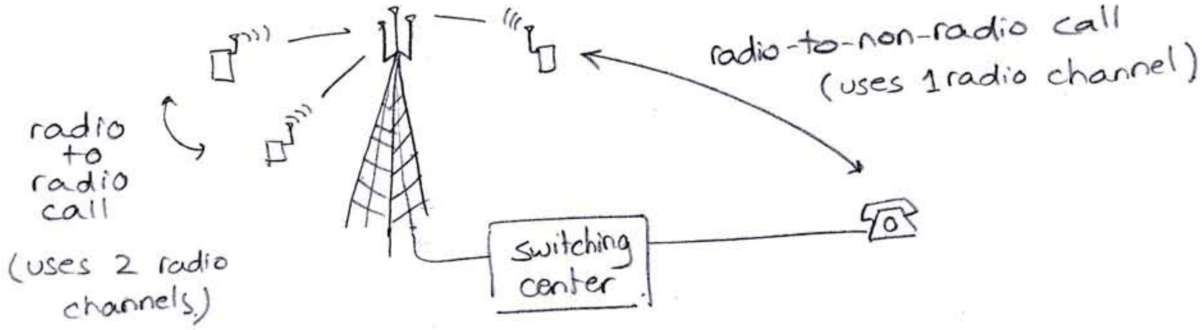
3.25

- Let n_1 be the number of radio-to-radio calls in the system
  n_2 be the number of radio-to-non-radio calls in the system

- The arrivals of radio-to-radio calls and radio-to-non-radio calls are independent
  - Arrivals of radio-to-radio calls $\rightarrow$ Poisson with rate λ_1
  - Arrivals of radio-to-non-radio calls $\rightarrow$ Poisson with rate λ_2
  - call durations $\rightarrow$ exponential with mean $= 1/\mu$



- Consider the Markov chain with the state defined as (n_1, n_2)

$$(n_1, n_2) \in S \triangleq \{(n_1,n_2): n_1 \geq 0, n_2 \geq 0, \underbrace{2n_1+n_2 \leq m}_{\text{only } m \text{ radio channels in the system}}, n_1 \in \mathbb{Z}, n_2 \in \mathbb{Z}\}$$

see section 3.4.4 (Multidimensional M.C.)

$$p(n_1,n_2) = \frac{(1-\rho_1)\rho_1^{n_1} \cdot (1-\rho_2)\rho_2^{n_2}}{G}, \quad (n_1,n_2) \in S$$

where $G = \sum_{(n_1,n_2) \in S} p(n_1,n_2)$, $\rho_1 = \frac{\lambda_1}{\mu}$ and $\rho_2 = \frac{\lambda_2}{\mu}$

$$\begin{aligned} P_{B1} &= P(\text{radio-to-radio call blocked}) \\ &= P(\text{no radio ch. available}) + P(\text{only 1 radio ch. available}) \\ &= \sum_{(n_1,n_2):\, 2n_1+n_2=m} p(n_1,n_2) + \sum_{(n_1,n_2):\, 2n_1+n_2=m-1} p(n_1,n_2) \end{aligned}$$

$$\begin{aligned} P_{B2} &= P(\text{radio-to-non-radio call blocked}) \\ &= P(\text{no radio ch. available}) \\ &= \sum_{(n_1,n_2):\, 2n_1+n_2=m} p(n_1,n_2) \end{aligned}$$

Assume m is even. $\frac{m}{2}$ is an integer

$$* \quad G = \sum_{n_1=0}^{\frac{m}{2}} \sum_{n_2=0}^{m-2n_1} (1-\rho_1)\rho_1^{n_1}(1-\rho_2)\rho_2^{n_2}$$

$$= \sum_{n_1=0}^{m/2} (1-\rho_1)\rho_1^{n_1}(1-\rho_2)\left(\sum_{n_2=0}^{m-2n_1} \rho_2^{n_2}\right)$$

$$= \sum_{n_1=0}^{m/2} (1-\rho_1)\rho_1^{n_1}(1-\rho_2)\left(\frac{1-\rho_2^{m-2n_1+1}}{1-\rho_2}\right)$$

$$= (1-\rho_1)\left[\left(\sum_{n_1=0}^{m/2} \rho_1^{n_1}\right) - \rho_2^{m+1}\left(\sum_{n_1=0}^{m/2}\left(\frac{\rho_1}{\rho_2^2}\right)^{n_1}\right)\right]$$

$$= (1-\rho_1)\left[\frac{1-\rho_1^{\frac{m}{2}+1}}{1-\rho_1} - \rho_2^{m+1}\left(\frac{1-(\rho_1/\rho_2^2)^{\frac{m}{2}+1}}{1-(\rho_1/\rho_2^2)}\right)\right]$$

$$= 1-\rho_1^{\frac{m}{2}+1} - (1-\rho_1)\frac{\rho_2\left(\rho_2^{m+2}-\rho_1^{\frac{m}{2}+1}\right)}{(\rho_2^2-\rho_1)}$$

$$* \quad P_{B2} = \sum_{2n_1+n_2=m} p(n_1,n_2)$$

$$= \sum_{n_1=0}^{m/2} p(n_1, m-2n_1)$$

$$= \sum_{n_1=0}^{m/2} \frac{(1-\rho_1)\rho_1^{n_1}(1-\rho_2)\rho_2^{m-2n_1}}{G}$$

$$= \frac{(1-\rho_1)(1-\rho_2)\rho_2^m}{G}\sum_{n_1=0}^{m/2}\left(\frac{\rho_1}{\rho_2^2}\right)^{n_1}$$

$$= \frac{(1-\rho_1)(1-\rho_2)\rho_2^m}{G}\left(\frac{1-(\rho_1/\rho_2^2)^{\frac{m}{2}+1}}{1-(\rho_1/\rho_2^2)}\right)$$

$$= \frac{(1-\rho_1)(1-\rho_2)}{G}\left(\frac{\rho_2^{m+2}-\rho_1^{\frac{m}{2}+1}}{\rho_2^2-\rho_1}\right) \qquad \frac{m-1}{2}+1$$

$$* \quad P_{B1} = \frac{(1-\rho_1)(1-\rho_2)}{G}\left(\frac{\rho_2^{m+2}-\rho_1^{\frac{m}{2}+1}}{\rho_2^2-\rho_1}\right) + \frac{(1-\rho_1)(1-\rho_2)}{G}\left(\frac{\rho_2^{m+1}-\rho_1^{\frac{m+1}{2}}}{\rho_2^2-\rho_1}\right)$$

$$= \frac{(1-\rho_1)(1-\rho_2)}{G}\left(\frac{(\rho_1+1)\rho_1^{m+1}-(\rho_2+1)\rho_2^{\frac{m+1}{2}}}{\rho_2^2-\rho_1}\right)$$