Expansions

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
4. $(a+b)^2 - (a-a)^2 = 4ab$
5. $(a+b)(a-b) = a^2 - b^2$
6. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
   $= a^3 + b^3 + 3a^2b + 3ab^2$
7. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
   $= a^3 - b^3 - 3a^2b + 3ab^2$
8. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
   $= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
9. $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$
10. $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$
11. $\left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2 = 2\left(a^2 + \frac{2}{a}\right)$
12. $(x+a)(x+b) = x^2 + (a+b)x\ ab$
13. $(x+a)(x-b) = x^2 + (a-b)x - ab$

14 $(x-a)(x-b) = x^3 + (b-b)x - ab$

15 $(x-a)(x-b) = x^2 - (a+b)x + ab$

17 $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2$
$ab - bc - ac)$

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Ex 4a

2 i $\left(\frac{7}{8}x+\frac{4}{5}y\right)^2$

using the identity $(a+b)^2=a^2+2ab+b^2$

$a=\frac{7}{8}x \quad b=\frac{4}{5}y$

$\therefore \left(\frac{7}{8}x+\frac{4}{5}y\right)^2=\left(\frac{7}{8}x\right)^2+\cancel{2}\times\frac{7}{\cancel{8}}x\times\frac{4}{5}y+\left(\frac{4}{5}y\right)^2$

$=\frac{49x^2}{64}+\frac{7}{5}xy+\frac{16y^2}{25}$

ii $\left(\frac{2x}{7}-\frac{7y}{4}\right)^2$

using identity $(a-b)^2=a^2+b^2-2ab$

$\left(\frac{2x}{7}-\frac{7y}{4}\right)^2=\left(\frac{2x}{7}\right)^2+\left(\frac{7y}{4}\right)^2-2\left(\frac{2x}{7}\times\frac{7y}{4}\right)$

$\therefore \left(\frac{2x}{7}+\frac{7y^2}{4}\right)^2=\left(\frac{2}{7}x\right)^2-\cancel{2}\times\frac{\cancel{2}}{\cancel{7}}x\times\frac{\cancel{7}}{\cancel{4}}y+\left(\frac{7}{4}y\right)^2$

$=\frac{4x^2}{49}-xy+\frac{49}{16}y^2$

3 using identity $(a+b)^2 - (a-b)^2$

$(a+b)^2 - (a-b)^2 = 4ab$

$a = \frac{a}{2b}$, $b = \frac{2b}{a}$

$\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2$

$= 4 \times \frac{\cancel{a}}{2b} \times \frac{2\cancel{b}}{\cancel{a}} = 4$

$\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 - 4$

$4 - 4 = 0$

i $(a+b)^2 - (a-b)^2 = 4ab$

$(x+y)^2 - (x-y)^2 = 4xy$

$\left(\frac{7}{2}\right)^2 - (x-y)^2 = \cancel{4}^2 \times \frac{5}{\cancel{2}_1}$

$\frac{49}{4} - (x-y)^2 = 10$

$\frac{49}{4} - \cancel{46}\frac{10^{\times 4}}{1 \times 4} = (x-y)^2$

$\frac{49-40}{4} = (x-y)^2$

ii $x^2 - y^2 = (x+y)(x-y)$

$= \pm \frac{7}{2} \times \frac{3}{2}$

$= \pm \frac{21}{4}$

PTO

5 i

$(a+b)^2-(a-b)^2=4ab$

$(a+b)^2-(0.9)^2=4\times 0.36$

$(a+b)^2-0.81=1.44$

$(a+b)^2=1.44+0.81$

$(a+b)^2=2.25$

$(a+b)=\pm 1.5$

ii $a^2-b^2=(a+b)(a-b)$

$=\pm 1.5\times 0.9$

$=\pm 1.35$

6

i $(a+b)^2+(a-b)^2=2(a^2+b^2)$

$6^2+4^2=2(a^2+b^2)$

$36+16=2(a^2+b^2)$

$52=2(a^2+b^2)$

$\frac{52}{2}=a^2+b^2$

$26=a^2+b^2$

ii

ii $(a+b)^2 = a^2 + b^2 + 2ab$

$6^2 = 26 + 2ab$

$36 - 26 = 2ab$

$10 = 2ab$

$\frac{10}{2} = ab$

$5 = ab$

7 $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$

$6^2 - \left(a - \frac{1}{a}\right)^2 = 4$

$36 - 4 = \left(a - \frac{1}{a}\right)^2$

$32 = \left(a - \frac{1}{a}\right)^2$

$\sqrt{32} = a - \frac{1}{a}$

$\pm\sqrt{16 \times 2} = a - \frac{1}{a}$

$\pm 4\sqrt{2} = a - \frac{1}{a}$

ii $(a+b)(a-b)=a^2-b^2$

$\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)=a^2-\frac{1}{a^2}$

$4\times\pm 4\sqrt{2}=a^2-\frac{1}{a^2}$

$\pm 24\sqrt{2}=a^2-\frac{1}{a^2}$

8

1 $a+\frac{1}{a}$

$\left(a+\frac{1}{a}\right)^2-\left(a-\frac{1}{a}\right)^2=4$

$\left(a+\frac{1}{a}\right)^2-8^2=4$

$\left(a+\frac{1}{a}\right)^2=4+64$

$\left(a+\frac{1}{a}\right)^2=68$

$\left(a+\frac{1}{a}\right)=\pm\sqrt{68}$

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$\left(a+\frac{1}{a}\right) = \pm\sqrt{68}$

$\left(a+\frac{1}{a}\right) = \pm\sqrt{4\times17}$

$\left(a+\frac{1}{a}\right) = \pm 2\sqrt{17}$

ii $\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right) = a^2-\frac{1}{a^2}$

$\pm 2\sqrt{17}\times 8 = a^2-\frac{1}{a^2}$

$\pm 16\sqrt{17} = a^2-\frac{1}{a^2}$

so $a^2-\frac{1}{a^2} = \pm 16\sqrt{17}$

4) $\frac{a^2}{a} - \frac{3a}{a} + \frac{1}{a} = \frac{0}{a}$

$a-3+\frac{1}{a}=0$

$a+\frac{1}{a}=3$

ii $\left(a+\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$3^2 = a^2 + \frac{1}{a^2} + 2$

$9-2 = a^2 + \frac{1}{a^2}$

$7 = a^2 + \frac{1}{a^2}$

10

i $\frac{a^2-5a-1}{a} = \frac{0}{a}$

$\frac{a^2}{a} - \frac{5a}{a} - \frac{1}{a} = 0$

$a - 5 - \frac{1}{a} = 0$

$a - \frac{1}{a} = 5$

iii $\left(a+\frac{1}{a}\right)^2 - \left(a-\frac{1}{a}\right)^2 = 4$

$\left(a+\frac{1}{a}\right)^2 - \left(a-\frac{1}{a}\right)^2 = 4$

$\left(a+\frac{1}{a}\right)^2 - 5^2 = 4$

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$\left(a+\frac{1}{a}\right)^2 = 4+25$

$\left(a+\frac{1}{a}\right)^2 = 4+25$

$\left(a+\frac{1}{a}\right)^2 = 29$

ii $a^2-b^2=(a+b)(a-b)$

$a^2-\frac{1}{a^2}=\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$

$= \pm\sqrt{29}\times 5$

$= \pm 5\sqrt{29}$

ii $(3x+4y)=16$

squarring on both sides

$(3x+4y)^2=16^2$

$(3x)^2+(4y)^2+2\times 3x\times 4y=256$

$9x^2+16y^2+24xy=256$

$9x^2+16y^2+24xy=256$

$9x^2+16y^2=256-96$

$9x^2+16y^2=\underline{160}$

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12 Let 2 numbers be x and y, $x > y$

$x - y = 5$

$x^2 + y^2 = 73$

To find :- xy

$(x-y)^2 = x^2 + y^2 - 2xy$

$5^2 = 73 - 2xy$

$2xy = 73 - 25$

$2xy = 48$

$xy = \frac{48}{2} = 24$

$\therefore$ product of numbers is 24

Ex 4B

2

i $(3a-2b)^3$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$a = 3a \quad b = 2b$

$\therefore (3a-2b)^3 = (3a)^3 - (2b)^3 - 3 \times 3a \times 2b(3a-2b)$

$= 27a^3 - 8b^3 - 18ab(3a-2b)$

$= 27a^3 - 8b^3 - 54a^2b^2 + 36ab^2$

ii $(5a+3b)^3$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a = 5a \quad b = 3b$

$(5a+3b)^3 = \cancel{(5a+3b)^3}\ a^3 + b^3 + 3ab(a+b)$

$a = 5a, \; b = 3b$

$\therefore (5a+3b)^3 = (5a)^3 + 3 \times 5a \times 3b (5a+3b)$

$125a^3 + 27b^3 + 225a^2b + 135ab^2$

iii $\left(2a + \frac{1}{2a}\right)^3$

$a = 2a, \; b = \frac{1}{2a}$

$\left(2a + \frac{1}{2a}\right) = (2a)^3 + \left(\frac{1}{2a}\right)^3 + 3 \times 2a \times \frac{1}{2a}\left(2a + \frac{1}{2a}\right)$

$8a^3 + \frac{1}{8a^3} + 3\left(2a + \frac{1}{2a}\right)$

$= 8a^3 + \frac{1}{8a^3} + 6a - \frac{3}{2a}$

D $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a+b)^3 = a^3 - 3ab(a-b)$

-2) 2 $\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} - 3 \times \frac{1}{a}\left(a - \frac{1}{a}\right)$

$\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$

3 $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$\left(a + \frac{1}{a}\right)^2 = 47 + 2 = 49$

$\left(a + \frac{1}{a}\right)^{\cancel{2}} = \sqrt{49}$

$a + \frac{1}{a} = \pm 7$

ii $\left(a + \frac{1}{a}\right)^2 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$

$7^3 = a^3 + 1 + 3 \times 7$ 6

$343 - 21 = a^3 + \frac{1}{a^3}$

$a^3 + \frac{1}{a^3} = 322$

4

i $\left(a - \frac{1}{a}\right)^3 = a^2 + \frac{1}{a^2} - 2$

$\left(a - \frac{1}{a}\right)^2 = 18 - 2 = 16$

$= \left(a - \frac{1}{a}\right) = \pm 4$

ii $\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$

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$4^3 = a^3 - \frac{1}{a^3} - 3 \times 4$

$64 = a^3 - \frac{1}{a^3} - 12$

$64 = 12 = a^3 - 1$

$64 + 12 = a^3 - \frac{1}{a^3}$

$= 76 = a^3 - \frac{1}{a^3}$

6 Cubing on both sides

$(a+2b)^3 = 5^3$

$a^3 + (2b)^3 a + 3 \times a \times 2a (a + 2ab = 125$

$a^3 + 8b^3 + 6ab \times 5 = 125$

$a^3 + 8b^3 + 30ab = 125$

7 given $\left(a + \frac{1}{a}\right)^2 = 3$

$\Rightarrow a + \frac{1}{a} = \pm\sqrt{3}$

$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$

$(\sqrt{3})^3 = a^3 + \frac{1}{a^3} + 3 \times \sqrt{3}$

$3\sqrt{3} = a^3 + \frac{1}{a^3}$

$\therefore a^3 + \frac{1}{a^3} = 0$

$(\sqrt{3})^3 = \sqrt{3} \times \sqrt{3} \times \sqrt{3}$
$= 3\sqrt{3}$

8 if $a+b+c=0$ then $a^3+b^3+c^3 = 3abc$
$a+2b+c=0$ then $a^3+(2b)^3+c^3 = 3 \times a \times$
$a^3+8b^3+c^3 = 6abc$

OR
given, $a+2b+c=0$
$a+2b = -c$
cubing on both sides $(a+2b)^3 = (-c)^3$
$a^3 + (2b)^3 + 3 \times a \times 2b(a+2b) = -c^3$

$a^3 + 8b^3 + 6ab \times -c = -c^3$

$a^3 + 8b^3 + c^3 = 6abc$

9
i $9^3 - 5^3 - 4^3$
$9^3 + (-5)^3 + (-4)^3$
$a = 9,\ b = -5,\ c = -4$
$a+b+c = 9-5-4 = 9-9 = 0$

if $a+b+c=0$ then $a^3+b^3+c^3 = 3abc$
$9^3 + (-5)^3 + (-4)^3 = 3 \times 9 \times -5 \times -4$
$= 540$

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ii $38^3 + (-26)^3 + (-12)^3$

$a = 38, b = -26, c = -12$

$a + b + c = 38 - 26 - 12 = 38 - 38 = 0$

$\therefore 38^3 + (-26)^3 + (-12)^3 = 3 \times 38 \times -26 \times -12$

$= 35568$

12

$x + \frac{1}{x} = 2$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$2^2 - 2 = x^2 + \frac{1}{x^2}$

$2^{\cancel{2}} \cancel{-} 2 = x^2 + \frac{1}{x^2}$

$2 = x^2 + \frac{1}{x^2}$

$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$2^3 = x^3 + \frac{1}{x^3} + 3 \times 2$

$8 - 6 = x^3 + \frac{1}{x^3}$

$\therefore x^3 + \frac{1}{x^3} = 2$

16 2

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x} + 2$

$\left(x^2 + \frac{1}{x^2}\right)^2 = (x^2)^2 + \frac{1}{(x^2)^2} + 2$

$2^2 - 2 = x^4 + \frac{1}{x^4}$

$\therefore x^4 + \frac{1}{x^4} = 2$

$\therefore x^2 + \frac{1}{x^2} = x^3 + \frac{1}{x^3} + x^4 + \frac{1}{x^4}$

i $(x+y)^2 = x^2 + y^2 + 2xy$

$9^2 = x^2 + y^2 + 2 \times 20$

$81 - 40 = x^2 + y^2$

$\therefore x^2 + y^2 = 41$

ii $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$9^3 = x^3 + y^3 + 3 \times 20 \times 9$

$729 - 540 = x^3$

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16 2 positive number