Introduction to Communications Engineering

IT4593E

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ONE LOVE. ONE FUTURE.

Thông tin chung

- Tên học phần: **Nhập môn kỹ thuật truyền thông**
- Mã học phần: **IT4593E**
- Khối lượng: **2 TC (2-1-0-4)**
- Lý thuyết và bài tập: **10 buổi lý thuyết, 5 buổi bài tập**
- Đánh giá học phần:

  30% QT (kiểm tra + bài tập/project + chuyên cần-quiz)

  70% CK (trắc nghiệm + tự luận)
- Tài liệu tham khảo:
  - Lecture slides
  - Lecture notes
  - Textbooks, ví dụ ***Communication Systems Engineering***, 2nd Edition, by John G. Proakis Masoud Salehi
  - Internet

Lec 04: Decision Theory
4.3 Reciever

Signal Space Receiver

Assume the received signal is $\rho(t)$ with $0 \le t < T$, the receiver needs to:

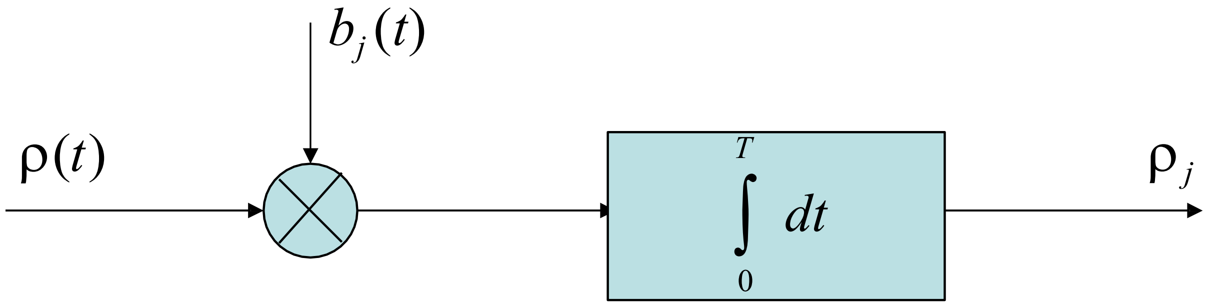
1. Compute d projections $$\rho_j = \int_0^T \rho(t) b_j(t) dt$$

2. Given the received vector $\underline{\rho} = (\rho_1, \ldots, \rho_j, \ldots, \rho_d)$ choose $\underline{s_R} \in M$ according to the ML criterion (minimum distance or Voronoi)

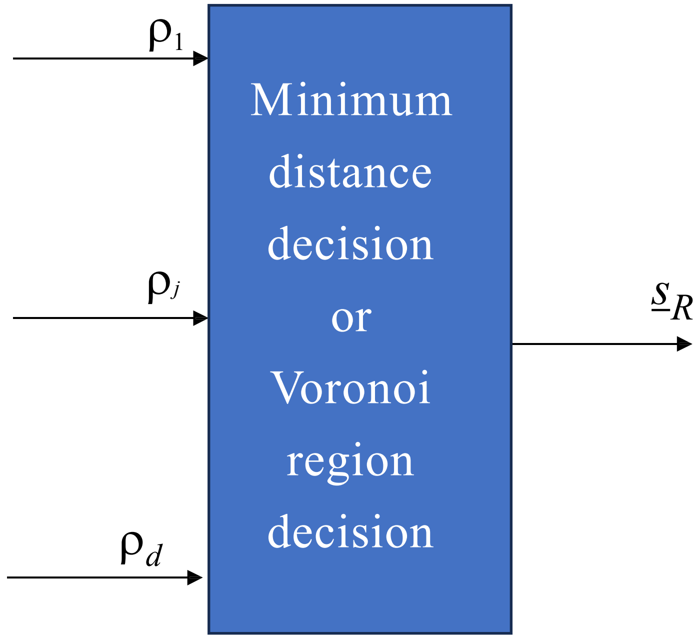
3. With $\underline{s_R}$, recover the binary information vector $\underline{u_R}$ via the inverse mapping: $\underline{u_R} = e^{-1}(\underline{s_R})$

Signal Space Receiver (with integrators)

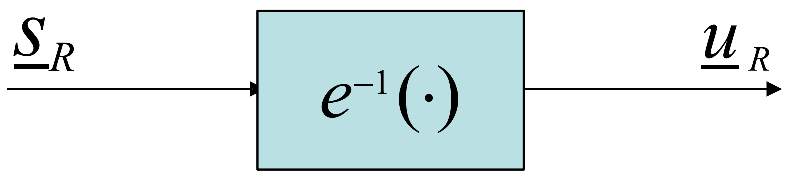
1. Given $\rho(t)$ compute d projections $\rho_j = \int_0^T \rho(t) b_j(t) dt$



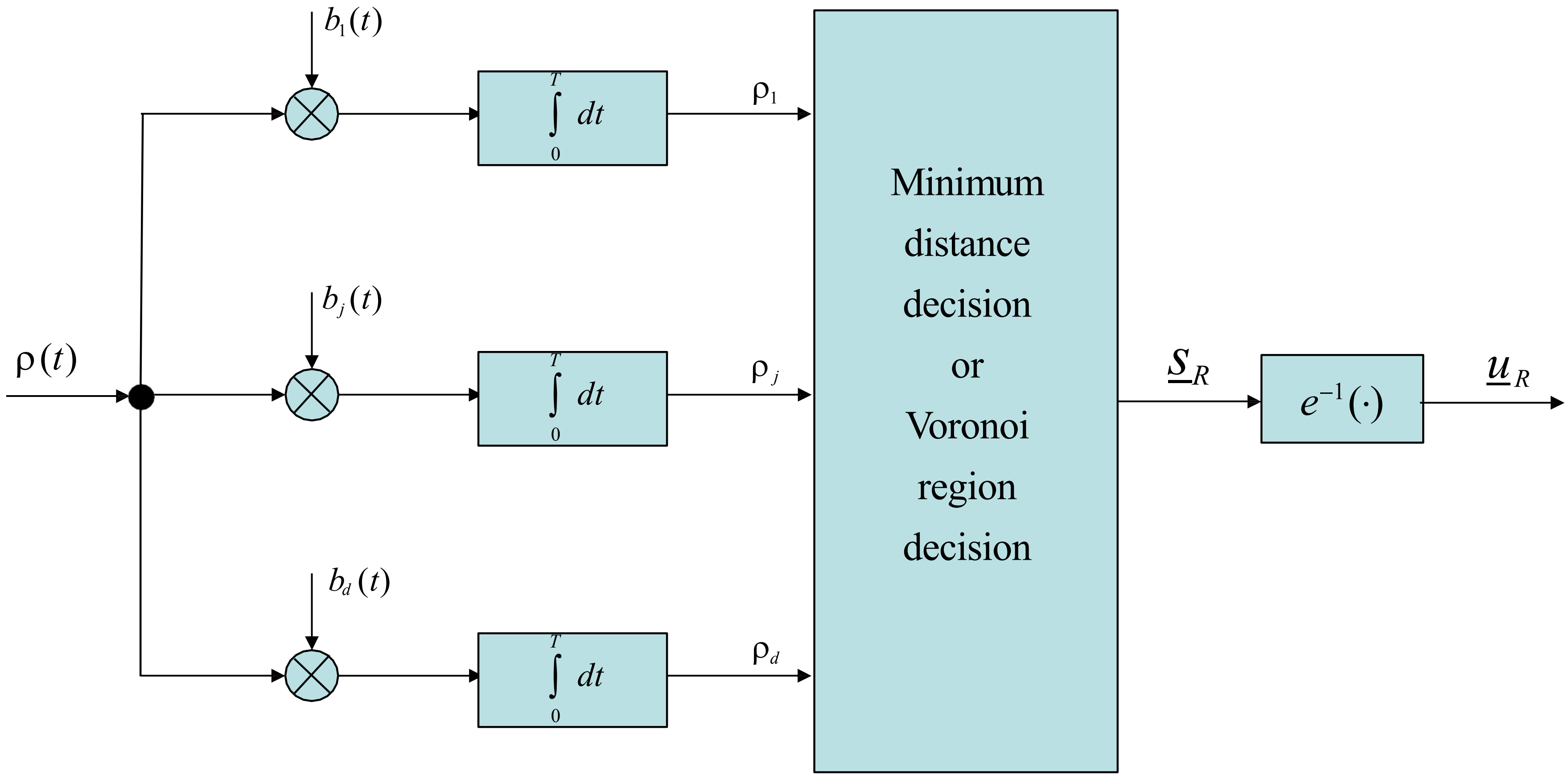
2. After obtaining $\underline{\rho}=(\rho_1,\ldots,\rho_j,\ldots\rho_d)$ apply the ML criterion to choose: $\underline{s}_R \in M$



3. Given $\underline{s}_R$, recover $\underline{u}_R$ via the inverse mapping:



Reciever's Complete Block Diagram



Matched Filter

A filter with impulse response: $h(t)$

The output signal $y(t)$ is determined by the input signal $x(t)$ as follows:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Assume:

- The filter input signal is the received signal $\rho(t)$
- The impulse response is: $h(t) = b_j(T - t)$

→ We have the **Matched Filter (MF)**

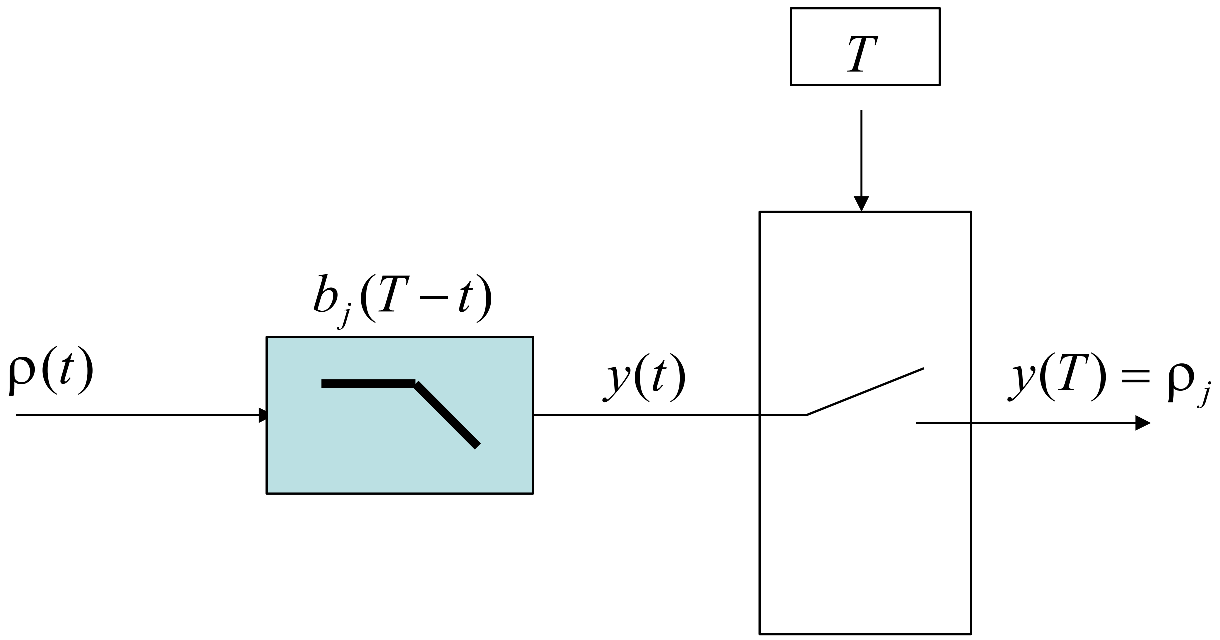
The output of the matched filter is:

$$y(t) = \int_{-\infty}^{+\infty} \rho(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} \rho(\tau)\, b_j(T-t+\tau)\, d\tau$$

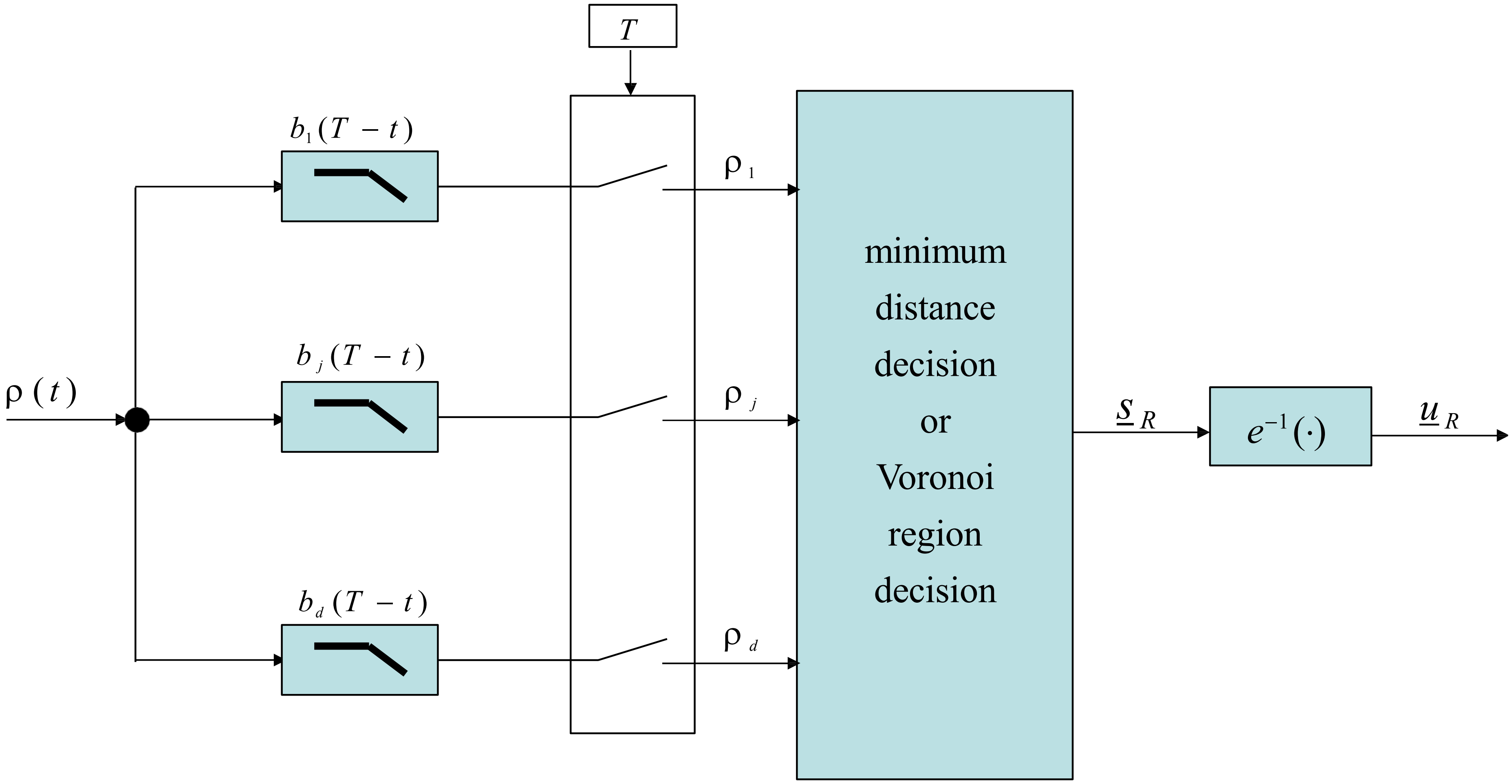
Assume sampling the output signal at time $t = T$

$$y(t=T) = \int_{-\infty}^{+\infty} \rho(\tau)\, b_j(\tau)\, d\tau = \int_{0}^{T} \rho(\tau)\, b_j(\tau)\, d\tau = \rho_j$$

Using the MF provides an alternative method to compute the projections $b_j(t)$ instead of using integrators → **Simpler**



Reciever’s Complete Block Diagram



Complete Receiver

Until now, we have focused on the first cycle: [0,T]

- Signal space $M = \{ s_1(t) , \ldots , s_i(t), \ldots, s_m(t) \}$ built by signals defined on the time domain $[0,T]$

- Orthonormal basis $B = \{ b_1(t) , \ldots , b_j(t), \ldots, b_d(t) \}$ built by signals defined on the time domain $[0,T]$

- Projections are defined: $$\rho_j = \rho_j[0] = \int_0^T \rho(t) b_j(t) dt$$

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Complete Receiver

How to handle other cycles? For example, the second cycle $[T,2T]$**?**

The signals used in computation in this cycle are similar to signals in space M, but shifted by T.

Similar to using the signal space

$$M' = \{ s'_1(t) , \; \ldots , \; s'_i(t), \; \ldots, \; s'_m(t) \},$$

which consists of signals in the domain $[T,2T]$ defined as follows:

$$s'_i(t) = s'_i(t - T)$$

In the second cycle $[T,2T]$

- Signal space is $M' = \{ s'_1(t), \ldots, s'_i(t), \ldots, s'_m(t) \}$ with:

$$s'_i(t) = s_i(t-T)$$

- Orthonormal basis $B' = \{ b'_1(t), \ldots, b'_j(t), \ldots, b'_d(t) \}$

with: $b'_i(t) = b_i(t-T)$

- Projections: $$\rho_j[1] = \int_T^{2T} \rho(t) b_j(t) dt$$

In any cycle $[nT,(n+1)T]$

- Signal space is $M' = \{ s'_1(t) , \ldots , s'_i(t), \ldots, s'_m(t) \}$ with:

$$s'_i(t) = s_i(t - nT)$$

- Corresponding orthonormal basis

$$B' = \{ b'_1(t) , \ldots , b'_j(t), \ldots, b'_d(t) \}$$

with: $b'_i(t) = b_i(t - nT)$

- Projections are computed as follows:

$$\rho_j[n] = \int_{nT}^{(n+1)T} \rho(t) b_j(t) dt$$

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With the outputs of the matched filter

$$y(t) = \int_{-\infty}^{+\infty} \rho(\tau)\, f(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} \rho(\tau)\, b_j(T-t+\tau)\, d\tau$$

Take the value at time $t=(n+1)T$

$$y(t=(n+1)T) = \int_{-\infty}^{+\infty} \rho(\tau)\, b_j(\tau - nT)\, d\tau = \int_{nT}^{(n+1)T} \rho(\tau)\, b_j(\tau - nT)\, d\tau = \rho_j[n]$$

The matched filter allows computing the projections: $\rho_j[n]$

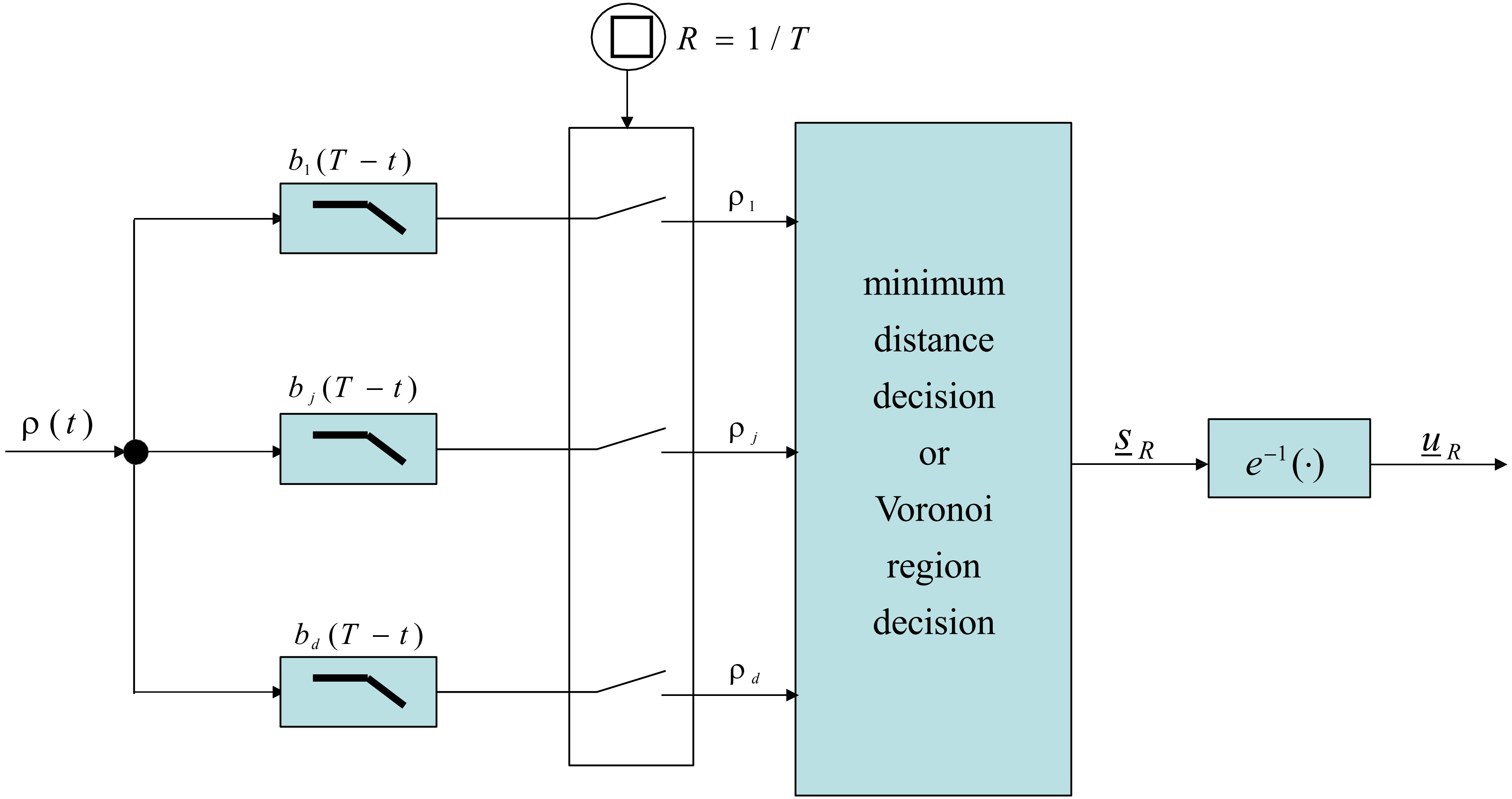
Not only for the first cycle but for any cycle $[nT,(n+1)T]$

→ What we need to do is:

to sample the filter output:

- At the rate $R=1/T$
- At $t=(n+1)T$

Complete Receiver with MF

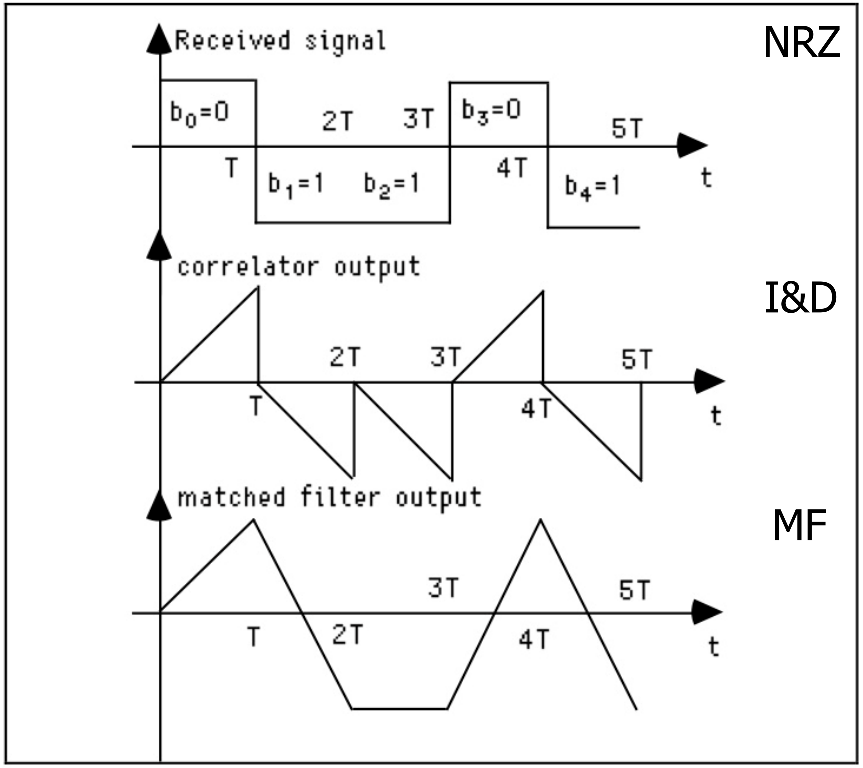


Symbol Synchronization - or the timing for sampling from the MF

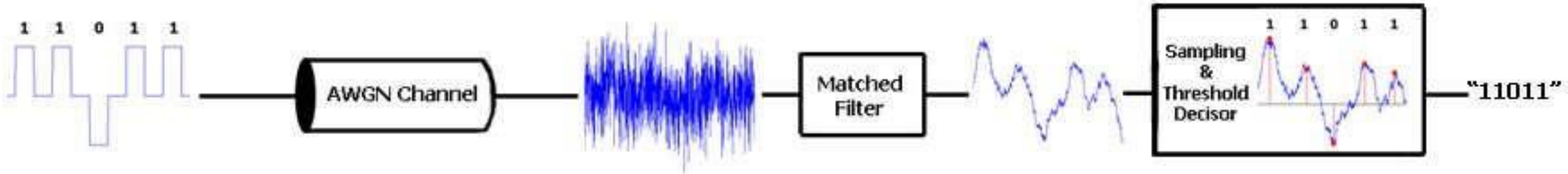
- A binary data stream is characterized by the bit rate: R_b.
- Each signal belonging to the signal space corresponds to k bits and exists in the time domain $T=kT_b$.
- The symbols (e.g., A=00, B=01, C=10, D=11, with k=2) are transmitted at the rate: $R=1/T=R_b/k$ (symbol rate).
- At the receiver, the filter output must be sampled at the same rate R.

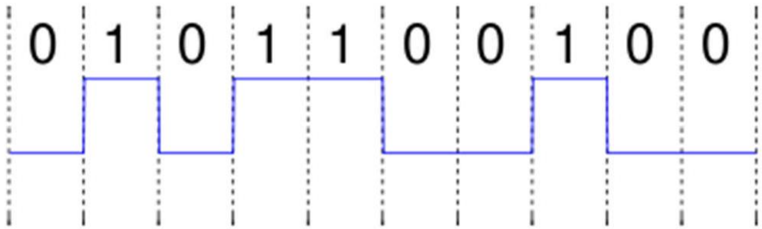
Note: The nominal value of R is known, but the actual value is not exactly that (due to physical factors)

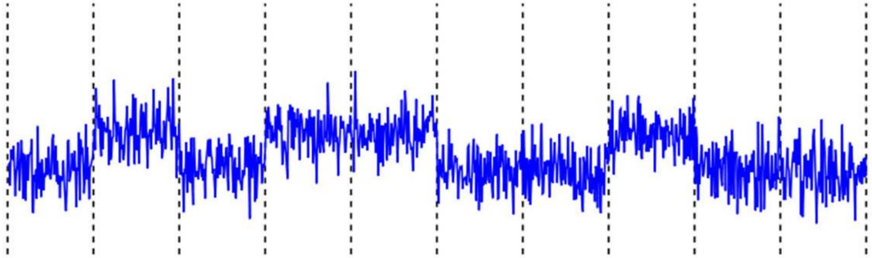
- In practice, it is very difficult for oscillators at the transmitter and receiver to have identical frequencies R: therefore we need to recover R from the signal.
- The MF output must be sampled exactly at $t=(n+1)T$: timing phase (time reference) must be recovered.
- **Concept of symbol synchronization: Starting from the received signal, the symbol rate and its phase must be accurately recovered.**
- This is important for accurately computing the projections and detecting the transmitted signal.

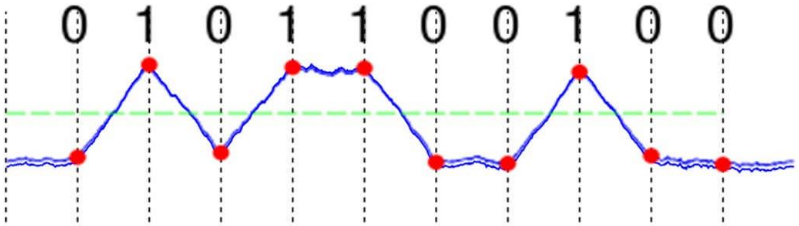


I&D: integrate-and-dump filter, a.k.a, correlator, correlation filter


1 1 0 1 1
AWGN Channel
Matched
Filter
Sampling
&
Threshold
Decisor
1 1 0 1 1
"11011"


0 1 0 1 1 0 0 1 0 0




0 1 0 1 1 0 0 1 0 0

Correlation Receiver

Starting from the Euclidean distance criterion:

$$\underline{s}_R = \arg\min_{\underline{s}_i \in M} d_E^2(\underline{\rho}, \underline{s}_i)$$

We have:

$$d_E^2(\underline{\rho}, \underline{s}_i) = \sum_{j=1}^{d} (\rho_j - s_{ij})^2 = \sum_{j=1}^{d} \rho_j^{\ 2} + \sum_{j=1}^{d} s_{ij}^{\ 2} - 2\sum_{j=1}^{d} \rho_j s_{ij}$$

We obtain:

$$\underline{s}_R = \arg\min_{\underline{s}_i \in M} d_E^2(\underline{\rho} - \underline{s}_i) = \arg\min_{\underline{s}_i \in M} \left[\sum_{j=1}^{d} \rho_j^{\ 2} + \sum_{j=1}^{d} s_{ij}^{\ 2} - 2\sum_{j=1}^{d} \rho_j s_{ij} \right]$$

$$\underline{s}_R = \arg\min_{\underline{s}\in M} \left[\sum_{j=1}^{d} s_j^{\,2} - 2\sum_{j=1}^{d} \rho_j s_{ij}\right] = \arg\max_{\underline{s}\in M} \left[\sum_{j=1}^{d} \rho_j s_{ij} - \frac{1}{2}\sum_{j=1}^{d} s_{ij}^{\,2}\right]$$

Since: $$E(s_i) = \sum_{j=1}^{d} s_{ij}^{\,2}$$

We have: $$\underline{s}_R = \arg\max_{\underline{s}_i\in M} \left[\sum_{j=1}^{d} \rho_j s_{ij} - \frac{1}{2} E(s_i)\right]$$

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$$\underline{S}_R = \arg\max_{\underline{s}_i \in M} \left[\sum_{j=1}^{d} \rho_j s_{ij} - \frac{1}{2} E(s_i) \right]$$

Note that

$$\int_0^T \rho(t)\, s_i(t)\, dt = \int_0^T \rho(t) \left[\sum_{j=1}^{d} s_{ij} b_j(t) \right] dt = \sum_{j=1}^{d} s_{ij} \int_0^T \rho(t)\, b_j(t)\, dt = \sum_{j=1}^{d} s_{ij} \rho_j$$

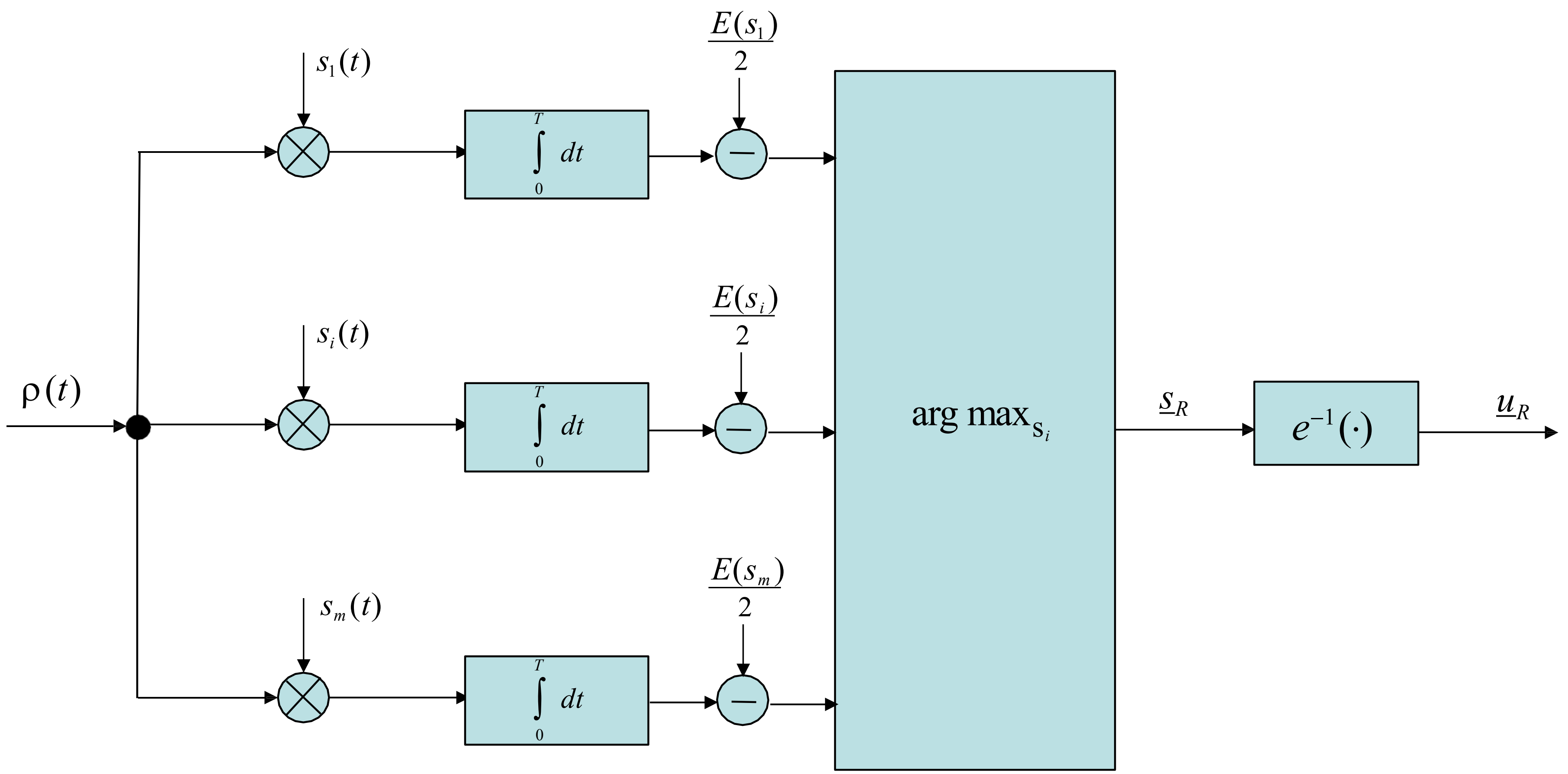
is the **correlation value** between the received signal and the signal in space M, $s_i(t)$:

$$\int_0^T \rho(t)\, s_i(t)\, dt$$

Therefore, the ML criterion based on computing the correlation value is:

$$\underline{s_R} = \arg\max_{\underline{s}_i \in M} \left[\int_0^T \rho(t)\, s_i(t)\, dt - \frac{1}{2} E(s_i) \right]$$

Correlation receiver



Comparison of the two receiver types

Receiver using MF

- d filters
- one decision unit based on Euclidean distance

Correlation receiver has:

- m integrators ($m>d$)
- one decision unit based on maximum value (max decisor)

Exercise

$$M = \{s_1(t) = P_T(t), s_2(t) = -P_T(t)\}$$

1. Draw the transmitted waveform for the bit sequence $\underline{u}_T$=101010….

2. Determine the matched filter

3. Draw the matched filter output (in the noiseless case)

4. Verify the sampled output value of the MF (at $t=(n+1)T$) with the transmitted symbols

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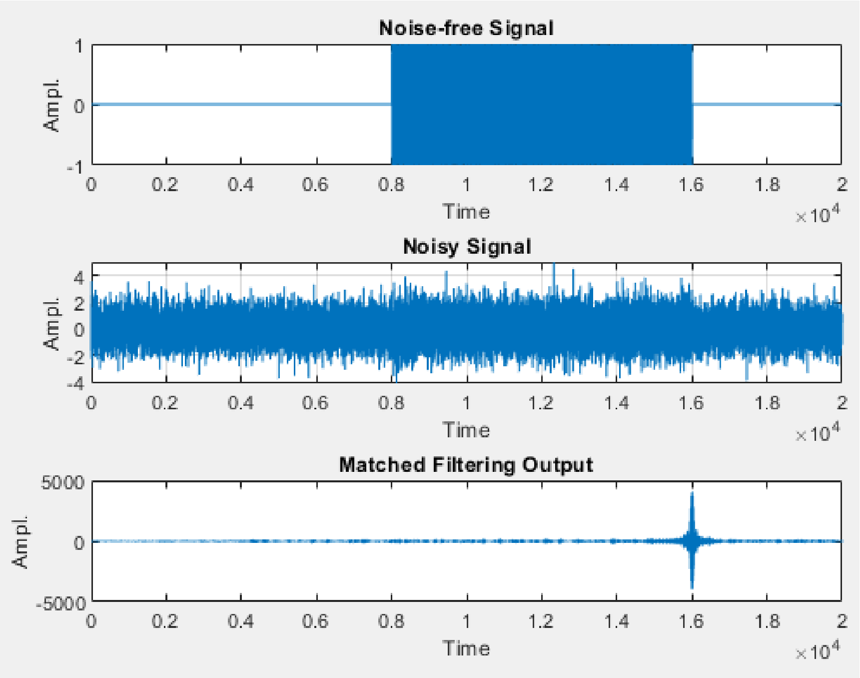
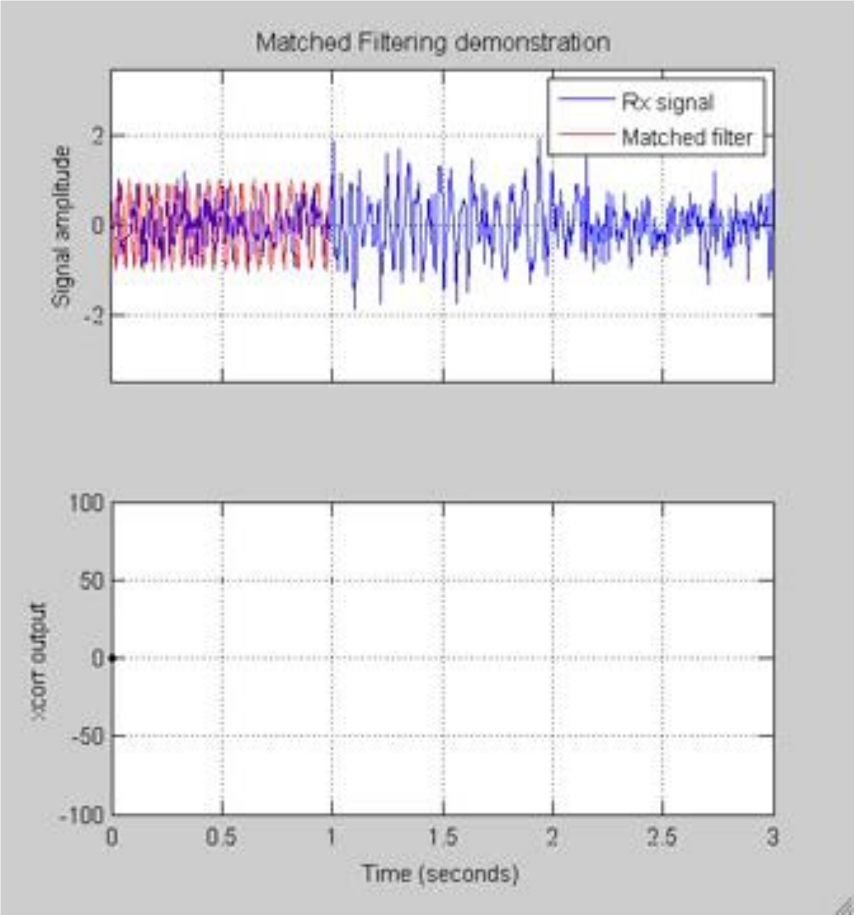
MF Demonstration



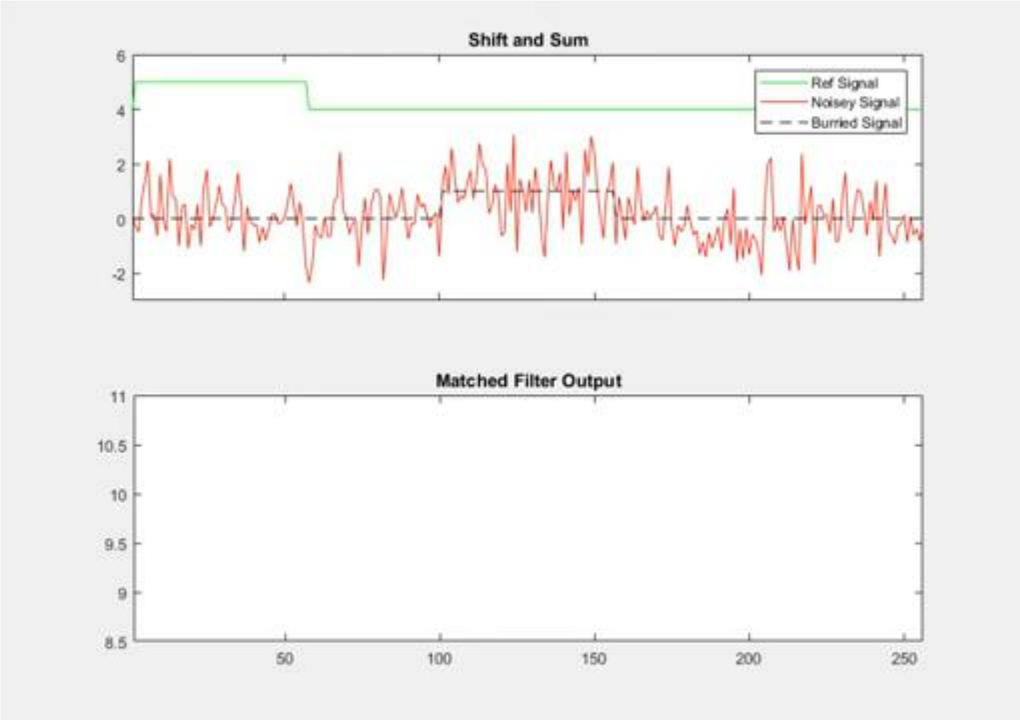

```
Fs = 8e3;
Ton = 1;
Fstart = 400;     %start frequency of chirp signal
Fstop = 500;     %stop frequency of chirp signal
t = 0:1/Fs:Ton-1/Fs;
x = sin(2*pi*(Fstart*t+(Fstop-Fstart)/(2*Ton)*t.^2)); %modulated signal
y = zeros(1,2.5*Fs);
y(1*Fs+1:1*Fs+1+length(x)-1) = x;
yn = y + randn(size(y));     %signal + additive white Gaussian noise
xmf = conj(fliplr(x));
yf = filter(xmf,1,yn);
figure
s(1) = subplot(3,1,1);
plot(y)
title(s(1),'Noise-free Signal')
xlabel("Time")
ylabel("Ampl.")
s(2) = subplot(3,1,2);
plot(yn)
grid on
title(s(2),'Noisy Signal')
ylabel("Ampl.")
xlabel("Time")
s(3) = subplot(3,1,3);
grid on
plot(yf)
title(s(3),'Matched Filtering Output')
ylabel("Ampl.")
xlabel("Time")
```

MF Demonstration



Source: Youtube

MF Demonstration



Source: Youtube