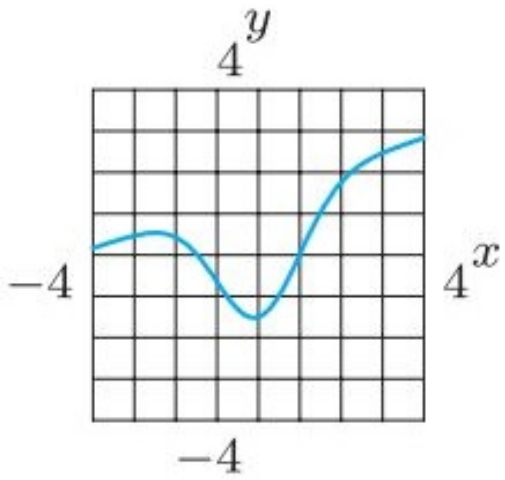
Math 115
Worksheet Section 2.3

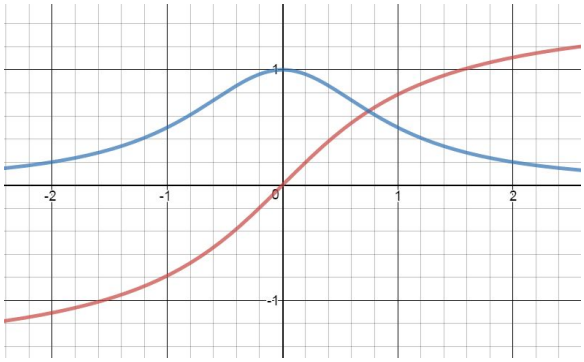
Warm-up questions

If $f'(x) < 0$ on an interval, then f is _____ on that same interval.
If $f'(x) > 0$ on an interval, then f is _____ on that same interval.
If f is constant on an interval, then $f'(x) =$_____ for all values of x in that interval.
If f is linear on an interval, then $f'(x) =$_____ for all values of x in that interval.

Problem 1. Given the graph below of a function $f(x)$, sketch $f'(x)$. (Note, this is a window from $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$).



Problem 2. One of these graphs is a function $g(x)$ and the other is $g'(x)$. Which is which?



Problem 3. Using the table below, estimate $f'(2)$. Where does $f'(x)$ appear to be positive, negative and zero? Where does it appear to be largest and smallest?

x	0	1	2	3	4	5	6	7	8
$f(x)$	18	13	10	9	9	11	15	21	30

Problem 4. Using the limit definition of the derivative, find the derivative of $f(x) = x^2$

Problem 5.

Figure 2.35 is the graph of f', the derivative of a function f. On what interval(s) is the function f

(a) Increasing? **(b)** Decreasing?

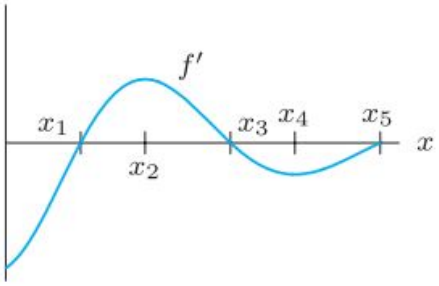
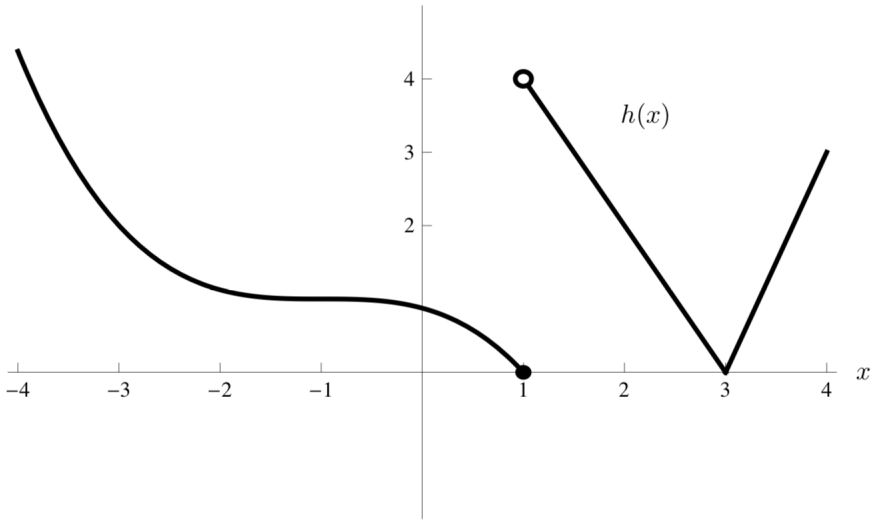


Figure 2.35: Graph of f', not f

Problem 6. Draw a graph of a continuous function $f(x)$ with

- $f'(x) > 0$ for $1 < x < 3$,
- $f'(x) < 0$ for $x < 1$ and $x > 3$ and
- $f'(x) = 0$ at $x = 1$ and $x = 3$.

Problem 7. (Winter 2013 Exam 1) Below is the graph of a function h.



Carefully draw a graph of $h'(x)$. Be sure to label important points or features on your graph.

Problem 8. (Fall 2009 Exam 1) If $f(x) = \frac{g(x)}{h(x)}$ and $h(3) = 0$, which of the following statements MUST be true?

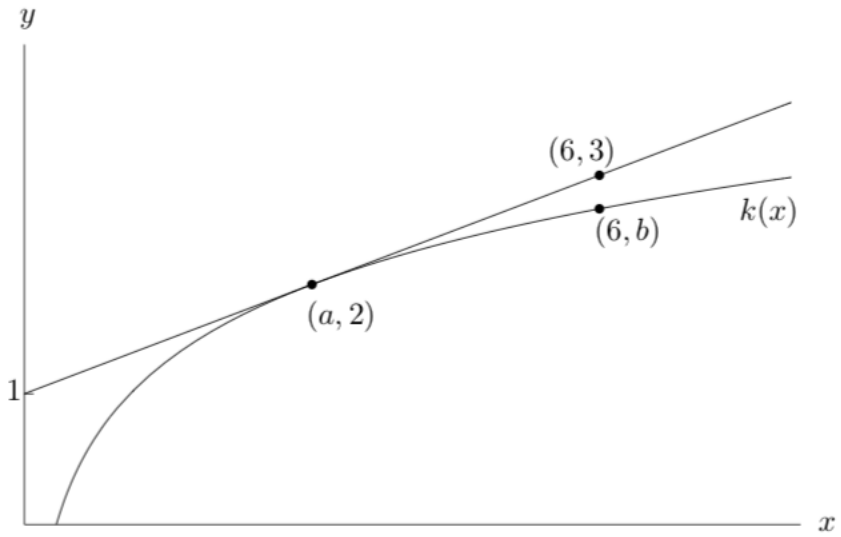
(a) The graph of f has a vertical asymptote at $x = 3$.

(b) 3 is not in the domain of f.

(c) f is not continuous on $[-2, 2]$.

(d) $\lim_{x \to 3} f(x)$ does not exist.

Problem 9. (Winter 2013 Exam 1) The figure shows the graph a function $k(x)$ and its tangent line at a point $(a, 2)$. The average rate of change of $k(x)$ between $x = a$ and $x = 6$ is $\frac{1}{6}$. Find exact numerical values for the following. If it is not possible to find a value, write **NP**.



(a) $a =$

(b) $b =$

(c) $k'(2) =$

(d) $k'(a) =$

(e) $k'(6) =$

Problem 10. (Fall 2017 Exam 1) On the board, sketch the graph of a function $y = R(x)$ verifying all of the following conditions:

- The function $R(x)$ is defined on $-8 \leqslant x \leqslant 9$.
- $R'(x) = 2$ for $-8 < x < -5$.
- $R(x)$ is concave down and decreasing on $-5 < x < -2$.
- $R(-2) = 1$.
- $R(x) = R(-x)$ for $-2 \leqslant x \leqslant 2$.
- The vertical intercept of $R(x)$ is $y = 3$.
- $\lim_{x \to 5^-} R(x) = -2$ but $\lim_{x \to 5^+} R(x)$ does not exist.
- $R(x)$ is not continuous at $x = 7$ but $\lim_{x \to 7} R(x)$ exists.

Make sure that your graph is large and unambiguous.

Problem 11. (Winter 2013 Exam 1) In each of the following problems, give a formula for a function whose domain is all real numbers, with all of the indicated properties. If there is no such function, then write "NO SUCH FUNCTION EXISTS". You do not need to show your work.

(a) A sinusoidal function $P(t)$ with the following three properties:

- The period of the graph of $P(t)$ is 7.
- The graph of $P(t)$ attains a maximum value at the point $(1, 20)$.
- The graph of $P(t)$ attains a minimum value at the point $(-2.5, 6)$.

(b) A function $h(x)$ with the following two properties:

- $h(x)$ is concave down for all x.
- $h(x) > 0$ for all x.

(c) A function $j(x)$ with the following two properties:

- $j(x)$ is decreasing for all x.
- $j(x)$ is concave up for all x.

(d) A rational function $l(x)$ with the following two properties:

- $l(0) = 2$.
- The line $y = 2$ is a horizontal asymptote to the graph of $l(x)$.