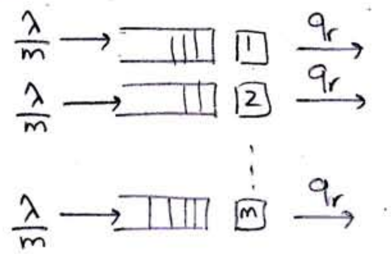
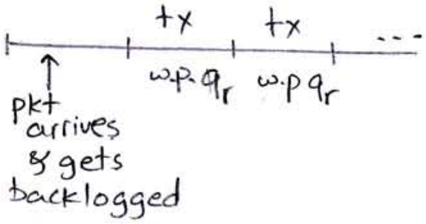
4.8



Assume that any given transmission is successful with probability p

(a)



Let X be the number of slots from the backlogged slot until a packet is transmitted successfully

$$P(X=k) = (1-q_r p)^{k-1} q_r p \qquad k=1,2,3,\dots$$

$$E[X] = \frac{1}{q_r p}$$

$$E[X^2] = \frac{2-q_r p}{(q_r p)^2}$$

(b) a node can be modeled by an M/G/1 queue with vacation

M : because arrival process is Poisson with rate $\frac{\lambda}{m}$

G : because service time is geometrically distributed

with vacation : because the system is slotted, and hence, an arrival in the middle of a slot when the queue is empty has to wait until the begining of the next slot

vacations are deterministic : $V=1$ slot $\quad \therefore \ \bar{V}=1$ and $\overline{V^2}=1$

$$\underbrace{T}_{\text{avg delay}} = \underbrace{\bar{X}}_{\text{avg service time}} + \underbrace{W}_{\text{avg. waiting time}}$$

$$W = \frac{R}{1-\rho} \quad \text{where} \quad \rho = \frac{\lambda}{m}\bar{X} \quad \text{and} \quad R = \rho\,\frac{\overline{X^2}}{2\bar{X}} + (1-\rho)\,\frac{\overline{V^2}}{2\bar{V}}$$

$$\therefore\ T = \frac{1}{q_r p} + \frac{\rho}{1-\rho}\,\frac{(2-q_r p)}{2 q_r p} + \frac{1}{2} \qquad \text{with } \rho = \frac{\lambda}{m}\,\frac{1}{q_r p}$$

$$T = \frac{2(1-\rho) + \rho(2 - q_r P) + (1-\rho) q_r P}{(1-\rho)\, 2 q_r P}$$

$$= \frac{2 - \cancel{2}\rho + \cancel{2}\rho - \rho q_r P + q_r P - \rho q_r P}{(1-\rho)\, 2 q_r P}$$

$$= \frac{2 + q_r P - 2\rho q_r P}{(1-\rho)\, 2 q_r P}$$

$$= \frac{1}{(1-\rho) q_r P} + \frac{(1-2\rho)}{2(1-\rho)}$$

(c) $P=1$, $q_r = \frac{1}{m}$ $\Rightarrow$ $\rho = \frac{\lambda}{m} \cdot \frac{1}{\frac{1}{m} \times 1} = \lambda$

$$\therefore T = \frac{m}{1-\lambda} + \frac{1-2\lambda}{2-2\lambda}$$

$$T_{TDM} = \frac{m}{2(1-\lambda)} + 1$$