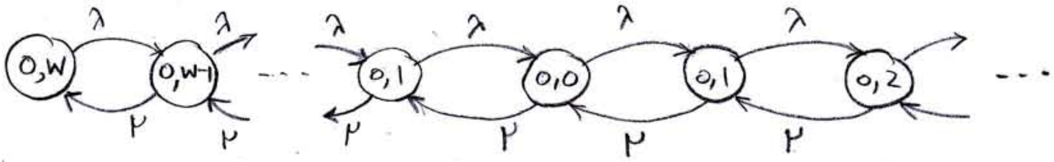
3.13

(a) Consider the Markov chain with state $= (n_{people}, n_{taxis})$

number of persons — number of taxis



arrival rate of persons $= \lambda$

arrival rate of taxis $= \mu$

define $\rho = \frac{\lambda}{\mu}$

steady state probabilities

$P(0,W) = 1-\rho$

$P(0,W-1) = (1-\rho)\rho$

$\vdots$

$P(0,1) = (1-\rho)\rho^{W-1}$

$P(0,0) = (1-\rho)\rho^{W}$

$P(1,0) = (1-\rho)\rho^{W+1}$

$\vdots$

$P(k,0) = (1-\rho)\rho^{W+k}$

Given: $W=5$, $\lambda=1$, $\mu=2$

$\therefore \rho = \frac{1}{2}$

Let P_i = number of waiting taxis

$P_5 = P(0,5) = \frac{1}{2}$

$P_4 = P(0,4) = \frac{1}{4}$

$P_3 = P(0,3) = \frac{1}{8}$

$P_2 = P(0,2) = \frac{1}{16}$

$P_1 = P(0,1) = \frac{1}{32}$

$P_0 = \sum_{k=0}^{\infty} P(k,0) = \frac{1}{64}\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k = \frac{1}{64}\times\frac{1}{1-\frac{1}{2}} = \frac{1}{32}$