Warsaw University of Technology

Introduction to Digital Systems

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Combinational Logic

Binary adder

bin2bcd decoder

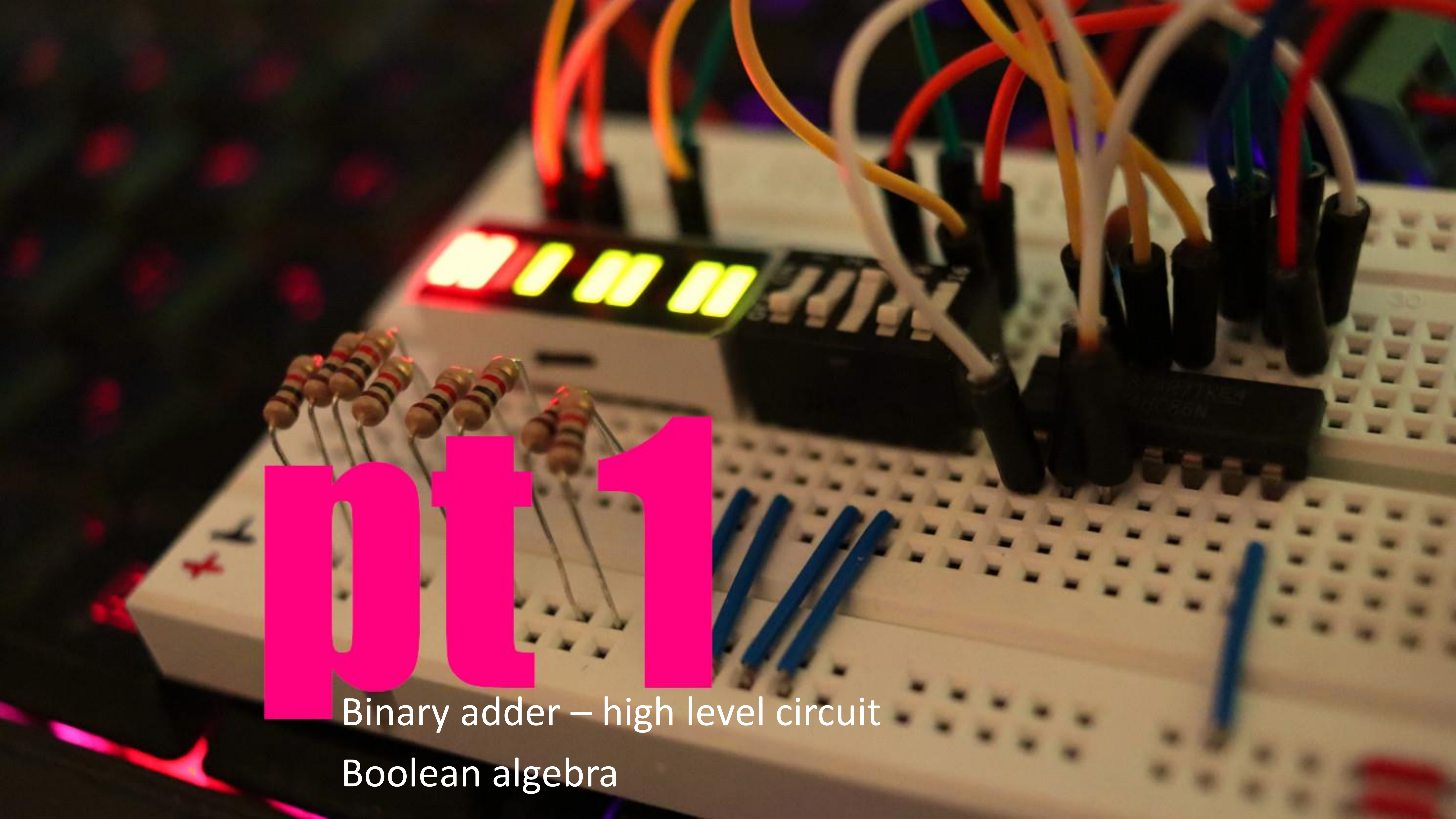
7-segment LED display driver

Truth table

Sum of Products & Product of Sums

Boolean algebra

Karnaugh map



pt1

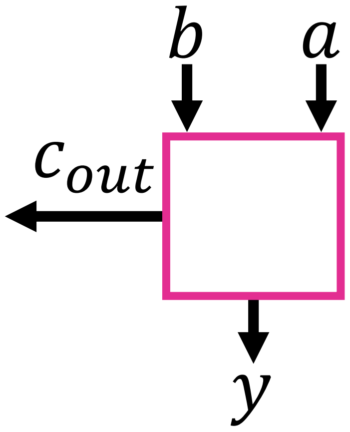
Binary adder – high level circuit

Boolean algebra

The idea of the Binary Adder

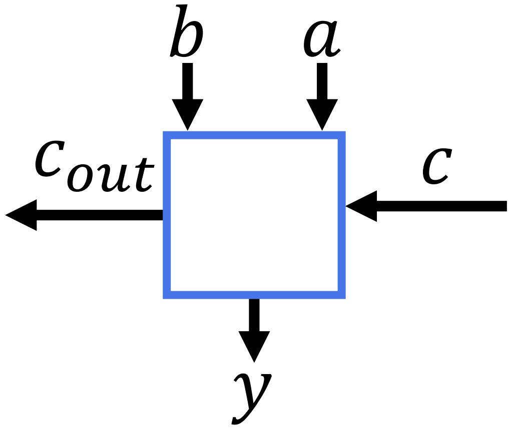
Half adder
sum of 2 bits

$$\begin{array}{rrr} & c_{out} & \\ & & a \\ + & & b \\ \hline & c_{out} & y \end{array}$$



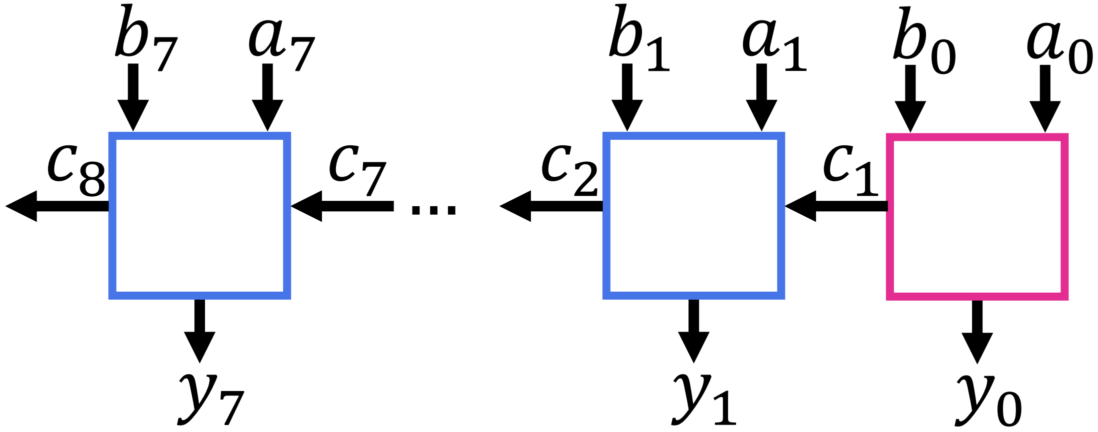
Full adder
sum of 3 bits

$$\begin{array}{rrr} & c_{out} & c \\ & & a \\ + & & b \\ \hline & c_{out} & y \end{array}$$



8-bit adder

$$\begin{array}{rccccccccc} & & & & & & & & c_2 & c_1 & \\ & & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ + & & b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\ \hline & c_8 & y_7 & y_6 & y_5 & y_4 & y_3 & y_2 & y_1 & y_0 \end{array}$$



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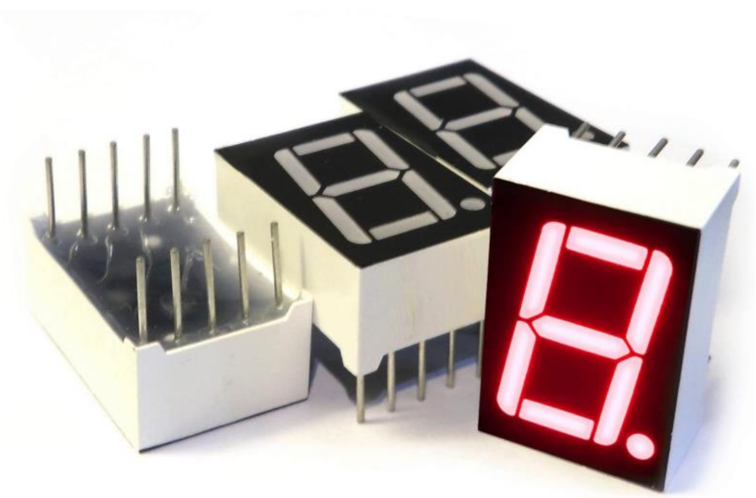
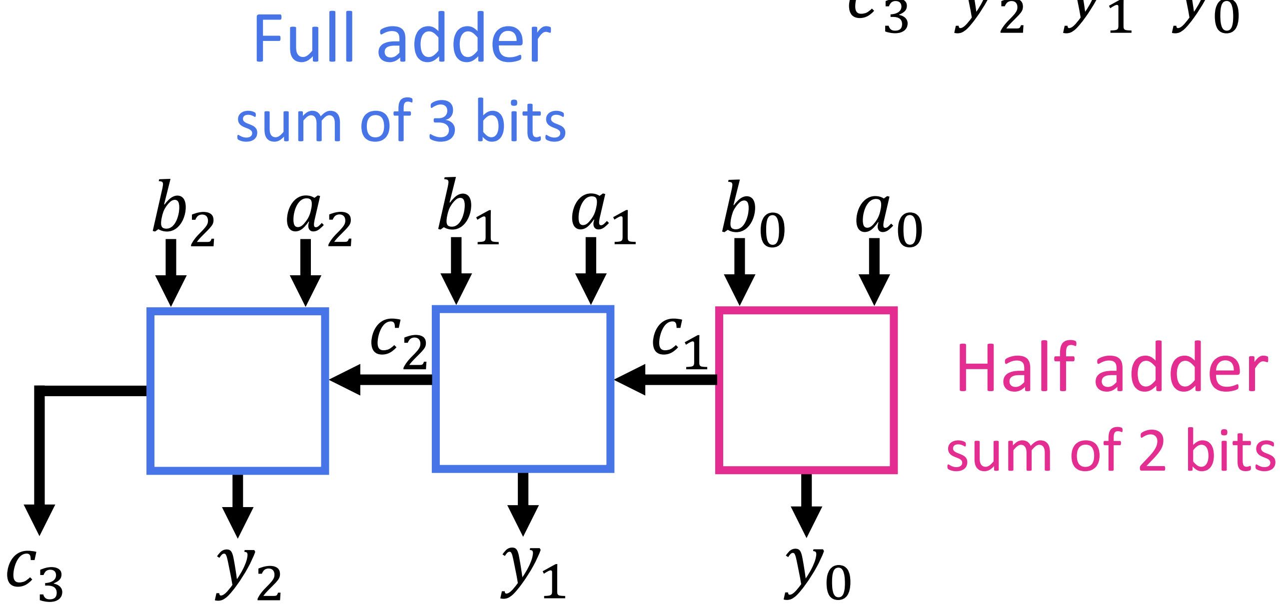
What will we do?

- Simulation
  Falstad circuit simulator
  https://www.falstad.com/circuit/circuitjs.html
- 3-bit adder
- with 7-segment LED display

$$
\begin{array}{ccccc}
 & c_3 & c_2 & c_1 & \\
 & & a_2 & a_1 & a_0 \\
+ & & b_2 & b_1 & b_0 \\
\hline
 & c_3 & y_2 & y_1 & y_0
\end{array}
$$





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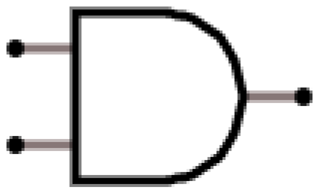
Truth Table + Sum of Products: AND



- AND gate

$$Y(A,B) = AB$$

	A	B	Y
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1

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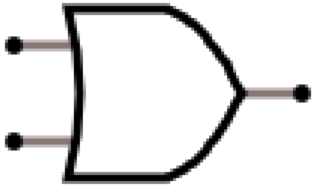
Truth Table + Sum of Products: OR



- OR gate

$$Y = A + B$$

- using the algorithm

$$Y = \bar{A}B + A\bar{B} + AB$$

- Minimization
- Because (idempotence)

$$1 + 1 = 1 \;,\; 0 + 0 = 0$$

$$Y = \bar{A}B + AB + A\bar{B} + AB$$

$$Y = A + B$$

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

$$\bar{A}B + AB = B(\bar{A} + A) = B \cdot 1 = B$$

$$A\bar{B} + AB = A(\bar{B} + B) = A$$

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Truth Table + Sum of Products: OR

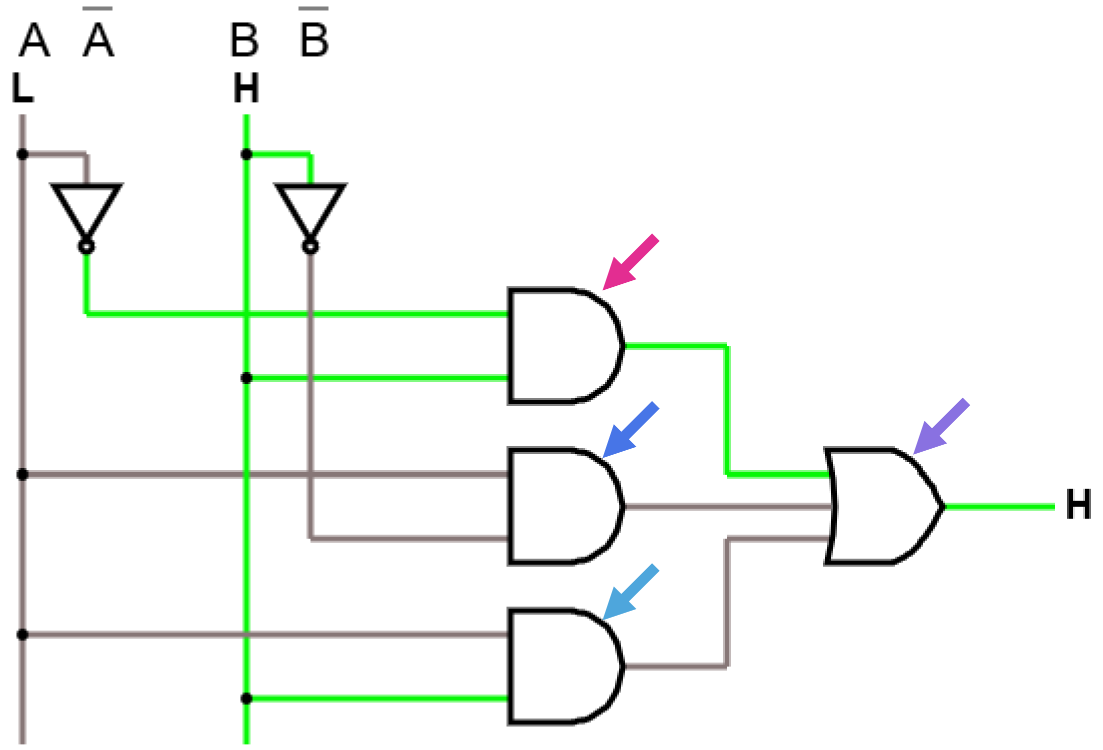
- OR gate

$$Y = A + B$$

- using the algorithm

$$Y = \boxed{\boxed{\bar{A}B} + \boxed{A\bar{B}} + \boxed{AB}}$$

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1



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Boolean algebra laws

$\bar{0} = 1$	$\bar{\bar{X}} = X$	$\bar{1} = 0$
$X \cdot 1 = X$	identity	$X + 0 = X$
$X + 1 = 1$	annihilation	$X \cdot 0 = 0$
$X \cdot \bar{X} = 0$	complementation	$X + \bar{X} = 1$
$X \cdot Y = Y \cdot X$	commutativity	$X + Y = Y + X$
$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$	associativity	$X + (Y + Z) = (X + Y) + Z$
$X \cdot X = X$	idempotence	$X + X = X$
$\overline{X + Y} = \bar{X} \cdot \bar{Y}$	De Morgan's laws	$\overline{X \cdot Y} = \bar{X} + \bar{Y}$
$X \cdot (X + Y) = X$	absorption	$X + X \cdot Y = X$

Distributivity of · over +

$$X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$$

Distributivity of + over ·

$$X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$$

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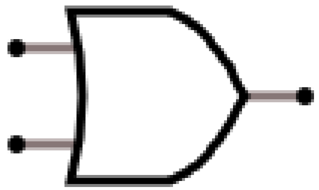
Truth Table + Product of Sums: OR



- OR gate

$$Y = A + B$$

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

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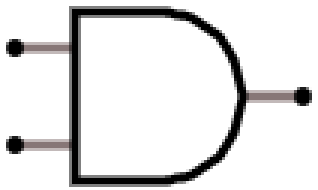
Truth Table + Product of Sums : AND



- AND gate

$$Y = AB$$

- using the algorithm

$$Y = (A + B) \cdot (A + \bar{B}) \cdot (\bar{A} + B)$$

- minimization

	A	B	Y
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1

$$Y = (AA + A\bar{B} + BA + \cancel{B\bar{B}})(\bar{A} + B)$$

$$= (A + \cancel{A\bar{B}} + \cancel{AB})(\bar{A} + B) = A(\bar{A} + B)$$

$$= \cancel{A\bar{A}} + AB = AB$$

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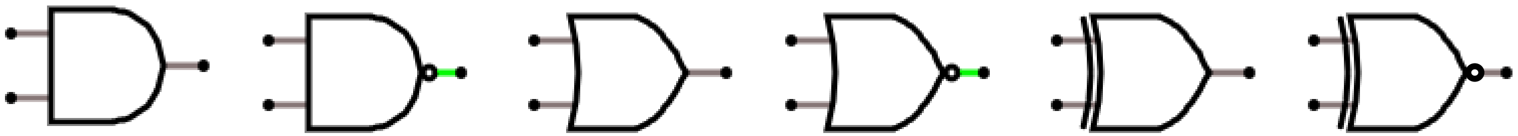
Logic gates



	A	B	AND	NAND	OR	NOR	XOR	XNOR
			AB	$\overline{AB}$	$A+B$	$\overline{A+B}$	$A\oplus B$	$\overline{A\oplus B}$
0	0	0	0	1	0	1	0	1
1	0	1	0	1	1	0	1	0
2	1	0	0	1	1	0	1	0
3	1	1	1	0	1	0	0	1

$$A\oplus B = \bar{A}B + A\bar{B}$$

$$\overline{A\oplus B} = AB + \bar{A}\bar{B}$$

$$\overline{A\oplus B} = \overline{\bar{A}B + A\bar{B}} = \overline{\bar{A}B}\cdot\overline{A\bar{B}} = (A+\bar{B})(\bar{A}+B) = \cancel{A\bar{A}} + AB + \bar{B}\bar{A} + \cancel{\bar{B}B}$$

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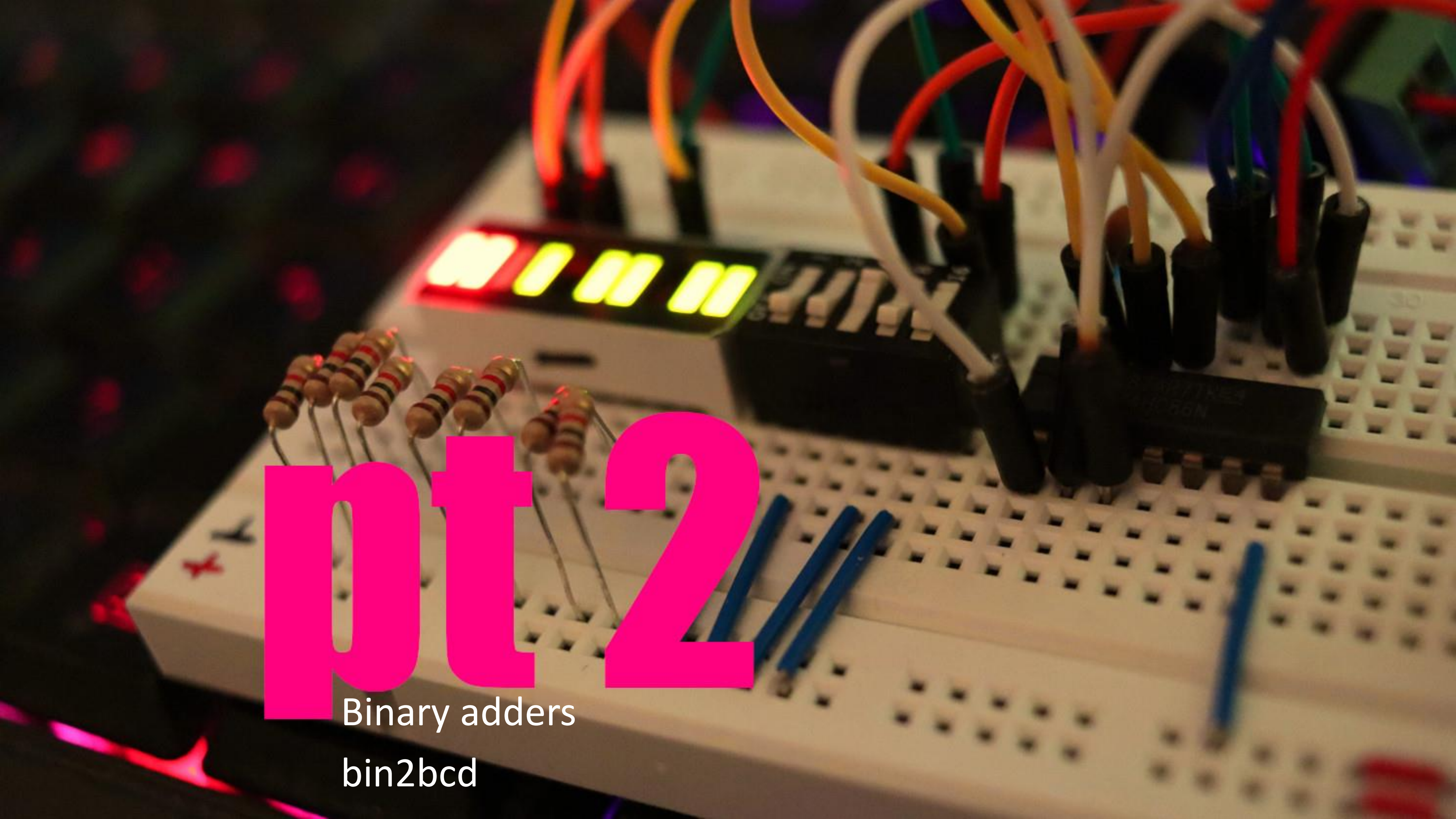
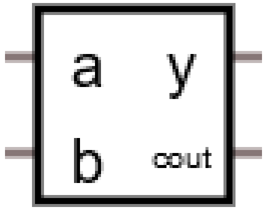


pt 2

Binary adders

bin2bcd

Half adder

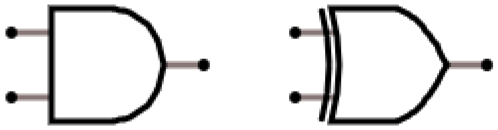


- We add two bits
- We expect 2-bit outcome
- Using Sum of Product method:

$$y = \bar{a}b + a\bar{b} = a \oplus b$$

$$c_{out} = ab$$

	a	b	c_{out}	y
0	0	0	0	0
1	0	1	0	1
2	1	0	0	1
3	1	1	1	0



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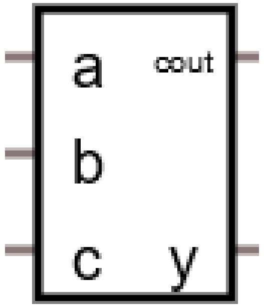
Full adder



- We add three bits
- We expect 2-bit outcome
- Using Sum of Product method:

$$y = \bar{c}\bar{a}b + \bar{c}a\bar{b} + c\bar{a}\bar{b} + cab$$

$$= \bar{c}\left(\bar{a}b + a\bar{b}\right) + c\left(\bar{a}\bar{b} + ab\right)$$

$$= \bar{c}(\underbrace{a \oplus b}_{d}) + c(\underbrace{\overline{a \oplus b}}_{\bar{d}})$$

$$= \bar{c}d + c\bar{d} = c \oplus d$$

$$= c \oplus (a \oplus b)$$

	c	a	b	c_{out}	y
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

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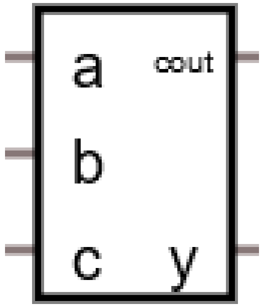
Full adder



- We add three bits
- We expect 2-bit outcome
- Using Sum of Product method:

$$y = c \oplus (a \oplus b)$$

$$c_{out} = \bar{c}ab + c\bar{a}b + ca\bar{b} + cab$$

$$= ab(\bar{c} + c) + c(\bar{a}b + a\bar{b})$$

$$= ab + c(a \oplus b)$$

	c	a	b	c_{out}	y
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

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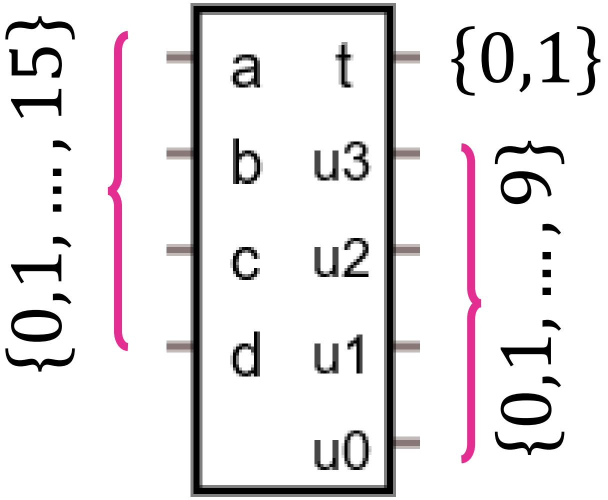
4-bit bin2bcd



	a	b	c	d	t	u_3	u_2	u_1	u_0
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	1
2	0	0	1	0	0	0	0	1	0
3	0	0	1	1	0	0	0	1	1
4	0	1	0	0	0	0	1	0	0
5	0	1	0	1	0	0	1	0	1
6	0	1	1	0	0	0	1	1	0
7	0	1	1	1	0	0	1	1	1
8	1	0	0	0	0	1	0	0	0
9	1	0	0	1	0	1	0	0	1
10	1	0	1	0	1	0	0	0	0
11	1	0	1	1	1	0	0	0	1
12	1	1	0	0	1	0	0	1	0
13	1	1	0	1	1	0	0	1	1
14	1	1	1	0	1	0	1	0	0
15	1	1	1	1	1	0	1	0	1

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4-bit bin2bcd

$$t = ab + a\bar{b}c =$$

$$= \overbrace{ab + abc}\; + a\bar{b}c =$$

$$= ab + ac\left(b + \bar{b}\right) =$$

$$= ab + ac$$

	a	b	c	d	t	
0	0	0	0	0	0	
1	0	0	0	1	0	
2	0	0	1	0	0	
3	0	0	1	1	0	
4	0	1	0	0	0	
5	0	1	0	1	0	
6	0	1	1	0	0	
7	0	1	1	1	0	
8	1	0	0	0	0	
9	1	0	0	1	0	
10	1	0	1	0	1	$a\bar{b}c\bar{d}$
11	1	0	1	1	1	$a\bar{b}cd$
12	1	1	0	0	1	$ab\bar{c}\bar{d}$
13	1	1	0	1	1	$ab\bar{c}d$
14	1	1	1	0	1	$abc\bar{d}$
15	1	1	1	1	1	$abcd$

$$\left.\begin{matrix} a\bar{b}c\bar{d} \\ a\bar{b}cd \end{matrix}\right\} a\bar{b}c\left(d + \bar{d}\right)$$

$$\left.\begin{matrix} ab\bar{c}\bar{d} \\ ab\bar{c}d \\ abc\bar{d} \\ abcd \end{matrix}\right\} ab$$

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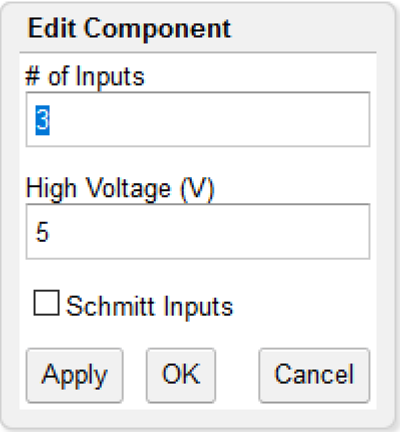
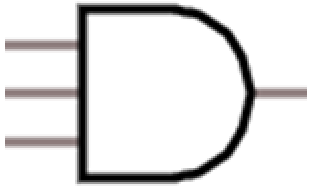
4-bit bin2bcd

$$t = ab + ac$$

$$u_3 = a\bar{b}\bar{c}$$





	a	b	c	d	u_3
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

$$\left.\begin{matrix} a\bar{b}\bar{c}\bar{d} \\ a\bar{b}\bar{c}d \end{matrix}\right\} \quad a\bar{b}\bar{c}(d + \bar{d}) = a\bar{b}\bar{c}$$

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4-bit bin2bcd

$$t = ab + ac$$

$$u_3 = a\bar{b}\bar{c}$$

$$u_2 = \bar{a}b + abc =$$

$$= \overbrace{\bar{a}b + \bar{a}bc} + abc =$$

$$= \bar{a}b + bc(\bar{a} + a) =$$

$$= \bar{a}b + bc$$

	a	b	c	d	u_2		
0	0	0	0	0	0		
1	0	0	0	1	0		
2	0	0	1	0	0		
3	0	0	1	1	0		
4	0	1	0	0	1	$\bar{a}b\bar{c}\bar{d}$	
5	0	1	0	1	1	$\bar{a}b\bar{c}d$	$\bar{a}b$
6	0	1	1	0	1	$\bar{a}bc\bar{d}$	
7	0	1	1	1	1	$\bar{a}bcd$	
8	1	0	0	0	0		
9	1	0	0	1	0		
10	1	0	1	0	0		
11	1	0	1	1	0		
12	1	1	0	0	0		
13	1	1	0	1	0		
14	1	1	1	0	1	$abc\bar{d}$	$abc(d + \bar{d})$
15	1	1	1	1	1	$abcd$	

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4-bit bin2bcd

$t = ab + ac$

$u_3 = a\bar{b}\bar{c}$

$u_2 = \bar{a}b + bc$

$u_1 = \bar{a}c + ab\bar{c}$

	a	b	c	d	u_1
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

$\bar{a}\bar{b}c\bar{d}$, $\bar{a}\bar{b}cd$ → $\bar{a}\bar{b}c$

$\bar{a}bc\bar{d}$, $\bar{a}bcd$ → $\bar{a}bc$

$\bar{a}c(\bar{b} + b)$
$= \bar{a}c$

$ab\bar{c}\bar{d}$, $ab\bar{c}d$ → $ab\bar{c}$

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4-bit bin2bcd

$$t = ab + ac$$

$$u_3 = a\bar{b}\bar{c}$$

$$u_2 = \bar{a}b + bc$$

$$u_1 = \bar{a}c + ab\bar{c}$$

$$u_0 = d$$

	a	b	c	d	u_0
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

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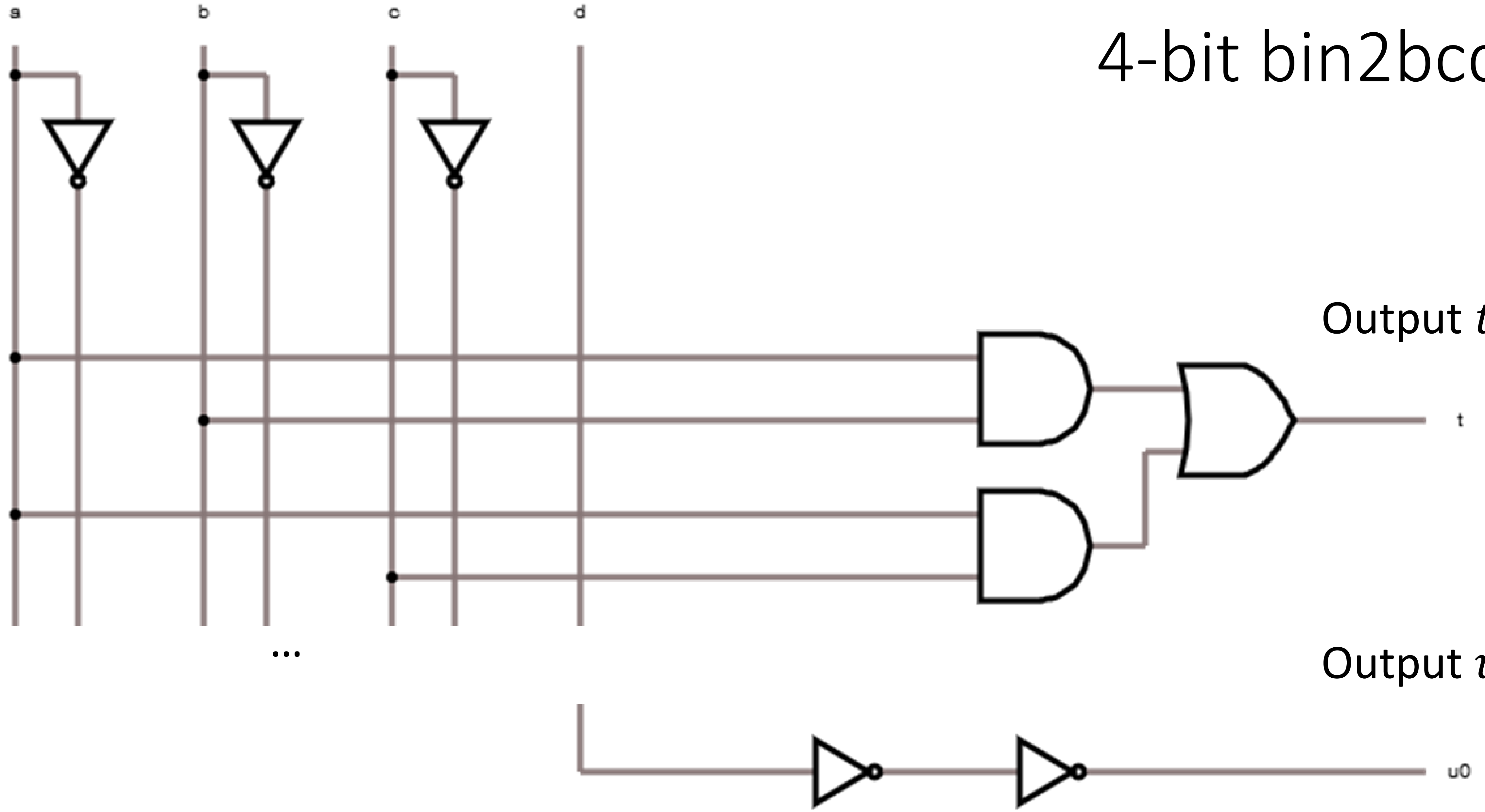
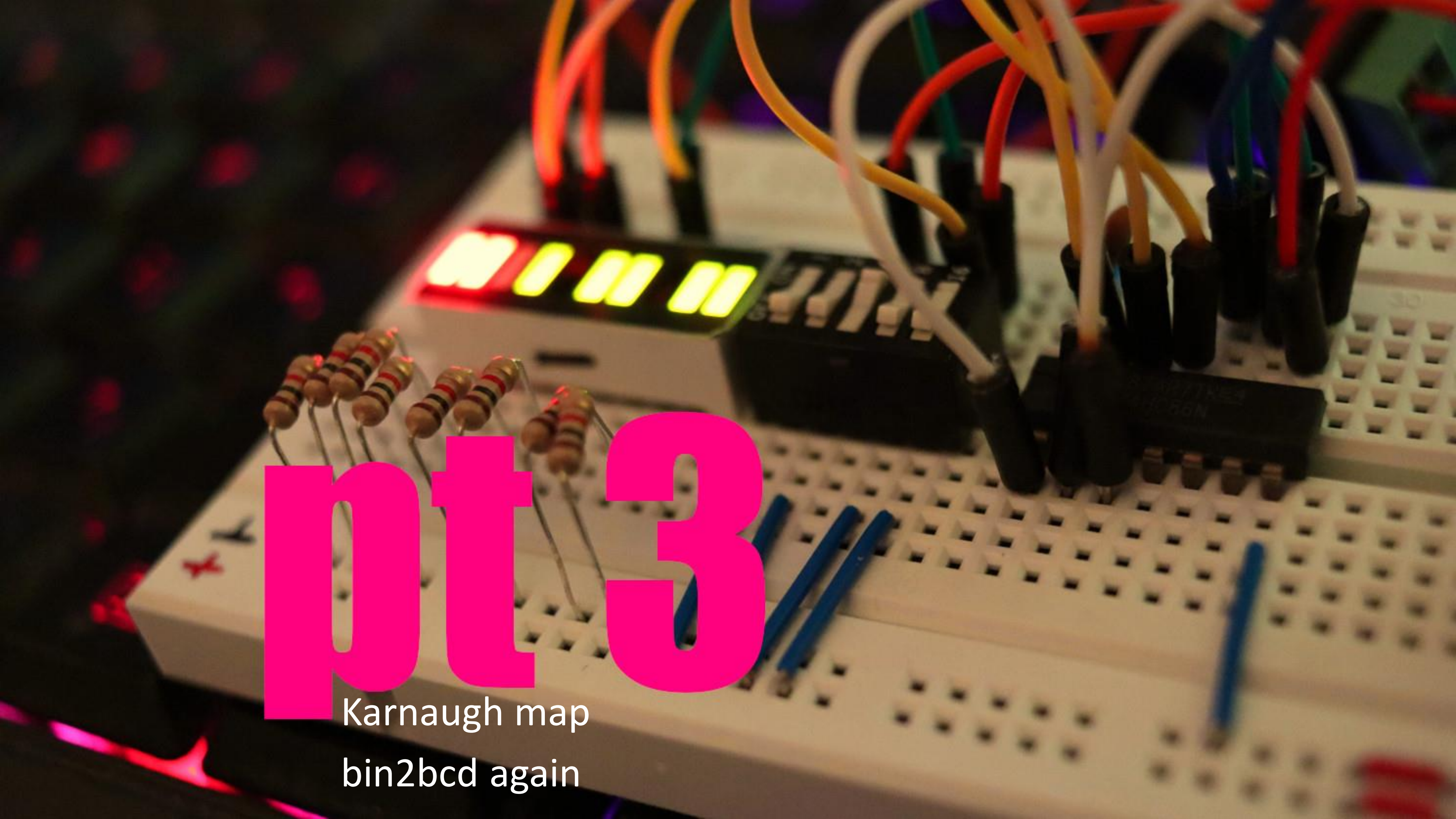
4-bit bin2bcd



Output t

Output $u_0 = d$



pt 3

Karnaugh map

bin2bcd again

What have we already learned?

Theory

- Truth tables
- Sum of Products, PoS
- Logic gates
- Boolean algebra laws
- Adder functions
- 4-bit bin2bcd

Practice

- Used for buiding circuits
- Minimization
- Designed
- Simulated

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4-bit bin2bcd
reprise

	a	b	c	d	u_1
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

$$\left.\begin{array}{l}\bar{a}\bar{b}c\bar{\boldsymbol{d}}\\ \bar{a}\bar{b}c\boldsymbol{d}\end{array}\right\}\ \bar{a}\bar{b}c$$

$$\left.\begin{array}{l}\bar{a}bc\bar{\boldsymbol{d}}\\ \bar{a}bc\boldsymbol{d}\end{array}\right\}\ \bar{a}bc$$

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4-bit bin2bcd

reprise

	a	b	c	d	u_1
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

$$\bar{a}\bar{b}c\bar{d}$$
$$\bar{a}\bar{b}cd$$
$$\bar{a}\bar{\boldsymbol{b}}c$$
$$\bar{a}c$$
$$\bar{a}bc\bar{d}$$
$$\bar{a}bcd$$
$$\bar{a}\boldsymbol{b}c$$

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4-bit bin2bcd
reprise

	a	b	c	d	u_0
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

$$u_0 = d$$

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Karnaugh map

AND

	A	B	Y
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1

A \ B	0	1
0	0 ${}_{0}$	0 ${}_{1}$
1	0 ${}_{2}$	$\boxed{1}$ ${}_{3}$

$$Y = AB$$

OR

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

A \ B	0	1
0	0 ${}_{0}$	1 ${}_{1}$
1	1 ${}_{2}$	1 ${}_{3}$

$$A\bar{B} + AB = A(\bar{B} + B) = A$$

$$Y = A + B$$

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Karnaugh map

ab \ cd	00	01	11	10
00	1 $_{0}$	$_{1}$	$_{3}$	1 $_{2}$
01	1 $_{4}$	1 $_{5}$	$_{7}$	$_{6}$
11	1 $_{12}$	1 $_{13}$	1 $_{15}$	1 $_{14}$
10	1 $_{8}$	$_{9}$	$_{11}$	1 $_{10}$

Gray code

$\bar{a}\bar{b}\bar{c}\bar{d}$

$\bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d = \bar{a}b\bar{c}(\bar{d} + d) = \bar{a}b\bar{c}$

ab

$b\bar{c}$

$\bar{b}\bar{d}$

Bits in loop		Variables in term
2^k	$\rightarrow$	$N - k$

N - no. of input bits

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4-bit bin2bcd K-map

ab \ cd	00	01	11	10
00	0 $_{0}$	0 $_{1}$	0 $_{3}$	0 $_{2}$
01	0 $_{4}$	0 $_{5}$	0 $_{7}$	0 $_{6}$
11	**1** $_{12}$	**1** $_{13}$	**1** $_{15}$	**1** $_{14}$
10	0 $_{8}$	0 $_{9}$	**1** $_{11}$	**1** $_{10}$

$t = ab + ac$

	a	b	c	d	t
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	**1**
11	1	0	1	1	**1**
12	1	1	0	0	**1**
13	1	1	0	1	**1**
14	1	1	1	0	**1**
15	1	1	1	1	**1**

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4-bit bin2bcd K-map

ab \ cd	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	0 (12)	0 (13)	0 (15)	0 (14)
10	**1** (8)	**1** (9)	0 (11)	0 (10)

$u_3 = a\bar{b}\bar{c}$

	a	b	c	d	u_3
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	**1**
9	1	0	0	1	**1**
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

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4-bit bin2bcd K-map

ab \ cd	00	01	11	10
00	0 $_{0}$	0 $_{1}$	0 $_{3}$	0 $_{2}$
01	**1** $_{4}$	**1** $_{5}$	**1** $_{7}$	**1** $_{6}$
11	0 $_{12}$	0 $_{13}$	**1** $_{15}$	**1** $_{14}$
10	0 $_{8}$	0 $_{9}$	0 $_{11}$	0 $_{10}$

$$u_2 = \bar{a}b + bc$$

	a	b	c	d	u_2
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	**1**
5	0	1	0	1	**1**
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	**1**
15	1	1	1	1	**1**

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4-bit bin2bcd K-map

ab \ cd	00	01	11	10
00	0_{0}	0_{1}	$\mathbf{1}_{3}$	$\mathbf{1}_{2}$
01	0_{4}	0_{5}	$\mathbf{1}_{7}$	$\mathbf{1}_{6}$
11	$\mathbf{1}_{12}$	$\mathbf{1}_{13}$	0_{15}	0_{14}
10	0_{8}	0_{9}	0_{11}	0_{10}

$$u_1 = \bar{a}c + ab\bar{c}$$

	a	b	c	d	u_1
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	**1**
3	0	0	1	1	**1**
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	**1**
13	1	1	0	1	**1**
14	1	1	1	0	0
15	1	1	1	1	0

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4-bit bin2bcd K-map

ab \ cd	00	01	11	10
00	0 (0)	**1** (1)	**1** (3)	0 (2)
01	0 (4)	**1** (5)	**1** (7)	0 (6)
11	0 (12)	**1** (13)	**1** (15)	0 (14)
10	0 (8)	**1** (9)	**1** (11)	0 (10)

$$u_0 = d$$

	a	b	c	d	u_0
0	0	0	0	0	0
1	0	0	0	1	**1**
2	0	0	1	0	0
3	0	0	1	1	**1**
4	0	1	0	0	0
5	0	1	0	1	**1**
6	0	1	1	0	0
7	0	1	1	1	**1**
8	1	0	0	0	0
9	1	0	0	1	**1**
10	1	0	1	0	0
11	1	0	1	1	**1**
12	1	1	0	0	0
13	1	1	0	1	**1**
14	1	1	1	0	0
15	1	1	1	1	**1**

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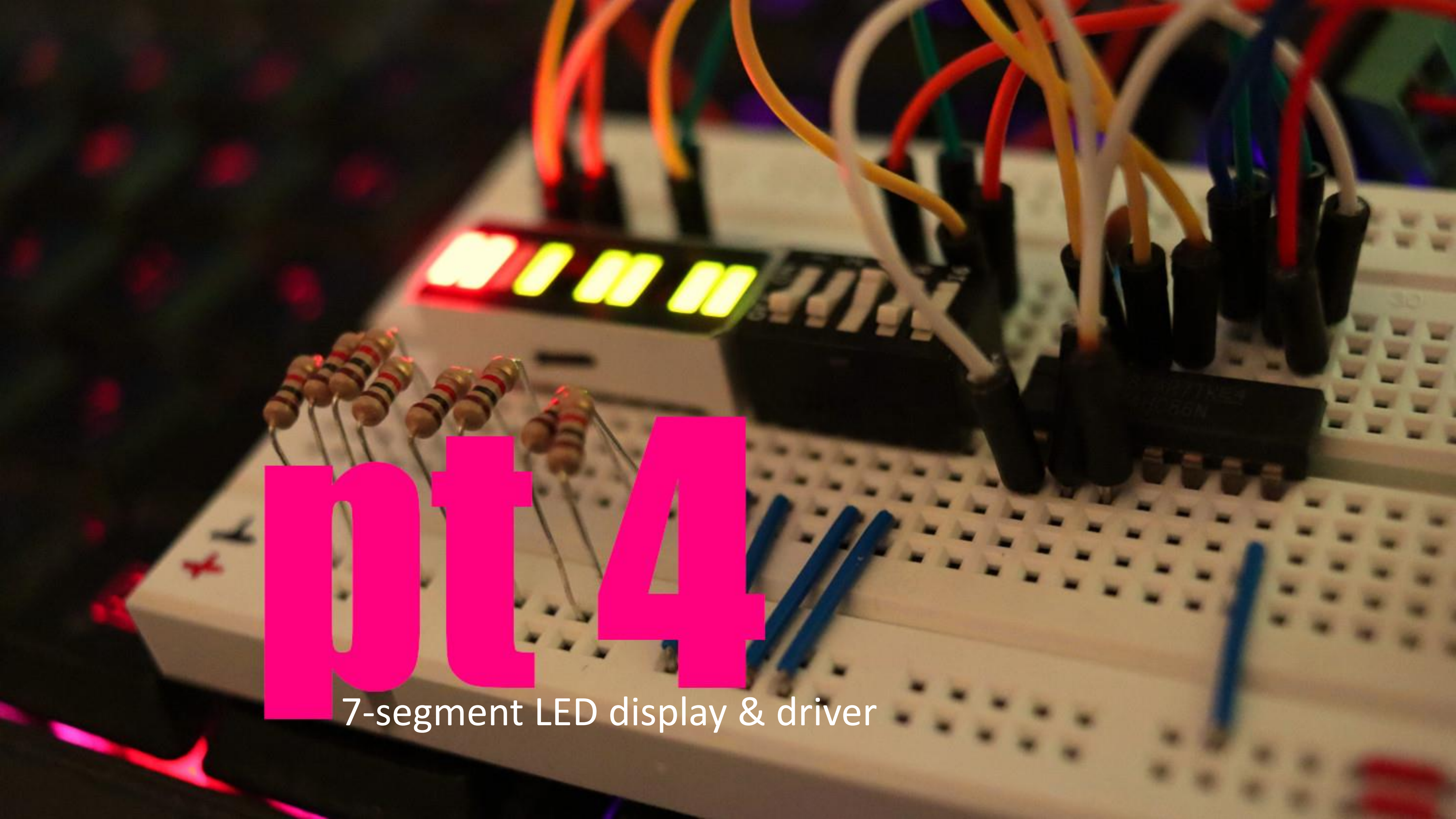
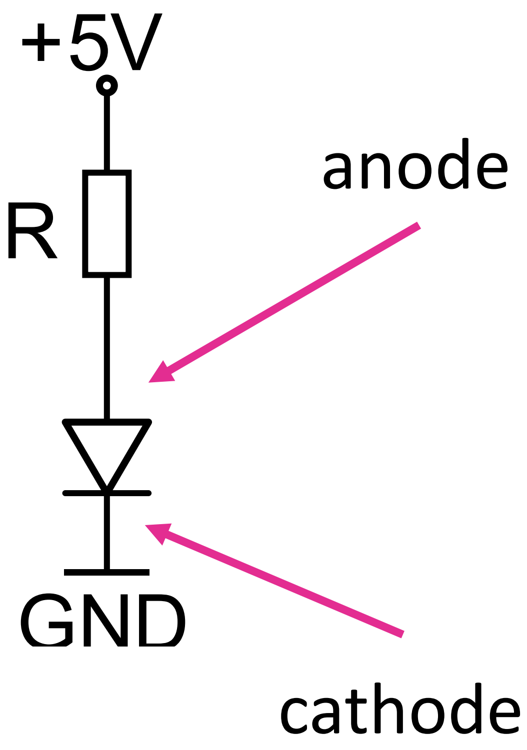
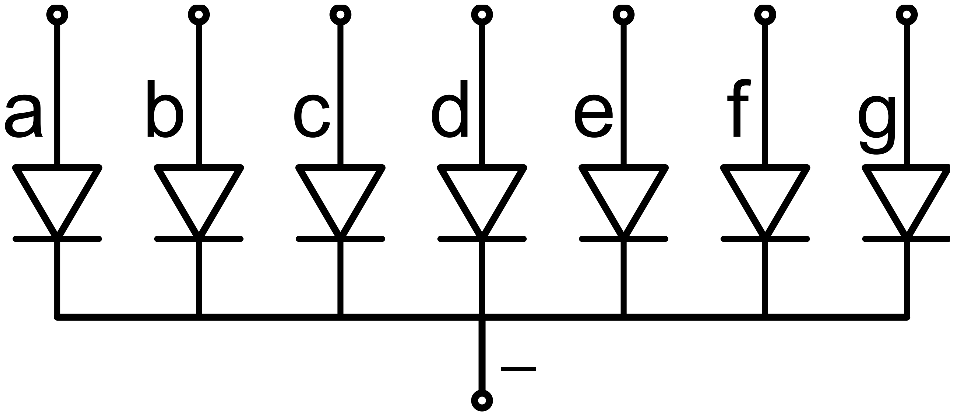


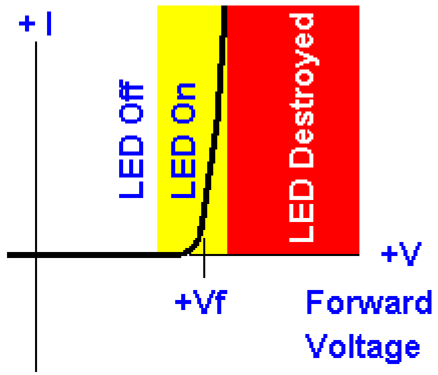
pt4

7-segment LED display & driver

7 segment LED display (common cathode)

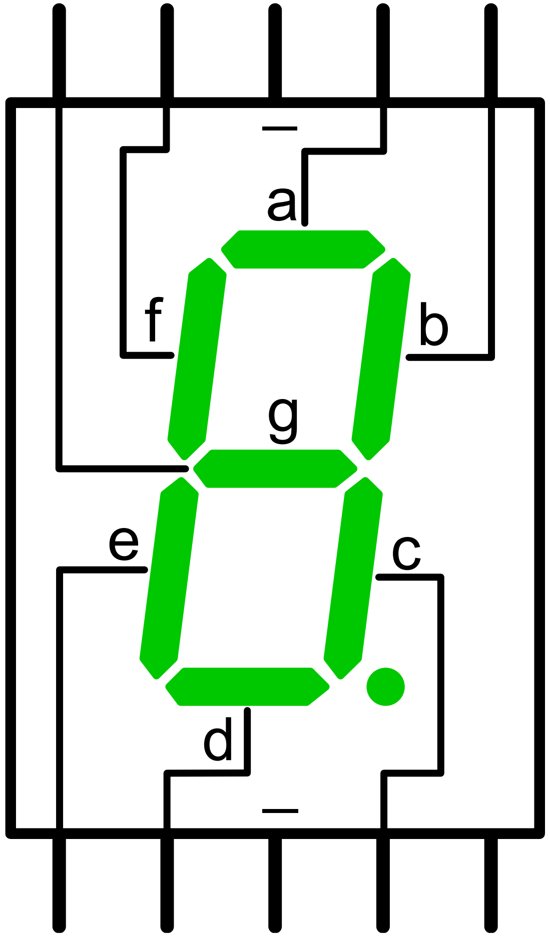






$$V_f \cong \begin{cases} 1.8\ \mathrm{V} \\ 2.0\ \mathrm{V} \end{cases}$$

$$I \cong \frac{5\ \mathrm{V} - V_f}{R}$$



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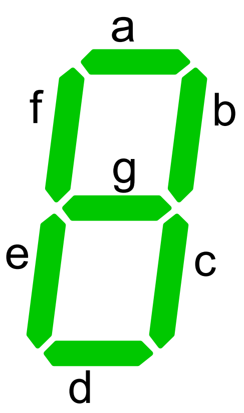
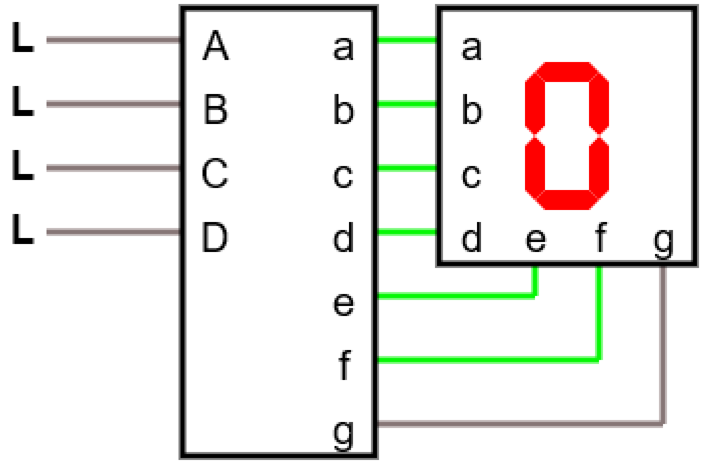
Driver





	A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	0	w	e		h	a	v	e
1	0	0	0	1	t	o		f	i	l	l
2	0	0	1	0	i	t		p	r	o	p
3	0	0	1	1	e	r	l	y			
4	0	1	0	0	w	e		h	a	v	e
5	0	1	0	1	t	o		f	i	l	l
6	0	1	1	0	i	t		p	r	o	p
7	0	1	1	1	e	r	l	y			
8	1	0	0	0	w	e		h	a	v	e
9	1	0	0	1	t	o		f	i	l	l
10	1	0	1	0	-	-	-	-	-	-	-
11	1	0	1	1	-	-	-	-	-	-	-
12	1	1	0	0	-	-	-	-	-	-	-
13	1	1	0	1	-	-	-	-	-	-	-
14	1	1	1	0	-	-	-	-	-	-	-
15	1	1	1	1	-	-	-	-	-	-	-

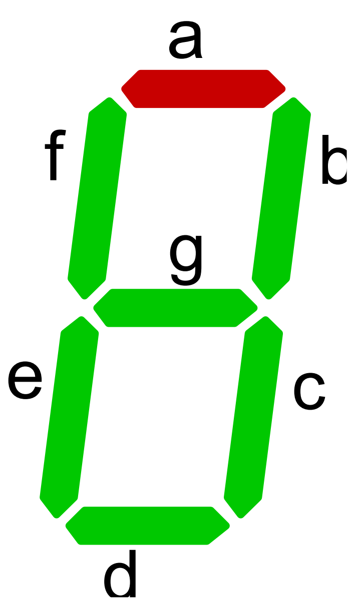
VALID DATA

NOT IMPORTANT

7 segment LED driver

AB \ CD	00	01	11	10
00	1 $_{0}$	0 $_{1}$	1 $_{3}$	1 $_{2}$
01	0 $_{4}$	1 $_{5}$	1 $_{7}$	1 $_{6}$
11	- $_{12}$	- $_{13}$	- $_{15}$	- $_{14}$
10	1 $_{8}$	1 $_{9}$	- $_{11}$	- $_{10}$

$$a = A + C + BD + \bar{B}\bar{D}$$



	A	B	C	D	a
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1

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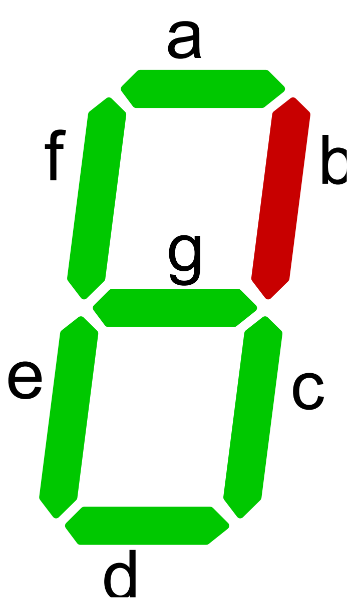
7 segment LED driver

AB \ CD	00	01	11	10
00	**1** $_{0}$	**1** $_{1}$	**1** $_{3}$	**1** $_{2}$
01	**1** $_{4}$	0 $_{5}$	**1** $_{7}$	0 $_{6}$
11	- $_{12}$	- $_{13}$	- $_{15}$	- $_{14}$
10	**1** $_{8}$	**1** $_{9}$	- $_{11}$	- $_{10}$

$$b = \bar{B} + CD + \bar{C}\bar{D}$$

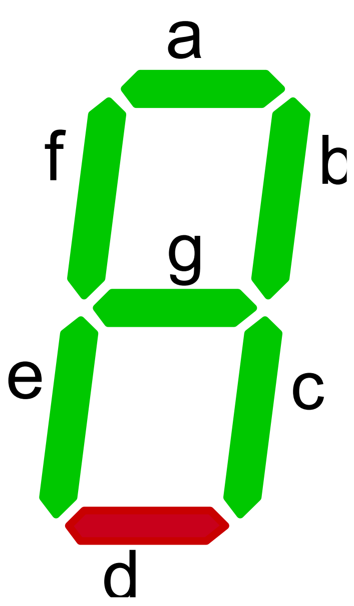


	A	B	C	D	b
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1

7 segment LED driver

AB \ CD	00	01	11	10
00	**1** 0	0 1	**1** 3	**1** 2
01	0 4	**1** 5	0 7	**1** 6
11	- 12	- 13	- 15	- 14
10	**1** 8	**1** 9	- 11	- 10

$$d = A + \bar{B}C + \bar{B}\bar{D} + C\bar{D} + B\bar{C}D$$



	A	B	C	D	d
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1

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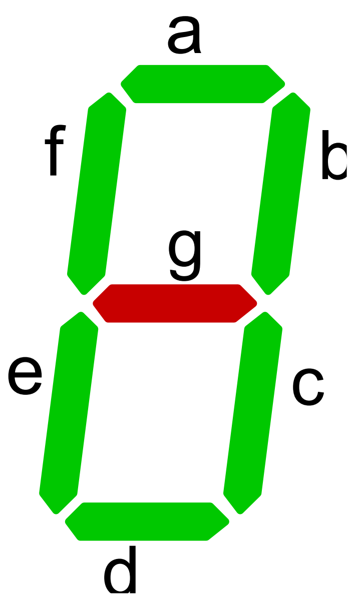
7 segment LED driver

CD \ AB	00	01	11	10
00	0_{0}	0_{1}	1_{3}	1_{2}
01	1_{4}	1_{5}	0_{7}	1_{6}
11	$-_{12}$	$-_{13}$	$-_{15}$	$-_{14}$
10	1_{8}	1_{9}	$-_{11}$	$-_{10}$

$$g = A + B\bar{C} + \bar{B}C + \cancel{B\bar{D}} + C\bar{D}$$



	A	B	C	D	g
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1

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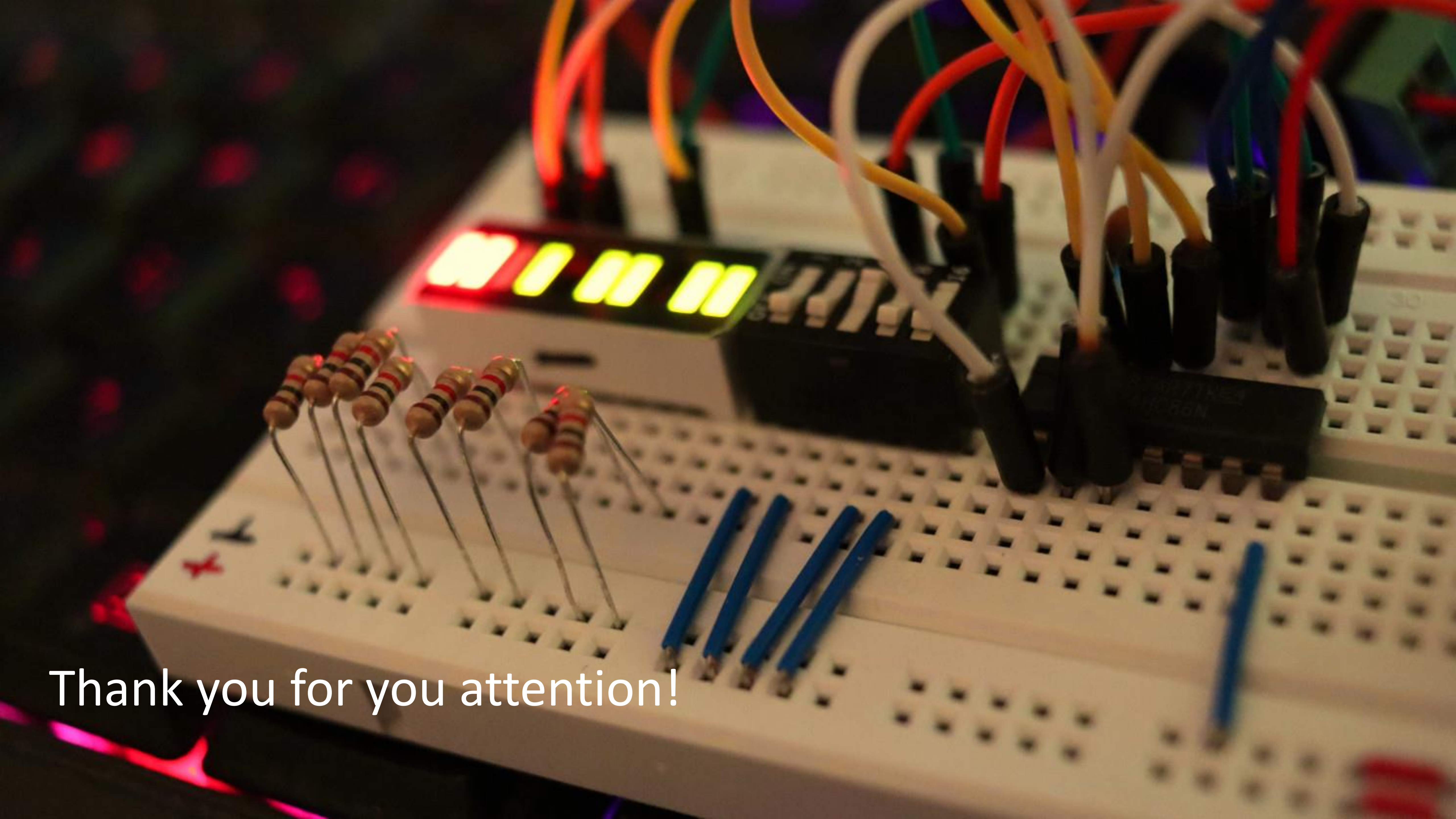

Thank you for you attention!