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Table of Contents

GRE Math Strategies

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0. Math Strategies

0. Verbal Strategies

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September 4, 2018

Dear Student,

Thank you for picking up a copy of *GRE Math Strategies*. I hope this book provides just the guidance you need to get the most out of your GRE studies.

At Manhattan Prep, we continually aspire to provide the best instructors and resources possible. We hope that you will find our commitment manifest in this book.

If you have any questions or comments in general, please email our Student Services team at gre@manhattanprep.com.

Or give us a shout at 212-721-7400 (or 800-576-4628 in the United States or Canada).

We try to keep all our books free of errors. But if you think we've goofed, please visit manhattanprep.com/GRE/errata.

I look forward to hearing from you. Thanks again, and best of luck preparing for the GRE!

Sincerely,

Chris Ryan
Executive Director, Product Strategy
Manhattan Prep

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INTRODUCTION

In This Chapter...

Introduction

We know that you're looking to succeed on the GRE so that you can go to graduate school and do the things you want to do in life.

We also know that you may not have done a lot of math since high school. It's going to take hard work on your part to get a top GRE score. That's why we've put together the only set of books that will take you from the basics all the way up to the material you need to master for a near-perfect score, or whatever your goal score may be. You've taken the first step. Now it's time to get to work!

HOW TO USE THIS BOOK

Manhattan Prep's GRE materials are comprehensive. Nevertheless. keep in mind, that, depending on your score goal, it may not be necessary to get absolutely everything. Grad schools only see your overall Quantitative, Verbal, and Writing scores—they don't see exactly which strengths and weaknesses went into creating those scores.

You may be enrolled in one of our courses, in which case you already have a syllabus telling you in what order you should approach this book. But if you bought this book online or at a bookstore, feel free to approach the units—and even the chapters within the units—in whatever order works best for you. For the most part, the units, and the chapters within them, are independent. You don't have to master one section before moving on to the next. So if you're having a hard time with one thing, you can make a note to come back to it later and move on to another section. Similarly, it may not be necessary to solve every practice problem for every section. As you go through the material, continually assess whether you understand and can apply the principles in each individual section and chapter. The best way to do this is to solve the Check Your Skills and Problem Sets throughout. If you're confident you have a concept or method down, feel free to move on. If you struggle with something, make note of it for further review. Stay active in your learning and stay oriented toward the test—it's easy to read something and think you understand it, only to have trouble applying it in the 1–2 minutes you have to solve a problem.

STUDY SKILLS

As you're studying for the GRE, try to integrate your learning into your everyday life. You're going to want to do at least a little bit of math every day. Try to learn and internalize a little bit at a time, switching up topics often to help keep things interesting.

Keep in mind that, while many of your study materials are on paper (including Education Testing Service's [ETS's] most recent source of official GRE questions, *The Official Guide to the GRE revised General Test, Third Edition*), your exam will be administered on a computer. Because this test is computer-based, you will *not* be able to write on diagrams of geometry figures or otherwise physically mark up problems. Get used to this now. Solve the problems in these books on scratch paper. (Each of our books talks specifically about what to write down for different problem types.)

Again, as you study, stay focused on the test-day experience. As you progress, work on timed drills and sets of questions. Eventually, you should be taking full practice tests (available at www.manhattanprep.com/gre) under actual timed conditions.

The GRE Exam

EXAM STRUCTURE

The revised test has six sections. You will get a 10-minute break between the third and fourth sections and a 1-minute break between the others. The Analytical Writing section, also known as the Essay, is always first. The other five sections can be seen in any order and will include:

- Two Verbal Reasoning sections (20 questions each in 30 minutes per section). We'll call these sections Verbal for short.
- Two Quantitative Reasoning sections (20 questions each in 35 minutes per section). We'll call these sections Math for short.
- Either an unscored section or a research section.

An unscored section will look just like a third Verbal or Math section. You will not be told which of the three sections doesn't count. If you get a research section, it will be identified as such, and it will be the last section you get.

USING THE CALCULATOR

The small, four-function calculator with a square root button found on the GRE means that re-memorizing times tables or square roots is less important than it used to be. However, the calculator is not a cure-all; in many problems, the difficulty is in figuring out what numbers to put into the calculator in the first place. In some cases, using a calculator will actually be less helpful than doing the problem some other way. Take a look at an example:

> If x is the remainder when (11)(7) is divided by 4 and y is the remainder when (14)(6) is divided by 13, what is the value of $x + y$?

This problem is designed so that the calculator won't tell the whole story. Certainly, the calculator will tell you that 11 × 7 = 77. When you divide 77 by 4, however, the calculator yields an answer of 19.25. The remainder is not 0.25 (a remainder is always a whole number).

You might just go back to your pencil and paper, and find the largest multiple of 4 that is less than 77. Because 4 does go into 76, you can conclude that 4 would leave a remainder of 1 when dividing into 77. (Notice that you don't even need to know how many times 4 goes into 76, just that it goes in. One way to mentally "jump" to 76 is to say that 4 goes into 40, so it goes into 80…that's a bit too big, so subtract 4 to get 76.)

However, it is also possible to use the calculator to find a remainder. Divide 77 by 4 to get 19.25. Thus, 4 goes into 77 nineteen times, with a remainder left over. Now use your calculator to multiply 19 (JUST 19, not 19.25) by 4. You will get 76. The remainder is 77 − 76, which is 1. Therefore, $x = 1$. You could also multiply the leftover 0.25 times 4 (the divisor) to find the remainder of 1.

Use the same technique to find y. Multiply 14 by 6 to get 84. Divide 84 by 13 to get 6.46. Ignore everything after the decimal, and just multiply 6 by 13 to get 78. The remainder is therefore 84 − 78, which is 6. Therefore, $y = 6$.

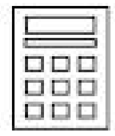
You are looking for $x + y$, and $1 + 6 = 7$, so the answer is 7.

You can see that blind faith in the calculator can be dangerous. Use it responsibly! And this leads us to …

PRACTICE USING THE CALCULATOR!

The on-screen calculator will slow you down or lead to incorrect answers if you're not careful! If you plan to use it on test day (which you should), you'll want to practice first.

We have created an online practice calculator for you to use. To access this calculator, go to www.manhattanprep.com/gre and sign in to the student center using the instructions on the "How to Access Your Online Resources" page found at the front of this book.

Throughout this book, you will see the  symbol. This symbol means "Use the calculator here!" As much as possible, have the online practice calculator up and running during your review of our math books. You'll have the chance to use the on-screen calculator when you take our practice exams as well.

NAVIGATING THE QUESTIONS IN A SECTION

The GRE offers you the ability to move freely around the questions in a section. You can go forward and backward one-by-one and can even jump directly to any question from the "review list." The review list provides a snapshot of which questions you have answered, which ones you have tagged for "mark and review," and which ones are incomplete.

You should double-check the review list for completion if you finish the section early. Using the review list feature will take some practice as well, which is why we've built it into our online practice exams.

The majority of test-takers will be pressed for time. Some people won't be able to go back to multiple problems at the end of the section. Generally, if you can't get a question the first time, you won't be able to get it the second time around either. With these points in mind, here's what we recommend.

0. Do the questions in the order in which they appear.
0. When you encounter a difficult question, do your best to eliminate answer

choices that you know are wrong.

0. If you're not sure of an answer, take an educated guess from the choices remaining. Do NOT skip it and hope to return to it later.
0. Using the "mark" button at the top of the screen, mark up to three questions per section that you think you might be able to solve with more time. Mark a question only after you have taken an educated guess.
0. Always click on the review list at the end of a section. This way, you can quickly make sure you have neither skipped nor incompletely answered any questions.
0. If you have time, identify any questions that you marked for review and return to them. If you do not have any time remaining, you will have already taken good guesses at the tough ones.

What you want to avoid is surfing—clicking forward and backward through the questions searching for "easy" ones. This will eat up valuable time. Of course, you'll want to move through the tough ones quickly if you can't get them, but try to avoid skipping around.

Following this guidance will take practice. Use our practice exams to fine-tune your approach.

Math Formats in Detail

The 20 questions in each Math section can be broken down by format as follows:

- 7 Quantitative Comparison questions—These ask you to compare two quantities and pick one of four choices (A, B, C, or D).
- 10 Discrete Quant questions—Most of these are standard multiple-choice questions, asking you to pick one of five choices (A, B, C, D, or E). A few will ask you to pick one *or more* choices from a list. Others will ask you to "fill in the blank," essentially.
- 3 Data Interpretation questions—All three questions will be related to a single set of data given in graph or table form. The formats of the questions themselves are the same as for Discrete Quant: most are standard multiple-choice, and the rest ask you to pick one or more choices or to fill in the blank.

QUANTITATIVE COMPARISON

The format of every Quantitative Comparison or "QC" question is the same. All QC questions contain a "Quantity A" and a "Quantity B." Some also contain common information that applies to both quantities.

Your job is to, well, compare the two quantities (surprise!) and decide which one of the following four statements is true:

0. Quantity A is *always* greater than Quantity B, in every possible case.
0. Quantity B is *always* greater than Quantity A.
0. Quantity A is *always* equal to Quantity B.

0. *None* of the above is *always* true.

For instance, it might be that most of the time, Quantity A is greater, but in just one case, Quantity B is greater, or the two quantities are equal. Then, as the GRE puts it, "the relationship cannot be determined."

On the actual GRE, these four choices are worded exactly as shown in the following example:

$$x \geq 0$$

<u>**Quantity A**</u>	<u>**Quantity B**</u>
x	x^2

(A) Quantity A is greater.

(B) Quantity B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the information given.

If $x = 0$, then the two quantities are equal. If $x = 2$, then Quantity (B) is greater. Thus, the relationship between the quantities can't be determined once and for all.

The answer is **(D)**.

Unit 6 in this book is all about Quantitative Comparisons.

SELECT ONE OR MORE ANSWER CHOICES

These are the Discrete Quant and Data Interpretation questions that ask you to pick one *or more* choices. According to the *Official Guide to the GRE revised General Test*, the official directions for "Select One or More Answer Choices" read as follows:

> *Directions:*
>
> Select one or more answer choices according to the specific question directions.
>
> If the question does not specify how many answer choices to indicate, indicate all that apply.
>
> The correct answer may be just one of the choices or as many as all of the choices, depending on the question.
>
> No credit is given unless you indicate all of the correct choices and no others.
>
> If the question specifies how many answer choices to indicate, indicate exactly that number of choices.

There is no partial credit. If three of six choices are correct and you indicate two of the three, no credit is given. If you are told to indicate two choices and you indicate three, no credit is given. Read the directions carefully.

On your screen, the answer choices for "Select One or More" will be *boxes*, not circles (as with standard "pick just one" multiple-choice questions). The boxes are a good visual reminder that you should be ready to pick more than one choice on these questions, just as you might check more than one box on a checklist.

Here's a sample question:

If $ab = |a| \times |b|$ and $ab \neq 0$, which of the following must be true?

Indicate <u>all</u> such statements.

☐ $a = b$
☐ $a > 0$ and $b > 0$
☐ $ab > 0$

Note that only one, only two, or all three of the choices may be correct. (Also note the word "must" in the question stem!)

If $ab = |a| \times |b|$, then you know *ab* is positive, since the right side of the equation must be positive. If *ab* is positive, however, that doesn't necessarily mean that *a* and *b* are each positive; it simply means that they have the same sign.

Answer choice (A) is not correct because it is not true that *a* must equal *b*; for instance, *a* could be 2 and *b* could be 3.

Answer choice (B) is not correct because it is not true that *a* and *b* must each be positive; for instance, *a* could be −3 and *b* could be −4.

Now look at choice (C). Because $|a| \times |b|$ must be positive, *ab* must be positive as well; that is, because two sides of an equation are, by definition, equal to one another, if one side of the equation is positive, the other side must be positive as well. Thus, answer **(C)** is correct.

Note that “indicate all that apply” didn’t make the problem fundamentally different. You just had to do a little more work by checking every answer choice.

NUMERIC ENTRY

These are the Discrete Quant and Data Interpretation questions that ask you to "fill in the blank." That is, you type a number into a box on the screen. You have to come up with that number on your own. You are not able to work backward from answer choices, and in many cases, it will be difficult to make a guess. However, the math principles being tested are the same as on the rest of the exam.

You'll be given one box if you are supposed to enter an integer (such as 12 or 0 or −3) or a decimal (such as 2.7 or −1.53). Click on the answer box and type your answer.

In contrast, you'll be given two boxes if you are supposed to enter a fraction. One box (the numerator) will be on top of a fraction line, while the other box (the denominator) will be underneath. Click on each box separately to enter your answer. You do not have to reduce to lowest terms.

Follow directions carefully. You may be asked to round your answer in a particular way, or the units of measurement may be different from what you assume.

Here is a sample question:

> If $x\Delta y = 2xy - (x - y)$, what is the value of $3\Delta 4$?

You are given a function involving two variables, *x* and *y*, and asked to substitute 3 for *x* and 4 for *y*:

$$
\begin{aligned}
x\Delta y &= 2xy - (x - y)\\
3\Delta 4 &= 2(3)(4) - (3 - 4)\\
3\Delta 4 &= 24 - (-1)\\
3\Delta 4 &= 25
\end{aligned}
$$

The answer is **25**.

So you would type 25 into the box.

Now that you've seen the structure and the math question formats of the GRE, it's time to begin fine-tuning your math skills.

Unit One: Algebra

This unit covers algebra in all its various forms (and disguises) on the GRE. Master fundamental techniques and nuanced strategies to help you solve for unknown variables of every type.

In This Unit...

Chapter 1

EQUATIONS

In This Chapter...

Chapter 1

Equations

The GRE will expect you to be proficient at manipulating and solving algebraic equations. If you haven't faced equations since you were last in school, this can be intimidating. In this chapter, the objective is to help you become comfortable setting up and solving equations. You'll start with some basic equations (without the variables at first), and then work your way up to some pretty tricky problems. Time to dive in.

The Order of Operations (PEMDAS)

$$3 + 4(5 - 1) - 3^2 \times 2 = ?$$

Before you start dealing with variables, spend a moment looking at expressions that are comprised of only numbers, such as the preceding example. The GRE probably won't ask you to compute something like this directly, but learning to use order of operations on numerical expressions will help you manipulate algebraic expressions and equations. So you have a string of numbers with mathematical symbols in between them. Which part of the expression should you focus on first?

Intuitively, most people think of going in the direction they read, from left to right. When you read a book, moving left to right is a wise move (unless you're reading a language such as Chinese or Hebrew). However, when you perform basic arithmetic, there is an order that is of greater importance: **the order of operations**.

The order in which you perform the mathematical functions should primarily be determined by the functions themselves. In the correct order, the six operations are **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, and **A**ddition/**S**ubtraction (or **PEMDAS**).

Before you solve a problem that requires PEMDAS, here's a quick review of the basic operations:

Parentheses can be written as ( ) or [ ] or even { }.

Exponents are $5^{2\leftarrow\text{ these numbers}}$. For example, 5^2 ("five squared") can be expressed as 5×5. In other words, it is 5 times itself, or two 5's multiplied together.

Likewise, 4^3 ("four cubed," or "four to the third power") can be expressed as $4 \times 4 \times 4$ (or three 4's multiplied together). The exponent 3 tells you how many 4's are in the product.

Roots are very closely related to exponents. For example, $\sqrt[3]{64}$ is the third root of 64 (commonly called the cube root). The cube root, in this case $\sqrt[3]{64}$, is basically asking the question, "What multiplied by itself three times equals 64?" This is written as $4 \times 4 \times 4 = 64$, so $\sqrt[3]{64} = 4$. The plain old square root $\sqrt{9}$ can be thought of as $\sqrt[2]{9}$. "What times itself equals 9?" The answer is $3 \times 3 = 9$, so $\sqrt{9} = 3$

.

Exponents and roots can also undo each other: $\sqrt{5^2} = 5$ and $= 7$.

Multiplication and **Division** can also undo each other: $2 \times 3 \div 3 = 2$ and $10 \div 5 \times 5 = 10$.

Multiplication can be expressed with a multiplication sign (×) or with parentheses: $(5)(4) = 5 \times 4 = 20$. Division can be expressed with a division sign (÷), a slash (/), or a fraction bar(—): $20 \div 5 = 20/5 = \dfrac{20}{5} = 4$. Also remember that multiplying or dividing by a negative number changes the sign:

$4 \times (-2) = -8$ $\qquad\qquad$ $-8 \div (-2) = 4$

Addition and **Subtraction** can also undo each other: $8 + 7 - 7 = 8$ and $15 - 6 + 6 = 15$.

PEMDAS is a useful acronym you can use to remember the order in which operations should be performed. Some people find it useful to write PEMDAS like this:

$$\xrightarrow{\mathrm{PE}^{\mathrm{M}}/_{\mathrm{D}}{}^{\mathrm{A}}/_{\mathrm{S}}}$$

For Multiplication/Division and Addition/Subtraction, perform whichever comes first from left to right. The reason that Multiplication and Division are at the same level of importance is that any Multiplication can be expressed as Division, and vice versa; for example, $7 \div 2$ is equivalent to $7 \times \frac{1}{2}$. In a sense, Multiplication and Division are two sides of the same coin.

Addition and Subtraction have this same relationship: $3 - 4$ is equivalent to $3 + (-4)$.

The correct order of steps to simplify this sample expression is as follows:

	$3 + 4(5 - 1) - 3^2 \times 2$
Parentheses	$3 + 4(4) - 3^2 \times 2$
Exponents	$3 + 4(4) - 9 \times 2$
Multiplication or **D**ivision (left to right)	$3 + 16 - 18$
Addition or **S**ubtraction (left to right)	$3 + 16 - 18 = 19 - 18 = 1$

Remember: If you have two operations of equal importance, you should do them in left-to-right order: $3 - 2 + 3 = 1 + 3 = 4$. The only instance in which you would override this order is when the operations are in parentheses: $3 - (2 + 3) = 3 - (5) = -2$.

Next are two problems together. Try them first on your own, then an explanation will follow:

$$5 - 3 \times 4^3 \div (7 - 1)$$

P

E

M/D

A/S

Your work should have looked like this:

$$5-3\times4^3\div(7-1)$$

$$5-3\times\underbrace{4^3}\div6 \longrightarrow 4^3=4\times4\times4=64\rightarrow \begin{array}{r} \overset{2}{1}6 \\ \times\ 4 \\ \hline 64 \end{array}$$

$$5-\underbrace{3\times64}\div6 \longrightarrow \begin{array}{r} \overset{1}{6}4 \\ \times\ 3 \\ \hline 192 \end{array}$$

$$5-\underbrace{192}\div6$$

$$\underbrace{5-}$$

$$-2$$

$$\begin{array}{r} 32 \\ 6\overline{)192} \\ -18 \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

Here’s one more:

$$32 \div 2^4 \times (5-3^2)$$

P

E

M/D

A/S

Here's the work you should have done:

$$32 \div 2^4 \times (5 - 3^2)$$
$$32 \div 2^4 \times (5 - 9)$$
$$32 \div 2^4 \times (-4)$$
$$32 \div 16 \times (-4)$$
$$2 \times (-4)$$
$$-8$$

Check Your Skills

1. $-4 + 12/3 =$

2. $(5 - 8) \times 10 - 7 =$

3. $-3 \times 12 \div 4 \times 8 + (4 - 6) =$

4. $2^4 \times (8 \div 2 - 1)/(9 - 3) =$

Solving for a Variable with One Equation

EXPRESSIONS VERSUS EQUATIONS

So far, you've been dealing only with expressions. Now you're going to be dealing with equations. An important structural difference between equations and expressions is that an equation consists of two expressions separated by an equals sign, while an expression lacks an equals sign altogether. An equation is a sentence: "Something equals something else." The somethings are each expressions.

Pretty much everything you will be doing with equations is related to one basic principle: You can do anything you want to one side of the equation, *as long as you also do the same thing to the other side of the equation*. Take the equation 3 + 5 = 8. You want to subtract 5 from the left side of the equation, but you still want the equation to be true. All you have to do is subtract 5 from the right side as well, and you can be confident that your new equation will still be valid:

$$\begin{array}{rcr} 3+5 & = & 8 \\ -5 & & -5 \\ \hline 3 & = & 3 \end{array}$$

Note that this would also work if you had variables in your equation:

$$\begin{array}{rcr} x+5 & = & 8 \\ -5 & & -5 \\ \hline x & = & 3 \end{array}$$

Next, you're going to see some of the many ways you can apply this principle to solving algebra problems.

SOLVING EQUATIONS

What does it mean to solve an equation? What are you really doing when you manipulate algebraic equations?

A solution to an equation is a number that, when substituted in for the variable, makes the equation *true*. Remember, an equation is a sentence: "Something equals something else." In general, a sentence like this can be true or false. You want a way to make it true.

Take the equation $2x + 7 = 15$. You are looking for the value of x that will make this equation true. What if you plugged in 3 for x? If you replaced x with the number 3, you would get $2(3) + 7 = 15$. This equation can be simplified to $6 + 7 = 15$, which further simplifies to $13 = 15$. However, 13 does *not* equal 15, so when $x = 3$, the equation is *not* true. Therefore, $x = 3$ is *not* a solution to the equation.

Now, if you replaced x with the number 4, you would get $2(4) + 7 = 15$. This equation can be simplified to $8 + 7 = 15$. Simplify it further, and you get $15 = 15$, which is a true statement.

That means that when $x = 4$, the equation is true. So $x = 4$ is a solution to the equation.

Now the question becomes, what is the best way to find these solutions? If you had to use trial and error, or guessing, the process could take a very long time. The following sections will talk about how you can efficiently and accurately manipulate equations so that solutions become easier to find.

Isolating a Variable

You know that you can make a change to an equation as long as you make the same change to both sides. This is called the **Golden Rule.** Now, look at the various changes you can make. Try to solve the following problem:

If $5(x - 1)^3 - 30 = 10$, then $x = ?$

To solve for a variable, you need to get it by itself on one side of the equals sign. To do that, you need to change the appearance of the equation, without changing the fact that it's true. The good news is that all of the changes you need to make to this equation to solve for *x* will actually be very familiar to you—PEMDAS operations!

To get *x* by itself, you want to move every term that *doesn't include* the variable to the other side of the equation. The easiest thing to move at this stage is the 30, so start there. If 30 is being subtracted on the left side of the equation, and you want to move it to the other side, then you need to do the opposite operation to cancel it out. So you're going to **add** 30 to both sides, like this:

$$\begin{array}{rcr} 5(x-1)^3 - \cancel{30} & = & 10 \\ + \cancel{30} & & +30 \\ \hline 5(x-1)^3 & = & 40 \end{array}$$

Now you've only got one term on the left side of the equation. The *x* is still inside the parentheses, and the expression in the parentheses is being multiplied by 5, so the next step will be to move that 5 over to the other side of the equation. Once again, you want to perform the opposite operation, so **divide** both sides of the equation by 5:

$$\frac{\cancel{5}(x-1)^3}{\cancel{5}} = \frac{40}{5} \quad \leftarrow \quad \text{These horizonta lines mean divis}$$

$$(x-1)^3 = 8$$

At this point, you could cube ($x - 1$), but that is going to involve a whole lot of multiplication. Instead, you can get rid of the exponent by performing the opposite operation. Roots are the opposite of exponents. So if the left side of the equation is raised to the third power, you can undo that by taking the third root of both sides, also known as the cube root, as shown here:

$$\begin{aligned} &= \sqrt[3]{8} \\ (x-1) &= 2 \end{aligned}$$

Now that nothing else is being done to the parentheses, you can just get rid of them. The equation is:

$$x - 1 = 2$$

Then, add 1 to both sides, and you get $x = 3$. This would have been hard to guess! If you plug 3 back in for x in the original equation (a step often worth doing, especially as you're learning), you'll find that this value makes the original equation true.

Take a look at the steps you took to isolate x. Do you notice anything? You *added* 30, then you *divided* by 5, then you got rid of the *exponent*, and then you simplified the *parentheses*. You did PEMDAS backward! And, in fact, when you're isolating a variable, it turns out that the simplest way to do so is to reverse the order of PEMDAS when deciding what order you will perform your operations. Start with addition/subtraction, then multiplication/division, then exponents, and finish with terms in parentheses.

Now that you know the best way to isolate a variable, go through one more example. Try it on your own first, then an explanation will follow.

If $4\sqrt{(x-6)}+7=19$, then x = ?

A/S

M/D

E

P

The equation you're simplifying is $4\sqrt{(x-6)}+7=19$. If there's anything to add or subtract, that will be the easiest first step. There is, so first get rid of the 7 by subtracting 7 from both sides:

$$
\begin{array}{r}
4\sqrt{(x-6)} + 7 = 19 \\
-7 \quad -7 \\
\hline
4\sqrt{(x-6)} \qquad = 12
\end{array}
$$

Now you want to see if there's anything being multiplied or divided by the term containing an *x*. The square root that contains the *x* is being multiplied by 4, so, your next step will be to get rid of the 4. You can do that by dividing both sides of the equation by 4:

$$
\begin{aligned}
\frac{\cancel{4}\sqrt{(x-6)}}{\cancel{4}} &= \frac{12}{4} \\
&= 3
\end{aligned}
$$

Now that you've taken care of multiplication and division, it's time to check for exponents. That means you need to check for exponents and roots, because they're so intimately related. There are no exponents in the equation, but the *x* is inside a square root. To cancel out a root, you can use an exponent. Squaring a square root will cancel it out, so your next step is to square both sides:

$$
\begin{aligned}
\text{Copyrighted image} &= 3 \\
\left(\text{Copyrighted image}\right)^2 &= 3^2 \\
x - 6 &= 9
\end{aligned}
$$

The final step is to add 6 to both sides, and you end up with $x = 15$.

Check Your Skills

5. $3(x + 4)^3 - 5 = 19$

6. $\dfrac{3x - 7}{2} + 20 = 6$

7. $\sqrt[3]{(x + 5)} - 7 = -8$

EQUATION CLEAN-UP MOVES

You've covered the basic operations that you'll be dealing with when solving equations. But what would you do if you were asked to solve for *x* in the following equation?

$$\frac{5x - 3(4 - x)}{2x} = 10$$

Now *x* appears in multiple parts of the equation, and your job has become more complicated. In addition to your PEMDAS operations, you also need to be able to simplify, or clean up, your equation. There are different ways you can clean up this equation. First, notice how you have an *x* in the denominator (the bottom of the fraction) on the left side of the equation. You're trying to find the value of *x*, not of some number divided by x. So your first clean-up move is to **always get variables out of denominators**. The way to do that is to multiply both sides of the equation by the *entire* denominator. Watch what happens:

$$\cancel{2x} \times \frac{5x - 3(4 - x)}{\cancel{2x}} = 10 \times 2x$$

If you multiply a fraction by its denominator, you can cancel out the entire denominator. Now you're left with:

$$5x - 3(4 - x) = 20x$$

No more fractions! What should you do next? At some point, if you want the value of x, you're going to have to get all the terms that contain an x together. Right now, however, that x sitting inside the parentheses seems pretty tough to get to. To make that x more accessible, you should **simplify grouped terms within the equation**. That 3 on the outside of the parentheses wants to multiply the terms inside, so you need to **distribute** it. What that means is you're going to multiply the 3 by each term inside, one at a time: 3 times 4 is 12, and 3 times $-x$ is $-3x$. The equation becomes:

$$5x - (12 - 3x) = 20x$$

Now, if you subtract what's in the parentheses from $5x$, you can get rid of the parentheses altogether. Just as you multiplied the 3 by *both* terms inside the parentheses, you also have to subtract both terms:

$$5x - (12) - (-3x) = 20x$$
$$5x - 12 + 3x = 20x$$

Remember, *subtracting a negative number is the same as adding a positive number; the negative signs cancel out.*

You're very close. You're ready to make use of your final clean-up move—**combine like terms**. "Like terms" are terms that can be combined into one term. For example, "$3x$" and "$5x$" are like terms because they can be combined into "$8x$." Ultimately, all the PEMDAS operations and clean-up moves have one goal: to get a variable by itself so you can determine its value. At this point, you have four terms in the equation: $5x$, -12, $3x$, and $20x$. You want to get all the terms with an x on one side of the equation, and all the terms that only contain numbers on the other side.

First, combine $5x$ and $3x$, because they're on the same side of the equation. That gives you:

$$8x - 12 = 20x$$

Now you want to get the $8x$ together with the $20x$, but which one should you move? The best move to make here is to move the $8x$ to the right side of the equation, because that way one side of the equation will have terms that contain only numbers (-12) and the right side will have terms that contain variables ($8x$ and $20x$). So now it's time for the PEMDAS operations again. To find x:

$$\begin{array}{rcr} 8x - 12 & = & 20x \\ -\,8x \qquad & & -8x \\ \hline -\,12 & = & 12x \\ \dfrac{-12}{12} & = & \dfrac{12x}{12} \\ -\,1 & = & x \end{array}$$

Before moving on to the next topic, review what you've learned:

- **Whatever you do to one side of the equation, you must do to the other side at the same time.**
- **To isolate a variable, you should perform the PEMDAS operations in *reverse order*:**

  0.
  Addition/Subtraction

  0.
  Multiplication/Division

  0.
  Exponents/Roots

  0.
  Parentheses

- **To clean up an equation:**

  0.
  Get variables out of denominators by multiplying both sides by that entire denominator.

  0.
  Simplify grouped terms by multiplying or distributing.

  0.
  Combine similar or like terms.

Check Your Skills

8. $\dfrac{11 + 3(x + 4)}{x - 3} = 7$

9. $\dfrac{-6 - 5(3 - x)}{2 - x} = 6$

10. $\dfrac{2x + 6(9 - 2x)}{x - 4} = -3$

Solving for Variables with Two Equations

Some GRE problems, including word problems, give you two equations, each of which has two variables. To solve such problems, you'll need to solve for one or each of those variables. At first glance, this problem may seem quite daunting:

If $3x + y = 10$ and $y = x - 2$, what is the value of y?

Maybe you've gotten pretty good at solving for one variable, but now you face two variables and two equations.

You might be tempted to test numbers, and indeed you could actually solve this problem that way. Could you do so in under two minutes? Maybe not. Fortunately, there is a much faster way.

SUBSTITUTION

One method for combining equations is called substitution. In substitution, you *insert the expression for one variable in one equation into that variable in the other equation.* The goal is to end up with one equation with one variable, because once you get a problem to that point, you know you can solve it.

There are four basic steps to substitution, which can be demonstrated with the previous question.

Step One is to isolate one of the variables in one of the equations. For this example, y is already isolated in the second equation: $y = x - 2$.

For **Step Two**, it is important to understand that the left and right sides of the equation are equivalent. This may sound obvious, but it has some interesting implications. If y equals $x - 2$, then that means you could replace the variable y with the expression $(x - 2)$ and the equation would have the same value. In fact, that's exactly what you're going to do. Step Two will be to go to the first equation and replace the variable y with its equivalent, $(x - 2)$. In other words, you're substituting $(x - 2)$ in for y. So:

$$3x + y = 10 \rightarrow 3x + (x - 2) = 10$$

Now for **Step Three**, you have one equation and one variable, so the next step is to solve for x:

$$\begin{aligned} 3x + x - 2 &= 10 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

Now that you have a value for x, **Step Four** is to substitute that value into either original equation to solve for your second variable, y:

$$y = x - 2 \rightarrow y = 3 - 2 = 1$$

So the answer to the question is $y = 1$. **It should be noted that Step Four will only be necessary if the variable you solve for in Step Three is not the variable the**

question asks for. The question asked for *y*, but you found *x*, so Step Four was needed to answer the question.

Now that you've gotten the hang of substitution, try a new problem:

If $2x + 4y = 14$ and $x - y = -8$, what is the value of x?

As you learned, the first step is to isolate your variable. Because the question asks for *x*, you should manipulate the second equation to isolate *y*. Taking this approach will make Step Four unnecessary and save you time:

$$\begin{aligned} x - y &= -8 \\ x &= -8 + y \\ x + 8 &= y \end{aligned}$$

Then, for Step Two, you can substitute for *y* in the first equation:

$$\begin{aligned} 2x + 4y &= 14 \\ 2x + 4(x + 8) &= 14 \end{aligned}$$

Now, for Step Three, isolate *x*:

$$\begin{aligned} 2x + 4x + 32 &= 14 \\ 6x &= -18 \\ x &= -3 \end{aligned}$$

So the answer to the question is $x = -3$.

Check Your Skills

11. $x = 10$
    $x + 2y = 26$

12. $x + 4y = 10$
    $y - x = -5$

13. $6y + 15 = 3x$
    $x + y = 14$

Subtraction of Expressions

One of the most common errors involving order of operations occurs when an expression with multiple terms is subtracted. The subtraction must occur across *every* term within the expression. Each term in the subtracted part must have its sign reversed. Here are several examples:

$x - (y - z) = x - y + z$ (Note that the signs of both y and $-z$ have been reversed.)

$x - (y + z) = x - y - z$ (Note that the signs of both y and z have been reversed.)

$x - 2(y - 3z) = x - 2y + 6z$ (Note that the signs of both y and $-3z$ have been reversed.)

What is $5x - [y - (3x - 4y)]$?

Both expressions in parentheses must be subtracted, so the signs of each term must be reversed for *each* subtraction. Note that the square brackets are just fancy parentheses, used so that you avoid having parentheses right next to each other. Work from the innermost parentheses outward:

$$\begin{aligned} 5x - [y - (3x - 4y)] &= \\ 5x - [y - 3x + 4y] &= \\ 5x - y + 3x - 4y &= \mathbf{8x - 5y} \end{aligned}$$

Check Your Skills

14. Simplify: $3a - [2a - (3b - a)]$

Fraction Bars as Grouping Symbols

Even though fraction bars do not fit into the PEMDAS hierarchy, they do take precedence. In any expression with a fraction bar, you should **pretend that there are parentheses around the numerator and denominator of the fraction**. This may be obvious as long as the fraction bar remains in the expression, but it is easy to forget if you eliminate the fraction bar or add or subtract fractions, so put parentheses in to remind yourself:

$$\text{Simplify:}\frac{x-1}{2}-\frac{2x-1}{3} \rightarrow \text{Write on your paper as:}\frac{(x-1)}{2}-\frac{(2x-1)}{3}$$

The common denominator for the two fractions is 6, so multiply the numerator and denominator of the first fraction by 3, and those of the second fraction by 2:

$$=\frac{(3x-3)}{6}-\frac{(4x-2)}{6}$$

Once you put all numerators over the common denominator, the parentheses remind you to reverse the signs of both terms in the second numerator:

$$\frac{(3x-3)-(4x-2)}{6}=\frac{3x-3-4x+2}{6}=\frac{-x-1}{6}=-\frac{x+1}{6}$$

In that last step, the minus sign was pulled out from both the x and the 1 and put in front of the fraction. You can do this because $-x-1$ is the same thing as $-(x+1)$. Again, the fraction bar is working as a grouping symbol.

Check Your Skills

15. Simplify: $\frac{a+4}{4}-\frac{2a-2}{3}$

Check Your Skills Answer Key

1.

0

$-4 + 12/3 =$	Divide first.
$-4 + 4 = 0$	Then add the two numbers.

2.

−37

$(5 - 8) \times 10 - 7 =$	
$(-3) \times 10 - 7 =$	First, combine what is inside the parentheses.
$-30 - 7 =$	Then multiply -3 by 10.
$-30 - 7 = -37$	Subtract the two numbers.

3.

−74

$-3 \times 12 \div 4 \times 8 + (4 - 6)$	
$-3 \times 12 \div 4 \times 8 + (-2)$	First, combine what's in the parentheses.
$-36 \div 4 \times 8 + (-2)$	Multiply -3 by 12.
$-9 \times 8 - 2$	Divide -36 by 4.
$-72 + (-2) = -74$	Multiply -9 by 8 and then subtract 2.

4.

8

$2^4 \times (8 \div 2 - 1)/(9 - 3) =$

$2^4 \times (4 - 1)/(6) =$	$8/2 = 4$ and $9 - 3 = 6$
$16 \times (3)/(6) =$	$4 - 1 = 3$ and $2^4 = 16$
$48/6 =$	Multiply 16 by 3.
$48/6 = 8$	Divide 48 by 6.

5.

***x* = −2**

$3(x + 4)^3 - 5 = 19$

$3(x + 4)^3 = 24$	Add 5 to both sides.
$(x + 4)^3 = 8$	Divide both sides by 3.
$(x + 4) = 2$	Take the cube root of both sides.
$x = -2$	Remove the parentheses and subtract 4 from both sides.

6.

***x* = −7**

$$\frac{3x - 7}{2} + 20 = 6$$

$\frac{3x - 7}{2} = -14$	Subtract 20 from both sides.
$3x - 7 = -28$	Multiply both sides by 2.
$3x = -21$	Add 7 to both sides.

$$x = -7$$

Divide both sides by 3.

7.

***x* = −6**

$$\sqrt[3]{(x+5)} - 7 = -8$$

$$\sqrt[3]{(x+5)} = -1$$ Add 7 to both sides.

$$x + 5 = -1$$ Cube both sides, remove parentheses.

$$x = -6$$ Subtract 5 from both sides.

8.

***x* = 11**

$$\frac{11 + 3(x+4)}{x-3} = 7$$

$$11 + 3(x+4) = 7(x-3)$$ Multiply both sides by the denominator (x – 3).

$$11 + 3x + 12 = 7x - 21$$ Simplify grouped terms by distributing.

$$23 + 3x = 7x - 21$$ Combine like terms (11 and 12.)

$$23 = 4x - 21$$ Subtract $3x$ from both sides.

$$44 = 4x$$ Add 21 to both sides.

$$11 = x$$ Divide both sides by 4.

9.

***x* = 3**

$$\frac{-6 - 5(3-x)}{2-x} = 6$$

$-6 - 5(3 - x) = 6(2 - x)$	Multiply both sides by the denominator (2 – x).
$-6 - 15 + 5x = 12 - 6x$	Simplify grouped terms by distributing.
$-21 + 5x = 12 - 6x$	Combine like terms (–6 and –15).
$-21 + 11x = 12$	Add 6x to both sides.
$11x = 33$	Add 21 to both sides.
$x = 3$	Divide both sides by 11.

10.

x = 6

$$\frac{2x + 6(9 - 2x)}{x - 4} = -3$$

$2x + 6(9 - 2x) = -3(x - 4)$	Multiply by the denominator (x – 4).
$2x + 54 - 12x = -3x + 12$	Simplify grouped terms by distributing.
$-10x + 54 = -3x + 12$	Combine like terms (2x and –12x).
$54 = 7x + 12$	Add 10x to both sides.
$42 = 7x$	Subtract 12 from both sides.
$6 = x$	Divide both sides by 7.

11.

x = 10, y = 8

$x = 10$

$x + 2y = 26$

$(10) + 2y = 26$	Substitute 10 for x in the second equation.

$2y = 16$	Subtract 10 from both sides.
$y = 8$	Divide both sides by 2.

12.

$x = 6, y = 1$

$$x + 4y = 10$$
$$y - x = -5$$

$y = x - 5$	Isolate y in the second equation.
$x + 4(x - 5) = 10$	Substitute $(x - 5)$ for y in the first equation.
$x + 4x - 20 = 10$	Simplify grouped terms within the equation.
$5x - 20 = 10$	Combine like terms (x and $4x$).
$5x = 30$	Add 20 to both sides.
$x = 6$	Divide both sides by 5.
$y - 6 = -5$	Substitute 6 for x in the second equation to solve for y.
$y = 1$	Add 6 to both sides.

13.

$x = 11, y = 3$

$$6y + 15 = 3x$$
$$x + y = 14$$

$2y + 5 = x$	Divide the first equation by 3.
$(2y + 5) + y = 14$	Substitute $(2y + 5)$ for x in the second equation.
$3y + 5 = 14$	Combine like terms ($2y$ and y).

$3y = 9$ Subtract 5 from both sides.

$y = 3$ Divide both sides by 3.

$x + 3 = 14$ Substitute 3 for y in the second equation to solve for x.

$x = 11$

14.

3b

$3a - [2a - (3b - a)]$ Rewrite the expression carefully.

$= 3a - [2a - 3b + a]$ Distribute the minus sign and drop interior parentheses.

$= 3a - [3a - 3b]$ Combine like terms ($2a$ and a).

$= 3a - 3a + 3b$ Distribute the minus sign and drop brackets.

$= 3b$ Perform the subtraction ($3a - 3a$) to cancel those terms.

15.

$$\frac{20 - 5a}{12}$$

$$\frac{a+4}{4} - \frac{2a-2}{3}$$

Rewrite the expression and identify the common denominator (12).

$$= \frac{3}{3} \times \frac{(a+4)}{4} - \frac{4}{4} \times \frac{(2a-2)}{3}$$

Multiply the first fraction by $\frac{3}{3}$ and the second fraction by $\frac{4}{4}$.

Show the products on top and bottom of

$$= \frac{3(a+4)}{12} - \frac{4(2a-2)}{12}$$

each fraction.

$$= \frac{3(a+4) - 4(2a-2)}{12}$$

Combine the two fractions into one fraction with the common denominator.

$$= \frac{3a + 12 - 8a + 8}{12}$$

Distribute the 3 and the −4 and drop parentheses.

$$= \frac{20 - 5a}{12}$$

Combine like terms ($3a$ and $-8a$), including constraints (12 and 8).

Problem Set

1. Evaluate $-3x^2$, $-3x^3$, $3x^2$, $(-3x)^2$, and $(-3x)^3$ if $x = 2$, and also if $x = -2$.

2. Evaluate $(4 + 12 \div 3 - 18) - [-11 - (-4)]$

3. Which of the parentheses in the following expressions are unnecessary and could thus be removed without any change in the value of the expression?

   (o)
   $-(5^2) - (12 - 7)$

   (o)
   $(x + y) - (w + z) - (a \times b)$

4. Evaluate $\left[\dfrac{4+8}{2-\ (-6)}\right] - \ [4 + 8 \div 2 - \ (-6)]$

5. Simplify: $x - (3 - x)$

6. Simplify: $(4 - y) - 2(2y - 3)$

7. Solve for x: $2(2 - 3x) - (4 + x) = 7$

8. Solve for z: $\frac{4z - 7}{3 - 2z} = -5$

9.

Quantity A	**Quantity B**
$3 \times (5 + 6) \div -1$	$3 \times 5 + 6 \div -1$

10.

$$(x - 4)^3 + 11 = -16$$

Quantity A	**Quantity B**
x	-4

11.

$$2x + y = 10$$

$$3x - 2y = 1$$

Quantity A	**Quantity B**
x	y

Solutions

1.

If $x = 2$:	If $x = -2$:
$-3x^2 = -3(4) = \mathbf{-12}$	$-3x^2 = -3(4) = \mathbf{-12}$
$-3x^3 = -3(8) = \mathbf{-24}$	$-3x^3 = -3(-8) = \mathbf{24}$
$3x^2 = 3(4) = \mathbf{12}$	$3x^2 = 3(4) = \mathbf{12}$
$(-3x)^2 = (-6)^2 = \mathbf{36}$	$(-3x)^2 = 6^2 = \mathbf{36}$
$(-3x)^3 = (-6)^3 = \mathbf{-216}$	$(-3x)^3 = 6^3 = \mathbf{216}$

Remember that exponents are evaluated *before* multiplication. Watch not only the order of operations, but also the signs in these problems.

2.

−3

$$\begin{aligned}(4 + 12 \div 3 - 18) - \ [-11 - \ (-4)] \ &= \\ (4 + 4 - 18) - \ (-11 + 4) \ &= \\ (-10) - \ (-7) \ &= \\ -10 + 7 \ &= \ \mathbf{-3}\end{aligned}$$

3.

(a): The parentheses around 5^2 are unnecessary, because this exponent is performed before the negation (which counts as multiplying by −1) and before the subtraction. The other parentheses are necessary because they cause the right-hand subtraction to be performed before the left-hand subtraction. Without them, the two subtractions would be performed from left to right.

(b): The first and last pairs of parentheses are unnecessary. The addition is performed before the neighboring subtraction by default, because addition and subtraction are performed from left to right. The multiplication is the first operation to be performed, so the right-hand parentheses are completely unnecessary. The middle parentheses are necessary to ensure that the addition is performed before the subtraction that comes to the left of it.

4.

$$-\frac{25}{2}$$

$$\begin{aligned}
\left[\frac{4+8}{2-(-6)}\right] - [4+8\div 2-(-6)] &= \\
\left(\frac{4+8}{2+6}\right) - (4+8\div 2+6) &= && \text{Subtraction of negative} = \text{addition.} \\
\left(\frac{12}{8}\right) - \left(4+4+6\right) &= && \text{Fraction bar acts as a grouping symbol.} \\
\frac{3}{2} - 14 &= && \text{Arithmetic} \\
\frac{3}{2} - \frac{28}{2} &= -\frac{25}{2} && \text{Arithmetic}
\end{aligned}$$

5.

$2x - 3$

Do not forget to reverse the signs of every term in a subtracted expression:

$x - (3 - x) = x - 3 + x = 2x - 3$

6.

−5*y* + 10 (or 10 − 5*y*)

Do not forget to reverse the signs of every term in a subtracted expression.

$(4 - y) - 2(2y - 3) = 4 - y - 4y + 6 = -5y + 10$ (or $10 - 5y$)

7.

−1

$$\begin{aligned} 2(2-3x) - (4+x) &= 7 \\ 4 - 6x - 4 - x &= 7 \\ -7x &= 7 \\ x &= -1 \end{aligned}$$

8.

$$\frac{4}{3}$$

$$\begin{aligned} \frac{4z-7}{3-2z} &= -5 \\ 4z - 7 &= -5(3-2z) \\ 4z - 7 &= -15 + 10z \end{aligned}$$

$$8 = 6z$$

$$z = \tfrac{8}{6} = \tfrac{4}{3}$$

9.

(B)

Evaluate Quantity A first:

$$3 \times (5 + 6) \div -1$$

$3 \times (11) \div -1$	Simplify the parentheses.
$33 \div -1$	Multiply and divide in order from left to right.
-33	

Now evaluate Quantity B:

$3 \times 5 + 6 \div -1$	
$15 + -6$	Multiply and divide in order from left to right.
9	Add.

Quantity A	**Quantity B**
-33	9

Quantity B is greater.

10.

(A)

Simplify the given equation to solve for *x*:

$$\begin{aligned} (x-4)^3 + 11 &= -16 \\ (x-4)^3 &= -27 \\ x - 4 &= -3 \\ x &= 1 \end{aligned}$$

Quantity A	**Quantity B**
$x = 1$	-4

Quantity A is greater.

11.

(B)

Use substitution to solve for the values of *x* and *y*:

$2x + y = 10 \rightarrow y = 10 - 2x$	Isolate y in the first equation.
$3x - 2y = 1 \rightarrow 3x - 2(10 - 2x) = 1$	Substitute $(10 - 2x)$ for y in the second equation.
$3x - 20 + 4x = 1$	Distribute
$7x = 21$	Group like terms ($3x$ and $4x$) and add 20 to both sides.
$x = 3$	Divide both sides by 7.
$2x + y = 10 \rightarrow 2(3) + y = 10$	Substitute 3 for x in the first equation.
$6 + y = 10$	
$y = 4$	

Quantity A	Quantity B
$x = 3$	$y = 4$

Quantity B is greater.

Chapter 2

QUADRATIC EQUATIONS

In This Chapter...

Chapter 2

Quadratic Equations

Identifying Quadratic Equations

This section begins with a question:

> If $x^2 = 4$, what is x?

You know what to do here. Simply take the square root of both sides:

$$\begin{aligned} \sqrt{x^2} &= \sqrt{4} \\ x &= 2 \end{aligned}$$

So $x = 2$. The question seems to be answered. But, what if x were equal to -2? What would be the result? Plug -2 in for x:

$$(-2)^2 = 4 \rightarrow 4 = 4$$

If plugging -2 in for x yields a true statement, then -2 must be a solution to the equation. But, from your initial work, you know that 2 is a solution to the equation.

So which one is correct?

As it turns out, they both are. An interesting thing happens when you start raising variables to exponents. The number of possible solutions increases. When a variable is squared, as in the example here, it becomes possible that there will be two solutions to the equation.

What this means is that whenever you see an equation with a squared variable, you need to:

- Recognize that the equation may have two solutions.
- Know how to find both solutions.

A quadratic equation is any equation for which the highest power on a variable is the second power (e.g., x^2).

For an equation such as $x^2 = 25$ or $x^2 = 9$, finding both solutions shouldn't be too challenging. Take a minute to find both solutions for each equation.

You should have found that x equals either 5 or −5 in the first equation, and 3 or −3 in the second equation. However, what if you are asked to solve for x in the following equation?

$$x^2 + 3x - 10 = 0$$

Unfortunately, you don't yet have the ability to deal with equations like this, which is why the next part of this chapter will deal with some more important tools for manipulating and solving **quadratic equations: distributing** and **factoring**.

Distributing

You first came across distributing when you were learning how to clean up equations and isolate a variable. Essentially, distributing is applying multiplication across a sum.

To review, if you are presented with the expression $3(x + 2)$, and you want to simplify it, you have to distribute the 3 so that it is multiplied by both the x and the 2:

$$3(x + 2) \rightarrow (3 \times x) + (3 \times 2) \rightarrow 3x + 6$$

But what if the first part of the multiplication is more complicated? Suppose you need to simplify $(a + b)(x + y)$?

Simplifying this expression is really an extension of the principle of distribution—every term in the first part of the expression must multiply every term in the second part of the expression. In order to do so correctly every time, you can use a handy acronym to remember the steps necessary: FOIL. The letters stand for **F**irst, **O**uter, **I**nner, **L**ast.

In this case, it looks like this:

$(\boldsymbol{a} + b)(\boldsymbol{x} + y)$	F – multiply the first term in each of the parentheses: $a \times x = ax$.
$(\boldsymbol{a} + b)(x + \boldsymbol{y})$	O – multiply the outer term in each: $a \times y = ay$.

$(a + \mathbf{b})(\boldsymbol{x} + y)$ I – multiply the inner term in each: $b \times x = bx$.

$(a + \mathbf{b})(x + \boldsymbol{y})$ L – multiply the last term in each: $b \times y = by$.

So you have $(a + b)(x + y) = ax + ay + bx + by$.

You can verify this system with numbers. Take the expression (3 + 4)(10 + 20). This is no different than multiplying (7)(30), which gives you 210. See what happens when you FOIL the numbers:

$(\mathbf{3} + 4)(\mathbf{10} + 20)$ F – multiply the first term in each of the parentheses: $3 \times 10 = 30$.

$(\mathbf{3} + 4)(10 + \mathbf{20})$ O – multiply the outer term in each: $3 \times 20 = 60$.

$(3 + \mathbf{4})(\mathbf{10} + 20)$ I – multiply the inner term in each: $4 \times 10 = 40$.

$(3 + \mathbf{4})(10 + \mathbf{20})$ L – multiply the last term in each: $4 \times 20 = 80$.

Finally, sum the four products: 30 + 60 + 40 + 80 = 210.

Now that you have the basics down, go through a more GRE-like situation. Take the expression $(x + 2)(x + 3)$. Once again, begin by FOILing it:

$(\boldsymbol{x} + 2)(\boldsymbol{x} + 3)$ F – multiply the first term in each of the parentheses: $x \times x = x^2$.

$(\boldsymbol{x} + 2)(x + \mathbf{3})$ O – multiply the outer term in each: $x \times 3 = 3x$.

$(x + \mathbf{2})(\boldsymbol{x} + 3)$ I – multiply the inner term in each: $2 \times x = 2x$.

$(x + \mathbf{2})(x + \mathbf{3})$ L – multiply the last term in each: $2 \times 3 = 6$.

The expression becomes $x^2 + 3x + 2x + 6$. Combine like terms, and you are left with $x^2 + 5x + 6$. The next section will discuss the connection between distributing, factoring, and solving quadratic equations. But for the moment, practice FOILing expressions.

Check Your Skills

1. $(x + 4)(x + 9)$

2. $(y + 3)(y - 6)$

3. $(x + 7)(3 + x)$

Factoring

What is factoring? *Factoring is the process of reversing the distribution of terms.*

For example, when you multiply y and $(5 - y)$, you get $5y - y^2$. Reversing this, if you're given $5y - y^2$, you can "factor out" a y to transform the expression into $y(5 - y)$. Another way of thinking about factoring is that you're *pulling out* a common factor that's in every term and rewriting the expression as a *product*.

You can factor out many different things on the GRE: variables, variables with exponents, numbers, and expressions with more than one term, such as $(y - 2)$ or $(x + w)$. Here are some examples:

$t^2 + t$ $= t(t + 1)$	Factor out a t. Notice that a 1 remains behind when you factor a t out of a t.
$5k^3 - 15k^2$ $= 5k^2(k - 3)$	Factor out a $5k^2$.
$21j + 35k$ $= 7(3j + 5k)$	Factor out a 7; because the variables are different, you can't factor out any variables.

If you ever doubt whether you've factored correctly, just distribute back. For instance, $t(t + 1) = t \times t + t \times 1 = t^2 + t$, so $t(t + 1)$ is the correct factored form of $t^2 + t$.

You might factor expressions for a variety of reasons. One common reason is to simplify an expression (the GRE complicates equations that are actually quite simple). The other reason, which will be discussed in more detail shortly, is to find possible values for a variable or combination of variables.

Check Your Skills

4. $4 + 8t$

5. $5x + 25y$

6. $2x^2 + 16x^3$

Applying This to Quadratics

If you were told that $7x = 0$, you would know that x must be 0. This is because the only way to make the product of two or more numbers equal 0 is to have at least one of those numbers equal 0. Clearly, 7 does not equal 0, which means that x must be 0.

What if you were told that $kj = 0$? Well, now you have two possibilities. If $k = 0$, then $0(j) = 0$, which is true, so $k = 0$ is a solution to the equation $kj = 0$. Likewise, if $j = 0$, then $k(0) = 0$, which is also true, so $j = 0$ is also a solution to $kj = 0$.

Either of these scenarios make the equation true, and are the only scenarios, in fact, that make the product $kj = 0$. (If this is not clear, try plugging in non-zero numbers for both k and j and see what happens.)

This is why you want to rewrite quadratic equations such as $x^2 + 3x - 10 = 0$ in factored form: $(x + 5)(x - 2) = 0$. The left side of the factored equation is a *product*, so it's really the same thing as $jk = 0$. Now you know that either $x + 5$ is 0, or $x - 2$ is 0. This means either $x = -5$ or $x = 2$. Once you've factored a quadratic equation, it's straightforward to find the solutions.

Check Your Skills

The way the diamonds work is that you multiply the two numbers on the sides to obtain the top number, and you add them to arrive at the bottom number.

Take another look at the connection between $(x + 2)(x + 3)$ and $x^2 + 5x + 6$:

F	O	I	L
$x \times x$	$x \times 3$	$2 \times x$	2×3

$$x^2 \quad + \quad 5x \quad + \quad 6$$

The 2 and the 3 play two important roles in building the quadratic expression:

- They multiply to give you 6, which is the final term in your quadratic expression.
- Multiplying the outer terms gives you $3x$, and multiplying the inner terms gives you $2x$. You can then add those terms to get $5x$, the middle term of your quadratic expression.

So when you are trying to factor a quadratic expression such as $x^2 + 5x + 6$, the key is to find the two numbers whose product equals the final term (6) and whose sum equals the coefficient of the middle term (the 5 in $5x$). In this case, the two numbers that multiply to 6 and add up to 5 are 2 and 3: $2 \times 3 = 6$ and $2 + 3 = 5$.

So the diamond puzzle is just a visual representation of this same goal. For any quadratic expression, take the final term (the **constant**) and place it in the top portion of the diamond. Take the **coefficient** of the middle term (in this case, the 5 in $5x$) and place it in the lower portion of the diamond. For instance, if the middle term is $5x$, take the 5 and place it at the bottom of the diamond. Now go through the entire process with a new example: $x^2 + 7x + 12$.

The final term is 12, and the coefficient of the middle term is 7, so the diamond will look like this:

When factoring quadratics (or solving the diamond puzzle), it is better to focus first on determining which numbers could multiply to the final term. The reason is that these problems typically deal only with integers, and there are far fewer pairs of integers that will multiply to a certain product than will add to a certain sum. For instance, in this problem, there are literally an infinite number of integer pairs that can add to 7 (remember, negative numbers are also integers: −900,000 and 900,007 sum to 7, for instance). On the other hand, there are only a few integer pairs that multiply to 12. You can actually list them all out: 1 and 12, 2 and 6, and 3 and 4. Because 1 and 12 sum to 13, they don't work; 2 and 6 sum to 8, so they don't work either. However, 3 and 4 sum to 7, so this pair of numbers is the one you want. So your completed diamond looks like this:

Now, because your numbers are **3** and **4**, the factored form of your quadratic expression becomes $(x + \mathbf{3})(x + \mathbf{4})$.

Note: if you are factoring $x^2 + 7x + 12 = 0$, you get $(x + 3)(x + 4) = 0$, so your solutions are *negative* 3 or *negative* 4, not 3 and 4 themselves. Remember, if you have $(x + 3)(x + 4) = 0$, then either $x + 3 = 0$ or $x + 4 = 0$.

Here's another example with one important difference. Solve the diamond puzzle for this quadratic expression: $x^2 - 9x + 18$. Your diamond looks like this:

Check Your Skills

10. $x^2 + 14x + 33$

11. $x^2 - 14x + 45$

The previous section dealt with quadratic equations in which the final term was positive. This section discusses how to deal with quadratics in which the final term is negative. The basic method is the same, although there is one important twist.

Take a look at the quadratic expression $x^2 + 3x - 10$. Start by creating your diamond:

You are looking for two numbers that will multiply to −10. The only way for the product of two numbers to be negative is for one of them to be positive and one of them to be negative. That means that in addition to figuring out pairs of numbers that multiply to 10, you also need to worry about which number will be positive and which will be negative. For the moment, disregard the signs. There are only two pairs of integers that multiply to 10: 1 and 10 and 2 and 5. Start testing the pair 1 and 10, and see what you can learn.

Try making 1 positive and 10 negative. If that were the case, the factored form of the expression would be $(x+1)(x-10)$. FOIL it and see what it would look like:

F	**O**	**I**	**L**
$x \times x$	$x \times -10$	$1 \times x$	1×-10
x^2	$-10x$	$1x$	-10

$$x^2 \quad - \quad 9x \quad - \quad 10$$

The sum of 1 and −10 is −9, but you want 3. That's not correct, so try reversing the signs. Now see what happens if you make 1 negative and 10 positive. The factored form would now be $(x-1)(x+10)$. Once again, FOIL it out:

F	**O**	**I**	**L**
$x \times x$	$x \times 10$	$-1 \times x$	$(-1) \times 10$
x^2	$10x$	$-x$	-10

$$x^2 \quad + \quad 9x \quad - \quad 10$$

Again, this doesn't match your target. The sum of −1 and 10 is not 3. Compare these examples to the examples in the last section. Notice that, with the examples in the

To recap, the way to factor *any* quadratic expression where the final term is negative is as follows:

0. Ignore the signs initially. Find a pair of numbers that multiply to the coefficient of the final term and whose *difference* is the coefficient of the middle term (for $x^2 + 3x - 10$, the numbers 2 and 5 multiply to 10 and $5 - 2 = 3$).
0. Now that you have the pair of numbers (2 and 5), you need to figure out which one will be positive and which one will be negative. As it turns out, this is straightforward to do. Pay attention to signs again. If the sign of the middle term is positive, then the greater of the two numbers will be positive and the smaller will be negative. This was the case in the previous example. The middle term was +3, so the pair of numbers was +5 and −2. On the other hand, when the middle term is negative, the greater number will be negative and the smaller number will be positive.

Work through one more example to see how this works. What is the factored form of $x^2 - 4x - 21$? Take some time to work through it for yourself before looking at the explanation.

First, start your diamond. It looks like this:

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Check Your Skills

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12. $x^2 + 3x - 18$

13. $x^2 - 5x - 66$

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Using FOIL with Square Roots

Some GRE problems ask you to solve factored expressions that involve roots. For example, the GRE might ask you to solve the following:

> **What is the value of $(\sqrt{8}-\sqrt{3})(\sqrt{8}+\sqrt{3})$?**

Even though these problems do not involve any variables, you can solve them as you would solve a pair of quadratic factors, using FOIL:

First:	$\sqrt{8}\times\sqrt{8}=8$	**Outer:**	$\sqrt{8}\times\sqrt{3}=\sqrt{24}$
Inner:	$\sqrt{8}\times\left(-\sqrt{3}\right)=-\sqrt{24}$	**Last:**	$\left(-\sqrt{3}\right)\left(\sqrt{3}\right)=-3$

The four terms are: $8+\sqrt{24}-\sqrt{24}-3$.

You can simplify this expression by removing the two middle terms (they cancel each other out) and subtracting: $8+\sqrt{24}-\sqrt{24}-3=8-3=5$. Although the problem looks complex, using FOIL reduces the entire expression to 5.

Check Your Skills

17. FOIL $\left(\sqrt{8}-\right.$ $\left(\sqrt{8}-\right.$

One-Solution Quadratics

Not all quadratic equations have two solutions. Some have only one solution. One-solution quadratics are also called **perfect square** quadratics, because both roots are the same. Consider the following examples:

$$\begin{aligned} x^2 + 8x + 16 &= 0 \qquad \text{Here, the one solution for } x \text{ is } -4. \\ (x+4)(x+4) &= 0 \\ (x+4)^2 &= 0 \\ x^2 - 6x + 9 &= 0 \qquad \text{Here, the one solution for } x \text{ is } 3. \\ (x-3)(x-3) &= 0 \\ (x-3)^2 &= 0 \end{aligned}$$

Be careful not to assume that a quadratic equation always has two solutions. Always factor quadratic equations to determine their solutions. In doing so, you will see whether a quadratic equation has one or two solutions.

Check Your Skills

18. Solve for x: $x^2 - 10x + 25 = 0$

The Three Special Products

Three quadratic expressions called *special products* come up so frequently on the GRE that it pays to memorize them. You should immediately recognize these three expressions and know how to factor (or distribute) each one automatically. This will usually put you on the path toward the solution to the problem:

Special Product #1:	$x^2 - y^2 = (x + y)(x - y)$
Special Product #2:	$x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$
Special Product #3:	$x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$

You should be able to identify these products when they are presented in disguised form. For example, $a^2 - 1$ can be factored as $(a + 1)(a - 1)$. Similarly, $(a + 1)^2$ can be distributed as $a^2 + 2a + 1$.

Avoid the following common mistakes with special products:

Wrong:	$(x + y)^2 = x^2 + y^2$?	Right:	$(x + y)^2 = x^2 + 2xy + y^2$
	$(x - y)^2 = x^2 - y^2$?		$(x - y)^2 = x^2 - 2xy + y^2$

Check Your Skills

20. $4a^2 + 4ab + b^2 = 0$

21. $x^2 + 22xy + 121y^2 = 0$

Check Your Skills Answer Key

1.

$\mathbf{x^2 + 13x + 36}$

$(x + 4)(x + 9)$

$(\boldsymbol{x} + 4)(\boldsymbol{x} + 9)$ F – multiply the first term in each parentheses: $x \times x = x^2$.

$(\boldsymbol{x} + 4)(x + \mathbf{9})$ O – multiply the outer term in each: $x \times 9 = 9x$.

$(x + \mathbf{4})(\boldsymbol{x} + 9)$ I – multiply the inner term in each: $4 \times x = 4x$.

$(x + \mathbf{4})(x + \mathbf{9})$ L – multiply the last term in each: $4 \times 9 = 36$.

$x^2 + 9x + 4x + 36 \rightarrow x^2 + 13x + 36$

2.

$\mathbf{y^2 - 3y - 18}$

$(y + 3)(y - 6)$

$(\boldsymbol{y} + 3)(\boldsymbol{y} - 6)$ F – multiply the first term in each parentheses: $y \times y = y^2$.

$(\boldsymbol{y} + 3)(y - \mathbf{6})$ O – multiply the outer term in each: $y \times -6 = -6y$.

$(y + \mathbf{3})(\boldsymbol{y} - 6)$ I – multiply the inner term in each: $3 \times y = 3y$.

$(y + \mathbf{3})(y - \mathbf{6})$ L – multiply the last term in each: $3 \times -6 = -18$.

$y^2 - 6y + 3y - 18 \rightarrow y^2 - 3y - 18$

$(x - 2) = 0 \rightarrow x = 2$ Remove the parentheses and solve for x.

OR $(x - 1) = 0 \rightarrow x = 1$ Remove the parentheses and solve for x.

8.

$x = -4$ OR -5

$(x + 4)(x + 5) = 0$

$(x + 4) = 0 \rightarrow x = -4$ Remove the parentheses and solve for x.

OR $(x + 5) = 0 \rightarrow x = -5$ Remove the parentheses and solve for x.

9.

$y = 3$ OR -6

$(y - 3)(y + 6) = 0$

$(y - 3) = 0 \rightarrow y = 3$ Remove the parentheses and solve for y.

OR $(y + 6) = 0 \rightarrow y = -6$ Remove the parentheses and solve for y.

10.

$(x + 3)(x + 11)$

$x^2 + 14x + 33$

⋮

The numbers 1 and 33 and 3 and 11 multiply to 33, and the numbers 3 and 11 sum to 14 $(x + 3)(x + 11)$

11.

$(x - 5)(x - 9)$

$x^2 - 14x + 45$

The numbers 1 and 45, 3 and 15, and 5 and 9 multiply to 45. The numbers 5 and 9 sum to 14. $(x - 5)(x - 9)$

12.

$(x + 6)(x - 3)$

$x^2 + 3x - 18$

The numbers 1 and 18, 2 and 9, and 3 and 6 multiply to 18. The difference of 3 and 6 is 3. The middle term is positive, so the greater of the two numbers (6) is positive.

$(x + 6)(x - 3)$

13.

$(x + 6)(x - 11)$

$x^2 - 5x - 66$

The numbers 5 and 7 multiply to 35 and their difference is 2. The middle term is positive, so the greater of the two numbers, 7, is positive. Thus, $(x - 5)(x + 7) = 0$.

16.

$x = 2$ OR 13

$x^2 - 15x = -26$

$x^2 - 15x + 26 = 0$ Add 26 to both sides so that the expression equals 0.

The numbers 2 and 13 multiply to 26 and sum to 15.

$(x - 2)(x - 13) = 0$

17.

2

$$\textbf{FOIL}\left(\sqrt{8} - \sqrt{2}\right)\left(\sqrt{8} - \sqrt{2}\right)$$

First: $\sqrt{8} \times \sqrt{8} = 8$

Outer: $\sqrt{8} \times (-\sqrt{2}) = -\sqrt{16} = -4$

Inner: $(-\sqrt{2}) \times \sqrt{8} = -\sqrt{16} = -4$

Last: $(-\sqrt{2}) \times (-\sqrt{2}) = 2$

Sum of FOIL terms: $8 - 4 - 4 + 2 = 2$

18.

***x* = 5**

$$\begin{aligned} x^2 - 10x + 25 &= 0 \\ (x-5)(x-5) &= 0 \end{aligned}$$

$$x = 5$$

19.

***x* = −1, 2; *x* ≠ 4**

$$\frac{(x+1)(x-2)}{x-4} = 0$$

The numerator is 0 if either $(x + 1)$ or $(x - 2)$ is 0. Thus, $x = -1$ or $x = 2$. However, $x \neq 4$ because $x = 4$ would make the fraction undefined.

20.

$(2a + b)^2 = 0$

$4a^2 + 4ab + b^2 = 0 \rightarrow (2a)^2 + 2(2a)(b) + b^2 = 0 \rightarrow (2a + b)(2a + b) = 0$

21.

$(x + 11y)^2 = 0$

$x^2 + 22xy + 121y^2 = 0 \rightarrow x^2 + 2x(11y) + (11y)^2 = 0 \rightarrow (x + 11y)(x + 11y) = 0$

Problem Set

1. If -4 is a solution for x in the equation $x^2 + kx + 8 = 0$, what is k?

2. If 8 and -4 are the solutions for x, which of the following could be the equation?

(A) $x^2 - 4x - 32 = 0$
(B) $x^2 - 4x + 32 = 0$
(C) $x^2 + 4x - 12 = 0$
(D) $x^2 + 4x + 32 = 0$
(E) $x^2 + 4x + 12 = 0$

3. If $16 - y^2 = 10(4 + y)$, what is y?

4. If $x^2 - 10 = -1$, what is x?

5. If $x^2 - 13x = 30$, what is x?

x 1

12.

$$x^2 - 12x + 36 = 0$$

Quantity A	**Quantity B**
x	6

13.

$$xy > 0$$

Quantity A	**Quantity B**
$(x + y)^2$	$(x - y)^2$

Solutions

1.

$k = 6$

If −4 is a solution, then you know that (x + 4) must be one of the factors of the quadratic equation. The other factor is (x + ?). You know that the product of 4 and ? must be equal to 8; thus, the other factor is (x + 2). You know that the sum of 4 and 2 must be equal to k. Therefore, $k = 6$.

2. **(A)**

If the solutions to the equation are 8 and −4, the factored form of the equation is:

$(x - 8)(x + 4) = 0$

Distributed, this equals: $x^2 - 4x - 32 = 0$.

3.

$y = \{-4, -6\}$

Simplify and factor to solve.

$$16 - y^2 = 10(4 + y)$$
$$16 - y^2 = 40 + 10y$$
$$y^2 + 10y + 24 = 0$$
$$(y + 4)(y + 6) = 0$$

$y + 4 = 0$	OR	$y + 6 = 0$
$y = -4$		$y = -6$

$$
\begin{array}{rclcrcll}
s+11 & = & 0 & & s-7 & = & 0 & \text{The edge of a square must be positive,} \\
s & = & -11 & or & s & = & 7 & \text{so discard the negative value for } s.
\end{array}
$$

7.

$t = 2$

$$
\begin{aligned}
H &= Vt + 5t^2 \\
60 &= 20t + 5t^2 \\
5t^2 + 20t - 60 &= 0 \\
5(t^2 + 4t - 12) &= 0 \\
5(t+6)(t-2) &= 0
\end{aligned}
$$

$$
\begin{array}{rclcrcll}
t+6 & = & 0 & & t-2 & = & 0 & \text{A time must be positive, so discard the} \\
t & = & -6 & or & t & = & 2 & \text{negative value for } t.
\end{array}
$$

8.

2

Use FOIL to simplify this product:

$$
\begin{aligned}
&\mathbf{F}:\ 3 \times 3 = 9 \\
&\mathbf{O}:\ 3 \times \sqrt{7} = 3\sqrt{7} \\
&\mathbf{I}: -\sqrt{7} \times 3 = -3\sqrt{7} \\
&\mathbf{L}: -\sqrt{7} \times \sqrt{7} = -7 \\
&9 + 3\sqrt{7} - 3\sqrt{7} - 7 = 2
\end{aligned}
$$

9.

19

Factor both quadratic equations. Then use the greatest possible values of *x* and *y* to find the maximum value of the sum $x + y$:

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The value of x could be greater than or less than 1. **The relationship cannot be determined.**

12.

(C)

First, factor the equation in the common information:

$x^2 - 12x + 36 = 0 \rightarrow (x - 6)(x - 6) = 0$
$x = 6$

$$x^2 - 12x + 36 = 0$$

Quantity A	Quantity B
$x = 6$	6

The two quantities are equal.

13.

(A)

Expand the expressions in both columns:

$$xy > 0$$

Quantity A	Quantity B
$(x+y)^2 =$ $x^2 + 2xy + y^2$	$(x-y)^2 =$ $x^2 - 2xy + y^2$

Now subtract $x^2 + y^2$ from both columns:

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Chapter 3

INEQUALITIES & ABSOLUTE VALUE

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Chapter 3

Inequalities & Absolute Value

Inequalities

Earlier you explored how to solve equations. Now look at how you can solve *inequalities*.

Inequalities are expressions that use <, >, ≤, or ≥ to describe the relationship between two values.

Examples of inequalities:

$5 > 4$ $\quad$ $y \leq 7$ $\quad$ $x < 5$ $\quad$ $2x + 3 \geq 0$

The following table illustrates how the various inequality symbols are translated. Notice that when inequalities are translated, you read from left to right:

$x < y$	*x* is less than *y*.	
$x > y$	*x* is greater than *y*.	
$x \leq y$	*x* is less than or equal to *y*.	*x* is at most *y*.
$x \geq y$	*x* is greater than or equal to *y*.	*x* is at least *y*.

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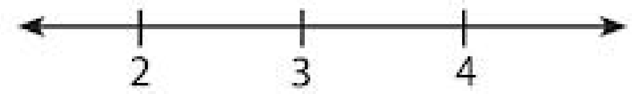
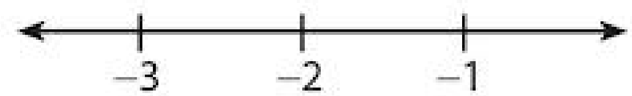
1.

$x > 3$



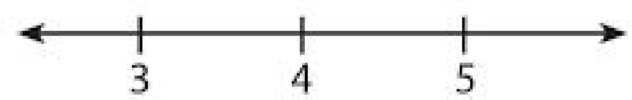
2.

$b \geq -2$



3.

$y = 4$



4. z is greater than v.

5. The total amount is greater than $2,000.

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make the
equation true.

Plug back in to
check:

(Any number less than 5) + 3 < 8. True.

5 + 3 = 8. True.

Check Your Skills

6. Which of the following numbers are solutions to the inequality $x < 10$?

   Indicate <u>all</u> that apply.

   - $\boxed{\text{E}}\,-3$
   - $\boxed{\text{E}}\,2.5$
   - $\boxed{\text{E}}\,-3/2$
   - $\boxed{\text{E}}\,9.999$

CLEANING UP INEQUALITIES

As with equations, your objective is to isolate the variable on one side of the inequality. When the variable is by itself, it is easiest to see what the solution (or range of solutions) really is. Although $2x + 6 < 12$ and $x < 3$ provide the same information (the second inequality is a simplified form of the first), you understand the full range of solutions much more easily when you look at the second inequality, which literally tells you that "*x* is less than 3."

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Check Your Skills

7. $x - 6 < 13$

8. $y + 11 \geq -13$

9. $x + 7 > 7$

INEQUALITY MULTIPLICATION AND DIVISION

You can also use multiplication and division to isolate the variables, as long as you recognize a very important distinction. *If you multiply or divide by a negative number, you must switch the direction of the inequality sign.* If you are multiplying or dividing by a positive number, the direction of the sign stays the same.

Here are a couple of examples to illustrate.

Multiplying or dividing by a *positive* number—the sign stays the same.

Example 1		Example 2	
$2x > 10$		$z/3 \leq 2$	
$2x/2 > 10/2$	Divide each side by 2.	$z/3 \times (3) \leq 2 \times (3)$	Multiply each side by 3.

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true.

What about multiplying or dividing an inequality by a *variable?* The short answer is…**try not to do it**! The issue is that you don't know the sign of the "hidden number" that the variable represents. If the variable logically can't be negative (e.g., it counts people or measures a length), then you can go ahead and multiply or divide.

If the variable must be negative, then you are also free to multiply or divide—just remember to flip the sign. However, if you don't know whether the variable is positive or negative, try to work through the problem with the inequality as is. (If the problem is a Quantitative Comparison, consider whether not knowing the sign of the variable you want to multiply or divide by means that the answer is (D).)

Check Your Skills

10. $x + 3 \geq -2$

11. $-2y < 8$

12. $a + 4 \geq 2a$

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Check Your Skills

13. $|3| = 3$

14. $|-3| = -3$

15. $|3| = -3$

16. $|-3| = 3$

17. $|3 - 6| = 3$

18. $|6 - 3| = -3$

SOLVING ABSOLUTE VALUE EQUATIONS

On the GRE, some absolute value equations place a variable inside the absolute value signs:

Example: $|y| = 3$

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Here's a slightly more difficult problem, using the same technique:

Example: $6 \times |2x + 4| = 30$

To solve this, you can use the same approach:

$6 \times |2x+4| = 30$ $|2x+4| = 5$	Step 1: Isolate the absolute value expression on one side of the equation or inequality.
$(2x+4) = 5$ or $-(2x+4) = 5$ $2x+4=5$ or $-2x-4=5$	Step 2: Set up two equations—the positive and the negative values are set equal to the other side.
$2x = 1$ or $-2x = 9$	Step 3: Solve both equations/inequalities.
$x = \frac{1}{2}$ or $x = -\frac{9}{2}$	Note: There are two possible values for *x*.

Check Your Skills

19. $|a| = 6$

20. $|x + 2| = 5$

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Putting Them Together: Inequalities and Absolute Values

Some problems on the GRE include both inequalities and absolute values. You can solve these problems by combining what you have learned about solving inequalities with what you have learned about solving absolute values:

Example 1: $|x| \geq 4$

Even though you're now dealing with an inequality, and not an equals sign, the basic process is the same. The absolute value is already isolated on one side, so now you need to set up your two equations or, in this case, inequalities. The first inequality replaces the absolute value with the *positive* of what's inside, and the second replaces the absolute value with the *negative* of what's inside:

$+(x) \geq 4$ *or* $-(x) \geq 4$

Now that you have your two equations, isolate the variable in each equation:

$$
\begin{array}{l|ll}
+(x) \geq 4 & -(x) \geq 4 & \\
x \geq 4 & -x \geq 4 & \text{Divide by } -1. \\
 & x \leq -4 & \text{Remember to flip the sign when dividing by a negative.}
\end{array}
$$

So the two solutions to the original equation are $x \geq 4$ and $x \leq -4$. Here it is represented on a number line:

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It seems like every number should be a solution to the equation However, if you start testing numbers, that isn't the case. Test $x = 5$, for example. Is $|5 + 3| < 5$? No, it isn't. As it turns out, the only numbers that make the original inequality true are those that are true for *both* inequalities. Your number line should look like this:

In the first example, it was the case that x could be greater than or equal to 4 *or* less than or equal to −4. For this example, however, it seems to make more sense to say that x is greater than −8 *and* less than 2.

The inequality you just graphed means "($x + 3$) is less than five units away from from 0, in either direction." The shaded segment indicates all numbers x for which this is true. As the inequalities become more complicated, don't worry about interpreting their meaning—simply solve them algebraically.

To summarize, when representing inequalities on the number line, absolute value expressions where variables are *greater than some quantity* will show up *as two ranges in opposite directions* (or "double arrows"); however, absolute value expressions where variables are *less than some quantity* will show up *as a single range* (or "line segment").

Check Your Skills

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Manipulating Compound Inequalities

Sometimes a problem with compound inequalities will require you to manipulate the inequalities to solve the problem. You can perform operations on a compound inequality as long as you remember to perform those operations on **every term** in the inequality, not just the outside terms. For example:

$x+3<y<x+5$	$\xrightarrow{\times}$	$x<y<x+2$	**Wrong**: You must subtract 3 from every term in the inequality.
$x+3<y<x+5$	$\xrightarrow{\checkmark}$	$x<y-3<x+2$	CORRECT
$\frac{c}{2} \leq b-3 \leq \frac{d}{2}$	$\xrightarrow{\times}$	$c \leq b-3 \leq d$	**Wrong:** You must multiply by 2 in every term in the inequality.
$\frac{c}{2} \leq b-3 \leq \frac{d}{2}$	$\xrightarrow{\checkmark}$	$c \leq 2b-6 \leq d$	Correct

If $1 > 1 - ab > 0$, which of the following must be true?

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Using Extreme Values

One effective technique for solving GRE inequality problems is to focus on the **Extreme Values** of a given inequality. This is particularly helpful when solving the following types of inequality problems:

0. Problems with multiple inequalities where the question involves the potential range of values for variables in the problem
0. Problems involving both equations and inequalities

INEQUALITIES WITH RANGES

Whenever a question asks about the possible range of values for a problem, consider using Extreme Values:

> If $0 \le x \le 3$ and $y < 8$, which of the following could NOT be the value of xy?
> (0)
> 0
> (0)
> 8
> (0)
> 12
> (0)
> 16
> (0)
> 24

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very negative. Likewise, because *x* can be positive and *y* can be infinitely negative, *xy* can be infinitely negative. Therefore, *xy* can equal any number.

Check Your Skills

27. If $-4 < a < 4$ and $-2 < b < -1$, which of the following could NOT be the value of ab?

(A) −3
(B) 0
(C) 4
(D) 6
(E) 9

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Max 6	Max 8	$6 \times 8 = 48$

This time, ab is maximized when you take the *negative* extreme values for both a and b, resulting in $ab = 49$. Notice that you could have focused right away on the first and fourth scenarios, because they are the only scenarios that produce positive products.

> If $-4 \le m \le 7$ and $-3 < n < 10$, what is the maximum possible integer value for $m - n$?

Again, you are looking for a maximum possible value, this time for $m - n$. You need to test the extreme values for m and for n to determine which combinations of extreme values will maximize $m - n$:

Extreme Values for *m*	**Extreme Values for *n***
The lowest value for m is −4.	The lowest value for n is greater than −3.
The highest value for m is 7.	The highest value for n is less than 10.

Now consider the different extreme value scenarios for m, n, and $m - n$:

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13.

True

14.

False *(Note that absolute value is always positive!)*

15.

False

16.

True

17.

True ($|3-6| = |-3| = 3$)

18.

False

19.

$|a| = 6$

$$\boldsymbol{a = 6} \qquad or \qquad \boldsymbol{a = -6}$$

20.

x = 3 or −7

$|x+2| = 5$

$+(x+2) = 5 \qquad or \qquad -(x+2) = 5$

$x + 2 = 5 \qquad or \qquad -x - 2 = 5$

$x = 3 \qquad or \qquad -x = 7$

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29.

2

$(x + 2)^2 \leq 2 - y$	
$y + (x + 2)^2 \leq 2$	Add y to both sides.
$y \leq 2 - (x + 2)^2$	Subtract $(x + 2)^2$ from both sides.

Note that y is maximized when $(x + 2)^2$ is minimized. The smallest possible value for $(x + 2)^2$ is 0, when $x = -2$. When $(x + 2)^2 = 0$, $y = 2$.

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<u>**Quantity A**</u>	<u>**Quantity B**</u>
$\lvert 2 - x \rvert$	2

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Chapter 4

FORMULAS & FUNCTIONS

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Strange Symbol Formulas

Another type of GRE formula problem involves the use of strange symbols. In these problems, the GRE introduces an arbitrary symbol, which defines a certain procedure. These problems may look confusing because of the unfamiliar symbols. However, the symbol is *irrelevant*. All that is important is that you carefully follow each step in the procedure that the symbol indicates.

A technique that can be helpful is to break the operations down one-by-one and say them aloud (or in your head)—to "hear" them explicitly. Here are some examples:

Formula Definition	**Step-by-Step Breakdown**
$x \heartsuit y = x^2 + y^2 - xy$	"The first number squared, plus the second number squared, minus the product of the two …"
$s \circ t = (s - 2)(t + 2)$	"Two less than the first number times two more than the second number …"
$\boxed{x}$ is defined as the product of all integers smaller than x but greater than 0 …	"… x minus 1, times x minus 2, times x minus 3 … Aha! So this is $(x - 1)$ factorial!"

Notice that it can be helpful to refer to the variables as "the first number," "the second number," and so on. In this way, you use the physical position of the numbers to keep them straight in relation to the strange symbol.

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You determine percent decrease as follows:

$$\frac{\text{change}}{\text{original}} = \frac{27 - 1}{27} = \frac{26}{27} \approx 0.963 = 96.3\%\text{decrease}$$

Check Your Skills

4. When Tom moved to a new home, his distance to work decreased by $\frac{1}{2}$ the original distance and the constant rate at which he travels to work increased by $\frac{1}{3}$ the original rate. By what percent has the time it takes Tom to travel to work decreased?

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8. If each number of a sequence is 4 more than the previous number, and the 3rd number in the sequence is 13, what is the 114th number in the sequence?

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as actually representing the rule that the magic box is using to transform your number.

The magic box analogy is a helpful way to conceptualize a function as a rule built on an independent variable. The value of a function changes as the value of the independent variable changes. In other words, the value of a function is dependent on the value of the independent variable. Examples of functions include:

$f(x) = 4x^2 - 11$	The value of the function *f* is *dependent* on the *independent* variable *x*.
$g(t) = t^3 + \sqrt{t} - \frac{2t}{5}$	The value of the function *g* is *dependent* on the *independent* variable *t*.

Think of functions as consisting of an "input" variable (the number you put into the magic box), and a corresponding "output" value (the number that comes out of the box). The function is simply the rule that turns the "input" variable into the "output" variable.

By the way, the expression $f(x)$ is pronounced "*f* of *x*", not "*fx*." It does not mean "*f* times *x*."The letter *f* does not stand for a variable; rather, it stands for the rule that dictates how the input *x* changes into the output $f(x)$.

The "domain" of a function indicates the set of possible inputs. The "range" of a function indicates the set of possible outputs. For instance, the function $f(x) = x^2$ can take any input but never produces a negative number. So the domain is all numbers, but the range is $f(x) \geq 0$.

NUMERICAL SUBSTITUTION

This is the most basic type of function problem. Input the numerical value (say, 5) in place of the independent variable *x* to determine the value of the function:

> If $f(x) = x^2 - 2$, what is the value of $f(5)$?

In this problem, you are given a rule for $f(x)$: square *x* and subtract 2. Then, you are asked to apply this rule to the number 5. Square 5 and subtract 2 from the result:

$$f(5) = (5)^2 - 2 = 25 - 2 = 23$$

VARIABLE SUBSTITUTION

This type of problem is slightly more complicated. Instead of finding the output value for a numerical input, you must find the output when the input is an algebraic expression:

> If $f(z) = z^2 - \frac{z}{3}$, what is the value of $f(w + 6)$?

Input the variable expression $(w + 6)$ in place of the independent variable (z) to determine the value of the function:

$$f(w+6) = (w+6)^2 - \frac{w+6}{3}$$

Compare this equation to the equation for $f(z)$. The expression $(w + 6)$ has taken the place of every *z* in the original equation. In a sense, you are treating the expression $(w + 6)$ as one thing, as if it were a single letter or variable.

The rest is algebraic simplification:

$$\begin{aligned} f(w+6) &= (w+6)(w+6) - \left(\frac{w}{3}+\frac{6}{3}\right) \\ &= w^2+12w+36-\frac{w}{3}-2 \\ &= w^2+11\frac{2}{3}w+34 \end{aligned}$$

COMPOUND FUNCTIONS

Imagine putting a number into one magic box, and then putting the output directly into another magic box. This is the situation you have with compound functions:

> $$\text{If}\, f(x) = x^3 + \sqrt{x}\,\text{and}\, g(x) = 4x - 3,\ \text{what is}\, f(g(3))?$$

The expression $f(g(3))$, pronounced "*f* of *g* of 3," looks ugly, but the key to solving compound function problems is to work from the inside out. In this case, start with $g(3)$. Notice that you put the number into *g*, not into *f*, which may seem backward at first:

$g(3) = 4(3) - 3 = 12 - 3 = 9$

Use the result from the *inner* function *g* as the new input variable for the *outer* function *f*:

$$f(g(3)) = f(9) = (9)^3 + \sqrt{9} = 729 + 3 = 732$$

The final result is 732.

Note that changing the order of the compound functions changes the answer:

> $$\text{If } f(x) = x^3 + \sqrt{x} \text{ and } g(x) = 4x - 3, \text{ what is } g(f(3))?$$

Again, work from the inside out. This time, start with *f*(3), which is now the inner function:

$$f(3) = (3)^3 + \sqrt{3} = 27 + \sqrt{3}$$

Use the result from the *inner* function *f* as the new input variable for the *outer* function *g*:

$$g(f(3)) = g\left(27 + \sqrt{3}\right) = 4\left(27 + \sqrt{3}\right) - 3 = 108 + 4\sqrt{3} - 3 = 105 + 4\sqrt{3}$$

Thus, $g(f(3)) = 105 + 4\sqrt{3}.$

In general, *f*(*g*(*x*)) and *g*(*f*(*x*)) are **not the same rule overall** and will often lead to different outcomes. As an analogy, think of "putting on socks" and "putting on shoes" as two functions: the order in which you perform these steps obviously matters!

You may be asked to find a value of *x* for which *f*(*g*(*x*)) = *g*(*f*(*x*)). In that case, use variable substitution, working as always from the inside out:

> If $f(x) = x^3 + 1$, and $g(x) = 2x$, for what value of x does $f(g(x)) = g(f(x))$?

Simply evaluate as you did in the problems above, using *x* instead of an input value:

$$f(g(x)) = g(f(x))$$
$$f(2x) = g(x^3 + 1)$$
$$(2x)^3 + 1 = 2(x^3 + 1)$$
$$8x^3 + 1 = 2x^3 + 2$$
$$6x^3 = 1$$
$$x = \sqrt[3]{\frac{1}{6}}$$

FUNCTIONS WITH UNKNOWN CONSTANTS

On the GRE, you may be given a function with an unknown constant. You will also be given the value of the function for a specific number. You can combine these pieces of information to find the complete function rule:

> If $f(x) = ax^2 - x$, and $f(4) = 28$, what is $f(-2)$?

Solve these problems in three steps. First, use the value of the input variable and the corresponding output value of the function to solve for the unknown constant:

$$\begin{aligned} f(4) = a(4)^2 - 4 &= 28 \\ 16a - 4 &= 28 \\ 16a &= 32 \\ a &= 2 \end{aligned}$$

Then, rewrite the function, replacing the constant with its numerical value:

$$f(x) = ax^2 - x = 2x^2 - x$$

Finally, solve the function for the new input variable:

$$f(-2) = 2(-2)^2 - (-2) = 8 + 2 = 10$$

Check Your Skills Answer Key

1.

***k* = 70**

Baking time in minutes $= \frac{Vk}{T}$

$$30 = \frac{150 \times k}{350}$$

$$k = \frac{30 \times 350}{150} = 70$$

2.

20

Deal with the formula in the parentheses first:

$3 \Delta 1 = 3^1 + 1 = 3 + 1 = 4$

$-2 \Delta (3 \Delta 1) = -2 \Delta 4$

$-2 \Delta 4 = (-2)^4 + 4 = 16 + 4 = 20$

3.

$\mathbf{8\frac{1}{8}}$

$$2\lambda16 = \frac{16}{2} + \frac{2}{16} = 8 + \frac{1}{8} = 8\frac{1}{8}$$

4. **62.5% decrease**

   No numbers are specified, so you should choose values for the original distance and the original rate. Good numbers to pick for the distance are multiples of 2, because the rate is decreased by $\frac{1}{2}$. Good numbers to pick for the rate are multiples of 3, because the rate is increased by $\frac{1}{3}$:

$$\text{Percent decrease in time} = \frac{\text{change in time}}{\text{original time}} = \frac{4 - 1.5}{4} = \frac{2.5}{4} = 62.5\% \text{ decrease.}$$

5. **17**

   $S_{11} = 2 \times (11) - 5 = 22 - 5 = 17$

6. **–6**

   $B_9 = (-1)^{(9)} \times 9 + 3 = -9 + 3 = -6$

7. **3**

You know the value of A_4, therefore, you can write the definition for A_4 to solve for A_3, write the definition for A_3 to solve for A_2, and so on:

$$\begin{aligned} A_4 &= 2A_3 + 3 &\rightarrow\quad 45 &= 2A_3 + 3 &\rightarrow\quad A_3 &= 21 \\ A_3 &= 2A_2 + 3 &\rightarrow\quad 21 &= 2A_2 + 3 &\rightarrow\quad A_2 &= 9 \\ A_2 &= 2A_1 + 3 &\rightarrow\quad 9 &= 2A_1 + 3 &\rightarrow\quad A_1 &= 3 \end{aligned}$$

8. **457**

   There are 114 − 3 = 111 "jumps" of 4 between the 3rd and the 114th terms. Because 111 × 4 = 444, there is an increase of 444 from the 3rd term to the 114th term: 13 + 444 = 457.

9. **6**

   The units digits of the powers of 7 follow a repeating pattern: **7**, 4**9**, 34**3**, 240**1**, 1680**7**, etc. Pattern = {7, 9, 3, 1}. There are 8 repeats of the pattern from A_1 to A_{32}, inclusive. The pattern begins again on A_{33}, so A_{33} has the same units digit as A_1, which is 7. The units digit of 7^{33} is 7, and 7 − 1 = 6.

10. **5**

    Simply plug in (−1) for each occurrence of x in the function definition and evaluate:

$$f(x) = \frac{1}{x+2} + (x-1)^2 \qquad f(-1) = \frac{1}{(-1)+2} + ((-1)-1)^2$$

$$f(1) = \frac{1}{1} + (-2)^2 = 1 + 4 = 5$$

11. **5**

    Plug in 3 for u in the definition of $t(u)$, set it equal to 37, and solve for a:

$$t(u) = au^2 - 3u + 1 \rightarrow t(3) = a(3)^2 - 3(3) + 1 = 37$$

$$9a - 9 + 1 = 37$$

$$9a = 45$$

$$a = 5$$

12. **100**

First, find the output value of the inner function: $f(4) = 3(4) - \sqrt{4} = 12 - 2 = 10.$

Then, find $g(10)$: $10^2 = 100$.

13.

$$\frac{-x^3 + x + 1}{x^3 + x^2}$$

Simply plug in $\left(\frac{1}{x}\right)$ for y in $g(y)$, and simplify the expression:

$$g(y) = y^2 - \frac{1}{y+1} \rightarrow g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 - \frac{1}{\left(\frac{1}{x}\right) + 1}$$

$$g\left(\frac{1}{x}\right) = \frac{1}{x^2} - \frac{1}{\frac{x+1}{x}} = \frac{1}{x^2} - \frac{x}{x+1}$$

$$g\left(\frac{1}{x}\right) = \frac{x + 1 - x^3}{x^2(x+1)} = \frac{-x^3 + x + 1}{x^3 + x^2}$$

Problem Set

1. If $A \diamond B = 4A - B$, what is the value of $(3 \diamond 2) \diamond 3$?

2. I $= \dfrac{\boldsymbol{u} + \boldsymbol{y}}{\boldsymbol{x} + \boldsymbol{z}}$, what i: ?

3. Life expectancy is defined by the formula $\dfrac{2SB}{G}$, where S = shoe size, B = average monthly electric bill in dollars, and G = GRE score. If Melvin's GRE score is twice his monthly electric bill, and his life expectancy is 50, what is his shoe size?

4. The formula for spring factor in a shoe insole is $\dfrac{w^2 + x}{3}$, where w is the width of the insole in centimeters and x is the grade of rubber on a scale of 1 to 9. What is the maximum spring factor for an insole that is 3 centimeters wide?

5. Cost is expressed by the formula tb^4. If b is doubled, by what factor has the cost increased?

(A)

(B)

(C)

(D)

17.

$A_n = 2^n - 1$ for all integers n

≥ 1

Quantity A **Quantity B**

The units digit of	The units digit of
A_{26}	A_{34}

18.

$P \blacksquare Q = P + 2Q$ for all integers P and Q

Quantity A	**Quantity B**
$11 \blacksquare 5$	$5 \blacksquare 11$

19. The length of a rectangle increased by a factor of 2, and at the same time its area increased by a factor of 6.

Quantity A	**Quantity B**
The factor by which the width of the rectangle increased	3

Solutions

1. **37**

   First, simplify 3 ◊ 2: $4(3) - 2 = 12 - 2 = 10$. Then, solve 10 ◊ 3: $4(10) - 3 = 40 - 3 = 37$.

2. **2**

   Plug the numbers in the grid into the formula, matching up the number in each section with the corresponding variable in the formula: $\frac{u + y}{x + z} = \frac{8 + 10}{4 + 5} = \frac{18}{9} = 2.$

3. **Size 50**

   $$\frac{2SB}{2B} = 50$$
   $$S = 50$$

   Substitute $2B$ for G in the formula. Note that the term $2B$ appears in both the numerator and denominator, so they cancel out.

4. **6**

   Determine the maximum spring factor by setting $x = 9$.

   Let s = spring factor:

   $$s = \frac{w^2 + x}{3}$$

   $$s = \frac{(3)^2 + 9}{3} = \frac{18}{3} = 6$$

5. **D**

   Pick numbers to see what happens to the cost when b is doubled. If the original value of b is 2, the cost is $16t$. When b is doubled to 4, the new cost value is $256t$. The cost has increased by a factor of $\frac{256}{16}$, or 16.

6. **8cm^3**

$$V = s^3 \rightarrow 64 = s^3 \rightarrow s = 4$$

The length of a side on the real sculpture is 4 m.

$$\frac{0.5\,\text{cm}}{1\,\text{m}} = \frac{x\,\text{cm}}{4\,\text{m}} \rightarrow x = 2$$

The length of a side on the model is 2 cm.

$V = s^3 = (2)^3 = 8$

The volume of the model is 8.

7. $\sqrt{6}$

Let c = competitive edge:

$$c =$$

Pick numbers to see what happens to the competitive edge when W is tripled and L is halved. If the original value of W is 2 and the original value of L is 2, the original value

14.

44

First, find the output value of the inner function: $g(4) = 16$. Then, find $f(16)$: $3(16) - \sqrt{16} = 48 - 4 = 44$.

15.

$x = \{-1, 2\}$

To find the values for which $f(x) = g(x)$, set the functions equal to each other:

$$\begin{aligned} 2x^2 - 4 &= 2x \\ 2x^2 - 2x - 4 &= 0 \\ 2\left(x^2 - x - 2\right) &= 0 \\ 2(x-2)(x+1) &= 0 \end{aligned}$$

$$\begin{aligned} x - 2 &= 0 & \quad x + 1 &= 0 \\ x &= 2 \text{ or} & x &= -1 \end{aligned}$$

16. **(C)**

$g(x) = |x - 1| - 1$. This function is an absolute value, which typically has a V-shape. You can identify the correct graph by trying $x = 0$, which yields $g(0) = 0$, the origin. Then, try $x = 1$, which yields $g(1) = -1$ and the point $(1, -1)$. Next, try $x = 2$: $g(2) = |2 - 1| - 1 = 1 - 1 = 0$. These three points fall on the V-shape.

17.

(C)

The powers of 2 have a repeating pattern of four terms for their units digits: {2, 4, 8, 6}. That means that every fourth term, the pattern repeats. For instance, the fifth term has the same units as the first term, because 5 − 1 = 4. So terms that are four terms apart, or a multiple of four terms apart, will have the same units digit.

The 34th term and the 26th term are 34 − 26 = 8 terms apart. Because 8 is a multiple of 4, the terms will have the same units digit. **The two quantities are equal.** Incidentally, the units digit of A_{26} and A_{34} is 3.

18.

(B)

$$P \blacksquare Q = P + 2Q \text{ for all integers } P \text{ and } Q$$

<u>Quantity A</u>	<u>Quantity B</u>
$11 \blacksquare 5 =$	$5 \blacksquare 11 =$
$(11) + 2 \times (5) =$	$(5) + 2 \times (11) =$
$11 + 10 = \mathbf{21}$	$5 + 22 = \mathbf{27}$

Quantity B is greater.

19.

(C)

Plug in numbers to answer this question. Use a table to organize the information:

$4 \times W = 12$

$W = 3$

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