$\boxed{14.9}$ $\mathbb{P}(n=k) = e^{-\hat{n}} \frac{\hat{n}^k}{k!} \qquad k=0,1,2,\ldots$

(a) $\mathbb{P}(\text{idle}) = \mathbb{P}\left(\{\text{no packets in the system}\} \text{ OR } \{\text{there are some packets} \text{ AND } \text{none of them is transmitted}\}\right)$

$$= \mathbb{P}\left(\bigcup_{k=0}^{\infty} \{n=k, \text{ none of the } k \text{ packets is transmitted}\}\right)$$

$$= \sum_{k=0}^{\infty} \mathbb{P}(n=k)\, \mathbb{P}(k \text{ packets are not tx})$$

$$= \sum_{k=0}^{\infty} e^{-\hat{n}} \frac{\hat{n}^k}{k!} \cdot \left(1-\frac{1}{\hat{n}}\right)^k$$

$$= e^{-\hat{n}} \sum_{k=0}^{\infty} \frac{(\hat{n}-1)^k}{k!}$$

$$= e^{-\hat{n}} \cdot e^{\hat{n}-1}$$

$$= e^{-1}$$

(b) $\mathbb{P}(n=k \mid \text{idle}) = \frac{\mathbb{P}(n=k, \text{ idle})}{\mathbb{P}(\text{idle})} = \frac{\mathbb{P}(n=k, \text{ the } k \text{ packets are not transmitted})}{\mathbb{P}(\text{idle})}$

$$= \frac{e^{-\hat{n}} \frac{\hat{n}^k}{k!} \cdot \left(1-\frac{1}{\hat{n}}\right)^k}{e^{-1}}$$

$$= e^{-(\hat{n}-1)} \frac{(\hat{n}-1)^k}{k!}$$

(c) $\mathbb{P}(\text{success}) = \mathbb{P}\left(\bigcup_{k=1}^{\infty} \{n=k, \text{ only 1 out of the } k \text{ packets is transmitted}\}\right)$

$$= \sum_{k=1}^{\infty} e^{-\hat{n}} \frac{\hat{n}^k}{k!} \cdot k\left(\frac{1}{\hat{n}}\right)\left(1-\frac{1}{\hat{n}}\right)^{k-1}$$

$$= e^{-\hat{n}} \sum_{k=1}^{\infty} \frac{(\hat{n}-1)^{k-1}}{(k-1)!}$$

$$= e^{-\hat{n}}\, e^{\hat{n}-1}$$

$$= e^{-1}$$

(d) $\mathbb{P}(n=k+1 \mid \text{success}) = \frac{\mathbb{P}(n=k+1, \text{ success})}{\mathbb{P}(\text{success})} = \frac{\mathbb{P}(n=k+1, \text{ only 1 out } k+1 \text{ tx})}{\mathbb{P}(\text{success})}$

$$= \frac{e^{-\hat{n}} \frac{\hat{n}^{k+1}}{(k+1)!} (k+1) \frac{1}{\hat{n}} \left(1-\frac{1}{\hat{n}}\right)^k}{e^{-1}}$$

$$= e^{-(\hat{n}-1)} \frac{(\hat{n}-1)^k}{k!}$$