CAPES

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPESP
FUNDAÇÃO DE AMPARO À PESQUISA DO ESTADO DE SÃO PAULO

IFT - UNESP
INSTITUTO DE FÍSICA TEÓRICA

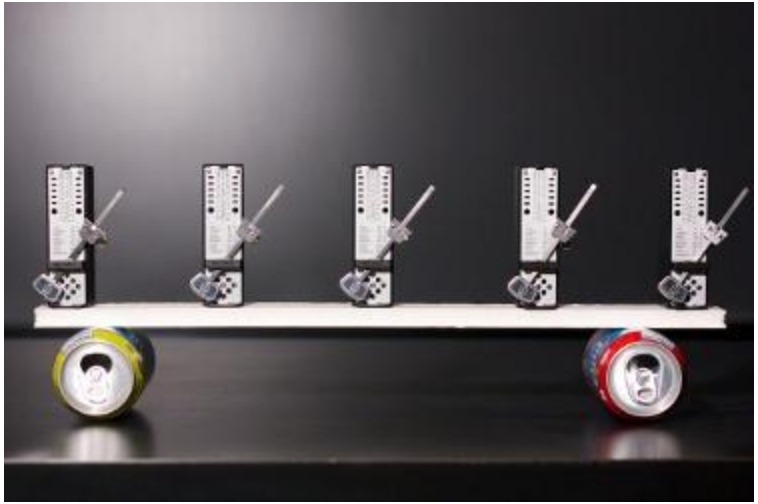
ICTP SAIFR

The influence of external drives on the Frustrated Kuramoto model

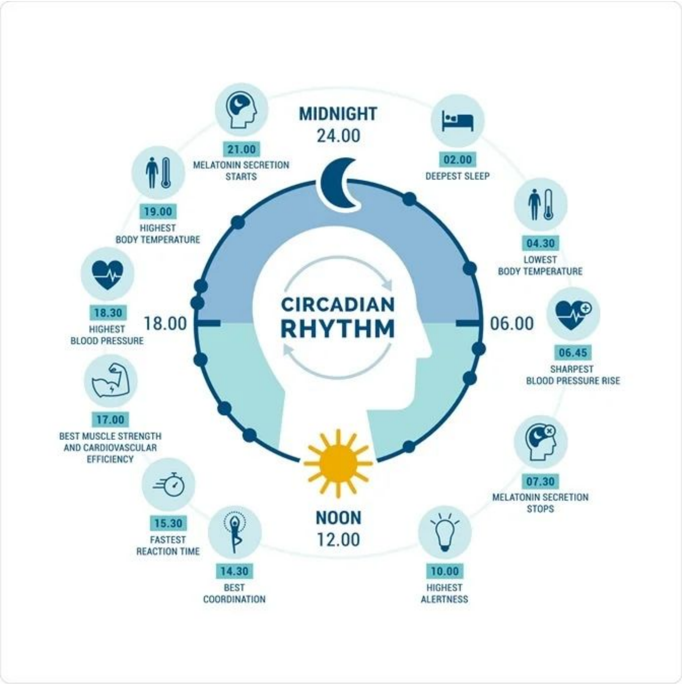
Guilherme S. Costa and Marcus A. M. de Aguiar

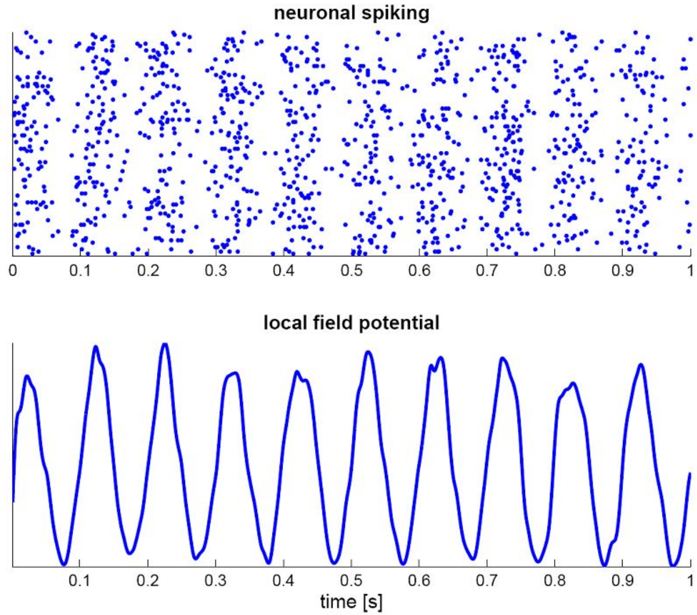
Workshop on Dynamical Processes on Complex Networks

Synchronization examples



More “complex” mechanisms





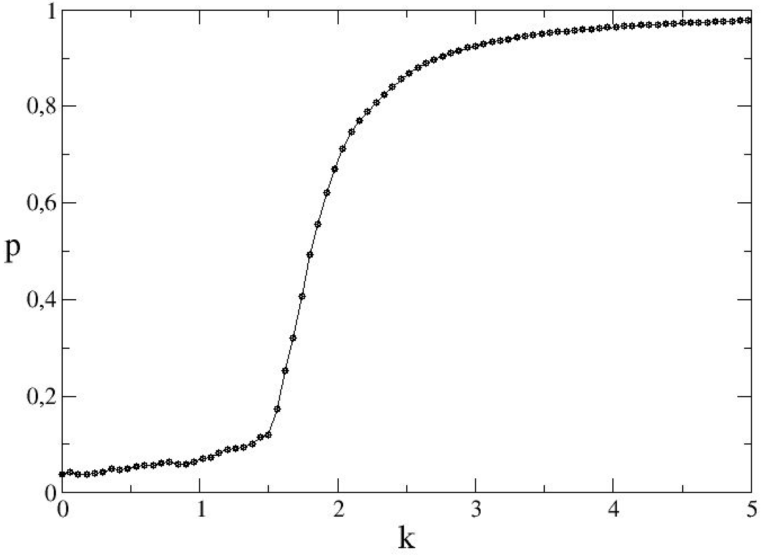
Original Kuramoto model

Population of N oscillators described by phases θ_i with natural frequencies ω_i:

$$\dot{\theta}_i = \omega_i + \frac{k}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i)$$

Complex order parameter:

$$z = pe^{i\psi} \equiv \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j}$$



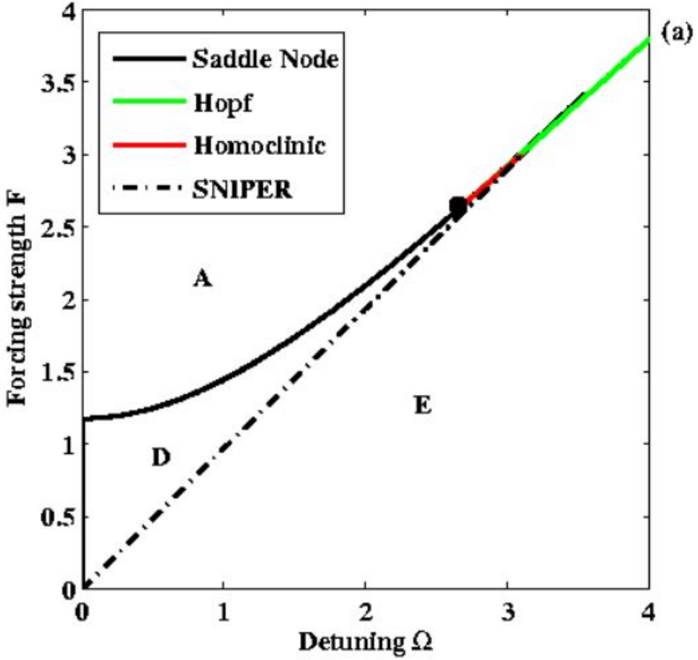
Y. Kuramoto, Springer Berlin Heidelberg, 1975.

Kuramoto with external drives

$$\dot{\theta}_i = \omega_i + \frac{k}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + F\sin(\Omega t)$$

$$\dot{p} = -\Delta p + \frac{kp}{2}(1-p^2) + \frac{F}{2}(1-p^2)\cos(\psi - \Omega t)$$

$$\dot{\psi} = \omega_0 - \Omega - \frac{F}{2}\left(p + \frac{1}{p}\right)\sin(\psi - \Omega t)$$



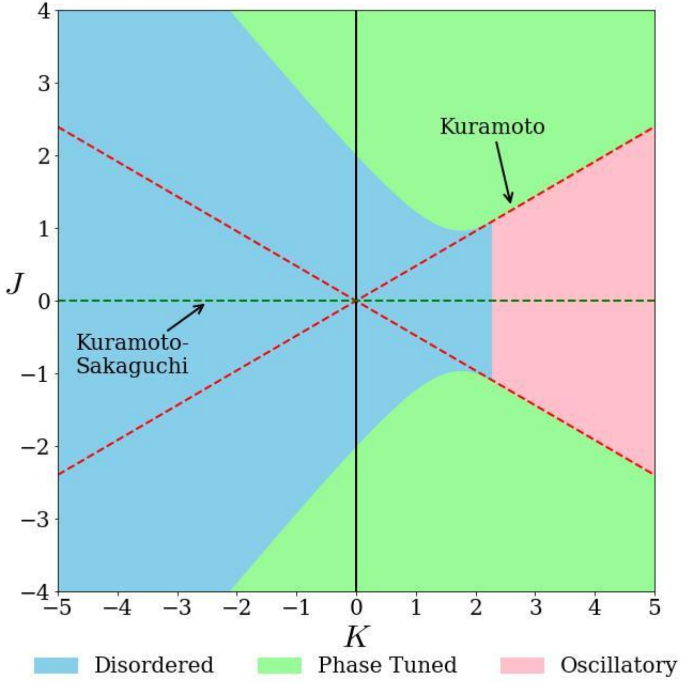
L. M. Childs and S. H. Strogatz, Chaos: An Interdisciplinary Journal of Nonlinear Science,2008

Matrix coupling and generalized frustration

$$\frac{d\vec{\sigma}_i}{dt} = \mathbf{W}_i\vec{\sigma}_i + \frac{1}{N}\sum_{j=1}^{N}[\mathbf{K}\vec{\sigma}_j - (\vec{\sigma}_i \cdot \mathbf{K}\vec{\sigma}_j)\vec{\sigma}_i]$$

$$\mathbf{K} \equiv \mathbf{K}_R + \mathbf{K}_S = K\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} + J\begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$$

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N}[K\sin(\theta_i - \theta_j - \alpha) + J\sin(\theta_i + \theta_j + \beta)]$$



G. L. Buzanello, A. E. D. Barioni, and M. A. M. de Aguiar, Chaos: An Interdisciplinary Journal of Nonlinear Science, 2022.

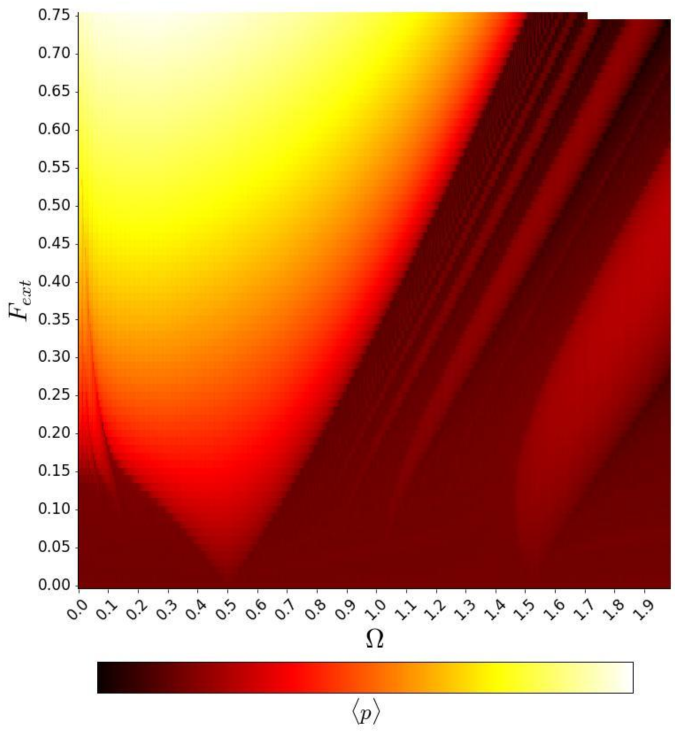
External drives on matrix coupled oscillators

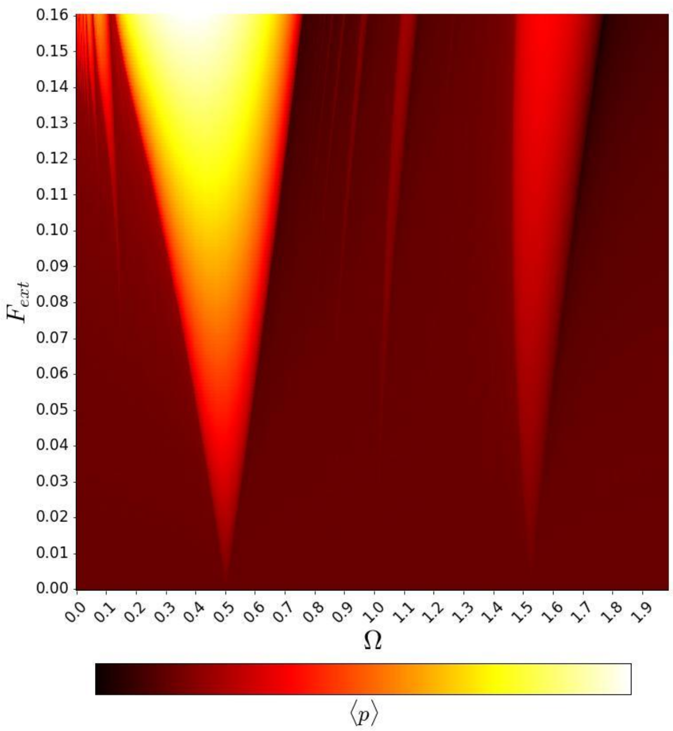
$$\dot{p} = -\Delta p + \frac{(1-p^2)}{2}\left[pK\cos\alpha - pJ\cos\left(2\phi + 2\Omega t + \beta\right) + F\cos\phi\right]$$

$$\dot{\phi} = (\omega_0 - \Omega) - \frac{(1+p^2)}{2}\left[K\sin\alpha - J\sin\left(2\phi + 2\Omega t + \beta\right) + \frac{F}{p}\sin\left(\phi\right)\right]$$

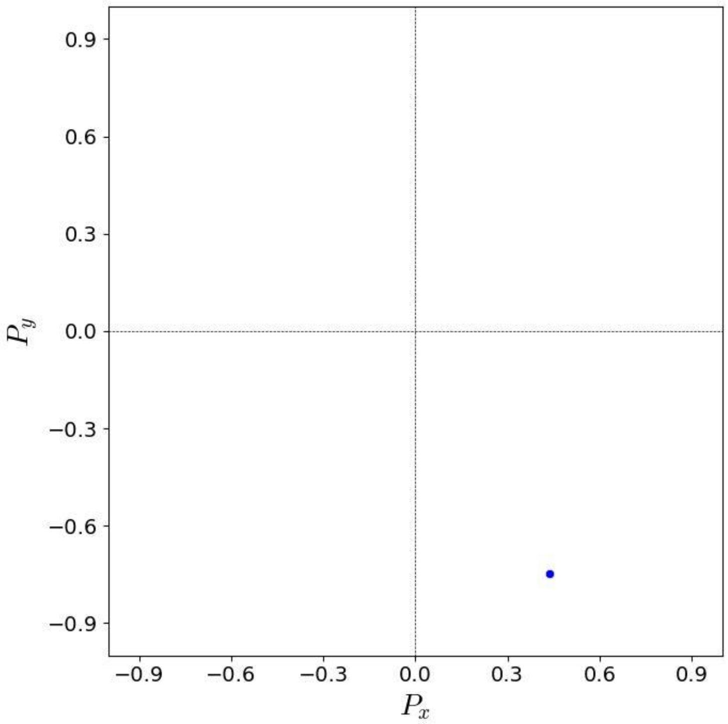
Equations are non-autonomous!

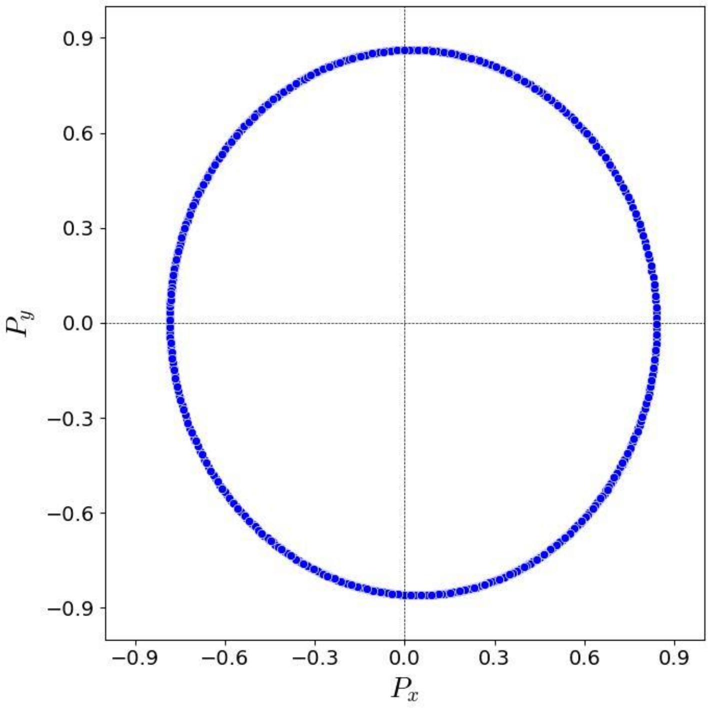
Oscillatory states (ω = 0.5 ; Δ=1.0)

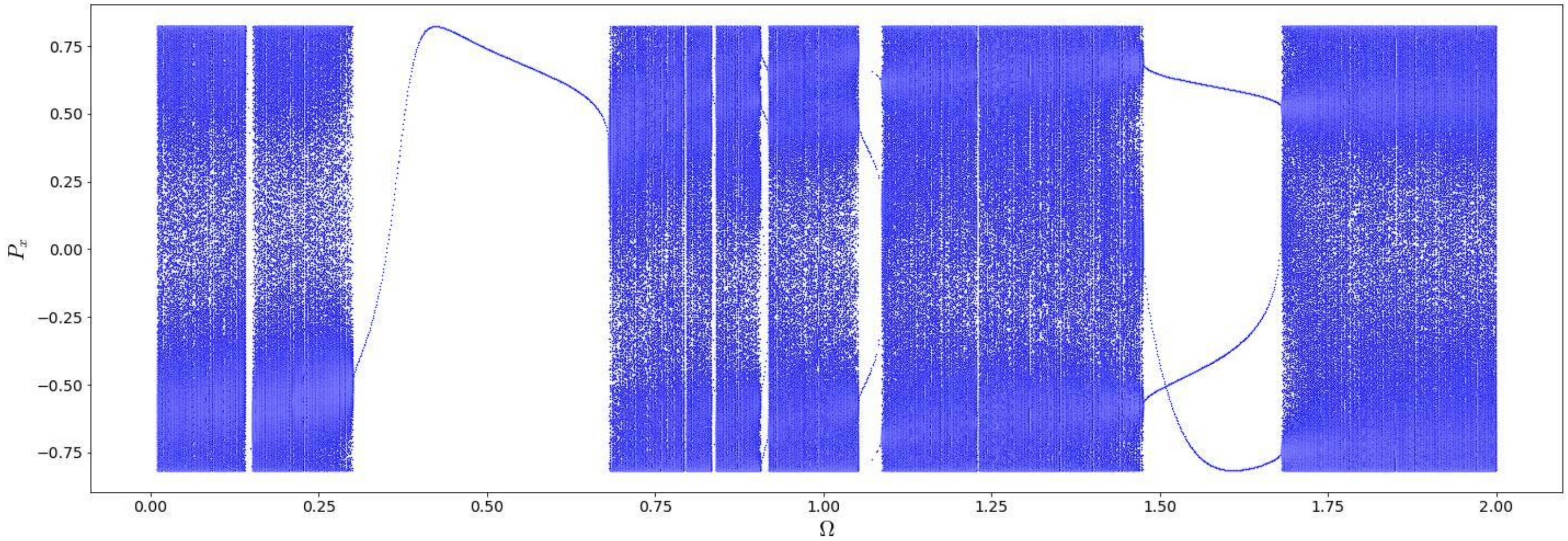




Poincaré maps

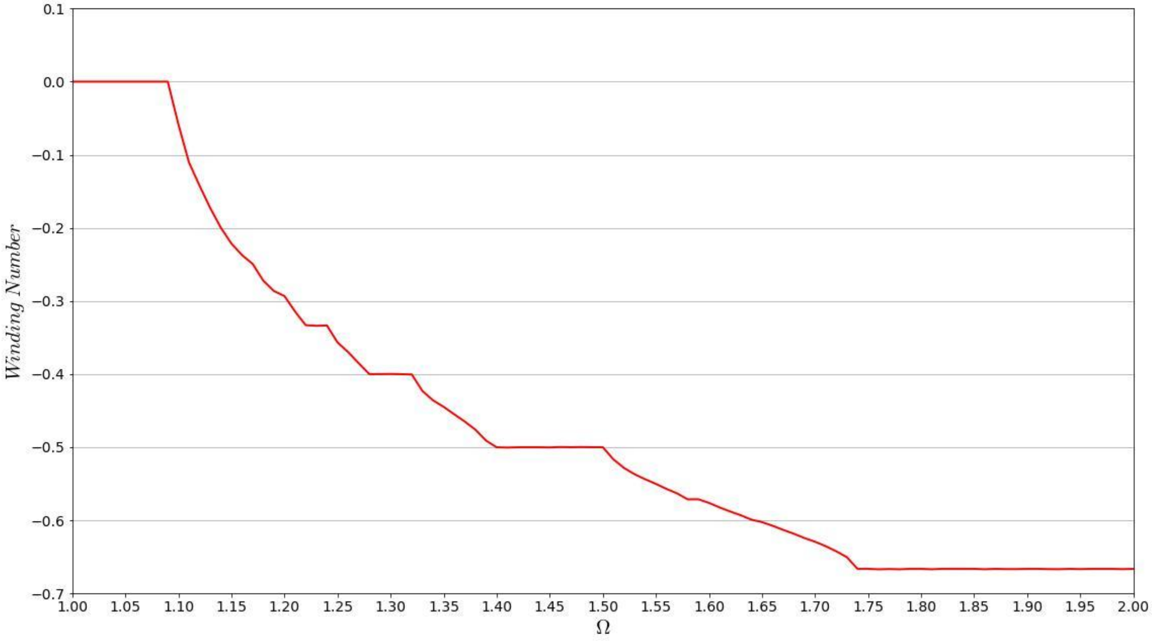


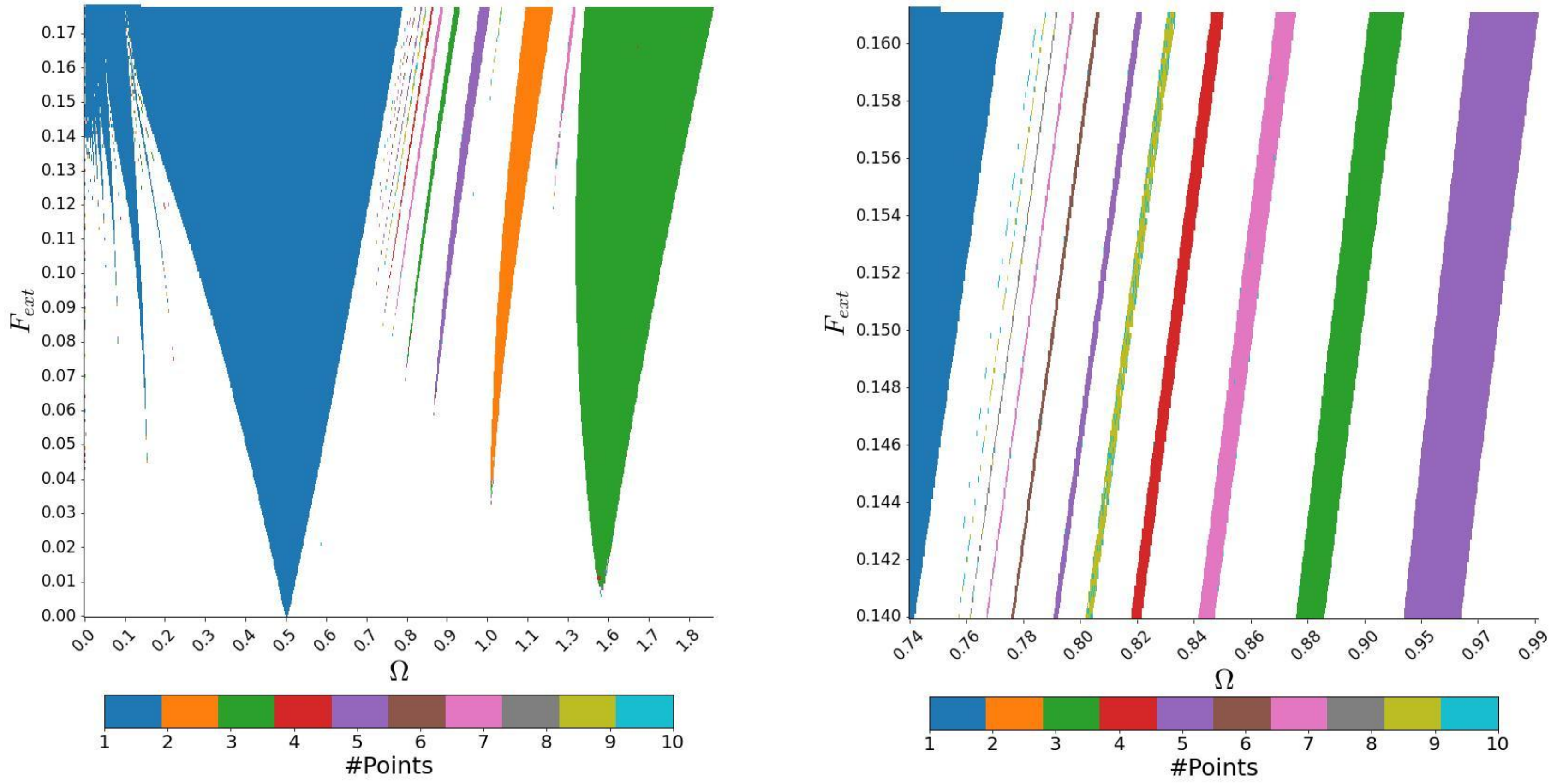



0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
P_x
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Ω

Winding Number

$$w = \lim_{n \to \infty} \frac{\theta_n}{n}$$





F_{ext}
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.3
1.6
1.7
1.8
Ω
1
2
3
4
5
6
7
8
9
10
#Points
F_{ext}
0.140
0.142
0.144
0.146
0.148
0.150
0.152
0.154
0.156
0.158
0.160
0.74
0.76
0.78
0.80
0.82
0.84
0.86
0.88
0.90
0.95
0.97
0.99
Ω
1
2
3
4
5
6
7
8
9
10
#Points

Final remarks and Prospects

- The matrix coupling Kuramoto model exhibits novel and interesting new synchronized states.
- Adding a global external drive to it we can obtain dynamical equations for the order parameter via Ott-Antonsen ansatz.
- The external driving acting on oscillatory synchronized states lead to the formation of Arnold tongues and mode-locking regions.
- What happens in the phase tuned region ?

Thanks for your attention!

e-mail: guilherme.h.silvacosta@gmail.com