$\boxed{4.3}$

(a) $$P_{success} = Q_a(1,n)\, Q_r(0,n) + Q_a(0,n)\, Q_r(1,n)$$
$$= (m-n)\, q_a (1-q_a)^{m-n-1} \cdot (1-q_r)^n + n\, q_r (1-q_r)^{n-1} \cdot (1-q_a)^{m-n}$$
$$= (1-q_a)^{m-n} (1-q_r)^n \left[\frac{(m-n) q_a}{1-q_a} + \frac{n q_r}{1-q_r}\right]$$
$$\approx e^{-q_a(m-n)}\, e^{-q_r n} \left[(m-n) q_a + n q_r\right]$$
$$= e^{-G(n)}\, G(n)$$

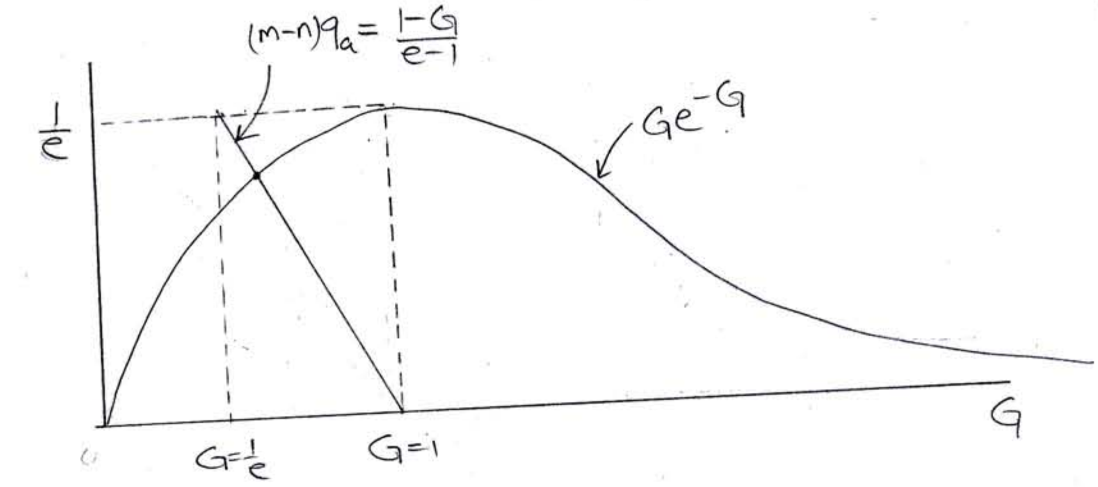
$G(n) = (m-n)\, q_a + n\, q_r$

for $q_r = \frac{1}{m}$ and $q_a = \frac{1}{me}$, we have $G(n) = (m-n)\frac{1}{me} + n\left(\frac{1}{m}\right)$

$$G = \frac{1}{e} + \frac{n}{m}\left(1 - \frac{1}{e}\right)$$

$$\Rightarrow \frac{n}{m} = \frac{G - 1/e}{1 - 1/e}$$

$$\therefore \quad (m-n)\, q_a = (m-n)\frac{1}{me} = \frac{1}{e}\left(1 - \frac{n}{m}\right) = \frac{1}{e}\left(\frac{1 - 1/e - G + 1/e}{1 - 1/e}\right) = \frac{1-G}{e-1}$$



(b) $$P_{success}\Big|_{n=m} = m\, q_r (1-q_r)^{m-1}$$

$$P_{success}\Big|_{n=m} \approx G(n)\, e^{-G(n)}\Big|_{n=m} = e^{-1} \qquad \left(G(m) = (m-m)\frac{1}{me} + m\left(\frac{1}{m}\right) = 1\right)$$

(c) $F \triangleq (m-n)q_a$

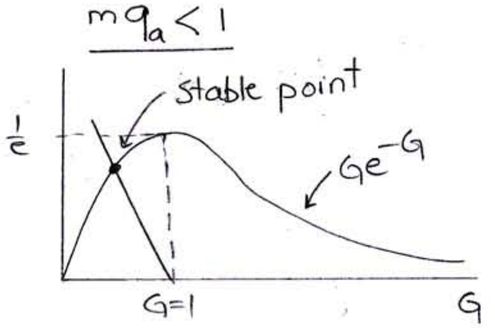
$G = (m-n)q_a + nq_r$

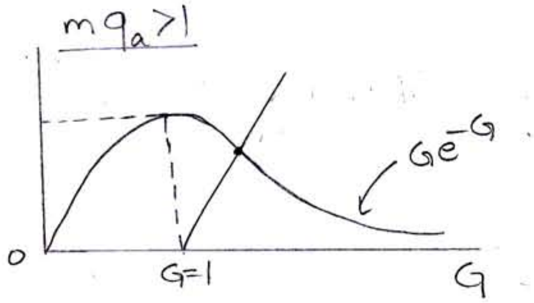
$\because$ F is a linear function of G $\therefore$ we only need two point to completely describe it

① $n=0$: $F = mq_a$, $G = mq_a$

② $n=m$: $F = 0$, $G = mq_r = m\left(\frac{1}{m}\right) = 1$

So we have two points : $(G_1, F_1) = (mq_a, mq_a)$ and $(G_2, F_2) = (1, 0)$





(d) Back to $q_a = \frac{1}{me}$ $\left(q_r = \frac{1}{m}\right)$

The stable point occurs at $(m-n)q_a = P_{success}$

$$\Rightarrow \quad \frac{1+G}{e-1} = Ge^{-G}$$

Rewrite the equation as $G = 1 - (e-1)Ge^{-G}$

Then solve iteratively : $G_{k+1} = 1 - (e-1)G_k e^{-G_k}$ with $G_0 = 0.5$ (some value between 0 & 1)

$G_1 = 0.4789$
$G_2 = 0.4902$
$G_3 = 0.4841$
$G_4 = 0.4874$
$\vdots$
$G^* = 0.4862$

(e) $\frac{n^*}{m} = \frac{G^* - 1/e}{1 - 1/e} = 0.1872$