# Math 115
# Worksheet Ch 5 Review

**Problem 1.** (Winter 2014 Final Exam) One of the ways Captain Christina likes to relax in her retirement is to go for long walks around her neighborhood. She has noticed that early every Tuesday morning, a truck delivers butter to a local bakery famous for its cookie dough. Consider the following functions:

- Let $C(b)$ be the bakery's cost, in dollars, to buy b pounds of butter.
- Let $K(b)$ be the amount of cookie dough, in cups, the bakery makes from b pounds of butter.
- Let $u(t)$ be the instantaneous rate, in pounds per hour, at which butter is being unloaded $t$ hours after 4 am.

(a) Interpret $K(C^{-1}(10)) = 20$ in the context of this problem.

(b) Interpret $\displaystyle\int_5^{12} K'(b)\,db = 40$ in the context of this problem.

(c) Give a single mathematical equality involving the derivative of C which supports the following claim: It costs the bakery approximately \$0.70 less to buy 14.8 pounds of butter than to buy 15 pounds of butter.

(d) Assume that $u(t) > 0$ and $u'(t) < 0$ for $0 \leqslant t \leqslant 4$ and that $u(2) = 800$. Rank the following quantities in order from least to greatest.

I. 0 II. 800 III. $\displaystyle\int_1^2 u(t)\,dt$ IV. $\displaystyle\int_2^3 u(t)\,dt$

**Problem 2.** (Fall 2013 Final Exam) True or false:

(a) If $f(x)$ is an odd function and the tangent line to the graph of $f(x)$ at $x = 2$ is $y = 4(x-2)+7$, then the tangent line to the graph of $f(x)$ at $x = -2$ is $y = -4(x+2)-7$.

(b) If $h$ is an even function, and $\displaystyle\int_{-3}^{8} h(x)\,dx = 17$, then $\displaystyle\int_{-8}^{3} h(x)\,dx = 17$.

(c) If $\displaystyle\int_3^7 p(t)\,dt = -5$, then $\displaystyle\int_{-1}^{3} p(t-4)\,dt = -5$.

(d) If $f$ is a twice differentiable function such that $f''$ is continuous, $f'(3) > 0$ and $f''(3) < 0$, then $f(3+\Delta x) \leqslant f(3) + f'(3)\Delta x$ for all sufficiently small values of $\Delta x$.

**Problem 3.** (Fall 2012 Final Exam) Let $f(x)$ and $g(x)$ be increasing continuous functions defined on the interval $[0, 10]$ with $f(0) = g(0) = 0$. Also suppose $f$ is always concave down and $g$ is always concave up. For each of the following statements, determine whether it is always true, sometimes true, or never true.

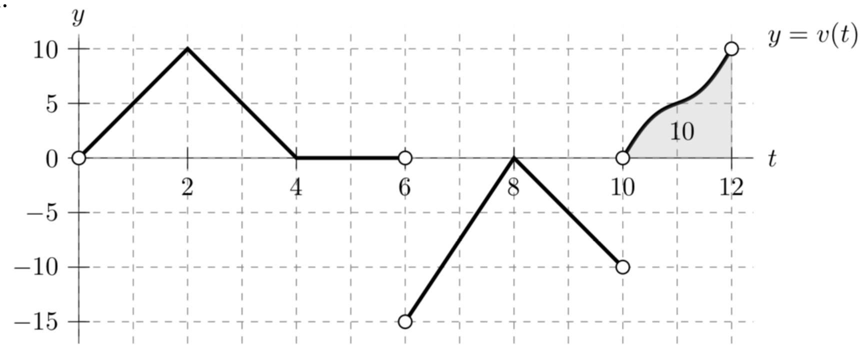
(a) $\displaystyle\int_0^{10} f(x)\,dx > \int_0^{10} g(x)\,dx$.

(b) $f'(10) < g'(10)$.

(c) $g'(0) > g'(2)$.

(d) $\displaystyle\int_0^{10} |f(x)|\,dx > \int_0^{10} f(x)\,dx$.

**Problem 4.** (Winter 2016 Final Exam) Elana goes on an amusement park ride that moves straight up and down. Let $v(t)$ model Elana's velocity (in meters/second) $t$ seconds after the ride begins (where $v(t)$ is positive when the ride is moving upwards, and negative when the ride is moving downwards). A graph of $v(t)$ for $0 < t < 12$ is shown below. Assume that $v(t)$ is piecewise linear for $0 < t < 6$ and $6 < t < 10$, and that the area of the shaded region is 10, as indicated on the graph.




(a) Write an integral that gives Elana's average velocity, in meters/second, from 2 seconds into the ride until 4 seconds into the ride. Then compute the exact value of this integral.

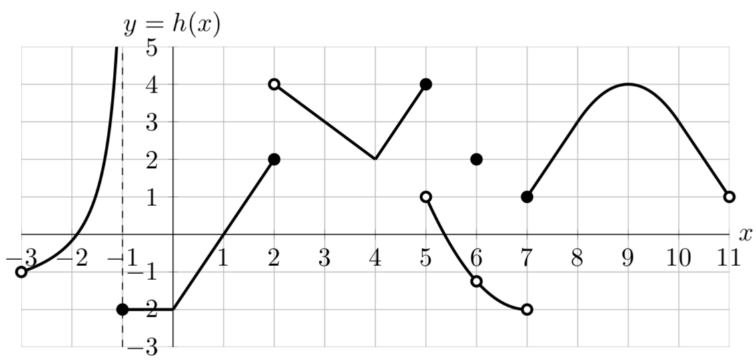
(b) Let $h(t)$ be Elana's height (in meters) above the ground $t$ seconds after the ride begins. Assume that $h$ is continuous, and suppose Elana is at a height of 10 meters above the ground when the ride begins. Fill in the exact values of $h(t)$ in the table below.

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $h(t)$ | | | | | | | |

(c) Sketch a detailed graph of $h(t)$ for $0 < t < 12$. In your sketch, be sure that you pay close attention to each of the following:

- where $h$ is increasing, decreasing, or constant;
- where h is/is not differentiable;
- the values of $h(t)$ you found above;
- the concavity of the graph of $y = h(t)$.

**Problem 5.** (Fall 2017 Final Exam) The graph of a portion of a function $y = h(x)$ is shown below. Note that the graph is linear where it appears to be linear, including on the intervals $[7, 8]$ and $[10, 11)$.




(a) Calculate the average value of $h(x)$ on $[-1, 1]$.

(b) Calculate $\displaystyle\int_{7.5}^{10.5} h''(x)$.

**Problem 6.** Are the following statements true for all continuous function $f(x)$ and $g(x)$? Give an explanation for you answer (e.g. reference any property of integrals that you may be using).

(a) If $\int_0^2 (f(x)+g(x))dx = 10$ and $\int_0^2 f(x)dx = 3$, then $\int_0^2 g(x)dx = 7$.

(b) If $\int_0^2 f(x)dx = 2$, then $\int_0^4 f(x)dx = 4$.

(c) If $\int_0^2 f(x)dx = 6$ and $g(x) = 2f(x)$, then $\int_0^2 g(x)dx = 12$.

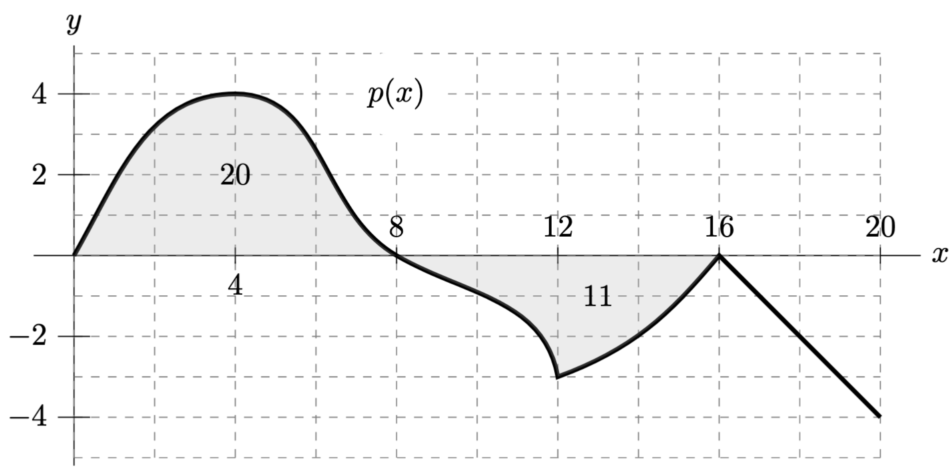
(d) If $a = b$, then $\int_a^b f(x)dx = 0$.

(e) $\int_1^2 f(x)dx + \int_2^3 g(x)dx = \int_1^3 (f(x)+g(x))dx$.

(f) If $f(x) \le g(x)$ on the inteval $[a, b]$, then the average value of $f$ is less than or equal to the average value of $g$ on the interval $[a, b]$.

(g) The average value of $f$ on the interval $[0, 10]$ is the average value of $f$ on $[0, 5]$ and the average value of $f$ on $[5, 10]$.

**Problem 7.** (Fall 2015 Final Exam) Recall that a function $h$ is odd if $h(-x) = -h(x)$ for all $x$. A portion of the graph of $p(x)$, an odd function, is shown below. Assume that the areas of the two shaded regions are 20 and 11, as indicated on the graph, and note that $p(x)$ is linear for $16 < x < 20$.




(a) Compute the exact value of $\int_0^{20} (5 - 3p(x))dx$.

(b) Compute the exact value of $\int_4^8 p'(x)dx$.

(c) Find the average value of $p(x)$ on the interval $-16 \le x \le 8$.

(d) Use a right Riemann sum with 3 equal subintervals to estimate $\int_{12}^{18} p(x)dx$. Write out all terms of the sum.

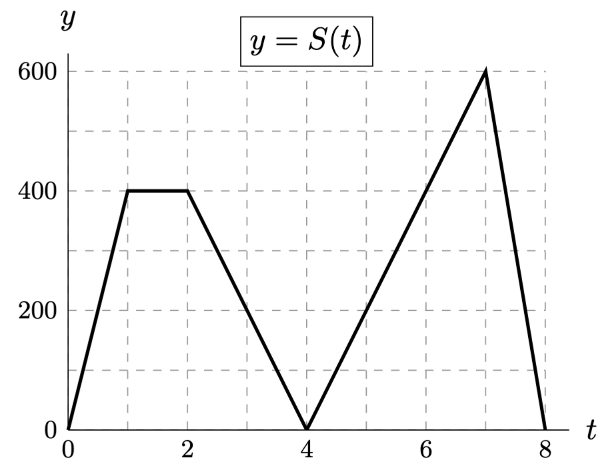
**Problem 8.** Compute the following integrals.

(a) If $f(x)$ is even and $\int_{-2}^{2}(f(x)-3)dx = 8$, find $\int_{0}^{2} f(x)dx$.

(b) Without any computation, find $\int_{-\pi/4}^{\pi/4} x^3\cos(x^2)dx$.

**Problem 9.** (Winter 2019 Final Exam)

Students from two rival universities had a competition to see who could clean up the most litter at a nature preserve.
University A went first, cleaning up litter from noon to 4pm. Each student from University A cleaned at a rate of 12 pounds of litter per hour. Then University B cleaned up litter from 4pm to 8pm. Each student from University B cleaned at a rate of 9 pounds of litter per hour.
Let $S(t)$ be the number of students cleaning up litter at time t hours past noon. The graph of $S(t)$ is shown to the right.




(a) Find the total amount of litter cleaned up by University A. Show your work.

(b) Find the total amount of litter cleaned up throughout the entire eight-hour competition. Show your work.

(c) The competition was broadcast live on TV. The number of people viewing the TV broadcast at time $t$ hours past noon is given by the function

$$B(t) = 4S(t) + 200$$

Find the average number of people viewing the TV broadcast during the eight-hour competition.

**Problem 10.** (Fall 2019 Final Exam) Given below is a table of values for a function $g(x)$ and its derivative $g'(x)$. The functions $g(x)$, $g'(x)$, and $g''(x)$ are all defined and continuous for all real numbers.

| $x$ | -3 | -2 | 0 | 2 | 3 | 4 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|
| $g(x)$ | 2 | 3 | 7 | 9 | 5 | 1 | -5 | -7 |
| $g'(x)$ | 0 | 4 | 1 | 0 | -2 | -4 | -1 | -3 |

Assume that between consecutive values of x given in the table above, g(x) is either always increasing or always decreasing. Find the quantities in a.–c. exactly, or write nei if there is not enough information provided to do so.

(a) $\int_{3}^{6} g(x)dx$ (b) $\int_{-2}^{2} 3g'(x)dx$ (c) $\int_{0}^{4}(g''(x)+x)dx$

(d) Use a right-hand Riemann sum with three equal subdivisions to estimate $\int_{2}^{8} g(x)dx$. Write out all the terms in your sum. Is this an underestimate or overestimate?