HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS
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MI2020E

EXERCISES OF PROBABILITY AND STATISTICS

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Chapter 3

VECTORS OF RANDOM VARIABLES

3.1 PAIRS OF DISCRETE RANDOM VARIABLES.

Problem 3.1.

Two random variables X and Y have the joint PDT as follows

$X \setminus Y$	1	2	3
1	0.12	0.15	0.03
2	0.28	0.35	0.07

(a) Find the marginal probability distribution (PDT) of X and of Y.
(b) Find the PDT of Z, where $Z = XY$.
(c) Find the conditional probability distribution of X given that $Y = 1$.
(d) Find the conditional probability distribution of Y given that $X = 1$.
(e) Are X and Y independent?
(f) Find the covariance matrix of (X, Y).
(g) Find the coefficient of correlation between X and Y.
(h) Are X and Y related?
(i) Find $P[X < 2; Y < 1.5]$.

Problem 3.2.

Random variables X and Y have the joint PDT as the following

$X \mid Y$	-1	0	1
-1	$\frac{4}{15}$	$\frac{1}{15}$	$\frac{4}{15}$
0	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
1	0	$\frac{2}{15}$	0

(a) Find the marginal probability distribution (PDT) of X and of Y.
(b) Find the PDT of Z, where $Z = X + Y$.
(c) Find the conditional probability distribution of X given that $Y = -1$.
(d) Find the conditional probability distribution of Y given that $X = 1$.
(e) Are X and Y independent?
(f) Find the covariance of (X, Y).
(g) Find the coefficient of correlation between X and Y.
(h) Are X and Y related?
(i) Find $P[X < 0; Y < 0.5]$.

Problem 3.3.

Vector (X, Y) has the following joint PMF

$$f(x,y) = P(X = x, Y = y) = \begin{cases} cxy, & x = 1,2,4;\ y = 1,3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?
(b) What is $P[Y < X]$?
(c) Find the marginal probability distribution (PDT) of X and of Y.
(d) Find the conditional probability distribution of X given that $Y = 3$.
(e) Find the covariance matrix of (X, Y).
(f) Find the coefficient of correlation between X and Y.
(g) Are X and Y related?

3.2 PAIRS OF CONTINUOUS RANDOM VARIABLES.

Problem 3.4.
Random variables X and Y have joint PDF

$$f(x,y) = \begin{cases} cxy^2, & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?
(b) Find $P[X > Y]$ and $P[Y < X^2]$.
(c) Find $P\left[\min(X,Y) \le \frac{1}{2}\right]$.
(d) Find the marginal PDF of X and of Y.
(e) Find the conditional pdf of X given that $Y = 1/2$.
(f) Find the conditional pdf of Y given that $X = 1/3$.
(g) Find the covariance of (X,Y).
(h) Find the coefficient of correlation between X and Y.
(i) Are X and Y related? independent?
(j) Find the joint cdf of (X,Y).

Problem 3.5.
Random variables X and Y have the joint PDF

$$f(x,y) = \begin{cases} c, & x+y \le 1,\ x \ge 0,\ y \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?
(b) What is $P[X \le Y]$?
(c) Find the marginal CDF of X and of Y.
(d) Find the conditional pdf of X given that $Y = 1/2$.
(e) Find the conditional pdf of Y given that $X = 1/3$.
(f) Find the covariance of (X,Y).
(g) Find the coefficient of correlation between X and Y.
(h) Are X and Y related? independent?
(i) Find the joint cdf of (X,Y).

Problem 3.6.

Random variables X and Y have the joint PDF

$$f(x,y) = \begin{cases} cy, & 0 \le y \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Draw the region of nonzero probability.
(b) What is the value of the constant c?
(c) Find the marginal CDF of X and of Y.
(d) Find the conditional pdf of X given that $Y = 1/2$.
(e) Find the covariance of (X, Y).
(f) Find the coefficient of correlation between X and Y.
(g) Are X and Y related? independent?
(h) What is $P[X + Y > 1/2]$?
(i) Find the joint cdf of (X, Y).