

Math 115 Fall 2024  
Worksheet 15 – MVT

**Warm-up questions**

What are the hypothesis of the mean value theorem?

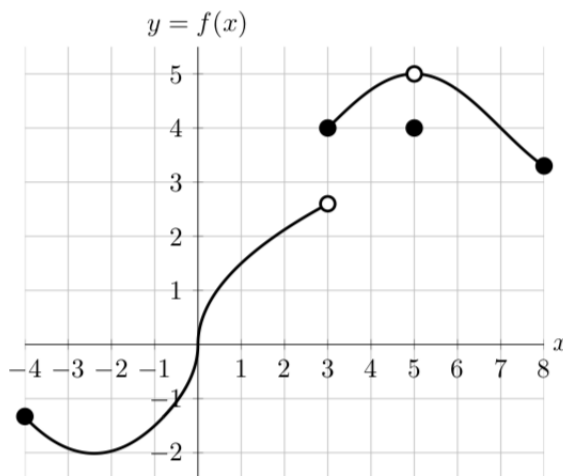
What is the conclusion of the mean value theorem?

**Problem 1.** Give an example of a function  $f$  that is...

- (a) continuous on the interval  $[-1, 1]$  but that does not satisfy the hypothesis of the Mean Value Theorem on that interval, and for which the conclusion of the Mean Value Theorem is false.
- (b) continuous on the interval  $[-1, 1]$  but that does not satisfy the hypothesis of the Mean Value Theorem on that interval, and for which the conclusion of the Mean Value Theorem is true.
- (c) differentiable on the interval  $(0, 2)$  but that does not satisfy the hypothesis of the Mean Value Theorem on  $[0, 2]$ .

**Problem 2.** (Winter 2018 Exam 2) The graph of the function  $f$  with domain  $-4 \leq x \leq 8$  is shown below. The function  $f(x)$  satisfies:

- $f(x) = 1.5x^{\frac{1}{3}}$  for  $-1 < x < 1$ , and
- $f(x) = 4 + \sin\left(\frac{\pi}{4}(x - 3)\right)$  for  $3 \leq x < 5$  and  $5 < x \leq 8$ .



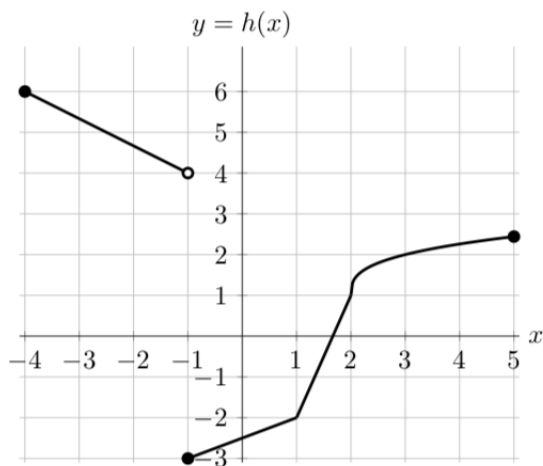
- (a) On which of the following intervals is the conclusion of the Mean Value Theorem true?

$[-4, 0]$      $[0, 5]$      $[1, 3]$      $[3, 7]$     none of these

- (b) On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, 0]$      $[0, 5]$      $[1, 3]$      $[3, 7]$     none of these

**Problem 3.** (Fall 2017 Exam 2) Consider the graph of  $h(x)$  below. Note that  $h$  is linear on the intervals  $[-4, -1)$ ,  $[-1, 1]$  and  $[1, 2]$ , differentiable on  $(2, 5)$ , and has a sharp corner at  $x = 2$ .



(a) On which of the following intervals is the conclusion of the Mean Value Theorem true?

$[-4, -1]$      $[-2, -1]$      $[0, 4]$      $[2, 5]$     none of these

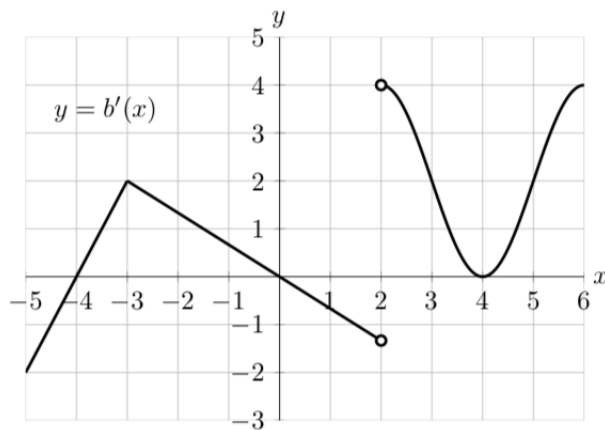
(b) On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, -1]$      $[-2, -1]$      $[0, 4]$      $[2, 5]$     none of these

(c) For which values given below is the function  $m(x) = h(h(x))$  not differentiable? Circle all that apply.

$x = -3$      $x = -1$      $x = 2$      $x = 3$      $x = 4$     none of these

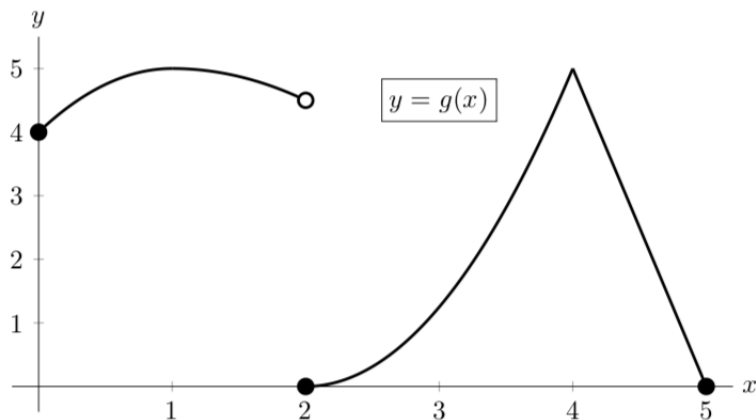
**Problem 4.** The graph of a portion of the derivative of  $b(x)$  is shown below. Assume that  $b(x)$  is defined and continuous on  $[-5, 6]$ .



On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, -2]$      $[-2, 2]$      $[1, 4]$      $[-5, 6]$     none of these

**Problem 5.** (Fall 2016 Exam 2) The entire graph of a function  $g(x)$  is shown below. Note that the graph of  $g(x)$  has a horizontal tangent line at  $x = 1$  and a sharp corner at  $x = 4$ .



- (a) On which of the following intervals does the function  $f(x)$  satisfy the hypotheses of the Mean Value Theorem? Circle the correct answer(s):

$[0, 2]$      $[1, 3]$      $[2, 4]$      $[3, 5]$     None of these

- (b) On the interval  $[8, 12]$  the hypotheses of the Mean Value Theorem are true for the function  $f(x)$ . What does the conclusion of this theorem say in this interval?

**Answer:**

**Problem 6.** (Winter 2016 Exam 2) Let  $h(x)$  be a twice differentiable function defined for all real numbers  $x$ . (So  $h$  is differentiable and its derivative  $h'$  is also differentiable.) Some values of the derivative of  $h$  are given in the table below.

$x$	-8	-6	-4	-2	0	2	4	6	8
$h'(x)$	3	7	0	-3	-5	-4	0	-2	6

- (a) Circle all the intervals which must contain a number  $c$  such that  $h''(c) = 2$ .

$-8 < x < -6$      $-4 < x < -2$      $-2 < x < 0$      $6 < x < 8$

- (b) Suppose that  $h''(x) < 0$  for  $x < -8$  and  $h(-8) = 7$ . Circle all the numbers below which could equal the value of  $h(-10)$ .

-2    -1    0    1    2    None of these