HUST

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HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

DATA STRUCTURES AND ALGORITHSMS

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DATA STRUCTURES AND ALGORITHMS

WEEK 3: RECURSIVE BACKTRACKING

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CONTENT

- General recursive backtracking diagram
- Listing binary strings
- Listing permutations
- Sudoku puzzles

GENERAL DIAGRAM

- Combination enumeration problem: List the sets x = (x[1], x[2],…, x[k], x[k+1],. . ., x[n]) with x[i] ∈ Ai , i = 1, 2, ..., n and satisfy the given set of constraints P.
- Example:
  - “The problem of listing a binary string of length n” leads to listing the sets x = (x[1], x[2],…, x[k], x[k+1],. . ., x [n]) with x[i] ? {0, 1}, i = 1, 2, ..., n
  - “The problem of listing a binary string of length n with an even number of 0 bits” leads to listing the sets x = (x[1], x[2],…, x[k], x[k +1],. . ., x[n]) with x[i] ? {0, 1}, i = 1, 2, ..., n and satisfying the constraint: number of elements x[i] = 0 with i = 1, 2, ..., n is an even number.
  - The backtracking algorithm allows solving combinatorial enumeration problems. There are two ways to implement the backtracking algorithm: recursive or non-recursive.

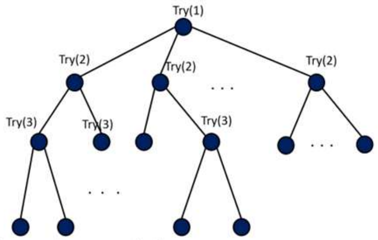
GENERAL DIAGRAM

- Combination enumeration problem: List the sets x = (x[1], x[2],…, x[k], x[k+1],. . ., x[n]) with x[i] ∈ Ai , i = 1, 2, ..., n and satisfy the given set of constraints P.
- The call to execute the backtracking recursive algorithm is: Try(1);
- If only need to find one solution, you need to find a way to terminate the nested recursive procedure calls generated by the Try(1) call after the first solution is recorded.
- If at the end of the algorithm we do not get any solution, it means the problem has no solution.

```
Try(k){// try values assignable to x[k]
  for v in candidates(k) do {
     if (check(v,k)) then {
        x[k] = v;
        [Update the data structure D]
        if (k == n) then solution();
        else Try(k+1);
        [Recover the data structure D]
     }
  }
}
```

GENERAL DIAGRAM

- Combination enumeration problem: List the sets x = (x[1], x[2],…, x[k], x[k+1],. . ., x[n]) with x[i] ∈ Ai , i = 1, 2, ..., n and satisfy the given set of constraints P.
- The call to execute the backtracking recursive algorithm is: Try(1);



```
Try(k){// try values assignable to x[k]
  for v in candidates(k) do {
     if (check(v,k)) then {
        x[k] = v;
        [Update the data structure D]
        if (k == n) then solution();
        else Try(k+1);
        [Recover the data structure D]
     }
  }
}
```

LISTING BINARY STRINGS

- Given a positive integer n ≥ 1. List all binary strings of length n in lexicographic order.
- For example: n = 3, we have binary strings of length 3 that need to be listed in the following order:
  - 000
  - 001
  - 010
  - 011
  - 100
  - 101
  - 110
  - 111

LISTING BINARY STRINGS

- Given a positive integer n ≥ 1. List all binary strings of length n in lexicographic order.
- Solution representation: each binary string is represented by the array (x[1], x[2], . . ., x[n]) in which x[k] ∈{0,1} is the k^{th} bit in binary string.

```
Try(k){// try values assignable to x[k]
  for v in candidates(k) do {
     if (check(v,k)) then {
        x[k] = v;
        [Update the data structure D]
        if (k == n) then solution();
        else Try(k+1);
        [Recover the data structure D]
     }
  }
}
```

Determine:
- candidates(k)
- check(v, k)

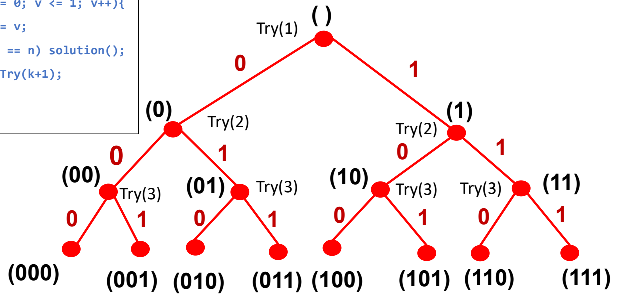
→

```
void Try(int k){
    for (int v = 0; v <= 1; v++){
        x[k] = v;
        if (k == n) solution();
        else Try(k+1);
    }
}
```

LISTING BINARY STRINGS

```
void Try(int k){
  for (int v = 0; v <= 1; v++){
        x[k] = v;
        if (k == n) solution();
        else Try(k+1);
  }
}
```

Try to list binary strings of length 3



LISTING PERMUTATIONS

- Given a positive integer n ≥ 1. List all permutations of n numbers 1, 2, ..., n in lexicographic order.
- For example: n = 3, we have the permutations of 1, 2, 3 in lexicographic order as follows :
  - (1, 2, 3)
  - (1, 3, 2)
  - (2, 1, 3)
  - (2, 3, 1)
  - (3, 1, 2)
  - (3, 2, 1)

LISTING PERMUTATIONS

- Solution representation: each permutation of n elements is represented by the array (x[1], x[2], . . ., x[n]) where :
  - x[k] ∈{1, 2,..., *n*} is the kth element in the permutation
  - x[k] ≠ x[1], x[2],... x[*k*-1], x[*k*+1], ...,x[*n*]

```
Try(k){// try values assignable to x[k]
  for v in candidates(k) do {
     if (check(v,k)) then {
        x[k] = v;
        [Update the data structure D]
        if (k == n) then solution();
        else Try(k+1);
        [Recover the data structure D]
     }
  }
}
```

Determine:
- candidates(k)
- check(v, k)

→

```
void Try(int k){
    for (int v = 1; v <= n; v++){
        if (check(v, k)) {
            x[k] = v;
            if (k == n) solution();
            else Try(k+1);
        }
    }
}
```

LISTING PERMUTATIONS

```
#include <stdio.h>
int n;
int x[100];
void solution(){
    for(int k = 1; k <= n; k++)
        printf("%d ",x[k]);
    printf("\n");
}
int check(int v, int k) {
  for (int i = 1; i <= k-1; i++)
    if (x[i] == v) return 0;
  return 1;
}
```

```
void Try(int k){
    for (int v = 1; v <= n; v++){
        if (check(v, k)) {
           x[k] = v;
           if (k == n) solution();
           else Try(k+1);
        }
    }
}
int main(){
   scanf("%d",&n);
   Try(1);
}
```

LISTING PERMUTATIONS

- Marking technique
  - used[v] = 1: v appear
  - used[v] = 0: v does not appear

```
try(k){//try values assignable to x[k]
  for v in candidates(k) do {
     if (check(v,k)) then {
        x[k] = v;
        [Update the data structure D]
        if (k == n) then solution();
        else try(k+1);
        [Recover the data structure D]
     }
  }
}
```

```
void Try(int k){
    for(int v = 1; v <= n; v++){
        if (used[v]==0){
            x[k] = v;
            used[v] = 1;
            if (k == n) solution();
            else Try(k+1);
            used[v] = 0;
        }
    }
}
int main(){
   scanf("%d",&n);
   for(int v = 1; v <= n; v++) used[v] = 0;
   Try(1);
}
```

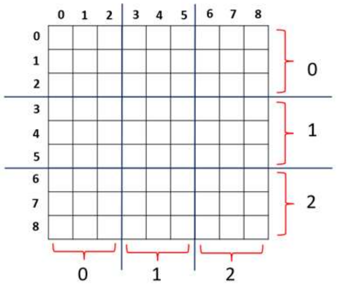
LISTING PERMUTATIONS

```
#include <stdio.h>
int n;
int x[100];
int used[100];
void solution(){
    for(int k = 1; k <= n; k++)
         printf("%d ",x[k]);
    printf("\n");
}
```

```
void Try(int k){
    for(int v = 1; v <= n; v++){
        if (used[v]==0){
            x[k] = v;
            used[v] = 1;
            if (k == n) solution();
            else Try(k+1);
            used[v] = 0;
        }
    }
}
int main(){
   scanf("%d",&n);
   for(int v = 1; v <= n; v++) used[v] = 0;
   Try(1);
}
```

SODOKU PUZZLES

- Problem statement:
  - Given a 9x9 square grid divided into 9 3x3 grids.
  - Rows and columns are numbered 0, 1, 2, ..., 8
  - List all the ways to fill in the numbers 1, 2. ..., 9 cells in a 9x9 square grid such that:
    - The numbers on each line are different,
    - The numbers on each column are different,
    - The numbers on each 3x3 grid are different.



SODOKU PUZZLES

- State the problem:
  - Given a 9x9 square grid divided into 9 3x3 grids.
  - Rows and columns are numbered 0, 1, 2, ..., 8
  - List all the ways to fill in the numbers 1, 2. ..., 9 cells in a 9x9 square grid such that:
    - The numbers on each line are different,
    - The numbers on each column are different,
    - The numbers on each 3x3 grid are different.

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	1	4	3	6	5	8	9	7
3	6	5	8	9	7	2	1	4
8	9	7	2	1	4	3	6	5
5	3	1	6	4	2	9	7	8
6	4	2	9	7	8	5	3	1
9	7	8	5	3	1	6	4	2

SODOKU PUZZLES

- Solution representation: X[i, j] is the numeric value filled in cell row i column j (i, j = 0, 1, 2, . . ., 8)
- Marking array:
  - markR[*r*, *v*] = 1: *v* appears in row *r* and markR[*r*, *v*] = 0 for others(*r* = 0, 1, ..., 8 và *v* = 1, 2, . . ., 9)
  - markC[*c*, *v*] = 1: v appears in column c and markC[*c*, *v*] = 0, for others (*c* = 0, 1, 2, . . ., 8 và *v* = 1, 2, . . ., 9)
  - markS[*i*, *j*, *v*] = 1: *v* appear in the grid 3x3 at row *i* and column *j* ,and markS[*i*, *j*, *v*] = 0 for others (*i*, *j* = 0, 1, 2 và *v* = 1, 2, . . ., 9)

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	1	4	3	6	5	8	9	7
3	6	5	8	9	7	2	1	4
8	9	7	2	1	4	3	6	5
5	3	1	6	4	2	9	7	8
6	4	2	9	7	8	5	3	1
9	7	8	5	3	1	6	4	2

SODOKU PUZZLES

- Order of browsing cells to test values: top to bottom and left to right
- Recursive function Try(r, c): tries the value for cell row r column c

```
check(v, r, c){
  if markR[r,v] = 1 then return 0;
  if markC[c,v] = 1 then return 0;
  if markS[r/3,c/3,v] = 1 then return 0;
  return 1;
}
```

```
Try(r, c){
  for v = 1 to 9 do {
    if (check(v,r,c)) then {
      X[r,c] = v;
      markR[r,v] = 1; markC[c,v] = 1; markS[r/3,c/3,v] = 1;
      if r = 8 and c = 8 then solution();
      else {
        if c = 8 then Try(r+1, 0); else Try(r, c+1);
      }
      markR[r,v] = 0; markC[c,v] = 0; markS[r/3,c/3,v] = 0;
    }
  }
}
```

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THANK YOU !