

COGS109
Homework 3
Gustav Sto. Tomas
A15358078

HW3

1. (4.4)

- a) With $p = 1$ and $n = 100$: $10/100$; $= 10\%$
- b) With $p = 2$ and $n = 100$: $10/100 * 10/100 = (10/100)^2 = 1\%$
- c) With $p = 100$ and $n = 100$: $((10/100)^{100}) * 100 = 1.e-98$
10% of *each feature's* (100 p) range, 100 times (100 n)
- d) The larger/higher the dimensionality, the larger the vector space = further between data points, in particular ones that are not 'noise.' Furthermore, with higher dimensions, data points start to 'look alike' as they expand away exponentially from the test observation. That is, the average distance between data points in space become more similar to the maximum possible distance between points in space with higher dimensionality.
- e) If I understand the question correctly (which, admittedly, I am not sure I do), we want to make a hypercube for *only* the 10% of the data that is nearest the test observation in hyperspace (similar to above). The volume (or area in case of lower dimensions) of the hypercube would then be $10/100$ (or 0.1) raised to p_n . Each side of the cube would then be 10% of one dimension p of dimensions $n = (10/100)^{(1/p_n)}$:
- | | | |
|-----------------|------------------------|------------|
| for $p = 1$: | $10/100$ | $= 0.1$ |
| for $p = 2$: | $(10/100)^{(1/2)}$ | $= 0.3162$ |
| for $p = 100$: | $((10/100)^{(1/100)})$ | $= 0.9772$ |

2. (4.6)

a)

$X1 = 40$;
 $X2 = 3.5$;
 $B0 = -6$;
 $B1 = 0.05$;
 $B2 = 1$;
 $x = B0 + B1*X1 + B2*X2$;
 $p = 1/(1+\exp(-x))$;

Probability of getting an A is 0.3775

b)

For a 50% change, 50 hours of studying are needed.

3. (4.7)

pi_1 = 0.8;
pi_2 = 0.2;
sigma = 36;
mu1 = 10;
mu2 = 0;
X = 4;

$$\begin{aligned} p_k(x) &= (p_i \cdot (1/\sqrt{2 \cdot \sigma^2}) \cdot \exp(-(1/(2 \cdot \sigma^2)) \cdot (x - \mu_i)^2)) / \\ &\quad \sum_{i=1}^K (p_i \cdot (1/\sqrt{2 \cdot \sigma^2}) \cdot \exp(-(1/(2 \cdot \sigma^2)) \cdot (x - \mu_i)^2)) \\ &= (p_1 \cdot \exp(-(1/(2 \cdot \sigma^2)) \cdot (x - \mu_1)^2)) / \\ &\quad (p_1 \cdot \exp(-(1/(2 \cdot \sigma^2)) \cdot (x - \mu_1)^2)) + (p_2 \cdot \exp(-(1/(2 \cdot \sigma^2)) \cdot (x - \mu_2)^2))); \end{aligned}$$

$$\begin{aligned} p &= (p_1 \cdot \exp(-(1/(2 \cdot 36^2)) \cdot (4 - 10)^2)) / \\ &\quad (0.80 \cdot \exp(-(1/(2 \cdot 36^2)) \cdot (4 - 10)^2)) + (0.20 \cdot \exp(-(1/(2 \cdot 36^2)) \cdot 4^2))); \\ &= 0.7519 \end{aligned}$$

The probability that a company will issue dividend is $0.75 = 75\%$.

("...but this is not a math class...")

4. See Jupyter Notebook