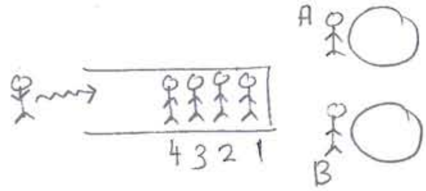
All service times are independent

<u>$m=2$</u>, <u>$N=6$</u>



server 1 $\quad T_A \mid T_1 \mid T_3 \mid$

server 2 $\quad T_B \mid T_2 \mid T_4 \mid$

the time the new customer waited for the customers ahead of him/her (in the queue) to be served is $T_2 + T_4$

<u>for general m</u>

$$Y_k \triangleq \sum_{i=1}^{N_Q} T_i \underbrace{1_{\{i^{\text{th}} \text{ customer goes to server } k\}}}_{\substack{\text{function of } T_1, \dots, T_{i-1} \\ \text{and hence, independent of } T_i}}$$

$$\begin{aligned}
E[Y_k] &= E\Big[E\Big[\sum_{i=1}^{N_Q} T_i \, 1_{\{i^{\text{th}} \text{ customer} \to \text{server } k\}} \Big| N_Q \Big] \Big] && \text{iterated expectation} \\
&= E\Big[\sum_{i=1}^{N_Q} E[T_i \, 1_{\{i^{\text{th}} \text{ customer} \to \text{server } k\}}] \Big] && \text{linearity of expectation} \\
&= E\Big[\sum_{i=1}^{N_Q} E[T_i] \, E[1_{\{i^{\text{th}} \text{ customer} \to \text{server } k\}}] \Big] && \text{independence} \\
&= E\Big[\sum_{i=1}^{N_Q} \frac{1}{\mu} \times \underbrace{\mathbb{P}(i^{\text{th}} \text{ customer} \to \text{server } k)}_{= \frac{1}{m} \text{ (symmetry)}} \Big] \\
&= \frac{1}{m\mu} E[N_Q]
\end{aligned}$$

$$\begin{aligned}
E[Y] &= \sum_{k=1}^{m} E[Y_k] \, \underbrace{\mathbb{P}(\text{new customer} \to \text{server } k)}_{= \frac{1}{m}} \\
&= \frac{1}{m\mu} E[N_Q]
\end{aligned}$$

avg. time the new customer wait for the customers ahead of him/her in the queue to be served