Tutorial 2 - Bayesian Networks

COMP9418 – Advanced Topics in Statistical Machine Learning

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Lecture: Bayesian Networks

Topic: Questions from lecture topics

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Question 1

Z	Y	P(Z|Y)
1	1	a
0	1	1-a
1	0	b
0	0	1-b

X	P(x)
1	a
0	1-a

X	Y	P(Y|X)
1	1	a
1	0	1-a
0	1	b
0	0	1-b

Consider the random variables X, Y, Z which have the following joint distribution:

$$P(X, Y, Z) = P(X)P(Y|X)P(Z|Y)$$

a. Show that X and Z are conditionally independent given Y. $P(x,Y,Z)=P(x)P(Y|X)P(Z|x,Y)$

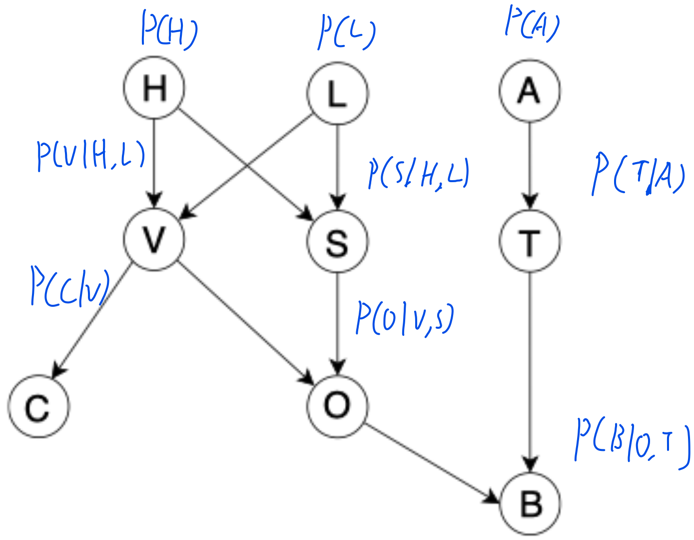
b. If X, Y and Z are binary variables, how many parameters are needed to specify a distribution of this form?

$2^3-1=7$ don't need the final one since it's just gonna be 1 - all the other parameters

But we only need $P(X)$, $P(Y|X)$ and $P(Z|Y)$, so we only need 5 parameters

Question 2

The Bayesian network shown below is a greatly simplified version of a network used for medical diagnosis in an intensive care unit. The diagnostic variables are the hypovolemia (H), the left ventricular failure (L) and the anaphylaxis (A). The intermediate variables are the left ventricular endiastolic volume (V), the stroke volume (S) and the total peripheral resistance (T). The measurement variables are the central venous pressure (C), the cardiac output (O) and the blood pressure (B). H, L and A take on the values $\{true, false\}$; V, S and T take on the values $\{low, high\}$; and C, O and B take on the values $\{low, medium, high\}$.



The Bayesian network is fully specified by its CPTs. We have:

$P(H = true) = 0.2$	
$P(L = true) = 0.05$	$P(A = true) = 0.01$
$P(V = low\|H = false, L = false) = 0.05$	$P(S = low\|H = false, L = false) = 0.05$
$P(V = low\|H = false, L = true) = 0.01$	$P(S = low\|H = false, L = true) = 0.95$
$P(V = low\|H = true, L = false) = 0.98$	$P(S = low\|H = true, L = false) = 0.5$
$P(V = low\|H = true, L = true) = 0.95$	$P(S = low\|H = true, L = true) = 0.98$
$P(T = low\|A = false) = 0.3$	$P(T = low\|A = true) = 0.98$
$P(C = low\|V = low) = 0.95$	$P(C = medium\|V = low) = 0.04$
$P(C = low\|V = high) = 0.01$	$P(C = medium\|V = high) = 0.29$
$P(O = low\|V = low, S = low) = 0.98$	$P(O = medium\|V = low, S = low) = 0.01$
$P(O = low\|V = low, S = high) = 0.3$	$P(O = medium\|V = low, S = high) = 0.69$
$P(O = low\|V = high, S = low) = 0.8$	$P(O = medium\|V = high, S = low) = 0.19$
$P(O = low\|V = high, S = high) = 0.01$	$P(O = medium\|V = high, S = high) = 0.01$
$P(B = low\|O = low, T = low) = 0.98$	$P(B = medium\|O = low, T = low) = 0.01$
$P(B = low\|O = low, T = high) = 0.3$	$P(B = medium\|O = low, T = high) = 0.6$
$P(B = low\|O = medium, T = low) = 0.98$	$P(B = medium\|O = medium, T = low) = 0.01$
$P(B = low\|O = medium, T = high) = 0.05$	$P(B = medium\|O = medium, T = high) = 0.4$
$P(B = low\|O = high, T = low) = 0.9$	$P(B = medium\|O = high, T = low) = 0.09$
$P(B = low\|O = high, T = high) = 0.01$	$P(B = medium\|O = high, T = high) = 0.09$

$\sum_B P(B|O,T) = 1$

$\sum_A P(A) \sum_T P(T|A)$

$P(H,L,V,S,C,O,A,T,B) = P(H)P(L)P(A)P(V|H,L)P(S|H,L)P(C|V)P(O|V,S)P(T|A)P(B|O,T)$

a. Write down the factorised joint distribution defined by the Bayesian network.

$\sum_{A,T,B,O,S,V,L,H,C} P(\cdots) = 1$

b. Show that the above factorised joint distribution is correctly normalised, using the rules of probability.

c. Let $X \perp\!\!\!\perp Y$ denote that X and Y are marginally independent and $X \perp\!\!\!\perp Y|Z$ denote that X and Y are conditionally independent given Z. Using the concept of d-separation, show or refute the following independence statements:

i. $H \perp\!\!\!\perp L$ Yes

ii. $H \perp\!\!\!\perp A$ Yes

iii. $C \perp\!\!\!\perp L$ No

iv. $V \perp\!\!\!\perp A|B$ No

d. For the cases where independence holds in item c, prove these independences using the rules of probability.

$P(H,L) = P(H)P(L)$

$P(H,A) = P(H)P(A)$ ← Independence

Question 3

Jack has three coins, C_1, C_2 and C_3, with p_1, p_2 and p_3 as their corresponding probabilities of landing heads. Jack flips coin C_1 twice and then decides, based on the outcome, whether to flip coin C_2 or C_3 next. In particular, if the two C_1 flips come out the same, Jack flips coin C_2 three times next. However, if the C_1 flips come out different, he flips coin C_3 three times next. Given the outcome of Jack's last three flips, we want to know whether his first two flips came out the same.

a. Show a Bayesian network structure (graph) for this problem.
b. Show the network conditional probability tables (CPTs) for all variables. If you have parameters that are shared among variables, define the CPT once and indicate which variables use that CPT.
c. Provide a query that solves this problem.

a)