Name: Sudhanshu Mishra
Roll no. 17807726

Q1.

After using Poincare-Linsteadt method on the modified Van der Pol oscillator, we get the following equation for A which is the amplitude of X_o at t=0;

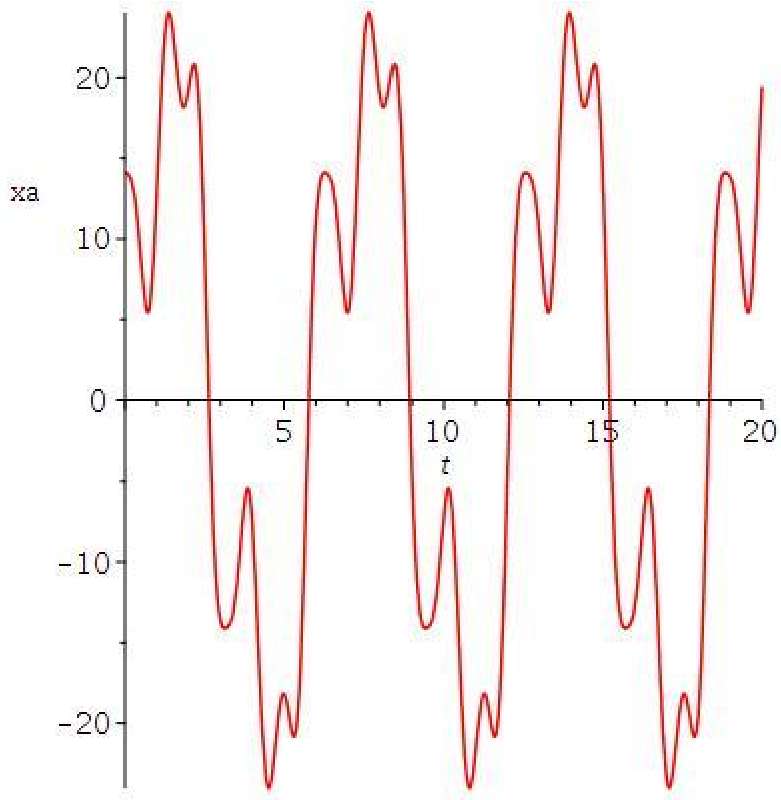
$$temp := \left\{A = \frac{\sqrt{c\left(1+\sqrt{1-8c}\right)}}{c},\ w[1] = 0\right\};$$

For the system to seek limit cycles, A must be real. Therefore,
0<c< 0.125;

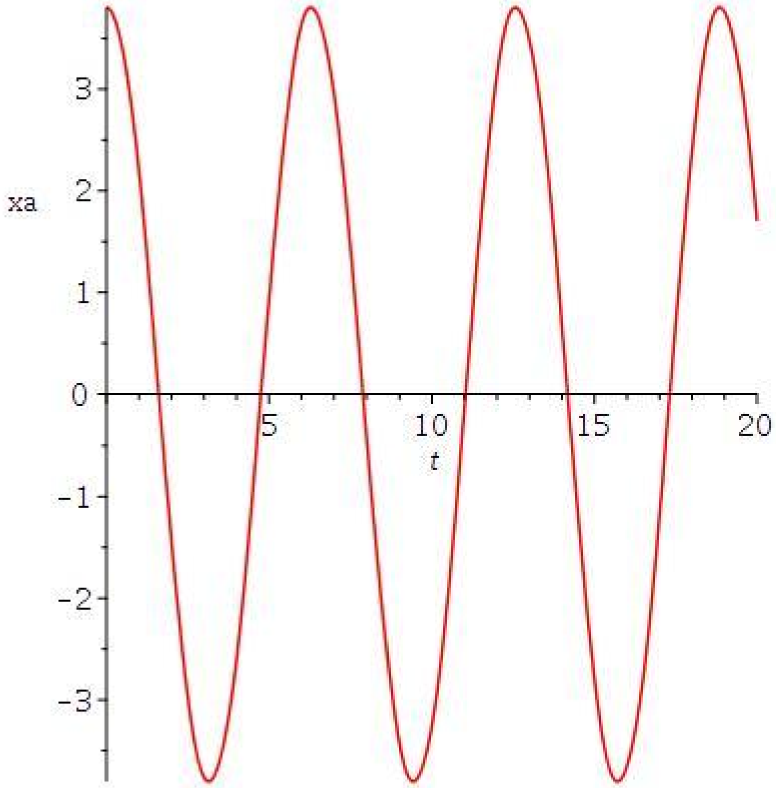
The final expression for omega is :

$$\varpi := 1 - \frac{\epsilon^2\left(-10c\sqrt{1-8c}+56c^2+\sqrt{1-8c}-14c+1\right)}{384c^2}$$

For epsilon = 0.1, c=0.01, the plot of xa vs t



For epsilon = 0.1, c=0.1, the plot of xa vs t



We see that the solution is indeed periodic.

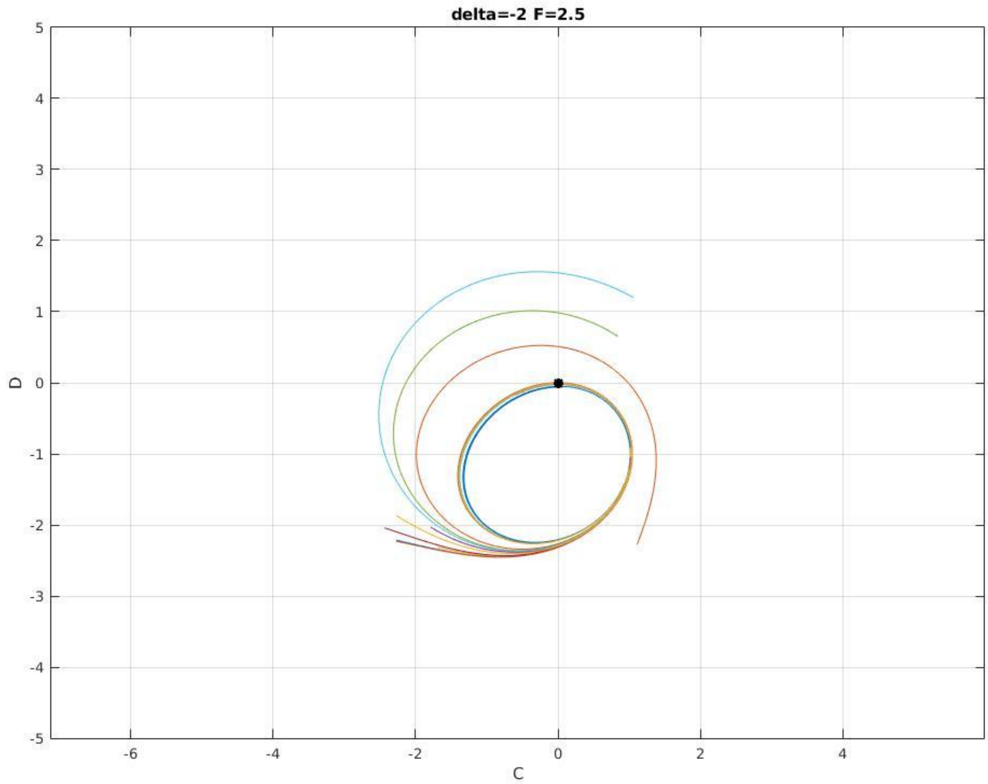
Q3.

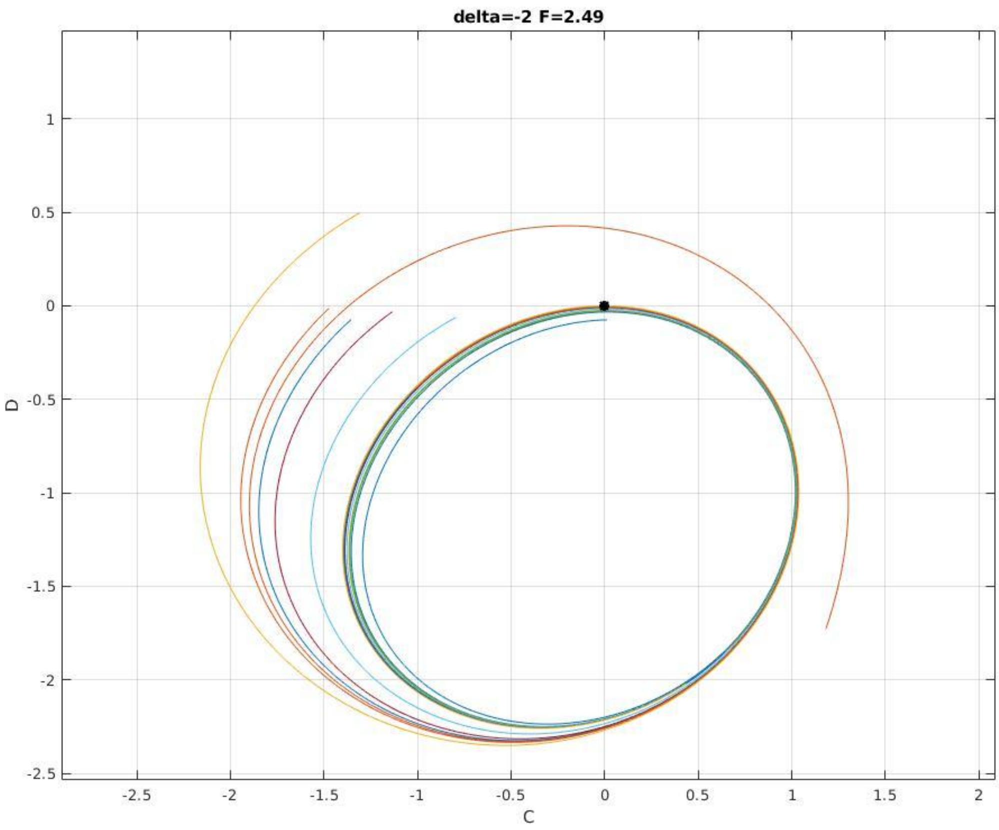
Using the equations for the derivatives of C and D from the notes.

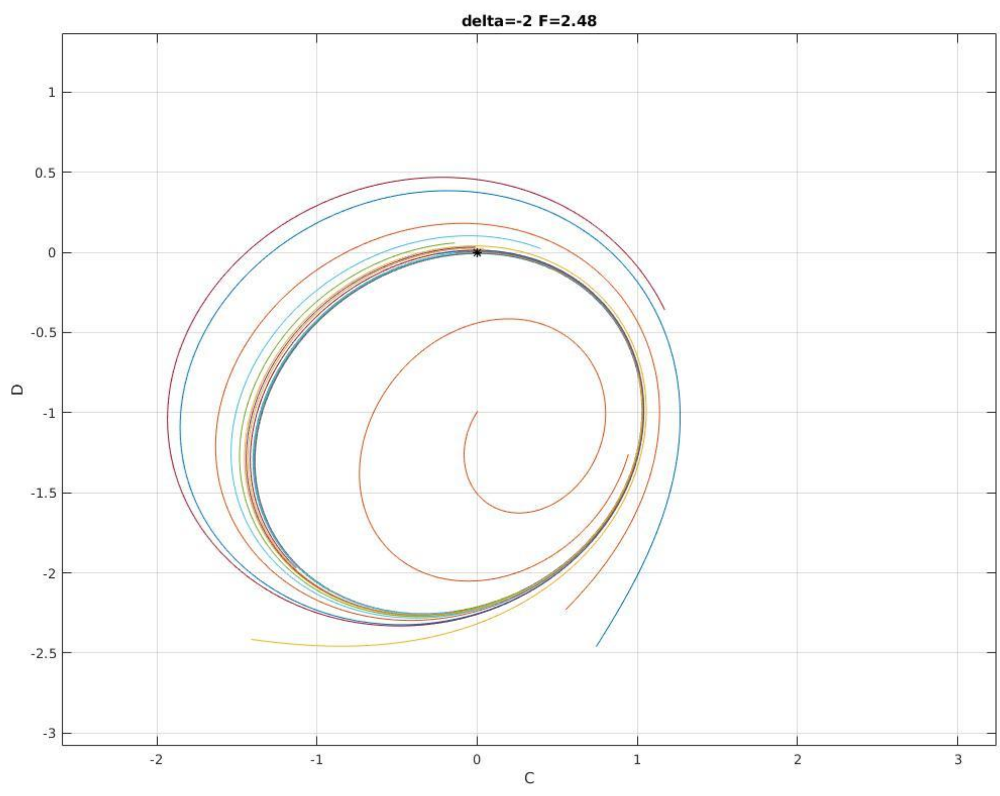
$$\dot{C} = \epsilon\left(\frac{C}{2} - \frac{C^3}{8} + \frac{\Delta D}{2} - \frac{CD^2}{8} - \frac{F}{2}\right), \text{ and}$$
$$\dot{D} = \epsilon\left(\frac{D}{2} - \frac{D^3}{8} - \frac{\Delta C}{2} - \frac{C^2 D}{8}\right).$$

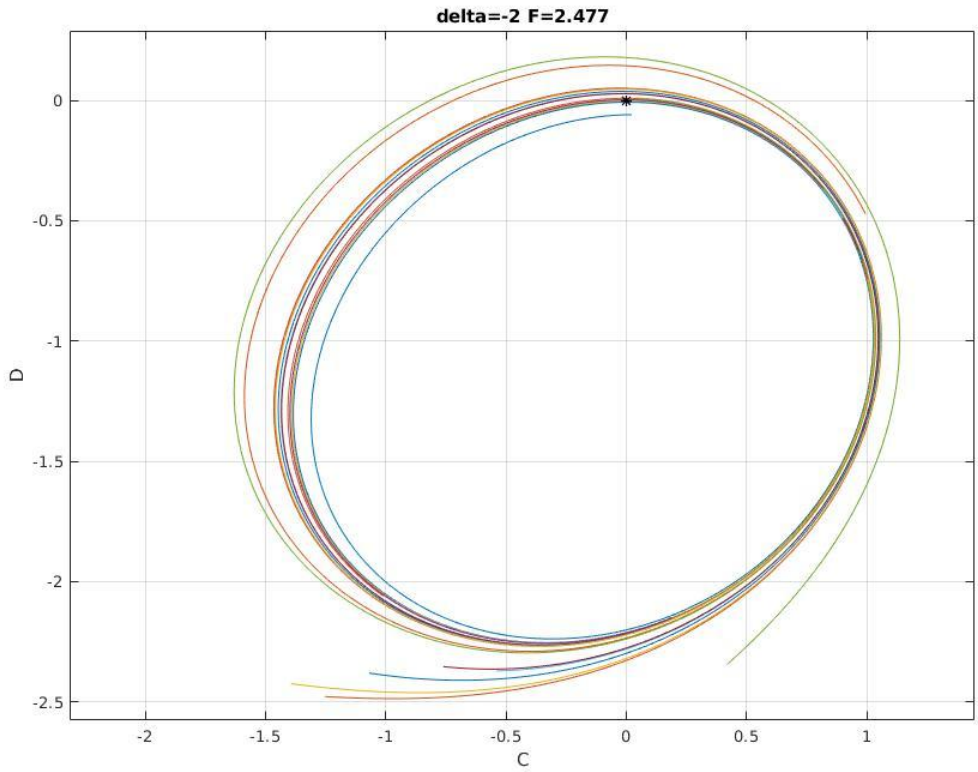
Taking epsilon =0.1 and delta=-2;

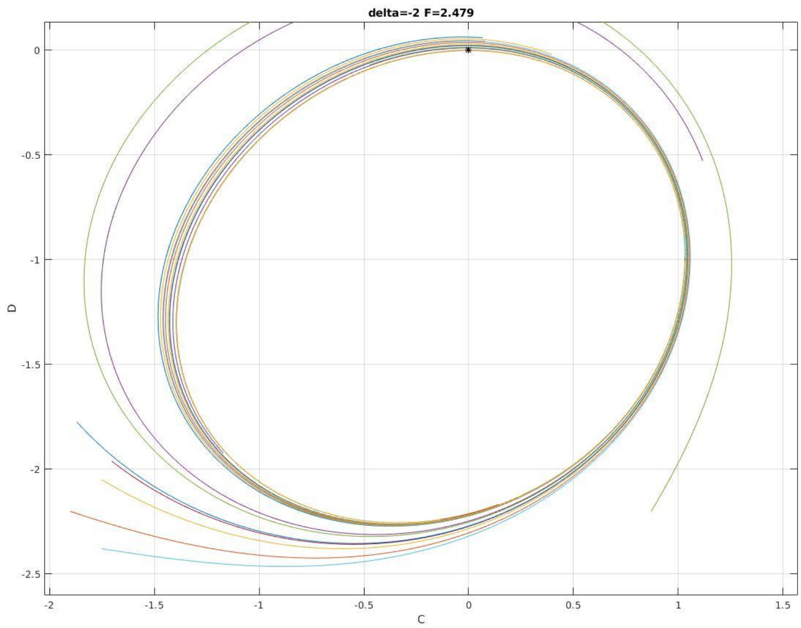
We obtain the following phase portraits for different values of F:




delta=-2 F=2.49
D
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
C


delta=-2 F=2.48
D
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-2
-1
0
1
2
3
C





We see that at F ~ 2.477 the transition from drift to weak entrainment happens, i.e the closed curve no longer contains the origin.

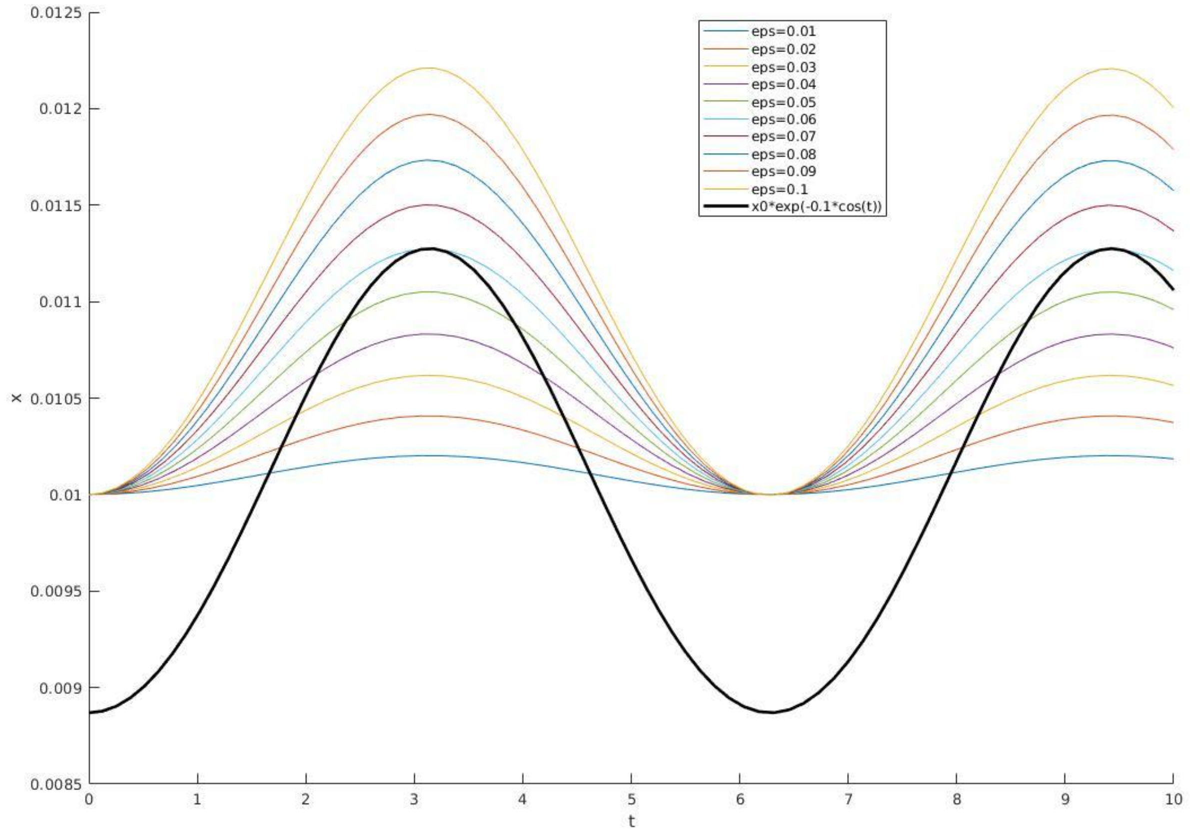
MATLAB Code:

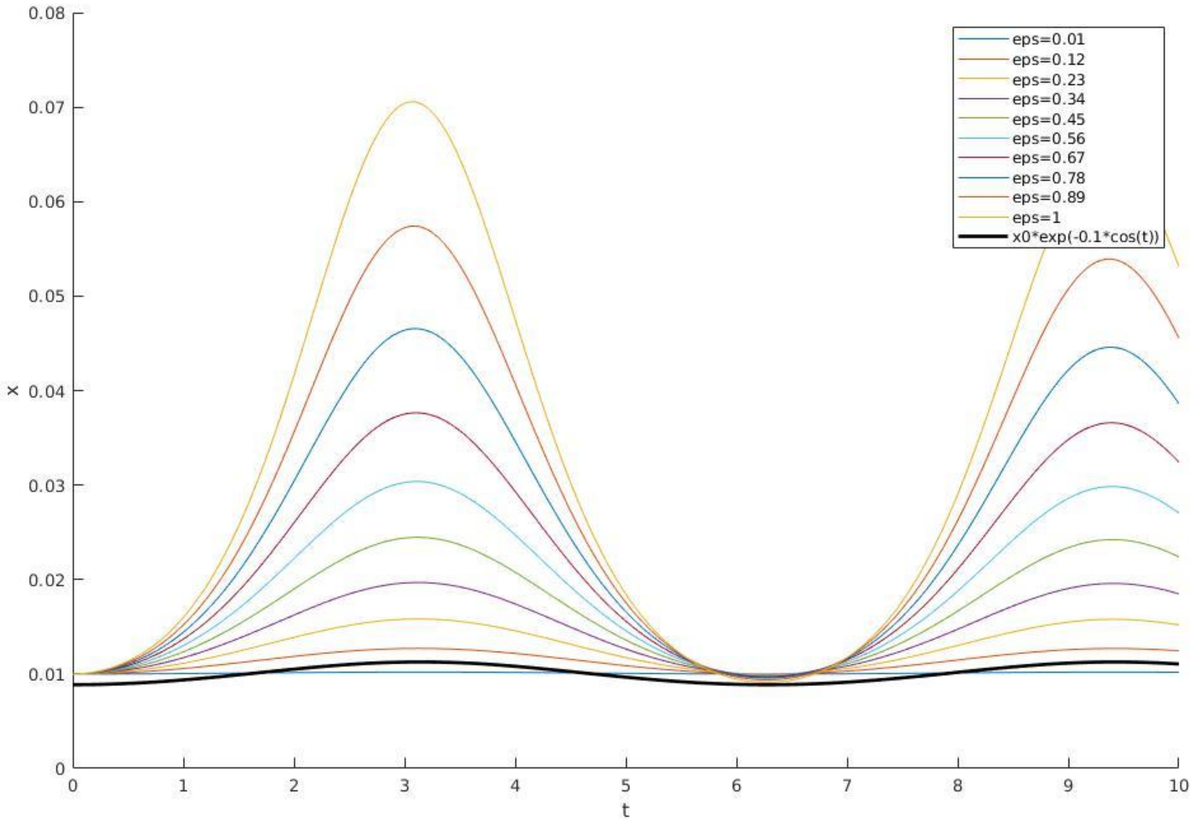
```
function qdot= entrain(t,x)
global epsilon;
global del;
global F;
qdot =
[epsilon*(0.5*x(1)-0.125*x(1)^3+0.5*del*x(2)-0.125*x(1)*x(2)^2-0.5*F);epsilon*(0.5*x(2)-0.12
5*x(2)^3-0.5*del*x(1)-0.125*(x(1)^2)*x(2))];

end

clc;
clear;
close all;

global epsilon;
global del;
global F;

del=-2;
F=2.477;
epsilon=0.1;
figure(1)
axis([-10,10,-10,10]);
axis 'manual'

op = odeset('reltol',1e-7,'abstol',1e-9);
for k=1:10
   grid
   x=ginput(1);
   [t,y]=ode45('entrain',[0,100],x,op);

   plot(y(:,1),y(:,2),'HandleVisibility','off');
   hold on;
end
plot(0,0,'k*','linewidth',1);
hold off
xlabel('C');
ylabel('D');
title('delta='+string(del)+' F='+string(F));
axis([-3,3,-3,3])
axis 'equal'

figure(2)
plot()
```

Q2.

For the initial condition x0 = 0.01;

The numerically integrated plots for different epsilons are as follows:

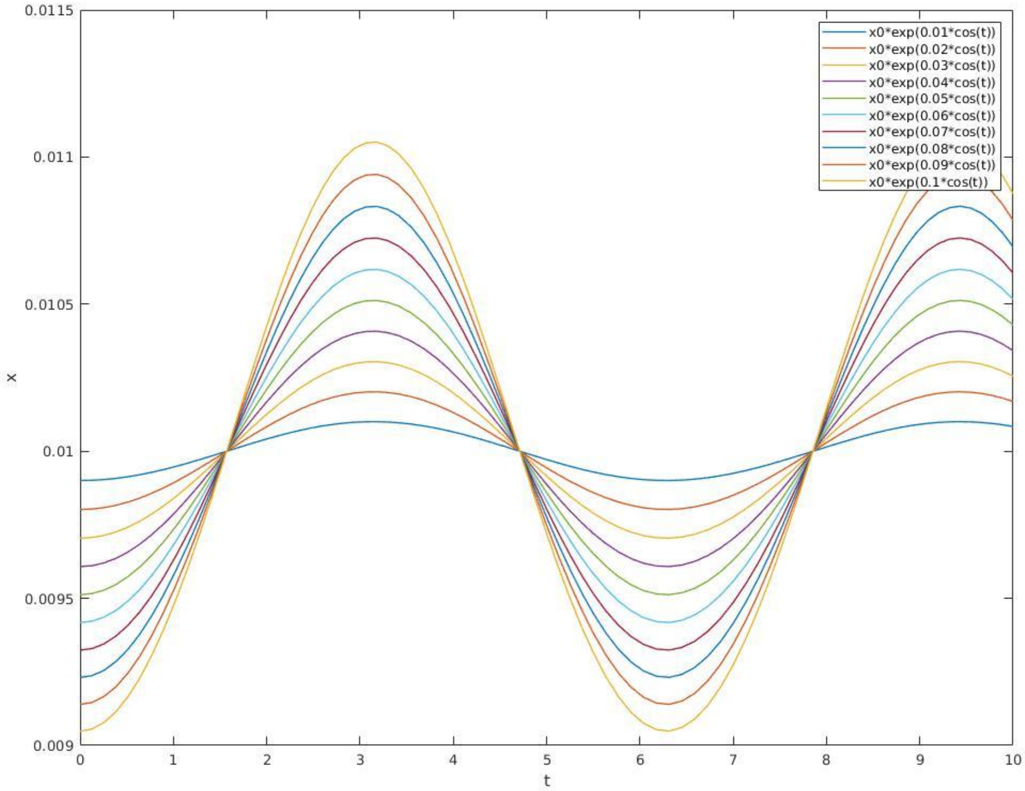




We see that during the upper cycle as epsilon increases the response of the system is perturbed more. However, during the lower cycle epsilon has a very weak effect on the response. As can be seen that when epsilon reaches 1, only then does the response in the lower cycle show minor oscillations.

The dark black line is the response when considering x_dot = epsilon*x*sin(t);

The response of this approximation for different epsilons is as follows:



We see that the approximation is able to match the upper cycle quite well however the match for the lower cycle is very bad.

On applying first order averaging,

We see that $$1/(2\pi) * \int_0^{2\pi} xsin(t + x)dt = 0$$

Therefore,

Xa_dot = 0;
Hence f(a)=0;
Therefore, m can take any value.

Another possible way to average this equation is to do the average over half the period.
From 0 to pi the average is = 2x*cos(x);
From pi to 2*pi the average is = -2x*cos(x);

This is the reason for the curves that we get above;
During the first 0 to pi, the solution increases rapidly and saturates. While during the second half it plummets to 0.
This is the reason that during the upper cycle the magnitude increases while during the lower cycle it almost stays constant, as the derivative is negative.

Thus if considering the averaging in this sense we get m=1;

MATLAB Code:

```
clc;
close all;
clear all;
global epsilon;
epsilon = 0.1;
%avg_sin(0.1,2);
op = odeset('reltol',1e-7,'abstol',1e-9);
x = 0.01;
%x = linspace(0.1,1,2);
eps = linspace(0.01,0.1,2);
figure(1)
for k=x
   for epsilon=eps
      [t,y]=ode45('avg_sin',[0,100],k,op);
      hold on;
      plot(t,y,'DisplayName','eps='+string(epsilon));
      avg = mean(y.*sin(t+y));
      davg = epsilon*avg;
      plot(t,davg,'k','HandleVisibility','off');
   end
end

%et = x*exp(-0.12*cos(t));
%plot(t,et,'-k','linewidth',2,'DisplayName','x0*exp(-0.1*cos(t))');
hold off;
xlabel('t')
ylabel('x')
legend
% figure(2)
% for epsilon=eps
%       et = x*exp(-epsilon*cos(t));
%       plot(t,et,'linewidth',1,'DisplayName','x0*exp('+string(epsilon)+'*cos(t))');
%       hold on;
% end
% hold off;
% xlabel('t')
% ylabel('x')
% legend
% axis([0,10,-0.01,0.02]);
% axis 'auto'
```