Name: Sudhanshu Mishra
Roll no: 17807726

Question 1:

1). Plot a closed curve in x-y plane that satisfies.

$$x^4 + 0.1x^3 + 2x^2 - x + y^4 - 0.1y^3 + \frac{y^2}{1+y^2} + 0.1xy = 100$$

All pts on the closed curve satisfy this eqn.

Plotting using the continuation method

1st initial pt $(-3.0059, 0)$ → Calculated from newton raphson

2nd initial pt $(-3.0061, 0.01)$ → Calculated from newton raphson.

Perimeter is calculated by adding the lengths of the chords b/w two consecutive pts.

Area is calculated by adding the area b/w the vectors of two cons. pts

i.e $\frac{1}{2}|(\bar{r}_1 \times \bar{r}_2)|$ where $\bar{r}_1$ is position vector of 1st pt.

Perimeter = ~~21.6~~ 21.4401

Area = ~~35.0563~~ 34.8190

Matlab Code for plotting the curve using continuation method

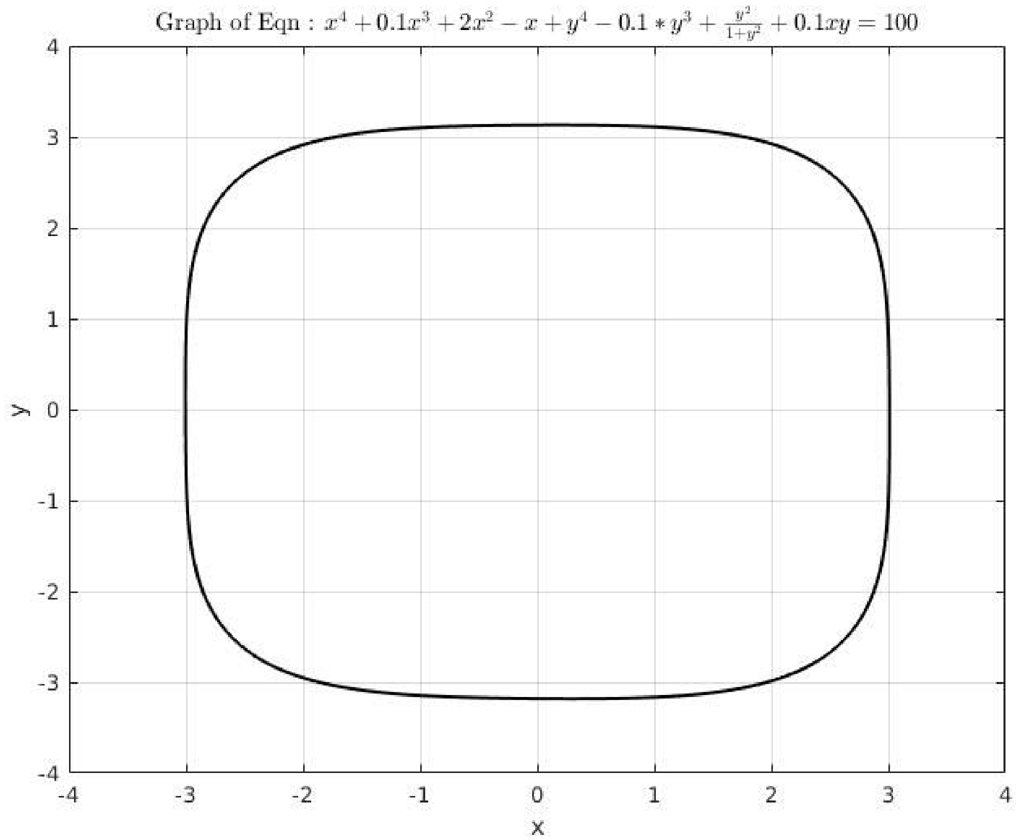
```matlab
clear all;
clc;
global alpha bigX;
alpha = 0;
bigX= [];
y = newton('junk',0);
bigX= [ bigX [y;alpha]];
alpha = 0.01;
y = newton('junk',0);
bigX = [bigX [y;alpha]];
continuation('junk',2143);
plot(bigX(1,:),bigX(2,:),'k','linewidth',1.5);
grid
%hold on;
%scatter(bigX(1,:),bigX(2,:),'r*');
xlabel('x');
ylabel('y');
title('Graph of Eqn : $x^{4}+0.1x^{3}+2x^{2}-x+y^{4}-0.1*y^{3}+\frac{y^2}{1+y^2}+0.1xy=100$', 'interpreter', 'latex')
%hold off;
```

Code for calculating Perimeter and Area

Editor - /home/sudhanshu/Desktop/ME627/Continuation lecture and files/perimeter.m

```matlab
l =size(bigX);
[m,i] = min(bigX,[],2); %min x coordinate
pmtr = 0;
area = 0;
j = 1;
cur_pt = bigX(:,j);
while j < l(2)
    nxt_pt = bigX(:,rem(j,l(2))+1);
    dr = norm(cur_pt-nxt_pt);
    pmtr = pmtr+dr;
    area = area + 0.5*norm(cross([cur_pt;0],[nxt_pt;0]));
    cur_pt = nxt_pt;
    %if j == i(1) %crossed the leftmost pt
    %    break
    %end
    j=j+1;
end
```

Plot of the Curve



Question 2:

2.)

$$\ddot{x}+x^3=0$$

$$x(0)=A$$

$$\dot{x}(0)=0$$

Let $x = A\sin(\omega t) + B\sin(2\omega t)$

$$[\qquad\qquad]$$

e1. $\cancel{\frac{3}{2}A^3 r^2 - \frac{3}{4}A^3 r + \frac{3}{4}A^3 - A\omega^2 = 0}$

~~e2~~ $\cancel{\frac{3}{4}}$

$\sin(\omega t)$

↳ $\frac{3}{4}A^3 + \frac{3}{2}A^3 r^2 - \frac{3}{4}A^3 r - A\omega^2 = 0$

$\sin(3\omega t)$

↳ $\frac{3}{4}A^3 r^3 - \frac{1}{4}A^3 + \frac{3}{2}A^3 r - 9rA\omega^2 = 0$

$$\boxed{\frac{3}{4}A^2 + \frac{3}{2}A^2 r^2 - \frac{3}{4}A^2 r = \omega^2} \rightarrow (1)$$

$$\boxed{\frac{3}{4}A^2 r^2 - \frac{1}{4}A^2 + \frac{3}{2}A^2 r - 9r\omega^2 = 0} \rightarrow (2)$$

solving for ω in 1

$$\omega = \left(\frac{1}{2}\sqrt{6r^2 - 3r + 3}\right)A \quad \left(-\frac{1}{2}\sqrt{6r^2-3r+3}\right)A$$

subs. ~~of~~ the +ve root in ~~eq~~ 2

We get

$$-\frac{3}{4}A^3r^3 - \frac{A^3}{4} + \frac{3}{4}A^3r - \frac{9r}{4}A^3(6r^2 - 3r + 3) = 0$$

Dividing above eqn by A^3

$$\Rightarrow -\frac{51}{4}r^3 - \frac{1}{4} - \frac{21}{4}r + \frac{27}{4}r^2 = 0$$

solving we get 1 real and 2 complex root

Taking the real root

$$r = -0.0448178799$$

Now as we can see that the amplitude of $x(t) = (1+r)A$

Now we can see that

On substituting $|r|$ in the eqn of ω

we get

~~$\omega = 0.8869187432$~~

$\omega = 0.8869195980 A$

Now lets see

$x(t) = A\sin(\omega t) + rA\sin(3\omega t)$

$|x(t)| \leq A(1+|r|)$

Now $|x(t)| = |A(1+|r|)|$

when $\sin(\omega t) = 1$ or -1

$\sin(3\omega t) = 1$ or -1

$\therefore \omega t = \frac{n\pi}{2}$ for $\omega t = \pi/2$

$3\omega t = 3\pi/2$

$3\omega t = \frac{n\pi}{2}$

$x(t) = A(1) + rA(-1)$

$= A(1-r)$

as r is $-$ve

$A(1-r) = A(1+|r|)$

$\therefore \omega_{actual} = \frac{0.8869195980\, A(1+r)}{(1+r)}$

$\therefore$

Taking $A(1+r) = \bar{A}$

$$T = \frac{2\pi}{\omega} = \frac{7.401780691}{\bar{A}}$$

Maple Code

```
> restart:
> de := diff(x(t), t, t) + x(t)^3
```

$$de := \frac{d^2}{dt^2} x(t) + x(t)^3 \tag{1}$$

```
> dea := combine(expand(subs(x(t) = A·sin(omega·t) + r·A·sin(3·omega·t), de)), trig)
```

$$dea := \frac{3 A^3 \sin(3 \omega t) r^3}{4} - \frac{r^3 A^3 \sin(9 \omega t)}{4} + \frac{3 A^3 \sin(\omega t)}{4} - \frac{A^3 \sin(3 \omega t)}{4} + \frac{3 A^3 \sin(\omega t) r^2}{2} + \frac{3 A^3 r^2 \sin(5 \omega t)}{4} - \frac{3 A^3 r^2 \sin(7 \omega t)}{4} - \frac{3 A^3 \sin(\omega t) r}{4} + \frac{3 A^3 \sin(3 \omega t) r}{2} - \frac{3 A^3 r \sin(5 \omega t)}{4} - A \omega^2 \sin(\omega t) - 9 r A \omega^2 \sin(3 \omega t) \tag{2}$$

```
> e1 := coeff(dea, sin(omega·t))
```

$$e1 := \frac{3}{4} A^3 + \frac{3}{2} A^3 r^2 - \frac{3}{4} A^3 r - A \omega^2 \tag{3}$$

```
> e2 := coeff(dea, sin(3·omega·t))
```

$$e2 := \frac{3}{4} A^3 r^3 - \frac{1}{4} A^3 + \frac{3}{2} A^3 r - 9 r A \omega^2 \tag{4}$$

```
> w := solve({e1}, {omega})
```

$$w := \left\{\omega = \frac{\sqrt{6 r^2 - 3 r + 3}\, A}{2}\right\}, \left\{\omega = -\frac{\sqrt{6 r^2 - 3 r + 3}\, A}{2}\right\} \tag{5}$$

```
> subs(w[1], e2)
```

$$\frac{3 A^3 r^3}{4} - \frac{A^3}{4} + \frac{3 A^3 r}{2} - \frac{9 r A^3 (6 r^2 - 3 r + 3)}{4} \tag{6}$$

```
> eqr := simplify(%/A^3)
```

$$eqr := -\frac{51}{4} r^3 - \frac{1}{4} - \frac{21}{4} r + \frac{27}{4} r^2 \tag{7}$$

```
> rsol := evalf(solve({eqr}, {r}))
```

$$rsol := \{r = -0.0448178799\}, \{r = 0.2871148223 - 0.5958737620\, I\}, \{r = 0.2871148223 + 0.5958737620\, I\} \tag{8}$$

```
> evalf(solve({eqr}, {r}))
```

$$\{r = -0.0448178799\}, \{r = 0.2871148223 - 0.5958737620\, I\}, \{r = 0.2871148223 + 0.5958737620\, I\} \tag{9}$$

```
> xsol := A·sin(omega·t) + r·A·sin(3·omega·t)
```

$$xsol := A \sin(\omega t) + r A \sin(3 \omega t) \tag{10}$$

```
> wsol := subs(r = -0.0448178799, w[1])
```

$$wsol := \{\omega = 0.8869195980\, A\} \tag{11}$$

```
> T = 2·Pi/wsol[1]
```

$$T = \left(\frac{2\pi}{\omega} = \frac{7.084278354}{A}\right) \tag{12}$$

```
> %·(1 + 0.0448178799)
```

$$1.044817880\, T = \left(\frac{6.564784353}{\omega} = \frac{7.401780691}{A}\right) \tag{13}$$

Question 3
Van der Pol Oscillator

Matlab Code

Newton.m

```
function x=newton(fun,x)

ep=1e-7;
n=length(x);

e=eye(n)*ep;

f0=feval(fun,x);
tol=1e-11;

iter=0;
while (iter < 60)*(norm(f0)>tol)
    iter=iter+1;

    D=zeros(n);
    for k=1:n
        D(:,k)=(feval(fun,x+e(:,k))-f0)/ep;
    end

    x=x-D\f0;
    f0=feval(fun,x);
end

if iter==60
    disp('did not converge')
end
```

Continuation_side.m

Editor - /home/sudhanshu/Desktop/ME627/Continuation lecture and files/continuation_side.m

```
function z=continuation_side(x)

global bigX alpha continuation_function

alpha=x(end);

z=feval(continuation_function,x(1:end-1));
Delta_S=norm(bigX(:,1)-bigX(:,2));

z=[z;norm(bigX(:,end)-x)-Delta_S];
```

Continuation.m

Editor - /home/sudhanshu/Desktop/ME627/Continuation lecture and files/continuation.m

```
function continuation(fun,N)

global bigX continuation_function

continuation_function=fun;

for n=1:N
    xg=2*bigX(:,end)-bigX(:,end-1);
    xg=newton('continuation_side',xg);
    bigX=[bigX,xg];
end
```

Van der pol ODE function
Vdp.m

Editor - /home/sudhanshu/Desktop/ME627/Continuation lecture and files/vdp.m

```
function qdot = vdp(t,q)

epsilon=1.1;

qdot = [q(2);-q(1)-q(2)*(q(1)*q(1)-1)*epsilon];

end
```

Van Der Pol Harmonic Balance function
Vdp_junk.m

Editor - /home/sudhanshu/Desktop/ME627/Continuation lecture and files/vdp_junk.m*

```matlab
function z=vdp_junk(x)

global alpha

epsilon=alpha;
A=x(1);B=x(2);C=x(3);P=x(4);Q=x(5);omega=x(6);

z = [ -A*omega^2+A-(1/2)*epsilon*P*B*C*omega+(1/4)*epsilon*B^2*Q*omega+(1/4)*epsilon*A^2*Q*omega-(1/4)*epsilon*C^2*Q*omega;
    (1/2)*epsilon*Q*C*B*omega+(1/2)*epsilon*B^2*A*omega+(1/2)*A*P^2*epsilon*omega-(1/4)*A^2*P*epsilon*omega+(1/4)*B^2*P*epsilon*omega
    +(1/2)*epsilon*Q^2*A*omega+(1/2)*epsilon*C^2*A*omega-(1/4)*epsilon*C^2*omega*P+(1/4)*epsilon*A^3*omega-A*omega*epsilon;
    -epsilon*Q*B*A*omega+epsilon*A*C*omega*P-4*B*omega^2-epsilon*C*A^2*omega-epsilon*P^2*C*omega
    -(1/2)*epsilon*B^2*C*omega-epsilon*Q^2*C*omega+B-(1/2)*epsilon*C^3*omega+2*epsilon*C*omega;
    C+epsilon*A*P*B*omega+epsilon*A*C*Q*omega+epsilon*P^2*B*omega+epsilon*A^2*B*omega+(1/2)*epsilon*C^2*B*omega
    +epsilon*Q^2*B*omega-4*C*omega^2-2*epsilon*B*omega+(1/2)*B^3*epsilon*omega;
    -(3/2*epsilon)*B^2*Q*omega-(3/2*epsilon)*C^2*Q*omega-(3/4*epsilon)*P^2*Q*omega-(3/2*epsilon)*A^2*Q*omega
    +3*Q*epsilon*omega-(3/4*epsilon)*Q^3*omega-9*omega^2*P-(3/2*epsilon)*A*C*B*omega+P;
    -(1/4)*epsilon*A^3*omega+(3/4*epsilon)*P^3*omega-3*epsilon*omega*P+(3/4*epsilon)*B^2*A*omega+(3/2*(A^2))*P*epsilon*omega
    +(3/2*(B^2))*P*epsilon*omega-(3/4*epsilon)*C^2*A*omega+(3/2*epsilon)*C^2*omega*P+(3/4*epsilon)*Q^2*omega*P+Q-9*Q*omega^2
];
```

Code for analyzing and implementing the Van Der Pol Oscillator using Harmonic Balance

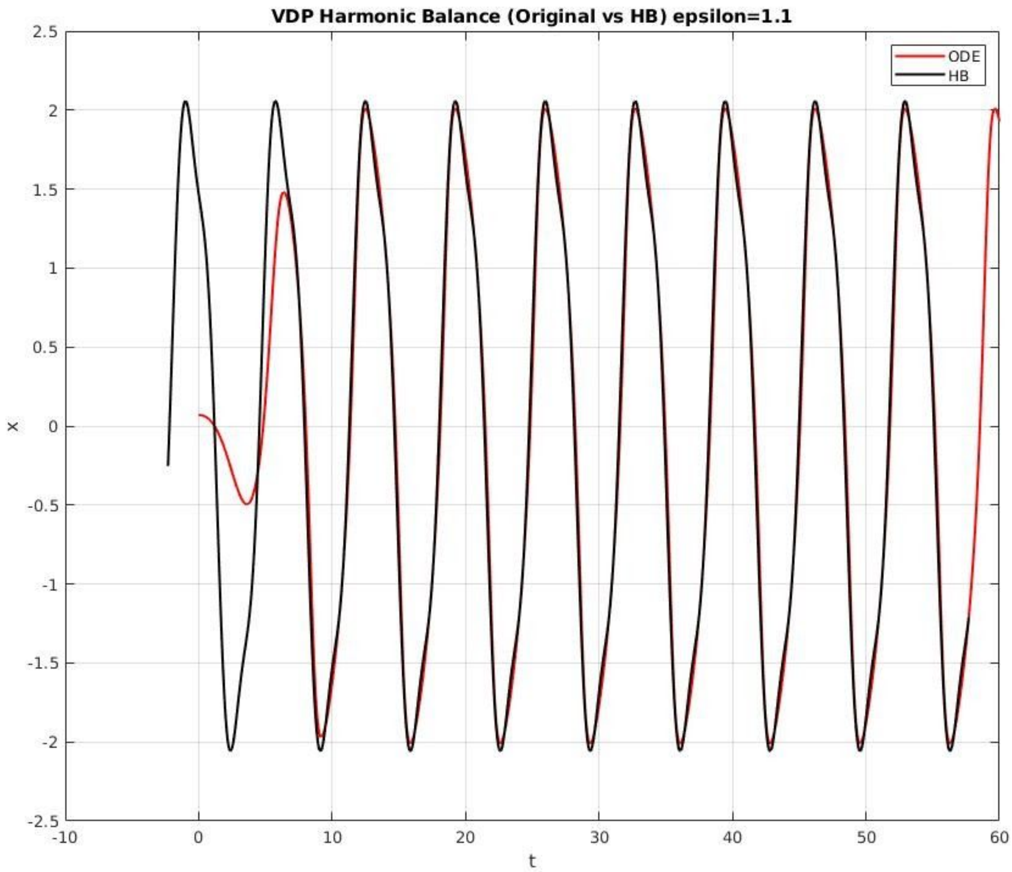
Editor - /home/sudhanshu/Desktop/ME627/Continuation lecture and files/analyze_vdp.m

```matlab
clear all;
clc;

options = odeset('AbsTol',1e-8,'RelTol',1e-8);
[t,q]=ode45('vdp',[0,240],[0.01;0],options);

%% Continuation for 60 steps
global alpha bigX;
alpha=0;
bigX=[];
y=newton('vdp_junk',[2;0;0;0;0;1]);
bigX=[bigX, [y;alpha]];
alpha=0.02;
y=newton('vdp_junk',[2;0;0;0;0;1]);
bigX=[bigX, [y;alpha]];
continuation('vdp_junk',60);

%% ODE solution for eps=1.1 for 60 steps
[t,q]=ode45('vdp',[0,60],[0.07;0],options);
%final solution from continuation
a=bigX(:,end);
tau=a(6)*t;
x = a(1)*sin(tau)+a(2)*sin(2*tau)+a(3)*cos(2*tau)+a(4)*sin(3*tau)+a(5)*cos(3*tau);

figure(1)
plot(t,q(:,1),t-2.6,x);
grid

xlabel('t')
ylabel('x')
title('VDP Harmonic Balance (Original vs HB) epsilon='+string(a(end)))
legend('ODE','HB')

figure(2)
plot(t,q);
grid

xlabel('t')
ylabel('x')
title('VDP ODE solution 60 steps with epsilon=1.1');
legend('ODE')
%% Harmonic Balance for epsilon =1.1

alpha=1.1;
y=newton('vdp_junk',[2.0225;0;0;0.1241;-0.2548;0.9308]);
```

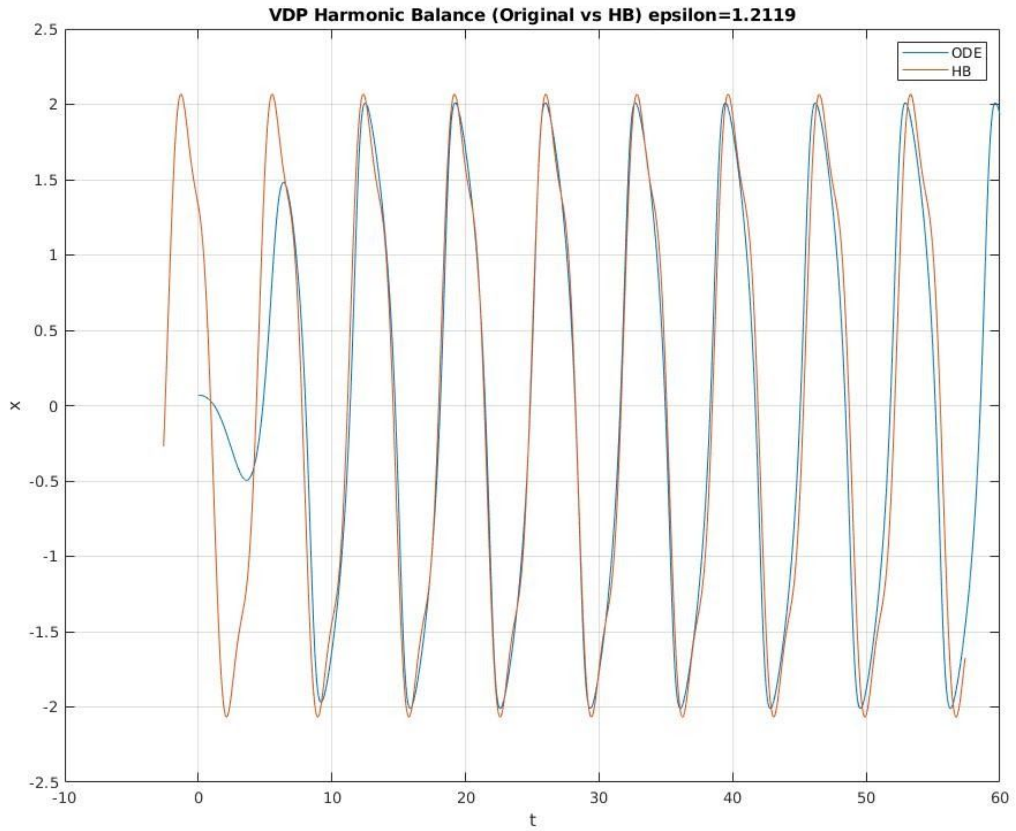
```
bigX=[bigX, [y;alpha]];
a=bigX(:,end);
tau=a(6)*t;
x = a(1)*sin(tau)+a(2)*sin(2*tau)+a(3)*cos(2*tau)+a(4)*sin(3*tau)+a(5)*cos(3*tau);

figure(3)
plot(t,q(:,1),'Color','r','linewidth',1.5);
hold on
plot(t-2.3, x,'Color','k','linewidth',1.5);
grid
xlabel('t')
ylabel('x')
title('VDP Harmonic Balance (Original vs HB) epsilon='+string(a(end)))
legend('ODE','HB')
```

Plots for the VDP oscillator

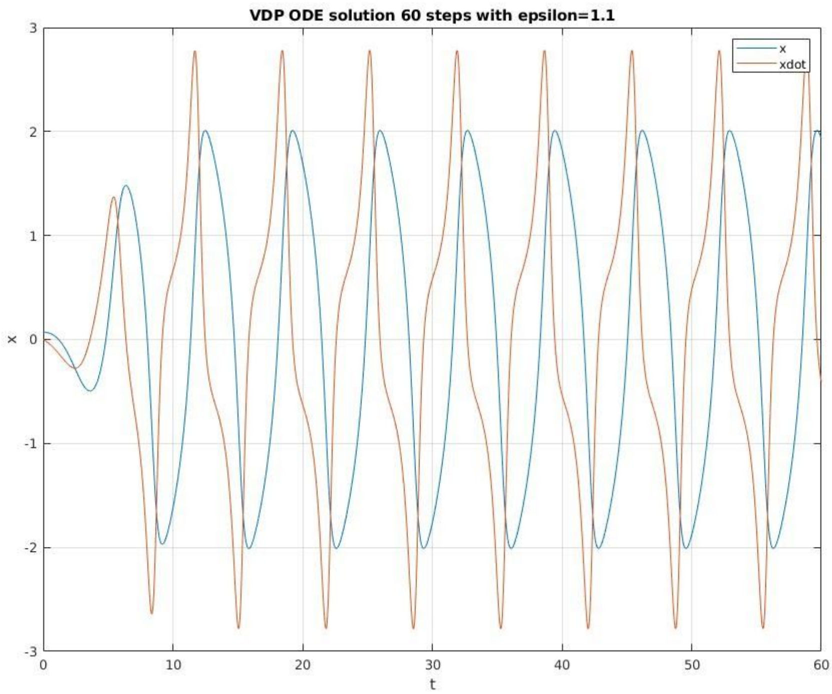
VDP ODE vs HB eps=1.1



VDP Harmonic Balance with Continuation plotting for epsilon=1.2119



VDP for 60 steps using ODE for 60 steps at eps=1.1 exactly



Using ginput

The chosen pts for Time Period calculation are

$x1 = 2.0025$ $t1 = 25.7921$
$X2 = 2.0025$ $t2 = 32.5062$

Time period = 32.5062-25.7921=6.7141