HUST

ĐẠI HỌC BÁCH KHOA HÀ NỘI

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

APPLIED ALGORITHMS

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DEPTH FIRST SEARCH (DFS) AND APPLICATIONS

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CONTENTS

- The longest path on a tree (Đường đi dài nhất trên cây)
- Total path length on the tree (Tổng đường đi trên cây)

THE LONGEST PATH ON THE TREE

- Given a tree $T = (V, E)$, each edge (u,v) has weight $w(u,v)$. Find the path with the longest length on T (the length of the path is the sum of weight on all edges of the path).
- Denote $A[v]$ is the set of vertices adjacent to vertex v on T
- The algorithm is based on depth-first-search (DFS)
  - Choose an arbitrary vertex s on T
  - Perform DFS(s) to find vertex x farthest from s
  - Perform DFS(x) to find the vertex y that is farthest from x
  - The path from x to y found will be the longest path on T

THE LONGEST PATH ON THE TREE

```
Init(V, A) {
   for v in V do d[v] = -1;
}
DFS(u) {
  for x in A[u] do {
    if d[x] < 0 then {
       d[x] = d[u] + w(u,x);
       DFS(x);
    }
  }
}
```

```
LongestPathOnTree(V, A){
  Init(V, A);
  s = select a node in V;
  DFS(s);
  x = select u in V such that d[u] is maximal;
  Init(V, A);
  DFS(x);
  y = select u in V such that d[u] is maximal;
  P = unique path between x and y in T;
  return P;
}
```

THE LONGEST PATH ON THE TREE

- The complexity: $O(|V| + |E|)$

TOTAL PATH LENGTH ON THE TREE

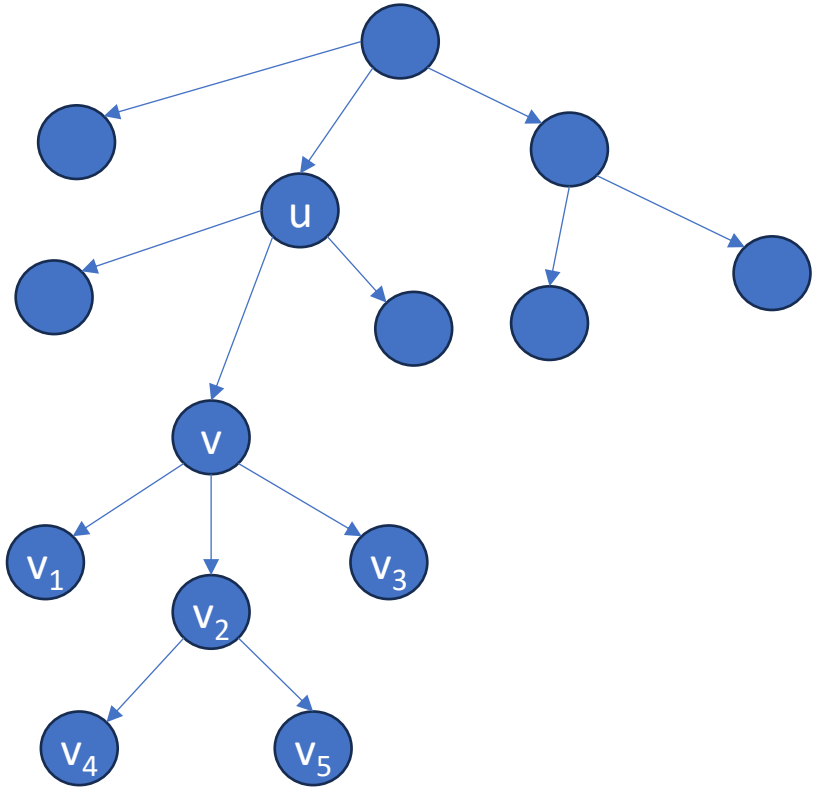
- Given a tree $T = (V, E)$, each edge (u,v) has weight $w(u,v)$. Vertex set V includes n vertices.
- Denote:
  - $A[v]$: is the set of vertices adjacent to vertex v on T
  - $c(u,v)$ is the length of the unique path between two vertices u and v on T
  - $f(u)$: total path length from other vertices to u on T: $f(u) = \sum_{v \in V} c(v,u)$
- Find $f(u)$ for every $u \in V$

TOTAL PATH LENGTH ON THE TREE

- Choose an arbitrary vertex s on T as the root, perform DFS on T starting from s:
  - $p(u)$: parent vertex of u (the vertex from which the algorithm visits u)
  - $d(u)$: total path length from descendant vertices of u to u
  - $N(u)$: number of descendant vertices of u (including vertex u)

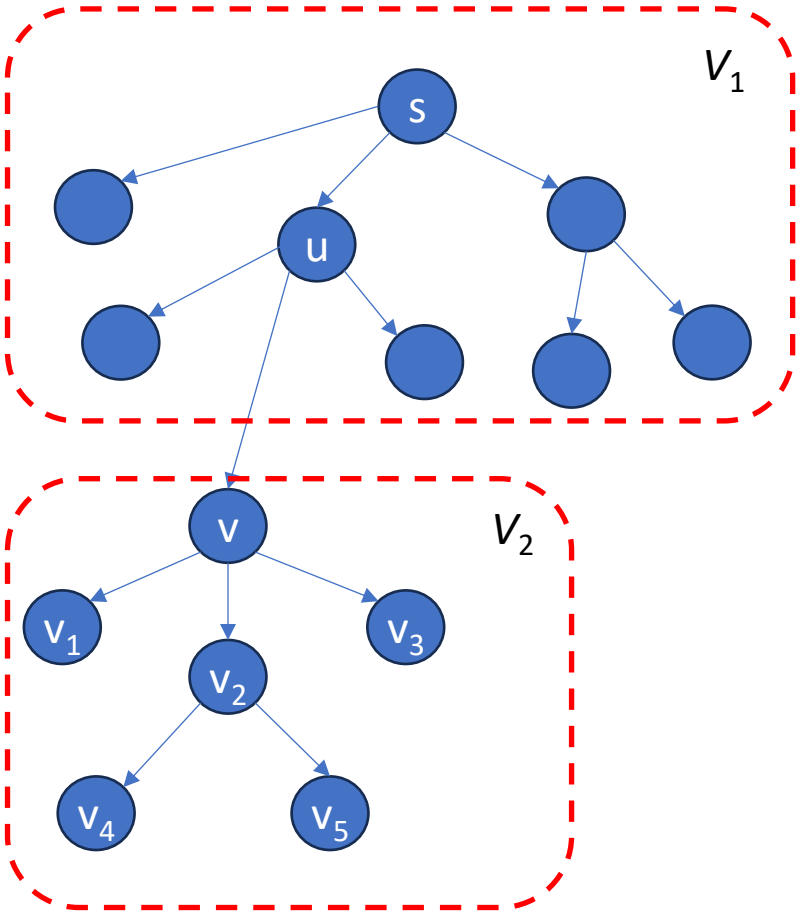
TOTAL PATH LENGTH ON THE TREE

- DFS1(u): depth-first search in the first phase
  - Purpose: calculate $d(x)$ and $N(x)$ for all vertices x that are descendants of u
  - When DFS1(u) is completed, $d(u)$ is calculated and it will be used to calculate $d(p(u))$
  - Do: for each vertex $v \in A[u]$:
    - Call DFS1(v)
    - Update: $d(u) = d(u) + d(v) + N(v)^*w(u,v)$
    - $N(u) = N(u) + N(v)$



TOTAL PATH LENGTH ON THE TREE

- DFS1(u): depth-first search in the first phase
  - Purpose: calculate $d(x)$ and $N(x)$ for all vertices x that are descendants of u
  - When DFS1(u) is completed, $d(u)$ is calculated and it will be used to calculate $d(p(u))$
  - Do: for each vertex $v \in A[u]$:
    - Call DFS1(v)
    - Update: $d(u) = d(u) + d(v) + N(v)*w(u,v)$
    - $N(u) = N(u) + N(v)$
- DFS2(u): depth-first search in the second phase
  - Purpose: When DFS2(u) is called, $f(u)$ has been already calculated and we will calculate $f(v)$ for each vertex v being a child of u
  - Do: for each vertex $v \in A[u]$ not has been visited
    - $F = f(u) - (d(v) + w(u,v)*N(v))$
    - $f(v) = F + d(v) + w(u,v)*(n - N(v))$
    - Call DFS2(v)



TOTAL PATH LENGTH ON THE TREE

```
DFS1(u){
  for v in A[u] do {
     if p(v) = 0 then {
        p(v) = u;
        DFS1(v);
        d(u) = d(u) + d(v) + N(v)*w(u,v);
        N(u) = N(u) + N(v);
     }
  }
}
Phase1(){
  for v in V do {
     p(v) = 0; d(v) = 0; N(v) = 1; f(v) = 0;
  }
  p(1) = 1; DFS1(1);
}
```

```
DFS2(u){
  for v in A[u] do {
     if p(v) = 0 then {
        F = f(u) – (d(v) + N(v)*w(u,v));
        f(v) = F + d(v) + w(u,v)*(n – N(v));
        p(v) = u; DFS2(v);
     }
  }
}
Phase2(){
  for v in V do { p(v) = 0; }
  f(1) = d(1); p(1) = 1; DFS2(1);
}
Main(){
   Phase1(); Phase2();
}
```

TOTAL PATH LENGTH ON THE TREE

- The complexity: $O(|V| + |E|)$

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THANK YOU !