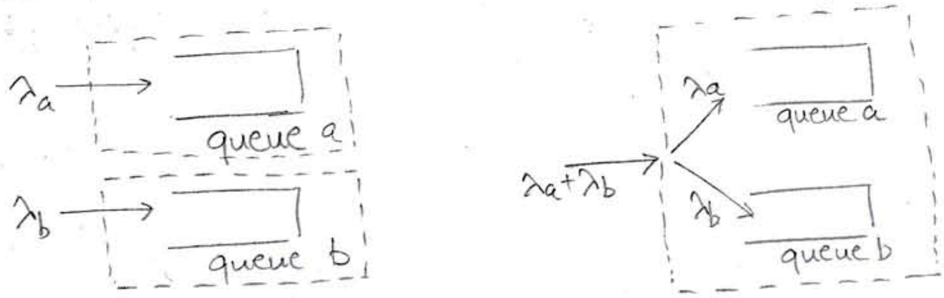
3.38

(b) Define $W_{(k)}$ = average time in queue averaged over the first k priorities

Obviously, $W_{(1)} = W_1$

<u>Note</u> Consider two types of traffic: type a and type b

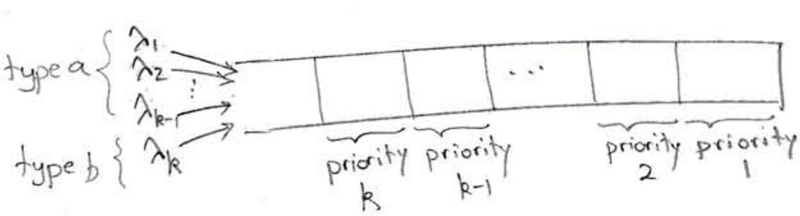


$N_{Qa} = \lambda_a W_a$, $N_{Qb} = \lambda_b W_b$ $\qquad N_Q = (\lambda_a + \lambda_b) W$

$\therefore \; N_Q = N_{Qa} + N_{Qb}$ $\qquad$ (Recall prob 3.9) $\qquad$ #

$\therefore \; (\lambda_a + \lambda_b) W = \lambda_a W_a + \lambda_b W_b$

We will use Little's theorem in a similar manner.



We have $\left(\sum_{i=1}^{k-1} \lambda_i + \lambda_k\right) W_{(k)} = \left(\sum_{i=1}^{k-1} \lambda_i\right) W_{(k-1)} + \lambda_k W_k \qquad k = 1, \ldots, n$

$\therefore \; W_k = \frac{1}{\lambda_k}\left[\left(\sum_{i=1}^{k} \lambda_i\right) W_{(k)} - \left(\sum_{i=1}^{k-1} \lambda_i\right) W_{(k-1)}\right]$

$W_{(k)}$ is the average waiting time of an M/M/m with arrival rate $\lambda = \sum_{i=1}^{k} \lambda_i$ and mean service time $\frac{1}{\mu}$