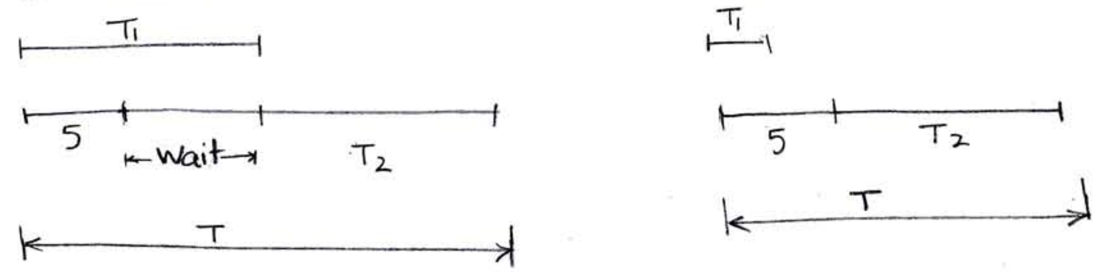
3.5 Requ'd: expected time bet. arrival of first student and departure of second student.

Let T_1 = duration of the first appointment.

T_2 = duration of the second appointment.

We have two cases:

$T_1 > 5$ min $\qquad\qquad$ $T_1 < 5$ min



We want to compute $E[T]$.

$$E[T] = P(T_1>5)\, E[T\mid T_1>5] + P(T_1<5)\, E[T\mid T_1<5]$$

$$P(T_1>5) = \int_5^\infty \frac{1}{30} e^{-t_1/30}\, dt_1 = e^{-5/30}$$

$$E[T\mid T_1<5] = E[5+T_2\mid T_1<5] = E[5+T_2] = 5 + \underbrace{E[T_2]}_{=30} = 35$$

$$E[T\mid T_1>5] = E[T_1+T_2\mid T_1>5] = \underbrace{E[T_1\mid T_1>5]}_{=35} + \underbrace{E[T_2]}_{=30} = 65$$

$$E[T_1\mid T_1>5] = \int_{t=5}^\infty t\, \frac{e^{-(t-5)/30}}{30}\, dt$$

$$= \underbrace{\int_{t=5}^\infty (t-5)\, \frac{e^{-(t-5)/30}}{30}\, dt}_{=30} + 5 \underbrace{\int_{t=5}^\infty \frac{e^{-(t-5)/30}}{30}\, dt}_{=1}$$

$$= 30 + 5$$

Therefore

$$E[T] = e^{-5/30} \times 65 + (1-e^{-5/30}) \times 35$$

$$= 35 + 30\, e^{-5/30}$$

$$= 60.39 \quad \text{min.}$$