

# Math 115

## Worksheet Section 1.8

**Problem 1.** (1.8 #26) Find the following limit exactly, using algebra (not a table or graph):

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 + 3x - 4}$$

**Problem 2.** First, find the left- and right-handed limits of the given function  $f(x)$  at both  $x = 3$  and  $x = 5$ . Then find all values where  $f(x)$  is not continuous.

$$f(x) = \begin{cases} \frac{6-x}{x^2-8x+17} & 0 \leq x \leq 3 \\ 12-2x & 3 < x < 5 \\ 12-2x & 5 \leq x \leq 6 \end{cases}$$

**Problem 3.** Find the following limits, or say they do not exist (DNE) and justify your answer.

(a)  $\lim_{x \rightarrow \infty} \frac{9x^3 - x + 83}{x^4} =$

(b)  $\lim_{x \rightarrow \infty} \frac{x^4}{9x^3 - x + 83} =$

(c)  $\lim_{x \rightarrow \infty} \frac{9x^3 - x + 83}{x^3} =$

(d)  $\lim_{x \rightarrow \infty} \frac{2e^x + 5}{3e^x + 7} =$

(e)  $\lim_{x \rightarrow \infty} \frac{2^{-x} + 5}{3^{-x} + 7} =$

(f)  $\lim_{x \rightarrow \infty} \frac{2^x + 5}{3^x + 7} =$

(g)  $\lim_{x \rightarrow -\infty} \frac{e^{-x}}{x} =$

**Problem 4.** On the board, draw the graph of a function  $f$  that has all the following properties:

- $\lim_{x \rightarrow 3} f(x) = 5$
- $f(3) = 0$
- $\lim_{x \rightarrow 0^+} f(x) = 4$
- $\lim_{x \rightarrow 0} f(x)$  DNE
- $\lim_{x \rightarrow -\infty} f(x) = +\infty$

**Problem 5.** (1.8 # 6–8) If  $\lim_{x \rightarrow 2} f(x) = 7$ ,  $\lim_{x \rightarrow 2} g(x) = 4$ , and  $\lim_{x \rightarrow 2} h(x) = \frac{1}{2}$ , find

(a)  $\lim_{x \rightarrow 2} (f(x) - 2h(x))$

(b)  $\lim_{x \rightarrow 2} (g(x))^2$

(c)  $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)h(x)}$

**Problem 6.** (Fall 2017 Exam 1) If  $q(x) = \frac{2e^{kx}}{1+2^x}$ , find all values of  $k$  such that  $\lim_{x \rightarrow \infty} q(x) = 0$ . If there are none, write NONE. Show your work or reasoning to justify your answer.

**Problem 7.** (Winter 2018 Exam 1)

(a) Let  $p(x)$  be a polynomial satisfying all the following properties:

(i)  $p(x) = 0$  only at  $x = -2, 0, 3$ .

(ii)  $\lim_{x \rightarrow -\infty} p(x) = -\infty$  and  $\lim_{x \rightarrow \infty} p(x) = -\infty$ .

Find one possible formula for  $p(x)$ . There may be more than one correct answer.

(b) Let  $h(x)$  be a rational function satisfying all the following properties:

(i)  $\lim_{x \rightarrow 2} h(x) = 0$  and  $h$  is not defined at  $x = 2$ .

(ii)  $\lim_{x \rightarrow \infty} h(x) = 0$ .

**Problem 8.** (Winter 2017 Exam 1) Find all real numbers  $B$  and positive integers  $k$  such that the rational function

$$H(x) = \frac{9 + x^k}{16 - Bx^3}$$

satisfies the following two conditions:

- $H(x)$  has a vertical asymptote at  $x = 2$ .
- $\lim_{x \rightarrow \infty} H(x)$  exists.

**Problem 9.** (Winter 2016 Exam 1) Consider the function  $f(x)$  defined by

$$f(x) = \begin{cases} xe^{Ax} + B & \text{if } x < 3 \\ C(x-3)^2 & \text{if } 3 \leq x \leq 5 \\ \frac{130}{x} & \text{if } x > 5 \end{cases}$$

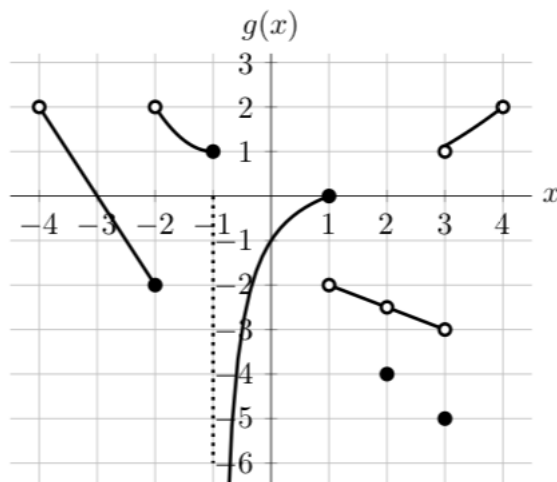
Suppose  $f(x)$  satisfies all of the following:

- $f(x)$  is continuous at  $x = 3$ .
- $\lim_{x \rightarrow 5^+} f(x) = 2 + \lim_{x \rightarrow 5^-} f(x)$ .
- $\lim_{x \rightarrow -\infty} f(x) = -4$ .

Find the values of  $A$ ,  $B$  and  $C$ . You must give exact answers.

**Problem 10.** (Winter 2018 Exam 1) Consider the functions  $f(x)$  and  $g(x)$  given by the formula and graph below.

$$f(x) = \begin{cases} 2x^3 - 2x^2 & \text{for } x \leq 1 \\ x^3 + 1 & \text{for } x > 1 \end{cases}$$



(a) Circle the correct answer(s) in each of the following questions.

(i) At which of the following values of  $x$  is the function  $g(x)$  not continuous?

$$x = -3 \quad x = -1 \quad x = 0 \quad x = 2 \quad x = 3.5$$

(ii) At which of the following values of  $x$  is the function  $f(x) + g(x)$  continuous?

$$x = -2 \quad x = -1 \quad x = 0 \quad x = 1 \quad x = 2$$

Note that  $g(x)$  is linear on the intervals  $(-4, -2)$ ,  $(1, 2)$  and  $(2, 3)$ . All your answers below should be exact. If any of the quantities do not exist, write DNE.

(b) Find  $\lim_{x \rightarrow 2} (2f(x) + g(x))$ .

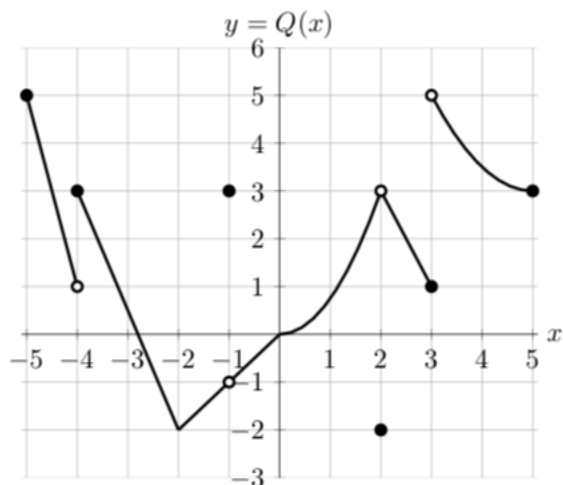
(c) Find  $\lim_{x \rightarrow \infty} \frac{f(2x)}{x^3}$ .

(d) Find  $\lim_{x \rightarrow \infty} g(x^2 e^{-x} + 3)$ .

(e) For which values of  $p$  does  $\lim_{x \rightarrow p^+} g(x) = 1$ ?

(f) Find  $\lim_{x \rightarrow -1^-} f(-x)$ .

**Problem 11.** (Fall 2017 Exam 1) The graph of a function  $Q(x)$  with domain  $[-5, 5]$  is shown below.



(a) Find the numerical value of the following mathematical expressions. If the answer cannot be determined with the information given, write “NI”. If any of the quantities does not exist, write “DNE”.

(i) Find  $\lim_{x \rightarrow -1} Q(x)$ .

(ii) Find  $\lim_{w \rightarrow 2} Q(Q(w))$ .

(iii) Find  $\lim_{x \rightarrow \infty} Q\left(\frac{1}{x} + 3\right)$ .

(iv) Find  $\lim_{x \rightarrow \frac{1}{3}} xQ(3x - 5)$ .

(b) For which values of  $-5 < p < 5$  is  $\lim_{x \rightarrow p^-} Q(x) \neq Q(p)$ ?