MIDTERM MOCK TEST - MI1016 - SEMESTER 20241

Questions with only one correct answer

Question 1. Which of the following functions is odd?

A. $y = \arccos x$.

B. $y = \arcsin x$.

C. $y = \cos x$.

D. $y = \sin x^2$.

Question 2. Determine the range of the function $y = \operatorname{arccot}(\tan^2 x)$.

A. $(0, \pi)$.

B. $\left(0, \dfrac{\pi}{2}\right]$.

C. $\left[\dfrac{\pi}{2}, \pi\right)$.

D. $\mathbb{R}$.

Question 3. Determine the value $a \in \mathbb{R}$ such that the function $y = \begin{cases} 2^{\frac{1}{\arcsin x}}, x \neq 0, \\ a, x = 0 \end{cases}$ is continuous from the left.

A. $a = -1$.

B. $a = 0$.

C. $a = 1$.

D. $a = 2$.

Question 4. Compute the following indefinite integral $\displaystyle\int \frac{dx}{(x+1)\ln(x+1)}, x > -1$.

A. $\ln(x+1) + C, C \in \mathbb{R}$.

B. $\ln|\ln(x+1)| + C, C \in \mathbb{R}$.

C. $\ln^2(x+1) + C, C \in \mathbb{R}$.

D. $\dfrac{1}{\ln(x+1)} + C, C \in \mathbb{R}$.

Question 5. Consider $f\colon \mathbb{R} \to \mathbb{R}, f(x) = \arctan x - \dfrac{\pi}{4} + x^3$ and let $g(x) = f^{-1}(x) + x^2$. Compute $g'(1)$.

A. $g'(1) = \dfrac{7}{2}$.

B. $g'(1) = \dfrac{11}{2}$.

C. $g'(1) = \dfrac{2}{7}$.

D. $g'(1) = \dfrac{16}{7}$.

Question 6. Suppose that the function $y = \begin{cases} \dfrac{mx - \sin(2x)}{x^2}, x \neq 0 \\ 0 \end{cases}$ is differentiable at $x = 0$ and $f'(0) = n$. Compute $\lambda = m \cdot n$?

A. $\lambda = \dfrac{4}{3}$.

B. $\lambda = 2$.

C. $\lambda = \dfrac{8}{3}$.

D. $\lambda = 0$.

Question 7. Consider the sequence $u_n = \dfrac{\cos n}{n!}, n \geq 1$. Which of the following statements is true?

A. (u_n) is increasing.

B. (u_n) is bounded.

C. $\lim\limits_{n\to\infty} u_n$ does not exists.

D. (u_n) is decreasing.

Question 8. Which of the following functions is bounded over its domain of definition?

A. $y = e^{x^2}$.

B. $y = \arctan \dfrac{1}{x}$.

C. $y = \tan x$.

D. $y = e^{\frac{1}{x^2}}$.

Questions with multiple correct answers

Question 9. Which of the following functions is an infinitesimal as $x \to 0^+$.

A. $y = x \ln x$.

B. $y = \dfrac{\ln x}{x}$.

C. $y = \dfrac{x}{\ln x}$.

D. $y = x^{\ln x}$.

Question 10. Given $f\colon [0;2] \to \mathbb{R}$ be a continuously differentiable function. Which of the following statements is always correct?

A. If $f(2)f(0) < 0$ then $\exists c \in (0;2)$ such that $f'(c) = 0$.

B. If $f(2)f(0) < 0$ then $\exists c \in (0;2)$ such that $f(c) = 0$.

C. If $f(0) = 0$ then $\exists c \in (0;2)$ such that $f(2) = 2f'(c)$.

D. Function f cannot attain its maximum in $[0;2]$.

Question 11. Which of the following functions is an infinitesimal of higher order than $\alpha(x) = e^{\sqrt{x}} - 1$ as $x \to 0^+$.

A. $y = \sqrt[3]{1+x} - 1$.

B. $y = \arctan \sqrt{x}$.

C. $y = \sin x$.

D. $y = \cos \sqrt{x}$.

E. $y = 1 - \cos \sqrt{x}$.

F. $y = \sqrt{1+\sqrt{x}} - \cos x$.

Question 12. Which of the following functions is convex over $(0, +\infty)$?

A. $y = \ln x$.

B. $y = e^x$.

C. $y = \sin^2 x$.

D. $y = -\ln(1+x^2)$.

E. $y = \arctan x$.

F. $y = \operatorname{arccot} x$.

Constructed-repsonse questions

Question 13. Compute the Maclaurin polynomial of order 6 of $\dfrac{1}{1+x^2}$.

Question 14. Determine the local extremes of $y = \sin x + \cos x$.

Question 15. Show that if n is odd then the equation $x^n + x - 10 = 0$ has at least one solution.