Chapter 5. Hypothesis Testing

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Contents

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- Past experience indicates that a monthly long-distance telephone bill is normally distributed with $\mu_0 = 17.85$ (USD) and a standard deviation $\sigma = 3.39$.

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- Do the data allow us to infer that the campaign was successful?

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- Past experience indicates that a monthly long-distance telephone bill is normally distributed with $\mu_0 = 17.85$ (USD) and a standard deviation $\sigma = 3.39$.
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- Do the data allow us to infer that the campaign was successful?

$\rightarrow$ We need to make a decision of accepting or rejecting the hypothesis that the campaign was successful ($\mu > 17.85$, the mean monthly bill μ after the advertising campaign is greater than 17.85).

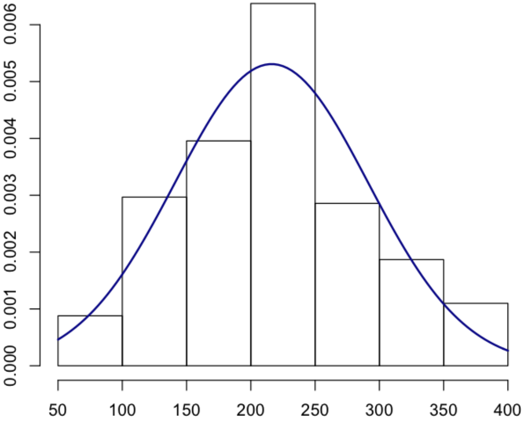
Introduction

Example 5.2: It is known that the mean monthly electricity bills of households in a region in March 2022 is 225.5 (thousands dong). Observed the monnthly electricity bills of 200 households in April 2022 and the data are given below:

196.65	468.75	320.50	300.50	213.05	140.60	290.00	216.95	360.50	317.95	195.55
220.50	255.60	289.00	194.55	374.25	382.05	185.55	219.10	215.60	220.00	186.75
97.80	340.50	88.50	209.50	234.04	333.00	291.10	108.50	245.00	184.00	153.50
219.50	214.15	155.20	140.40	108.50	410.00	125.50	220.30	160.00	300.50	310.20
244.40	194.50	210.20	360.00	456.50	237.40	235.00	203.25	109.20	240.15	260.50
275.50	101.55	455.50	246.25	291.55	262.00	378.65	194.50	248.00	262.92	85.75
248.00	204.75	310.70	213.10	320.50	125.60	110.25	77.35	119.50	313.50	222.00
388.10	110.50	160.00	210.00	310.30	380.10	281.00	105.35	280.15	188.80	272.50
103.40	213.50	280.50	119.50	166.10	180.50	212.00	154.75	100.50	452.60	436.35
225.00	124.30	170.00	127.35	107.90	140.00	195.00	315.10	241.05	168.00	120.50
223.95	237.05	285.45	100.50	228.55	248.70	175.80	466.05	219.00	216.00	425.50
390.00	176.85	240.50	226.00	108.70	160.00	470.50	225.00	440.00	265.00	162.80
260.50	175.80	73.05	460.50	263.60	59.50	198.00	416.50	315.50	155.00	190.00
158.50	225.00	266.70	153.60	238.00	297.60	201.75	240.50	270.90	196.65	299.20
70.50	125.60	100.40	240.00	240.00	224.05	194.00	247.00	325.40	102.20	166.10
361.00	430.00	240.00	250.50	470.00	157.75	98.40	236.50	230.85	317.65	200.70
165.00	350.50	319.15	275.88	203.05	234.50	220.75	180.50	436.50	403.00	460.50
220.00	103.50	222.15	170.50	224.15	460.00	260.40	200.50	311.40	260.00	251.55
100.60	212.20									

Introduction

The distribution of data can be approximated by a normal distribution:



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- The sample mean $\bar{x} = 236.78 > \mu_0 = 225.5$ (the mean monnthly electricity bills of 200 households in April 2022 is greater than that in March 2022).

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- Question: Is there enough evidence to infer that the mean monnthly electricity bills μ of all households in April 2022 is greater than that in March 2022?
- We need to make a decision of accepting or rejecting the hypothesis "$\mu > 225.5$".

Hypothesis testing: Definitions

- Research hypothesis: "$\mu > 225.5$" is called the research hypothesis: need to be accepted or rejected.

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- The alternative hypothesis, labeled by H_1: is the research hypothesis ($\mu > 225.5$).
- Problem of testing: test the null hypothsis $H_0 : \mu \leq 225.5$ versus the alternative hypothesis $H_1 : \mu > 225.5$.

Hypothesis testing: Definitions

- 04 outcomes of a test:

Possible Hypothesis Test Outcomes		
Decision	Accept H_o	Reject H_o
H_o is true	Correct Decision (No error)	Type I Error
	Probability = $1 - \alpha$	Probability = α
H_o is false	Type II Error	Correct Decision (No error)
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- Type I error: we reject H_0 when it is true (we infer that $\mu > 225.5$ but μ is really ≤ 225.5)
- Type II error: we accept H_0 when it is false (we infer that $\mu \leq 225.5$ but μ is really > 225.5)

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- If α decreases, then β will increase and and vice versa.
- We control the P(Type I error), α, at a given threshold, called the level of significance, and the test is called the test of significance.
- We usually choose the level of significance α that equals to $1\%, 5\%$ or 10%.

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- Step 5: Make a decision: if the test-statistic $Z \in W_\alpha$ then we reject H_0, otherwise we fail to reject H_0. Give a conclusion about the research hypothesis.

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  - So if the test-statistic $Z \in W_\alpha$ then we reject H_0, otherwise we fail to reject H_0 .

Four problems of testing

We study on the following four problems of testing:

- Test on a population mean μ.
- Test on a population proportion p.
- Test on the difference between 2 population means $\mu_1 - \mu_2$.
- Test on the difference between 2 population proportions $p_1 - p_2$.

Test on a population mean μ.

Problem 1: We observe a poppulation X where $\mu = E(X)$ and $\sigma^2 = V(X)$. Consider a random sample $(X_1, X_2, ..., X_n)$ taken from the population X.

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- We test a research hypothesis on μ that $\mu > \mu_0$ or $\mu < \mu_0$ or $\mu \neq \mu_0$.

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- It can be proved that the null hypotheses $H_0 : \mu = \mu_0$ and $H_0 : \mu \leq \mu_0$ against the alternative hypothesis $H_1 : \mu > \mu_0$ have the same rejection region W_α.

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- It can be proved that the null hypotheses $H_0 : \mu = \mu_0$ and $H_0 : \mu \leq \mu_0$ against the alternative hypothesis $H_1 : \mu > \mu_0$ have the same rejection region W_α.
- Thus, we only consider the null hypothesis $H_0 : \mu = \mu_0$ against one of three types of alternative:
  - $H_1 : \mu > \mu_0$: right-tailed test
  - or $H_1 : \mu < \mu_0$: left-tailed test
  - or $H_1 : \mu \neq \mu_0$: two-tailed test.

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$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1) \text{ if } H_0 \text{ is true}$$

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- Find the rejection region W_α: We reject H_0 and accept H_1 if $\bar{X} >> \mu_0$ or $\bar{X} - \mu_0$ is large enough $\Leftrightarrow Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} > x$, for a critical value x to be found.

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- The rejection region $W_\alpha = \{Z > x\} = (x; +\infty)$ satisfies:

$$P(\text{Type I error}) = P(\text{reject } H_0 | \text{if } H_0 \text{ is true}) = P(Z \in W_\alpha | H_0 \text{ is true})$$

$$P(\text{Type I error}) = P(Z > x | H_0 \text{ is true}) \leq \alpha \Leftrightarrow x = Z_\alpha$$

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- The rejection region $W_\alpha = \{Z > Z_\alpha\} = (Z_\alpha; +\infty)$.
- The decision rule: If $Z > Z_\alpha$, then we reject H_0, otherwise we fail to reject H_0.

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Case 1.2: We consider the left-tailed test $H_1 : \mu < \mu_0$.

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- Find the rejection region W_α: We reject H_0 and accept H_1 if $\bar{X} << \mu_0$ or $\bar{X} - \mu_0$ is small enough $\Leftrightarrow Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} < x$, for a critical value x to be found.

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- The rejection region $W_\alpha = \{Z < x\} = (\infty; x)$ satisfies:

$$P(\text{Type I error}) = P(\text{reject } H_0|\text{if } H_0 \text{ is true}) = P(Z \in W_\alpha|H_0 \text{ is true})$$

$$P(\text{Type I error}) = P(Z < x|H_0 \text{ is true}) \leq \alpha \Leftrightarrow x = -Z_\alpha$$

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- The decision rule: If $Z < -Z_\alpha$, then we reject H_0, otherwise we fail to reject H_0.

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Case 1: The population $X \sim N(\mu; \sigma^2)$ where σ^2 is known.
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Case 1.3: We consider the two-tailed test $H_1 : \mu \neq \mu_0$.

- We choose the test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1) \text{ if } H_0 \text{ is true}$$

- The rejection region
$W_\alpha = \{|Z| > Z_{\alpha/2}\} = (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty)$.

Test on a population mean μ.

Case 1: The population $X \sim N(\mu; \sigma^2)$ where σ^2 is known.
Case 1.3: We consider the two-tailed test $H_1 : \mu \neq \mu_0$.

- We choose the test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0, 1) \text{ if } H_0 \text{ is true}$$

- The rejection region
$W_\alpha = \{|Z| > Z_{\alpha/2}\} = (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty)$.
- The decision rule: If $|Z| > Z_{\alpha/2}$, then we reject H_0, otherwise we fail to reject H_0.

Test on a population mean μ.

Case 2: The population $X \sim N(\mu; \sigma^2)$ where σ^2 is unknown.

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$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \text{ if } H_0 \text{ is true}$$

Test on a population mean μ.

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- We choose the test statistic

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \text{ if } H_0 \text{ is true}$$

- Similarly, the rejection region is as the following

$$W_\alpha = \begin{cases} (t_{n-1;\alpha}; +\infty), & \text{if } H_1 : \mu > \mu_0 \\ (-\infty; -t_{n-1;\alpha}), & \text{if } H_1 : \mu < \mu_0 \\ (-\infty; -t_{n-1;\alpha/2}) \cup (t_{n-1;\alpha/2}; +\infty), & \text{if } H_1 : \mu \neq \mu_0 \end{cases}$$

.

Test on a population mean μ.

Case 3: We do not know $X \sim N(\mu; \sigma^2)$ where σ^2 is known and n is large enough $(n > 30)$.

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- Similarly, the rejection region is as the following

$$W_\alpha = \begin{cases} (Z_\alpha; +\infty), & \text{if } H_1 : \mu > \mu_0 \\ (-\infty; -Z_\alpha), & \text{if } H_1 : \mu < \mu_0 \\ (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty), & \text{if } H_1 : \mu \neq \mu_0 \end{cases}$$

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Test on a population mean μ.

Case 4: We do not know $X \sim N(\mu; \sigma^2)$ where σ^2 is unknown and n is large enough ($n > 30$).

- We choose the test statistic

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \approx N(0;1) \text{ if } H_0 \text{ is true}$$

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$$W_\alpha = \begin{cases} (Z_\alpha; +\infty), & \text{if } H_1 : \mu > \mu_0 \\ (-\infty; -Z_\alpha), & \text{if } H_1 : \mu < \mu_0 \\ (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty), & \text{if } H_1 : \mu \neq \mu_0 \end{cases}$$

.

Case 5: We do not know $X \sim N(\mu; \sigma^2)$ and n is small. We do not study this case (the nonparametric test).

Test on a population mean μ.

Example 5.1: Test the hypothesis that the average content of containers of a particular lubricant differs from 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.05 level of significance and assume that the distribution of contents is normal.

Test on a population mean μ.

Example 5.1: Test the hypothesis that the average content of containers of a particular lubricant differs from 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.05 level of significance and assume that the distribution of contents is normal.

- Let X be the content of containers of lubricant. It is supposed that $X \sim N(\mu; \sigma^2)$ where σ^2 is unknown. We test the null hypothesis $H_0 : \mu = 10$ against $H_1 : \mu \neq 10$ at $\alpha = 0.05$ level of significance.

Test on a population mean μ.

Example 5.1: Test the hypothesis that the average content of containers of a particular lubricant differs from 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.05 level of significance and assume that the distribution of contents is normal.

- Let X be the content of containers of lubricant. It is supposed that $X \sim N(\mu; \sigma^2)$ where σ^2 is unknown. We test the null hypothesis $H_0 : \mu = 10$ against $H_1 : \mu \neq 10$ at $\alpha = 0.05$ level of significance.
- The test statistic is

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \text{ if } H_0 \text{ is true}$$

Test on a population mean μ.

- For the given sample: $n = 10; \bar{x} = (10.2 + ... + 9.8)/10 = 10.06; s^2 = (10.2^2 + ... + 9.8^2 - 10 * 10.06^2)/9 = 0.06; s = 0.245$. Then

$$T_{obs} = \frac{10.06 - 10}{0.245/\sqrt{10}} = 0.774$$

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$$T_{obs} = \frac{10.06 - 10}{0.245/\sqrt{10}} = 0.774$$

- The rejection region is $W_\alpha = (-\infty; -t_{n-1;\alpha/2}) \cup (t_{n-1;\alpha/2}; +\infty) = (-\infty; -t_{9;0.025}) \cup (t_{9;0.025}; +\infty) = (-\infty; -2.26) \cup (2.26; +\infty)$.

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- Since $T_{obs} \notin W_\alpha$ then we fail to reject H_0.

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- For the given sample: $n = 10; \bar{x} = (10.2 + ... + 9.8)/10 = 10.06; s^2 = (10.2^2 + ... + 9.8^2 - 10 * 10.06^2)/9 = 0.06; s = 0.245$. Then

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- Since $T_{obs} \notin W_\alpha$ then we fail to reject H_0. Thus, there is not enough evidence to infer that the average content of containers of a lubricant differs from 10 liters at 5% level of signifiance.

Introduction

Example 5.2: Past experience indicates that the mean of monthly long-distance telephone bills is $\mu_0 = 17.85$ (USD). After an advertising campaign aimed at increasing long-distance telephone usage, the manager of the telephone company took a random sample of 25 household bills and recorded the mean of their monthly usage $\bar{x} = 19.93$. It is supposed that the monthly long-distance telephone bill follows a normal distribution of a standard deviation $\sigma = 3.39$.

- Do the data allow us to infer that the campaign was successful at 5% level of signifiance?
- Find the probabillity of type II error of the test if the true mean of telephone bills after the advertising is 20 (USD).

Test on a population mean μ.

Solution of Example 5.2:

- Let X be the monthly long-distance telephone bill of households after the advertising campaign. It is supposed that $X \sim N(\mu; \sigma^2)$ where $\sigma = 3.39$. We test the null hypothesis $H_0 : \mu = 17.85$ against $H_1 : \mu > 17.85$ at $\alpha = 0.05$ level of significance.

Test on a population mean μ.

Solution of Example 5.2:

- Let X be the monthly long-distance telephone bill of households after the advertising campaign. It is supposed that $X \sim N(\mu; \sigma^2)$ where $\sigma = 3.39$. We test the null hypothesis $H_0 : \mu = 17.85$ against $H_1 : \mu > 17.85$ at $\alpha = 0.05$ level of significance.
- The test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0; 1) \text{ if } H_0 \text{ is true}$$

Test on a population mean μ.

Solution of Example 5.2:

- Let X be the monthly long-distance telephone bill of households after the advertising campaign. It is supposed that $X \sim N(\mu; \sigma^2)$ where $\sigma = 3.39$. We test the null hypothesis $H_0 : \mu = 17.85$ against $H_1 : \mu > 17.85$ at $\alpha = 0.05$ level of significance.
- The test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0; 1) \text{ if } H_0 \text{ is true}$$

For the given sample, we have $n = 25; \bar{x} = 19.93$ then

$$Z_{obs} = \frac{19.93 - 17.85}{3.39/\sqrt{25}} = 3.07$$

Test on a population mean μ.

- The rejection region is
  $W_\alpha = (Z_\alpha; +\infty) = (Z_{0.05}; +\infty) = (1.645; +\infty)$.

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  $W_\alpha = (Z_\alpha; +\infty) = (Z_{0.05}; +\infty) = (1.645; +\infty)$.
- Since $T_{obs} \in W_\alpha$ then we reject H_0. Thus, there is enough evidence to infer that the campaign was successful at 5% level of signifiance.

Test on a population mean μ.

- The rejection region is
$W_\alpha = (Z_\alpha; +\infty) = (Z_{0.05}; +\infty) = (1.645; +\infty)$.
- Since $T_{obs} \in W_\alpha$ then we reject H_0. Thus, there is enough evidence to infer that the campaign was successful at 5% level of signifiance.
- The probabillity of type II error of the test is

$$\beta = \text{P(type II error)} = \text{P(accept } H_0 |\ H_0 \text{ is false)} = P(Z \notin W_\alpha | \mu = 20)$$

$$= P(Z \leq Z_\alpha | \mu = 20) = P(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \leq Z_\alpha | \mu = 20)$$

$$= P(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq Z_\alpha + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}} | \mu = 20)$$

Test on a population mean μ.

- Since

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0; 1)$$

for all $\mu = E(X)$,

Test on a population mean μ.

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$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0;1)$$

for all $\mu = E(X)$, then

$$\beta = \Phi(Z_\alpha + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}) = \Phi(1.645 + \frac{17.85 - 20}{3.39/\sqrt{25}}) = \Phi(-1.53) = 0.063$$

Test on a population mean μ.

- Since
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0;1)$$
for all $\mu = E(X)$, then
$$\beta = \Phi(Z_\alpha + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}) = \Phi(1.645 + \frac{17.85 - 20}{3.39/\sqrt{25}}) = \Phi(-1.53) = 0.063$$
- The power of test is $1 - \beta = 1 - 0.063 = 0.937$.

Test on a population proportion p.

Example: Suppose that the proportion of defective items in a production line is 1.2%. A technical improvement was made to reduce the rate of defective items. Test 1000 items from the production line after the improvement, 8 of them were found to be defective. Do the data allow us to infer that the improvement was successful at 10% level of signifiance?

Test on a population proportion p.

Example: Suppose that the proportion of defective items in a production line is 1.2%. A technical improvement was made to reduce the rate of defective items. Test 1000 items from the production line after the improvement, 8 of them were found to be defective. Do the data allow us to infer that the improvement was successful at 10% level of signifiance?

- Let p be the proportion of defective items in the production line after the technical improvement.

Test on a population proportion p.

Example: Suppose that the proportion of defective items in a production line is 1.2%. A technical improvement was made to reduce the rate of defective items. Test 1000 items from the production line after the improvement, 8 of them were found to be defective. Do the data allow us to infer that the improvement was successful at 10% level of signifiance?

- Let p be the proportion of defective items in the production line after the technical improvement.
- We test the research hypthesis that $p < 0.012$

Test on a population proportion p.

Problem 2: Let p be a poppulation proportion (for example, the proportion of defective items in a production line).

Test on a population proportion p.

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- We test the null hypotheses $H_0 : p = p_0$ against the alternative $H_1 : p > p_0$ (or $<, \neq$) at level of significance α.

Test on a population proportion p.

Problem 2: Let p be a popppulation proportion (for example, the proportion of defective items in a production line).

- We test the null hypotheses $H_0 : p = p_0$ against the alternative $H_1 : p > p_0$ (or $<, \neq$) at level of significance α.
- Take a random sample of size n (large enough) from the population. Let $\hat{p}$ be the sample proportion (point estimator of p). The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \sim N(0;1) \text{ if } H_0 \text{ is true}$$

Test on a population proportion p.

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- We test the null hypotheses $H_0 : p = p_0$ against the alternative $H_1 : p > p_0$ (or $<, \neq$) at level of significance α.
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$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \sim N(0; 1) \text{ if } H_0 \text{ is true}$$

- The rejection region is as the following

$$W_\alpha = \begin{cases} (Z_\alpha; +\infty), & \text{if } H_1 : p > p_0 \\ (-\infty; -Z_\alpha), & \text{if } H_1 : p < p_0 \\ (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty), & \text{if } H_1 : p \neq p_0 \end{cases}$$

.

Test on a population proportion p.

Example 5.3: Suppose that the proportion of defective items in a production line is 1.2%. A technical improvement was made to reduce the rate of defective items. Test 1000 items from the production line after the improvement, 8 of them were found to be defective.

- Do the data allow us to infer that the improvement was successful at 10% level of signifiance?
- Find the power of test if the true proportion of defective items after the technical improvement is $p_1 = 0.5\%$.

Test on a population proportion p.

Solution:

- Let p be the proportion of defective items in the production line after the technical improvement. We test the null hypotheses $H_0 : p = 0.012$ against the alternative $H_1 : p < 0.012$ at level of significance $\alpha = 0.1$.

Test on a population proportion p.

Solution:

- Let p be the proportion of defective items in the production line after the technical improvement. We test the null hypotheses $H_0 : p = 0.012$ against the alternative $H_1 : p < 0.012$ at level of significance $\alpha = 0.1$.
- The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \sim N(0;1) \text{ if } H_0 \text{ is true}$$

Test on a population proportion p.

Solution:

- Let p be the proportion of defective items in the production line after the technical improvement. We test the null hypotheses $H_0 : p = 0.012$ against the alternative $H_1 : p < 0.012$ at level of significance $\alpha = 0.1$.
- The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \sim N(0;1) \text{ if } H_0 \text{ is true}$$

For the given sample: $n = 1000$; $\hat{p} = 8/1000 = 0.008$, $p_0 = 0.012$ then

$$Z_{obs} = \frac{0.008 - 0.012}{\sqrt{0.008 * 0.992/1000}} = -1.42$$

Test on a population proportion p.

Solution:

- Let p be the proportion of defective items in the production line after the technical improvement. We test the null hypotheses $H_0 : p = 0.012$ against the alternative $H_1 : p < 0.012$ at level of significance $\alpha = 0.1$.
- The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \sim N(0; 1) \text{ if } H_0 \text{ is true}$$

For the given sample: $n = 1000$; $\hat{p} = 8/1000 = 0.008$, $p_0 = 0.012$ then

$$Z_{obs} = \frac{0.008 - 0.012}{\sqrt{0.008 * 0.992/1000}} = -1.42$$

- The rejection region is $W_\alpha = (-\infty; -Z_\alpha) = (-\infty; -1.28)$.

Test on a population proportion p.

Solution:

- Let p be the proportion of defective items in the production line after the technical improvement. We test the null hypotheses $H_0 : p = 0.012$ against the alternative $H_1 : p < 0.012$ at level of significance $\alpha = 0.1$.
- The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \sim N(0; 1) \text{ if } H_0 \text{ is true}$$

For the given sample: $n = 1000$; $\hat{p} = 8/1000 = 0.008$, $p_0 = 0.012$ then

$$Z_{obs} = \frac{0.008 - 0.012}{\sqrt{0.008 * 0.992/1000}} = -1.42$$

- The rejection region is $W_\alpha = (-\infty; -Z_\alpha) = (-\infty; -1.28)$.
- Since $Z_{obs} \in W_\alpha$ then we reject H_0. Thus, there is enough evidence to infer that the technical improvement was successful at 10% level of signifiance

Test on a population proportion p.

- The power of the test is

$$1-\beta = \mathrm{P}(\text{reject } H_0|\ H_0 \text{ is false}) = P(Z \in W_\alpha | p = p_1)$$
$$= P(Z < -Z_\alpha | p = p_1) = P\left(\frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} < -Z_\alpha | p = p_1\right)$$

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$$= P\left(\frac{\hat{p} - p_1}{\sqrt{p_1(1-p_1)/n}} < -Z_\alpha \sqrt{\frac{p_0(1-p_0)}{p_1(1-p_1)}} + \frac{p_0 - p_1}{\sqrt{p_1(1-p_1)/n}} | p = p_1\right)$$

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$$= P\left(\frac{\hat{p}-p_1}{\sqrt{p_1(1-p_1)/n}} < -Z_\alpha\sqrt{\frac{p_0(1-p_0)}{p_1(1-p_1)}} + \frac{p_0-p_1}{\sqrt{p_1(1-p_1)/n}} | p = p_1\right)$$

- Since $Z = \frac{\hat{p}-p_1}{\sqrt{p_1(1-p_1)/n}} \sim N(0,1)$ if $p = p_1$, thus

$$1-\beta = \Phi\left(-Z_\alpha\sqrt{\frac{p_0(1-p_0)}{p_1(1-p_1)}} + \frac{p_0-p_1}{\sqrt{p_1(1-p_1)/n}}\right)$$

Test on a population proportion p.

- When $p_1 = 0.005, n = 1000, p_0 = 0.012, Z_\alpha = 1.28$, the power of the test is

$$1 - \beta = \Phi(-1.28\sqrt{\frac{0.012 * 0.988}{0.005 * 0.995}} + \frac{0.012 - 0.005}{\sqrt{0.005 * 0.995/1000}})$$
$$= \Phi(1.16) = 0.88$$

Test on the difference between two means $\mu_1 - \mu_2$.

Example: A high school math teacher claims that students in her class will score higher on the math portion of the ACT than students in a colleague's math class. The mean ACT math score for 49 students in her class is 22.1 and the sample standard deviation is 4.8. The mean ACT math score for 44 of the colleague's students is 19.8 and the sample standard deviation is 5.4. At $\alpha = 0.1$, can the teacher's claim be supported?

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- Let μ_1, μ_2 be the mean ACT math scores of students in her class and in her colleague's class, respectively.
- We want to test whether $\mu_1 - \mu_2 > 0$ at $\alpha = 0.1$ level of significance.

Test on the difference between two means $\mu_1 - \mu_2$.

Problem 3: We observe 2 poppulations X_1, X_2 where $\mu_1 = E(X_1)$, $\mu_2 = E(X_2)$ and $\sigma_1^2 = V(X_1)$, $\sigma_2^2 = V(X_2)$.

Test on the difference between two means $\mu_1 - \mu_2$.

Problem 3: We observe 2 poppulations X_1, X_2 where $\mu_1 = E(X_1)$, $\mu_2 = E(X_2)$ and $\sigma_1^2 = V(X_1)$, $\sigma_2^2 = V(X_2)$.

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- We test on the difference between two means $\mu_1 - \mu_2$.
- The null hypothesis is $H_0 : \mu_1 - \mu_2 = 0$ against the alternative $H_1 : \mu_1 - \mu_2 > 0$ (or $>, \neq$).

Test on the difference between two means $\mu_1 - \mu_2$.

Problem 3: We observe 2 poppulations X_1, X_2 where $\mu_1 = E(X_1)$, $\mu_2 = E(X_2)$ and $\sigma_1^2 = V(X_1)$, $\sigma_2^2 = V(X_2)$.

- We test on the difference between two means $\mu_1 - \mu_2$.
- The null hypothesis is $H_0 : \mu_1 - \mu_2 = 0$ against the alternative $H_1 : \mu_1 - \mu_2 > 0$ (or $>, \neq$).
- Consider 2 independent random sample of size n_1, n_2 from 2 populations. Let $\bar{X}_1, \bar{X}_2$ be the sample means and S_1^2, S_2^2 be the sample variances of 2 samples, respectively. The sampling distribution of $\bar{X}_1 - \bar{X}_2$ is the following:

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$$

Test on the difference between two means $\mu_1 - \mu_2$

Case 1: Suppose that 2 populations X_1, X_2 are normal and 2 variances σ_1^2, σ_2^2 are known.

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- The rejection region is as the following

$$W_\alpha = \begin{cases} (Z_\alpha; +\infty), & \text{if } H_1 : \mu_1 - \mu_2 > 0 \\ (-\infty; -Z_\alpha), & \text{if } H_1 : \mu_1 - \mu_2 < 0 \\ (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty), & \text{if } H_1 : \mu_1 - \mu_2 \neq 0 \end{cases}$$

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Test on the difference between two means $\mu_1 - \mu_2$

Case 2: Two populations X_1, X_2 are not supposed to be normal, 2 variances σ_1^2, σ_2^2 are unknown and the sample sizes n_1, n_2 are large enough (> 30).

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Case 3: Two populations X_1, X_2 are supposed to be normal, 2 variances σ_1^2, σ_2^2 are unknown and equal ($\sigma_1^2 = \sigma_2^2$).

Test on the difference between two means $\mu_1 - \mu_2$

Case 3: Two populations X_1, X_2 are supposed to be normal, 2 variances σ_1^2, σ_2^2 are unknown and equal ($\sigma_1^2 = \sigma_2^2$).

- The test statistic is

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S^2}{n_1} + \frac{S^2}{n_2}}} \sim t_{d.f} \text{ if } H_0 \text{ is true,}$$

where $d.f = n_1 + n_2 - 2$ and

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \text{ is called the pooled variance.}$$

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- The rejection region is as the following

$$W_\alpha = \begin{cases} (t_{d.f,\alpha}; +\infty), & \text{if } H_1 : \mu_1 - \mu_2 > 0 \\ (-\infty; -t_{d.f,\alpha}), & \text{if } H_1 : \mu_1 - \mu_2 < 0 \\ (-\infty; -t_{d.f,\alpha/2}) \cup (t_{d.f,\alpha/2}; +\infty), & \text{if } H_1 : \mu_1 - \mu_2 \neq 0. \end{cases}$$

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Test on the difference between two means $\mu_1 - \mu_2$.

Example 5.4: A high school math teacher claims that students in her class will score higher on the math portion of the ACT than students in a colleague's math class. The mean ACT math score for 49 students in her class is 22.1 and the sample standard deviation is 4.8. The mean ACT math score for 44 of the colleague's students is 19.8 and the sample standard deviation is 5.4. At $\alpha = 0.1$, can the teacher's claim be supported?

Test on the difference between two means $\mu_1 - \mu_2$.

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Solution:

- Let μ_1, μ_2 be the mean ACT math scores of students in her class and in her colleague's class, respectively. We test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_1 : \mu_1 - \mu_2 > 0$ at $\alpha = 0.1$ level of significance.

Test on the difference between two means $\mu_1 - \mu_2$.

- It is not supposed that the populations are normal and the two population variances are unknown, but the sample sizes $n_1 = 49, n_2 = 44$ are large enough. Thus, the test statistic is

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \approx N(0,1) \text{ if } H_0 \text{ is true}$$

Test on the difference between two means $\mu_1 - \mu_2$.

- It is not supposed that the populations are normal and the two population variances are unknown, but the sample sizes $n_1 = 49, n_2 = 44$ are large enough. Thus, the test statistic is

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \approx N(0,1) \text{ if } H_0 \text{ is true}$$

For the given sample, we have: $n_1 = 49, n_2 = 44; \bar{x}_1 = 22.1$, $\bar{x}_2 = 19.8, s_1 = 4.8, s_2 = 5.4$, then

$$T_{obs} = \frac{22.1 - 19.8}{\sqrt{\frac{4.8^2}{49} + \frac{5.4^2}{44}}} = 2.16$$

Test on the difference between two means $\mu_1 - \mu_2$.

- The rejection region is $W_\alpha = (Z_\alpha; +\infty) = (Z_{0.1}; +\infty) = (1.28; +\infty)$.

Test on the difference between two means $\mu_1 - \mu_2$.

- The rejection region is $W_\alpha = (Z_\alpha; +\infty) = (Z_{0.1}; +\infty) = (1.28; +\infty)$.
- Since $T_{obs} = 2.16 \in W_\alpha$ then we reject H_0. Thus, there is enough evidence at the 10% level to support the teacher's claim that her students score better on the ACT.

Test on the difference between two means $\mu_1 - \mu_2$.

Example 5.5: To find out whether a new serum will arrest leukemia, 9 mice, all with an advanced stage of the disease, are selected. Five mice receive the treatment and 4 do not. Survival times, in years, from the time the experiment commenced are as follows:

- Treatment: 2.1, 5.3, 1.4, 4.6, 0.9
- No Treatment: 1.9, 0.5, 2.8, 3.1

At the 0.05 level of significance, can the serum be said to be effective? Assume the two populations to be normally distributed with equal variances.

Test on the difference between two means $\mu_1 - \mu_2$.

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At the 0.05 level of significance, can the serum be said to be effective? Assume the two populations to be normally distributed with equal variances.

Solution:

- Let X_1, X_2 be the survival times, in years, of mice with and without treatment of the new serum. To test whether the serum is said to be effective, we test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_1 : \mu_1 - \mu_2 > 0$ at $\alpha = 0.05$ level of significance.

Test on the difference between two means $\mu_1 - \mu_2$.

- It is supposed that $X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$ where σ_1, σ_2 are unknown but equal. Thus, the test statistic is

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S^2}{n_1} + \frac{S^2}{n_2}}} \sim t_{n_1+n_2-2} \text{ if } H_0 \text{ is true}$$

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For the given sample, we have: $n_1 = 5, n_2 = 4$,
$\bar{x}_1 = (2.1 + 5.3 + 1.4 + 4.6 + 0.9)/5 = 2.86$,
$\bar{x}_2 = (1.9 + 0.5 + 2.8 + 3.1)/4 = 2.075$,
$s_1^2 = (2.1^2 + .. + 0.9^2 - 5 * 2.86^2)/4 = 3.883$,
$s_2^2 = (1.9^2 + ... + 3.1^2 - 4 * 2.075^2)/3 = 1.3625$, then

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{4 * 3.883 + 3 * 1.3625}{7} = 2.803$$

Test on the difference between two means $\mu_1 - \mu_2$.

- Thus

$$T_{obs} = \frac{2.86 - 2.075}{\sqrt{\frac{2.803}{5} + \frac{2.803}{4}}} = 0.699$$

Test on the difference between two means $\mu_1 - \mu_2$.

- Thus

$$T_{obs} = \frac{2.86 - 2.075}{\sqrt{\frac{2.803}{5} + \frac{2.803}{4}}} = 0.699$$

- The rejection region is
$W_\alpha = (t_{n_1+n_2-2;\alpha}; +\infty) = (t_{7;0.05}; +\infty) = (1.89; +\infty)$.

Test on the difference between two means $\mu_1 - \mu_2$.

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$W_\alpha = (t_{n_1+n_2-2;\alpha}; +\infty) = (t_{7;0.05}; +\infty) = (1.89; +\infty)$.
- Since $T_{obs} = 0.699 \notin W_\alpha$ then we fail to reject H_0. Thus, there is not enough evidence at the 5% level to infer that the new serum is said to be effective.

Test on the difference between two proportions $p_1 - p_2$.

Problem 4: Let p_1, p_2 be two population proportions (for example, proportions of defecctive items in 2 different production lines).

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- Draw 2 independent random samples of size n_1, n_2 from 2 populations.

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- Let $\hat{p}_1, \hat{p}_2$ be two sample proportions, the point estimates of p_1, p_2, respectively.

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- Draw 2 independent random samples of size n_1, n_2 from 2 populations.
- Let $\hat{p}_1, \hat{p}_2$ be two sample proportions, the point estimates of p_1, p_2, respectively.
- The test statistic is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \approx N(0,1) \text{ if } H_0 \text{ is true,}$$

where $\hat{p}$ is the weighted estimate of p_1 and p_2:

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

Test on the difference between two proportions $p_1 - p_2$

- Remark that

$$\hat{p}_1 = \frac{m_1}{n_1}; \hat{p}_2 = \frac{m_2}{n_2}; \hat{p} = \frac{m_1 + m_2}{n_1 + n_2},$$

where m_1, m_2 are the number of successes in 2 samples, respectively.

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where m_1, m_2 are the number of successes in 2 samples, respectively.

- The rejection region is as the following

$$W_\alpha = \begin{cases} (Z_\alpha; +\infty), & \text{if } H_1 : p_1 - p_2 > 0 \\ (-\infty; -Z_\alpha), & \text{if } H_1 : p_1 - p_2 < 0 \\ (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty), & \text{if } H_1 : p_1 - p_2 \neq 0 \end{cases}$$

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$$W_\alpha = \begin{cases} (Z_\alpha; +\infty), & \text{if } H_1 : p_1 - p_2 > 0 \\ (-\infty; -Z_\alpha), & \text{if } H_1 : p_1 - p_2 < 0 \\ (-\infty; -Z_{\alpha/2}) \cup (Z_{\alpha/2}; +\infty), & \text{if } H_1 : p_1 - p_2 \neq 0 \end{cases}$$
.
- The sample sizes n_1, n_2 must be large enough (we usually suppose that $n_1 p_1, n_1(1 - p_1), n_2 p_2, n_2(1 - p_2)$ are ≥ 5).

Test on the difference between two proportions $p_1 - p_2$.

Example 5.6: A recent survey stated that male college students smoke less than female college students. In a survey of 1245 male students, 361 said they smoke at least one pack of cigarettes a day. In a survey of 1065 female students, 341 said they smoke at least one pack a day. At $\alpha = 0.1$, can you support the claim that the proportion of male college students who smoke at least one pack of cigarettes a day is lower than the proportion of female college students who smoke at least one pack a day?

Test on the difference between two proportions $p_1 - p_2$.

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Solution:

- Let p_1, p_2 be the proportions of male and of female college students who smoke at least one pack of cigarettes a day, respectively. We test $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 < 0$ at $\alpha = 0.1$ level of significance.

Test on the difference between two proportions $p_1 - p_2$.

- The test statistic is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \approx N(0,1) \text{ if } H_0 \text{ is true.}$$

Test on the difference between two proportions $p_1 - p_2$.

- The test statistic is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \approx N(0,1) \text{ if } H_0 \text{ is true.}$$

For the given samples, we have: $n_1 = 1245, n_2 = 1065$;
$m_1 = 361, m_2 = 341$,

Test on the difference between two proportions $p_1 - p_2$.

- The test statistic is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \approx N(0,1) \text{ if } H_0 \text{ is true.}$$

For the given samples, we have: $n_1 = 1245, n_2 = 1065$; $m_1 = 361, m_2 = 341$, then

$$\hat{p}_1 = \frac{361}{1245} = 0.29; \hat{p}_2 = \frac{341}{1065} = 0.32; \hat{p} = \frac{361 + 341}{1245 + 1065} = 0.304$$

Test on the difference between two proportions $p_1 - p_2$.

- The test statistic is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \approx N(0,1) \text{ if } H_0 \text{ is true.}$$

For the given samples, we have: $n_1 = 1245, n_2 = 1065$; $m_1 = 361, m_2 = 341$, then

$$\hat{p}_1 = \frac{361}{1245} = 0.29; \hat{p}_2 = \frac{341}{1065} = 0.32; \hat{p} = \frac{361+341}{1245+1065} = 0.304$$

Thus,

$$Z = \frac{0.29 - 0.32}{\sqrt{0.304 * 0.694\left(\frac{1}{1245} + \frac{1}{1065}\right)}} = -1.56$$

Test on the difference between two proportions $p_1 - p_2$.

- The rejection region is
  $W_\alpha = (-\infty; -Z_\alpha) = (-\infty; Z_{0.1}) = (-\infty; -1.28)$.

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- The rejection region is
  $W_\alpha = (-\infty; -Z_\alpha) = (-\infty; Z_{0.1}) = (-\infty; -1.28)$.
- Since $Z_{obs} = -1.56 \in W_\alpha$ then we reject H_0. Thus, there is enough evidence at the 10% level to claim that the proportion of male college students who smoke at least one pack of cigarettes a day is lower than the proportion of female college students who smoke at least one pack a day.