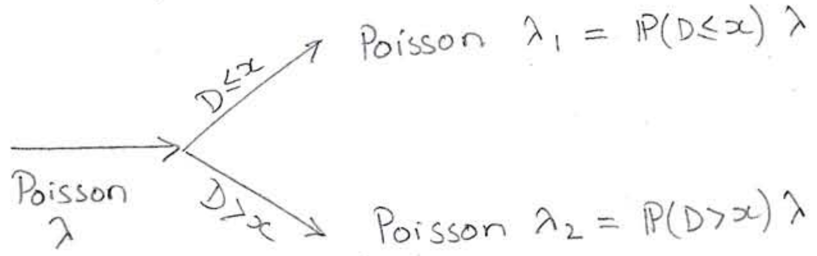
$\boxed{3.29}$

* Arrival of customers is a Poisson process with rate $\lambda = 1/\text{min}$
* Each customer carries D items where $D \sim \text{Uniform}[1:40]$
* Processing rate at the counters 15 items/min

* If a customer has x items or less, the customer goes to counter 1 otherwise, the customer goes to counter 2



Recall that the splitting of a Poisson process results in Poisson processes ("routing" decisions have to be independent, which is the case at hand)

$$\mathbb{P}(D=k \mid D \leq x) = \begin{cases} \dfrac{1/40}{x/40} & k \leq x \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{P}(D=k \mid D > x) = \begin{cases} \dfrac{1/40}{(40-x)/40} & k > x \\ 0 & \text{otherwise} \end{cases}$$

* Service time $T = \dfrac{D}{15}$ (minutes)

<u>At counter 1</u> : T is uniform on $\left\{\frac{1}{15}, \frac{2}{15}, \dots, \frac{x}{15}\right\}$

conditioned on $\{D \leq x\}$

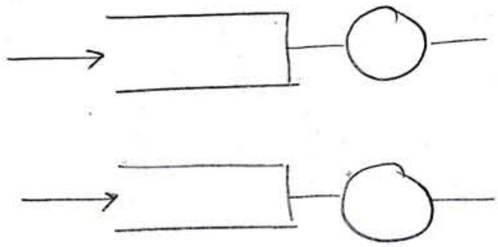
$$\begin{cases} \mathbb{P}(T = k/15) = \dfrac{1}{x}, & k = 1, \dots, x \\ E[T] = \displaystyle\sum_{k=1}^{x} \frac{1}{x}\left(\frac{k}{15}\right) \\ E[T^2] = \displaystyle\sum_{k=1}^{x} \frac{1}{x}\left(\frac{k}{15}\right)^2 \end{cases}$$

<u>At counter 2</u> :

conditioned on $\{D > x\}$

$$\begin{cases} \mathbb{P}\left(T = \dfrac{k}{15}\right) = \dfrac{1}{40-x} & k = x+1, \dots, 40 \\ E[T] = \displaystyle\sum_{k=x+1}^{40} \frac{1}{40-x}\left(\frac{k}{15}\right) \\ E[T^2] = \displaystyle\sum_{k=x+1}^{40} \frac{1}{40-x}\left(\frac{k}{15}\right)^2 \end{cases}$$

Therefore, we have two queues



Arrival:
Poisson λ_1

Service time
uniform on $\{\frac{1}{15}, \cdots, \frac{x}{15}\}$

Arrival
Poisson λ_2

service time
Uniform on $\{\frac{x+1}{15}, \cdots, \frac{40}{15}\}$

i.e. we have two M/G/1

Recall that for M/G/1, the mean waiting time $W = \frac{R}{1-\rho}$
where $R = \frac{\lambda \overline{X^2}}{2}$ and $\rho = \lambda \overline{X}$ #

Let W_1 and W_2 be the mean waiting times of customers at counters 1 and 2 respectively.

The mean waiting time W (averaged over all users) is :

$$W = \mathbb{P}(D \leq x)\, W_1 + \mathbb{P}(D > x)\, W_2$$

$$W_1 = \frac{\lambda_1 \overline{T_1^2}}{2(1-\lambda_1 \overline{T_1})} \qquad \overline{T_1} = \sum_{k=1}^{x} \frac{1}{x}\left(\frac{k}{15}\right), \quad \overline{T_1^2} = \sum_{k=1}^{x} \frac{1}{x}\left(\frac{k}{15}\right)^2$$

$$W_2 = \frac{\lambda_2 \overline{T_2^2}}{2(1-\lambda_2 \overline{T_2})} \qquad \overline{T_2} = \sum_{k=x+1}^{40} \frac{1}{40-x}\left(\frac{k}{15}\right), \quad \overline{T_2^2} = \sum_{k=x+1}^{40} \frac{1}{40-x}\left(\frac{k}{15}\right)^2$$