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8. If $a - \frac{1}{a} = 8$ and $\neq 0$ find

i $a + \frac{1}{a}$

ii $a^2 - \frac{1}{a^2}$

2	68
2	34
17	17
	1

i ⇒ $\left(a+\frac{1}{a}\right)^2 - \left(a-\frac{1}{a}\right)^2 = 4$

$$\left(a+\frac{1}{a}\right)^2 - (8^2) = 4$$

$$\left(a+\frac{1}{a}\right)^2 = 4 + 64$$

$$\sqrt{\left(a+\frac{1}{a}\right)^2} = \sqrt{68}$$

$$\left(a+\frac{1}{a}\right) = \pm\sqrt{4 \times 17}$$

$$\left(a+\frac{1}{a}\right) = \pm 2\sqrt{17}$$

ii $\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right) = a^2 - \frac{1}{a^2}$

$$\rightarrow \pm 2\sqrt{17} \times 8 = a^2 - \frac{1}{a^2}$$

$$\pm 16\sqrt{17} = a^2 - \frac{1}{a^2}$$

so $a^2 - \frac{1}{a^2} = \pm 16\sqrt{17}$

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ii $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

$\rightarrow$ Using the identity $(a-b)^2 = a^2 - 2ab + b^2$

$a = \frac{2}{7}x$, $b = \frac{7}{4}y$

$\therefore \left(\frac{2x}{7} - \frac{7y}{4}\right)^2 = \left(\frac{2}{7}x\right)^2 + \left(\frac{7}{4}y\right)^2 - \cancel{2}^{2} \times \frac{\cancel{2}^{1}}{\cancel{7}} x \times \frac{\cancel{7}}{\cancel{4}_{1}} y$

$= \frac{4\cancel{x^2}x^2}{49} + \frac{49}{16}y^2 - xy$

3. Evaluate $\left(\overset{a}{\frac{a}{2b}} + \overset{b}{\frac{2b}{a}}\right)^2 = \left(\overset{a}{\frac{a}{2b}} - \overset{b}{\frac{2b}{a}}\right)^2 - 4$

solve this first

– Using the identity $(a+b)^2 - (a-b)^2 = 4ab$

$a = \frac{a}{2b}$, $b = \frac{2b}{a}$

$\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 = 4ab$

$= 4 \times \frac{\cancel{a}}{\cancel{2b}} \times \frac{\cancel{2b}}{\cancel{a}} - 4$

$= 4 - 4$

$= 0$

4. If $x+y = \frac{7}{2}$ and $xy = \frac{5}{2}$, find

i $x-y$

ii $x^2 - y^2$

i. Using the identity $(a+b)^2 - (a-b)^2 = 4ab$

$(x+y)^2 - (x-y)^2 = 4xy$

$\Rightarrow \left(\frac{7}{2}\right)^2 - (x-y)^2 = \cancel{4}^{2} \times \frac{5}{\cancel{2}_{1}}$

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$$\Rightarrow \frac{49}{4} - (x-y)^2 = 10$$

$$\Rightarrow \frac{49}{4} - \frac{10 \times 4}{1 \times 4} = (x+y)^2$$

$$\frac{49-40}{4} = (x+y)^2$$

$$(x-y)^2 = \frac{9}{4}$$

$$(x-y) = \sqrt{\frac{9}{4}}$$

$$(x-y) = \pm\frac{3}{2}$$

ii Using the identity $(a+b)(a-b) = a^2 - b^2$

$(x+y)(x-y) = x^2 - y^2$

$$\rightarrow x^2 - y^2 = \frac{7}{2} \times \frac{3}{2}$$

$$x^2 - y^2 = \pm\frac{21}{4}$$

5. If $a-b = 0.9$ and $ab = 0.36$, find

i $a+b$.

ii $a^2 - b^2$

i Using the identity $(a+b)^2 - (a-b)^2 = 4ab$

$$\rightarrow (a+b)^2 - (0.9)^2 = 4 \times 0.36$$

$$(a+b)^2 - 0.81 = 1.44$$

$$(a+b)^2 = 1.44 + 0.81$$

$$(a+b)^2 = 2.25$$

$a+b = \sqrt{2.25}$ → 15 square is 225 so 1.5

$$a+b = \pm 1.5$$

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ii Using the identity $(a+b)(a-b) = a^2-b^2$

$\rightarrow$ $1.5 \times 0.9 = a^2-b^2$

$a^2-b^2 = 1.35$

$$\begin{array}{r} {}^{4}1.5 \\ \times 0.9 \\ \hline 135 \\ +000 \\ \hline 1.35 \end{array}$$

6. If $a-b=4$ and $a+b=6$, find

i a^2+b^2.

ii ab

i $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

$6^2 + 4^2 = 2(a^2+b^2)$

$36 + 16 = 2(a^2+b^2)$

$\frac{\cancel{52}^{26}}{\cancel{2}_1} = a^2+b^2$

$a^2+b^2 = 26$

ii $(a+b)(a-b)$ (we can use many identities)

$(a+b)^2 = a^2+b^2+2ab$

$6^2 = 26 + 2\times ab$

$36-26 = 2ab$

$\frac{\cancel{10}^{5}}{\cancel{2}_1} = ab$

$ab = 5$

7. If $a+\frac{1}{a} = 6$ and $a \neq 0$ find

i $a - \frac{1}{a}$

ii $a^2 - \frac{1}{a^2}$

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i $\cancel{\left(a - \frac{1}{a}\right)^2 = a^2}$

i $\left(a+\frac{1}{a}\right)^2 - \left(a-\frac{1}{a}\right)^2 = 4$

$6^2 - \left(a-\frac{1}{a}\right)^2 = 4$

$36 - 4 = \left(a-\frac{1}{a}\right)^2$

$32 = \left(a-\frac{1}{a}\right)^2$

$\pm\sqrt{32} = \sqrt{\left(a-\frac{1}{a}\right)^2}$ → make squares on sides to interrupt the squares as is not a perfect square

$\pm\sqrt{32} = a - \frac{1}{a}$

$\pm\sqrt{16\times 2} = a - \frac{1}{a}$

$\pm 4\sqrt{2} = a - \frac{1}{a}$

ii If $(a+b)(a-b) = a^2 - b^2$

$\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right) = \cancel{a^2 - b^2}$ $\quad a^2 - \frac{1}{a^2}$

$6 \times \pm 4\sqrt{2} = \cancel{a^2 - b^2}$ $\quad a^2 - \frac{1}{a^2}$

$\pm 24\sqrt{2} = \cancel{a^2 - b}$ $\quad a^2 - \frac{1}{a^2}$

So $a^2 - \frac{1}{a^2} = \pm 24\sqrt{2}$