ences in the satellite area are [JBH78], [CRW73], [Bin75], and [WiE80]. For local area networks, [MeB76] is the source work on Ethernet, and [FaN69] and [FaL72] are source works on ring nets. [CPR78] and [KuR82] are good overview articles and [Ros86] provides a readable introduction to FDDI. [FiT84] does an excellent job of comparing and contrasting the many approaches to implicit and explicit tokens and polling on buses. Finally, DQDB is treated in [NBH88] and [HCM90].

Section 4.6. [KGB78] provides a good overview of packet radio. Busy tones are described in [ToK75] and [SiS81]. Transmission radii are discussed in [TaK85].

PROBLEMS

4.1 **(a)** Verify that the steady-state probabilities p_n for the Markov chain in Fig. 4.3 are given by the solution to the equations

$$p_n = \sum_{i=0}^{n+1} p_i P_{in}$$

$$\sum_{n=0}^{m} p_n = 1$$

(b) For $n < m$, use part (a) to express p_{n+1} in terms of $p_0, p_1, \ldots, p_n$.

(c) Express p_1 in terms of p_0 and then p_2 in terms of p_0.

(d) For $m = 2$, solve for p_0 in terms of the transition probabilities.

4.2 **(a)** Show that P_{succ} in Eq. (4.5) can be expressed as

$$P_{succ} = \left[\frac{(m-n)q_a}{1-q_a} + \frac{nq_r}{1-q_r}\right](1-q_a)^{m-n}(1-q_r)^n$$

(b) Use the approximation $(1-x)^y \approx e^{-xy}$ for small x to show that for small q_a and q_r,

$$P_{succ} \approx G(n)e^{-G(n)}$$

where $G(n) = (m-n)q_a + nq_r$.

(c) Note that $(1-x)^y = e^{y \ln(1-x)}$. Expand $\ln(1-x)$ in a power series and show that

$$\frac{(1-x)^y}{e^{-xy}} = \exp\left(-\frac{x^2y}{2} - \frac{x^3y}{3}\cdots\right)$$

Show that this ratio is close to 1 if $x \ll 1$ and $x^2y \ll 1$.

4.3 **(a)** Redraw Fig. 4.4 for the case in which $q_r = 1/m$ and $q_a = 1/me$.

(b) Find the departure rate (*i.e.*, P_{succ}) in the fully backlogged case $n = m$.

(c) Note that there is no unstable equilibrium or undesired stable point in this case and show (graphically) that this holds true for any value of q_a.

(d) Solve numerically (using $q_a = 1/me$) for the value of G at which the stable point occurs.

(e) Find n/m at the stable point. Note that this is the fraction of the arriving packets that are not accepted by the system at this typical point.

4.4 Consider the idealized slotted multiaccess model of Section 4.2.1 with the no-buffering assumption. Let n_k be the number of backlogged nodes at the beginning of the k^{th} slot and let $\overline{n}$ be the expected value of n_k over all k. Note that $\overline{n}$ will depend on the particular way in which collisions are resolved, but we regard $\overline{n}$ as given here (see [HIG81]).

(a) Find the expected number of *accepted* arrivals per slot, $\overline{N}_a$, as a function of $\overline{n}, m$, and q_a, where m is the number of nodes and q_a is the arrival rate per node.

(b) Find the expected departure rate per slot, $\overline{P}_{succ}$, as a function of $\overline{n}, m$, and q_a. *Hint:* How is $\overline{N}_a$ related to $\overline{P}_{succ}$? Recall that both are averages over time.

(c) Find the expected number of packets in the system, $\overline{N}_{sys}$, immediately after the beginning of a slot (the number in the system is the backlog plus the accepted new arrivals).

(d) Find the expected delay T of an accepted packet from its arrival at the beginning of a slot until the completion of its successful transmission at the end of a slot. *Hint:* Use Little's theorem; it may be useful to redraw the diagram used to prove Little's theorem.

(e) Suppose that the strategy for resolving collisions is now modified and the expected backlog $\overline{n}$ is reduced to $\overline{n}' < \overline{n}$. Show that $\overline{N}_a$ increases, $\overline{P}_{succ}$ increases, $\overline{N}_{sys}$ decreases, and T decreases. Note that this means that improving the system with respect to one of these parameters improves it with respect to all.

4.5 Assume for simplicity that each transmitted packet in a slotted Aloha system is successful with some fixed probability p. New packets are assumed to arrive at the beginning of a slot and are transmitted immediately. If a packet is unsuccessful, it is retransmitted with probability q_r in each successive slot until successfully received.

(a) Find the expected delay T from the arrival of a packet until the completion of its successful transmission. *Hint:* Given that a packet has not been transmitted successfully before, what is the probability that it is both transmitted and successful in the i^{th} slot $(i > 1)$ after arrival?

(b) Suppose that the number of nodes m is large, and that q_a and q_r are small. Show that in state n, the probability p that a given packet transmission is successful is approximately $p = e^{-G(n)}$, where $G(n) = (m - n)q_a + nq_r$.

(c) Now consider the stable equilibrium state n^* of the system where $G = G(n^*)$; $Ge^{-G} = (m - n^*)q_a$. Substitute (b) into your expression for T for (a), using $n = n^*$, and show that

$$T = 1 + \frac{n^*}{q_a(m - n^*)}$$

(Note that if n^* is assumed to be equal to $\overline{n}$ in Problem 4.4, this is the same as the value of T found there.)

(d) Solve numerically for T in the case where $q_a m = 0.3$ and $q_r m = 1$; show that $n^* \approx m/8$, corresponding to 1/8 loss of incoming traffic, and $T \approx m/2$, giving roughly the same delay as TDM.

4.6 **(a)** Consider P_{succ} as given exactly in Eq. (4.5). For given $q_a < 1/m$, $n > 1$, show that the value of q_r that maximizes P_{succ} satisfies

$$\frac{1}{1 - q_r} - \frac{q_a(m - n)}{1 - q_a} - \frac{q_r n}{1 - q_r} = 0$$

(b) Consider the value of q_r that satisfies the equation above as a function of q_a, say $q_r(q_a)$. Show that $q_r(q_a) > q_a$ (assume that $q_a < 1/m$).

(c) Take the total derivative of P_{succ} with respect to q_a, using $q_r(q_a)$ for q_r, and show that this derivative is negative. *Hint*: Recall that $\partial P_{succ}/\partial q_r$ is 0 at $q_r(q_a)$ and compare $\partial P_{succ}/\partial q_a$ with $\partial P_{succ}/\partial q_r$.

(d) Show that if q_r is chosen to maximize P_{succ} and $q_r < 1$, then P_{succ} is greater if new arrivals are treated immediately as backlogged than if new arrivals are immediately transmitted. *Hint:* In the backlog case, a previously unbacklogged node transmits with probability $q_a q_r < q_a$.

4.7 Consider a slotted Aloha system with "perfect capture." That is, if more than one packet is transmitted in a slot, the receiver "locks onto" one of the transmissions and receives it correctly; feedback immediately informs each transmitting node about which node was successful and the unsuccessful packets are retransmitted later.

(a) Give a convincing argument why expected delay is minimized if all waiting packets attempt transmission in each slot.

(b) Find the expected system delay assuming Poisson arrivals with overall rate λ. *Hint:* Review Example 3.16.

(c) Now assume that the feedback is delayed and that if a packet is unsuccessful in the slot, it is retransmitted on the k^{th} subsequent slot rather than the first subsequent slot. Find the new expected delay as a function of k. *Hint:* Consider the system as k subsystems, the i^{th} subsystem handling arrivals in slots j such that $j \bmod k = i$.

4.8 Consider a slotted system in which all nodes have infinitely large buffers and all new arrivals (at Poisson rate λ/m per node) are allowed into the system, but are considered as backlogged immediately rather than transmitted in the next slot. While a node contains one or more packets, it independently transmits one packet in each slot, with probability q_r. Assume that any given transmission is successful with probability p.

(a) Show that the expected time from the beginning of a backlogged slot until the completion of the first success at a given node is $1/pq_r$. Show that the second moment of this time is $(2 - pq_r)/(pq_r)^2$.

(b) Note that the assumption of a constant success probability allows each node to be considered independently. Assume that λ/m is the Poisson arrival rate at a node, and use the service-time results of part (a) to show that the expected delay is

$$T = \frac{1}{q_r p(1-\rho)} + \frac{1-2\rho}{2(1-\rho)}$$

$$\rho = \frac{\lambda}{mpq_r}$$

(c) Assume that $p = 1$ (this yields a smaller T than any other value of p, and corresponds to very light loading). Find T for $q_r = 1/m$; observe that this is roughly twice the delay for TDM if m is large.

4.9 Assume that the number of packets n in a slotted Aloha system at a given time is a Poisson random variable with mean $\hat{n} \geq 1$. Suppose that each packet is independently transmitted in the next slot with probability $1/\hat{n}$.

(a) Find the probability that the slot is idle.

(b) Show that the a posteriori probability that there were n packets in the system, given an idle slot, is Poisson with mean $\hat{n} - 1$.

(c) Find the probability that the slot is successful.

(d) Show that the a posteriori probability that there were $n + 1$ packets in the system, given a success, is $e^{-(\hat{n}-1)}(\hat{n}-1)^n/n!$ (*i.e.*, the number of remaining packets is Poisson with mean $\hat{n} - 1$).

4.10 Consider a slotted Aloha system satisfying assumptions 1 to 6a of Section 4.2.1 *except* that each of the nodes has a limitless buffer to store all arriving packets until transmission *and* that nodes receive immediate feedback only about whether or not their own packets were successfully transmitted. Each node is in one of two different modes. In mode 1, a node transmits (with probability 1) in each slot, repeating unsuccessful packets, until its buffer of waiting packets is exhausted; at that point the node goes into mode 2. In mode 2, a node transmits a "dummy packet" in each slot with probability q_r until a successful transmission occurs, at which point it enters mode 1 (dummy packets are used only to simplify the mathematics). Assume that the system starts with all nodes in mode 2. Each node has Poisson arrivals of rate λ/m.

(a) Explain why at most one node at a time can be in mode 1.

(b) Given that a node is in mode 1, find its probability, p_1, of successful transmission. Find the mean time $\overline{X}$ between successful transmissions and the second moment $\overline{X^2}$ of this time. *Hint:* Review the ARQ example in Section 3.5.1, with $N = 1$.

(c) Given that all nodes are in mode 2, find the probability p_2 that some dummy packet is successfully transmitted in a given slot. Find the mean time $\overline{v}$ until the completion of such a successful transmission and its second moment $\overline{v^2}$.

(d) Regard the intervals of time when all nodes are in mode 2 as reservation intervals. Show that the mean time a packet must wait in queue before first attempting transmission is

$$W = \frac{R + E\{S\}\overline{v}}{1-\rho}, \qquad \rho = \lambda\overline{X}$$

where R is the mean residual time until completion of a service in mode 1 or completion of a reservation interval, and S is the number of whole reservation intervals until the node at which the packet arrived is in mode 1.

(e) Show that

$$W = \frac{\lambda(2-p_1)}{2p_1^2(1-\rho)} + \frac{2-p_2}{2p_2} + \frac{m-1}{p_2(1-\rho)}$$

Show that W is finite if $q_r < (1-\lambda)^{1/(m-1)}$.

4.11 Consider the somewhat unrealistic feedback assumption for unslotted Aloha in which all nodes are informed, precisely τ time units after the beginning of each transmission whether or not that transmission was successful. Thus, in the event of a collision, each node knows how many packets were involved in the collision, and each node involved in the collision knows how many other nodes started transmission before itself. Assume that each transmission lasts one time unit and assume that $m = \infty$. Consider a retransmission strategy in which the first node involved in a collision waits one time unit after receiving feedback on its collision and then transmits its packet. Successive nodes in the collision retransmit in order spaced one time unit apart. All new arrivals to the system while these retransmissions are taking place wait until the retransmissions are finished. At the completion of the retransmissions, each backlogged node chooses a time to start its transmission uniformly distributed over the next time unit. All new arrivals after the end of the retransmissions above start transmission immediately.

(a) Approximate the system above as a reservation system with reservation intervals of duration $1 + \tau$ (note that this is an approximation in the sense that successful transmissions will sometimes occur in the reservation intervals, but the approximation becomes more accurate as the loading becomes higher). Find the expected packet delay for this approximation (assume Poisson arrivals at rate λ).

(b) Show that the delay above remains finite for all $\lambda < 1$.

4.12 This problem illustrates that the maximum throughput of unslotted Aloha can be increased up to e^{-1} at an enormous cost in delay. Consider a finite but large set m of nodes with unlimited buffering at each node. Each node waits until it has accumulated k packets and then transmits them one after the other in the next k time units. Those packets involved in collisions, plus new packets, are then retransmitted a random time later, again k at a time. Assume that the starting time of transmissions from all nodes collectively is a Poisson process with parameter G (*i.e.*, ignore stability issues).

(a) Show that the probability of success on the j^{th} of the k packets in a sequence is $e^{-G(k+1)}$. *Hint:* Consider the intervals between the initiation of the given sequence and the preceding sequence and subsequent sequence.

(b) Show that the throughput is $kGe^{-G(k+1)}$, and find the maximum throughput by optimizing over G.

4.13 **(a)** Consider a CRP that results in the feedback pattern $e, 0, e, e, 1, 1, 0$ when using the tree algorithm as illustrated in Fig. 4.9. Redraw this figure for this feedback pattern.

(b) Which collision or collisions would have been avoided if the first improvement to the tree algorithm had been used?

(c) What would the feedback pattern have been for the CRP if both improvements to the tree algorithms had been used?

4.14 Consider the tree algorithm in Fig. 4.9. Given that k collisions occur in a CRP, determine the number of slots required for the CRP. Check your answer with the particular example of Fig. 4.9. *Hint 1:* Note that each collision corresponds to a nonleaf node of the rooted tree. Consider "building" any given tree from the root up, successively replacing leaf nodes by internal nodes with two upward-going edges. *Hint 2:* For another approach, consider what happens in the stack for each collision, idle, or success.

4.15 Consider the tree algorithm in Fig. 4.9. Assume that after each collision, each packet involved in the collision flips an unbiased coin to determine whether to go into the left or right subset.

(a) Given a collision of k packets, find the probability that i packets go into the left subset.

(b) Let A_k be the expected number of slots required in a CRP involving k packets. Note that $A_0 = A_1 = 1$. Show that for $k \geq 2$,

$$A_k = 1 + \sum_{i=0}^{k} \binom{k}{i} 2^{-k}(A_i + A_{k-i})$$

(c) Simplify your answer in part (b) to the form

$$A_k = c_{kk} + \sum_{i=0}^{k-1} c_{ik} A_i$$

and find the coefficients c_{ik}. Evaluate A_2 and A_3 numerically. For more results on A_k for large k, and the use of A_k in evaluating maximum throughput, see [Mas80].

4.16 **(a)** Consider the first improvement to the tree algorithm as shown in Fig. 4.10. Assume that each packet involved in a collision flips an unbiased coin to join either the left or right subset. Let B_k be the expected number of slots required in a CRP involving k packets; note that $B_0 = B_1 = 1$. Show that for $k \geq 2$,

$$B_k = 1 + \sum_{i=1}^{k} \binom{k}{i} 2^{-k}(B_i + B_{k-i}) + 2^{-k} B_k$$

(b) Simplify your answer to the form

$$B_k = C'_{kk} + \sum_{i=1}^{k-1} C'_{ik} B_i$$

and evaluate the constants C'_{ik}. Evaluate B_2 and B_3 numerically (see [Mas80]).

4.17 Let X_L and X_R be independent Poisson distributed random variables, each with mean G. Use the definition of conditional probabilities to evaluate the following:

(a) $P\{X_L = 0 \mid X_L + X_R \geq 2\}$
(b) $P\{X_L = 1 \mid X_L + X_R \geq 2\}$
(c) $P\{X_L \geq 2 \mid X_L + X_R \geq 2\}$
(d) $P\{X_R = 1 \mid X_L = 1, X_L + X_R \geq 2\}$
(e) $P\{X_R = i \mid X_L = 0, X_L + X_R \geq 2\} \quad (i \geq 2)$
(f) $P\{X_R = i \mid X_L \geq 2, X_L + X_R \geq 2\}$

4.18 Suppose that at time k, the FCFS splitting algorithm allocates a new interval from $T(k)$ to $T(k) + \alpha_0$. Suppose that this interval contains a set of packets with arrival times $T(k) + 0.1\alpha_0, T(k) + 0.6\alpha_0, T(k) + 0.7\alpha_0$, and $T(k) + 0.8\alpha_0$.

(a) Find the allocation intervals for each of the subsequent times until the CRP is completed.
(b) Indicate which of the rules of Eqs. (4.15) to (4.18) are used in determining each of these allocation intervals.
(c) Indicate the path through the Markov chain in Fig. 4.13 for this sequence of events.

4.19 **(a)** Show that $\overline{n}$, the expected number of packets successfully transmitted in a CRP of the FCFS splitting algorithm, is given by

$$\overline{n} = 1 - e^{-G_0} + \sum_{i=1}^{\infty} p(R, i)$$

(Assume that the initial allocation interval is α_0, with $G_0 = \alpha_0 \lambda$.)

(b) Show that

$$\overline{n} = \lambda \alpha_0 (1 - E\{f\})$$

where $E\{f\}$ is the expected fraction of α_0 returned to the waiting interval in a CRP. (This provides an alternative way to calculate $E\{f\}$.)

4.20 Show, for Eq. (4.41), that if n_k is a Poisson random variable with mean $\hat{n}_k$, then the a posteriori distribution of n_k, given an idle, is Poisson with mean $\hat{n}_k[1 - q_r(\hat{n}_k)]$. Show that the a posteriori distribution of $n_k - 1$, given a success, is Poisson with mean $\hat{n}_k[1 - q_r(\hat{n}_k)]$.

4.21 *Slotted CSMA with Variable Packet Lengths.* Assume that the time to transmit a packet is a random variable X; for consistency with the slotted assumption, assume that X is discrete, taking values that are integer multiples of β. Assume that all transmissions are independent and identically distributed (IID) with mean $\overline{X} = 1$.

(a) Let Y be the longer of two IID transmissions X_1 and X_2 [*i.e.*, $Y = \max(X_1, X_2)$]. Show that the expected value of Y satisfies $\overline{Y} \leq 2\overline{X}$. Show that if X takes the value of β with large probability and $k\beta$ (for a large k) with small probability, this bound is close to equality.

(b) Show that the expected time between state transitions, given a collision of two packets, is at most $2 + \beta$.

(c) Let $g(n) = \lambda\beta + q_r n$ be the expected number of attempted transmissions following a state transition to state n, and assume that this number of attempts is Poisson with mean $g(n)$. Show that the expected time between state transitions in state n is at most

$$\beta e^{-g(n)} + (1+\beta)g(n)e^{-g(n)} + (1+\beta/2)g^2(n)e^{-g(n)}$$

Ignore collisions of more than two packets as negligible.

(d) Find a lower bound to the expected number of departures per unit time in state n [see Eq. (4.39)].

(e) Show that this lower bound is maximized (for small β) by

$$g(n) \approx \sqrt{\beta}$$

with a corresponding throughput of approximately $1 - 2\sqrt{\beta}$.

4.22 *Pseudo-Bayesian Stabilization for Unslotted CSMA*. Assume that at the end of a transmission, the number n of backlogged packets in the system is a Poisson random variable with mean $\hat{n}$. Assume that in the idle period until the next transmission starts, each backlogged packet attempts transmission at rate x and each new arrival starts transmission immediately. Thus, given n, the time τ until the next transmission starts has probability density $p(\tau \mid n) = (\lambda + xn)e^{-(\lambda+xn)\tau}$.

(a) Find the unconditional probability density $p(\tau)$.

(b) Find the a posteriori probability $P\{n, b \mid \tau\}$ that there were n backlogged packets and one of them started transmission first, given that the transmission starts at τ.

(c) Find the a posteriori probability $P\{n, a \mid \tau\}$ that there were n backlogged packets and a new arrival started transmission first, given that this transmission starts at τ.

(d) Let n' be the number of backlogged packets immediately after the next transmission starts (not counting the packet being transmitted); that is, $n' = n - 1$ if a backlogged packet starts and $n' = n$ if a new arrival starts. Show that, given τ, n' is Poisson with mean $\hat{n}' = \hat{n}e^{-\tau x}$.

This means that the pseudo-Bayesian rule for updating estimated backlog (assuming unit time transmission) is to estimate the backlog at the end of the $(k+1)^{\text{st}}$ transmission in terms of the estimate at the end of the k^{th} transmission and the idle period τ_k between the transmissions by

$$\hat{n}_{k+1} = \begin{cases} \hat{n}_k e^{-\tau_k x_k} + \lambda(1+\beta); & \text{success} \\ \hat{n}_k e^{-\tau_k x_k} + 2 + \lambda(1+\beta); & \text{collision} \end{cases}$$

$$x_k = \beta^{-\frac{1}{2}} \min\left(\frac{1}{\hat{n}_k}, 1\right)$$

4.23 Give an intuitive explanation of why the maximum throughput, for small β, is approximately the same for CSMA slotted Aloha and FCFS splitting with CSMA. Show that the optimal expected number of packets transmitted after a state transition in Aloha is the same as that at the beginning of a CRP for FCFS splitting. Note that after a collision, the expected numbers are slightly different in the two systems, but the difference is unimportant since collisions are rare.

4.24 *Delay of Ideal Slotted CSMA/CD.* Assume that for all positive backlogs, the number of packets attempting transmission in an interval is Poisson with mean g.

(a) Start with Eq. (4.42), which is valid for CSMA/CD as well as CSMA. Show that for CSMA/CD,

$$E\{t\} = \frac{\beta e^{-g} + (1+\beta)ge^{-g} + 2\beta[1-(1+g)e^{-g}]}{ge^{-g}}$$

Show that $E\{t\}$ is minimized over $g > 0$ by $g = 0.77$, and

$$\min_g E\{t\} = 1 + 3.31\beta$$

(b) Show that for this g and mean packet length 1,

$$W = \frac{\overline{R} + \overline{y}}{1 - \lambda(1 + 3.31\beta)}$$

(c) Evaluate $\overline{R}$ and $\overline{y}$ to verify Eq. (4.67) for small β.

(d) Discuss the assumption of a Poisson-distributed number of packets attempting transmission in each interval, particularly for a backlog of 1.

4.25 Show that for unslotted CSMA/CD, the maximum interval of time over which a transmitting node can hear a collision is 2β. (Note in Fig. 4.20 that the time when a collision event starts at one node until it ends at another node can be as large as 3β.)

4.26 Consider an unslotted CSMA/CD system in which the propagation delay is negligible compared to the time β required for a node to detect that the channel is idle or busy. Assume that each packet requires one time unit for transmission. Assume that β time units after either a successful transmission or a collision ends, all backlogged nodes attempt transmission after a random delay and that the composite process of initiation times is Poisson of rate G (up to time β after the first initiation). For simplicity, assume that each collision lasts for β time units.

(a) Find the probability that the first transmission initiation after a given idle detection is successful.

(b) Find the expected time from one idle detection to the next.

(c) Find the throughput (for the given assumptions).

(d) Optimize the throughput numerically over G.

4.27 Modify Eq. (4.71) for the case in which a node, after transmitting a packet, waits for the packet to return before transmitting the free token. *Hint:* View the added delay as an increase in the packet transmission time.

4..28 Show by example that a node in a register insertion ring might have to wait an arbitrarily long time to transmit a packet once its transit buffer is full.

4.29 Suppose that two nodes are randomly placed on a bus; that is, each is placed independently, and the position of each is chosen from a uniform distribution over the length of the bus. Assuming that the length of the bus is 1 unit, show that the expected distance between the nodes is $1/3$.

4.30 Assume, for the analysis of generalized polling in Section 4.5.6, that each node has a packet to send with probability q.

(a) Find the probability $P\{i\}$ that node i, $i \geq 0$, is the lowest-numbered node with a packet to send.

(b) Assume that the CRP tests only the first 2^j lowest-numbered nodes at a time for some j. Represent the lowest-numbered node with a packet as $i = k2^j + r$, where k is an integer, and $0 \leq r < 2^j$. Show that, given i, the number of reservation slots needed to find i is $k + 1 + j$.

(c) Assume that the total number of nodes is infinite and approximate k above by $i2^{-j}$. Find the expected number of reservation slots to find the lowest-numbered node i containing a packet.

(d) Find the integer value of j that minimizes your answer in part (c). *Hint:* Find the smallest value of j for which the expected number of reservation slots is less than the expected number for $j + 1$.

4.31 Consider the simple packet radio network shown in Fig. 4.37 and let f_ℓ be the throughput on link ℓ ($\ell = 1, 2, 3$); the links are used only in the direction shown.

(a) Note that at most one link can be in any collision-free set. Using generalized TDM [as in Eq. (4.86)], show that any set of throughputs satisfying $f_1 + f_2 + f_3 \leq 1, f_\ell \geq 0$ ($\ell = 1, 2, 3$) can be achieved.

(b) Suppose that $f_1 = f_2 = f$ and $f_3 = 2f$ for some f. Use Eqs. (4.88) and (4.89) to relate f_1, f_2, f_3 to the attempt rates q_1, q_2, q_3 for slotted Aloha. Show that $q_1 = q_2$ and $q_3 = 2f$.

(c) Find the maximum achievable value of f for part (b).

4.32 Consider the packet radio network shown in Fig. 4.38 in which the links are used only in the direction shown. Links 5 and 6 carry no traffic but serve to cause collisions for link 7 when packets are transmitted on links 3 and 4, respectively.

(a) Show that throughputs $f_1 = f_2 = f_3 = f_4 = 1/3, f_7 = 4/9$ are feasible for the assumption of heavy loading and independent attempts on each link. Find the corresponding attempt rates and success rates from Eqs. (4.88) and (4.89).

(b) Now assume that links 3 and 4 are used to forward incoming traffic and always transmit in the slot following a successful incoming packet. Show that with $f_1 = f_2 = f_3 = f_4 = 1/3$, f_7 is restricted to be no more than 1/3.

(c) Assume finally that packet attempts on links 1 and 2 are in alternating order and links 3 and 4 operate as in part (b). Show that $f_1 = f_2 = f_3 = f_4 = 1/2$ is feasible, but that f_7 must then be 0.

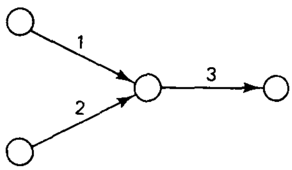

Figure 4.37

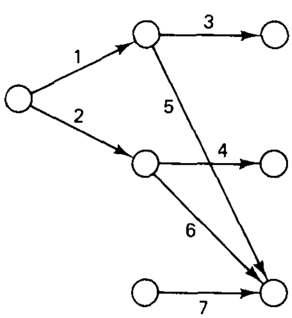

Figure 4.38

4.33 Suppose that an FDDI ring has two users. The target token rotation time is 3 msec and $\alpha_1 = \alpha_2 = 1$ msec. Assume that neither node has any traffic to send up to time 0, and then both nodes have an arbitrarily large supply of both high- and low-priority traffic. Node 0 is the first node to send traffic on the ring starting at time 0. Find the sequence of times t_i at which each node captures the token; ignore propagation delays. Explain any discrepancy between your solution and the upper bound of Eq. (4.81).

4.34 **(a)** Consider an FDDI ring with m nodes, with target token rotation time τ and high-priority allocations $\alpha_0, \ldots, \alpha_{m-1}$. Assume that every node has an arbitrarily large backlog of both high-priority and low-priority traffic. In the limit of long-term operation, find the fraction of the transmitted traffic that goes to each node for each priority. Ignore propagation and processing delay and use the fact that the bound in Eq. (4.81) is met with equality given the assumed initial conditions and given arbitarily large backlogs. Assume that $T = \alpha_0 + \alpha_1 + \cdots + \alpha_{m-1}$ is strictly less than τ.

(b) Now assume that each node k can fill only a fraction α_k/τ of the ring with high-priority traffic and find the fractions in part (a) again.

4.35 Give the rules that the two counters implementing the virtual queue in DQDB must follow. Assume there is also a binary variable F that is one when a frame is in the virtual queue. Assume that when $F = 0$, the first counter is kept at value 0.