* Series

$$\sum_{r=0}^{\infty} a^r = \frac{1}{1-a}$$

$$\sum_{r=0}^{\infty} r a^r = a\left(\sum_{r=1}^{\infty} r a^{r-1}\right) = a \frac{d}{da}\left(\sum_{r=0}^{\infty} a^r\right) = a \frac{d}{da}\left(\frac{1}{1-a}\right) = \frac{a}{(1-a)^2}$$

$$\sum_{r=0}^{\infty} r^2 a^r = \sum_{r=0}^{\infty} r(r-1)a^r + \sum_{r=0}^{\infty} r a^r$$

$$= a^2\left(\sum_{r=2}^{\infty} r(r-1)a^{r-2}\right) + \sum_{r=0}^{\infty} r a^r$$

$$= a^2 \frac{d^2}{da^2}\left(\frac{1}{1-a}\right) + \frac{a}{(1-a)^2}$$

$$= a^2 \frac{2}{(1-a)^3} + \frac{a}{(1-a)^2}$$

$$= \frac{a(1+a)}{(1-a)^3}$$

* Geometric distribution $\quad P_k = (1-p)^{k-1} p \qquad k = 1, 2, \ldots$

$$\sum_{k=1}^{\infty} k(1-p)^{k-1} p = \frac{p}{1-p}\left(\sum_{k=0}^{\infty} k(1-p)^k\right) = \frac{p}{1-p} \frac{1-p}{(1-(1-p))^2} = \frac{1}{p}$$

$$\sum_{k=1}^{\infty} k^2(1-p)^{k-1} p = \frac{p}{1-p}\left(\sum_{k=0}^{\infty} k^2(1-p)^k\right) = \frac{p}{1-p} \frac{(1-p)(1+1-p)}{(1-(1-p))^3} = \frac{2-p}{p^2}$$