## Math 115 Worksheet Section 3.2

## Warm-up question

 $(e^x)' = \underline{\qquad} (a^x)' = \underline{\qquad} (\sin(x))' = \underline{\qquad} (\cos(x))' = \underline{\qquad}$ 

**Problem 1.** An animal population is given by  $P(t) = 300(1.044)^t$  where t is the number of years since the study of the population began. Find P'(5) and provide a practical interpretation of your result.

**Problem 2.** Find the quadratic polynomial  $g(x) = ax^2 + bx + c$  which best fits the function  $f(x) = e^x$  at x = 0, in the sense that

$$g(0) = f(0)$$
 and  $g'(0) = f'(0)$  and  $g''(0) = f''(0)$ 

Using a computer or calculator, sketch the graphs of f and g on the same axes. What do you notice?

**Problem 3.** Using the equation of the tangent line to the graph of  $e^x$  at x=0, show that

$$e^x \ge 1 + x$$

for all values of x. A sketch may be helpful.

**Problem 4.** Find the 50<sup>th</sup> derivative of  $y = \cos(x)$ .

**Problem 5.** Find the tangent lines to  $f(x) = \sin(x)$  at  $x = \frac{4\pi}{3}$  and at  $x = \frac{10\pi}{3}$ . Graph them, along with  $\sin(x)$ . What do you notice? Express the second as a transformation of the first.

**Problem 6.** Are the following statements true or false? Give an explanation for your answer.

- (a) If f(x) is increasing, then f'(x) is increasing.
- (b) There is no function such that f'(x) = f(x) for all x besides the constant function f(x) = 0.
- (c) There is no function such that f'(x) = -f(x) for all x besides the constant function f(x) = 0.
- (d) There is no function such that f''(x) = -f(x) for all x besides the constant function f(x) = 0.
- (e) If f(x) is defined for all x, then f'(x) is defined for all x.

**Problem 7.** For what value(s) of a are  $y = a^x$  and y = x + 1 tangent at x = 0?

**Problem 8.** (Winter 2017 Exam 3) A Math 115 coordinator is trying to create functions with certain properties in order to test students' understanding of various calculus concepts. She wants a function f(x) of the form

$$f(x) = \begin{cases} ax^2 + ax + be^x & \text{for } x < 0\\ a + 2\cos(x) & \text{for } x \ge 0 \end{cases}$$

where a and b are constants. Find all value(s) of a and b for which f(x) be differentiable at x = 0. Show enough work to justify your answer.

**Problem 9.** Explain for which values of a > 0 the function  $a^x$  is increasing and for which values it is decreasing. Is this consistent with your formula for  $(a^x)'$ ?

**Problem 10.** Give an example of:

- (a) An exponential function for which the derivative is always negative.
- (b) A function f such that f'''(x) = f(x).

**Problem 11.** In 2009, the population of Mexico was 111 million and growing 1.13% annually, while the population of the US was 307 million and growing 0.975% annually. If we measure growth rates in people/year, which population was growing faster in 2009? Note  $\ln(1.0113) \approx 0.01124$  and  $\ln(1.00975) \approx 0.009702$ .