

Identities & Expansions

$$1 \quad (a+b)^2 = a^2 + 2ab + b^2$$

$$2 \quad (a-b)^2 = a^2 - 2ab + b^2$$

$$3 \quad (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$4 \quad (a+b)^2 - (a-b)^2 = 4ab$$

$$5 \quad (a+b)(a-b) = a^2 - b^2$$

$$6 \quad (a+b)^3 = a^3 + b^3 + 3ab(a+b) \\ = a^3 + b^3 + 3a^2b + 3ab^2$$

$$7 \quad (a-b)^3 = a^3 - b^3 - 3ab(a-b) \\ = a^3 - b^3 - 3a^2b + 3ab^2$$

$$8 \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\ = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$9 \quad \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$10 \quad \left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$11 \quad \left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2 = 2\left(a^2 + \frac{1}{a^2}\right)$$

$$12 \quad (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$13 \quad (x+a)(x-b) = x^2 + (a-b)x - ab$$

$$14 \quad (x-a)(x-b) = x^2 + (b-a)x - ab$$

$$15 \quad (x-a)(x-b) = x^2 - (a+b)x + ab$$

$$17 \quad a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

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Ex 4a

2
i $\left(\frac{7}{8}x + \frac{4}{5}y\right)^2$
using the identity $(a+b)^2 = a^2 + 2ab + b^2$

$$a = \frac{7}{8}x \quad b = \frac{4}{5}y$$

$$\therefore \left(\frac{7}{8}x + \frac{4}{5}y\right)^2 = \left(\frac{7}{8}x\right)^2 + 2 \times \frac{7}{8}x \times \frac{4}{5}y + \left(\frac{4}{5}y\right)^2$$

$$= \frac{49x^2}{64} + \frac{7}{5}xy + \frac{16y^2}{25}$$

ii $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

using identity $(a-b)^2 = a^2 + b^2 - 2ab$

$$\left(\frac{2x}{7} - \frac{7y}{4}\right)^2 = \left(\frac{2x}{7}\right)^2 + \left(\frac{7y}{4}\right)^2 - 2\left(\frac{2x}{7} \times \frac{7y}{4}\right)$$

$$\therefore \left(\frac{2x}{7} - \frac{7y}{4}\right)^2 = \left(\frac{2}{7}x\right)^2 - 2 \times \frac{2}{7}x \times \frac{7}{4}y + \left(\frac{7}{4}y\right)^2$$

$$= \frac{4x^2}{49} - xy + \frac{49}{16}y^2$$

3 using identity $(a+h)^2 - (a-h)^2$

$$(a+h)^2 - (a-h)^2 = 4ah$$

$$a = \frac{a}{2h} \quad h = \frac{2h}{a}$$

$$\left(\frac{a}{25} + \frac{2h}{a}\right)^2 - \left(\frac{a}{25} - \frac{2h}{a}\right)^2$$

$$= 4 \times \frac{a}{25} \times \frac{2h}{a} = 4$$

$$\left(\frac{a}{25} + \frac{2h}{a}\right)^2 - \left(\frac{a}{25} - \frac{2h}{a}\right)^2 = 4$$

$$4 - 4 = 0$$

8. If $a - \frac{1}{a} = 8$ and $a \neq 0$ find

i) $a + \frac{1}{a}$

ii) $a^2 - \frac{1}{a^2}$

i) $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$

$$\left(a + \frac{1}{a}\right)^2 - (8^2) = 4$$

$$\left(a + \frac{1}{a}\right)^2 = 4 + 64$$

$$\sqrt{\left(a + \frac{1}{a}\right)^2} = \sqrt{68}$$

$$\left(a + \frac{1}{a}\right) = \pm \sqrt{4 \times 17}$$

$$\left(a + \frac{1}{a}\right) = \pm 2\sqrt{17}$$

ii) $\left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) = a^2 - \frac{1}{a^2}$

$$\rightarrow \pm 2\sqrt{17} \times 8 = a^2 - \frac{1}{a^2}$$

$$\pm 16\sqrt{17} = a^2 - \frac{1}{a^2}$$

$$\text{So } a^2 - \frac{1}{a^2} = \pm 16\sqrt{17}$$

$$\begin{array}{r} 2 \overline{) 68} \\ 2 \overline{) 34} \\ 17 \overline{) 17} \\ 1 \end{array}$$

ii $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

→ Using the identity $(a-b)^2 = a^2 - 2ab + b^2$

$a = \frac{2}{7}x$, $b = \frac{7}{4}y$

$$\begin{aligned} \therefore \left(\frac{2x}{7} - \frac{7y}{4}\right)^2 &= \left(\frac{2}{7}x\right)^2 + \left(\frac{7}{4}y\right)^2 - 2 \times \frac{2}{7}x \times \frac{7}{4}y \\ &= \frac{4x^2}{49} + \frac{49y^2}{16} - xy \end{aligned}$$

3. Evaluate $\left(\frac{a+b}{2b} - \frac{a-b}{a}\right)^2 = \left(\frac{a-2b}{2b} - \frac{a-b}{a}\right)^2 - 4$

solve this first

— Using the identity $(a+b)^2 - (a-b)^2 = 4ab$

$a = \frac{a}{2b}$, $b = \frac{2b}{a}$

$$\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 = 4ab$$

$$= 4 \times \frac{a}{2b} \times \frac{2b}{a} - 4$$

$$= 4 - 4$$

$$= 0$$

4. If $x+y = \frac{7}{2}$ and $xy = \frac{5}{2}$ find

i $x-y$

ii $x^2 - y^2$

i. Using the identity $(a+b)^2 - (a-b)^2 = 4ab$
 $(x+y)^2 - (x-y)^2 = 4xy$

$$\Rightarrow \left(\frac{7}{2}\right)^2 - (x-y)^2 = 4 \times \frac{5}{2}$$

$$\Rightarrow \frac{49}{4} - (x-y)^2 = 40$$

$$\Rightarrow \frac{49}{4} - \frac{10 \times 4}{1 \times 4} = (x+y)^2$$

$$\frac{49-40}{4} = (x+y)^2$$

$$(x+y)^2 = \frac{9}{4}$$

$$(x+y) = \sqrt{\frac{9}{4}}$$

$$(x+y) = \pm \frac{3}{2}$$

ii Using the identity $(a+b)(a-b) = a^2 - b^2$
 $(x+y)(x-y) = x^2 - y^2$

$$\Rightarrow x^2 - y^2 = \frac{7}{2} \times \frac{3}{2}$$

$$x^2 - y^2 = \pm \frac{21}{4}$$

5. If $a-b = 0.9$ and $ab = 0.36$, find

i $a+b$

ii $a^2 - b^2$

i Using the identity $(a+b)^2 - (a-b)^2 = 4ab$

$$\Rightarrow (a+b)^2 - (0.9)^2 = 4 \times 0.36$$

$$(a+b)^2 - 0.81 = 1.44$$

$$(a+b)^2 = 1.44 + 0.81$$

$$(a+b)^2 = 2.25$$

$$a+b = \sqrt{2.25}$$

$$a+b = \pm 1.5$$

\rightarrow 15 square is 225
 10
 1.5

ii Using the identity $(a+b)(a-b) = a^2 - b^2$

$$\rightarrow 1.5 \times 0.9 = a^2 - b^2$$

$$a^2 - b^2 = 1.35$$

$$\begin{array}{r} 1.5 \\ \times 0.9 \\ \hline 135 \\ 000 \\ \hline 1.35 \end{array}$$

6. If $a-b=4$ and $a+b=6$, find

i $a^2 + b^2$

ii ab

i

$$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$6^2 + 4^2 = 2(a^2 + b^2)$$

$$36 + 16 = 2(a^2 + b^2)$$

$$52 = 2(a^2 + b^2)$$

$$26 = a^2 + b^2$$

ii $(a+b)(a-b)$ (we can use many identities)

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$6^2 = 26 + 2ab$$

$$36 - 26 = 2ab$$

$$10 = 2ab$$

$$5 = ab$$

$$ab = 5$$

7. If $a + \frac{1}{a} = 6$ and $a \neq 0$ find

i $a - \frac{1}{a}$

ii $a^2 - \frac{1}{a^2}$

$$i \quad \left(\frac{a-1}{a} \right)^2 = a^2$$

$$ii \quad \left(a + \frac{1}{a} \right)^2 - \left(a - \frac{1}{a} \right)^2 = 4$$

$$6^2 - \left(a - \frac{1}{a} \right)^2 = 4$$

$$36 - 4 = \left(a - \frac{1}{a} \right)^2$$

$$32 = \left(a - \frac{1}{a} \right)^2$$

$$\pm \sqrt{32} = \sqrt{\left(a - \frac{1}{a} \right)^2} \rightarrow \text{make squares on$$

sides to enter

the squares as

is not a perfect

square

$$\pm \sqrt{32} = a - \frac{1}{a}$$

$$\pm \sqrt{16 \times 2} = a - \frac{1}{a}$$

$$\pm 4\sqrt{2} = a - \frac{1}{a}$$

$$ii \quad 4(a+b)(a-b) = a^2 - b^2$$

$$\left(a + \frac{1}{a} \right) \left(a - \frac{1}{a} \right) = a^2 - b^2$$

$$a^2 - \frac{1}{a^2}$$

$$6 \times \pm 4\sqrt{2} = a^2 - b^2$$

$$a^2 - \frac{1}{a^2}$$

$$\pm 24\sqrt{2} = a^2 - b^2$$

$$a^2 - \frac{1}{a^2}$$

$$\text{So } a^2 - \frac{1}{a^2} = \pm 24\sqrt{2}$$