Tutorial 3 - Variable Elimination

COMP9418 – Advanced Topics in Statistical Machine Learning

Lecturer: Gustavo Batista

Lecture: Variable Elimination

Topic: Questions from lecture topics

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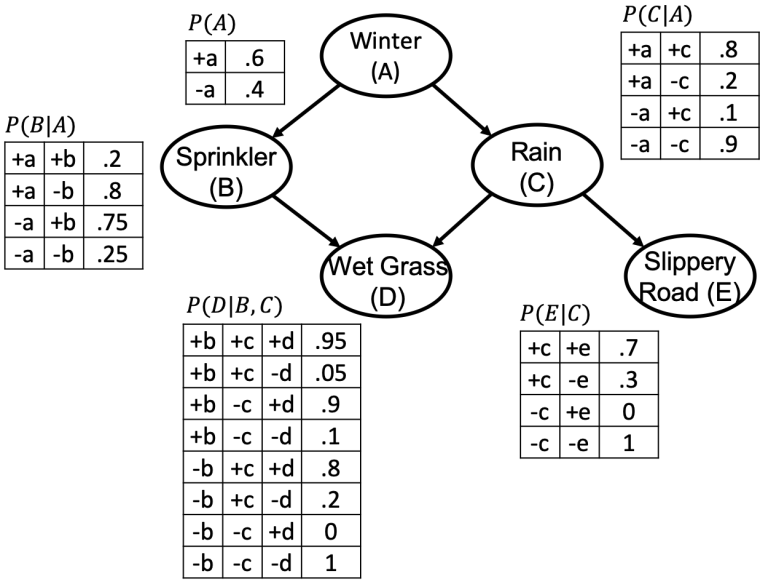
Question 1

Consider the following Bayesian network.



$P(A)$

+a	.6
-a	.4

$P(C|A)$

+a	+c	.8
+a	-c	.2
-a	+c	.1
-a	-c	.9

$P(B|A)$

+a	+b	.2
+a	-b	.8
-a	+b	.75
-a	-b	.25

$P(D|B,C)$

+b	+c	+d	.95
+b	+c	-d	.05
+b	-c	+d	.9
+b	-c	-d	.1
-b	+c	+d	.8
-b	+c	-d	.2
-b	-c	+d	0
-b	-c	-d	1

$P(E|C)$

+c	+e	.7
+c	-e	.3
-c	+e	0
-c	-e	1

Use variable elimination to computer the marginals $P(Q, e)$ and $P(Q|e)$ where $\mathbf{Q} = \{E\}$ and $\mathbf{e} : D = false$. Use the min-degree heuristic for determining a complete elimination order, breaking ties by choosing variables that come first in the alphabet.

Use the following algorithm for min-degree order:

```
Data: PGM: probabilistic graphical model
Data: X: variables in the PGM
Result: an ordering π of variables X
begin
    G ← induced graph of the factors in PGM;
    for i = 1 to number of variables in X do
        π(i) ← a variable in X with smallest number of neighbours in G;
        add an edge between every pair of non-adjacent neighbours of π(i) in G;
        delete variable π(i) from G and from X;
    end
end
return π
```

Question 2

Consider a chain network $C_0 \rightarrow C_1 \rightarrow \ldots \rightarrow C_n$. Suppose that variable C_t, for $t \geq 0$, denotes the health state of a component at time t. In particular, let each C_t take on states ok and faulty. Let C_0 denote component birth where $P(C_0 = ok) = 1$ and $P(C_0 = faulty) = 0$. For each $t > 0$, let the CPT of C_t be $P(C_t = ok|C_{t-1} = ok) = \lambda$ and $P(C_t = faulty|C_{t-1} = faulty) = 1$. If a component is healthy at time $t-1$, it remains healthy at time t with probability λ. If a component is faulty at time $t-1$, it remains faulty at time t with probability 1.

a. Using variable elimination with variable ordering C_0, C_1 compute $P(C_2)$.
b. Using variable elimination with variable ordering $C_0, C_1, \ldots, C_{n-1}$ compute $P(C_n)$.

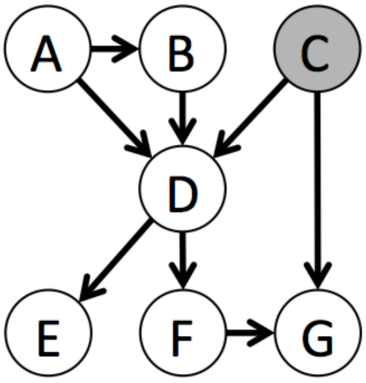
Question 3

Consider a Naive Bayes structure with edges $X \rightarrow Y_1, \ldots, X \rightarrow Y_n$.

a. What is the size of the largest factor computed in the variable elimination order $Y_1, \ldots, Y_n, X$?
b. What is the size of the largest factor computed in the variable elimination order $X, \ldots, Y_n$?

Question 4

For the Bayesian network below, all variables are binary. Assume we run variable elimination to compute the answer to the query $P(A, E| + c)$, with the following elimination order: B, D, G, F.



a. What is the size of the largest computed factor?
b. Can the min-degree heuristic help to find an ordering that generates a smaller largest factor?