HUST

ĐẠI HỌC BÁCH KHOA HÀ NỘI
HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

DATA STRUCTURES AND ALGORITHMS

ĐẠI HỌC BÁCH KHOA HÀ NỘI
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DATA STRUCTURES AND ALGORITHMS

WEEK 14 : GRAPH (PART 1)

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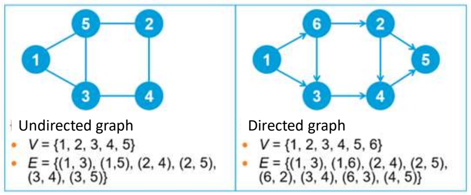
CONTENT

- Recall the basic concepts of graphs
- Graph representation data structure
- Graph traversal
- Find connected components
- Check bipartite graphs

RECALL THE BASIC CONCEPTS OF GRAPHS

- A graph is a structure consisting of nodes (also called vertices) and connections between nodes (also called edges or arcs).
- Graph notation: $G = (V, E)$, where V is a set of nodes and E is a set of edges (arcs)
- $(u,v) \in E$: *v is adjacent to u.*



RECALL THE BASIC CONCEPTS OF GRAPHS

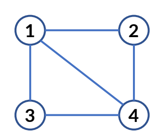
- Given $G = (V, E)$ is a graph
  - The path from vertex s to vertex t on the graph is a sequence of vertices $v_1, v_2, \ldots, v_k$, *where*
    - $s = v_1$, $t = v_k$
    - $(v_i, v_{i+1}) \in E$
  - A cycle is a path in which the first and last vertices coincide
  - An undirected graph G is called connected if there is always a path between any two vertices of G

GRAPH REPRESENTATION DATA STRUCTURE

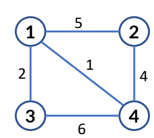
- **Adjacency matrix, weight matrix**
- **Adjacency list**
- **Edge list**

GRAPH REPRESENTATION DATA STRUCTURE

- **Adjacency matrix, weight matrix**



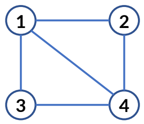
	1	2	3	4
1	0	1	1	1
2	1	0	0	1
3	1	0	0	1
4	1	1	1	0



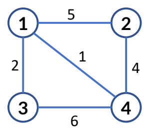
	1	2	3	4
1	0	5	2	1
2	5	0	0	4
3	2	0	0	6
4	1	4	6	0

GRAPH REPRESENTATION DATA STRUCTURE

- **Adjacency list**
  - **A[v]: list of vertices (or edges/arcs for weighted graphs) adjacent to vertex v**



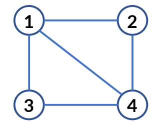
- A[1] = [2, 3, 4]
- A[2] = [1, 4]
- A[3] = [1, 4]
- A[4] = [1, 2, 3]



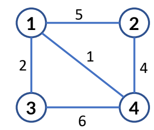
- A[1] = [(1,2,5), (1,3,2), (1,4,1)]
- A[2] = [(1,2,5), (2,4,4)]
- A[3] = [(1,3,2), (3,4,6)]
- A[4] = [(1,4,1), (2,4,4), (3,4,6)]

GRAPH REPRESENTATION DATA STRUCTURE

- **Edge list**
  - **E: list of edges/arcs of the graph**



Edge list: E = [(1,2), (1,3), (1,4), (2, 4), (3,4)]



List of edges with weights on edges
E = [(1,2, 5), (1,3, 2), (1,4, 1), (2, 4, 4), (3,4, 6)]

GRAPH TRAVERSAL

- Traverse the graph in depth: visit the vertices of the graph, each vertex exactly once
- A[v]: list of vertices adjacent to v

```
DFS(u, A) {// depth first search from u
  visited[u] = true; //Visit u;
  for v in A[u] do {
     if visited[v] = false then {
        DFS(v, A);
     }
  }
}
```

```
DFS(G = (V, A)){ // depth first search on G
  for v in V do visited[v] = false;
  for v in V do {
    if visited[v] = false then {
       DFS(v, A);
    }
  }
}
```


GRAPH TRAVERSAL

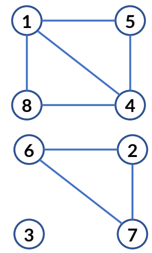
- Traverse the graph breadthwise: visit the vertices of the graph, each vertex exactly once
- A[v]: list of vertices adjacent to v

```
BFS(G = (V, A)){ // breadth first search on G
  for v in V do visited[v] = false;
  for v in V do {
    if visited[v] = false then {
       BFS(v, A);
    }
  }
}
```

```
BFS(u, A) {// breadth first search from u
  Q = Empty;
  Q.push(u);  visited[u] = true; // visit u
  while Q not empty do {
    v = Q.pop();
    for x in A[v] do
      if visited[x] = false then {
         Q.push(x); visited[x] = true; // visit x
      }
    }
}
```

FIND CONNECTED COMPONENTS

- Description of the problem
  - Given an undirected graph G = (V, A) in it
    - V is the set of vertices
    - A is an adjacency list structure: A[v] is a list of vertices adjacent to v
  - Need to find connected components of G
- Algorithm:
  - Apply depth-first search (or breadth-first search) on G
  - DFS(u) allows visiting all vertices belonging to the same connected component with u



FIND CONNECTED COMPONENTS

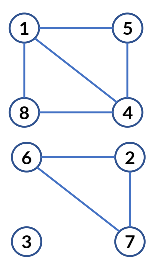
- nbCC: number of connected components of G
- C[v]: index (running from 1 to nbCC) of the connected component containing vertex v

```
DFS(u, A) {// DFS from u
  visited[u] = true; //Visit nh u;
  C[u] = nbCC;
  for v in A[u] do {
     if visited[v] = false then {
        DFS(v, A);
     }
  }
}
```

```
DFS(G = (V, A)){ // DFS on G
  for v in V do visited[v] = false;
  nbCC = 0;
  for v in V do {
    if visited[v] = false then {
       nbCC = nbCC + 1;
       DFS(v, A);
    }
  }
}
```


FIND CONNECTED COMPONENTS

- Illustrate with C language
- Data
  - Line 1: contains two positive integers n and m representing the number of vertices and edges, respectively
  - Line i + 1 (i = 1, 2, ..., m): contains 2 positive integers u and v which are the 2 endpoints of the ith edge
- Result
  - Write a list of vertices of each connected component found on one line



stdin	stdout
8 8 1 4 1 5 1 8 2 6 2 7 4 5 4 8 6 7	C[1]: 1 4 5 8 C[2]: 2 6 7 C[3]: 3

FIND CONNECTED COMPONENTS

```
#include <stdio.h>
#define N 100001
typedef struct Node{
  int id;
  struct Node* next;
}Node;
int n,m; // #nodes and #edges of G
Node* A[N]; // A[v]: pointer to an adj list
int nbCC; // #connected components
int C[N]; // C[v]: connected component
         //containing v
```

```
Node* makeNode(int id){
         Node* p = (Node*)malloc(sizeof(Node));
         p->id = id; p->next = NULL;
         return p;
}
Node* insert(int id, Node* h){
         Node* p = makeNode(id);
         p->next = h;
         return p;
}
```

FIND CONNECTED COMPONENTS

```
void input(){
  scanf("%d %d",&n,&m);
  for(int v = 1; v <= n; v++) A[v] = NULL;
  for(int i = 1; i <= m; i++){
    int u,v; scanf("%d%d",&u,&v);
    A[u] = insert(v,A[u]);
    A[v] = insert(u,A[v]);
  }
}
```

```
void DFS(int u){
  C[u] = nbCC;
  for(Node* p = A[u]; p != NULL; p = p->next){
    int v = p->id;
    if(C[v] == -1){
      DFS(v);
    }
  }
}
```


FIND CONNECTED COMPONENTS

```
void DFSG(){
  for(int u = 1; u <= n; u++) C[u] = -1;
  nbCC = 0;
  for(int u = 1; u <= n; u++){
    if(C[u] == -1){
      nbCC = nbCC + 1;
      DFS(u);
    }
  }
}
```

```
void printCC(){
  for(int k = 1; k <= nbCC; k++){
    printf("C[%d]: ",k);
    for(int v = 1; v <= n; v++){
      if(C[v] == k) printf("%d ",v);
    }
    printf("\n");
  }
}
```


FIND CONNECTED COMPONENTS

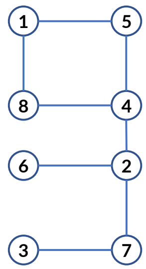
```
int main(){
  input();
  DFSG();
  printCC();
  return 0;
}
```


CHECK BIPARTITE GRAPHS

- Problem description
  - Given an undirected graph G = (V, A) in it
    - V is the set of vertices
    - A is an adjacency list structure: A[v] is a list of vertices adjacent to v
  - Check if G is a bipartite graph or not?

CHECK BIPARTITE GRAPHS

- Data
  - Line 1: contains two positive integers n and m representing the number of vertices and edges, respectively
  - Line i + 1 (i = 1, 2, ..., m): contains 2 positive integers u and v which are the 2 endpoints of the Ith edge
- Result
  - Write 1 if the graph is bipartite and 0 otherwise



stdin	stdout
8 8 1 5 1 8 2 4 2 6 2 7 3 7 4 5 4 8	1

CHECK BIPARTITE GRAPHS

- Algorithm:
  - Apply breadth-first search on G
  - d[v]: level (path length from starting vertex to v in BFS) of vertex v
  - BFS(u): visit all vertices with the same connected component with u
    - If the connected component containing u is a bipartite graph, then vertices v with even d[v] will be on the side containing u, and vertices v with odd d[v] will be on the other side.
    - If it is detected that edge (u,v) has d[u] and d[v] with the same parity, then the given graph is not a bipartite graph.

CHECK BIPARTITE GRAPHS - PSEUDOCODE

```
BFS(u) {
  Q = empty queue;  Q.push(u);  d[u] = 0;
  while Q not empty do {
    v = pop();
    for x in A[v] do {
      if(d[x] > -1){
          if  d[v] + d[x]  is even then return false;
      }
      else {  d[x] = d[v] + 1;   Q. push(x);  }
    }
  }
  return true;
}
```

```
solve(){
  for u = 1 to n do  d[u] = -1;
  for u = 1 to n do if(d[u] = -1){
    if(BFS(u) = false) {
       return false;
    }
  }
  return true;
}
```


CHECK BIPARTITE GRAPHS – COMPLETE CODE

```
#include <stdio.h>
#include <stdlib.h>
#define N 100001
typedef struct Node{
  int id;
  struct Node* next;
}Node;
Node* makeNode(int id){
  Node* p = (Node*)malloc(sizeof(Node));
  p->id = id; p->next = NULL;
  return p;
}
```

```
Node* head;
Node* tail;
void initQueue(){
  head = NULL; tail = NULL;
}
int queueEmpty(){
  return head == NULL && tail == NULL;
}
void push(int id){
  Node* p = makeNode(id);
  if(queueEmpty()){ head = p; tail = p; }
  else { tail->next = p; tail = p; }
}
```

CHECK BIPARTITE GRAPHS – COMPLETE CODE

```
int pop(){
  if(queueEmpty()) return -1;
  int r = head->id;  Node* tmp = head;
  head = head->next;
  if(head == NULL) tail = NULL;
  free(tmp);
  return r;
}
Node* add(int id, Node* h){
  Node* p = makeNode(id);
  p->next = h;  return p;
}
```

```
int n,m;
Node* A[N];
int d[N];// d[v] is the level of d
void input(){
  scanf("%d %d",&n,&m);
  for(int v = 1; v <= n; v++) A[v] = NULL;
  for(int i = 1; i <= m; i++){
     int u,v;
     scanf("%d%d",&u,&v);
     A[u] = add(v,A[u]);  A[v] = add(u,A[v]);
  }
}
```

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CHECK BIPARTITE GRAPHS – COMPLETE CODE

```
int BFS(int u){
  initQueue();  push(u);  d[u] = 0;
  while(!queueEmpty()){
    int v = pop();
    for(Node* p = A[v]; p != NULL; p = p->next){
       int x = p->id;
       if(d[x] > -1){ if(d[v] % 2 == d[x] % 2) return 0;  }
       else{  d[x] = d[v] + 1;   push(x);  }
    }
  }
  return 1;
}
```

```
void solve(){
  for(int v = 1; v <= n; v++)  d[v] = -1;
  int ans = 1;
  for(int v= 1; v <= n; v++) if(d[v]== -1){
    if(!BFS(v)){  ans = 0; break;  }
  }
  printf("%d",ans);
}
int main(){
  input();
  solve();
  return 0;
}
```

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THANK YOU !