ĐẠI HỌC BÁCH KHOA HÀ NỘI
VIỆN CÔNG NGHỆ THÔNG TIN VÀ TRUYỀN THÔNG

Discrete Mathematics

Course preparation group:
Nguyễn Khánh Phương
Đỗ Phan Thuận
Phạm Quang Dũng
Huỳnh Thanh Bình
Trần Vĩnh Đức
Bùi Quốc Trung
Đinh Viết Sang
Ban Hà Bằng

Content of Part 2

PART 1
COMBINATORIAL THEORY
(Lý thuyết tổ hợp)

PART 2
GRAPH THEORY
(Lý thuyết đồ thị)

Graph Representation

1. Incidence matrix
2. Adjacency matrix
3. Weight matrix
4. Adjacency list

Graph Representation

1. **Incidence matrix**
2. Adjacency matrix
3. Weight matrix
4. Adjacency list

1.Incidence Matrix

Matrix $M_{|V| \times |E|} = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i \\ 0 & \text{otherwise} \end{cases}$$

Can also be used to represent :

- **Multiple edges:** by using columns with identical entries, since these edges are incident with the same pair of vertices
- **Loops:** by using a column with exactly one entry equal to 1, corresponding to the vertex that is incident with the loop

1. Incidence Matrix

G = (V, E) is an unditected graph:

- $V = \{v_1, v_2, v_3, \ldots, v_n\}$
- $E = \{e_1, e_2, \ldots, e_m\}$

Then the incidence matrix with respect to this ordering of V and E is the $n \times m$ matrix $M = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i \\ 0 & \text{otherwise} \end{cases}$$

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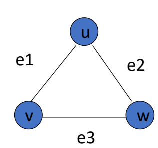
1.Incidence Matrix

Example: G = (V, E)



	e_1	e_2	e_3
v	1	0	1
u	1	1	0
w	0	1	1

Graph Representation

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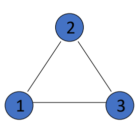
2. Adjacency Matrix

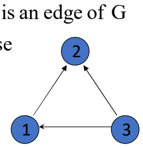
The Adjacency Matrix (NxN) $A = [a_{ij}]$ where $|V| = N$

For undirected graph $a_{ij} = \begin{cases} 1 \text{ if } \{v_i, v_j\} \text{ is an edge of G} \\ 0 \text{ otherwise} \end{cases}$



$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

For directed graph $a_{ij} = \begin{cases} 1 \text{ if } (v_i, v_j) \text{ is an edge of G} \\ 0 \text{ otherwise} \end{cases}$



$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

This makes it easier to find subgraphs, and to reverse graphs if needed.

3. Weight matrix

- **Weighted** graphs have values associated with edges.
- In the case weighted graphs, instead of adjacency matrix, we use weight matrix to represent the graph

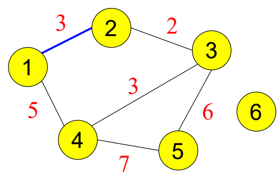
$$C = c[i, j], \; i, j = 1, 2, \ldots, n,$$

where

$$c[i,j] = \begin{cases} c(i,j), & \text{if } (i,j) \in E \\ \theta, & \text{if } (i,j) \notin E, \end{cases}$$

- θ: special value to identify (i, j) is not an edge; depends on the case, the value of θ could be: 0, $+\infty$, $-\infty$.

Weight matrix of undirected graph



$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 3 & 0 & 5 & 0 & 0 \\
2 & 3 & 0 & 2 & 0 & 0 & 0 \\
3 & 0 & 2 & 0 & 3 & 6 & 0 \\
4 & 5 & 0 & 3 & 0 & 7 & 0 \\
5 & 0 & 0 & 6 & 7 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

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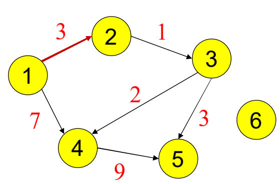
Weight matrix of directed graph



$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 3 & 0 & 7 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 2 & 3 & 0 \\
4 & 0 & 0 & 0 & 0 & 9 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

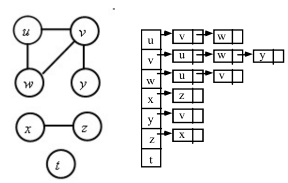
3. Adjacency List

Adjacency list: each vertex has a list of which vertices it is adjacent

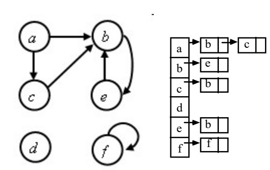
- Is an array Adjacency consiststing of $|V|$ list
- Each vertex has 1 list
- Each vertex $u \in V$: Adjacency[u] consists of nodes that are adjacent to u.

Example:

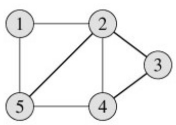
Undirected graph

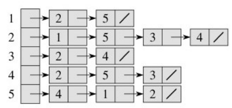


Directed graph



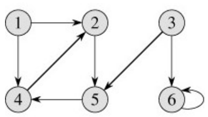
Graph representation

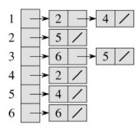




$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 0 & 1 & 0 \\ \hline \end{array}$$

graph   Adjacency list   Adjacency matrix





$$\begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 & 1 \\ 4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$$