Introduction to Communications Engineering

IT4593E

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ONE LOVE. ONE FUTURE.

Thông tin chung

- Tên học phần: **Nhập môn kỹ thuật truyền thông**
- Mã học phần: **IT4593E**
- Khối lượng: **2 TC (2-1-0-4)**
- Lý thuyết và bài tập: **10 buổi lý thuyết, 5 buổi bài tập**
- Đánh giá học phần:

  30% QT (kiểm tra + bài tập/project + chuyên cần-quiz)

  70% CK (trắc nghiệm + tự luận)
- Tài liệu tham khảo:
  - Lecture slides
  - Lecture notes
  - Textbooks, ví dụ ***Communication Systems Engineering***, 2nd Edition, by John G. Proakis Masoud Salehi
  - Internet

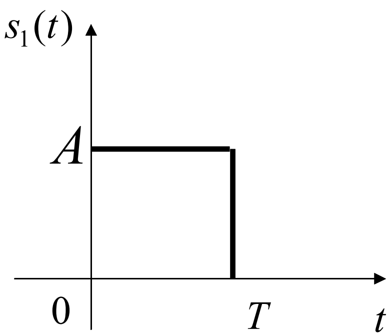
Part 2: Digital Modulations

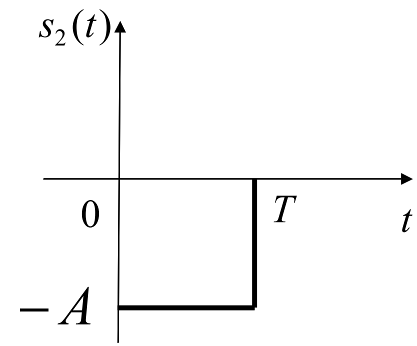
Lec 9: Pulse Amplitude Modulation (PAM)

(cont'd)

Bipolar NRZ (Non Return to Zero)

Signal set $\boxed{M = \{s_1(t) = +AP_T(t)\,,\, s_2(t) = -AP_T(t)\,\}}$





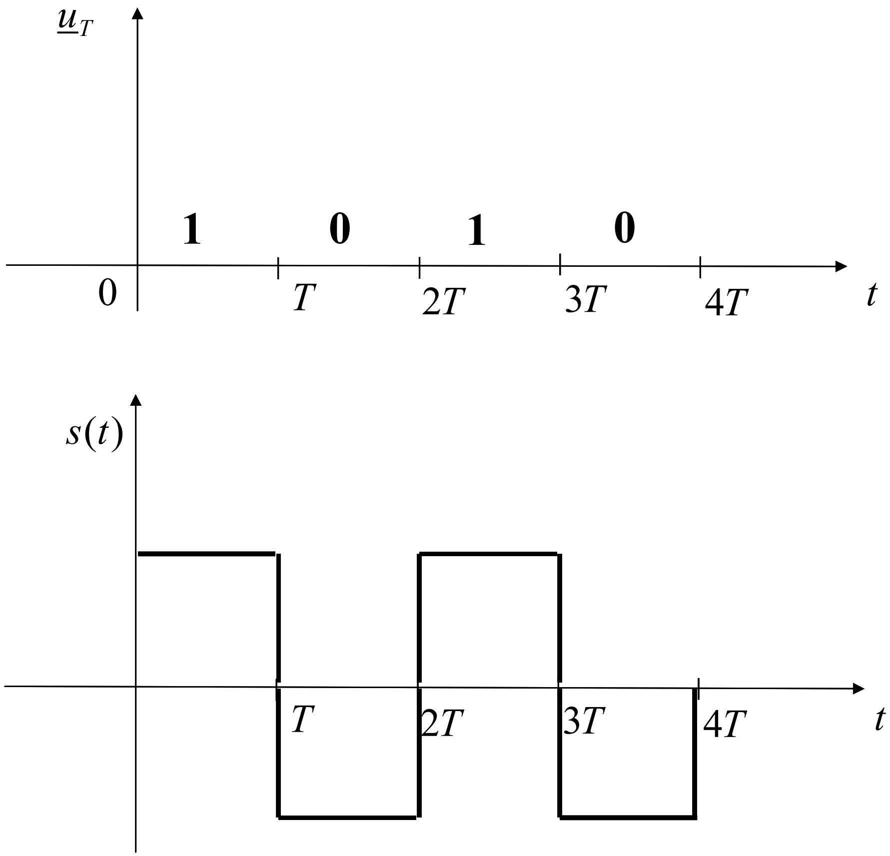
Vector $b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$

Vector set $M = \{\underline{s_1} = (+\alpha)\,,\, \underline{s_2} = (-\alpha)\}$

(it coincides with a <u>2-PAM with rectangular pulse</u>)

Bipolar NRZ

Transmitted waveform

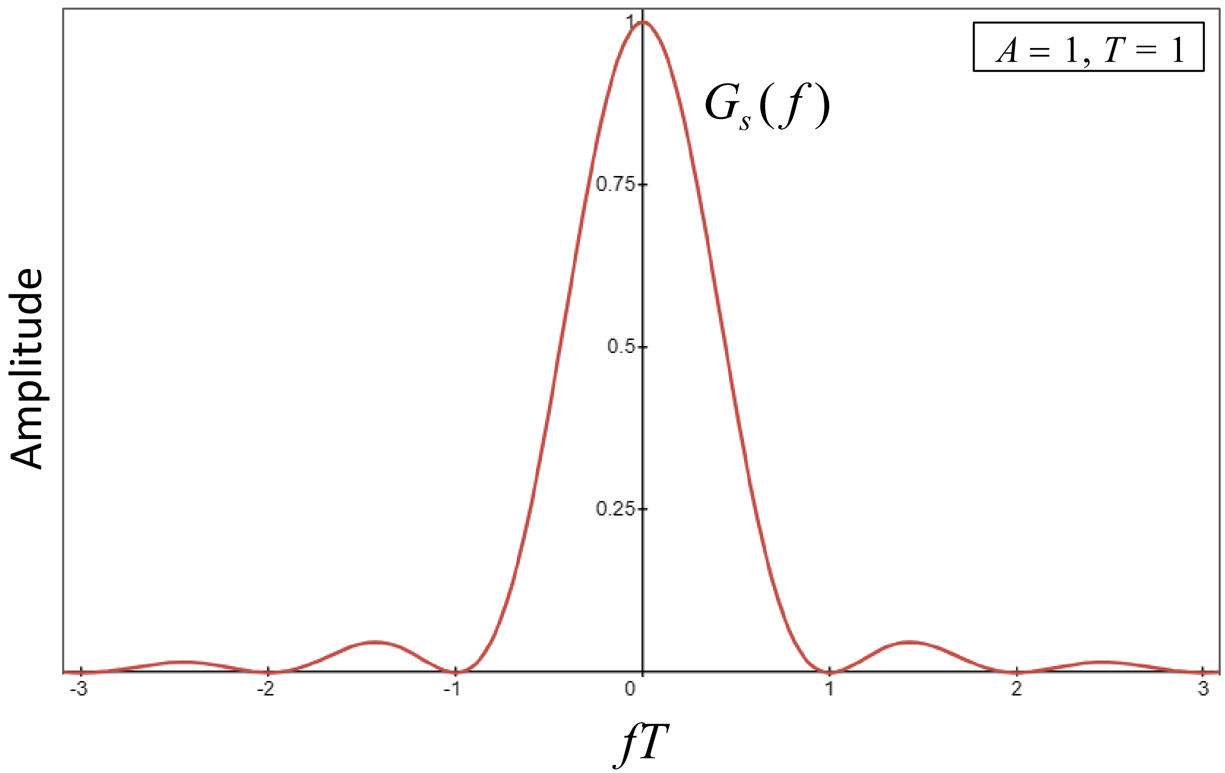


$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, -\alpha\}$$

Bipolar NRZ

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \operatorname{sinc}^2(fT)$$



Bipolar NRZ

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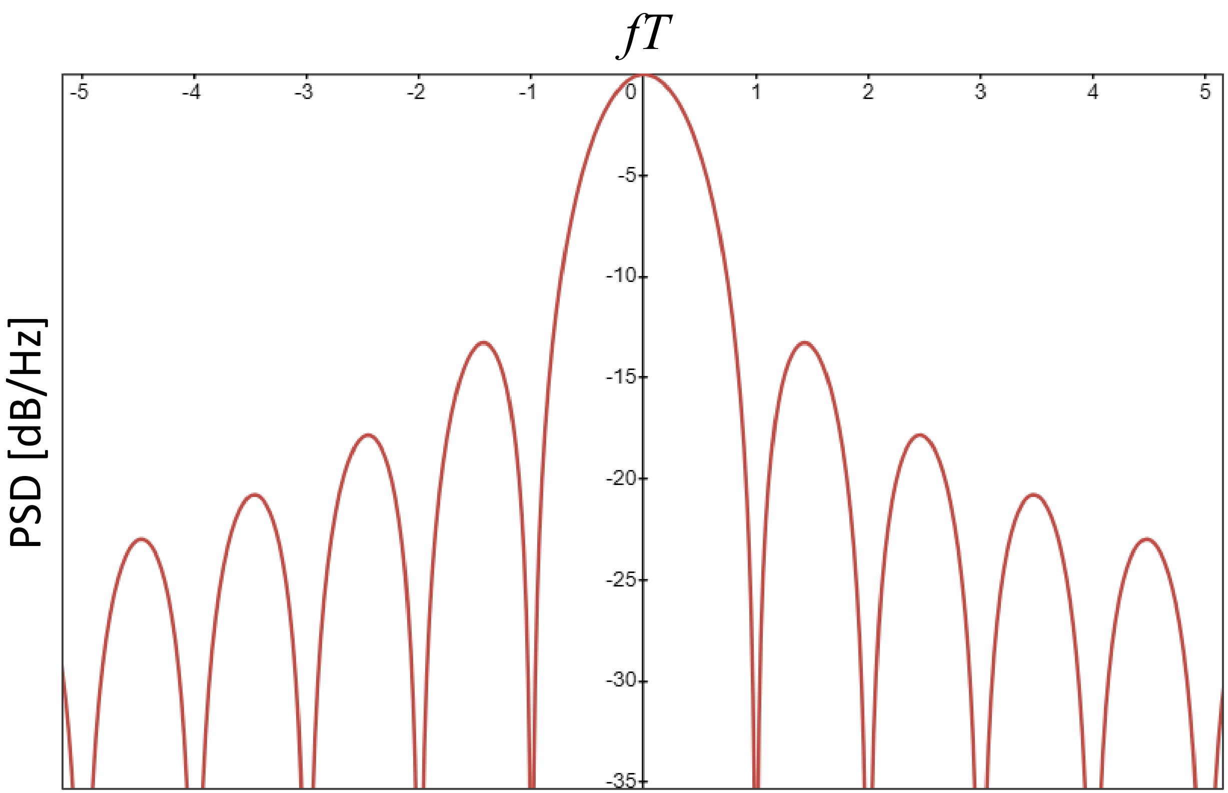
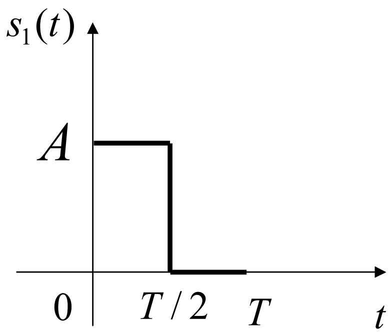
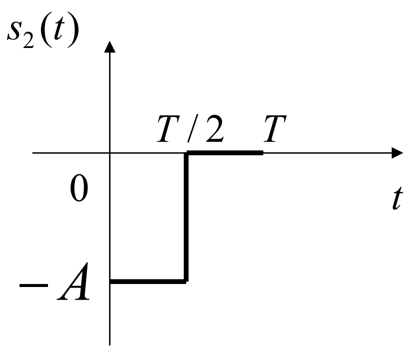


Bipolar RZ (Return to Zero)

Signal set $$\boxed{M=\{s_1(t)=+AP_{T/2}(t)\,,\,s_2(t)=-AP_{T/2}(t)\,\}}$$





Vector $$b_1(t)=\sqrt{\frac{2}{T}}P_{T/2}(t)$$

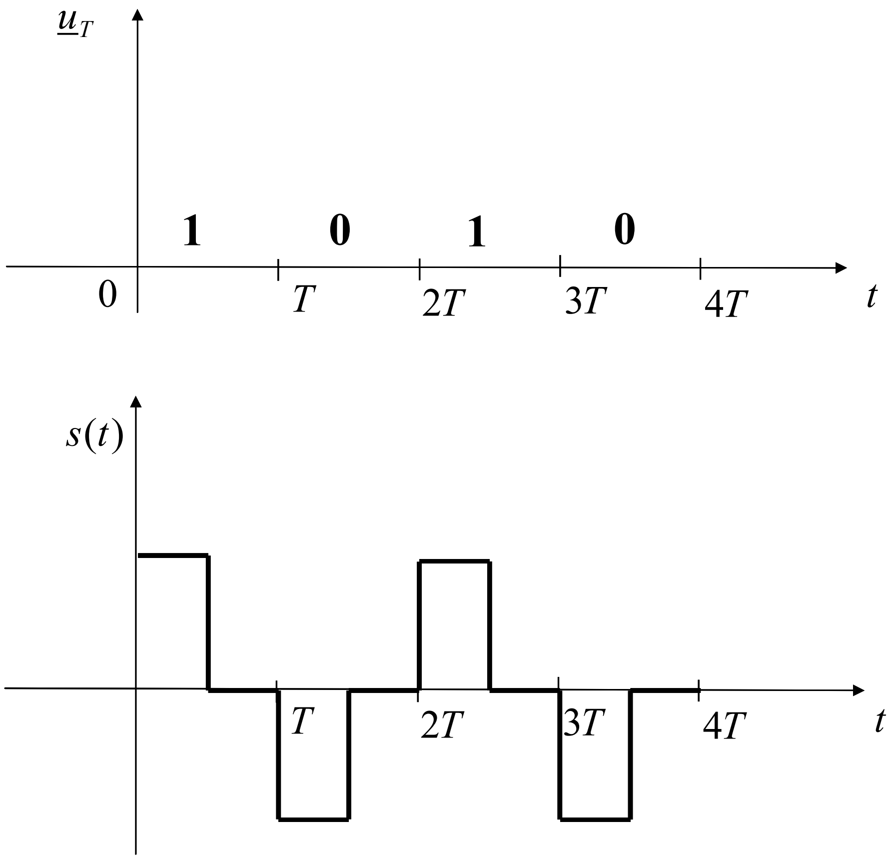
Vector set $$M=\{\underline{s_1}=(+\alpha)\,,\,\underline{s_2}=(-\alpha)\}$$

Bipolar RZ

Transmitted waveform

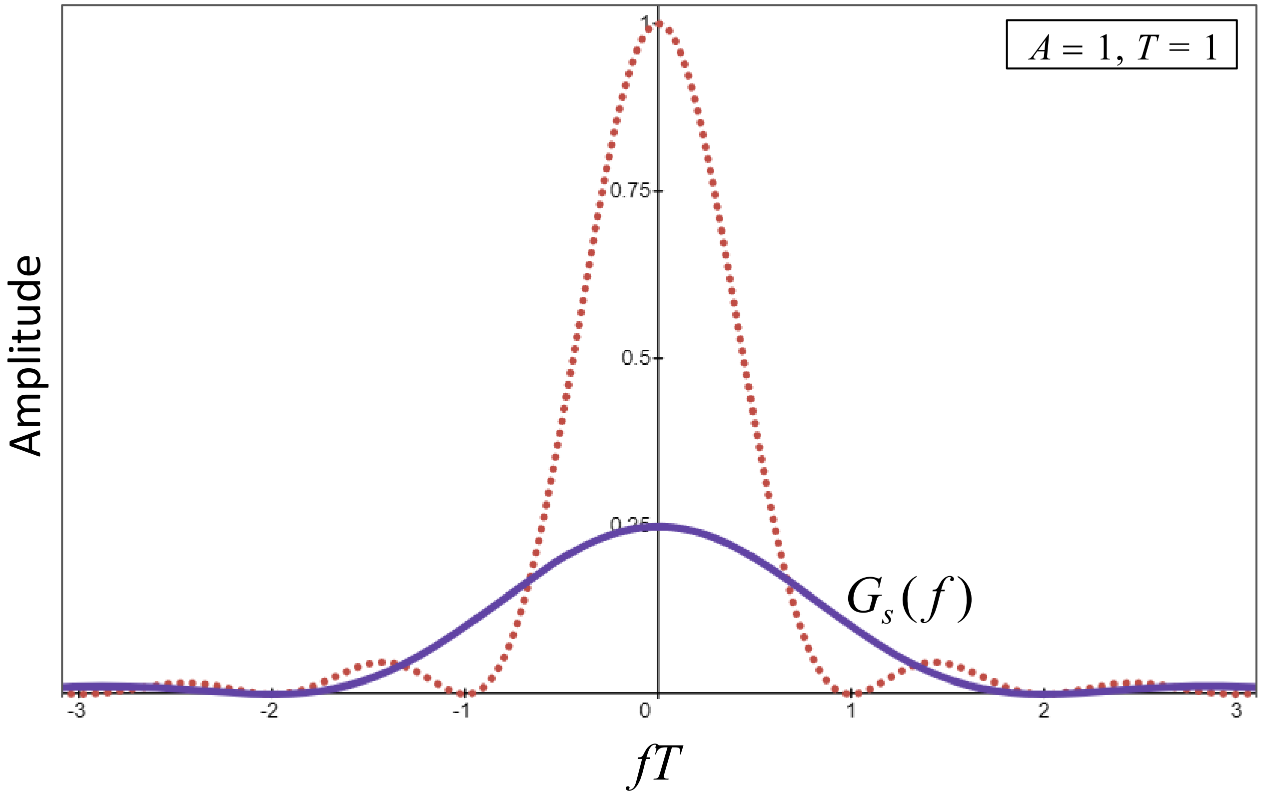
$$s(t) = \sum_n a[n] p(t - nT)$$

$$a[n] \in \{+\alpha, -\alpha\}$$



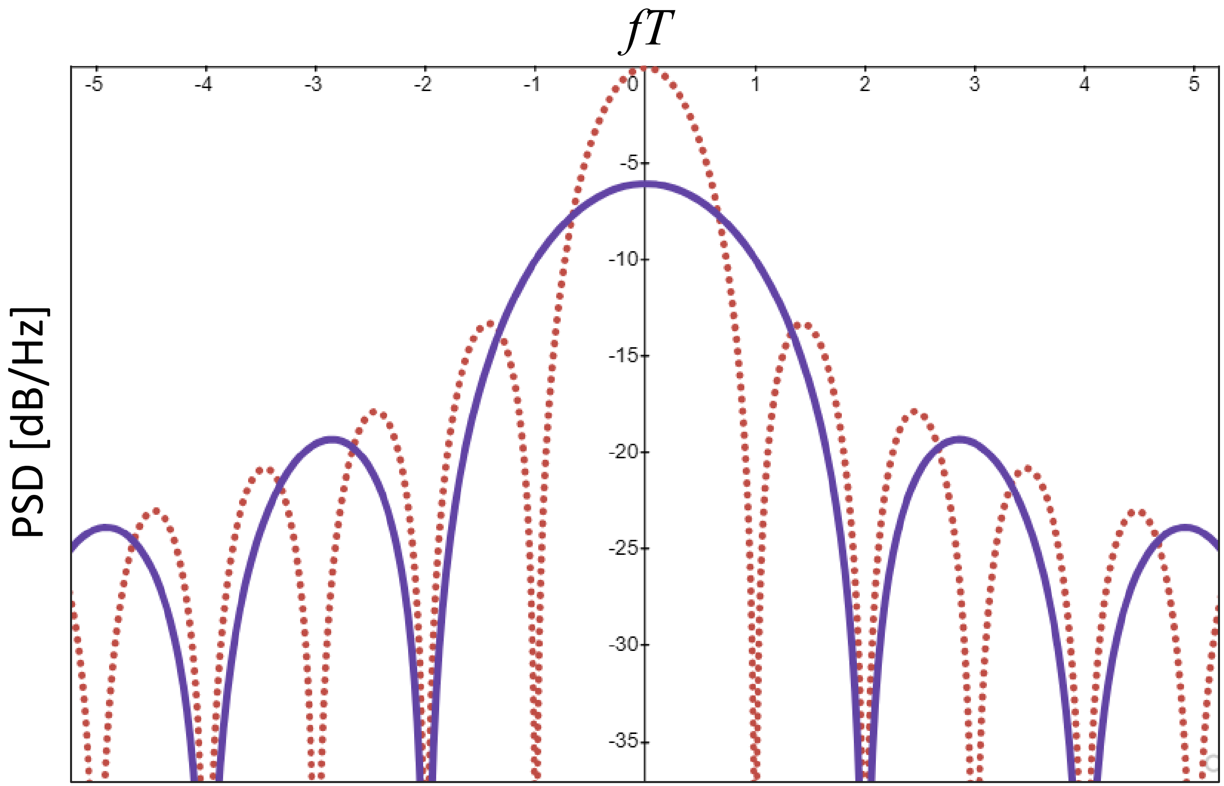
Bipolar RZ

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \operatorname{sinc}^2(fT/2)$$



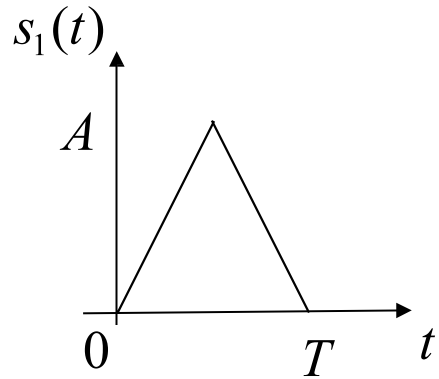
Bipolar RZ

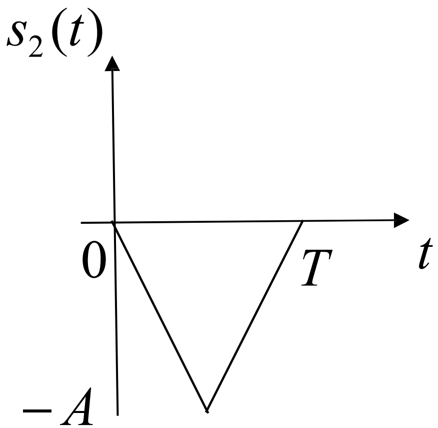
Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \mathrm{sinc}^2(fT/2)$$



Example: Bipolar triangular

Signal set $\boxed{M = \{s_1(t) = +A\Delta_T(t)\,,\, s_2(t) = -A\Delta_T(t)\,\}}$



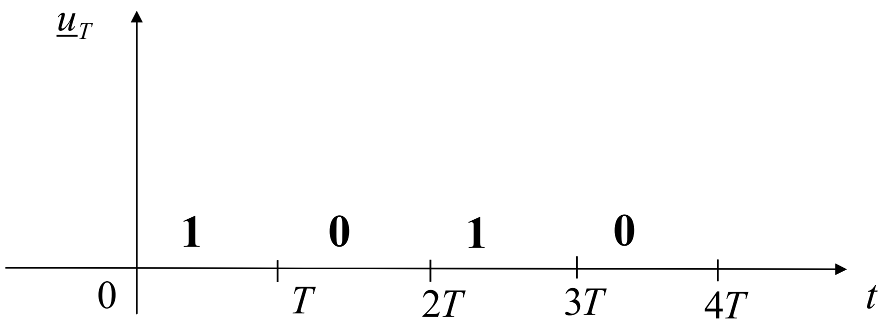


Vector $b_1(t) = \sqrt{\dfrac{3}{T}}\Delta_T(t)$

Vector set $M = \{\underline{s_1} = (+\alpha)\,,\, \underline{s_2} = (-\alpha)\}$

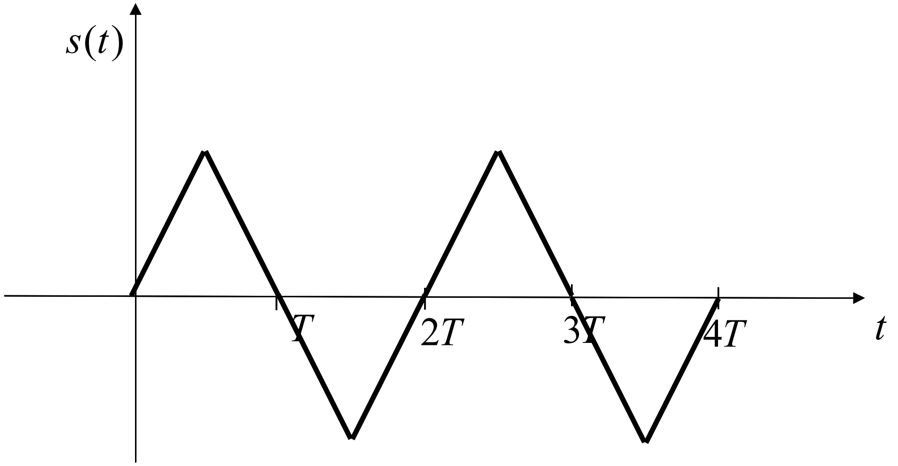
Example: Bipolar triangular

Transmitted waveform



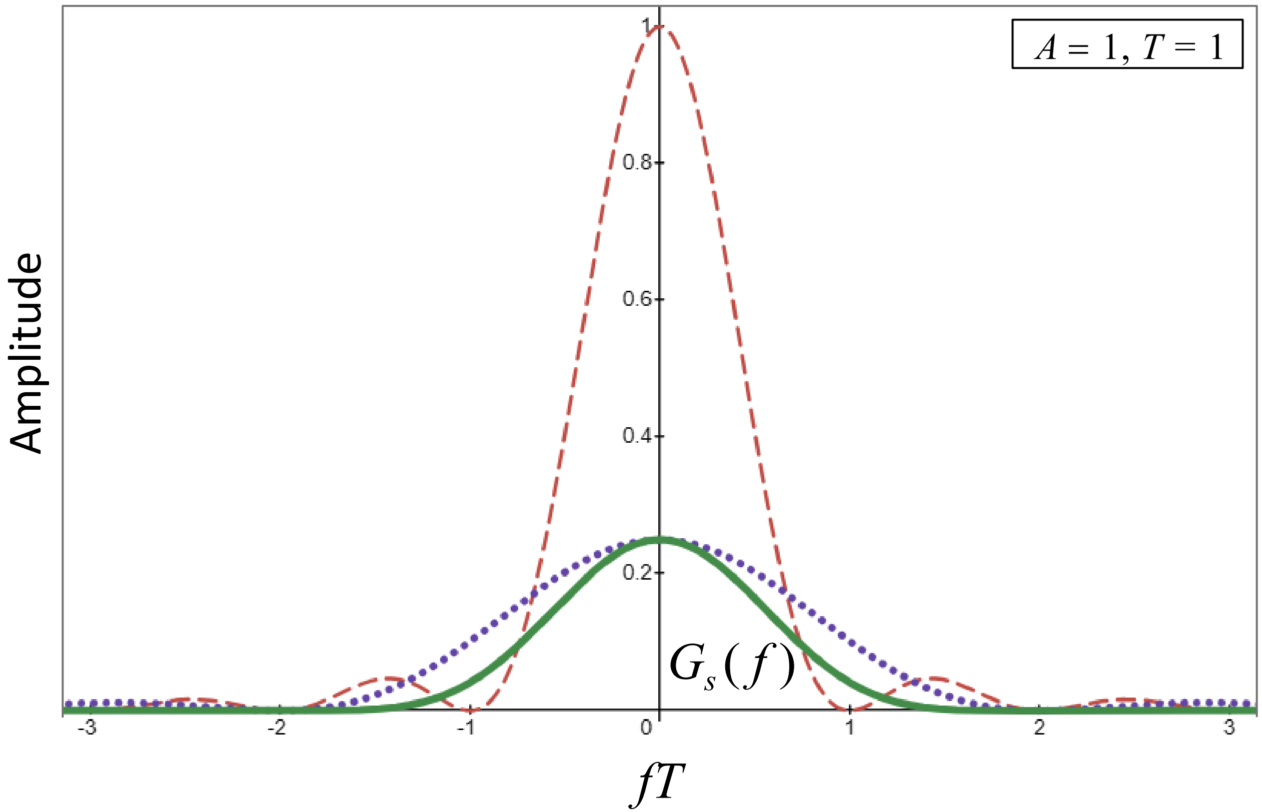
$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, -\alpha\}$$



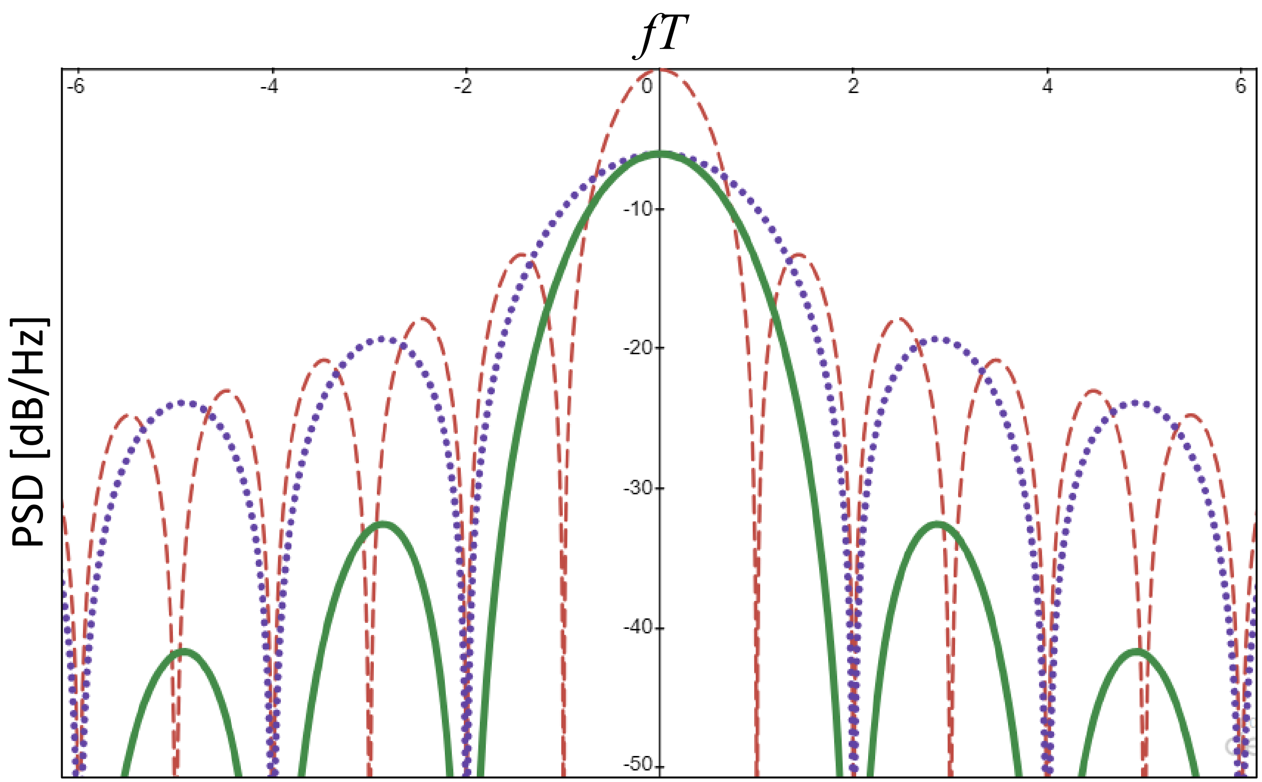
Example: Bipolar triangular

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \operatorname{sinc}^4(fT/2)$$



Example: Bipolar triangular

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \mathrm{sinc}^4(fT/2)$$



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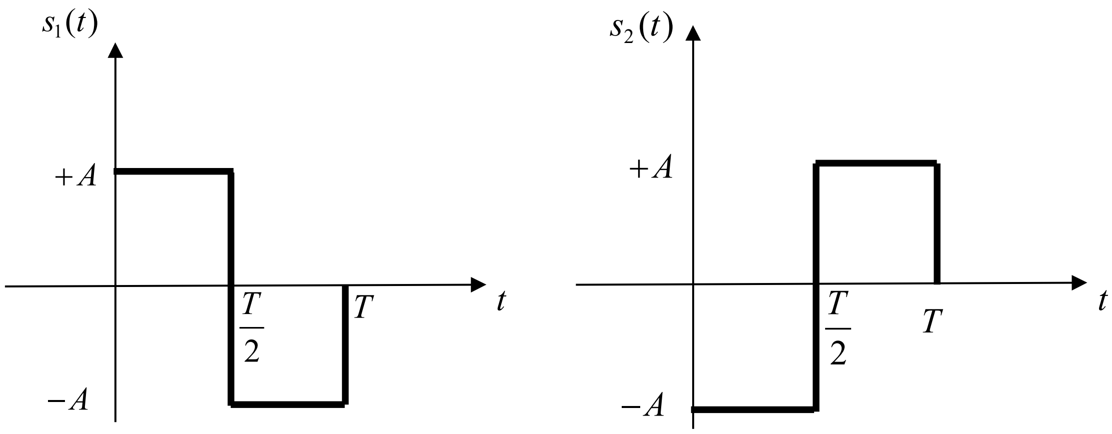
Manchester (biphase)

Signal set $$\boxed{M=\{s_1(t)=+Ax(t)\,,\,s_2(t)=-Ax(t)\,\}}$$

$$x(t)=\left[+P_{T/2}(t)-P_{T/2}(t-T/2)\right]$$

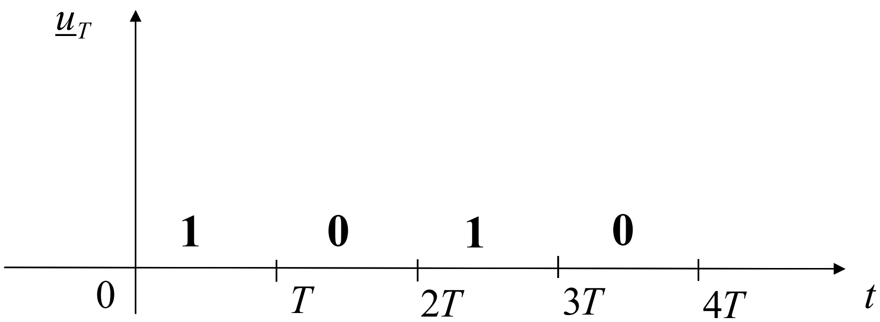


Vector $$b_1(t)=\frac{1}{\sqrt{T}}\left[+P_{T/2}(t)-P_{T/2}(t-T/2)\right]$$

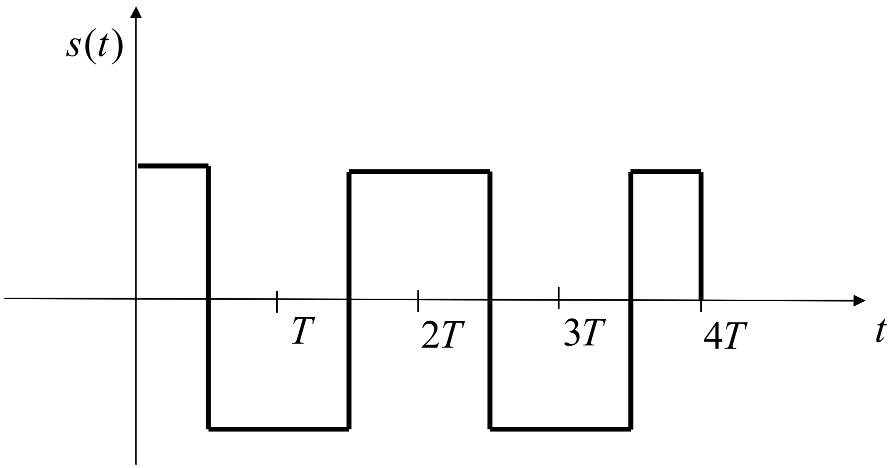
Vector set $$M=\{\underline{s}_1=(+\alpha)\,,\,\underline{s}_2=\left(-\alpha\right)\}$$

Manchester (biphase)

Transmitted waveform



$$s(t) = \sum_n a[n]p(t - nT)$$

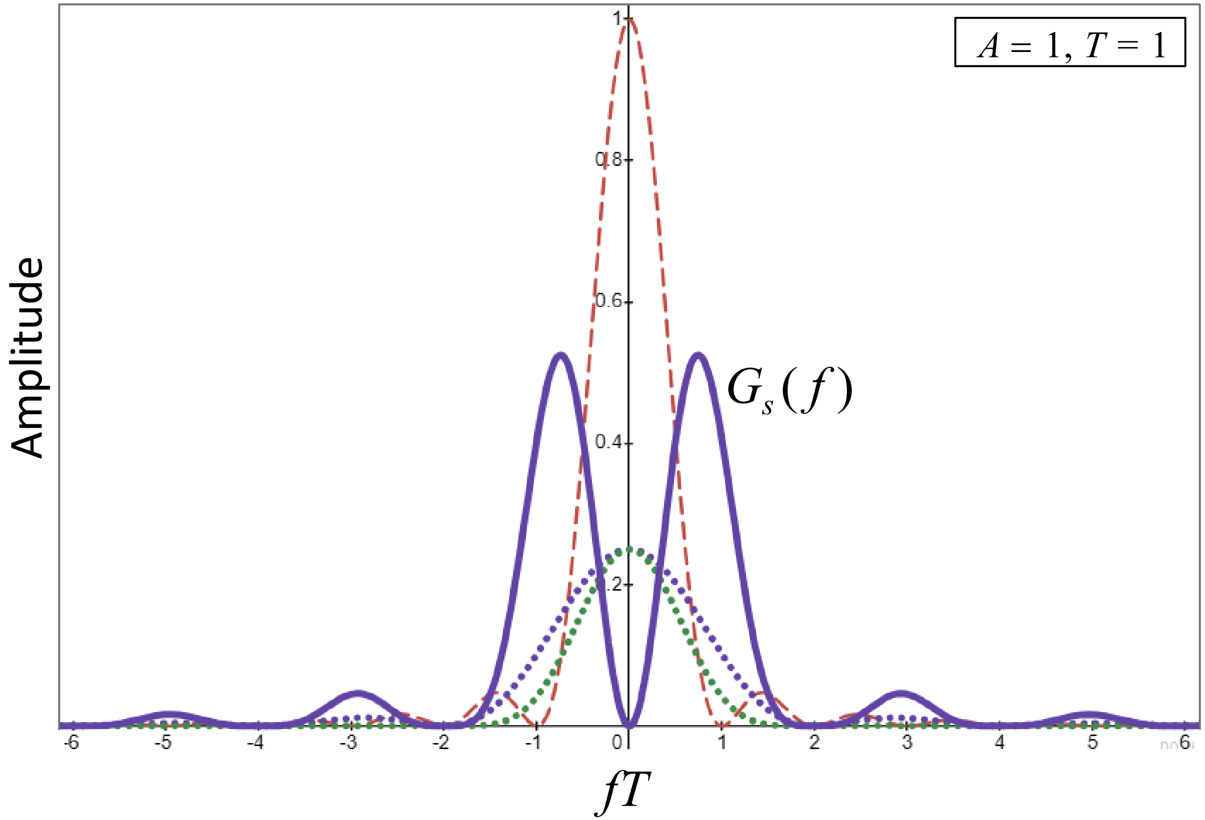


$$a[n] \in \{+\alpha, -\alpha\}$$

Manchester (biphase)

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \frac{\sin^4(\pi fT/2)}{(\pi fT/2)^2}$$

(maximum at $f \approx 0.74/T$)



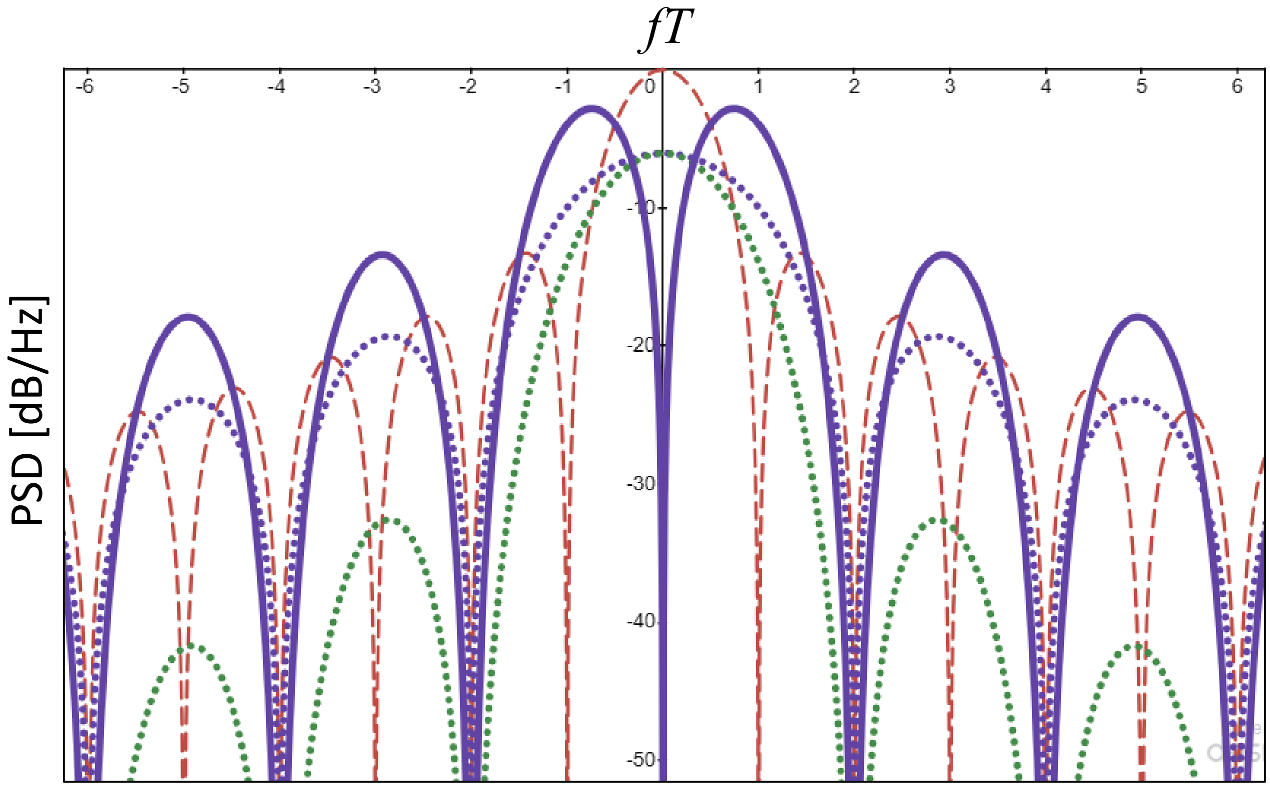
SOICT

Manchester (biphase)

Signal spectrum

(maximum at $f \approx 0.74/T$)

$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \frac{\sin^4(\pi fT/2)}{(\pi fT/2)^2}$$



SOICT

Manchester (biphase)

$$p(t) = b_1(t) = \frac{1}{\sqrt{T}}\left[+P_{T/2}(t) - P_{T/2}\left(t - \frac{T}{2}\right)\right]$$

$$P(f) = \frac{1}{\sqrt{T}}\left[+\frac{T}{2}\operatorname{sinc}\left(f\frac{T}{2}\right)\exp\left(-j2\pi f\frac{T}{4}\right) - \frac{T}{2}\operatorname{sinc}\left(f\frac{T}{2}\right)\exp\left(-j2\pi f\frac{3T}{4}\right)\right] =$$

$$= \left[+\frac{\sqrt{T}}{2}\operatorname{sinc}\left(f\frac{T}{2}\right)\exp\left(-j2\pi f\frac{T}{4}\right)\right]\left[1 - \exp\left(-j\pi fT\right)\right]$$

$$\left|P(f)\right|^2 = \frac{T}{4}\operatorname{sinc}^2\left(f\frac{T}{2}\right)\left|1 - \cos\left(-\pi fT\right) - j\sin\left(-\pi fT\right)\right|^2 =$$

$$= \frac{T}{4}\operatorname{sinc}^2\left(f\frac{T}{2}\right)\left|1 - \cos\left(\pi fT\right) + j\sin\left(\pi fT\right)\right|^2 =$$

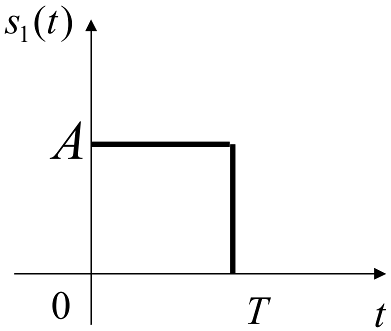
$$= \frac{T}{4}\operatorname{sinc}^2\left(f\frac{T}{2}\right)\left[1 + \cos^2\left(\pi fT\right) - 2\cos\left(\pi fT\right) + \sin^2\left(\pi fT\right)\right] =$$

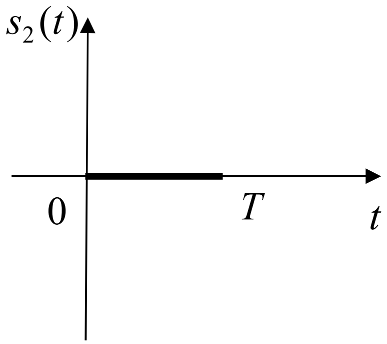
$$= \frac{T}{2}\operatorname{sinc}^2\left(f\frac{T}{2}\right)\left[1 - \cos\left(\pi fT\right)\right] = T\operatorname{sinc}^2\left(f\frac{T}{2}\right)\sin^2\left(\pi f\frac{T}{2}\right)$$

$$\boxed{\sin\left(\frac{A}{2}\right) = \sqrt{\frac{1 - \cos A}{2}}}$$

Unipolar NRZ

Signal set $\boxed{M = \{s_1(t) = +AP_T(t)\,,\, s_2(t) = 0\,\}}$



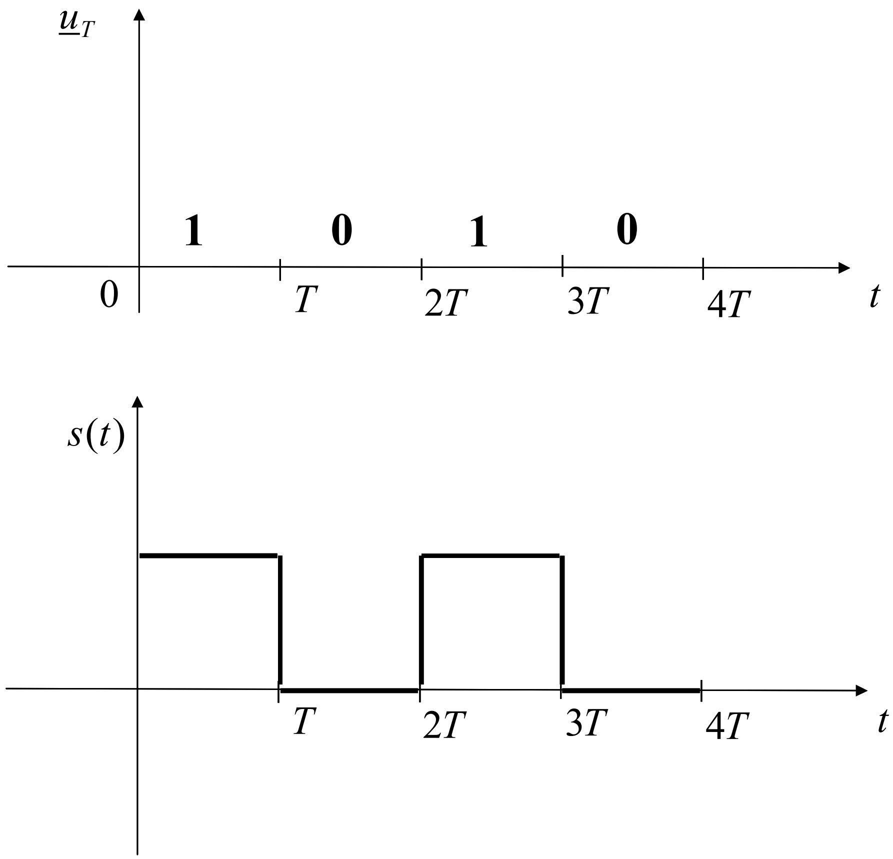


Vector $b_1(t) = \dfrac{1}{\sqrt{T}} P_T(t)$

Vector set $M = \{\underline{s_1} = (+\alpha)\,,\, \underline{s_2} = (0)\}$

Unipolar NRZ

Transmitted waveform



$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, 0\}$$

Unipolar NRZ

Signal spectrum

$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} + \frac{\mu_a^2}{T^2} \sum_{n=-\infty}^{+\infty} \left| P\left(\frac{n}{T}\right) \right|^2 \delta\left(f - \frac{n}{T}\right)$$

$$|P(f)|^2 = x\,\mathrm{sinc}^2(\pi f T) \qquad\qquad x \in R$$

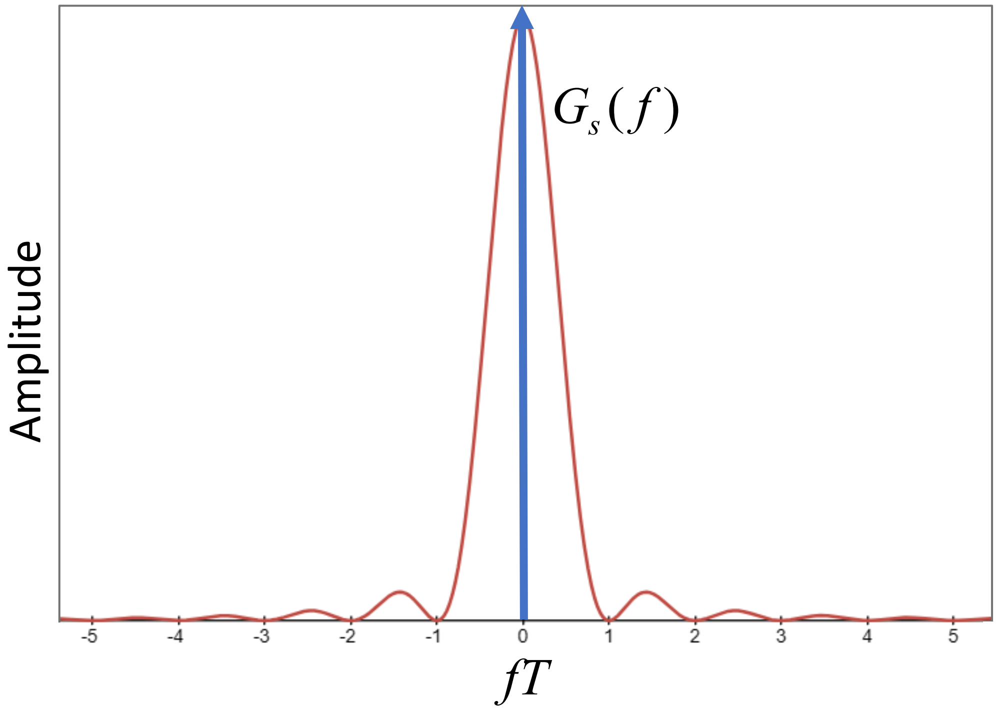
A Dirac delta at zero frequency

$$G_s(f) = \frac{A^2}{4} T \mathrm{sinc}^2(f\mathrm{T}) + \frac{A^2}{4} \delta(f)$$

Unipolar NRZ

Signal spectrum

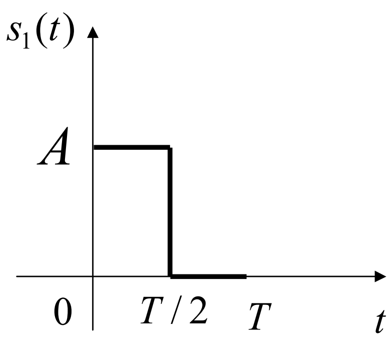
$$G_s(f) = \frac{A^2}{4} T \operatorname{sinc}^2(fT) + \frac{A^2}{4} \delta(f)$$

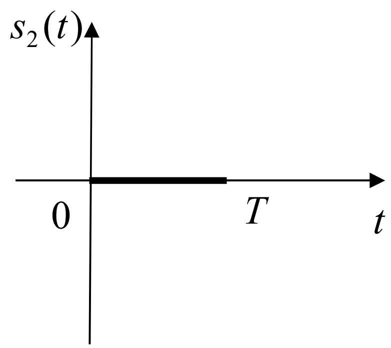


Unipolar RZ

Signal set

$$\boxed{M = \{s_1(t) = +AP_{T/2}(t)\,,\, s_2(t) = 0\,\}}$$





Vector

$$b_1(t) = \sqrt{\frac{2}{T}}P_{T/2}(t)$$

Vector set

$$M = \{\underline{s_1} = (+\alpha)\,,\, \underline{s_2} = (0)\}$$

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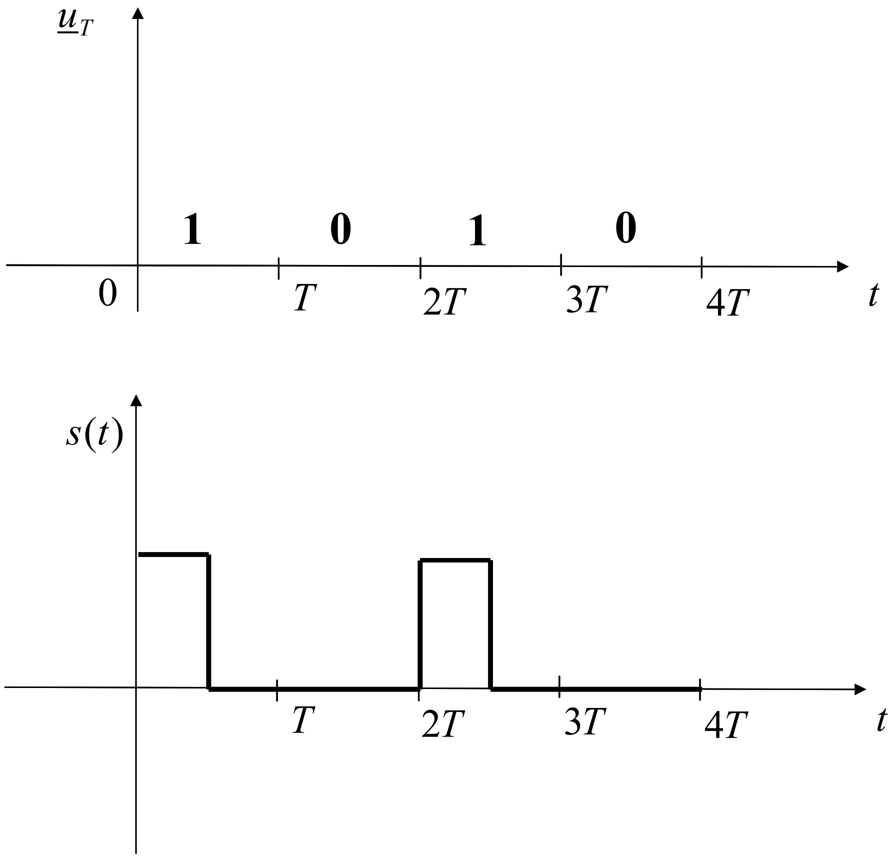
Unipolar RZ

Transmitted waveform



$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, 0\}$$

SOICT

Unipolar RZ

Signal spectrum $$G(f)=\sigma_a^2\frac{|P(f)|^2}{T}+\frac{\mu_a^2}{T^2}\sum_{n=-\infty}^{+\infty}\left|P\left(\frac{n}{T}\right)\right|^2\delta\left(f-\frac{n}{T}\right)$$

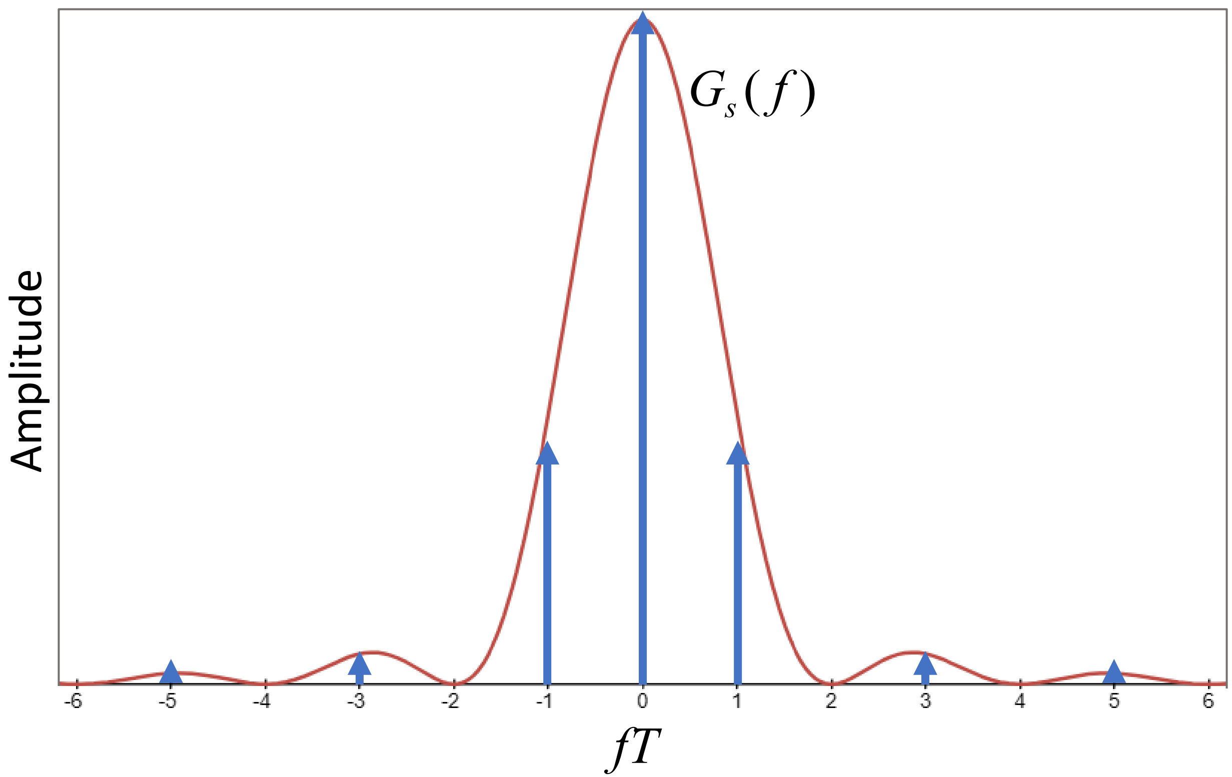
$$\left|P(f)\right|^2=z\left[\frac{\sin(\pi fT/2)}{(\pi fT/2)}\right]^2 \qquad (z\in R)$$

Dirac deltas at zero frequency and at odd multiples of $1/T$

$$G_s(f)=\frac{A^2}{16}T\mathrm{sinc}^2(fT/2)+\frac{A^2}{16}\sum_{i=-\infty}^{+\infty}\mathrm{sinc}^2\left(\frac{(2i+1)}{2}\right)\delta\left(f-\frac{(2i+1)}{T}\right)$$

Unipolar RZ

Signal spectrum $G_s(f) = \frac{A^2}{16} T\text{sinc}^2(fT/2) + \frac{A^2}{16} \sum_{i=-\infty}^{+\infty} \text{sinc}^2\left(\frac{(2i+1)}{2}\right) \delta\left(f - \frac{(2i+1)}{T}\right)$



m-PAM constellation: characteristics

1. Base-band modulation
2. One-dimensional signal space
3. m signals, symmetrical with respect to the origin
4. Information associated to the impulse amplitude
   PAM=Pulse Amplitude Modulation

SOICT

constellation: chòm sao

m-PAM constellation: constellation

SIGNAL SET $\boxed{M = \{s_i(t) = \alpha_i p(t)\}_{i=1}^{m}}$

Versor $b_1(t)=p(t) \qquad (d=1)$

VECTOR SET

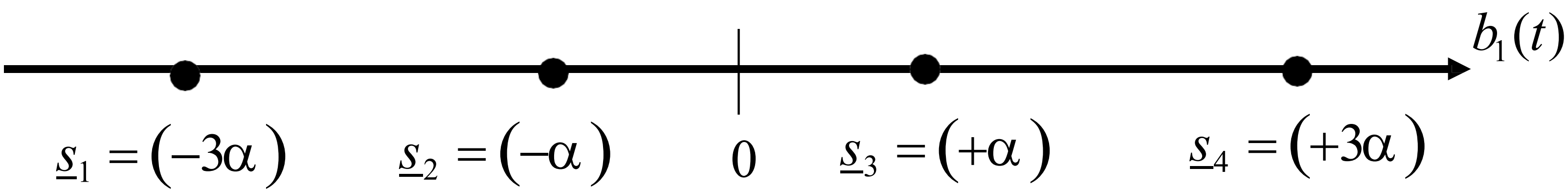
$$\boxed{M = \{\underline{s}_1 = (-(m-1)\alpha)\ ,\ \underline{s}_2 = (-(m-3)\alpha),\ldots,\underline{s}_{m-1} = (+(m-3)\alpha)\ ,\ \underline{s}_m = (+(m-1)\alpha)\} \subseteq R}$$

$$k = \log_2(m) \qquad \boxed{T = kT_b} \qquad \boxed{\boxed{R = \frac{R_b}{k}}}$$

m-PAM constellation: constellation

Example: 4-PAM constellation

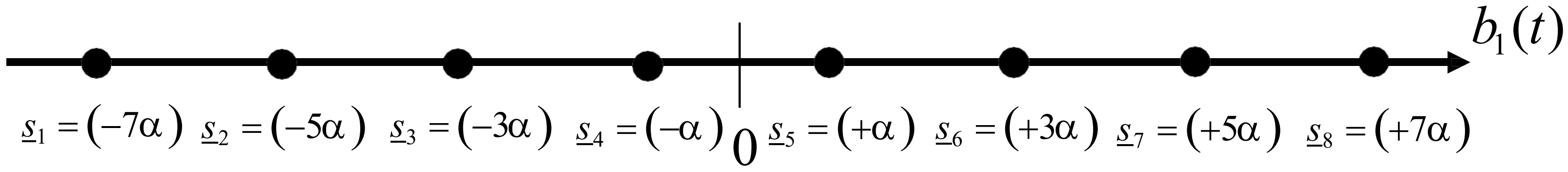
$$M = \{\underline{s}_1 = (\mathbf{-3\alpha})\ ,\ \underline{s}_2 = (\mathbf{-\alpha}), \underline{s}_3 = (+\boldsymbol{\alpha})\ ,\ \underline{s}_4 = (\mathbf{+3\alpha})\} \subseteq R$$



m-PAM constellation: constellation

Example: 8-PAM constellation

$$M = \{\underline{s}_1 = (\mathbf{-7\alpha}),\ \underline{s}_2 = (\mathbf{-5\alpha}), \underline{s}_3 = (\mathbf{-3\alpha}),\ \underline{s}_4 = (\mathbf{-\alpha}), \underline{s}_5 = (+\boldsymbol{\alpha}),\ \underline{s}_6 = (\mathbf{+3\alpha}), \underline{s}_7 = (\mathbf{+5\alpha}),\ \underline{s}_8 = (\mathbf{+7\alpha})\} \subseteq R$$

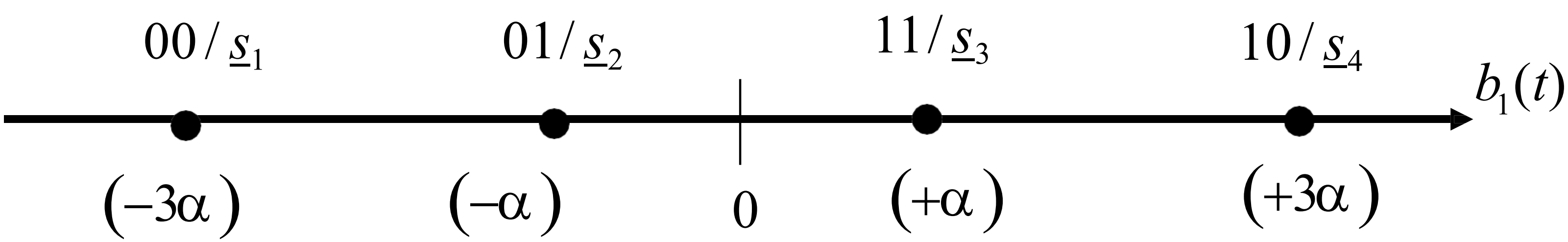


m-PAM constellation: binary labelling

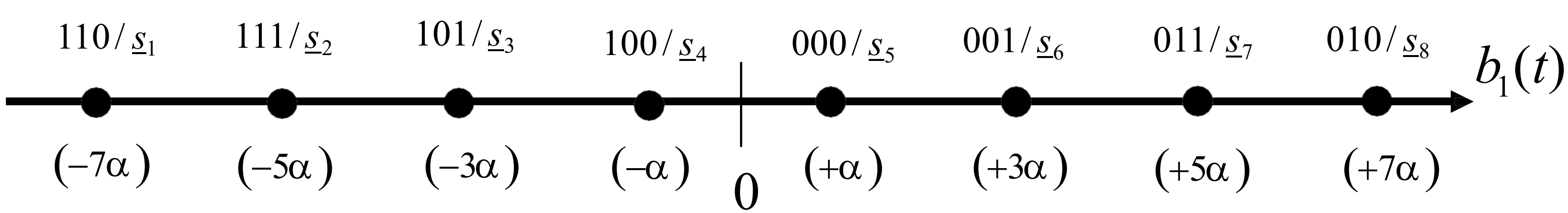
$$e : H_k \leftrightarrow M$$

It is always possible to build a Gray labeling

4-PAM:



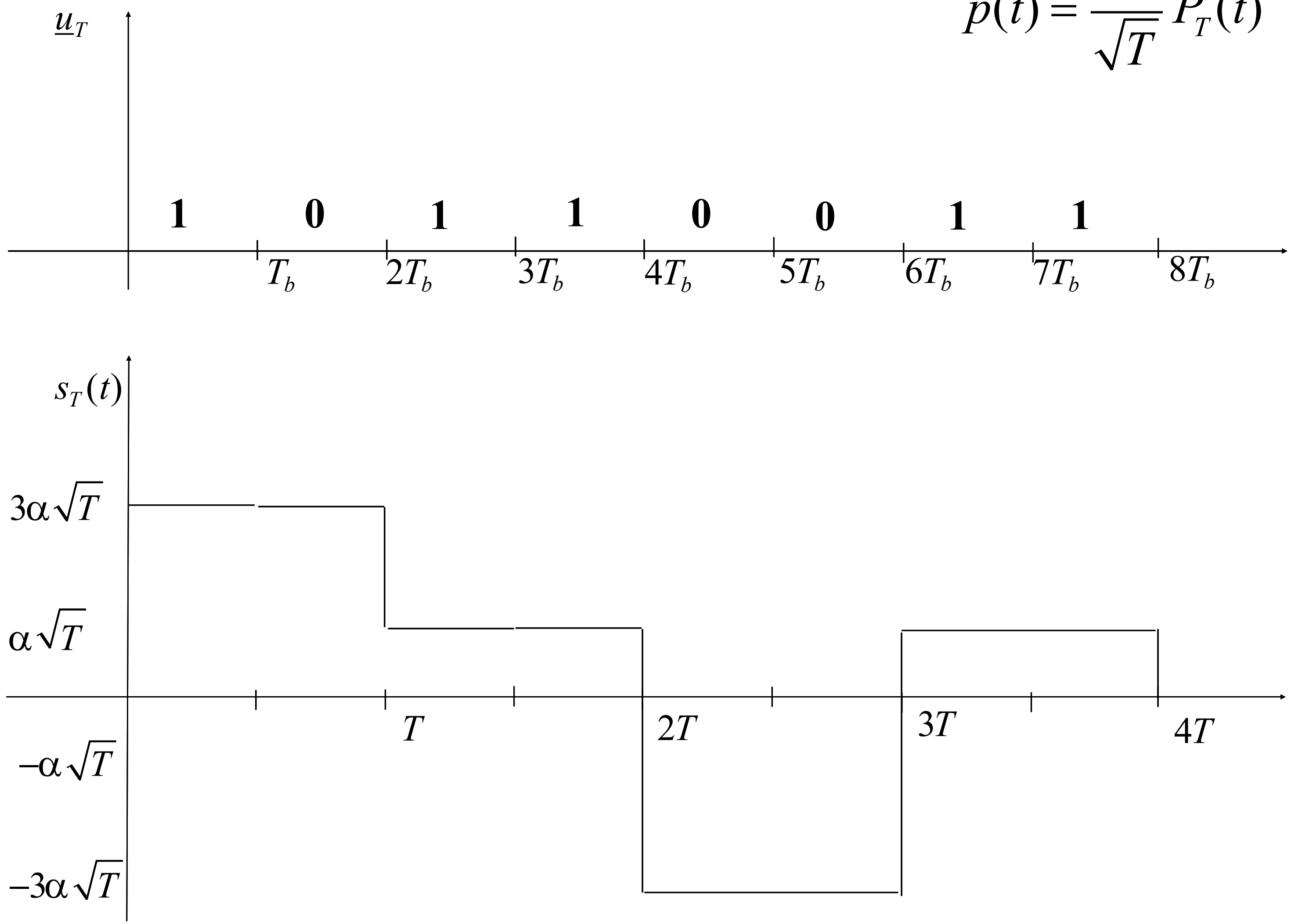
8-PAM:



m-PAM constellation: transmitted waveform

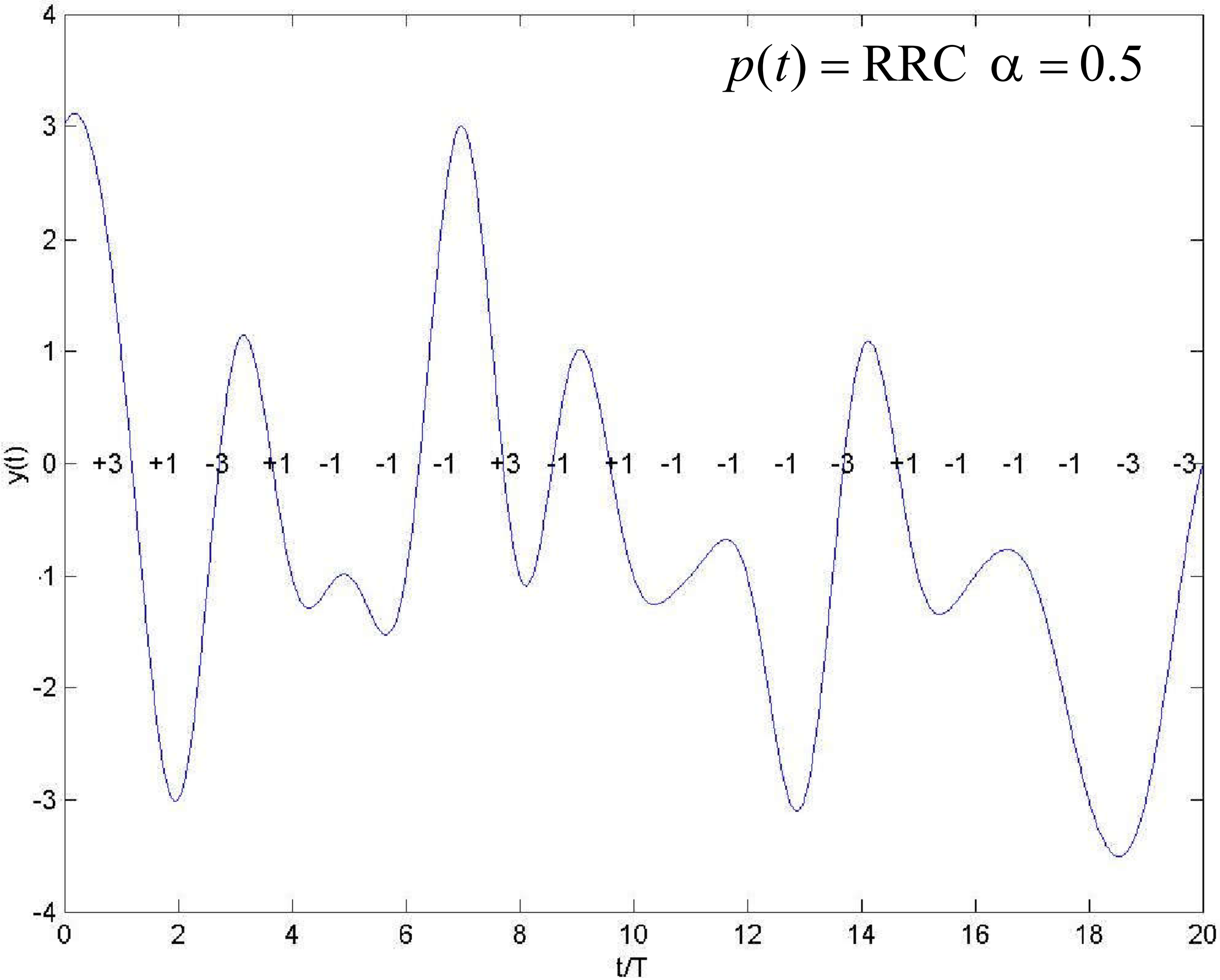
Example: 4-PAM

$$p(t) = \frac{1}{\sqrt{T}} P_T(t)$$



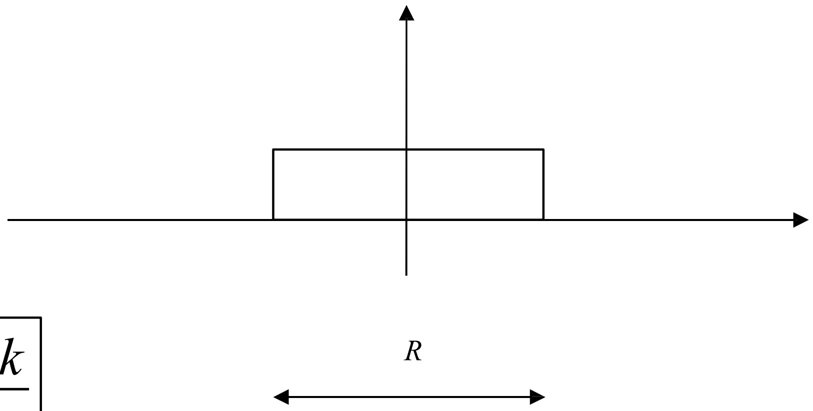
m-PAM constellation: transmitted waveform

Example: 4-PAM



m-PAM constellation: bandwidth and spectral efficiency

Case 1: $p(t)$ = ideal low pass filter



Total bandwidth
(ideal case)

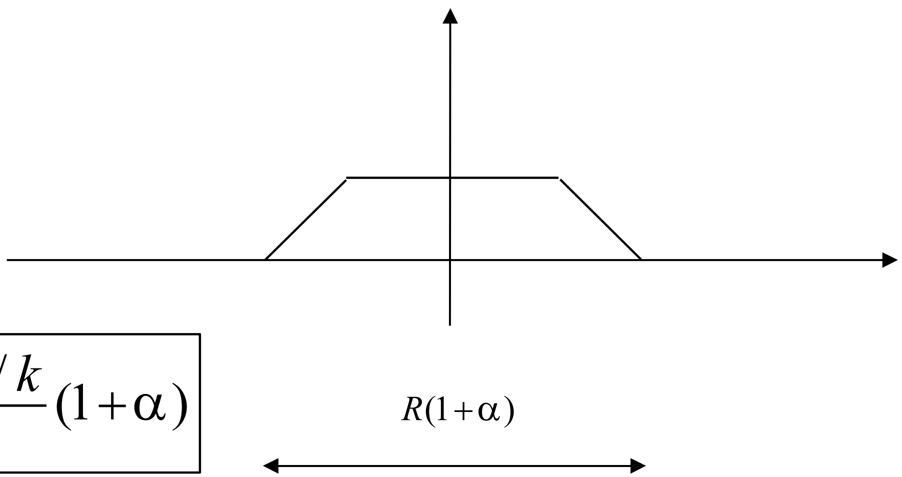
$$\boxed{B_{id} = \frac{R}{2} = \frac{R_b / k}{2}}$$

Spectral efficiency
(ideal case)

$$\boxed{\eta_{id} = \frac{R_b}{B_{id}} = 2k \; bps / Hz}$$

m-PAM constellation: bandwidth and spectral efficiency

Case 2: $p(t)$ = RRC filter roll off α



Total bandwidth $$\boxed{B = \frac{R}{2}(1+\alpha) = \frac{R_b / k}{2}(1+\alpha)}$$

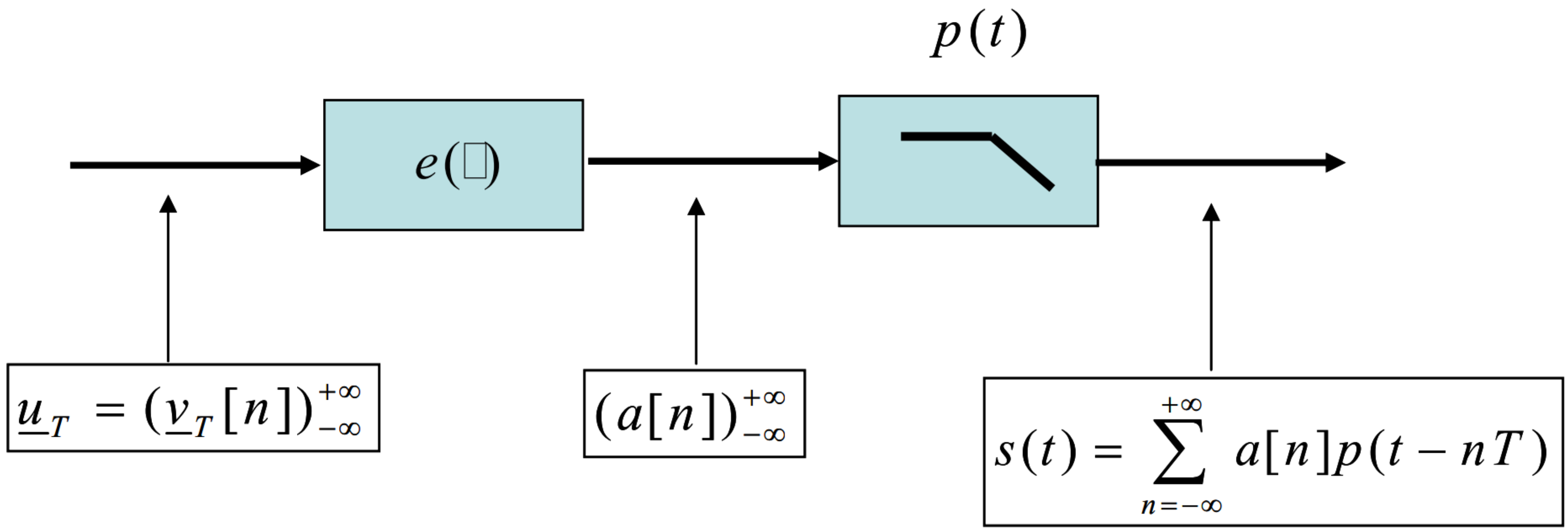
Spectral efficiency $$\boxed{\eta = \frac{R_b}{B} = \frac{2k}{(1+\alpha)}\ bps / Hz}$$

Exercise

Given a baseband channel with bandwidth B up to 4000 Hz, compute the maximum bit rate R_b we can transmit over it with a 256-PAM constellation in the two cases:

- Ideal low pass filter
- RRC filter with $\alpha = 0.25$

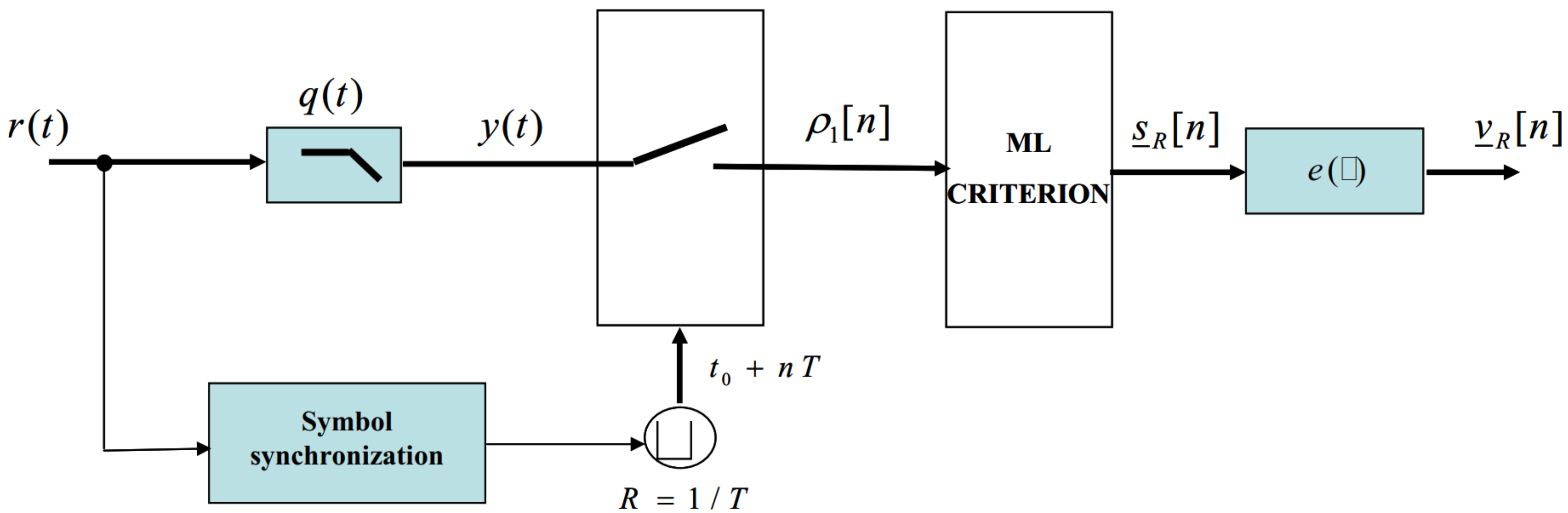
m-PAM constellation: modulator



Equal to 2-PAM, but we have m possible levels:

$$a[n] \in \{-(m-1)\alpha, -(m-3)\alpha, \ldots, +(m-3)\alpha, +(m-1)\alpha\}$$

m-PAM constellation: demodulator

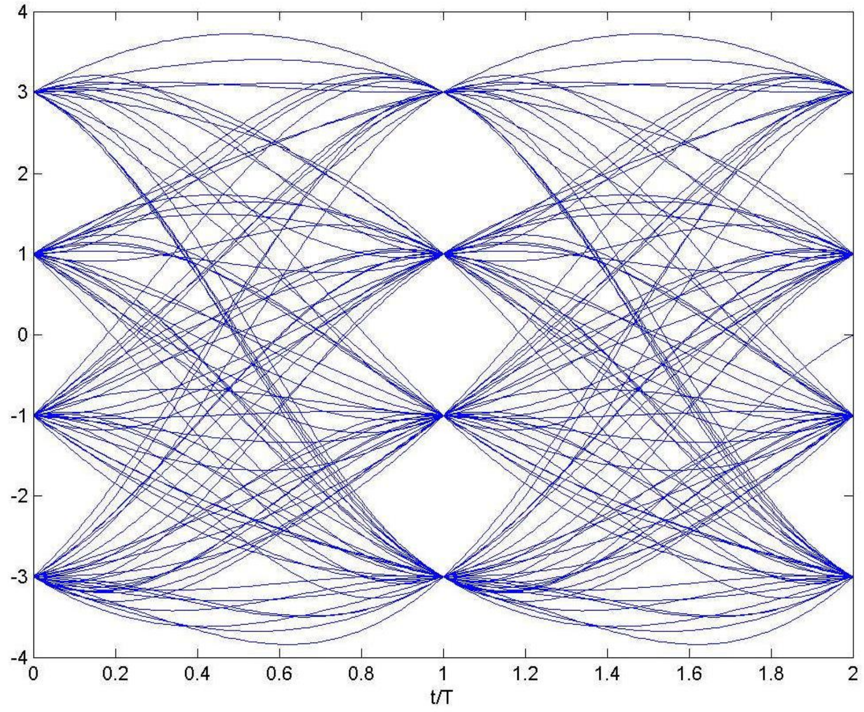


Equal to 2-PAM, but we have m possible levels:

$$a[n] \in \{-(m-1)\alpha, -(m-3)\alpha, \dots, +(m-3)\alpha, +(m-1)\alpha\}$$

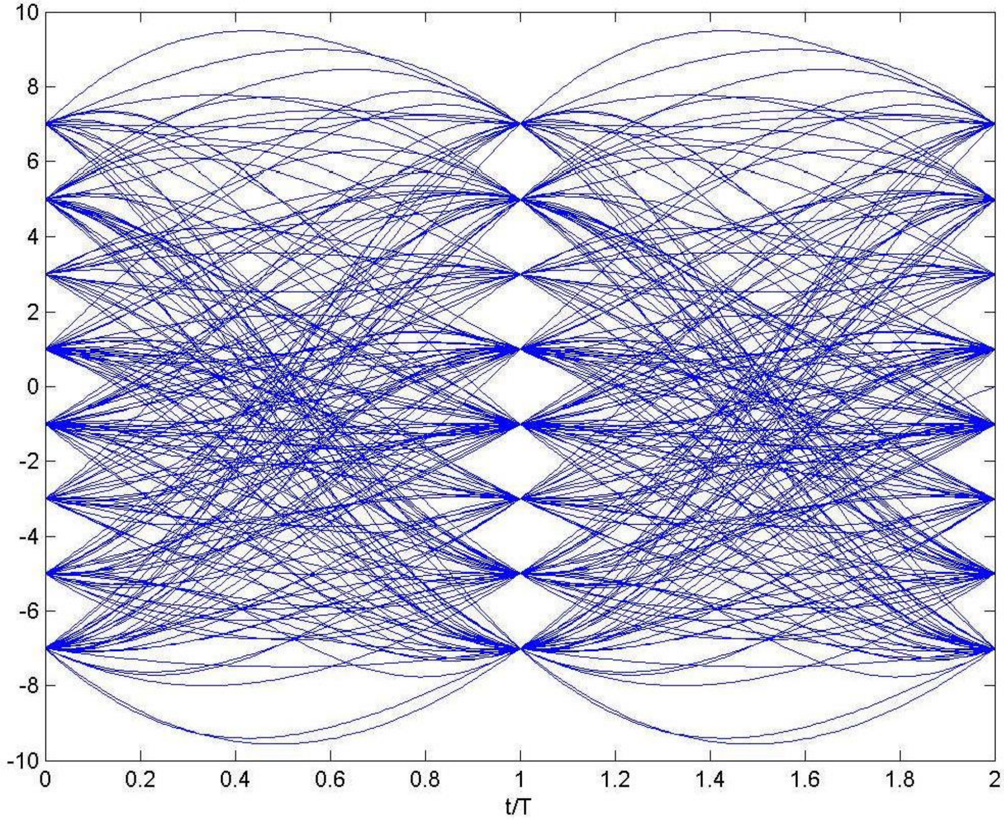
m-PAM constellation: eye diagram

4-PAM, $p(t)$ = RRC with =0.5



m-PAM constellation: eye diagram

8-PAM, $p(t)$ = RRC with =0.5



m-PAM constellation: error probability

By applying the asymptotic approximation we can obtain:

$$P_b(e) \approx \frac{m-1}{mk} erfc\left(\sqrt{\frac{3k}{m^2-1}\frac{E_b}{N_0}}\right)$$

m-PAM constellation: error probability

Comparison: 2-PAM vs. 4-PAM

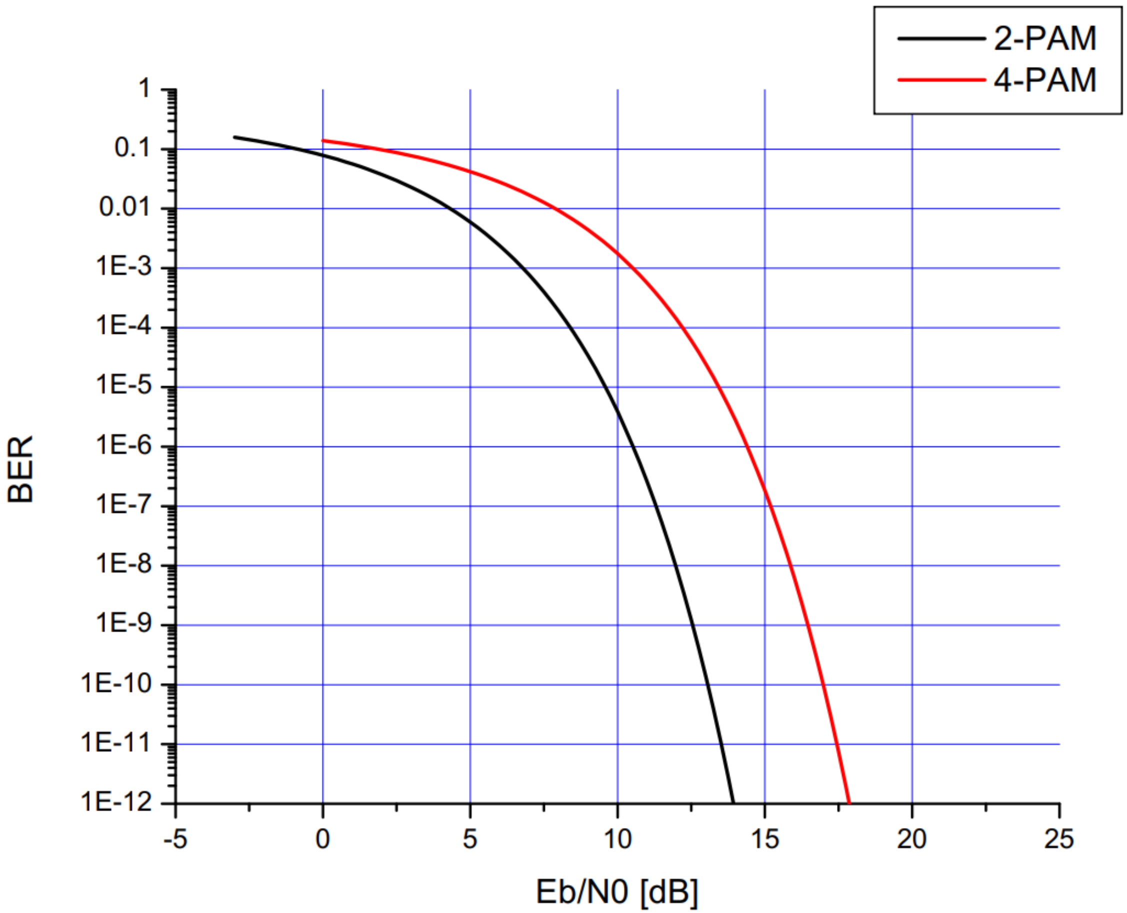
$$2-\text{PAM:} \qquad P_b(e) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$4-\text{PAM:} \qquad P_b(e) \approx \frac{3}{8} erfc\left(\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right)$$

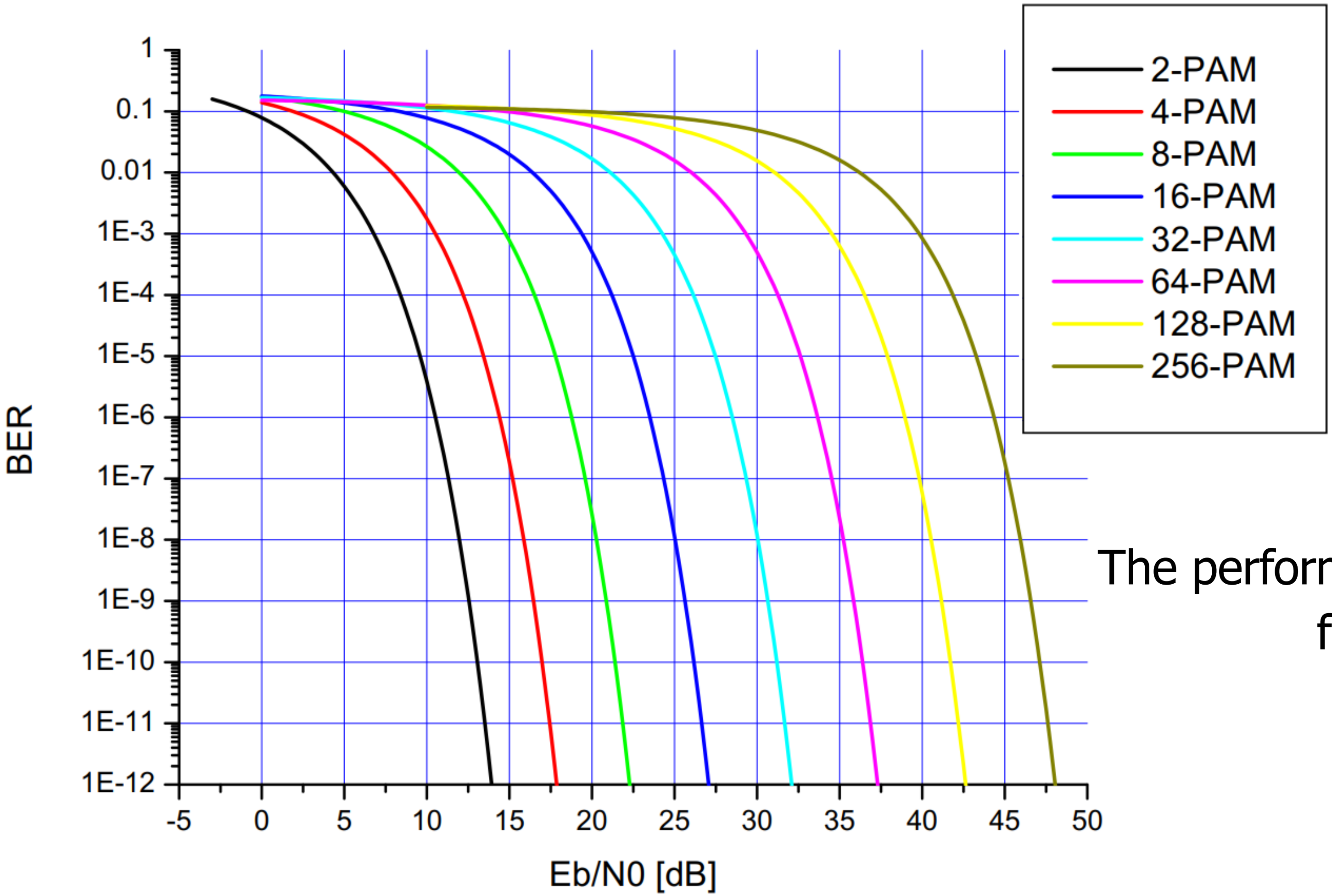
The 2-PAM constellation has better performance
The constellation gain is in the order of 10 log(5/2) = 4 dB

m-PAM constellation: error probability

Comparison: 2-PAM vs. 4-PAM



m-PAM constellation: error probability



The performance decrease for increasing m

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m-PAM constellation:
performance/spectral efficiency trade-off

Given a baseband channel with bandwidth B and an m-PAM constellation, by increasing the number of signals $m=2^k$, we **increase the spectral efficiency**

$$\eta_{id} = R_b / B = 2k \quad bps / Hz$$

then we **can transmit a higher bit rate** R_b.

Unfortunately, the performance decreases:
fixed a BER value, the signal-to-noise ratio E_b/N_0 necessary to achieve it increases with m.

Example

Suppose B=4kHz.

With a (ideal) 2-PAM we transmit R_b = 8 kbps

With a (ideal) 256-PAM we transmit R_b = 64 kbps

However, fixed a target BER (e.g. BER=1e-10), a 256-PAM requires a larger ratio E_b/N_0 (34 dB of difference!).

As an example, at the parity of transmitted power, the link distance is very lower (by a factor of 50!)

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Linear modulation

An m-PAM constellation is a base-band modulation characterized by a low pass TX filter $p(t)$.

Let us suppose to change this TX filter from $p(t)$ to $p(t)cos\,(2\pi f_0\, t)$

- The constellation stays unchanged → **the BER performance are the same**
- **The signal spectrum changes**

Linear modulation

$$s(t) = \sum_n a[n] p(t - nT)$$

$$G(f) = \sigma_a^2 \frac{|P(f)|^2}{T}$$

$$\left.\begin{aligned} s'(t) &= \sum_n a[n] p'(t - nT) \\ p'(t) &= p(t)\cos(2\pi f_0 t) \end{aligned}\right\}$$

$$G'(f) = \frac{1}{4}[G(f - f_0) + G(f + f_0)]$$

The signal spectrum is translated around frequency f_0

Linear modulation

A linear modulation simply **translates the spectrum around frequency f_0** (carrier frequency or Intermediate Frequency (IF))

The modulation formats obtained by **applying a linear modulation to m-PAM modulations are called m-ASK** (Amplitude Shift Keying).

The only one really important is 2-ASK, which is always called 2-PSK (Phase Shift Keying).

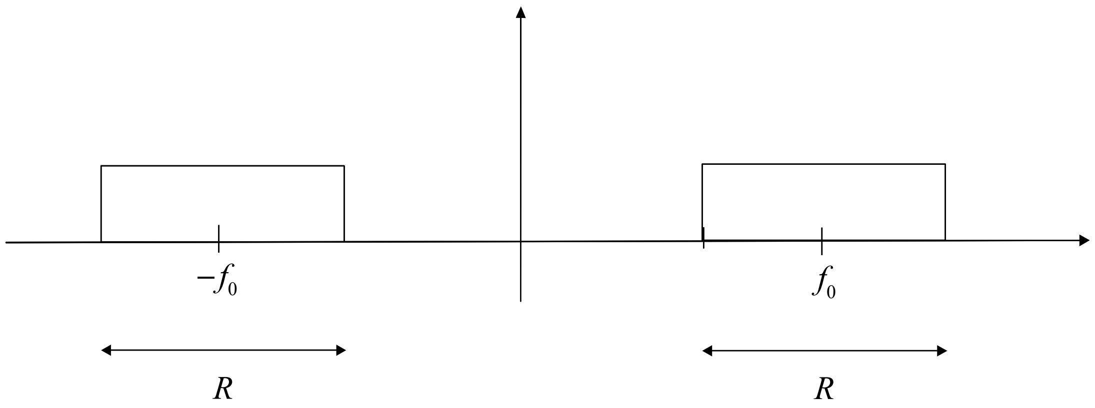
m-ASK constellation: characteristics

1. One-dimensional constellation identical to m-PAM
2. Vector $b_1(t) = p'(t) = p(t)\cos(2\pi f_0 t)$
3. Signal spectrum centred around f_0 → bandpass modulations
4. ASK (Amplitude Shift Keying)

m-ASK constellation: signal spectrum

$$G_s(f) = x\left[\left|P(f-f_0)\right|^2 + \left|P(f+f_0)\right|^2\right] \qquad x \in R$$

Example: $p(t)$ = ideal low pass filter



$$B_{id} = R = \frac{R_b}{k}$$

$$\boxed{\eta_{id} = \frac{R_b}{B_{id}} = k \ bps/Hz}$$

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m-ASK constellation: properties

<u>Properties</u>

- **Spectral efficiency halved with respect to m-PAM**
- BER performance identical to m-PAM
- No practical applications
  (only exception 2-ASK which is always called 2-PSK)