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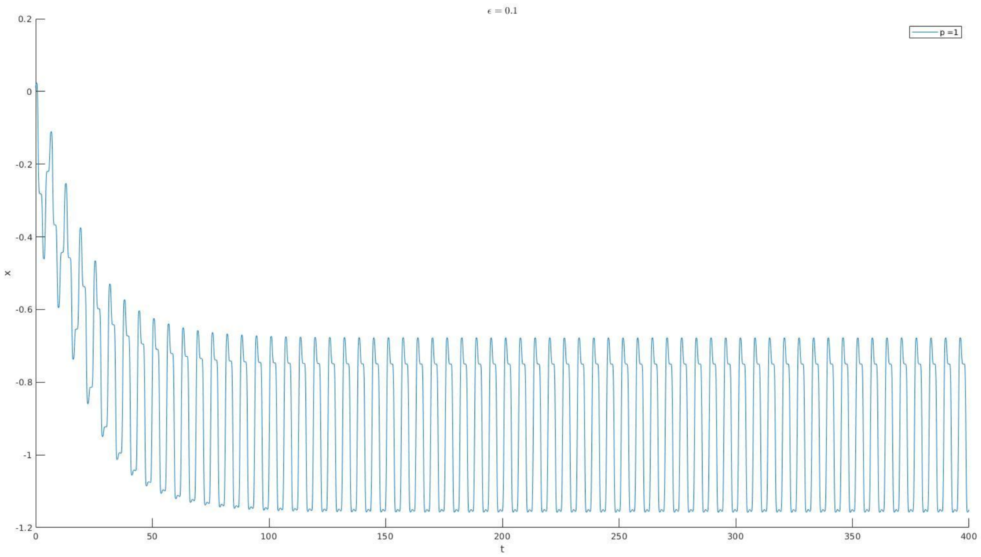
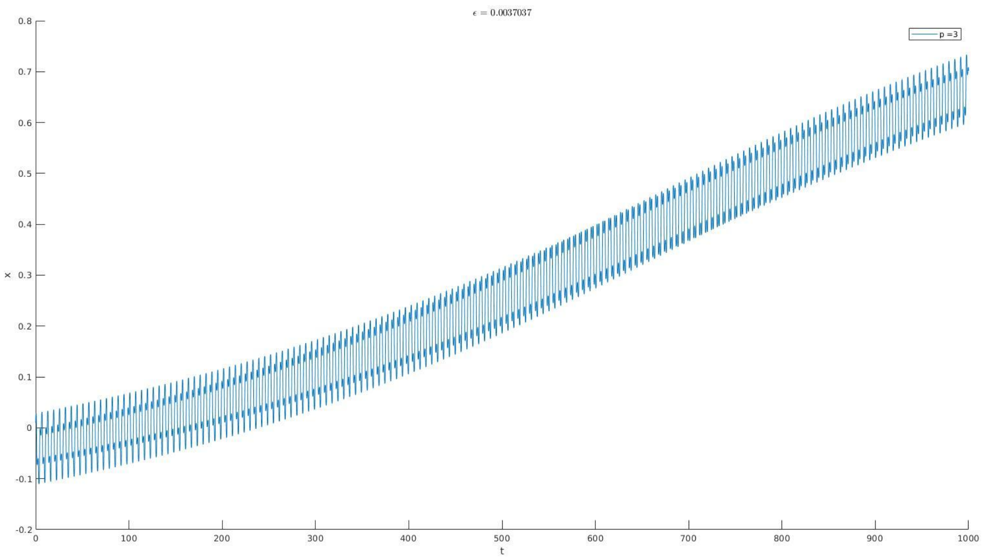
Q1.
Numerical Simulation where avg value stabilizes.

P = 1



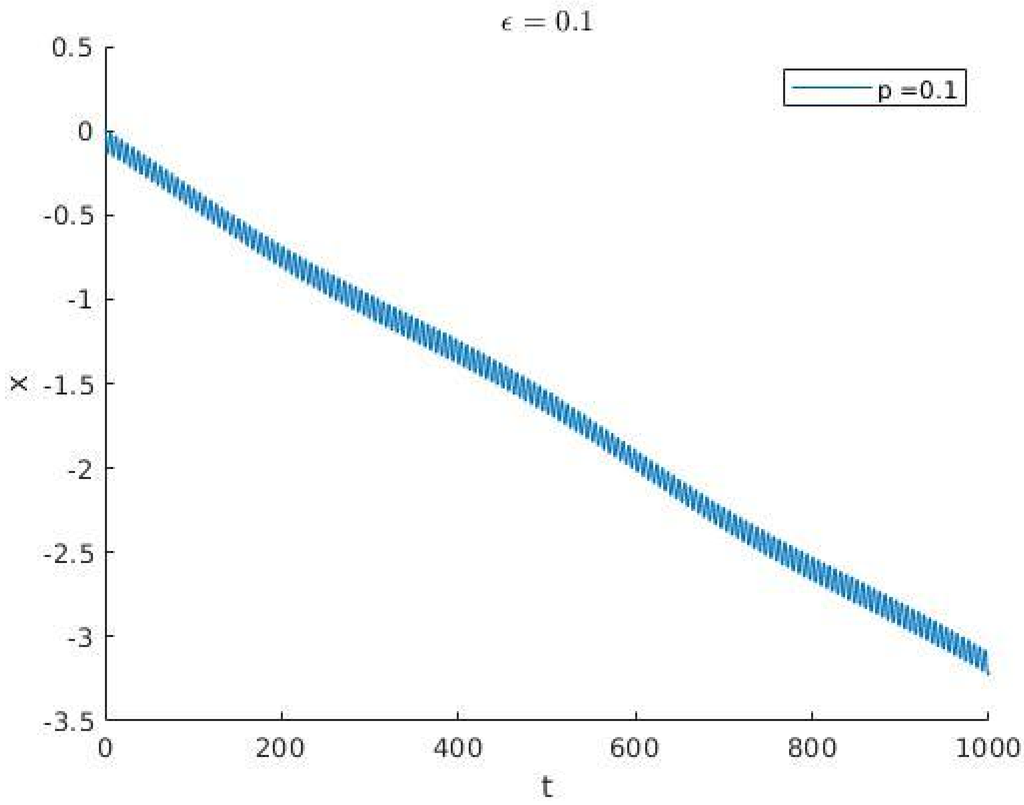
Numerical Simulation where it grows without bound

P = 3



Numerical Simulation where it grows negatively without bound

P =0.1



After applying MMS : The avgd eqn is

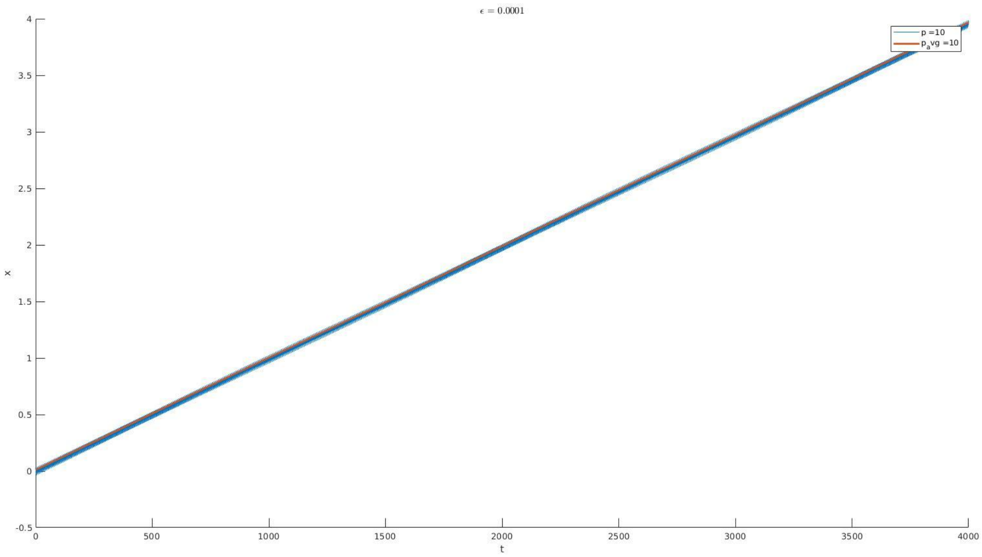
$$repreq := \frac{d}{dT_2} X(T_2) + \frac{(-15792\,p^3 - 5880\,p)\sin(X(T_2))\cos(X(T_2))^3}{1680} + \frac{(7896\,p^3 + 2940\,p)\sin(X(T_2))\cos(X(T_2))}{1680} - \frac{5\,p^6}{48} + \frac{303\,p^4}{560} - \frac{3\,p^2}{80} + \frac{5}{16}$$

Choosing epsilon = $0.1/p^3$

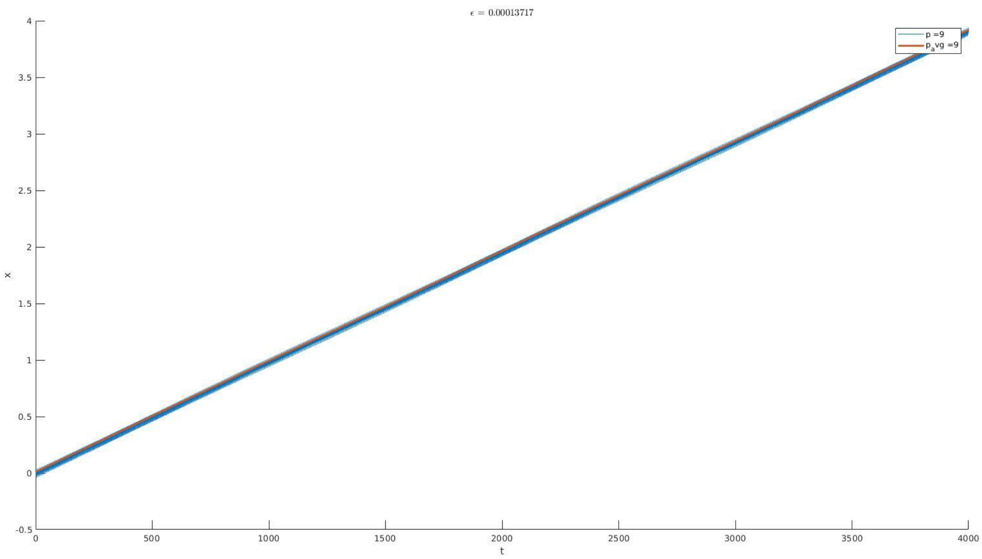
For different values of p:

P ranging from 1 to 10 :

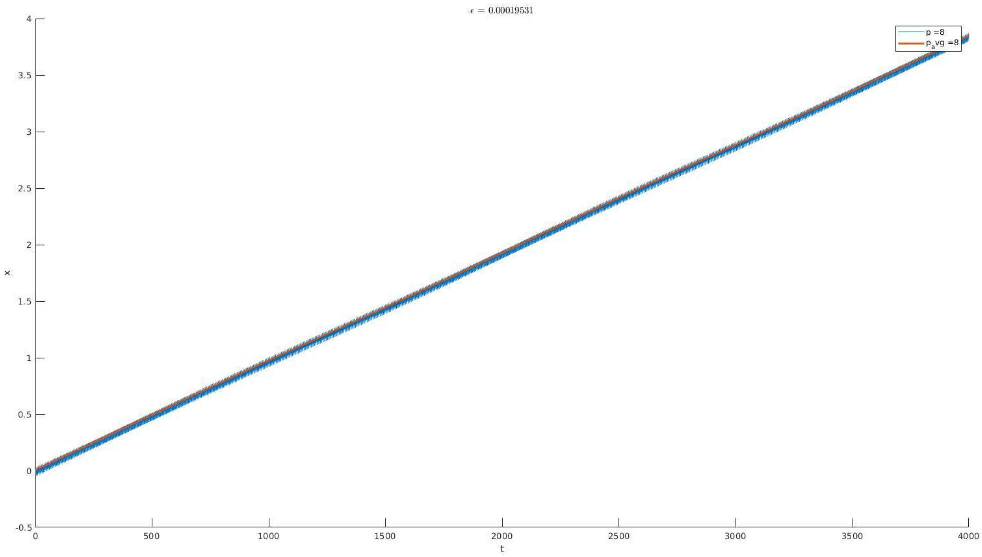
P =10



P = 9

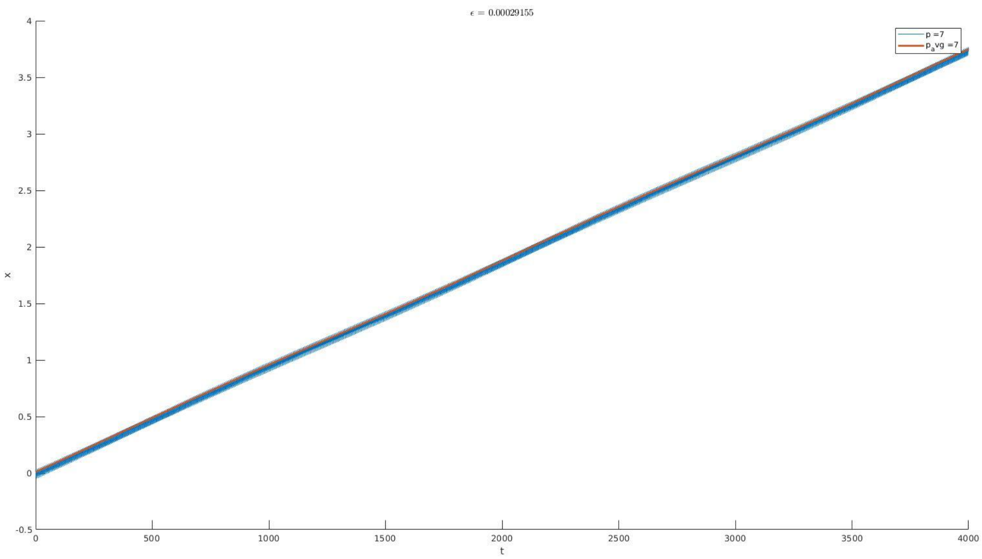


P = 8



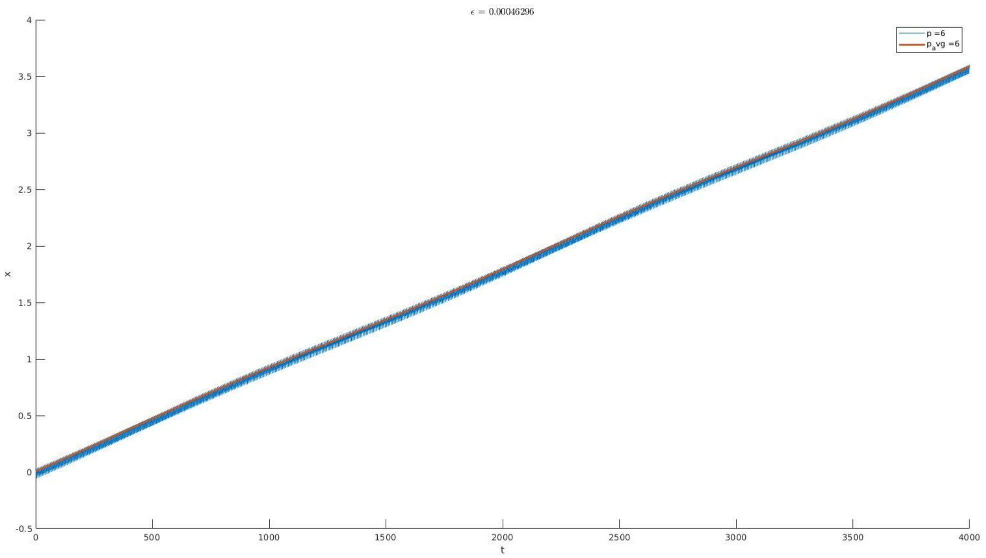
$\epsilon = 0.00019531$
p =8
p_avg =8
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P = 7

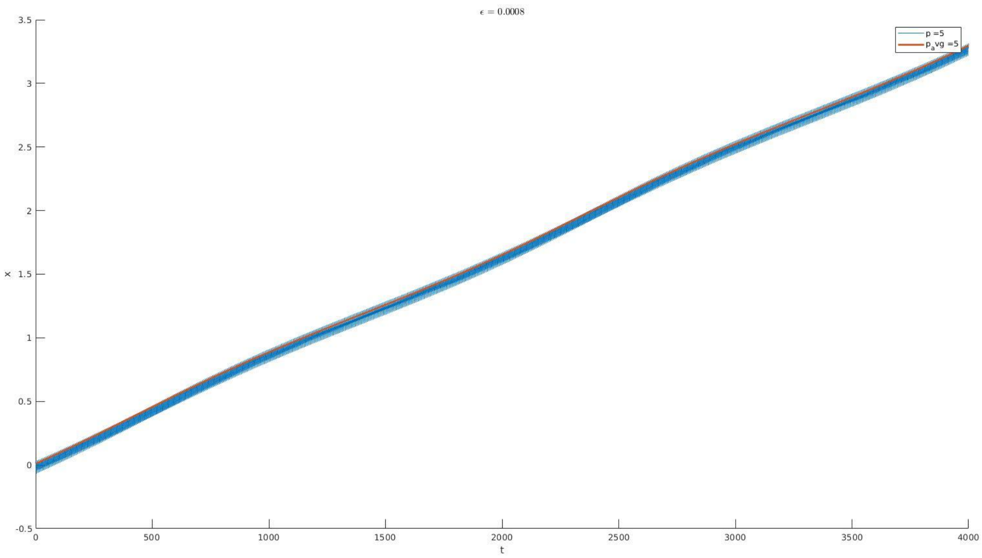


$\epsilon = 0.00029155$
p =7
p_avg =7
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P = 6

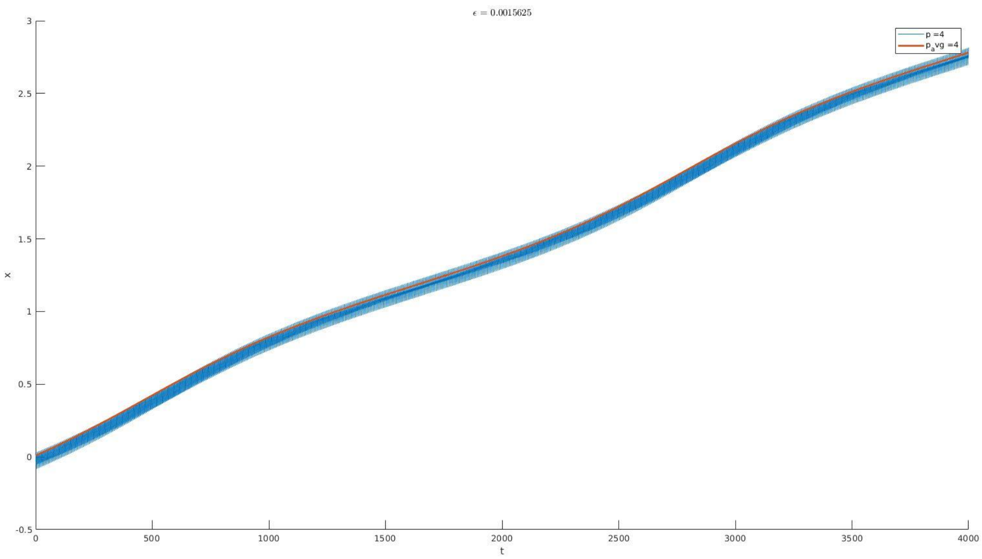


P = 5



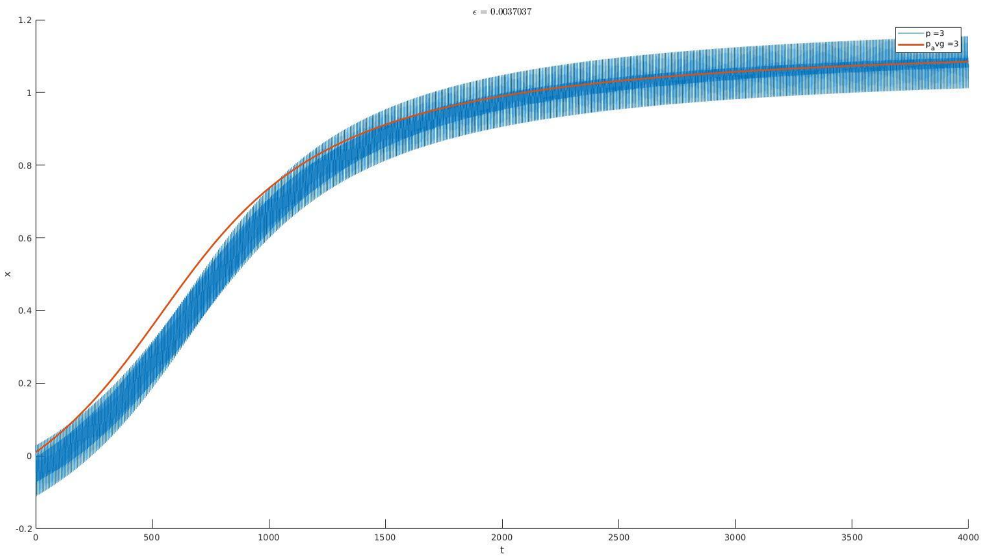
ϵ = 0.0008
p =5
p_avg =5
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P = 4

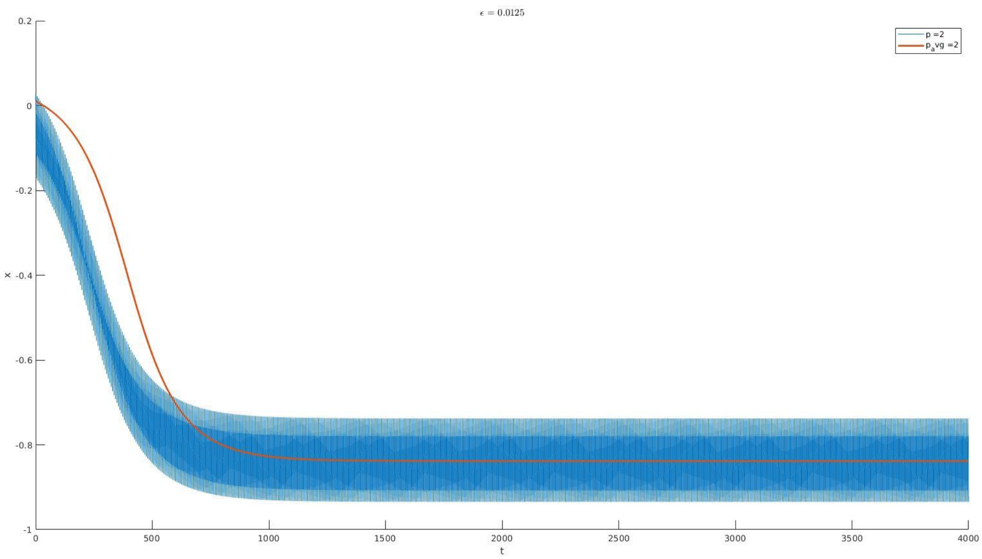


ϵ = 0.0015625
p =4
p_avg =4
3
2.5
2
1.5
1
0.5
0
-0.5
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

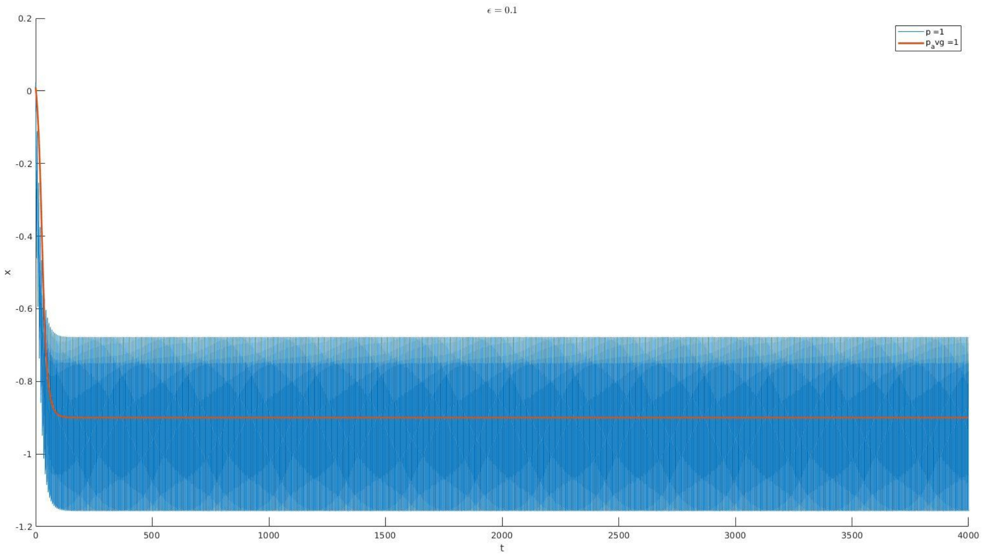
P = 3



P = 2

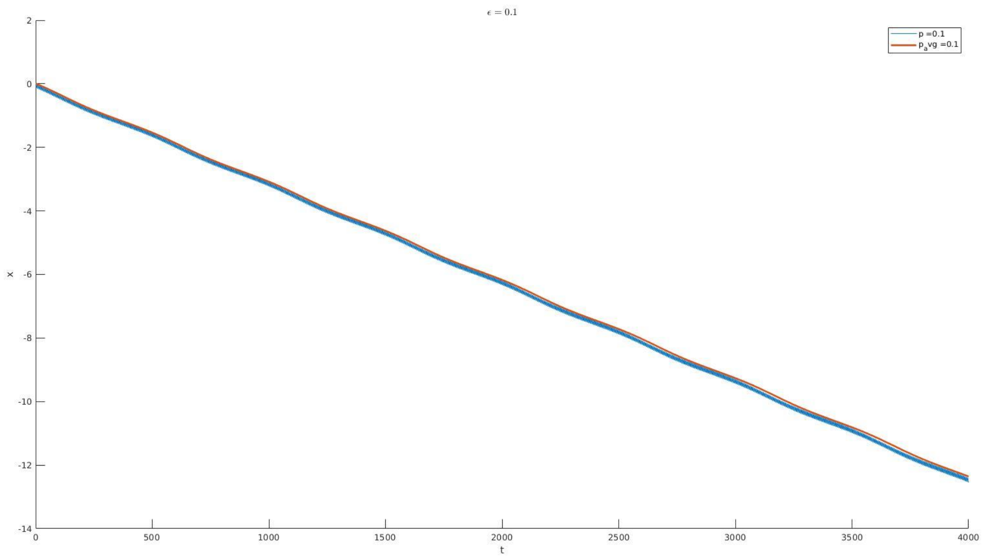


P = 1

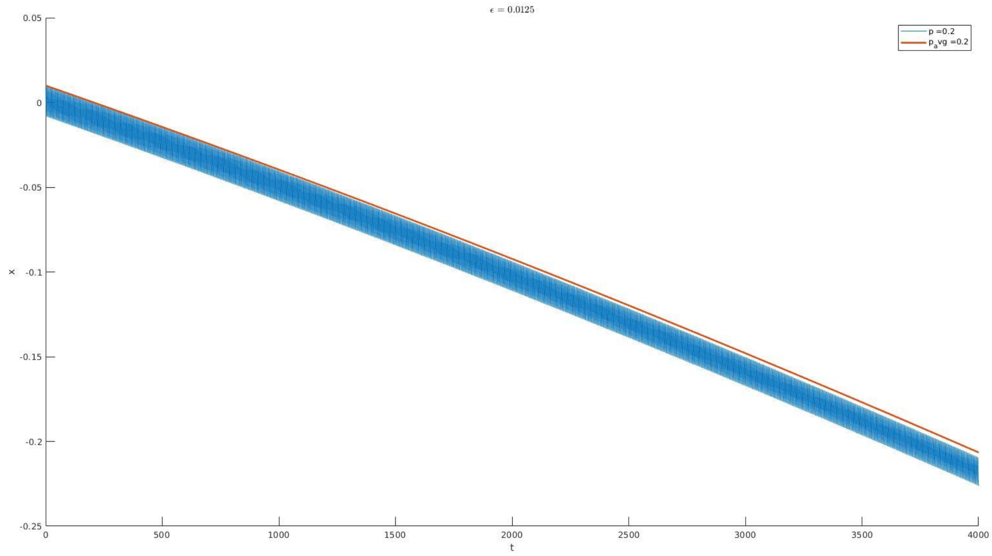


Different values of P ranging from 0.1 to 0.9
Choosing epsilon = $0.0001/p^3$

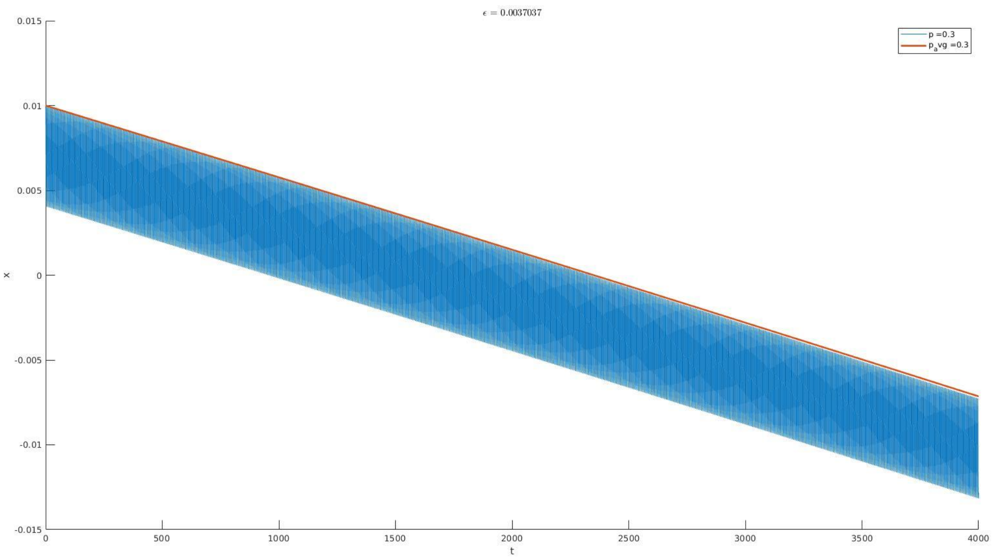
P = 0.1



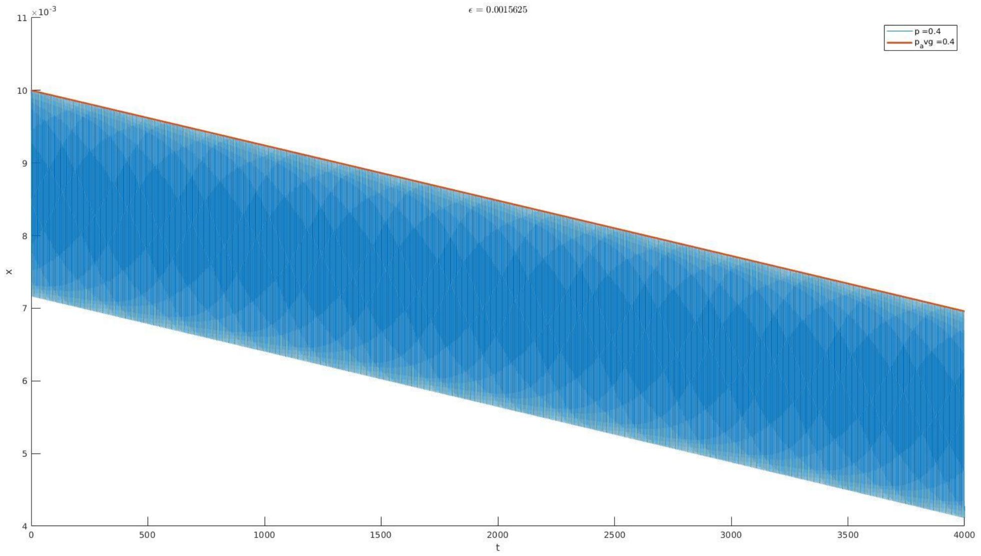
P = 0.2


$\epsilon = 0.0125$
p =0.2
$p_a$vg =0.2
x
t
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
500
1000
1500
2000
2500
3000
3500
4000

P = 0.3

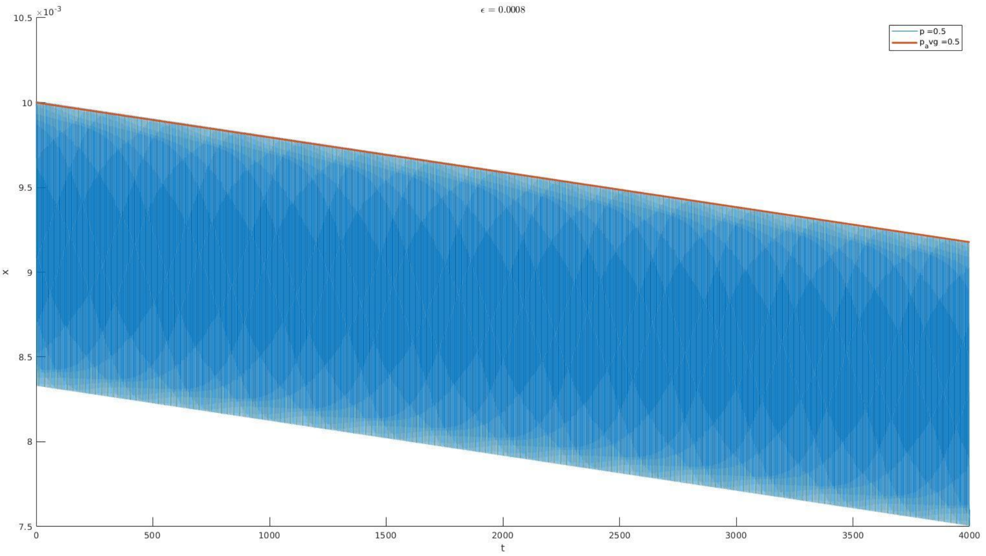

$\epsilon = 0.0037037$
p =0.3
$p_a$vg =0.3
x
t
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0
500
1000
1500
2000
2500
3000
3500
4000

P = 0.4



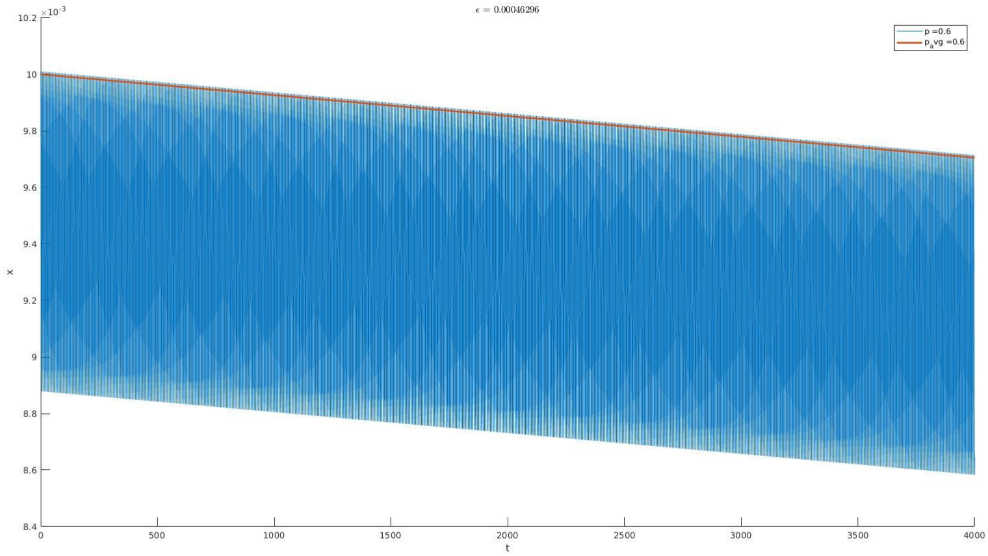
$\epsilon = 0.0015625$
p =0.4
p_avg =0.4
×10^-3
11
10
9
8
7
6
5
4
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P = 0.5



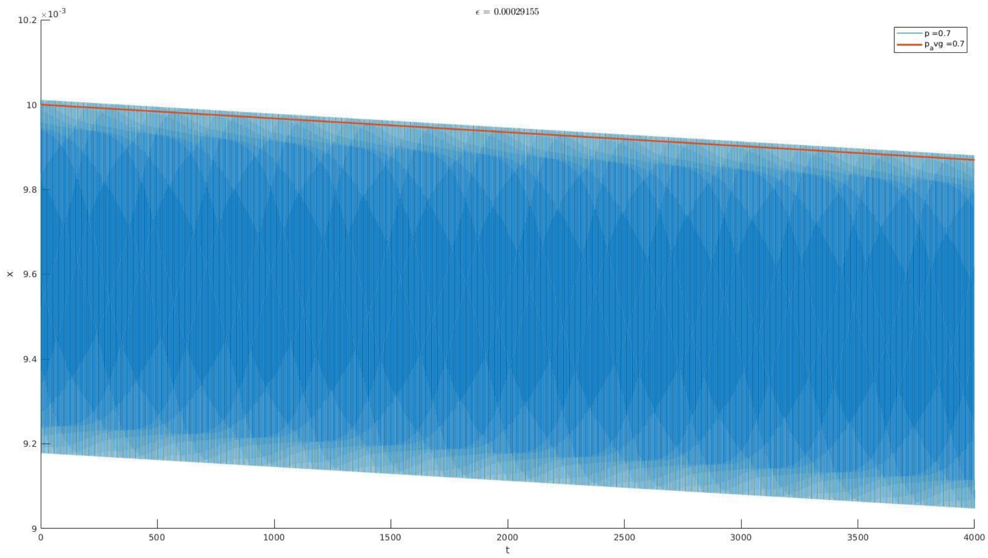
$\epsilon = 0.0008$
p =0.5
p_avg =0.5
×10^-3
10.5
10
9.5
9
8.5
8
7.5
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P = 0.6



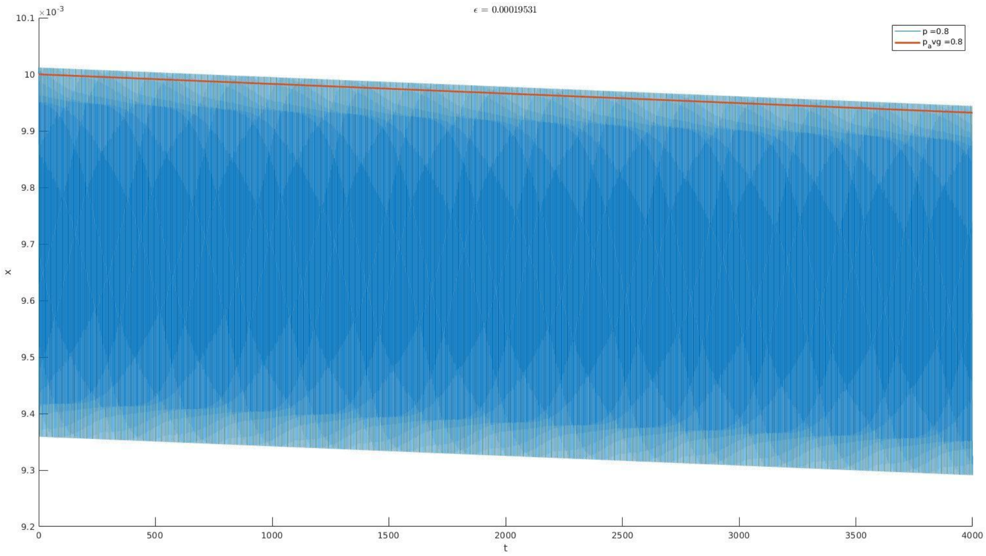
ϵ = 0.00046296
p =0.6
$p_a$vg =0.6
$\times 10^{-3}$
10.2
10
9.8
9.6
9.4
9.2
9
8.8
8.6
8.4
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P = 0.7

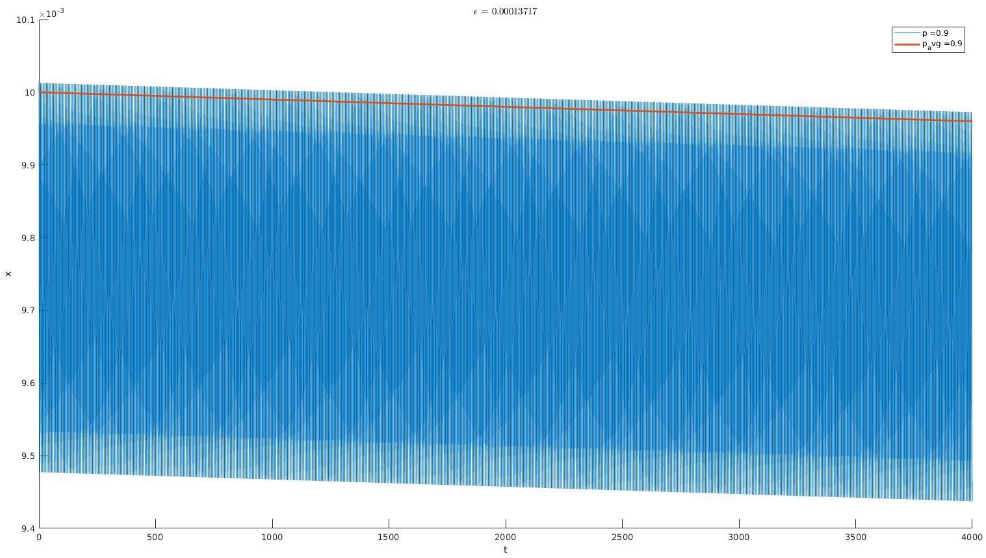


ϵ = 0.00029155
p =0.7
$p_a$vg =0.7
$\times 10^{-3}$
10.2
10
9.8
9.6
9.4
9.2
9
x
0
500
1000
1500
2000
2500
3000
3500
4000
t

P=0.8

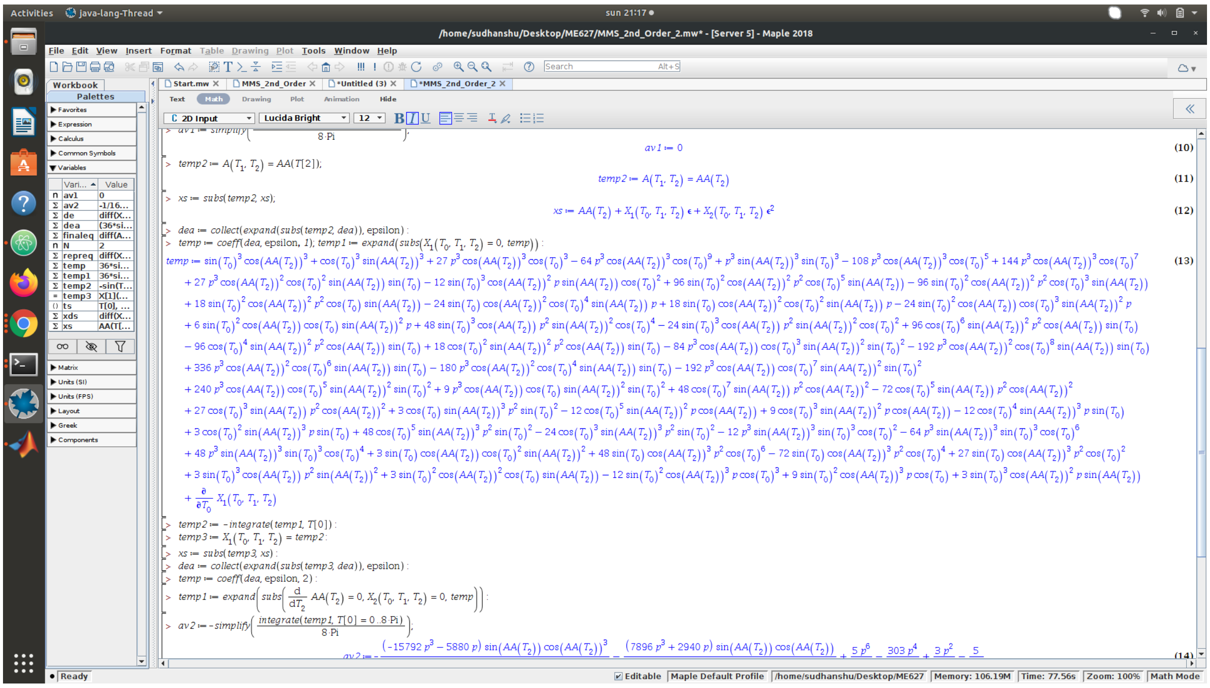


P = 0.9

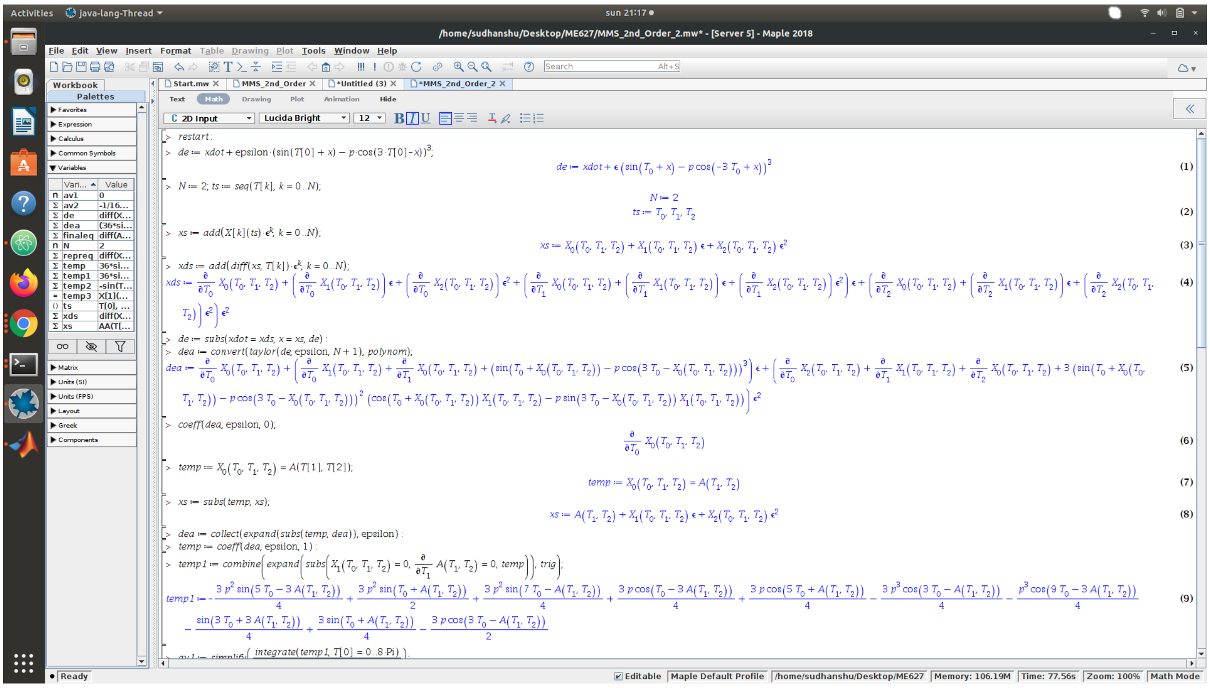


Discussion: The MMS avgd solution matches the numerical solution (in blue) for most cases. There is significant mismatch for P =3 and P =2 cases. For other P>1 cases, the solution is a good averaged version of the original curve. For P<1 cases, the averaged curve is able to follow the original solution, but it does not follow the mean value, rather it follows the peaks of the curve.

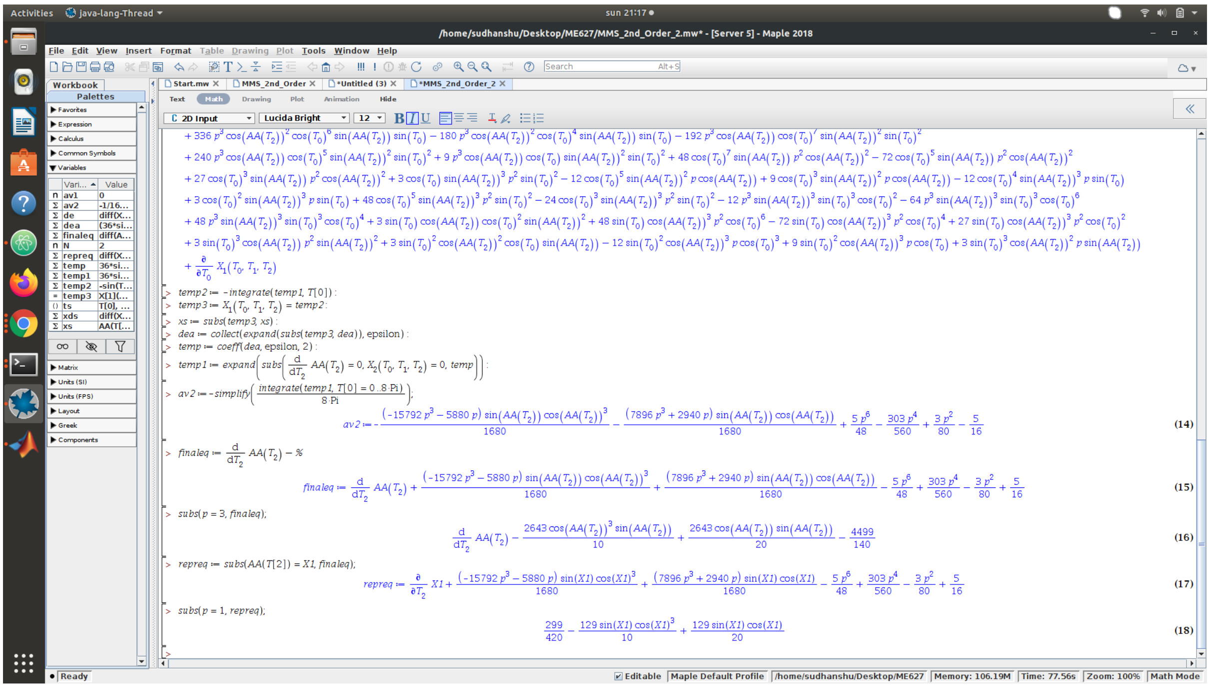
Maple Code Below:



/home/sudhanshu/Desktop/ME627/MMS_2nd_Order_2.mw* - [Server 5] - Maple 2018
File Edit View Insert Format Table Drawing Plot Tools Window Help
Start.mw MMS_2nd_Order *Untitled (3) *MMS_2nd_Order_2
av1 := 0 (10)
> temp2 := A(T_1, T_2) = AA(T[2]);
temp2 := A(T_1, T_2) = AA(T_2) (11)
> xs := subs(temp2, xs);
xs := AA(T_2) + X_1(T_0, T_1, T_2) ϵ + X_2(T_0, T_1, T_2) ϵ^2 (12)
> dea := collect(expand(subs(temp2, dea)), epsilon) :
> temp := coeff(dea, epsilon, 1); temp1 := expand(subs(X_1(T_0, T_1, T_2) = 0, temp)) :
(13)
> temp2 := -integrate(temp1, T[0]) :
> temp3 := X_1(T_0, T_1, T_2) = temp2 :
> xs := subs(temp3, xs) :
> dea := collect(expand(subs(temp3, dea)), epsilon) :
> temp := coeff(dea, epsilon, 2) :
> temp1 := expand(subs(d/dT_2 AA(T_2) = 0, X_2(T_0, T_1, T_2) = 0, temp)) :
> av2 := -simplify(integrate(temp1, T[0] = 0..8·Pi) / 8·Pi);
Ready
Editable Maple Default Profile /home/sudhanshu/Desktop/ME627 Memory: 106.19M Time: 77.56s Zoom: 100% Math Mode



/home/sudhanshu/Desktop/ME627/MMS_2nd_Order_2.mw* - [Server 5] - Maple 2018
File Edit View Insert Format Table Drawing Plot Tools Window Help
Start.mw MMS_2nd_Order *Untitled (3) *MMS_2nd_Order_2
> restart:
> de := xdot + epsilon·(sin(T[0] + x) − p·cos(3·T[0]−x))^3;
de := xdot + ϵ (sin(T_0 + x) − p cos(−3 T_0 + x))^3 (1)
> N := 2; ts := seq(T[k], k = 0..N);
N := 2
ts := T_0, T_1, T_2 (2)
> xs := add(X[k](ts)·ϵ^k, k = 0..N);
xs := X_0(T_0, T_1, T_2) + X_1(T_0, T_1, T_2) ϵ + X_2(T_0, T_1, T_2) ϵ^2 (3)
> xds := add(diff(xs, T[k])·ϵ^k, k = 0..N);
(4)
> de := subs(xdot = xds, x = xs, de) :
> dea := convert(taylor(de, epsilon, N + 1), polynom);
(5)
> coeff(dea, epsilon, 0);
∂/∂T_0 X_0(T_0, T_1, T_2) (6)
> temp := X_0(T_0, T_1, T_2) = A(T[1], T[2]);
temp := X_0(T_0, T_1, T_2) = A(T_1, T_2) (7)
> xs := subs(temp, xs);
xs := A(T_1, T_2) + X_1(T_0, T_1, T_2) ϵ + X_2(T_0, T_1, T_2) ϵ^2 (8)
> dea := collect(expand(subs(temp, dea)), epsilon) :
> temp := coeff(dea, epsilon, 1) :
> temp1 := combine(expand(subs(X_1(T_0, T_1, T_2) = 0, ∂/∂T_1 A(T_1, T_2) = 0, temp)), trig);
(9)
Ready
Editable Maple Default Profile /home/sudhanshu/Desktop/ME627 Memory: 106.19M Time: 77.56s Zoom: 100% Math Mode


/home/sudhanshu/Desktop/ME627/MMS_2nd_Order_2.mw* - [Server 5] - Maple 2018
+ 336 p^3 cos(AA(T2))^2 cos(T0)^6 sin(AA(T2)) sin(T0) − 180 p^3 cos(AA(T2))^2 cos(T0)^4 sin(AA(T2)) sin(T0) − 192 p^3 cos(AA(T2)) cos(T0)^7 sin(AA(T2))^2 sin(T0)^2
+ 240 p^3 cos(AA(T2)) cos(T0)^5 sin(AA(T2))^2 sin(T0)^2 + 9 p^3 cos(AA(T2)) cos(T0) sin(AA(T2))^2 sin(T0)^2 + 48 cos(T0)^7 sin(AA(T2)) p^2 cos(AA(T2))^2 − 72 cos(T0)^5 sin(AA(T2)) p^2 cos(AA(T2))^2
+ 27 cos(T0)^3 sin(AA(T2)) p^2 cos(AA(T2))^2 + 3 cos(T0) sin(AA(T2))^3 p^2 sin(T0)^2 − 12 cos(T0)^5 sin(AA(T2))^2 p cos(AA(T2)) + 9 cos(T0)^3 sin(AA(T2))^2 p cos(AA(T2)) − 12 cos(T0)^4 sin(AA(T2))^3 p sin(T0)
+ 3 cos(T0)^2 sin(AA(T2))^3 p sin(T0) + 48 cos(T0)^5 sin(AA(T2))^3 p^2 sin(T0)^2 − 24 cos(T0)^3 sin(AA(T2))^3 p^2 sin(T0)^2 − 12 p^3 sin(AA(T2))^3 sin(T0)^3 cos(T0)^2 − 64 p^3 sin(AA(T2))^3 sin(T0)^3 cos(T0)^6
+ 48 p^3 sin(AA(T2))^3 sin(T0)^3 cos(T0)^4 + 3 sin(T0) cos(AA(T2)) cos(T0)^2 sin(AA(T2))^2 + 48 sin(T0) cos(AA(T2))^3 p^2 cos(T0)^6 − 72 sin(T0) cos(AA(T2))^3 p^2 cos(T0)^4 + 27 sin(T0) cos(AA(T2))^3 p^2 cos(T0)^2
+ 3 sin(T0)^3 cos(AA(T2)) p^2 sin(AA(T2))^2 + 3 sin(T0)^2 cos(AA(T2))^2 cos(T0) sin(AA(T2)) − 12 sin(T0)^2 cos(AA(T2))^3 p cos(T0)^3 + 9 sin(T0)^2 cos(AA(T2))^3 p cos(T0) + 3 sin(T0)^3 cos(AA(T2))^2 p sin(AA(T2))
+ ∂/∂T0 X1(T0, T1, T2)
> temp2 := −integrate(temp1, T[0]) :
> temp3 := X1(T0, T1, T2) = temp2 :
> xs := subs(temp3, xs) :
> dea := collect(expand(subs(temp3, dea)), epsilon) :
> temp := coeff(dea, epsilon, 2) :
> temp1 := expand(subs(d/dT2 AA(T2) = 0, X2(T0, T1, T2) = 0, temp)) :
> av2 := −simplify(integrate(temp1, T[0] = 0..8·Pi) / 8·Pi);
av2 := −(−15792 p^3 − 5880 p) sin(AA(T2)) cos(AA(T2))^3 / 1680 − (7896 p^3 + 2940 p) sin(AA(T2)) cos(AA(T2)) / 1680 + 5 p^6/48 − 303 p^4/560 + 3 p^2/80 − 5/16 (14)
> finaleq := d/dT2 AA(T2) − %
finaleq := d/dT2 AA(T2) + (−15792 p^3 − 5880 p) sin(AA(T2)) cos(AA(T2))^3 / 1680 + (7896 p^3 + 2940 p) sin(AA(T2)) cos(AA(T2)) / 1680 − 5 p^6/48 + 303 p^4/560 − 3 p^2/80 + 5/16 (15)
> subs(p = 3, finaleq);
d/dT2 AA(T2) − 2643 cos(AA(T2))^3 sin(AA(T2)) / 10 + 2643 cos(AA(T2)) sin(AA(T2)) / 20 − 4499/140 (16)
> repreq := subs(AA(T[2]) = X1, finaleq);
repreq := ∂/∂T2 X1 + (−15792 p^3 − 5880 p) sin(X1) cos(X1)^3 / 1680 + (7896 p^3 + 2940 p) sin(X1) cos(X1) / 1680 − 5 p^6/48 + 303 p^4/560 − 3 p^2/80 + 5/16 (17)
> subs(p = 1, repreq);
299/420 − 129 sin(X1) cos(X1)^3 / 10 + 129 sin(X1) cos(X1) / 20 (18)
Editable | Maple Default Profile | /home/sudhanshu/Desktop/ME627 | Memory: 106.19M | Time: 77.56s | Zoom: 100% | Math Mode

Q2.

Problem 2.

$$\ddot{x} + \varepsilon c\dot{x} + (1+\varepsilon\cos(2t))x = 0 \qquad 0<\varepsilon<1$$

$$\boxed{\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \end{aligned}}$$

$\cancel{\dot{x}_1 = x_2}$

$\cancel{\dot{x}_2 = -\varepsilon c x_2 + \varepsilon}$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\varepsilon c x_2 - x_1(1+\varepsilon\cos(2t))$$

$\cancel{\ddot{x} + x = -\varepsilon c\dot{x} - \varepsilon}$

$$\boxed{\ddot{x} + x = -\varepsilon\underbrace{(c\dot{x} + \cos(2t)x)}_{f(x,\dot{x})}}$$

$$x_1 = x$$
$$x_2 = \dot{x}$$

$$\dot{x}_2 = -x - \varepsilon(c x_2 + \cos(2t)x_1) \rightarrow (1)$$
$$\dot{x}_1 = x_2 \rightarrow (2)$$

$$x_1 = A\cos(t+\phi)$$

$$x_2 = -A\sin(t+\phi)$$

$$\dot{x}_1 = x_2$$

$$\cancel{-A\sin(t+\phi) = -A\sin(t+\phi)}$$

$$\dot{x}_1 = \dot{A}\cos(t+\phi) - A\sin(t+\phi)\cdot(1+\dot{\phi})$$

In ② $\dot{x}_1 = x_2$

$$\dot{A}\cos(t+\phi) - A\sin(t+\phi)\cdot(1+\dot{\phi}) = -A\sin(t+\phi)$$

for brevity $\cos(t+\phi) = C$, $\sin(t+\phi) = S$

$$\Rightarrow \dot{A}C - \cancel{AS} - AS\dot{\phi} = \cancel{-AS}$$

$$\Rightarrow \boxed{\dot{A}C - AS\dot{\phi} = 0} \longrightarrow ③$$

In ①

$$\dot{x}_2 = -\dot{A}S - AC(1+\dot{\phi})$$

$$= -AC + \varepsilon f$$

$$\Rightarrow -\dot{A}S - AC\dot{\phi} = \varepsilon f \rightarrow ④$$

$\therefore$ (3)

$$\dot{A}c - As\dot{\phi} = 0 \quad \times c$$

(4) $\quad \dot{A}s + Ac\dot{\phi} = -\varepsilon f \quad \times s$

$\therefore$ Adding $c \times (3) + s \times (4)$

$$\dot{A} = -\varepsilon f \sin(t+\phi) \quad (5)$$

Adding $(4) \times c + (3) \times (-s)$

$$A\dot{\phi} = -\varepsilon f \cos(t+\phi)$$

$$\dot{\phi} = -\frac{\varepsilon f}{A}\cos(t+\phi) \longrightarrow (6)$$

$\therefore$

$$\boxed{\begin{aligned} \dot{A} &= -\varepsilon f \sin(t+\phi) \rightarrow (5) \\ \dot{\phi} &= -\frac{\varepsilon f}{A}\cos(t+\phi) \rightarrow (6) \end{aligned}}$$

Now sys. is in Lagrange Standard form.

damping const. c not cos(t+φ)

$$\boxed{f = -c x_2 - \cos(2t) x_1}$$

$x_1 = \cancel{A\cos(t)}\ AC$

$x_2 = -AS$

$$\therefore f = -c\left(-A\sin(t+\phi)\right) - \cos 2t \left(A\cos(t+\phi)\right)$$

$$\boxed{f = Ac\sin(t+\phi) - A\cos(t+\phi)\cos(2t)}$$

$\therefore$ Finally the diff. Eqn becomes.

(5)

$$\dot{A} = -\varepsilon\left(\sin(t+\phi)\right)\left[Ac\sin(t+\phi) - A\cos(t+\phi)\cos(2t)\right] \quad (7)$$

$$\dot{\phi} = -\varepsilon\left(\cos(t+\phi)\right)\left[c\sin(t+\phi) - \cos(t+\phi)\cos(2t)\right] \quad (8)$$

Apply MMS first order

So given Eqn is in Lag. std. form. can be seen from above.

i.e. $\dot{x} = -\varepsilon f' \rightarrow (9)$ where f'

Here x is a vector $\begin{Bmatrix} A \\ \phi \end{Bmatrix}$

So assume

$x = X(T_0, T_1)$ assuming 1^{st} order.

$T_0 = t \quad T_1 = \varepsilon t$

$$\dot{x} = \frac{\partial X}{\partial T_0}(1) + \frac{\partial X}{\partial T_1}(\varepsilon)$$

↳ In 1D case

also we take $x = X_0(T_0, T_1) + \varepsilon X_1(T_0, T_1)$

$$\therefore \dot{x} = \frac{\partial X_0}{\partial T_0} + \varepsilon \frac{\partial X_1}{\partial T_0} + \varepsilon \frac{\partial X_0}{\partial T_1} + \varepsilon^2 \frac{\partial X_1}{\partial T_1}$$

↳ (10)

$\therefore$ ~~Eq~~ Putting (10) in (9)

we get

$$\frac{\partial X_0}{\partial T_0} + \varepsilon\left(\frac{\partial X_1}{\partial T_0} + \frac{\partial X_0}{\partial T_1}\right) + \varepsilon^2\left(\frac{\partial X_1}{\partial T_1}\right) = -\varepsilon f'$$

$$\therefore \frac{\partial X_0}{\partial T_0} = 0 \Rightarrow X = A(T_1)$$

$$\therefore \frac{\partial X_1}{\partial T_0} + \frac{\partial A}{\partial T_1} = -\varepsilon f'$$

Now

$$\frac{\partial X_1}{\partial T_0} = -\frac{\overset{f(T_1)}{\partial A}}{\partial T_1} - \varepsilon \underbrace{f'}_{f(t, x) = f(T_0, x)}$$

Now lets evaluate

T_0 avg of f

f here is a vector

where $f_1 = \sin(t+\phi)\left[A\varepsilon \sin(t+\phi) - A\cos(t+\phi)\cos(2t)\right]$

$f_2 = \cos(t+\phi)\left[A\varepsilon \sin(t+\phi) - A\cos(t+\phi)\cos(2t)\right]$

T_0 avg of f_1 keeping A, ϕ const.

$\sin^2(t+\phi) = 1/2$

$\frac{1}{2} \cdot 2\sin(t+\phi)\cos(t+\phi)\cos(2t) = \frac{1}{2}\sin(2t+2\phi)\cos(2t)$

$= \frac{1}{4}\left(\sin(4t+2\phi) + \sin(2\phi)\right)$

T_0 avg becomes $= 1/4 \sin(2\phi)$

$\sin(t+\phi)\cos(t+\phi) = \frac{1}{2}\sin(2t+\phi) = 0$

$\cos(t+\phi)^2 \cos(2t) = \frac{\cos(2\phi)}{4}$

To avging

$$\frac{\partial X_1}{\partial T_0} = -\frac{\partial A}{\partial T_1} - \varepsilon\begin{bmatrix} Ac(1/2) - \frac{A}{4}\sin(2\phi) \\ 0 - \frac{\cos(2\phi)}{4} \end{bmatrix} = 0$$

∴ ~~[illegible]~~

We choose A s.t. $\frac{\partial X_1}{\partial T_0} = 0$

i.e we remove secular terms.

Using MMS, Avgd Eqn is

$$\therefore \frac{\partial A}{\partial t} = -\varepsilon\begin{bmatrix} \frac{Ac}{2} - \frac{A}{4}\sin 2\phi \\ -\frac{\cos(2\phi)}{4} \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{\partial A}{\partial t} \\ \frac{\partial \phi}{\partial t} \end{bmatrix} = -\varepsilon\begin{bmatrix} \frac{Ac}{2} - \frac{A}{4}\sin 2\phi \\ -\frac{\cos(2\phi)}{4} \end{bmatrix}$$

~~Now Eq. pts~~

$\cancel{A \cdot (2c - \sin 2\phi) = 0}$

$\cancel{2c = \sin 2\phi}$

$\cancel{\boxed{\phi = \frac{1}{2}\sin^{-1}(2c) + k\pi}}$

Now finding Eq. pts.

$$\frac{A}{4}\left(2c - \sin(2\phi)\right) = 0$$

$\Rightarrow A = 0$ or $\sin 2\phi = 2c$

$\Rightarrow \phi = \frac{1}{2}\sin^{-1}(2c) + k\pi$

$\hookrightarrow \phi = -\pi + (\,),\ \frac{-\sin^{-1}(2c)}{2},\ \frac{\sin^{-1}(2c)}{2},\ \pi + (\,)$

also $\cos(2\phi) = 0$ $\quad k \in \mathbb{Z}$

$\Rightarrow 2\phi = (2k+1)\pi/2$

$\Rightarrow \phi = \frac{(2k+1)\pi}{4}$

$\hookrightarrow \phi \in \{\ldots, -3\pi/4, -\pi/4, \pi/4, 3\pi/4, 5\pi/4, \ldots\}$

$\frac{1}{2}\sin^{-1}(2c) = \pi/4$	$\frac{1}{2}\sin^{-1}(2c) = -\pi/4$
$\sin^{-1}(2c) = \pi/2$	$c = -1/2$
$c = 1/2$	

~~[illegible]~~

Now linearizing around eq. pt.

Jacobian

$$\text{Matrix} = \begin{bmatrix} -\frac{c}{2} + \frac{\sin 2\phi}{4} & \frac{A}{4}\cos(2\phi).2 \\ 0 & \frac{1}{4}(-\sin 2\phi).2 \end{bmatrix}$$

~~for $c > 1/2$ Eq. pts $\phi = \frac{\pi}{4}$~~

for Eq. pts corresponding to $A=0$ $\therefore$, the Eq. pt has no dependence on c.

$\therefore$ Second set of Eq. pts must be chosen.

Now for $|c| > 1/2$ Eqn pts correspond to $A=0$ case

$$\therefore \phi = \frac{(2k+1)}{4}\pi = \frac{1}{2}\sin^{-1}(2c) + k\pi$$

$$\frac{1}{2}\sin^{-1}(2c) = \frac{\pi}{4}2$$

$$c = 1/2$$

$$\frac{1}{2}\sin^{-1}(2c) - \pi = \frac{\pi}{4}$$

~~$\frac{1}{2}\sin^{-1}(2c) = \frac{5\pi}{4}2$~~

~~$\sin^{-1}$~~

$\sin^{-1}(2c) - 2k\pi = (2k+1)\frac{\pi}{4}$

$\Rightarrow 2c = \sin\left((2k+1)\frac{\pi}{2}\right)$

$c = \frac{1}{2}\sin\left((2k+1)\frac{\pi}{2}\right)$

$c = 1/2, -1/2$

for $c = 1/2$, $\phi = \pi/2$

Jacobian Matrix $= \begin{bmatrix} -1/4 + 0 & \frac{\pi}{2}(-1) \\ 0 & -\frac{1}{2}(0) \end{bmatrix}$

$= \begin{bmatrix} -1/4 & -\frac{\pi}{2} \\ 0 & 0 \end{bmatrix}$

$-1/4 - \lambda$

λ $\lambda(\lambda + 1/4) = 0$

$[\lambda = -1/4, 0]$

for $c = -1/2$, $\phi = -\pi/2$ $\therefore$ for $c = 1/2$

$[\lambda = 1/4, 0]$ the eqn is stable around eq pts.

for Eq pts $A=0$ and $\phi=(2k+1)\frac{\pi}{4}$

Jacobian Matrix : $\begin{bmatrix} -\frac{c}{2}+\frac{\sin(2\phi)}{4} & \frac{A}{4}\cos(2\phi)\cdot 2 \\ 0 & -\frac{1}{2}(\sin 2\phi) \end{bmatrix} = H$

for the Eq. pt. $A=0$ $\phi=(2k+1)\pi/4$.

$$H = \begin{bmatrix} -c/2+1/4(\pm 1) & 0 \\ 0 & -\frac{1}{2}(\pm 1) \end{bmatrix}$$

Eigen values.

$-\frac{1}{2}(\pm 1)$, $-\frac{c}{2} \pm \frac{1}{4}$

for $\phi = \pi/4$

$\lambda = -\frac{c}{2}+1$, $-1/2$

for st. $-\frac{c}{2}+\frac{1}{4}<0$

$c/2 > 1/4$

$\boxed{c > 1/2}$

for $\phi = -\pi/4$

$\lambda = +\frac{1}{2}$, $-\frac{c}{2}-\frac{1}{4}$

$-\frac{c}{2}-1/4>0 \Rightarrow c/2 > -1/4$

$c > -1/2$ but second e.v. is +ve. so unstable

Thus, c = 0.5 is the minimum value for stability