Math 115
Worksheet Section 1.7

Continuous Functions: Recall that, roughly speaking, a function is continuous on an interval if it has no breaks, jumps, or holes on that interval.

Many of the functions we've seen are continuous on the interval $(-\infty, \infty)$, like polynomials, exponential functions, and sinusoidal functions. What about power functions and rational functions? Might there be points where those functions are not continuous?

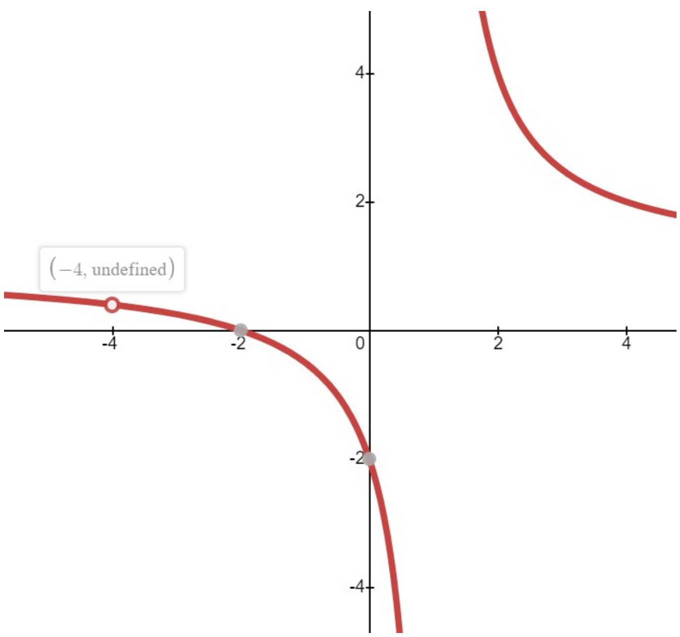
Problem 1. (a) Graph the piecewise function below and determine some intervals where the function is or is not continuous.

$$f(x) = \begin{cases} x & x \leq 2 \\ x^2 & x > 2 \end{cases}$$

(b) What value(s) of k, if any, make(s) the following function continuous on $(-\infty, \infty)$?

$$g(x) = \begin{cases} x+k & x \leq 2 \\ x^2 & x > 2 \end{cases}$$

Problem 2. Consider $h(x) = \dfrac{(x+2)(x+4)}{x^2+3x-4}$, whose graph is below. It has a hole at $x = -4$ (why?). How could we figure out what the y-coordiante of the hole is?



How does the function $k(x) = \dfrac{x+2}{x-1}$ compare to $h(x)$?

Limits: We write $\lim_{x \to c} f(x) = L$ if the values of $f(x)$ approach L as x approaches c.

Note that the value of the function <u>at</u> c is not relevant, and does not even need to be defined!

Problem 3. Draw the graphs of $f(x) = \dfrac{x}{x}$ and $g(x) = \dfrac{x}{|x|}$ on the board and consider whether each has a limit at 0.

Problem 4. Do these functions have limits at 0?

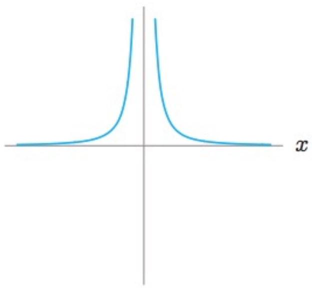


Figure 1.91: Graph of $1/x^2$

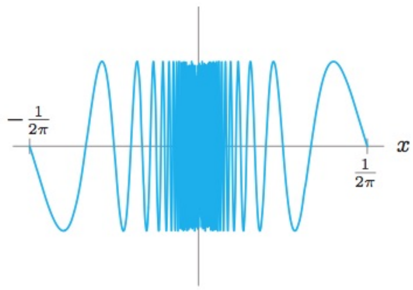


Figure 1.92: Graph of $\sin(1/x)$

Problem 5. (1.8 #1)

Use Figure 1.94 to give approximate values for the following limits (if they exist).

(a) $\lim_{x \to -2} f(x)$ **(b)** $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to 2} f(x)$ **(d)** $\lim_{x \to 4} f(x)$

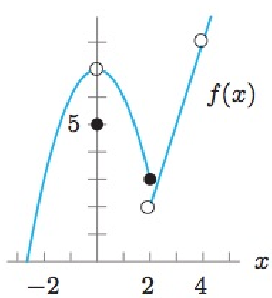


Figure 1.94

Problem 6. Think again about the last problem. Are there intervals on which f is not continuous? Then see if you can finish the definition in the box below.

Continuity: The function f is continuous at c if f is defined at c and

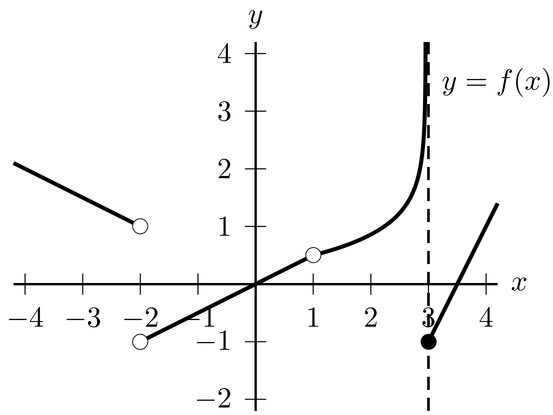
The function f is continuous on an interval $[a, b]$ if it continuous at every point in the interval.

Problem 7. Consider the function

$$N(u) = \begin{cases} e + 3^{u^2+k} & \text{if } u < 1. \\ 5e\ln(e + u - 1) & \text{if } u \geqslant 1. \end{cases}$$

Find all values of k so that $N(u)$ is continuous at $u = 1$. Show your work carefully, and leave your answer(s) in exact form.

6. [11 points] Below is the graph of a portion of a function $f(x)$.



a. [2 points] Give all values of a in the interval $-4 < a < 4$ that are not in the domain of $f(x)$. If there are none, write NONE.

Answer: ________________________

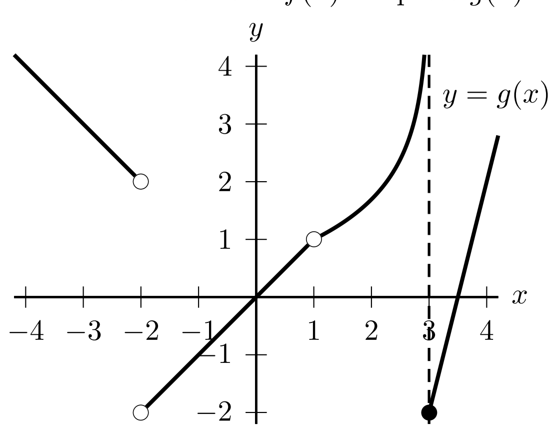
b. [2 points] Give all values of a in the interval $-4 < a < 4$ where $f(x)$ is not continuous at $x = a$. If there are none, write NONE.

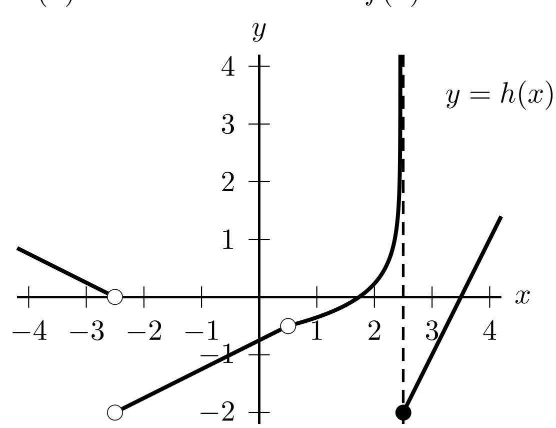
Answer: ________________________

c. [2 points] Give all values of a in the interval $-4 < a < 4$ where $\lim_{x \to a} f(x)$ does not exist. If there are none, write NONE.

Answer: ________________________

d. [5 points] The graphs below show portions of two other functions $g(x)$ and $h(x)$ which are transformations of $f(x)$. Express $g(x)$ and $h(x)$ as transformations of $f(x)$.





Answer: $g(x) =$ ________________ and $h(x) =$ ________________

3. [9 points] Consider the function h defined by

$$h(x) = \begin{cases} \dfrac{60(x^2 - x)}{(x^2+1)(3-x)} & \text{for } x < 2 \\ c & \text{for } x = 2 \\ 5e^{ax} - 1 & \text{for } x > 2 \end{cases}$$

where a and c are constants.

a. [5 points] Find values of a and c so that both of the following conditions hold.

- $\lim_{x \to 2} h(x)$ exists.
- $h(x)$ is not continuous at $x = 2$.

Note that this problem may have more than one correct answer. You only need to find one value of a and one value of c so that both conditions above hold. Remember to show your work clearly.

Answer: $a =$ ____________________ and $c =$ ____________________

b. [2 points] Determine $\lim_{x \to -\infty} h(x)$. If the limit does not exist, write DNE.

Answer: $\lim_{x \to -\infty} h(x) =$ ____________________

c. [2 points] Find all vertical asymptotes of the graph of $h(x)$. If there are none, write NONE.

Answer: Vertical asymptote(s): ____________________