PRACTICE 5

PROPOSITIONAL LOGIC

1. Which of the following are correct?

a. False |= True.

b. True |= False.

c. $(A \wedge B) \models (A \Leftrightarrow B)$.

d. $A \Leftrightarrow B \models A \vee B$.

e. $A \Leftrightarrow B \models \neg A \vee B$.

f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$.

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.

i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.

j. $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.

k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

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2. Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \vee C$
b. $\neg A \vee \neg B \vee \neg C \vee \neg D$
c. $(A \Rightarrow B)$

3. Determine whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules.

a. $Smoke \Rightarrow Smoke$

b. $Smoke \Rightarrow Fire$

c. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

d. $Smoke \lor Fire \lor \neg Fire$

e. $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

f. $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)$

3. Determine whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules.

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Figure 7.11 Standard logical equivalences. The symbols α, β, and γ stand for arbitrary sentences of propositional logic.

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4. Given the following expressions. (a) Convert them to Conjunctive Normal Form (CNF) (b) Prove $\neg A \wedge \neg B$ by resolution.

S1: $A \Leftrightarrow (B \vee E)$

S2: $E \Rightarrow D$

S3: $C \wedge F \Rightarrow \neg B$

S4: $E \Rightarrow B$

S5: $B \Rightarrow F$

S6: $B \Rightarrow C$

5. Given the following expressions. (a) Convert them to Conjunctive Normal Form (CNF) (b) Prove a7 by resolution.

1. a1 ∨ a2 → a3 ∨ a4
2. 2. a1 → a5
3. a2 ∧ a3 → a5
4. a2 ∧ a4 → a6 ∧ a7
5. a5 → a7
6. a1 ∧ a3 → a6 ∨ a7

Suppose that a1 and a2 are true.

6. Given the following knowledge bases, use forward reasoning to prove d

a)

(1) a

(2) b

(3) $b \rightarrow (c \rightarrow d)$

(4) c

b)

(1) $a \wedge b \rightarrow c$

(2) $b \wedge c \rightarrow d$

(3) a

(4) b

7. Given the following knowledge bases, use backward reasoning to prove (a) t (b) g

a)

(1) p

(2) $p \rightarrow q$

(3) $q \wedge r \wedge s \rightarrow t$

(4) $p \rightarrow u$

(5) $v \rightarrow w$

(6) $u \rightarrow v$

(7) $v \rightarrow t$

(8) r

(9) s

b)

(1) $(a \vee b) \wedge c \rightarrow (c \wedge d)$

(2) $a \wedge m \wedge d \rightarrow f$

(3) $m \rightarrow b \wedge c$

(4) $a \rightarrow c$

(5) $(a \wedge f) \rightarrow (\neg e \vee g)$

(6) $(m \wedge f) \rightarrow g$

(7) a

(8) m