EX1: Calculate Z-Transform and Region of Convergence

1. $x(n) = (\frac{1}{2})^n u(n)$
   - Using the causal pair with $a = 1/2$: $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$
   - ROC: $|z| > \frac{1}{2}$
2. $x(n) = -(\frac{1}{2})^n u(-n-1)$
   - Using the anti-causal pair with $a = 1/2$: $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$
   - ROC: $|z| < \frac{1}{2}$
3. $x(n) = (\frac{1}{2})^n u(-n)$
   - By definition: $X(z) = \sum_{n=-\infty}^{0} (\frac{1}{2})^n z^{-n} = \sum_{k=0}^{\infty} (\frac{1}{2})^{-k} z^k = \sum_{k=0}^{\infty} (2z)^k$.
   - This geometric series converges for $|2z| < 1$.
   - $X(z) = \frac{1}{1-2z}$
   - ROC: $|z| < \frac{1}{2}$
4. $x(n) = \delta(n)$
   - $X(z) = 1$
   - ROC: Entire z-plane.
5. $x(n) = \delta(n-1)$
   - $X(z) = z^{-1}$
   - ROC: Entire z-plane except $z = 0$.
6. $x(n) = \delta(n+1)$
   - $X(z) = z$
   - ROC: Entire z-plane except $z = \infty$.
7. $x(n) = (\frac{1}{2})^n (u(n) - u(n-10))$
   - This is a finite-duration signal, non-zero for $n = 0, 1, \ldots, 9$.
   - $X(z) = \sum_{n=0}^{9} (\frac{1}{2}z^{-1})^n = \frac{1-(\frac{1}{2}z^{-1})^{10}}{1-\frac{1}{2}z^{-1}} = \frac{1-\frac{1}{1024}z^{-10}}{1-\frac{1}{2}z^{-1}}$
   - ROC: Since it is a finite-duration signal, the ROC is the entire z-plane except for poles at $z = 0$ (due to z^{-10}) and possibly $z = \infty$. The pole at $z = 1/2$ is cancelled by a zero.
   - ROC: Entire z-plane except $z = 0$.

EX2: Calculate Z-Transform

$x(n) = \begin{cases} n, & 0 \le n \le N-1 \\ 0, & \textbf{otherwise} \end{cases}$

- Use the differentiation property: $ng(n) \leftrightarrow -z\frac{dG(z)}{dz}$.
- Let $g(n)$ be a rectangular pulse from $n = 0$ to $N-1$. Its transform is $G(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1-z^{-N}}{1-z^{-1}}$.
- Differentiating $G(z)$:

$$\frac{dG(z)}{dz} = \frac{(Nz^{-N-1})(1-z^{-1}) - (1-z^{-N})(z^{-2})}{(1-z^{-1})^2}$$

- Multiplying by $-z$:

$$X(z) = -z\frac{dG(z)}{dz} = \frac{-Nz^{-N}(1-z^{-1}) + z^{-1}(1-z^{-N})}{(1-z^{-1})^2} = \frac{z^{-1} + (N-1)z^{-N-1} - Nz^{-N}}{(1-z^{-1})^2}$$

- ROC: The signal is finite-duration. The ROC is the entire z-plane except for poles at $z = 0$ and $z = 1$.

EX3: Calculate Z-Transform and Region of Convergence

1. $x(n) = a^{|n|}$, $0 < |a| < 1$
   - Decompose the signal: $x(n) = a^n u(n) + a^{-n}u(-n-1)$.
   - The transform is the sum of a causal part and an anti-causal part.
   - $Z\{a^n u(n)\} = \frac{1}{1-az^{-1}}$, ROC: $|z| > |a|$.
   - $Z\{a^{-n}u(-n-1)\} = Z\{(1/a)^n u(-n-1)\} = -\frac{1}{1-(1/a)z^{-1}}$, ROC: $|z| < |1/a|$.
   - $X(z) = \frac{1}{1-az^{-1}} - \frac{1}{1-a^{-1}z^{-1}} = \frac{(a-a^{-1})z^{-1}}{1-(a+a^{-1})z^{-1}+z^{-2}}$
   - The overall ROC is the intersection: $|a| < |z| < 1/|a|$.
2. $x(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \textbf{otherwise} \end{cases}$
   - This is a finite geometric series: $X(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1-z^{-N}}{1-z^{-1}}$
   - ROC: Entire z-plane except $z = 0$.

EX4: Find the Inverse Z-Transform

1. $X(z) = (1+2z)(1+3z^{-1})(1-z^{-1})$
   - Expand the polynomial: $X(z) = (1+2z)(1+2z^{-1}-3z^{-2}) = 1+2z^{-1}-3z^{-2}+2z+4-6z^{-1} = 2z+5-4z^{-1}-3z^{-2}$
   - Inverse transform term-by-term: $x(n) = 2\delta(n+1)+5\delta(n)-4\delta(n-1)-3\delta(n-2)$
2. $X(z) = \frac{1}{1+\frac{1}{2}z^{-1}}$, **ROC:** $|z| > 1/2$
   - This matches the causal form with $a = -1/2$.
   - $x(n) = (-\frac{1}{2})^n u(n)$
3. $X(z) = \frac{1}{1+\frac{1}{2}z^{-1}}$, **ROC:** $|z| < 1/2$
   - This matches the anti-causal form with $a = -1/2$.
   - $x(n) = -(-\frac{1}{2})^n u(-n-1)$
4. $X(z) = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$, **ROC:** $|z| > 1/2$
   - Factor the denominator: $1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2} = (1+\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})$.
   - Use partial fraction expansion: $X(z) = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{4}z^{-1}}$.
   - $A = \left.\frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{4}z^{-1}}\right|_{z^{-1}=-2} = 4$. $B = \left.\frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}\right|_{z^{-1}=-4} = -3$.
   - $X(z) = \frac{4}{1+\frac{1}{2}z^{-1}} - \frac{3}{1+\frac{1}{4}z^{-1}}$. Since ROC is $|z| > 1/2$, both terms are causal.
   - $x(n) = 4(-\frac{1}{2})^n u(n) - 3(-\frac{1}{4})^n u(n)$
5. $X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}}$, **ROC:** $|z| > 1/2$
   - Factor and cancel: $X(z) = \frac{1-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{1}{1+\frac{1}{2}z^{-1}}$.
   - Given ROC $|z| > 1/2$, the signal is causal.
   - $x(n) = (-\frac{1}{2})^n u(n)$
6. $X(z) = \frac{1-az^{-1}}{z^{-1}-a}$, **ROC:** $|z| > |1/a|$
   - Rewrite: $X(z) = \frac{1-az^{-1}}{-a(1-\frac{1}{a}z^{-1})}$.
   - Perform long division: $\frac{1-az^{-1}}{1-\frac{1}{a}z^{-1}} = a + \frac{1-a^2}{1-\frac{1}{a}z^{-1}}$.
   - $X(z) = -\frac{1}{a}\left[a + \frac{1-a^2}{1-\frac{1}{a}z^{-1}}\right] = -1 - \frac{1-a^2}{a}\frac{1}{1-\frac{1}{a}z^{-1}}$.
   - The ROC $|z| > |1/a|$ makes the second term causal.
   - $x(n) = -\delta(n) - \frac{1-a^2}{a}(\frac{1}{a})^n u(n)$

EX7: An LTI and Causal System

Given:

- $x(n) = u(-n-1) + (\frac{1}{2})^n u(n)$
- $Y(z) = \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})}$
- System is causal.

1. **Find H(z) and its ROC:**
   - First find $X(z)$: $X(z) = Z\{u(-n-1)\} + Z\{(\frac{1}{2})^n u(n)\} = \frac{-1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$.
   - Combining terms: $X(z) = \frac{-(1-\frac{1}{2}z^{-1})+(1-z^{-1})}{(1-z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{-\frac{1}{2}z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}$.
   - The ROC of X(z) is the intersection of $|z| < 1$ and $|z| > 1/2$, so $1/2 < |z| < 1$.
   - Now find $H(z) = Y(z)/X(z)$:

   $$H(z) = \frac{\frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})}}{\frac{-\frac{1}{2}z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}} = \frac{1-z^{-1}}{1+z^{-1}}$$

   - Since the system is causal, its ROC must be outside the outermost pole. The pole is at $z = -1$.
   - So, the ROC of H(z) is $|z| > 1$.
2. **What is the ROC of Y(z)?**
   - The ROC of the output is the intersection of the ROCs of the input and the system: $ROC(Y) = ROC(X) \cap ROC(H) = \{z \mid 1/2 < |z| < 1\} \cap \{z \mid |z| > 1\}$.
   - The intersection is the empty set, $\emptyset$.
3. **Calculate y(n):**
   - Since the ROC is the empty set, the Z-transform does not converge, and the output signal is zero for all n.
   - $y(n) = 0$.

EX8: Causal LTI System with Transfer Function

Given:

- $H(z) = \frac{1-z^{-1}}{1+\frac{3}{4}z^{-1}}$
- $x(n) = (\frac{1}{3})^n u(n) + u(-n-1)$

1. **Find h(n) and y(n):**
   - **h(n):** System is causal, pole at $z = -3/4$. ROC is $|z| > 3/4$. $H(z) = \frac{1}{1+\frac{3}{4}z^{-1}} - \frac{z^{-1}}{1+\frac{3}{4}z^{-1}}$. $h(n) = (-\frac{3}{4})^n u(n) - (-\frac{3}{4})^{n-1} u(n-1)$.

- **y(n):** First find $X(z)$ as in the previous problem: $X(z) = \frac{-\frac{2}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-z^{-1})}$, with ROC $1/3 < |z| < 1$.
- $Y(z) = H(z)X(z) = \frac{1-z^{-1}}{1+\frac{3}{4}z^{-1}} \cdot \frac{-\frac{2}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-z^{-1})} = \frac{-\frac{2}{3}z^{-1}}{(1+\frac{3}{4}z^{-1})(1-\frac{1}{3}z^{-1})}$.
- The ROC of Y(z) is $ROC(H) \cap ROC(X) = \{|z| > 3/4\} \cap \{1/3 < |z| < 1\} = \{3/4 < |z| < 1\}$.
- Partial fraction expansion: $Y(z) = \frac{A}{1+\frac{3}{4}z^{-1}} + \frac{B}{1-\frac{1}{3}z^{-1}}$. We find $A = 8/13$ and $B = -8/13$.
- $Y(z) = \frac{8/13}{1+\frac{3}{4}z^{-1}} - \frac{8/13}{1-\frac{1}{3}z^{-1}}$.
- Based on the ROC $3/4 < |z| < 1$: The first term (pole at $-3/4$) is causal. The second term (pole at $1/3$) is also causal.
- $y(n) = \frac{8}{13}(-\frac{3}{4})^n u(n) - \frac{8}{13}(\frac{1}{3})^n u(n)$.

2. **Is the system stable?**

   - The ROC of H(z) is $|z| > 3/4$. Since this region includes the unit circle $|z| = 1$, the system is stable.

EX9: Causal LTI System with Transfer Function

Given:

- $H(z) = \frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$ and system is causal.
- $y(n) = -\frac{1}{3}(-\frac{1}{4})^n u(n) - \frac{4}{3}(2)^n u(-n-1)$

1. **Find ROC of H(z):** System is causal, poles at $z = 1/2$ and $z = -1/4$. ROC is outside the outermost pole. **ROC of H(z) is** $|z| > 1/2$.
2. **Is the system stable?** Yes, the ROC $|z| > 1/2$ includes the unit circle $|z| = 1$.
3. **Find the Z-Transform of x(n):**
   - First find $Y(z)$ from $y(n)$. It is a sum of a causal and anti-causal part. $Y(z) = -\frac{1}{3}\frac{1}{1+\frac{1}{4}z^{-1}} + \frac{4}{3}\frac{1}{1-2z^{-1}}$. The ROC is $1/4 < |z| < 2$.
   - Combining terms gives $Y(z) = \frac{1+z^{-1}}{(1+\frac{1}{4}z^{-1})(1-2z^{-1})}$.
   - Find $X(z) = Y(z)/H(z)$:

$$X(z) = \frac{\frac{1+z^{-1}}{(1+\frac{1}{4}z^{-1})(1-2z^{-1})}}{\frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}} = \frac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}}$$

4. **Find h(n):**
   - Use partial fractions on $H(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{4}z^{-1}}$.
   - We find $A = 2$ and $B = -1$. So, $H(z) = \frac{2}{1-\frac{1}{2}z^{-1}} - \frac{1}{1+\frac{1}{4}z^{-1}}$.
   - Since the system is causal, $h(n) = 2(\frac{1}{2})^n u(n) - (-\frac{1}{4})^n u(n)$.

EX10: Find H(z) and ROC for an LTI system

Given:

- $x(n) = (\frac{1}{3})^n u(n) + 2^n u(-n-1)$
- $y(n) = 5(\frac{1}{3})^n u(n) - 5(\frac{2}{3})^n u(n)$

1. **Find H(z) and ROC:**
   - $X(z) = \frac{1}{1-1/3z^{-1}} - \frac{1}{1-2z^{-1}} = \frac{-5/3z^{-1}}{(1-1/3z^{-1})(1-2z^{-1})}$. ROC(X): $1/3 < |z| < 2$.
   - $Y(z) = \frac{5}{1-1/3z^{-1}} - \frac{5}{1-2/3z^{-1}} = \frac{-5/3z^{-1}}{(1-1/3z^{-1})(1-2/3z^{-1})}$. ROC(Y): $|z| > 2/3$.
   - $H(z) = Y(z)/X(z) = \frac{1-2z^{-1}}{1-2/3z^{-1}}$.
   - The ROC of H(z) must satisfy $ROC(X) \cap ROC(H) \subseteq ROC(Y)$. The only possibility is **ROC(H):** $|z| > 2/3$.
2. **Calculate h(n):** Since H(z) is causal, $h(n) = (\frac{2}{3})^n u(n) - 2(\frac{2}{3})^{n-1} u(n-1)$.
3. **Determine the Difference Equation:** From $Y(z)(1-2/3z^{-1}) = X(z)(1-2z^{-1})$, we get $y(n) - \frac{2}{3}y(n-1) = x(n) - 2x(n-1)$.
4. **Is the system stable, causal?** The system is **causal** (ROC is outside pole). It is **stable** because the ROC $|z| > 2/3$ includes the unit circle.

EX11: Causal LTI System with Transfer Function

Given:

- $H(z) = \frac{1+2z^{-1}+z^{-2}}{(1+\frac{1}{2}z^{-1})(1-z^{-1})}$ and system is causal.
- Input $x(n) = e^{j(\pi/2)n}$.

1. **Find h(n):**
   - $H(z) = \frac{(1+z^{-1})^2}{(1+\frac{1}{2}z^{-1})(1-z^{-1})}$. The degree of numerator and denominator are equal, so we can use long division or PFE with a constant term.
   - $H(z) = A + \frac{B}{1+1/2z^{-1}} + \frac{C}{1-z^{-1}}$.
   - $A = H(\infty) = -2$. $B = H(z)(1+1/2z^{-1})|_{z^{-1}=-2} = 1/3$. $C = H(z)(1-z^{-1})|_{z^{-1}=1} = 8/3$.
   - $H(z) = -2 + \frac{1/3}{1+1/2z^{-1}} + \frac{8/3}{1-z^{-1}}$.
   - Since the system is causal (ROC is $|z| > 1$), $h(n) = -2\delta(n) + \frac{1}{3}(-\frac{1}{2})^n u(n) + \frac{8}{3}(1)^n u(n)$.
2. **Calculate y(n):**
   - For an LTI system, a complex exponential input is an eigenfunction. $y(n) = H(e^{j\omega_0})x(n)$. Here $\omega_0 = \pi/2$.

- We need to evaluate $H(z)$ at $z = e^{j\pi/2} = j$.
- $H(j) = \frac{(1+j^{-1})^2}{(1+\frac{1}{2}j^{-1})(1-j^{-1})} = \frac{(1-j)^2}{(1-j/2)(1+j)}$.
- $H(j) = \frac{1-j}{1-j/2} = \frac{2(1-j)}{2-j} = \frac{2(1-j)(2+j)}{|2-j|^2} = \frac{2(2-j-j^2)}{5} = \frac{2(3-j)}{5} = \frac{6}{5} - \frac{2}{5}j$.
- $y(n) = (\frac{6}{5} - \frac{2}{5}j)e^{j(\pi/2)n}$.

EX15: Determine the ROC of H(z)

Rule: $ROC(Y) \supseteq ROC(X) \cap ROC(H)$. If no pole-zero cancellation, equality holds.

1. Given: $ROC(X) : |z| > 3/4$, $ROC(Y) : |z| > 2/3$. $H(z) = Y(z)/X(z) = \frac{1-3/4z^{-1}}{1+2/3z^{-1}}$. No pole-zero cancellation occurred. Thus, we must have $ROC(Y) = ROC(X) \cap ROC(H)$. $\{|z| > 2/3\} = \{|z| > 3/4\} \cap ROC(H)$. This equality is impossible, because $\{|z| > 3/4\}$ is a subset of $\{|z| > 2/3\}$, so their intersection cannot be equal to the larger set. The problem is stated incorrectly. **Invalid problem**.

2. Given: $ROC(X) : |z| < 1/3$, $ROC(Y) : 1/6 < |z| < 1/3$. $H(z) = Y(z)/X(z) = \frac{1}{1-1/6z^{-1}}$. We need $\{1/6 < |z| < 1/3\} = \{|z| < 1/3\} \cap ROC(H)$. This equality holds if and only if $ROC(H) = \{|z| > 1/6\}$. **ROC of H(z) is** $|z| > 1/6$.

EX16: LTI System Problem

Given: $h(n) = a^n u(n)$ and $x(n) = u(n) - u(n-N)$.

1. **Calculate y(n) using convolution:** $y(n) = \sum_{k=0}^{N-1} h(n-k) = \sum_{k=0}^{N-1} a^{n-k} u(n-k)$.
   - For $n < 0$: $y(n) = 0$.
   - For $0 \le n < N$: $y(n) = \sum_{k=0}^{n} a^{n-k} = \frac{1-a^{n+1}}{1-a}$.
   - For $n \ge N$: $y(n) = \sum_{k=0}^{N-1} a^{n-k} = a^{n-(N-1)} \frac{1-a^N}{1-a}$.

2. **Calculate y(n) using Z-transform:** $Y(z) = H(z)X(z) = \frac{1}{1-az^{-1}} \cdot \frac{1-z^{-N}}{1-z^{-1}}$. Let $G(z) = \frac{1}{(1-az^{-1})(1-z^{-1})} = \frac{1/(1-a)}{1-az^{-1}} - \frac{1/(1-a)}{1-z^{-1}}$. $g(n) = \frac{1}{1-a}(a^n - 1)u(n)$. $y(n) = g(n) - g(n-N) = \frac{1}{1-a}[(a^n - 1)u(n) - (a^{n-N} - 1)u(n-N)]$.

EX17: LTI System Problem

Given: $H(z) = \frac{3}{1+\frac{1}{3}z^{-1}}$ and input is a unit impulse, $x(n) = \delta(n)$.

- **Using convolution:** $y(n) = x(n) * h(n) = \delta(n) * h(n) = h(n)$. We need to find $h(n)$. Assuming a causal system, ROC is $|z| > 1/3$. Then $h(n) = 3(-\frac{1}{3})^n u(n)$. So, $y(n) = 3(-\frac{1}{3})^n u(n)$.

- **Using Z-transform:** $X(z) = 1$. $Y(z) = H(z)X(z) = H(z)$. $Y(z) = \frac{3}{1+\frac{1}{3}z^{-1}}$. The inverse transform gives $y(n) = 3(-\frac{1}{3})^n u(n)$ (assuming causality).

EX18: LTI System Problem

Given: $H(z) = \frac{1-\frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$, ROC: $|z| > 1/2$.

1. **Find the impulse response h(n):**
   - Partial fraction expansion: $H(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}}$.
   - $A = \frac{1-1/2z^{-2}}{1-1/4z^{-1}}\Big|_{z^{-1}=2} = -2$.
   - $B = \frac{1-1/2z^{-2}}{1-1/2z^{-1}}\Big|_{z^{-1}=4} = 7$.
   - $H(z) = \frac{-2}{1-\frac{1}{2}z^{-1}} + \frac{7}{1-\frac{1}{4}z^{-1}}$. Since ROC is $|z| > 1/2$, both terms are causal.
   - $h(n) = -2(\frac{1}{2})^n u(n) + 7(\frac{1}{4})^n u(n)$.
2. **Find the difference equation:**
   - $H(z) = \frac{Y(z)}{X(z)} = \frac{1-1/2z^{-2}}{1-3/4z^{-1}+1/8z^{-2}}$.
   - Cross-multiply: $Y(z)(1 - 3/4z^{-1} + 1/8z^{-2}) = X(z)(1 - 1/2z^{-2})$.
   - Inverse transform: $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) - \frac{1}{2}x(n-2)$.

EX19: Find Z-Transform and ROC

1. $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-4k)$
   - This is an impulse train. Its Z-transform is $X(z) = \sum_{k=-\infty}^{\infty} z^{-4k}$.
   - This two-sided geometric series only converges if $|z^{-4}| = 1$.
   - ROC: $|z| = 1$. The transform does not converge in a region, only on the unit circle.
2. $x(n) = \frac{1}{2}(e^{j\pi n} + \cos(\frac{\pi}{2}n) + \sin(\frac{\pi}{2} + 2\pi n))u(n)$
   - Simplify: $\sin(\pi/2 + 2\pi n) = 1$.
   - $x(n) = [\frac{1}{2}(-1)^n + \frac{1}{4}(e^{j\pi/2})^n + \frac{1}{4}(e^{-j\pi/2})^n + \frac{1}{2}(1)^n]u(n)$.
   - This is a sum of four causal exponential terms. The poles are at $-1, e^{j\pi/2}, e^{-j\pi/2}, 1$. All have magnitude 1.
   - The ROC is outside the outermost pole. ROC: $|z| > 1$.
   - $X(z) = \frac{1/2}{1+z^{-1}} + \frac{1/4}{1-e^{j\pi/2}z^{-1}} + \frac{1/4}{1-e^{-j\pi/2}z^{-1}} + \frac{1/2}{1-z^{-1}}$.

EX20: Find the Inverse Z-Transform

$X(z) = \ln(1-2z)$**, ROC:** $|z| < 1/2$

- Use the Maclaurin series for logarithm: $\ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}$ for $|x| < 1$.
- With $x = 2z$: $X(z) = -\sum_{k=1}^{\infty} \frac{(2z)^k}{k} = -\sum_{k=1}^{\infty} \frac{2^k}{k} z^k$.

- The Z-transform definition is $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$. Let $k = -n$.
- $X(z) = -\sum_{n=-\infty}^{-1} \frac{2^{-n}}{-n} z^{-n} = \sum_{n=-\infty}^{-1} \frac{2^{-n}}{n} z^{-n}$.
- By comparing coefficients, we get:

$$x(n) = \begin{cases} \frac{2^{-n}}{n}, & n \leq -1 \\ 0, & n \geq 0 \end{cases}$$

EX21: A signal x(n) has the following poles and zeros

- **From the plot:** $X(z)$ has one pole at $p = 1/2$ and two zeros at $z = \pm j$.
- **New signal:** $y(n) = (\frac{1}{2})^n x(n)$.
- **Scaling Property:** $a^n x(n) \leftrightarrow X(z/a)$.
- **Z-Transform of y(n):** With $a = 1/2$, we have $Y(z) = X(z/(1/2)) = X(2z)$.
- This means all pole and zero locations of $X(z)$ are scaled by $a = 1/2$.
  - **New pole of Y(z):** $p' = p \cdot a = (1/2) \cdot (1/2) = 1/4$.
  - **New zeros of Y(z):** $z' = (\pm j) \cdot a = \pm j/2$.
- **Pole-Zero Plot of Y(z):**
  - A pole ('x') at position 1/4 on the real axis.
  - Two zeros ('o') at positions $+j/2$ and $-j/2$ on the imaginary axis.