Introduction to Communications Engineering

IT4593E

Đỗ Công Thuần, Ph.D.

Dept. of CE, SoICT, HUST

Email: thuandc@soict.hust.edu.vn

ONE LOVE. ONE FUTURE.

Thông tin chung

- Tên học phần: **Nhập môn kỹ thuật truyền thông**
- Mã học phần: **IT4593E**
- Khối lượng: **2 TC (2-1-0-4)**
- Lý thuyết và bài tập: **10 buổi lý thuyết, 5 buổi bài tập**
- Đánh giá học phần:

  30% QT (kiểm tra + bài tập/project + chuyên cần-quiz)

  70% CK (trắc nghiệm + tự luận)
- Tài liệu tham khảo:
  - Lecture slides
  - Lecture notes
  - Textbooks, ví dụ ***Communication Systems Engineering***, 2nd Edition, by John G. Proakis Masoud Salehi
  - Internet

Lec 07: InterSymbol Interference (ISI)

Signals with Infinite Time Domain

Consider a 1-D signal space with zero mean. The transmitted signal is defined as

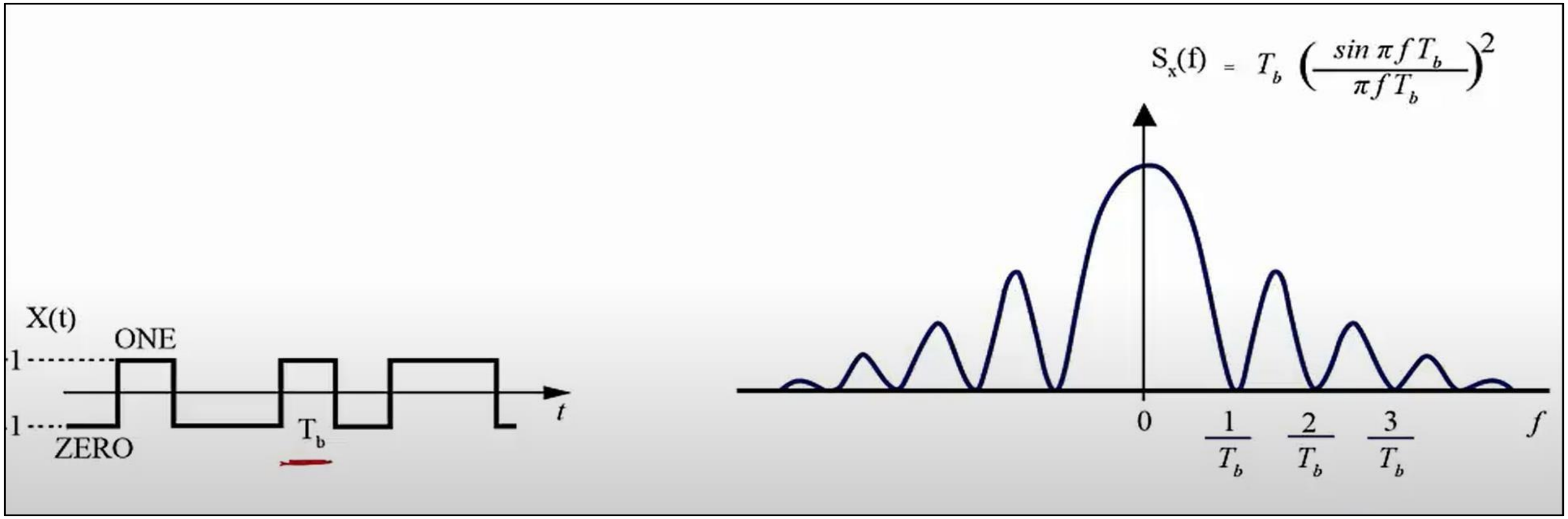
$$s(t) = \sum_n a[n]\, p(t - nT)$$

And the power spectral density (PSD) is calculated as follows:

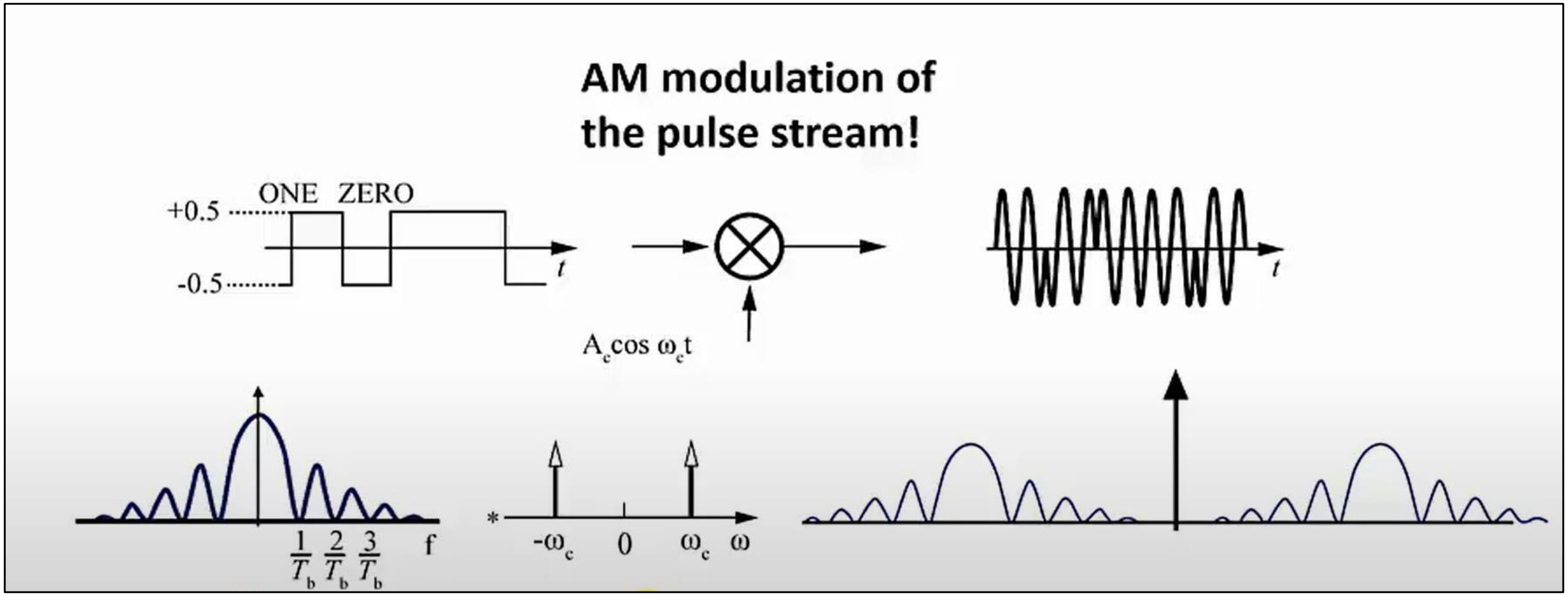
$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T}$$

Therefore, if $p(t) = b_1(t)$ has a finite time domain, the transmitted signal $s(t)$ will have an infinite frequency domain.

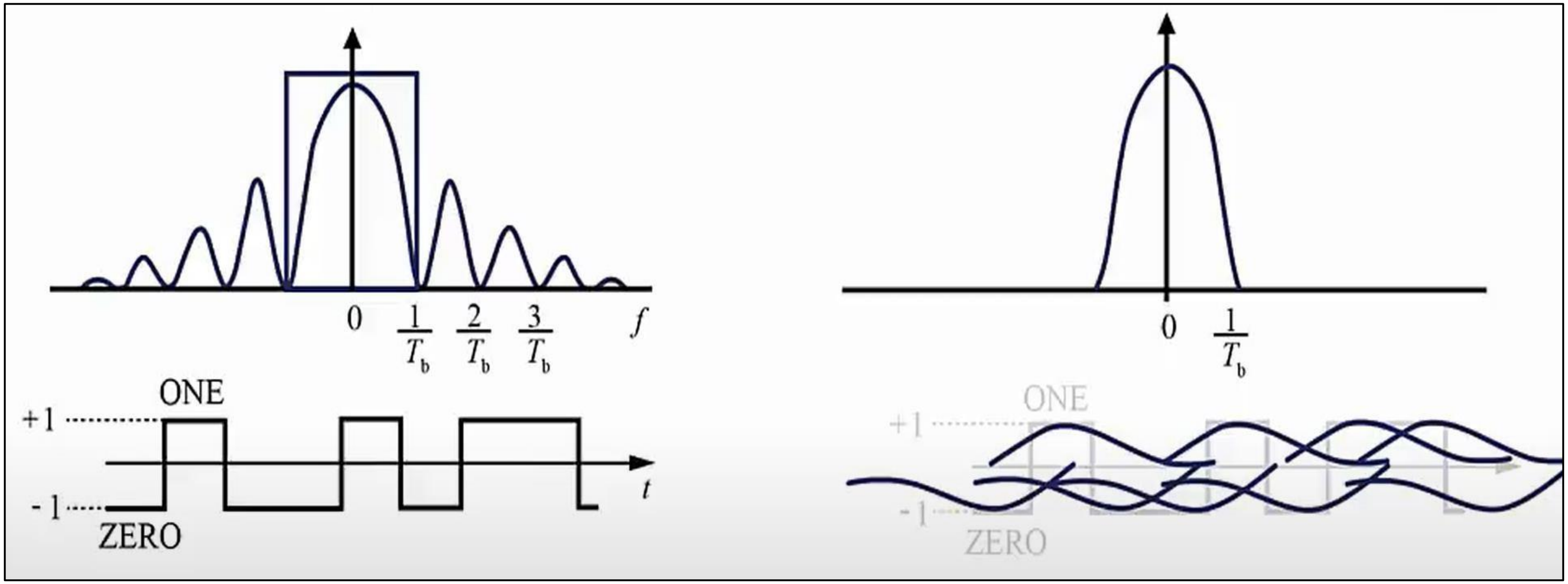
- Using a rectangular pulse to transmit symbols {0,1}



- If modulated to the frequency corresponding to ω_0



- The ISI phenomenon occurs in the time domain after filtering in the frequency domain:



Signals with Infinite Time Domain

To address this issue, we can use signals with an infinite time domain in order to obtain a finite frequency domain.

Consider the signal space M consisting of signals with an infinite time domain (but finite energy), and assume that M is a 1-D space with an orthonormal basis:

$$B = \{b_1(t)\}$$

Assume transmitting only one symbol $a[0]$.

Given the received signal $r(t)=\rho(t)$ (assuming ideal channel), we calculate the projection onto the orthonormal vector $b_1(t)$:

$$\rho[1] = \int_{-\infty}^{+\infty} \rho(t) b_1(t) dt$$

(Note: the integration interval is no longer only from 0 to T)

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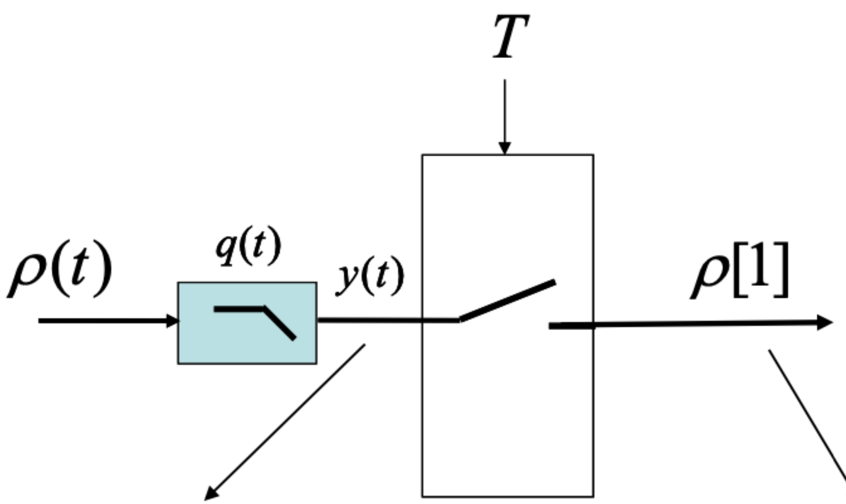
As learned in the previous lecture, the projection can be calculated using the matched filter (MF): $q(t) = b_1(T - t)$ (impulse response)



$$y(t) = \int_{-\infty}^{+\infty} r(\tau) q(t-\tau) d\tau = \int_{-\infty}^{+\infty} r(\tau) b_1(T - t + \tau) d\tau$$

$$y(t = T) = \int_{-\infty}^{+\infty} \rho(t) b_1(t) dt = \rho[1]$$

Now let us consider the case of transmitting an infinite sequence of symbols $(a[n])$

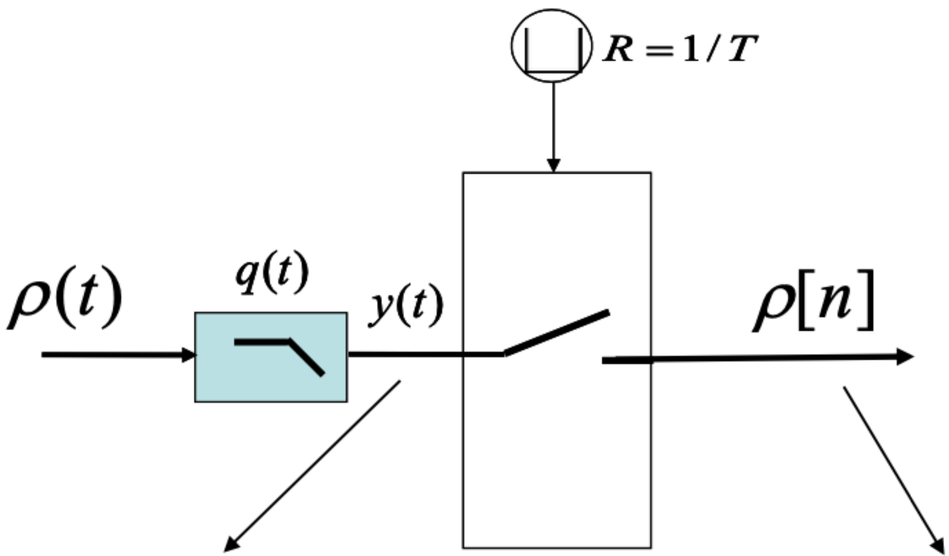
$$s(t) = \sum_{n=-\infty}^{+\infty} a[n]p(t - nT)$$

Assume an ideal channel with properties:

- $H(f)=1$
- $n(t)=0$

$$\Rightarrow \quad \rho(t) = s(t)$$

At the receiver side, the projections $\rho[n]$ are computed by using a matched filter (MF) and sampling at the appropriate instant $(n+1)T$



$$y(t) = \int_{-\infty}^{+\infty} r(\tau) q(t-\tau) d\tau = \int_{-\infty}^{+\infty} r(\tau) b_1(T-t+\tau) d\tau$$

$$y(t = (n+1)T) = \rho[n]$$

At the Sampling Instant

In the ideal case we have $\rho[n] = y((n+1)T) = y(T + nT)$

Write concisely as $\rho[n] = y(t_0 + nT)$

In the ideal case $t_0 = T$

In practice $t_0 = T + D$

Where the delay D may include:

- Propagation delay
- Processing delay
- ...

(The receiver's symbol synchronization blocks can determine the exact timing t_0)

Intersymbol interference (ISI)

Output signal of the MF:

$$y(t) = \rho(t) * q(t)$$

Since the channel is ideal, we have $\rho(t) = s(t)$, therefore:

$$y(t) = s(t) * q(t) = \left(\sum_{n=-\infty}^{+\infty} a[n] p(t - nT) \right) * q(t) =$$

$$= \sum_{n=-\infty}^{+\infty} a[n] x(t - nT)$$

where $\boxed{x(t) = p(t) * q(t)}$

Intersymbol interference

The received signal:

$$\rho[n] = y(t_0 + nT) = \sum_{m=-\infty}^{+\infty} a[m]x(t_0 + nT - mT) = \sum_{i=-\infty}^{+\infty} a[n-i]x(t_0 + iT) =$$

$$= \sum_{i=-\infty}^{+\infty} x[i]a[n-i]$$

where $x[i] = x(t_0 + iT)$

The received symbol $\rho[n]$ is computed from the transmitted symbol $a[n]$ as follows:

$$\boxed{\rho[n] = \sum_{i=-\infty}^{+\infty} x[i]a[n-i]}$$

We can write this as:

$$\rho[n] = x[0]a[n] + \sum_{i=1}^{+\infty} x[i]a[n-i] + \sum_{i=-\infty}^{-1} x[i]a[n-i]$$

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Contribution from previous transmitted symbols

Received symbol

$$\rho[n] = x[0]a[n] + \sum_{i=1}^{+\infty} x[i]a[n-i] + \sum_{i=-\infty}^{-1} x[i]a[n-i]$$

Contribution from the desired transmitted symbol

Contribution from subsequent transmitted symbols

It can be seen that the received symbol $\rho[n]$ depends not only on the transmitted symbol $a[n]$, but also on other transmitted symbols.

→ This means the phenomenon of **Intersymbol Interference (ISI)** has occurred.

Received symbol

$$\rho[n] = \sum_{i=-\infty}^{+\infty} x[i]a[n-i] =$$

$$= x[0]a[n] +$$

Transmitted symbol

$$+ x[1]a[n-1] + x[2]a[n-2] + \ldots.$$

previous transmitted symbols

$$+ x[-1]a[n+1] + x[-2]a[n+2] + \ldots.$$

subsequent transmitted symbols

Since we assume the transmission channel is ideal, it follows that

$$\rho[n] = a[n]$$

(Received symbol = Transmitted symbol)

This is achieved if and only if

$$\boxed{\begin{array}{lll} x[i]=1 & \text{if} & i=0 \\ x[i]=0 & \text{if} & i\neq 0 \end{array}}$$

$$\boxed{\begin{array}{lll} x\left(t_0+iT\right)=1 & \text{if} & i=0 \\ x\left(t_0+iT\right)=0 & \text{if} & i\neq 0 \end{array}}$$

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Signals with Finite Time Domain [0,T]

For signal spaces with finite time domain $[0,T]$ we can prove that the ISI phenomenon does not occur.

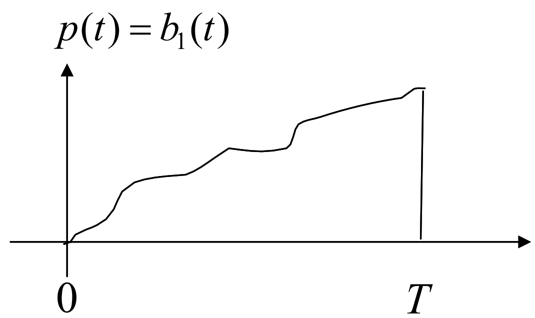
For an ideal channel we have:

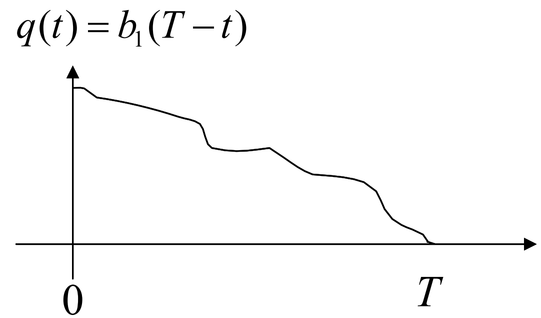
$$\rho[n] = a[n]$$

Meaning that in this case the function $x(t)$ automatically satisfies the NO ISI condition

Assumptions:

- $b_1(t)$ is an orthonormal vector with finite time domain $[0,T]$
- $p(t) = b_1(t)$
- $q(t) = p(T-t)$





Consider $$x(t) = p(t) * q(t) = \int_{-\infty}^{+\infty} p(\tau)\, q(t-\tau)\, d\tau$$

We have

1. for $t \le 0 \quad x(t) = 0$

2. $x(T) = \int_{-\infty}^{+\infty} p(\tau)\, q(T-\tau)\, d\tau = \int_{-\infty}^{+\infty} b_1(\tau)\, b_1(T-T+\tau)\, d\tau = 1$

3. for $t \ge 2\mathrm{T} \quad x(t) = 0$

(The convolution of two finite duration [0,T] signals has duration [0, 2T])

Therefore we have:

$$x(t_0 + iT) = 1 \quad \text{if} \quad i = 0$$
$$x(t_0 + iT) = 0 \quad \text{if} \quad i \neq 0 \qquad \text{for } t_0 = T$$

$$\boxed{\begin{aligned} x[i] &= 1 \quad \text{if} \quad i = 0 \\ x[i] &= 0 \quad \text{if} \quad i \neq 0 \end{aligned}}$$

The function x(t) automatically satisfies the NO ISI condition when the signal space consists of finite time domain signals $[0,T]$.

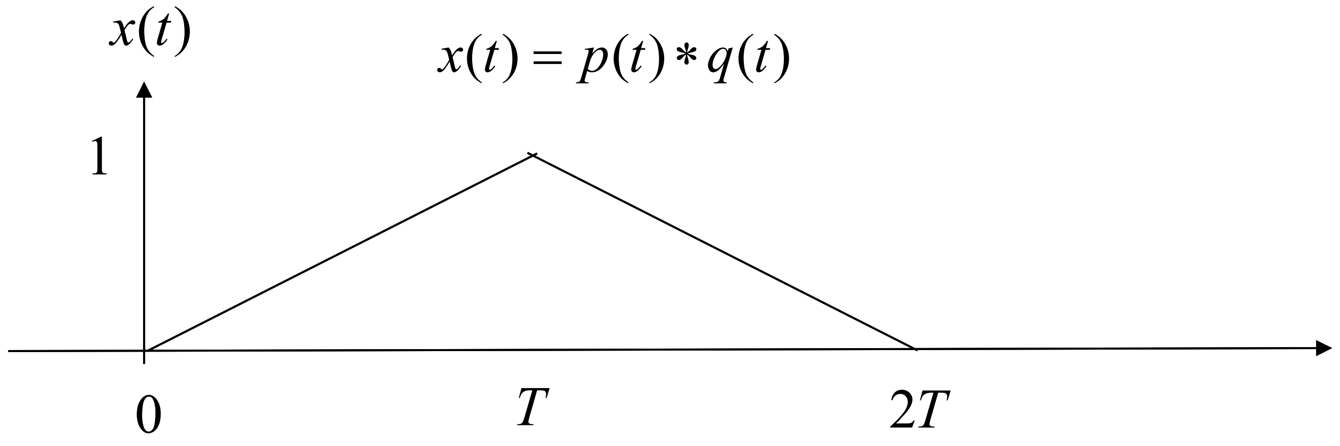
Example 1

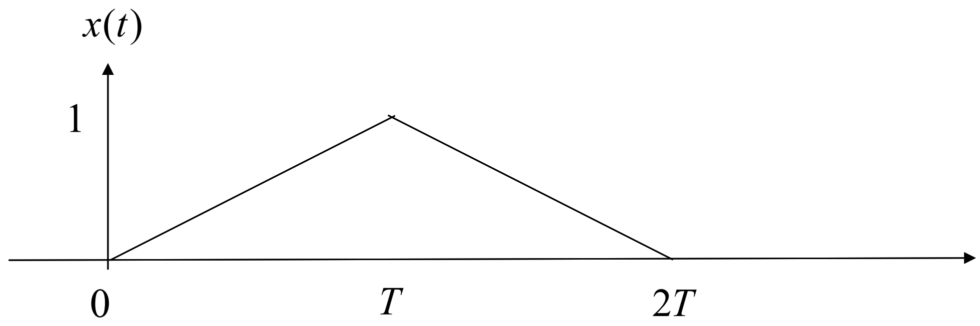
1-D space with vector $b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$

$$p(t) = b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$$

$$q(t) = p(T-t) = \frac{1}{\sqrt{T}} P_T(t)$$

$$x(t) = p(t) * q(t)$$





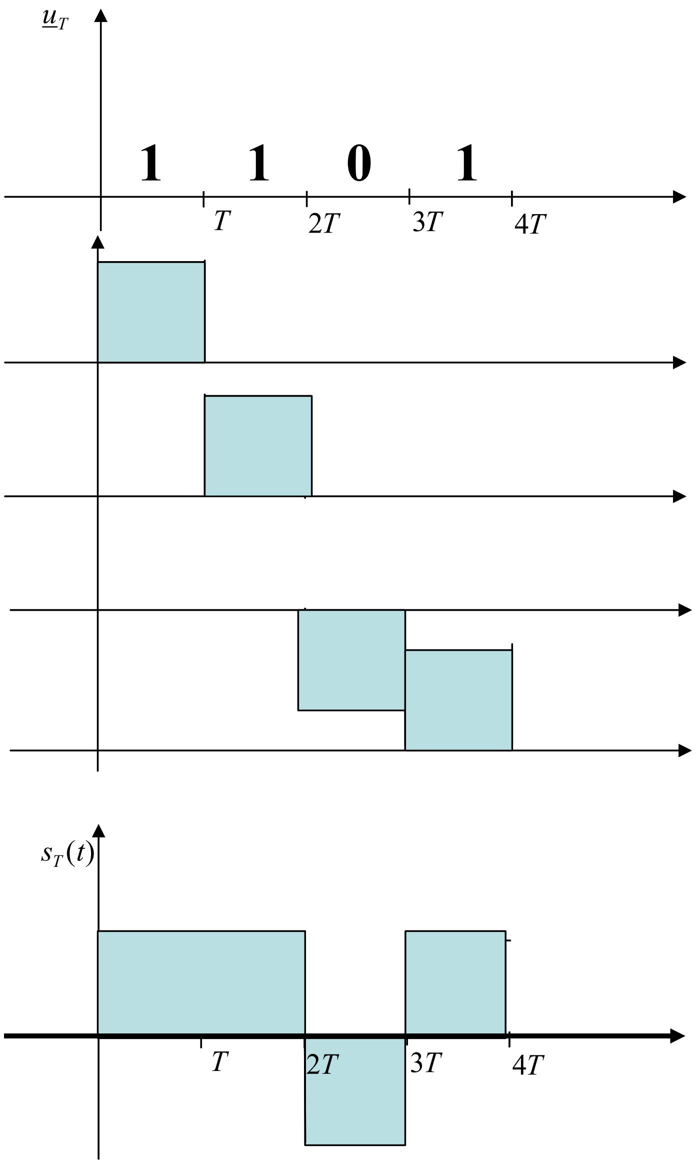
The function $x(t)$ satisfies

$$x(t_0 + iT) = 1 \quad \text{if} \quad i = 0$$
$$x(t_0 + iT) = 0 \quad \text{if} \quad i \neq 0$$

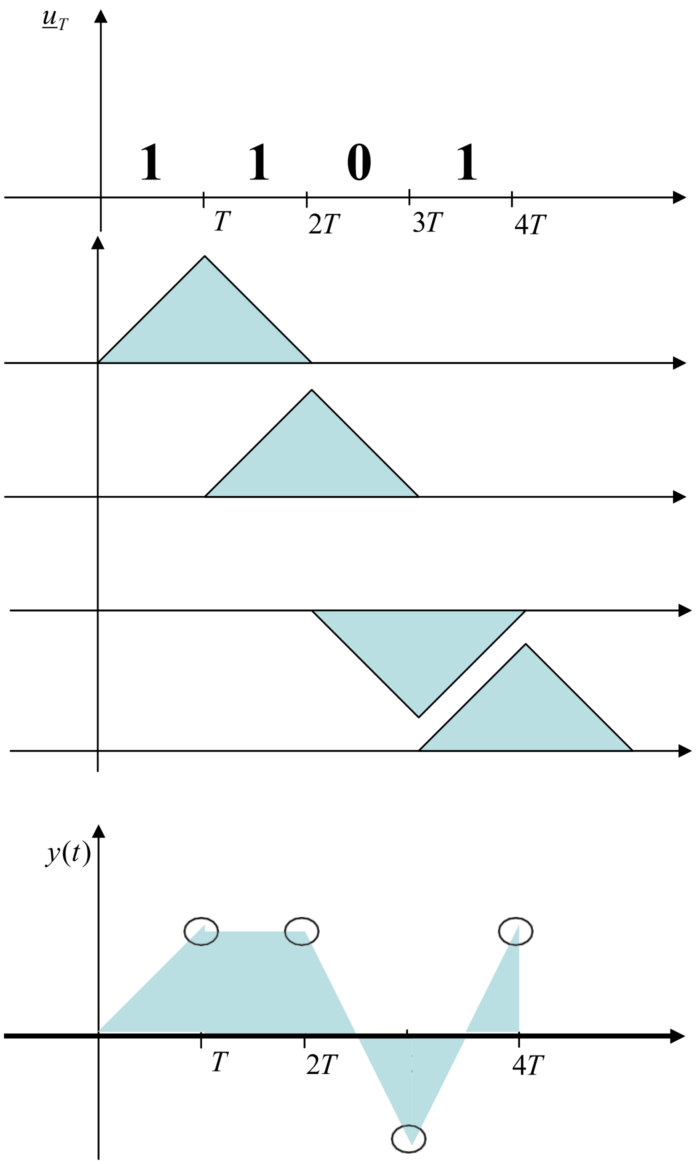
for $t_0 = T$

$$\Rightarrow \boxed{\begin{array}{lll} x[i] = 1 & \text{if} & i = 0 \\ x[i] = 0 & \text{if} & i \neq 0 \end{array}}$$

therefore, NO ISI: $\rho[n] = y(T + nT) = a[n]$



$$s(t) = \sum_n a[n]p(t - nT)$$



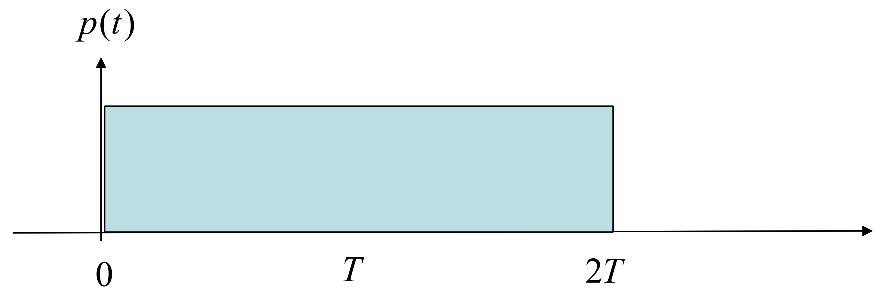
$$y(t) = \sum_{n} a[n]x(t - nT)$$

$$\rho[n] = y(T + nT) = a[n]$$

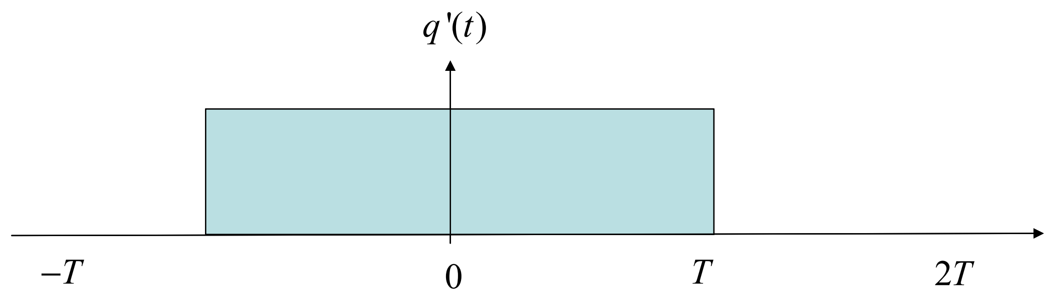
Example 2: Verifying the NO-ISI property of the following signal space

1-D space with vector $$b_1(t) = \frac{1}{\sqrt{2T}} P_{2T}(t)$$

$$p(t) = b_1(t)$$



$$q'(t) = p(T - t)$$



Assumption of a time-domain delay $D' = T$

$$q(t) = q'(t - T)$$