## PROBLEMS

**6.1** Consider a window flow controlled virtual circuit going over a satellite link. All packets have a transmission time of 5 msec. The round-trip processing and propagation delay is 0.5 sec. Find a lower bound on the window size for the virtual circuit to be able to achieve maximum speed transmission when there is no other traffic on the link.

**6.2** Suppose that the virtual circuit in Problem 6.1 goes through a terrestrial link in addition to the satellite link. The transmission time on the terrestrial link is 20 msec, and the processing and propagation delay is negligible. What is the maximum transmission rate in packets/sec that can be attained for this virtual circuit assuming no flow control? Find a lower bound to an end-to-end window size that will allow maximum transmission rate assuming no other traffic on the links. Does it make a difference whether the terrestrial link is before or after the satellite link?

**6.3** Suppose that node-by-node windows are used in the two-link system of Problem 6.2. Find lower bounds on the window size required along each link in order to achieve maximum speed transmission, assuming no other traffic on the links.

**6.4** The three-node network of Fig. 6.26 contains only one virtual circuit from node 1 to 3, and uses node-by-node windows. Each packet transmission on link (1,2) takes 1 sec, and on link (2,3) takes 2 sec; processing and propagation delay is negligible. Permits require 1 sec to travel on each link. There is an inexhaustible supply of packets at node 1. The system starts at time 0 with $W$ permits at node 1, $W$ permits at node 2, and no packets stored at nodes 2 and 3. For $W = 1$, find the times, from 0 to 10 sec at which a packet transmission starts at node 1 and node 2. Repeat for $W = 2$.

**6.5** In the discussion of node-by-node window flow control, it was assumed that node $i$ can send a permit back to its predecessor $(i-1)$ once it releases a packet to the DLC of link $(i, i+1)$. The alternative is for node $i$ to send the permit when it receives the DLC acknowledgment that the packet has been received correctly at node $i+1$. Discuss the relative merits of the two schemes. Which scheme requires more memory? What happens when link $(i, i+1)$ is a satellite link?

**6.6** Consider a combined optimal routing and flow control problem involving the network of Fig. 6.27 (cf. Section 6.4.1). The cost function is

$$(F_{13})^2 + (F_{23})^2 + (F_{34})^2 + \frac{a}{r_1} + \frac{a}{r_2}$$

where $a$ is a positive scalar parameter. Find the optimal values of the rates $r_1$ and $r_2$ for each value of $a$.

**6.7** Consider six nodes arranged in a ring and connected with unit capacity bidirectional links $(i, i+1)$, $i = 1, 2, 3, 4, 5$, and (6,1). There are two sessions from nodes 1, 2, 3, 4, and 5 to node 6, one in the clockwise and one in the counterclockwise direction. Similarly, there are two sessions from nodes 2, 3, and 4 to node 5, and two sessions from node 3 to node 4. Find the max-min fair rates for these sessions.

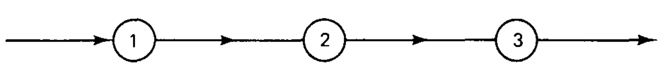


**Figure 6.26**

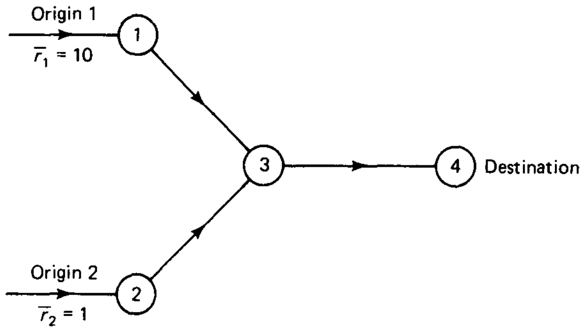


Figure 6.27

**6.8** Suppose that the definition of a fair rate vector of Section 6.4.2 is modified so that, in addition to Eq. (6.10), there is a constraint $r_p \leq b_p$ that the rate of a session $p$ must satisfy. Here $b_p$ is the maximum rate at which session $p$ is capable of transmitting. Modify the algorithm given in Section 6.4.2 to solve this problem. Find the fair rates for the example given in this section when $b_p = 1$, for $p = 2, 4, 5$ and $b_p = 1/4$, for $p = 1, 3$. *Hint:* Add a new link for each session.

**6.9** The purpose of this problem is to illustrate how the relative throughputs of competing sessions can be affected by the priority rule used to serve them. Consider two sessions $A$ and $B$ sharing the first link $L$ of their paths as shown in Fig. 6.28. Each session has an end-to-end window of two packets. Permits for packets of $A$ and $B$ arrive $d_A$ and $d_B$ seconds, respectively, after the end of transmission on link $L$. We assume that $d_A$ is exponentially distributed with mean of unity, while (somewhat unrealistically) we assume that $d_B = 0$. Packets require a transmission time on $L$ which is exponentially distributed with mean of unity. Packet transmission times and permit delays are all independent. We assume that a new packet for $A$ ($B$) enters the transmission queue of $L$ immediately upon receipt of a permit for $A$ ($B$).

**(a)** Suppose that packets are transmitted on $L$ on a first-come first-serve basis. Argue that the queue of $L$ can be represented by a Markov chain with the 10 queue states BB, BBA, BAB, ABB, BBAA, BABA, BAAB, ABBA, ABAB, and AABB (each letter stands for a packet of the corresponding session). Show that all states have equal steady-state probability and that the steady-state throughputs of sessions A and B in packets/sec are 0.4 and 0.6, respectively.

**(b)** Now suppose that transmissions on link $L$ are scheduled on a round-robin basis. Between successive packet transmissions for session B, session A transmits one packet if it has one waiting. Draw the state transition diagram of a five-state Markov chain which

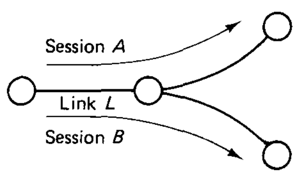


Figure 6.28

models the queue of $L$. Solve for the equilibrium state probabilities. What are the steady-state throughputs of sessions A and B?

**(c)** Finally, suppose that session A has nonpreemptive priority over session B at link $L$. Between successive packet transmissions for session B, session A transmits as many packets as it can. Session B regains control of the link only when A has nothing to send. Draw the state transition diagram of a five-state Markov chain which models $L$ and its queue. Solve for the equilibrium state probabilities. What are the steady-state throughputs of sessions A and B?

**6.10** Consider a window between an external source and the node by which that source is connected to the subnet. The source generates packets according to a Poisson process of rate $\lambda$. Each generated packet is accepted by the node if a permit is available. If no permit is available, the packet is discarded, never to return. When a packet from the given source enters the DLC unit of an outgoing link at the node, a new permit is instantaneously sent back to the source. The source initially has two permits and the window size is 2. Assume that the other traffic at the node imposes a random delay from the time a packet is accepted at the node to the time it enters the DLC unit. Specifically, assume that in any arbitrarily small interval $\delta$, there is a probability $\mu\delta$ that a waiting packet from the source (or the first of two waiting packets) will enter the DLC; this event is independent of everything else.

**(a)** Construct a Markov chain for the number of permits at the source.

**(b)** Find the probability that a packet generated by the source is discarded.

**(c)** Explain whether the probability in part (b) would increase or decrease if the propagation and transmission delay from source to node and the reverse were taken into account.

**(d)** Suppose that a buffer of size $k$ is provided at the source to save packets for which no permit is available; when a permit is received, one of the buffered packets is instantly sent to the network node. Find the new Markov chain describing the system, and find the probability that a generated packet finds the buffer full.

**6.11** Consider a network with end-to-end window flow control applied to each virtual circuit. Assume that the data link control operates perfectly and that packets are never thrown away inside the network; thus, packets always arrive at the destination in the order sent, and all packets eventually arrive.

**(a)** Suppose that the destination sends permits in packets returning to the source; if no return packet is available for some time-out period, a special permit packet is sent back to the source. These permits consist of the number modulo $m$ of the next packet awaited by the destination. What is the restriction on the window size $W$ in terms of the modulus $m$? Why?

**(b)** Suppose next that the permits contain the number modulo $m$ of *each* of the packets in the order received since the last acknowledgment was sent. Does this change your answer to part (a)? Explain.

**(c)** Is it permissible for the source to change the window size $W$ without prior agreement from the destination? Explain.

**(d)** How can the destination reduce the effective window size below the window size used by the source without prior agreement from the source? (By effective window size we mean the maximum number of packets for the source–destination pair that can be in the network at one time.)

**6.12** Consider a node-by-node window scheme. In an effort to reduce the required buffering, the designers associated the windows with destinations rather than with virtual circuits. Assume that all virtual circuit paths to a given destination $j$ use a directed spanning tree so that each node $i \neq j$ has only one outgoing link for traffic to that destination. Assume that each node $i \neq j$ originally has permits for two packets for $j$ that can be sent over the outgoing link to $j$. Each time that node $i$ releases a packet for $j$ into its DLC unit on the outgoing link, it sends a new permit for one packet back over the incoming link over which that packet arrived. If the packet arrived from a source connected to $i$, the permit is sent back to the source (each source also originally has two permits for the given destination).

**(a)** How many packet buffers does node $i$ have to reserve for packets going to destination $j$ to guarantee that every arriving packet for $j$ can be placed in a buffer?

**(b)** What are the pros and cons of this scheme compared with the conventional node-by-node window scheme on a virtual circuit basis?

**6.13** Consider a network using node-by-node windows for each virtual circuit. Describe a strategy for sending and reclaiming permits so that buffer overflow never occurs regardless of how much memory is available for packet storage at each node and of how many virtual circuits are using each link. *Hint*: You need to worry about too many permits becoming available to the transmitting node of each link.

**6.14** Consider a network using a node-by-node window for each session. Suppose that the transmission capacity of all links of the networks is increased by a factor $K$ and that the number of sessions that can be served also increases by a factor $K$. Argue that in order for the network to allow for full-speed transmission for each session under light traffic conditions, the total window of the network should increase by a factor $K$ if propagation delay is dominated by packet transmission time and by a factor $K^2$ if the reverse is true.

**6.15** Consider the variation of the leaky bucket scheme where $W > 1$ permits are allocated initially to a session and the count is restored back to $W$ every $W/r$ seconds. Develop a Markov chain model for the number of packets waiting to get a permit. Assume a Poisson arrival process.

**6.16** Describe how the gradient projection method for optimal routing can be used to solve in distributed fashion the combined optimal routing and flow control problem of Section 6.5.1.

**6.17** Let $r$ be a max-min fair rate vector corresponding to a given network and set of sessions.

**(a)** Suppose that some of the sessions are eliminated and let $\bar{r}$ be a corresponding max-min fair rate vector. Show by example that we may have $\bar{r}_p < r_p$ for some of the remaining sessions $p$.

**(b)** Suppose that some of the link capacities are increased and let $\bar{r}$ be a corresponding max-min fair rate vector. Show by example that we may have $\bar{r}_p < r_p$ for some sessions $p$.

**6.18** *Alternative Formulation of Max-Min Fair Flow Control ([Jaf81] and [GaB84b])*. Consider the max-min flow control problem where the rate vector $r$ is required, in addition, to satisfy $r_p \leq (C_a - F_a)q_a$ for each session $p$ and link $a$ crossed by session $p$. Here $q_a$ are given positive scalars.

**(a)** Show that a max-min fair rate vector exists and is unique, and give an algorithm for calculating it.

**(b)** Show that for a max-min fair rate vector, the utilization factor $\rho_a = F_a/C_a$ of each link $a$ satisfies

$$\rho_a \leq \frac{n_a q_a}{1 + n_a q_a}$$

where $n_a$ is the number of sessions crossing link $a$.

**(c)** Show that additional constraints of the form $r_p \leq R_p$, where $R_p$ is a given positive scalar for each session $p$, can be accommodated in this formulation by adding to the network one extra link per session.

**6.19** *Guaranteed Delay Bounds Using Leaky Buckets [PaG91a],[PaG91b].* In this problem we show how guaranteed delay bounds for the sessions sharing a network can be obtained by appropriately choosing the sessions' leaky bucket parameters. We assume that each session $i$ has a fixed route and is constrained by a leaky bucket with parameters $W_i$ and $r_i$, as in the scheme of Fig. 6.16. We assume a fluid model for the traffic, *i.e.*, the packet sizes are infinitesimal. In particular, if $A_i(\tau, t)$ is the amount of session $i$ traffic entering the network during an interval $[\tau, t]$,

$$A_i(\tau, t) \leq W_i + r_i(t - \tau).$$

Let $T_i^l(\tau, t)$ be the amount of session $i$ traffic transmitted on link $l$ in an interval $[\tau, t]$. Assume that each link $l$ transmits at its maximum capacity $C_l$ whenever it has traffic in queue and operates according to a priority discipline, called *Rate Proportional Processor Sharing*, whereby if two sessions $i$ and $j$ have traffic in queue throughout the interval $[\tau, t]$, then

$$\frac{T_i^l(\tau, t)}{T_j^l(\tau, t)} = \frac{r_i}{r_j} \tag{6.11}$$

We also assume that bits of the same session are transmitted in the order of their arrival. (For practical approximations of such an idealized scheme, see [PaG91a].) Let $S(l)$ be the set of sessions sharing link $l$ and let

$$\rho(l) = \frac{\sum_{i \in S(l)} r_i}{C_l}$$

be the corresponding link utilization. Assume that $\rho(l) < 1$ for all links $l$.

**(a)** Let $l_i$ be the first link crossed by session $i$. Show that the queue length at link $l_i$ for session $i$ is never more than $W_i$. Furthermore, at link $l_i$, each bit of session $i$ waits in queue no more than $W_i \rho(l_i)/r_i$ time units, while for each interval $[\tau, t]$ throughout which session $i$ has traffic in queue,

$$T_i^{l_i}(\tau, t) \geq \frac{r_i(t - \tau)}{\rho(l_i)}$$

*Hint*: The rate of transmission of session $i$ in an interval throughout which session $i$ has traffic in queue is at least $r_i/\rho(l_i)$. If $\bar{\tau}$ and $\bar{t}$ are the start and end of a busy period for session $i$, respectively, the queue length at times $t \in [\bar{\tau}, \bar{t}]$ is

$$Q_i(t) = A_i(\bar{\tau}, t) - T_i^{l_i}(\bar{\tau}, t) \leq W_i + \left(r_i - \frac{r_i}{\rho(l_i)}\right)(t - \bar{\tau})$$

**(b)** Let $\rho_{max}^i$ be the maximum utilization over the links crossed by session $i$ and assume that processing and propagation delay are negligible. Show that the amount of traffic of session $i$ within the entire network never exceeds $W_i$. Furthermore, the time spent inside the network by a bit of session $i$ is at most $W_i \rho_{max}^i / r_i$. *Hint*: Argue that while some link on session $i$'s path has some session $i$ traffic waiting in queue, the rate of departure of session $i$ traffic from the network is at least $r_i/\rho_{max}^i$.

**(c)** Consider now a generalization of the preceding scheme called Generalized Processor Sharing. In particular, suppose that at each link $l$, instead of Eq. (6.11), the following relation holds for all sessions $i, j \in S(l)$

$$\frac{T_i^l(\tau, t)}{T_j^l(\tau, t)} = \frac{\phi_i^l}{\phi_j^l}$$

where $\phi_i^l, \varphi_j^l$ are given positive numbers. For any session $i$ define

$$g_i = \min_{\substack{\text{all links } l\\ \text{crossed by } i}} \frac{1}{\rho(l)} \frac{\varphi_i^l}{\sum_{j \in S(l)} \varphi_j^l}$$

Show that if $g_i \geq r_i$, then the amount of traffic of session $i$ within the entire network never exceeds

$$\frac{W_i r_i}{g_i}$$

Furthermore, the time spent by a bit of session $i$ inside the network is at most $W_i/g_i$.