

Math 115

Worksheet Section 2.2

Problem 1. (a) What is the definition of the derivative of the function $f(x)$ at $x = c$?

(b) Compute the derivative of $g(x) = 3x^2$ at $x = 10$ **algebraically**. In other words, use algebra to find the limit from the definition exactly using limit computations we learned from the previous sections.

Problem 2. (Winter 2018 Exam 1) Let $m(x) = (1+x^2)^{3x-4}$. Which of the limits below represents $m'(2)$? There is only one correct answer. Be sure to explain your reasoning on the board.

(a) $\lim_{h \rightarrow 0} \frac{(1+x^2)^{3x-4} + h - 25}{h}$

(d) $\lim_{h \rightarrow 0} \frac{(1+(2+h)^2)^{3h+2} - 25}{h}$

(b) $\lim_{h \rightarrow 0} \frac{(1+h^2)^{3h-4} - 25}{h}$

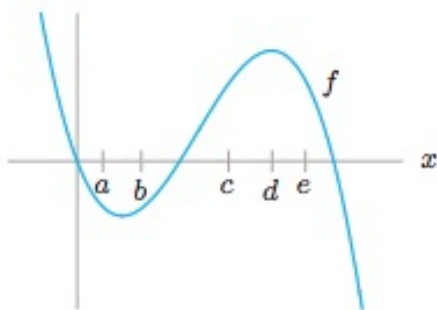
(e) $\lim_{h \rightarrow 0} \frac{(5+h^2)^{3h+2} - 25}{h}$

(c) $\lim_{h \rightarrow 0} \frac{(1+(2+h)^2)^{3h-4} - 25}{h}$

(f) $\lim_{h \rightarrow 0} \frac{(1+h^2)^{3h+2} - 25}{h}$

Problem 3. (2.2 #27) Create a table using difference quotients to approximate the derivative of x^x at $x = 2$ to one decimal place. You may use a calculator for this problem.

Problem 4. (2.2 #11) Match the derivatives in the table with the points a, b, c, d, e .



x	$f'(x)$
0	
0.5	
2	
-0.5	
-2	

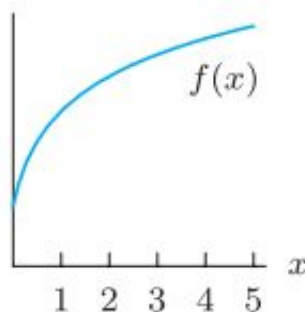
Problem 5. (2.2 # 15) For each of the following pairs, use the graph to decide which is larger. Explain.

(a) $f(3)$ or $f(4)$?

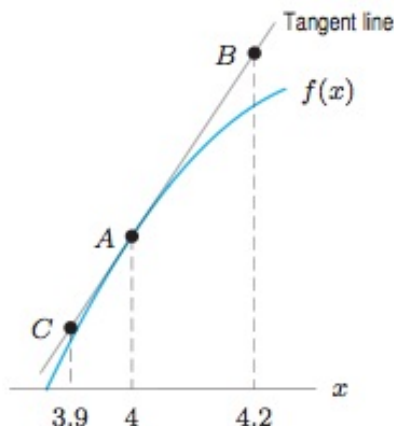
(b) $f(3) - f(2)$ or $f(2) - f(1)$?

(c) $\frac{f(2) - f(1)}{2 - 1}$ or $\frac{f(3) - f(1)}{3 - 1}$?

(d) $f'(1)$ or $f'(4)$?



Problem 6. (2.2 #17) The given function f has $f(4) = 25$ and $f'(4) = 1.5$. Find the coordinates



of the points A, B, C .

Problem 7. (Winter 2016 Exam 1) Consider the function g defined by

$$g(x) = \begin{cases} \frac{1}{e^x - 1} & \text{if } x < \frac{1}{2} \\ \cos(x^x) & \text{if } \frac{1}{2} \leq x < 5 \\ \frac{x^2}{(x-1)(6-x)} & \text{if } x \geq 5 \end{cases}$$

- Use the limit definition of the derivative to write an explicit expression for $g'(3)$. Your answer should not involve the letter g . Do not attempt to evaluate or simplify the limit.
- Find all vertical asymptotes of g , if there are any.

Problem 8. (Winter 2018 Exam 1) Sketch the graph of a single function $y = f(x)$ satisfying all of the following conditions:

- The domain of $f(x)$ is the interval $-8 < x \leq 6$.
- $f(x)$ is continuous on the interval $-8 < x < -2$.
- $f'(-7) = 0$.
- $f(x)$ is decreasing and concave up for all x in the interval $-6 < x < -4$.
- The average rate of change of $f(x)$ is equal to 0.5 between $x = -5$ and $x = -2$.
- $f(0) = 2$ and $f'(0) = -1$.
- $\lim_{x \rightarrow 2^-} f(x) = f(2)$ and $\lim_{x \rightarrow 2^+} f(x) < \lim_{x \rightarrow 2^-} f(x)$.
- $f(x)$ has constant rate of change on the interval $3 \leq x \leq 6$.

Make sure that your graph is large and unambiguous.