LESSON 19

LINEAR PHASE FIR DIGITAL FILTER

PhD. Nguyen Hong Quang
Assoc. Prof. Trinh Van Loan
PhD. Doan Phong Tung
Computer Engineering Department

❑ CONTENT

1. Digital filter design objective
2. Impulse response characteristic h(n) of linear phase FIR filter

❑ Lesson Objectives

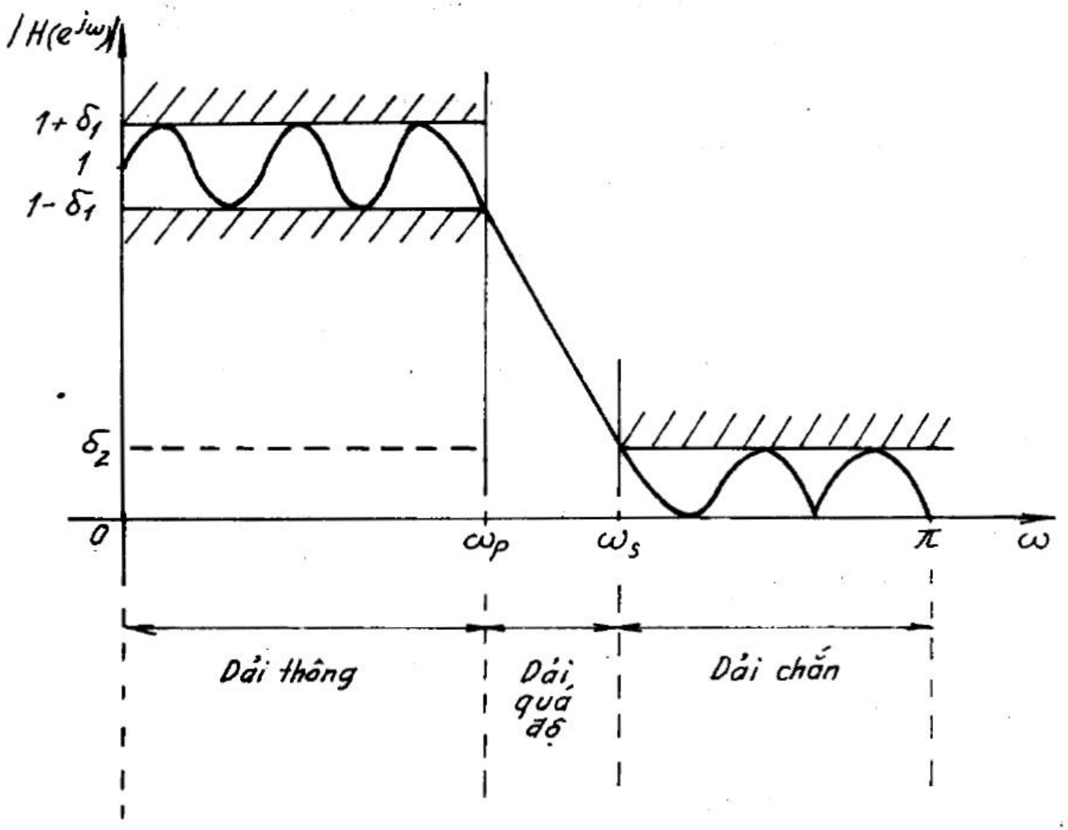
After completing this lesson, you will be able to understand the following topics:

- The objective is to design a linear phase FIR digital filter, the synthesis stages as well as the pros and cons of the FIR filter.
- Condition of impulse response for FIR filter to have linear phase.

1. Design goal

Step 1. Determine the filter coefficients that satisfy the given specifications:

$\delta_1, \delta_2, \omega_p, \omega_s$



- δ_1: ripple in pass-band
- δ_2: ripple in the stop-band
- ω_p: limiting frequency (bandwidth) pass-band
- ω_s: limiting frequency (bandwidth) stop-band

FIR digital filter synthesis stages

Step 2. Choose the structure to quantize the coefficients of the filter according to the finite number of bits allowed

Step 3. Quantize the filter variables, i.e. choose the word length for: input, output, intermediate memories

Step 4. Check by computer simulation that the final filter meets the specifications.

Advantages and disadvantages of the FIR filter

- Advantages: the filter is always causal, stable, satisfying linear phase, capable of receiving relatively small computational noise.
- Disadvantage: the order of the filter is quite high compared to the IIR filter with the same specifications.

2. Impulse response condition for FIR filter to have linear phase

- Differential Equation :

$$y(n) = h(0).x(n) + h(1).x(n-1) + \cdots + h(N-1).x(n-N+1)$$

- Transfer function H(z):

$$H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n} = h(0) + h(1)Z^{-1} + \cdots + h(N-1)Z^{-(N-1)}$$

- Frequency response (periodic with period 2π):

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \sum_{n=0}^{N-1} h(n)\cos\omega n + j\left[-\sum_{n=0}^{N-1} h(n)\sin\omega n\right]$$

- Amplitude response : $H\left(e^{j\omega}\right) = \left|H(e^{j\omega})\right|e^{j\varphi(\omega)}$

- Phase response : $\varphi(\omega) = \arg\left[H(e^{j\omega})\right]$

Group delay and linear phase FIR filter

- Delay:

$$x(n) \xrightarrow{F} X(e^{j\omega})$$
$$x(n-n_0) \xrightarrow{F} ?$$

$$F\{x(n-n_0)\} = \sum_{n=-\infty}^{\infty} x(n-n_0)e^{-j\omega n}$$
$$= e^{-j\omega n_0}X(e^{j\omega})$$

- Comment: The delay signal has a constant amplitude spectrum while the phase spectrum is shifted by $-\omega n_0$
- Definition of group delay: $\tau(\omega) = -\frac{d\varphi(\omega)}{d\omega}$, $\varphi(\omega)$: phase response
- For a filter to have a constant group delay, it must have a linear phase
- Then the signal through the passband of the filter will appear exactly at the output with the given delay.

Condition $h(n)$ for linear phase FIR

- Frequency response:

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega+j\beta}$$

- In there:
  - α, β: constants
  - $A(e^{j\omega})$: real function of ω
  - $\arg[H(e^{j\omega})] = \beta - \alpha\omega, 0 < \omega < \pi$
- Definition of group delay: $\tau = -\frac{d\varphi}{d\omega}$, φ: phase response

Condition $h(n)$ for linear phase FIR

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega + j\beta}$$

$$H(e^{j\omega}) = A(e^{j\omega})\cos(\beta - \omega\alpha) + jA(e^{j\omega})\sin(\beta - \omega\alpha)$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h(n)e^{-j\omega n} = \sum_{n=-\infty}^{+\infty} h(n)\cos\omega n - j\sum_{n=-\infty}^{-\infty} h(n)\sin\omega n$$

- Giả thiết $h(n)$ thực:

$$A(e^{j\omega})\cos(\beta - \omega\alpha) = \sum_{n=-\infty}^{+\infty} h(n)\cos\omega n$$

$$A(e^{j\omega})\sin(\beta - \omega\alpha) = -\sum_{n=-\infty}^{-\infty} h(n)\sin\omega n$$

$$\frac{\sin(\beta - \omega\alpha)}{\cos(\beta - \omega\alpha)} = \frac{-\sum_{n=-\infty}^{\infty} h(n)\sin\omega n}{\sum_{n=-\infty}^{\infty} h(n)\cos\omega n}$$

$$\sum_{n=-\infty}^{\infty} h(n)[\cos\omega n.\sin(\beta - \omega\alpha)$$
$$+\sin\omega n.\cos(\beta - \omega\alpha)] = 0$$

$$\sum_{n=-\infty}^{\infty} h(n)\sin[\omega(n - \alpha) + \beta] = 0 \quad \forall\omega$$

4 types of linear phase FIR filters

❖ With $\beta = 0$: $\alpha = \frac{N-1}{2}, h(n) = h\,(N-1-n)$

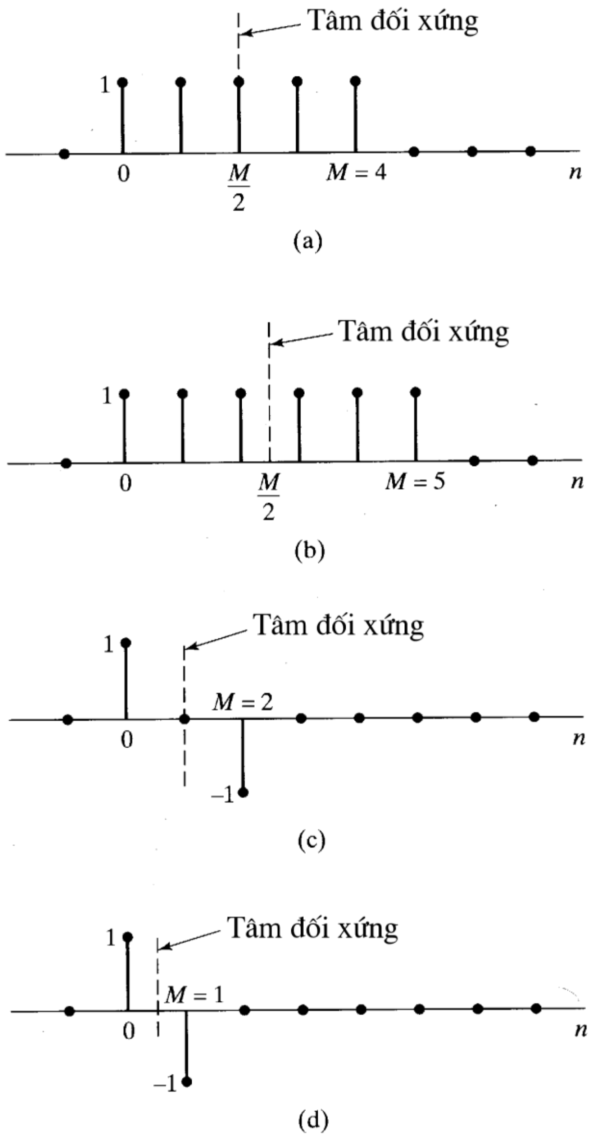
1. Filter type 1: h(n) symmetric, N odd

2. Type 2 filter: h(n) symmetric, N even

❖ For $\beta \neq 0$: $\alpha = \frac{N-1}{2}, \beta = \pm\frac{\pi}{2}, h(n) = -h(N-1-n)$

3. Type 3 filter: h(n) antisymmetric, N odd

4. Type 4 filter: h(n) antisymmetric, N even



4. Summary

- FIR filters are always causal, stable, and satisfy linear phase. However, the order of the filter is quite high compared to IIR filters with the same specifications.
- A linear phased FIR filter will pass the signal through the bandpass at the output with exactly a given delay.
- There are four types of FIR filters that satisfy the linear phase condition, which are class 1, 2, 3, and 4 filters.

5. Exercises

- Exercise 1
  - ❑ Prove that when h(n) is symmetric, $N = 6, \alpha = 2.5$ and $\beta = 0$ then:

$$\sum_{n=-\infty}^{\infty} h(n)\sin[\omega(n-\alpha)+\beta] = 0 \quad \forall\omega$$

Homework

- Exercise 2
  - Prove that when h(n) is symmetric, $N = 5, \alpha = 2$ and $\beta = 0$ then:

$$\sum_{n=-\infty}^{\infty} h(n)\sin[\omega(n-\alpha)+\beta] = 0 \quad \forall\omega$$

Homework

- Exercise 3

  - Prove that when h(n) is antisymmetric, $N = 5, \alpha = 2$ and $\beta = \frac{\pi}{2}$ then:

$$\sum_{n=-\infty}^{\infty} h(n)\sin[\omega(n-\alpha)+\beta] = 0 \quad \forall\omega$$

Homework

- Exercise 4
  - Prove that when h(n) is antisymmetric, $N = 6, \alpha = 2.5$ and $\beta = \frac{\pi}{2}$ then:

$$\sum_{n=-\infty}^{\infty} h(n)\sin[\omega(n-\alpha)+\beta] = 0 \quad \forall\omega$$

Next lesson. Lesson 20

ĐẶC ĐIỂM VÀ ỨNG DỤNG CỦA TỪNG LOẠI BỘ LỌC FIR

References :

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

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Wish you all good study!