $b = 1$ f) $a = \frac{-11}{2}$, $b = 1$

Answer: $\underline{a = 11,\ b = -2}$.

43. If $z = -(1 - i)^4$ and $w = 2 - i$, then $|zw| =$?

a) $16\sqrt{5}$ b) 80 c) $4\sqrt{5}$

d) $4\sqrt{10}$ e) 10 none of the above

Answer: $\underline{4\sqrt{5}}$.

44. If $z = (1 - i)^2$ and $w = 1 - 2i$, then $|zw| =$?

a) $4\sqrt{5}$ b) 20 c) $2\sqrt{5}$ d) 10 e) 40 f) $\sqrt{5}$

Answer: $\underline{2\sqrt{5}}$.

45. If $z_1 = 3 + 2i$, $z_2 = -1 + i$, and $z_3 = 1 - 2i$, then $\dfrac{z_1 + \bar{z}_2}{z_2 z_3} =$?

a) $\frac{-1}{10}(1 + 7i)$ b) $5 - 5i$ c) $\frac{1}{2}(1 + i)$

d) $\frac{5}{\sqrt{10}}(1 + i)$ e) $\frac{1}{10}(1 - 7i)$ f) $\frac{1}{2}(1 - i)$

Answer: $\underline{\frac{1}{2}(1 - i)}$.

46. If $z_1 = 3 - 2i$, $z_2 = 1 + i$, and $z_3 = -2 - i$, then $\dfrac{\bar{z}_1 + z_2^2 - z_3}{z_1 z_3}$ is equal to?

a) $\frac{7+9i}{13}$ b) $\frac{7-9i}{13}$ c) $\frac{-37-29i}{65}$

d) $\frac{-7-9i}{13}$ e) $\frac{37+29i}{65}$ f) $\frac{37-29i}{13}$

Answer: $\underline{\frac{-7-9i}{13}}$.

47. If $z_1 = 2 + 2i$, $z_2 = 1 - i$ and $z_3 = 1 + 2i$, then $\dfrac{\bar{z}_1 - z_2}{z_2 z_3}$ is:

a) $\frac{1+2i}{5}$ b) $\frac{2-4i}{\sqrt{10}}$ c) $\frac{1-2i}{5}$

d) $\frac{-1+2i}{5}$ e) $\frac{1+2i}{\sqrt{10}}$ f) $\frac{-2+4I}{5}$

Answer: $\underline{\frac{1-2i}{5}}$.

48. If A is a matrix such that the inverse of $3A$ is $\begin{bmatrix} -i & 2i \\ 4 & -5 \end{bmatrix}$, then A is:

a) $\frac{1}{9}\begin{bmatrix} -5 & -2i \\ -4 & -i \end{bmatrix}$ b) $\frac{1}{9}\begin{bmatrix} 5i & -2i \\ -4i & 1 \end{bmatrix}$ c) $\frac{1}{9}\begin{bmatrix} -5i & 2 \\ -4i & 1 \end{bmatrix}$

d) $\frac{1}{9}\begin{bmatrix} -5i & -2 \\ -4i & -1 \end{bmatrix}$ e) $\frac{1}{9}\begin{bmatrix} 5 & 2i \\ -4 & -i \end{bmatrix}$ f) none of the above

Answer: $\frac{1}{9}\begin{bmatrix} -5i & 2 \\ -4i & 1 \end{bmatrix}$.

49. If $A = \begin{bmatrix} 2+i & 1+2i \\ 2-i & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2i & -1 \\ 3+3i & 4+2i \end{bmatrix}$, compute $\det AB$.

    a) $-17+19i$ b) $17-19i$ c) $17+19i$

    d) $37-41i$ e) $-37+41i$ f) $-37-41i$

    Answer: $37-41i$.

50. Which of the following is a principal argument of any of the roots of the equation $x^5+x^4+x^3+x^2+x+1=0$?

    a) -1 b) π c) $\frac{4\pi}{3}$ d) $\frac{4\pi}{5}$ e) $\frac{5\pi}{6}$ f) $\frac{\pi}{2}$

    Answer: $\frac{4\pi}{3}$.

51. Which of the following are true for any complex numbers z and w?

    a) If $|z| = |w|$, then $z = w$. b) $\bar{z}+\bar{w} = \overline{z+w}$ c) If $z = w$, then $|z| = |w|$

    d) $|w+z| = |w|+|z|$ e) $\overline{\bar{w}+z} = w+\bar{z}$ f) $|\bar{w}+\bar{z}| = |w+z|$

    Answer: c), e), and f).

52. Which of the following are true for any complex numbers z and w?

    a) $z=\bar{z}$ if and only if z is real b) $\bar{z}\bar{w} = \overline{zw}$ c) $(1+i)^n = (1-i)^n$ for any $n = 1, 2, 3, \ldots$

    d) $2(z-\bar{z}) = \text{im } z$ e) $\overline{z^n} = \bar{z}^n$ f) $\overline{\frac{z}{w}} = \frac{\bar{z}}{\bar{w}}$

    Answer: a), b), e), and f).

53. Which of the following are true?

    a) $\dfrac{3i+4}{4i-3} = \dfrac{i}{25}\det\begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$

    b) $\|(1,2)\|^2 = (1+2i)^2$

    c) $12(1-i) = \det\begin{bmatrix} 4-i & -i \\ 2-i & 2+3i \end{bmatrix}$

    d) $(2+i)\overline{(1-2i)} = (2,1)\cdot(1,-2)$

    e) $\overline{(a-bi)^2} = \overline{a-bi}^2$ for any a, b in $\mathbb{R}$.

    Answer: a), c), and e).

APPENDIX B: Proofs

1. If m and n are integers, which of the following are false (select all correct responses):

   a) $6m + 8n$ is even

   b) $4r + 6s^2 + 2$ is odd

   c) $\frac{1}{m} + \frac{1}{n}$ is an integer for some m and n

   d) $12mn + 7$

   Answer: b).

2. If m and n are integers, which of the following are true (select all correct responses):

   a) There are integers m and n such that $m > 1$ and $n > 1$ and $\frac{1}{m} + \frac{1}{n}$ is an integer.

   b) There is an integer n such that $2n^2 - 5n + 2$ is prime.

   c) For all integers n, if n is odd then $\frac{n-1}{2}$ is odd.

   d) There is an integer $n > 5$ such that $2n - 1$ is prime.

   e) If $2m + n$ is odd then m and n are both odd.

   Answer: c), e).

3. If m and n are integers, which of the following are true (select all correct responses):

   a) If $n - m$ is even then $n^3 - m^3$ is even.

   b) If n is prime then $(-1)^n = -1$.

   c) For all integers n, n^2, $n^2 - n + 11$ is a prime number.

   d) For all integers n, $4(n^2 + n + 1) - 3n^2$ is a perfect square.

   e) If mn is a perfect square, then m and n are perfect squares.

   Answer: a), d).

4. Which of the following are true (select all correct responses):

   a) The product of any two odd integers is odd.

   b) The negative of any odd integer is odd.

   c) The difference of any two odd integers is odd.

   d) The product of any even integer and any integer is even.

   e) If a sum $a + b$ if two integers is even, then one of the summands (i.e. a or b) is even.

   Answer: a), b), and d).

5. Given that two integers are consecutive if and only if one is one more than the other, which of the following are true (select all correct responses):

   a) The difference of any two even integers is even.

   b) The difference of any two odd integers is even.

   c) The difference of the squares of any two consecutive integers is odd.

   d) Every positive integer can be expressed as a sum of three or fewer perfect squares.

   e) Any product of four consecutive integers is one less than a perfect square.

   Answer: a), b), c), and e).

APPENDIX C: Induction

1. For which of the following is the statement S_n not true for the base case (the lowest allowable value of n)?

   a) For all integers $n \geq 0$, $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$.

   b) For all integers $n \geq 1$, $2 + 4 + 6 + \cdots + 2n = n^2 + n$.

   c) For all integers $n \geq 1$, $1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{1}{2}n(5n + 3)$.

   d) For all integers $n \geq 1$, $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$.

   e) For all integers $n \geq 1$, $4^2 + 4^3 + 4^4 + \cdots + 4^n = \frac{4}{3}(4^n - 16)$.

   Answer: c), e).

2. Which of the following series has sum closest to 5000?

   a) $4 + 8 + 2 + 16 + \cdots + 200$

   b) $5 + 10 + 15 + 20 + \cdots + 220$

   c) $3 + 4 + 5 + 6 + \cdots + 100$

   d) $7 + 8 + 9 + 10 + \cdots + 101$

   e) $1 + 2 + 2^2 + 2^3 + \cdots + 2^{11}$

   Answer: c).

3. If, for each positive integer n, S_n is the statement $1 + 3 + 5 + \cdots + (2n - 1) = n^2$, which of the following are correct statements for S_{k+1}? (There may be more than one).

   a) $1 + 3 + 5 + \cdots + (2k - 1) = k^2 + 2k + 1$

   b) $1 + 3 + 5 + \cdots + (2k + 1) = (k + 1)^2$

   c) $1 + 3 + 5 + \cdots + (2k - 1) = (k + 1)^2$

   d) $1 + 3 + 5 + \cdots + (2k - 1) = k^2 - 2k + 1$

   e) $1 + 3 + 5 + \cdots + (2k + 1) = k^2 + 2k + 1$

   Answer: b), e).

4. Observe that:

$$\begin{aligned}
1 &= 1 \\
1 - 4 &= -(1 + 2) \\
1 - 4 + 9 &= 1 + 2 + 3 \\
1 - 4 + 9 - 16 &= -(1 + 2 + 3 + 4) \\
&\vdots
\end{aligned}$$

   A general formula for this is:

   a) $1 - 4 + 9 - 16 + \cdots + (-1)^{n-1}n^2 = (-1)^n(1 + 2 + 3 + 4 + \cdots + n)$

   b) $1 - 4 + 9 - 16 + \cdots + (-1)^n n^2 = (-1)^n(1 - 2 + 3 - 4 + \cdots + (-1)^{n+1}n)$

   c) $1 - 4 + 9 - 16 + \cdots + (-1)^n n^2 = (-1)^{n-1}(1 + 2 + 3 + 4 + \cdots + n)$

d) $1-4+9-16+\cdots+(-1)^{n-1}n^2=(-1)^{n-1}(1+2+3+4+\cdots+n)$

e) $1-4+9-16+\cdots+(-1)^{n-1}n^2=(-1)^{n-1}(1-2+3-4+\cdots+(-1)^{n+1}n)$

Answer: d).

5. Observe that:

$$\begin{aligned}
\tfrac{1}{1\cdot 3} &= \tfrac{1}{3}\\
\tfrac{1}{1\cdot 3}+\tfrac{1}{3\cdot 5} &= \tfrac{2}{5}\\
\tfrac{1}{1\cdot 3}+\tfrac{1}{3\cdot 5}+\tfrac{1}{5\cdot 7} &= \tfrac{3}{7}\\
\tfrac{1}{1\cdot 3}+\tfrac{1}{3\cdot 5}+\tfrac{1}{5\cdot 7}+\tfrac{1}{7\cdot 9} &= \tfrac{4}{9}\\
&\vdots
\end{aligned}$$

A general formula for this is:

a) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n-1)\cdot(2n+1)}=\frac{n}{2n+1}$

b) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n+1)\cdot(2n+3)}=\frac{n}{2n+3}$

c) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n-1)\cdot(2n+1)}=\frac{n+1}{2n+1}$

d) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n+1)\cdot(2n+3)}=\frac{n}{2n+1}$

e) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\cdots+\frac{1}{(2n-3)\cdot(2n-1)}=\frac{n+1}{2n+3}$

Answer: a).