4.10

(a) It is assumed that the system starts with all nodes in mode 2. All nodes attempt to transmit dummy packets with some probability until one succeeds. The successful node then switches to mode 1 while all other nodes remain in mode 2. The node in mode 1 attemps transmission every slot thus preventing all other nodes from succeeding and entring mode 1. The situation persists until the node in mode 1 succeeds then switches back to mode 2. At that point, all nodes are in mode 2. The whole scenario repeats without ever having more than one node in mode 1.

(b) Let M_j denote the mode of node j

$$P_1 \triangleq \mathbb{P}(\text{successful tx of node } j \mid M_j = 1)$$

$$= \mathbb{P}\left(\{\text{node } j \text{ tx}\} \text{ AND } \left\{\begin{matrix}\text{all other nodes (which are} \\ \text{in mode 2) do not tx}\end{matrix}\right\} \middle| M_j = 1\right)$$

$$= \underbrace{\mathbb{P}(\text{node } j \text{ tx} \mid M_j = 1)}_{1} \left[\underbrace{\mathbb{P}(\text{a node in mode 2 does not tx})}_{1-P_r}\right]^{m-1}$$

$$= (1-P_r)^{m-1}$$

$$\boxed{P_1 = (1-P_r)^{m-1}}$$

Let X be the time between successful tx given $\{M=1\}$

$$\mathbb{P}(X=1) = P_1$$

$$\mathbb{P}(X=2) = (1-P_1)\,P_1$$

$$\mathbb{P}(X=3) = (1-P_1)(1-P_1)\,P_1$$

$$\vdots$$

$$\mathbb{P}(X=k) = (1-P_1)^{k-1}\,P_1 \qquad \text{(Geometric distrib)}$$

$$E[X] = \frac{1}{P_1}$$

$$E[X^2] = \frac{2-P_1}{P_1^2}$$

(c)

$$P_2 \triangleq \mathbb{P}\left(\text{successful tx of any node} \mid \overbrace{M_1=2, \dots M_m=2}^{\text{all nodes in mode 2}}\right)$$

$$= \mathbb{P}\left(\text{only one node tx} \mid M_1=2, \dots, M_m=2\right)$$

$$= \sum_{j=1}^{m} \mathbb{P}\left(\text{node } j \text{ tx, all other nodes do not tx} \mid M_1=2, \dots, M_m=2\right)$$

$$= m \quad q_r (1-q_r)^{m-1}$$

$$\boxed{P_2 = m q_r (1-q_r)^{m-1}}$$

Let V be the time between successful tx given $\bigcap_{j=1}^{m} \{M_j = 2\}$

$$\mathbb{P}(Y=1) = P_2$$

$$\mathbb{P}(Y=2) = (1-P_2)\,P_2$$

$$\mathbb{P}(Y=3) = (1-P_2)(1-P_2)\,P_2$$

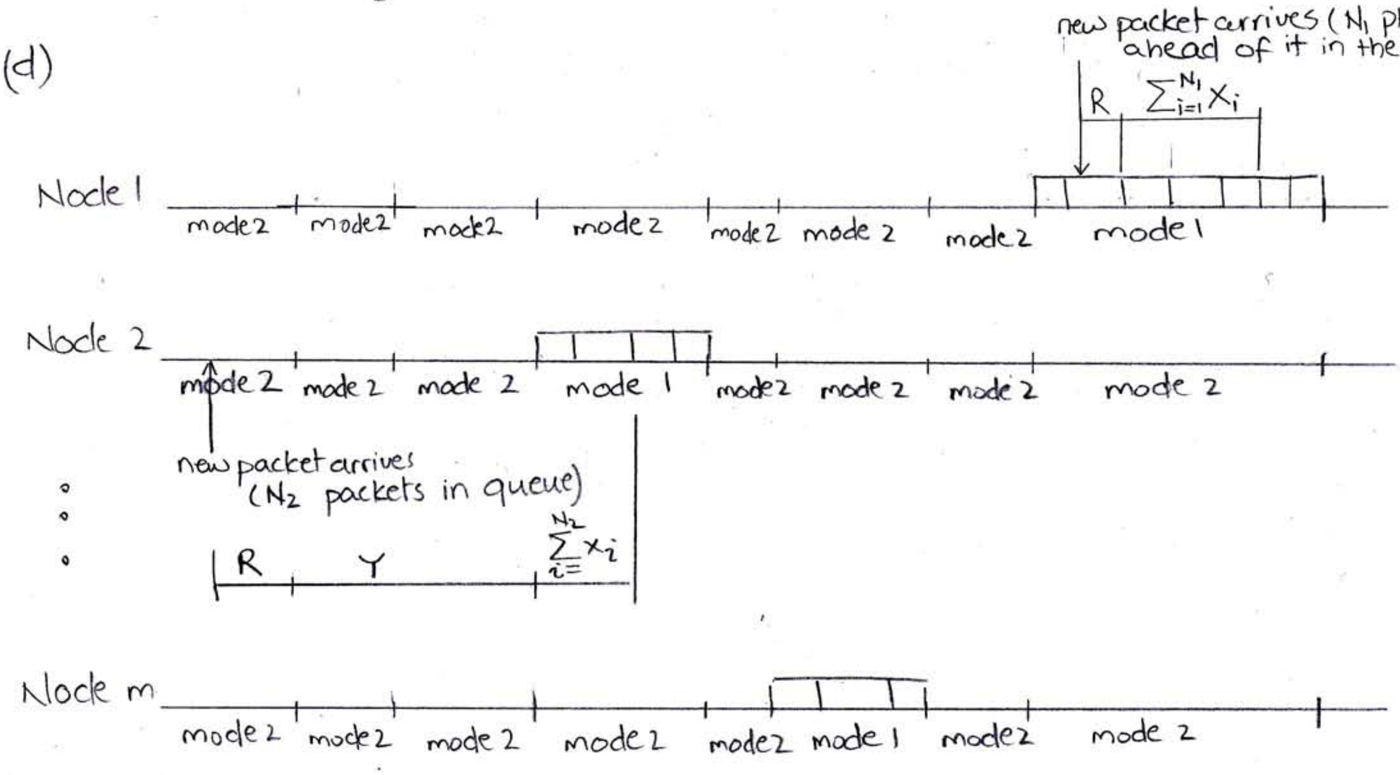
$$\vdots$$

$$\mathbb{P}(V=k) = (1-P_2)^{k-1} P_2$$

$$E[Y] = \frac{1}{P_2}$$

$$E[V^2] = \frac{2-P_2}{P_2^2}$$

(d)



$$W = R + Y + \sum_{i=1}^{N} X_i$$

W: Waiting time

R: Residual time

Y: total "mode 2" time until the first "mode 1" slot

$\sum_{i=1}^{N} X_i$: the sum is the total service time of users ahead in the queue

Let S be the number of whole "mode 2" intervals until the first "mode 1" slot

$$Y = \sum_{j=1}^{S} V_i \qquad (V_1, \dots, V_S \text{ are i.i.d.})$$

$$E[Y] = E\Big[E[Y|S]\Big] = E\Big[\sum_{j=1}^{S} E[V_i]\Big] = E\Big[S\,E[V]\Big] = E[S]\,E[V]$$

$$E\Big[\sum_{i=1}^{N} X_i\Big] = \underbrace{E[N]}_{=\lambda E[W]} E[X] \quad \text{(Little's theorem)}$$

Therefore

$$E[W] = E[R] + E[S]\,E[V] + \lambda E[W]\,E[X]$$

$$E[W] = \frac{E[R] + E[S]\,E[V]}{1 - \lambda E[X]}$$

Define $\rho = \lambda E[X]$

$$E[R] = \rho\,\frac{\overline{X^2}}{2\overline{X}} + (1-\rho)\,\frac{\overline{V^2}}{2\overline{V}}$$