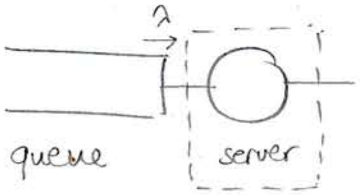
3.30

① Let N_s be the number of customers at the server (either 0 or 1)



Use Little's law on the server

$$E[N_s] = \underbrace{\lambda}_{\text{arrival rate}} \; \underbrace{\bar{X}}_{\text{avg. service time}}$$

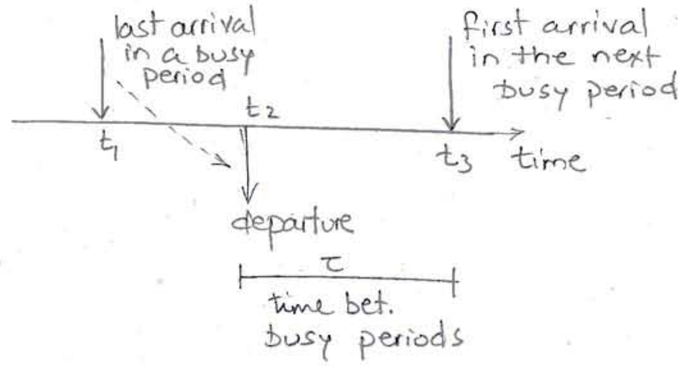
$$E[N_s] = 0 \cdot \underbrace{\mathbb{P}(N_s=0)}_{=\mathbb{P}(\text{empty system})} + 1 \cdot \underbrace{\mathbb{P}(N_s=1)}_{=\mathbb{P}(\text{busy system})}$$

$\therefore \; \mathbb{P}(\text{busy system}) = \lambda \bar{X}$

but $\mathbb{P}(\text{busy system}) + \mathbb{P}(\text{empty system}) = 1$

$\therefore \; \mathbb{P}(\text{empty system}) = 1 - \lambda \bar{X}$

② Consider the figure →



$$\mathbb{P}(\tau < t) = 1 - e^{-\lambda t}$$

$$E[\tau] = \frac{1}{\lambda}$$

③ $$\mathbb{P}(\text{busy}) = \frac{\text{total busy time}}{\text{total busy time} + \text{total idle time}}$$

$$= \frac{T_B}{T_B + T_I}$$

where T_B : mean length of a busy period
T_I : mean length of an idle period
(= mean length between busy periods)

$\mathbb{P}(\text{busy}) = \lambda \bar{X}$ and $T_I = \frac{1}{\lambda}$

$\therefore \; \frac{1}{\lambda \bar{X}} = 1 + \frac{1}{\lambda T_B}$ $\qquad \therefore \; T_B = \frac{1}{\lambda} \frac{\lambda \bar{X}}{1 - \lambda \bar{X}} = \frac{\bar{X}}{1 - \lambda \bar{X}}$

④ Average number of customers served in a busy period

$$= \frac{\text{avg. length of a busy period}}{\text{avg service time}} = \frac{T_B}{\bar{X}} = \frac{1}{1 - \lambda \bar{X}}$$