LESSON 18

IDEAL DIGITAL FILTERS

PhD. Nguyen Hong Quang
Assoc. Prof. Trinh Van Loan
PhD. Doan Phong Tung
Computer Engineering Department

❑ CONTENT

1. Ideal low pass filter.
2. Ideal high-pass filter.
3. Ideal bandpass filter.
4. Ideal band-pass filter.
5. The actual filter specifications.

❑ Lesson Objectives

After completing this lesson, you will be able to understand the following topics:

- The basic concepts and parameters of ideal filters include: low-pass filter, high-pass filter, band-pass filter and band-pass filter.
- Basic filter concepts and parameters.

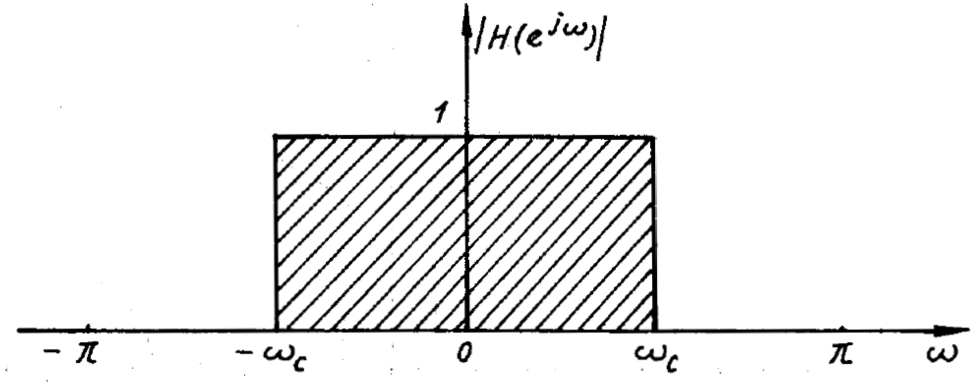
1. Ideal low-pass filter

- LPF: Low Pass Filter

$$H(\omega) = \begin{cases} 1, & |\omega| \le \omega_C \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

- Basic parameters of the filter

ω_c: cutoff frequency
$-\omega_c \le \omega \le \omega_c$: pass band
$\omega_c < |\omega| < \pi$: stop band



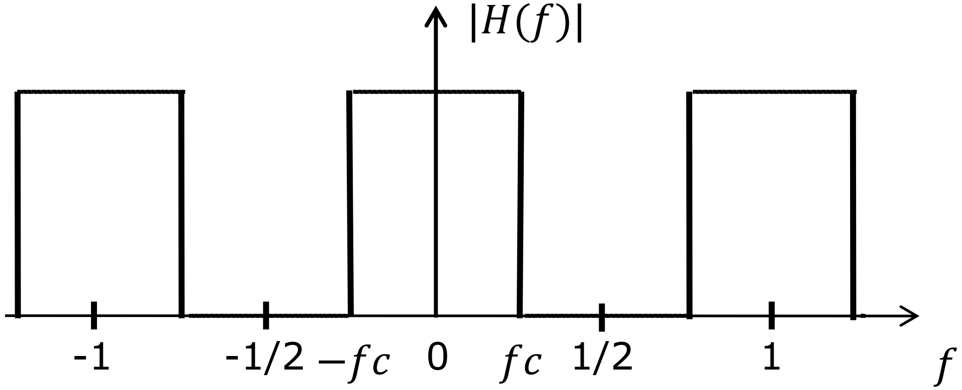
- Impulse response

$$h_{lp}(n) = \frac{1}{2\pi}\int_{-\omega_C}^{\omega_C} e^{j\omega n}\, d\omega = \frac{1}{2\pi j n} e^{j\omega n}\Big|_{-\omega_C}^{\omega_C}$$

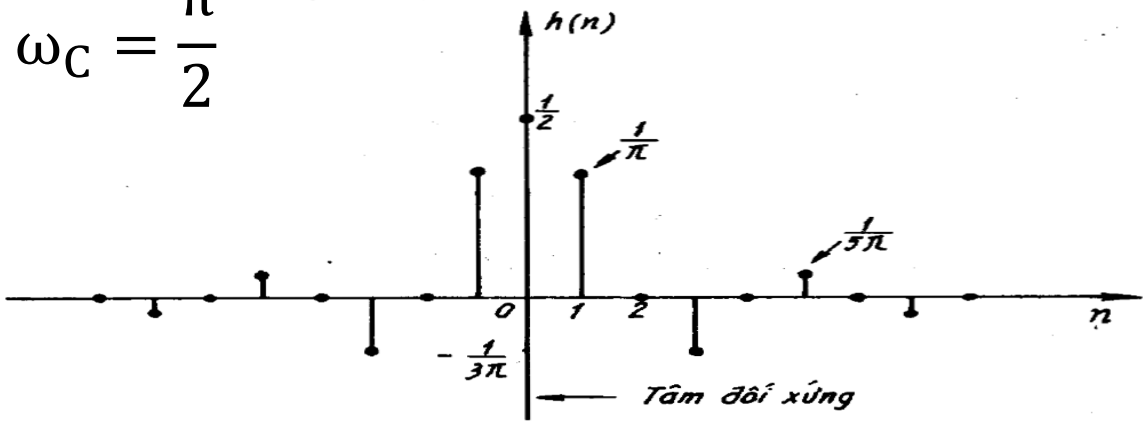
$$= \frac{1}{2\pi j n}\left(e^{j\omega_C n} - e^{-j\omega_C n}\right) = \frac{\sin \omega_C n}{\pi n}$$

$$n = 0 \to h(0) = \frac{1}{2\pi}\omega\Big|_{-\omega_C}^{\omega_C} = \frac{\omega_C}{\pi}$$

Characteristics of an ideal low-pass filter



$$\omega_C = \frac{\pi}{2}$$



- At all samples are integer times of 2 (even samples) except at $n = 0$ then $h(n) = 0$ because $\omega_C = \frac{\pi}{2}$
- With cutoff frequency $\omega_C = \frac{\pi}{M}$ thì $h(nM) = 0$
- The system is not causal so it is not physically possible

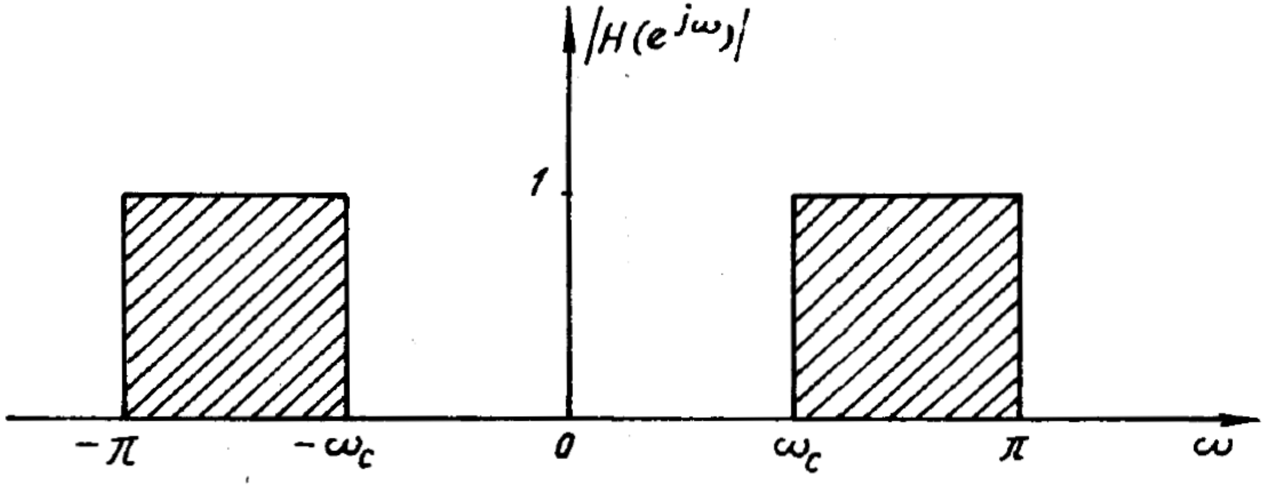
2. Ideal high-pass filter

- HPF: High Pass Filter

$$\left|H(e^{j\omega})\right| = \begin{cases} 1 & \begin{cases} -\pi \le \omega \le -\omega_C \\ \omega_C \le \omega \le \pi \end{cases} \\ 0 & \omega \text{ còn lại} \end{cases}$$

- Basic parameters of the filter

ω_c: cutoff frequency
$-\omega_c < \omega < \omega_c$: stop band
$\omega_c \le |\omega| \le \pi$: pass band



Impulse response of an ideal high-pass filter

- $n \neq 0$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{-\omega_C} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_C}^{\pi} e^{j\omega n}\, d\omega = \frac{1}{2\pi jn} e^{j\omega n}\Big|_{-\pi}^{-\omega_C} + \frac{1}{2\pi jn} e^{j\omega n}\Big|_{\omega_C}^{\pi}$$

$$= -\frac{\sin \omega_C n}{\pi n}$$

- $n = 0$

$$h(0) = \frac{1}{2\pi}\omega\Big|_{-\pi}^{\omega_C} + \frac{1}{2\pi}\omega\Big|_{\omega_C}^{\pi} = 1 - \frac{\omega_C}{\pi}$$

- The relationship between low-pass filter and zero-phase high-pass filter:

$$h_{hp}(n) = \begin{cases} 1 - h_{lp}(0) & n = 0 \\ -h_{lp}(n) & n \neq 0 \end{cases}$$

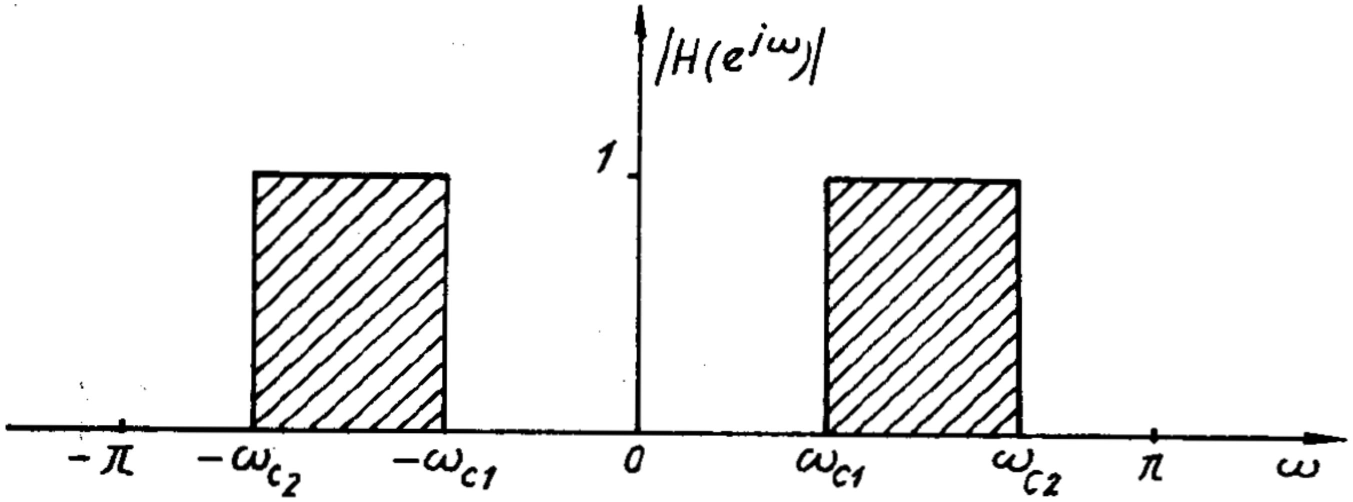
3. Ideal Bandpass Filter

- Ideal Band Pass Filter

$$\left|H(e^{j\omega})\right| = \begin{cases} 1 & \begin{cases} -\omega_{C2} \le \omega \le -\omega_{C1} \\ \omega_{C1} \le \omega \le \omega_{C2} \end{cases} \\ 0 & \omega \text{ còn lại} \end{cases}$$

- Basic parameters of the filter :

ω_{c1}: lower cut-off frequency
ω_{c2}: upper cut-off frequency
$\omega_{C1} \le |\omega| \le \omega_{C2}$: pass band
$|\omega| \le \omega_{C1}$: stop band
$\omega_{C2} \le |\omega| \le \pi$: stop band



Impulse response of an ideal bandpass filter

- $n \neq 0$

$$h(n) = \frac{1}{2\pi}\int_{-\omega_{C2}}^{-\omega_{C1}} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_{C1}}^{\omega_{C2}} e^{j\omega n}\, d\omega = \frac{1}{2\pi jn} e^{j\omega n}\Big|_{-\omega_{C2}}^{-\omega_{C1}} + \frac{1}{2\pi jn} e^{j\omega n}\Big|_{\omega_{C2}}^{\omega_{C1}}$$

$$= \frac{1}{\pi n}\left[\sin(\omega_{C2} n) - \sin(\omega_{C1} n)\right]$$

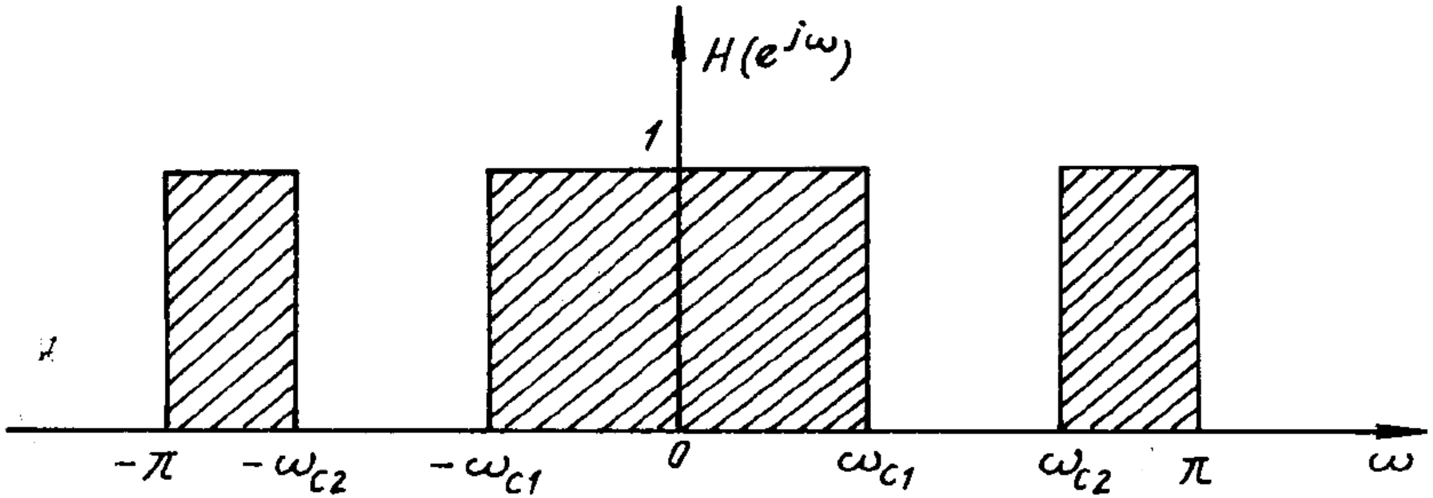
- $n = 0$

$$h(0) = \frac{1}{2\pi}\omega\Big|_{-\omega_{C2}}^{-\omega_{C1}} + \frac{1}{2\pi}\omega\Big|_{\omega_{C2}}^{\omega_{C1}} = \frac{1}{2\pi}(-\omega_{C1} + \omega_{C2} + \omega_{C2} - \omega_{C1}) = \frac{\omega_{C2} - \omega_{C1}}{\pi}$$

4. Ideal band-pass filter

- Ideal Band Stop Filter

$$\left|H(e^{j\omega})\right| = \begin{cases} 1 & \begin{cases} -\pi \le \omega \le -\omega_{C2} \\ -\omega_{C1} \le \omega \le \omega_{C1} \\ \omega_{C2} \le \omega \le \pi \end{cases} \\ 0 & \omega \text{ còn lại} \end{cases}$$



Impulse response of an ideal band-pass filter

- $n \neq 0$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{-\omega_{C2}} e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{-\omega_{C1}}^{\omega_{C1}} e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{\omega_{C2}}^{\pi} e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi jn} e^{j\omega n}\Big|_{-\pi}^{-\omega_{C2}} + \frac{1}{2\pi jn} e^{j\omega n}\Big|_{-\omega_{C1}}^{\omega_{C1}} + \frac{1}{2\pi jn} e^{j\omega n}\Big|_{-\omega_{C2}}^{\pi}$$

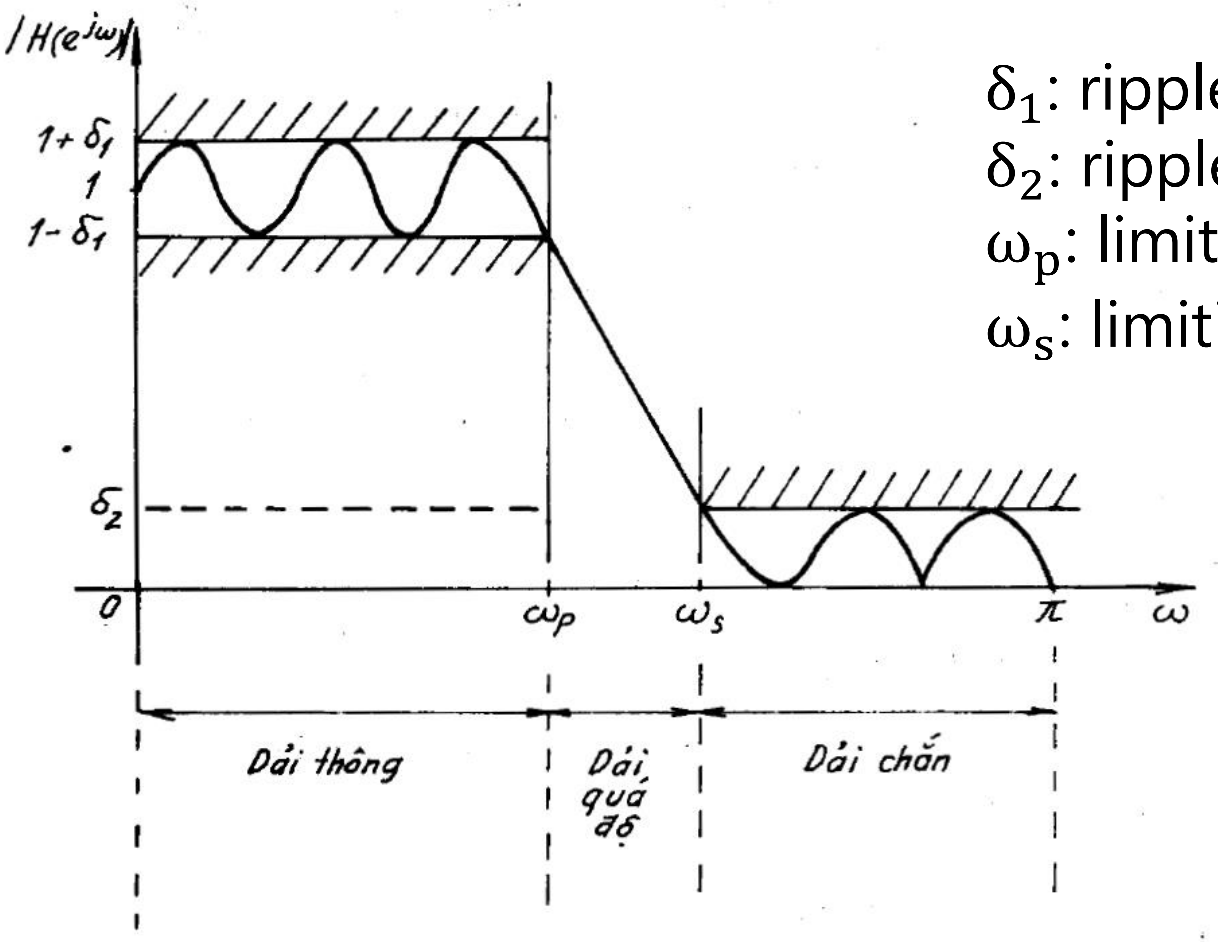
$$= \frac{1}{\pi n}[\sin(\omega_{C1}n) - \sin(\omega_{C2}n)]$$

- $n = 0$

$$h(0) = \frac{1}{2\pi}\omega\Big|_{-\pi}^{-\omega_{C2}} + \frac{1}{2\pi}\omega\Big|_{-\omega_{C1}}^{\omega_{C1}} + \frac{1}{2\pi}\omega\Big|_{-\omega_{C1}}^{\omega_{C1}} = 1 + \frac{\omega_{C1} - \omega_{C2}}{\pi}$$

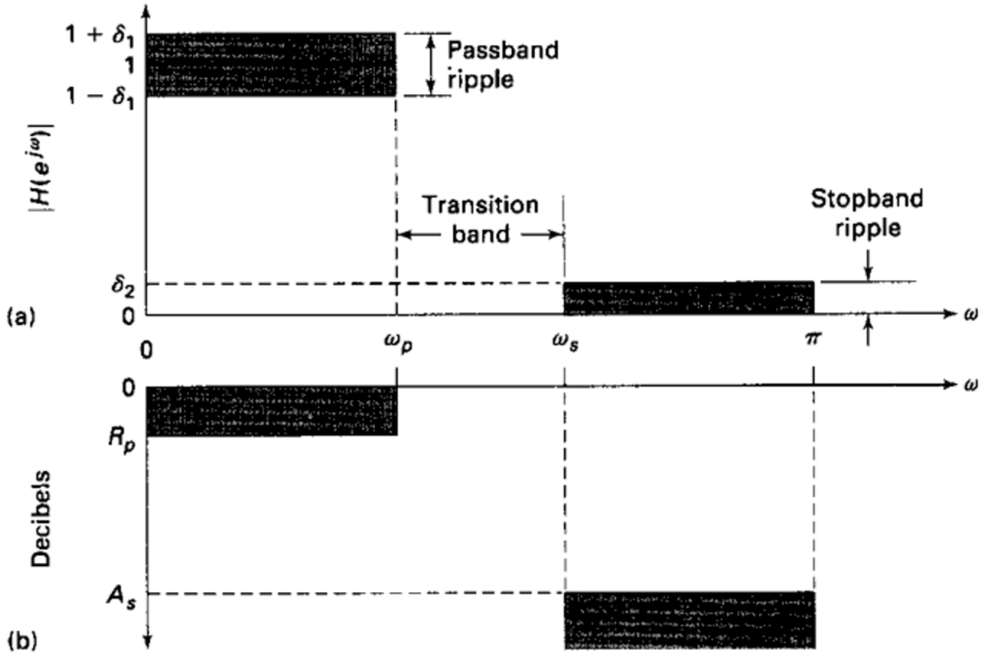
5. Realistic Filters

- Parameters of the actual filter:



δ_1: ripple in pass-band
δ_2: ripple in the stop-band
ω_p: limiting frequency (bandwidth) pass-band
ω_s: limiting frequency (bandwidth) stop-band

Parameters of the actual filter



R_p: ripple in the pass-band in dB

A_s: attenuation in the stop-band in dB

$$R_p = -20\log_{10}(1-\delta_1) > 0\ (\approx 0)$$

$$A_S = -20\log_{10}\delta_2 > 0\ (1)$$

Absolute scale: $|H(e^{j\omega})|$

Relative scale: $\mathrm{dB} = -20\log_{10}\frac{|H(e^{j\omega})|}{|H(e^{j\omega})|_{max}} \geq 0$

$[0, \omega_p]$: pass-band, δ_1 : amplitude tolerance in pass-band

$[\omega_s, \pi]$: stop-band, δ_2 amplitude tolerance in stop-band

$[\omega_p, \omega_s]$: transient range

4. Summary

- Digital filters are characterized by their amplitude response.
- Ideal filters are proposed to study the theoretical properties. However, these filters cannot be implemented in practice without satisfying causality.
- The actual filter revises the specifications and adjusts the parameters to approximate the ideal filter.

5. Exercises

- Determine and plot the impulse response of:

a. The ideal low-pass filter has a cutoff frequency of $\frac{\pi}{3}$

b. The ideal high-pass filter has a cutoff frequency of $\frac{\pi}{3}$

c. The ideal bandpass filter has a cutoff frequency of $\frac{\pi}{3}$ and $\frac{\pi}{2}$

d. The ideal band-pass filter has a cutoff frequency of $\frac{\pi}{3}$ and $\frac{\pi}{2}$

Next lesson. Lesson 19

LINEAR PHASE FIR DIGITAL FILTER

References :

- ***Nguyễn Quốc Trung (2008), Xử lý tín hiệu và lọc số, Tập 1, Nhà xuất bản Khoa học và Kỹ thuật, Chương 1 Tín hiệu và hệ thống rời rạc.***
- ***J.G. Proakis, D.G. Manolakis (2007), Digital Signal Processing, Principles, Algorithms, and Applications, 4th Ed, Prentice Hall, Chapter 1 Introduction.***

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Wish you all good study!