

## Question 1

### Topic

Matlab Commands

### Original Question

*Which command is used to clear the Matlab command window?*

### Explanation

In Matlab, there are several commands for clearing different parts of the environment:

- **clear**: Removes all variables from the current workspace.
- **close all**: Closes all open figure windows.
- **clc**: Clears all text from the Command Window, providing a "clean screen".
- **clear all**: A more powerful version of **clear** that also removes functions, MEX-links, etc., from memory.

The command specifically used to clear the text display in the command window is **clc**.

### Correct Answer

3. **clc**

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## Question 2

### Topic

Linear Regression

### Original Question

*Use the linear regression method to find the straight line ( $y = a_0 + a_1x$ ) that fits the data set  $(0,3)$ ,  $(1,6)$ ,  $(2,8)$ ,  $(3,11)$ ,  $(4,13)$ ,  $(5,14)$ ?*

### Explanation

Linear regression finds the best-fitting line by minimizing the sum of squared errors. The formulas for the coefficients  $a_0$  (intercept) and  $a_1$  (slope) are:

$$a_1 = \frac{n \sum(x_i y_i) - \sum x_i \sum y_i}{n \sum(x_i^2) - (\sum x_i)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x} \quad (\text{where } \bar{x} \text{ and } \bar{y} \text{ are the means})$$

**Calculations** (with  $n = 6$  points):

- $\sum x_i = 0 + 1 + 2 + 3 + 4 + 5 = 15$
- $\sum y_i = 3 + 6 + 8 + 11 + 13 + 14 = 55$
- $\sum x_i^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 0 + 1 + 4 + 9 + 16 + 25 = 55$
- $\sum (x_i y_i) = (0 \cdot 3) + (1 \cdot 6) + (2 \cdot 8) + (3 \cdot 11) + (4 \cdot 13) + (5 \cdot 14) = 177$

Now, we substitute these sums into the formulas:

$$a_1 = \frac{6 \cdot 177 - 15 \cdot 55}{6 \cdot 55 - 15^2} = \frac{1062 - 825}{330 - 225} = \frac{237}{105} \approx 2.2571$$

$$a_0 = \frac{\sum y_i}{n} - a_1 \frac{\sum x_i}{n} = \frac{55}{6} - (2.2571 \cdot \frac{15}{6}) \approx 9.1667 - 5.6428 \approx 3.5239$$

The resulting equation is  $y = 3.5239 + 2.2571x$ .

## Correct Answer

1.  $y=3.5238+2.2571x$

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## Question 3

### Topic

Polynomial Regression

### Original Question

Use non-linear regression to find the parameters  $(a,b,c)$  for the curve  $y = a + bx + cx^2$  that fits the data set  $(1,2), (2,4), (3,6), (4,7), (5,9)$ .

### Explanation

For a quadratic fit, we solve the following system of normal equations for  $a, b, c$ :

$$\begin{aligned} na + (\sum x_i)b + (\sum x_i^2)c &= \sum y_i \\ (\sum x_i)a + (\sum x_i^2)b + (\sum x_i^3)c &= \sum x_i y_i \\ (\sum x_i^2)a + (\sum x_i^3)b + (\sum x_i^4)c &= \sum y_i x_i^2 \end{aligned}$$

**Calculations** (with  $n = 5$  points):

- $\sum x = 15, \sum y = 28, \sum x^2 = 55, \sum x^3 = 225, \sum x^4 = 979$
- $\sum xy = 101, \sum x^2y = 409$

The system of equations becomes:

$$\begin{aligned} 5a + 15b + 55c &= 28 \\ 15a + 55b + 225c &= 101 \\ 55a + 225b + 979c &= 409 \end{aligned}$$

Solving this system (e.g., using a matrix solver) yields:  $a = 0, b \approx 2.1286, c \approx -0.0714$ .

## Correct Answer

2. (-0.0000, 2.1286, -0.0714)

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## Question 4

### Topic

Root Finding (Secant Method)

### Original Question

Use the Secant method to find the pair (root, actual error) for  $y = 3x^3 + 2x - 2$  with a tolerance of 0.01, starting with the interval  $[0.6, 1]$ .

### Explanation

The Secant method formula is:

$$x_{i+1} = x_i - f(x_i) \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

**Calculations:** Let  $f(x) = 3x^3 + 2x - 2$ .

- Initial points:  $x_0 = 0.6$  and  $x_1 = 1$ .
- $f(x_0) = f(0.6) = 3(0.6)^3 + 2(0.6) - 2 = -0.152$
- $f(x_1) = f(1) = 3(1)^3 + 2(1) - 2 = 3$

**Iteration 1:**

$$x_2 = 1 - 3 \frac{1 - 0.6}{3 - (-0.152)} \approx 0.6192$$

$f(x_2) \approx -0.0492$ . Since  $|f(x_2)| > 0.01$ , we continue.

**Iteration 2:**

$$x_3 = 0.6192 - (-0.0492) \frac{0.6192 - 1}{-0.0492 - 3} \approx 0.6254$$

$f(x_3) \approx -0.0151$ . Since  $|f(x_3)| > 0.01$ , we continue.

**Iteration 3:**

$$x_4 = 0.6254 - (-0.0151) \frac{0.6254 - 0.6192}{-0.0151 - (-0.0492)} \approx 0.6279$$

$f(x_4) \approx f(0.6279) \approx -0.0015$ . Since  $|f(x_4)| < 0.01$ , we stop. The result is root  $x \approx 0.6279$  with an actual error  $f(x) \approx -0.0015$ .

## Correct Answer

1. (0.6279, -0.0015)

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## Question 5

### Topic

Root Finding (Bisection Method)

### Original Question

Use the bisection method to find (root, actual error) for  $y = 2x^3 + x - 2$  with a tolerance of 0.15, starting with the interval  $[0.5, 1]$ .

### Explanation

The bisection method sets the new approximation to the midpoint  $c = (a + b)/2$ . **Calculations:** Let  $f(x) = 2x^3 + x - 2$ .

- Initial interval:  $a = 0.5, b = 1$ .
- $f(a) = f(0.5) = -1.25$  (negative)
- $f(b) = f(1) = 1$  (positive)

#### Iteration 1:

- $c = (0.5 + 1)/2 = 0.75$
- $f(0.75) = -0.40625$  (negative).  $|f(c)| > 0.15$ .
- New interval is  $[0.75, 1]$ .

#### Iteration 2:

- $c = (0.75 + 1)/2 = 0.875$
- $f(0.875) \approx 0.2148$  (positive).  $|f(c)| > 0.15$ .
- New interval is  $[0.75, 0.875]$ .

#### Iteration 3:

- $c = (0.75 + 0.875)/2 = 0.8125$
- $f(0.8125) \approx -0.1152$ .
- $|f(c)| \approx 0.1152 < 0.15$ . We stop.

The approximate root is 0.8125, and the actual error is  $\approx -0.1152$ .

### Correct Answer

4. (0.8125, -0.1147)

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## Question 6

### Topic

Root Finding (Newton's Method)

### Original Question

Use Newton's method for  $y = x^2 - \cos(x)$  with an initial guess  $x_0 = 2$  and a tolerance of 0.01. Find the pair (root, actual error).

### Explanation

Newton's method formula is  $x_{i+1} = x_i - f(x_i)/f'(x_i)$ . Note: all calculations must be in radians.

- $f(x) = x^2 - \cos(x)$
- $f'(x) = 2x + \sin(x)$
- $x_0 = 2, \varepsilon = 0.01$

#### Iterations:

- **Step 1:**  $x_1 = 2 - \frac{2^2 - \cos(2)}{2(2) + \sin(2)} = 2 - \frac{4.4161}{4.9093} \approx 1.1005$ .  $|f(x_0)| > 0.01$ .
- **Step 2:**  $x_2 = 1.1005 - \frac{1.1005^2 - \cos(1.1005)}{2(1.1005) + \sin(1.1005)} = 1.1005 - \frac{0.7579}{3.0924} \approx 0.8554$ .  $|f(x_1)| > 0.01$ .
- **Step 3:**  $x_3 = 0.8554 - \frac{0.8554^2 - \cos(0.8554)}{2(0.8554) + \sin(0.8554)} = 0.8554 - \frac{0.0758}{2.4662} \approx 0.8247$ .  $|f(x_2)| > 0.01$ .
- **Step 4:**  $f(0.8247) = 0.8247^2 - \cos(0.8247) \approx 0.0011$ .  $|f(x_3)| < 0.01$ . Stop.

The root is  $x \approx 0.8247$  with an error  $f(x) \approx 0.0011$ . This matches option 1.

### Correct Answer

1. (0.8247, 0.0013)
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## Question 7

### Topic

Numerical Differentiation

### Original Question

Use the backward finite difference formula to find the derivative of  $f(x) = x^4 + 2x^3 + 1$  at  $x = 2$  with  $h = 0.01$ .

## Explanation

The backward finite difference formula for the first derivative is:

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

**Calculations:**

- $f(x) = x^4 + 2x^3 + 1$
- $x = 2, h = 0.01$ , so  $x - h = 1.99$
- $f(2) = (2)^4 + 2(2)^3 + 1 = 16 + 16 + 1 = 33$
- $f(1.99) = (1.99)^4 + 2(1.99)^3 + 1 \approx 15.6824 + 15.7612 + 1 = 32.4436$
- $f'(2) \approx \frac{33 - 32.4436}{0.01} = \frac{0.5564}{0.01} = 55.64$

## Correct Answer

2. 55.6410

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## Question 8

### Topic

Numerical Integration (Trapezoidal Rule)

### Original Question

Use the (single-application) trapezoidal rule to integrate  $f(x) = 2x^4 + x^3 + 1$  on the interval  $[1, 2]$ .

## Explanation

The single-application trapezoidal rule formula is:

$$I \approx (b-a) \frac{f(a) + f(b)}{2}$$

**Calculations:**

- $f(x) = 2x^4 + x^3 + 1$
- $a = 1, b = 2$
- $f(a) = f(1) = 2(1)^4 + 1^3 + 1 = 4$
- $f(b) = f(2) = 2(2)^4 + 2^3 + 1 = 32 + 8 + 1 = 41$
- $I \approx (2-1) \frac{4+41}{2} = 1 \cdot \frac{45}{2} = 22.5$

## Correct Answer

3. 22.5

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## Question 9

### Topic

Numerical Integration (Composite Trapezoidal Rule)

### Original Question

Use the composite trapezoidal rule with  $N = 3$  subintervals to integrate  $f(x) = x^4 + 2x^3 + 1$  on the interval  $[0, 2]$ .

### Explanation

The composite trapezoidal rule formula is:

$$I \approx \frac{h}{2} \left[ f(x_0) + 2 \sum_{i=1}^{N-1} f(x_i) + f(x_N) \right]$$

#### Calculations:

- $a = 0$ ,  $b = 2$ ,  $N = 3$ . Step size  $h = (2 - 0)/3 = 2/3$ .
- Points:  $x_0 = 0$ ,  $x_1 = 2/3$ ,  $x_2 = 4/3$ ,  $x_3 = 2$ .
- Function values:
  - $f(0) = 1$
  - $f(2/3) = (2/3)^4 + 2(2/3)^3 + 1 \approx 1.7901$
  - $f(4/3) = (4/3)^4 + 2(4/3)^3 + 1 \approx 8.9012$
  - $f(2) = 33$
- $I \approx \frac{2/3}{2} [1 + 2(1.7901 + 8.9012) + 33] = \frac{1}{3} [1 + 2(10.6913) + 33] = \frac{55.3826}{3} \approx 18.4609$

## Correct Answer

3. 18.4609

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## Question 10

### Topic

Numerical Integration (Simpson's Rule)

## Original Question

Use Simpson's rule to integrate  $f(x) = 2x^4 + x^3 + 1$  on the interval  $[0, 1]$ .

## Explanation

This implies the single-application Simpson's 1/3 rule ( $N = 2$  subintervals). The formula is:

$$I \approx \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$$

### Calculations:

- $a = 0, b = 1, N = 2$ . Step size  $h = (1 - 0)/2 = 0.5$ .
- Points:  $x_0 = 0, x_1 = 0.5, x_2 = 1$ .
- Function values:  $f(0) = 1, f(0.5) = 1.25, f(1) = 4$ .
- $I \approx \frac{0.5}{3}[1 + 4(1.25) + 4] = \frac{1}{6}[1 + 5 + 4] = \frac{10}{6} \approx 1.6667$

## Correct Answer

4. 1.6667

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## Question 11

### Topic

Ordinary Differential Equations (Euler's Method)

## Original Question

Given  $y'(t) = y + t^2y + 1$  with  $y(0) = 2$ , find  $y(0.2)$  using a step size  $h = 0.1$ .

## Explanation

The Forward Euler method is  $y_{i+1} = y_i + hf(t_i, y_i)$ . Let  $f(t, y) = (1 + t^2)y + 1$ .

### • Step 1 (Find y at t=0.1):

- $f(t_0, y_0) = f(0, 2) = (1 + 0^2) \cdot 2 + 1 = 3$
- $y_1 = y_0 + hf(t_0, y_0) = 2 + 0.1 \cdot 3 = 2.3$

### • Step 2 (Find y at t=0.2):

- $f(t_1, y_1) = f(0.1, 2.3) = (1 + 0.1^2) \cdot 2.3 + 1 = 1.01 \cdot 2.3 + 1 = 3.323$
- $y_2 = y_1 + hf(t_1, y_1) = 2.3 + 0.1 \cdot 3.323 = 2.3 + 0.3323 = 2.6323$



## Correct Answer

2. 2.6323

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## Question 12

### Topic

Ordinary Differential Equations (Euler's Method)

### Original Question

Given  $y'(t) = y + t^2y + 1$  with  $y(0) = 1$ , find  $y(0.4)$  using a step size  $h = 0.2$ .

### Explanation

The Forward Euler method is  $y_{i+1} = y_i + hf(t_i, y_i)$ . Let  $f(t, y) = (1 + t^2)y + 1$ .

- **Step 1 (Find y at t=0.2):**

- $f(t_0, y_0) = f(0, 1) = (1 + 0^2) \cdot 1 + 1 = 2$
  - $y_1 = y_0 + hf(t_0, y_0) = 1 + 0.2 \cdot 2 = 1.4$

- **Step 2 (Find y at t=0.4):**

- $f(t_1, y_1) = f(0.2, 1.4) = (1 + 0.2^2) \cdot 1.4 + 1 = 1.04 \cdot 1.4 + 1 = 2.456$
  - $y_2 = y_1 + hf(t_1, y_1) = 1.4 + 0.2 \cdot 2.456 = 1.4 + 0.4912 = 1.8912$

## Correct Answer

3. 1.8912

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## Question 13

### Topic

ODE Methods (Theory)

### Original Question

*What is the advantage of the Backward Euler method compared to the Forward Euler method when solving differential equations?*

## Explanation

- **Forward Euler** is an explicit method. It is simple but only conditionally stable. For stiff ODEs, it requires an extremely small step size to remain stable.
- **Backward Euler** is an implicit method. It is more computationally expensive per step but is unconditionally stable (A-stable). This means it remains stable for any step size when applied to a stable problem, making it far superior for stiff equations.

The primary advantage is its much larger stability region.

## Correct Answer

1. **Backward Euler has a larger stability region.**
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## Question 14

### Topic

Optimization (Theory)

### Original Question

*When using Newton's method for minimizing a single-variable function, if the initial value is closer to the exact solution, does the problem converge faster?*

## Explanation

Yes. Newton's method for optimization exhibits quadratic convergence, meaning the error at each step is proportional to the square of the previous error ( $\text{error}_{k+1} \approx C \cdot (\text{error}_k)^2$ ). This is extremely fast, but it is only guaranteed if the initial guess is "sufficiently close" to the true solution. A closer initial guess means a smaller initial error, which leads to much faster convergence.

## Correct Answer

1. **True**
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## Question 15

### Topic

Linear Programming (Formulations)

## Original Question

What form is this linear programming problem in? minimize  $f(x) = 2x_1 - 4x_2 - x_3 + 6x_4$   
Subject to:  $x_1 + x_4 + x_5 \geq 12$ ,  $12x_1 + x_3 + x_6 \geq 3$ ,  $x_1 + x_2 - x_3 - x_4 \geq 6$ ,  $x_j \geq 0$  for all  $j$

## Explanation

- **Canonical Form:** A *minimize* problem where all constraints are  $\geq$  inequalities, and all variables are non-negative.
- **Standard Form:** A problem where all constraints are equalities ( $=$ ), and all variables are non-negative.

The given problem is a minimization problem, all constraints are of the form ' $\geq$ ', and all variables are non-negative. This perfectly matches the definition of Canonical Form.

## Correct Answer

### 2. Canonical form

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## Question 16

### Topic

Linear Programming (Theory)

## Original Question

Is the statement "a linear programming problem in the plane always has an optimal solution that is a vertex of the feasible region" true or false?

## Explanation

This statement is **false** because it uses the word "always". A linear program does not always have an optimal solution. It can be:

1. **Infeasible:** The feasible region is empty, so there are no solutions.
2. **Unbounded:** The objective function can be made arbitrarily large (for max) or small (for min). No finite optimal solution exists.

The Fundamental Theorem of Linear Programming states that *if* an optimal solution exists, then at least one optimal solution must be a vertex. The initial statement is false because it does not account for the cases where no optimal solution exists.

## Correct Answer

### 2. Sai (False)

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## Question 17

### Topic

Linear Programming (Theory)

### Original Question

*Is the statement "A linear programming problem always finds a feasible basic solution" true or false?*

### Explanation

This statement is **false**. A feasible basic solution corresponds to a vertex of the feasible region. However, a linear program can be **infeasible**, meaning its feasible region is empty. If there are no feasible points at all, there can be no feasible basic solutions (vertices). Phase I of the Simplex method is designed specifically to find a feasible basic solution or prove that none exist.

### Correct Answer

2. Sai (False)

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## Question 18

### Topic

Linear Programming (Graphical Method)

### Original Question

*Find the maximum value of  $Z = 3x + 5y$  subject to  $x + 4y \leq 24$ ,  $3x + y \leq 21$ ,  $x + y \leq 9$ ,  $x \geq 0$ ,  $y \geq 0$ .*

### Explanation

We find the vertices of the feasible region by finding the intersection points of the boundary lines.

- **Vertex A (Origin):**  $(0, 0)$
- **Vertex B (L2 & x-axis):**  $3x + y = 21, y = 0 \implies x = 7 \implies (7, 0)$
- **Vertex C (L2 & L3):**  $3x + y = 21, x + y = 9 \implies 2x = 12, x = 6, y = 3 \implies (6, 3)$
- **Vertex D (L1 & L3):**  $x + 4y = 24, x + y = 9 \implies 3y = 15, y = 5, x = 4 \implies (4, 5)$
- **Vertex E (L1 & y-axis):**  $x + 4y = 24, x = 0 \implies y = 6 \implies (0, 6)$

Evaluate  $Z = 3x + 5y$  at each vertex:

- $Z(0,0) = 0$
- $Z(7,0) = 21$
- $Z(6,3) = 3(6) + 5(3) = 18 + 15 = 33$
- $Z(4,5) = 3(4) + 5(5) = 12 + 25 = 37$  (**Maximum**)
- $Z(0,6) = 30$

## Correct Answer

2. 37

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## Question 19

### Topic

Matlab Scripting

### Original Question

*What is the result of the Matlab script: `j=0; i=1; while(j>5) for i=1:8 j=j+i; end end?`*

### Explanation

1. `j=0;`: Variable `j` is initialized to 0.
2. `i=1;`: Variable `i` is initialized to 1.
3. `while(j>5)`: The condition for the `while` loop is checked. Since  $j = 0$ , the condition  $0 > 5$  is **false**.
4. The body of the `while` loop is never executed.
5. The script terminates with the variables at their last assigned values.

The final values are  $j = 0$  and  $i = 1$ .

## Correct Answer

1. `j=0 & i=1`
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## Question 20

### Topic

Matlab Scripting (Logical Operators)

## Original Question

What is the result of the script: `if((-10 && 0) || (20134 && 900)) fprintf("%s", "True.") else fprintf("%s", "False.") end?`

## Explanation

In Matlab, any non-zero number is treated as 'true', and 0 is 'false'. We evaluate the condition:

`((-10 && 0) || (20134 && 900))`

- `(-10 && 0)`: 'true && false' evaluates to 'false'.
- `(20134 && 900)`: 'true && true' evaluates to 'true'.
- Combined: `(false || true)` evaluates to 'true'.

Since the overall condition is 'true', the 'if' block is executed, printing "True."

## Correct Answer

1. True

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## Question 21

### Topic

Matlab Indexing

## Original Question

What is the result of `A = [1, 2, 3, 4; 5, 6, 7, 8; 9, 10, 11, 12]; B = A(2, :);`?

## Explanation

The command `B = A(2, :)` selects a submatrix from A.

- The first index, '2', specifies the **2nd row**.
- The second index, ':', is a wildcard that specifies **all columns**.

Therefore, B is assigned the entire second row of A, which is '[5, 6, 7, 8]'.

## Correct Answer

2. `B = [5,6,7,8]`

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## Question 22

### Topic

Systems of Linear Equations (Theory)

### Original Question

Choose the most correct statement about a system  $Ax = b$ .

### Explanation

Let's analyze the statements for a general linear system  $Ax = b$ :

- **PA1:** "A system  $Ax = b$  has a unique solution if and only if  $\text{rank}(A) = \text{rank}(Ab)$ ". This is **false**. This condition (Rouché–Capelli theorem) only guarantees that the system is *consistent* (has at least one solution). For a unique solution, the rank must also equal the number of variables,  $n$ .
- **PA2:** "A system  $Ax = b$  has a unique solution if and only if  $\det(A) \neq 0$ ". This is **false** for a general system, as the determinant is only defined for square matrices. This statement is true only for square systems.

Since both statements are incorrect for the general case, both are considered wrong.

### Correct Answer

3. Both PA1 and PA2 are wrong

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## Question 23

### Topic

Interpolation

### Original Question

Use interpolation to find the curve that fits the data:  $(0.1, -2.9750)$ ,  $(0.2, -2.9400)$ ,  $(0.3, -2.8950)$ .

### Explanation

With three points, we can find a unique quadratic polynomial  $P(x) = ax^2 + bx + c$ . Since the x-values are equally spaced ( $h = 0.1$ ), we can use finite differences.

- **First Differences:**
  - $\Delta y_1 = -2.9400 - (-2.9750) = 0.0350$
  - $\Delta y_2 = -2.8950 - (-2.9400) = 0.0450$

- **Second Difference:**  $\Delta^2 y = \Delta y_2 - \Delta y_1 = 0.0100$

The coefficient  $a$  is given by  $a = \frac{\Delta^2 y}{2!h^2} = \frac{0.01}{2 \cdot 0.1^2} = 0.5$ . The polynomial is  $y = 0.5x^2 + bx + c$ . We plug in two points to find  $b$  and  $c$ :

$$-2.9750 = 0.5(0.1)^2 + b(0.1) + c \implies -2.98 = 0.1b + c$$

$$-2.9400 = 0.5(0.2)^2 + b(0.2) + c \implies -2.96 = 0.2b + c$$

Subtracting the equations gives  $0.02 = 0.1b \implies b = 0.2$ . Substituting back gives  $c = -3$ . The polynomial is  $y = 0.5x^2 + 0.2x - 3$ .

## Correct Answer

4.  $0.5 \cdot x^2 + 0.2 \cdot x - 3$

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## Question 24

### Topic

Systems of ODEs (Euler's Method)

### Original Question

Given the system  $y' = (z - y)x$ ,  $z' = (z + y)x$  with  $y(0) = 1$ ,  $z(0) = 1$ , find the solution at  $x = 0.6$  using Euler's method with  $h = 0.1$ .

### Explanation

We apply Euler's method iteratively:  $y_{i+1} = y_i + hf(x_i, y_i, z_i)$  and  $z_{i+1} = z_i + hg(x_i, y_i, z_i)$ . The results of each step are shown in the table below:

Step (i)	$x_i$	$y_i$	$z_i$	$y_{i+1}$	$z_{i+1}$
0	0.0	1.0000	1.0000	1.0000	1.0000
1	0.1	1.0000	1.0000	1.0000	1.0200
2	0.2	1.0000	1.0200	1.0004	1.0604
3	0.3	1.0004	1.0604	1.0022	1.1222
4	0.4	1.0022	1.1222	1.0070	1.2072
5	0.5	1.0070	1.2072	1.0170	1.3179
6	0.6	1.0170	1.3179	—	

## Correct Answer

b.  $y=1.0170$ ;  $z=1.3179$

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## Question 25

### Topic

Numerical Integration (Simpson's Rules)



## Original Question

Approximate  $\int_0^1 \frac{dx}{1+x}$  using Simpson's rules with  $n = 5$  subintervals.

## Explanation

Since  $n = 5$  is odd, we cannot use a single composite rule. We combine Simpson's 1/3 rule for the first 2 intervals and Simpson's 3/8 rule for the last 3 intervals.

- $f(x) = 1/(1+x)$ ,  $h = (1-0)/5 = 0.2$ .
- Points:  $f(0) = 1$ ,  $f(0.2) \approx 0.8333$ ,  $f(0.4) \approx 0.7143$ ,  $f(0.6) = 0.625$ ,  $f(0.8) \approx 0.5556$ ,  $f(1) = 0.5$ .

**Part 1: Simpson's 1/3 on  $[0, 0.4]$  ( $n = 2$ )**

$$I_1 = \frac{h}{3}[f(0) + 4f(0.2) + f(0.4)] = \frac{0.2}{3}[1 + 4(0.83333) + 0.71429] \approx 0.3365$$

**Part 2: Simpson's 3/8 on  $[0.4, 1.0]$  ( $n = 3$ )**

$$I_2 = \frac{3h}{8}[f(0.4) + 3f(0.6) + 3f(0.8) + f(1.0)] = \frac{3(0.2)}{8}[0.7143 + 3(0.625) + 3(0.5556) + 0.5] \approx 0.3567$$

**Total Integral:**

$$I = I_1 + I_2 = 0.3365 + 0.3567 = 0.6932$$

The exact value is  $\ln(2) \approx 0.69315$ . The calculated value matches this very closely.

## Correct Answer

**c. 0.69315**