

## Exercises

$$1 \quad (a+b)^2 = a^2 + 2ab + b^2$$

$$2 \quad (a-b)^2 = a^2 - 2ab + b^2$$

$$3 \quad (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$4 \quad (a+b)^2 - (a-b)^2 = 4ab$$

$$5 \quad (a+b)(a-b) = a^2 - b^2$$

$$6 \quad (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$= a^3 + b^3 + 3a^2b + 3ab^2$$

$$7 \quad (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$= a^3 - b^3 - 3a^2b + 3ab^2$$

$$8 \quad (a+bx+c)^2 = a^2 + b^2 + c^2 + 2abx + 2ac + 2bc$$

$$= a^2 + b^2 + c^2 + 2(ab + ac + bc)$$

$$9 \quad \left(\frac{a+b}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$10 \quad \left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$11 \quad \left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2 = 2\left(1 + \frac{1}{a^2}\right)$$

$$12 \quad (x+a)(x+b) = x^2 + (a+b)x + ab$$

$$13 \quad (x+a)(x-b) = x^2 + (a-b)x - ab$$

$$14 (a-b)(x-h) = x^3 + (h-h)x - ah$$

$$15 (a-b)(x-h) = x^2 - (a+b)x + ah$$

$$17 a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$18 \omega_1 = (a-b) - (b-a)$$

$$\omega_2 = (-a-\omega) + (\omega+a)$$

$$(a+b)\omega_1 + (b-a) = (a+b)$$

$$(a-b)\omega_2 + (b-a) = (a-b)$$

$$(\omega_1 + \omega_2) + (\omega_1 - \omega_2) = (0+1+0)$$

$$-1 + \omega = \omega$$

Ex 4a

ii  $\left(\frac{7}{8}x + \frac{4}{5}y\right)^2$

using the identity  $(a+b)^2 = a^2 + 2ab + b^2$

$$a = \frac{7}{8}x, b = \frac{4}{5}y$$

$$\therefore \left(\frac{7}{8}x + \frac{4}{5}y\right)^2 = \left(\frac{7}{8}x\right)^2 + 2 \times \frac{7}{8}x \times \frac{4}{5}y + \left(\frac{4}{5}y\right)^2$$

$$\left(\frac{4}{5}y\right)$$

$$= \frac{49x^2}{64} + \frac{7}{5}xy + \frac{16y^2}{25}$$

iii  $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

using identity  $(a-b)^2 = a^2 + b^2 - 2ab$

$$\left(\frac{2x}{7} - \frac{7y}{4}\right)^2 = \left(\frac{2x}{7}\right)^2 + \left(\frac{7y}{4}\right)^2 - 2 \left(\frac{2x}{7}\right) \left(\frac{7y}{4}\right)$$

$$\therefore \left(\frac{2x}{7} + \frac{7y}{4}\right)^2 = \left(\frac{2x}{7}\right)^2 - 2 \times \frac{2}{7}x \times \frac{7}{4}y + \left(\frac{7y}{4}\right)^2$$

$$= \frac{4x^2}{49} - xy + \frac{49}{16}y^2$$

3) using identity  $(a+b)^2 - (a-b)^2$

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$a = \frac{a}{2b}, b = \frac{2b}{a}$$

$$\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2$$

$$= 4 \cancel{\left(\frac{a}{2b} \times \frac{2b}{a}\right)} = 4$$

$$\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2$$

$$4 - 4 = 0$$

8. If  $a - \frac{1}{a} = 8$  and  $\neq 0$  find

i)  $a + \frac{1}{a}$

ii)  $a^2 - \frac{1}{a^2}$

$$i) \left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = ?$$

$$\left(a + \frac{1}{a}\right)^2 - (8^2) = 4$$

$$\left(a + \frac{1}{a}\right)^2 = 4 + 64$$

$$\sqrt{\left(a + \frac{1}{a}\right)^2} = \pm \sqrt{68}$$

$$\left(a + \frac{1}{a}\right) = \pm \sqrt{4 \times 17}$$

$$\left(a + \frac{1}{a}\right) = \pm 2\sqrt{17}$$

ii)  $\left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) = a^2 - \frac{1}{a^2}$

$$\rightarrow \pm 2\sqrt{17} \times 8 = a^2 - \frac{1}{a^2}$$

$$\pm 16\sqrt{17} = a^2 - \frac{1}{a^2}$$

$$\text{So } a^2 - \frac{1}{a^2} = \pm 16\sqrt{17}$$

$$\text{ii} \quad \left(\frac{2x}{7} - \frac{7y}{4}\right)^2$$

→ Using the identity  $(a-b)^2 = a^2 - 2ab + b^2$

$$a = \frac{2}{7}x, \quad b = \frac{7}{4}y$$

$$\begin{aligned} \therefore \left(\frac{2x}{7} - \frac{7y}{4}\right)^2 &= \left(\frac{2x}{7}\right)^2 + \left(\frac{7y}{4}\right)^2 - 2 \times \frac{2}{7}x \times \frac{7}{4}y \\ &= \frac{4x^2}{49} + \frac{49y^2}{16} - xy // \end{aligned}$$

$$3. \text{ Evaluate } \left(\frac{a+2b}{2b} - \frac{a}{a}\right)^2 = \left(\frac{a-2b}{2b} - \frac{a}{a}\right)^2 - 4$$

Solve this first

- Using the identity  $(a+b)^2 - (a-b)^2 = 4ab$

$$a = \frac{a}{2b}, \quad b = \frac{2b}{a}$$

$$\begin{aligned} \left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 &= 4ab \\ &= 4 \times \cancel{\frac{a}{2b}} \times \cancel{\frac{2b}{a}} - 4 \\ &= 4 - 4 \\ &= 0 // \end{aligned}$$

$$4. \text{ If } x+y = \frac{7}{2} \text{ and } xy = \frac{5}{2}, \text{ find }$$

$$\text{i} \quad x-y$$

$$\text{ii} \quad x^2 - y^2$$

$$\begin{aligned} \text{i.} \quad \text{Using the identity } (a+b)^2 - (a-b)^2 &= 4ab \\ (x+y)^2 - (x-y)^2 &= 4xy \end{aligned}$$

$$\Rightarrow \left(\frac{7}{2}\right)^2 - (x-y)^2 = 4 \times \frac{5}{2}$$

$$\Rightarrow \frac{49}{4} - (x-y)^2 = 10$$

$$\Rightarrow \frac{49}{4} - 10 = (x+y)^2$$

$$\frac{49-40}{4} = (x+y)^2$$

$$(x+y)^2 = \frac{9}{4}$$

$$(x+y) = \pm \frac{3}{2}$$

$$(x-y) = \pm \frac{3}{2}$$

ii Using the identity  $(a+b)(a-b) = a^2 - b^2$   
 $(x+y)(x-y) = x^2 - y^2$

$$\rightarrow x^2 - y^2 = \frac{7}{2} \times \frac{3}{2}$$

$$x^2 - y^2 = \frac{21}{4}$$

5. If  $a-b=0.9$  and  $ab=0.36$ , find

i  $a+b$

ii  $a^2 - b^2$

i Using the identity  $(a+b)^2 - (a-b)^2 = 4ab$

$$\rightarrow (a+b)^2 - (0.9)^2 = 4 \times 0.36$$

$$(a+b)^2 - 0.81 = 1.44$$

$$(a+b)^2 = 1.44 + 0.81$$

$$a+b = \sqrt{2.25}$$

$$a+b = \pm 1.5$$

15 square is 225  
 $\sqrt{2.25} = 1.5$

ii Using the identity  $(a+b)(a-b) = a^2 - b^2$

$$\rightarrow 1.5 \times 0.9 = a^2 - b^2$$

$$a^2 - b^2 = 1.35$$

$$\begin{array}{r} \cancel{1.5} \\ \times 0.9 \\ + 135 \\ \hline \cancel{0} \end{array}$$

$$1.35$$

6. If  $a-b=4$  and  $a+b=6$ , find

i  $a^2 + b^2$ .

ii  $ab$

$$\begin{aligned} i \quad & (a+b)^2 + (a-b)^2 = 2(a^2 + b^2) \\ & 6^2 + 4^2 = 2(a^2 + b^2) \\ & 36 + 16 = 2(a^2 + b^2) \\ & \frac{52}{2} = a^2 + b^2 \\ & a^2 + b^2 = 26 \end{aligned}$$

ii  $(a+b)(a-b)$  (we can use many identities)

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$6^2 = 26 + 2 \times ab$$

$$36 - 26 = 2ab$$

$$\frac{10}{2} = ab$$

$$ab = 5$$

7. If  $a + \frac{1}{a} = 6$  and  $a \neq 0$  find

i  $a - \frac{1}{a}$

ii  $a^2 - \frac{1}{a^2}$

$$\text{i} \quad \left( a + \frac{1}{a} \right)^2 = a^2 + 1$$

$$\text{i} \quad \left( a + \frac{1}{a} \right)^2 - \left( a - \frac{1}{a} \right)^2 = 4$$

$$6^2 - \left( a - \frac{1}{a} \right)^2 = 4$$

$$36 - 4 = \left( a - \frac{1}{a} \right)^2$$

$$32 = \left( a - \frac{1}{a} \right)^2$$

$$\pm \sqrt{32} = \sqrt{\left( a - \frac{1}{a} \right)^2} \rightarrow \begin{array}{l} \text{make square} \\ \text{side to way} \\ \text{the square is} \end{array}$$

$$\pm \sqrt{32} = a - \frac{1}{a}$$

is not a perfect square

$$\pm \sqrt{16 \times 2} = a - \frac{1}{a}$$

$$\pm 4\sqrt{2} = a - \frac{1}{a}$$

$$\text{ii } 4(a+b)(a-b) = a^2 - b^2$$

$$\left( a + \frac{1}{a} \right) \left( a - \frac{1}{a} \right) = a^2 - \frac{1}{a^2}$$

$$6 \times \pm 4\sqrt{2} = a^2 - b^2$$

$$\pm 24\sqrt{2} = a^2 - \frac{1}{a^2}$$

$$\text{So } a^2 - \frac{1}{a^2} = \pm 24\sqrt{2}$$