$\boxed{4.4}$

n_k : number of backlogged nodes at the begining of the k^{th} slot

$\bar{n} = E[n_k]$

N_a = number of packets accepted in a slot

(a) $\mathbb{P}(N_a = j \mid n \text{ nodes are backlogged}) = \binom{m-n}{j} q_a^j (1-q_a)^{m-n-j}$

$$E(N_a \mid n \text{ nodes are back logged}) = (m-n)\, q_a$$

$$E(N_a) = (m - E(n))\, q_a$$

$$\overline{N_a} = (m-\bar{n})\, q_a$$

(b) $n_{k+1} = n_k + \text{number of accepted packets in slot } k - \underbrace{\text{number of successfully transmitted packets in slot } k}_{\text{either 0 or 1}}$.

by taking expectation

$$\bar{n} = \bar{n} + \overline{N_a} - \overline{P_{succ}}$$

$$\therefore \overline{P_{succ}} = \overline{N_a}$$

(c) N_{sys} in slot k = $n_k + N_a$ in slot k

$$\overline{N_{sys}} = \bar{n} + \overline{N_a}$$

(d) Little's thm: $\lambda_a T = \overline{N_{sys}}$

λ_a = arrival rate of packets into the system

= average number of accepted packets per slot

$= \overline{N_a}$

$$\therefore T = \frac{\overline{N_{sys}}}{\overline{N_a}} \qquad T = 1 + \frac{\bar{n}}{(m-\bar{n})\, q_a}$$

(e) $\bar{n}' < \bar{n}$

part (a) $\Rightarrow$ $\overline{N_a'} > \overline{N_a}$

part (b) $\Rightarrow$ $\overline{P_{succ}'} > \overline{P_{succ}}$

part (c) $\Rightarrow$ $\overline{N'_{sys}} < \overline{N_{sys}}$

part (d) $\Rightarrow$ $T' < T$