

Worksheet Section 1.6

- Problem 1.** (a) Every exponential _____ function eventually dominates every _____.
- (b) Consider the rational function $r(x) = \frac{p(x)}{q(x)}$. If the polynomials $p(x)$ and $q(x)$ have no common zeroes, then:
- Zeros of $p(x)$ give rise to _____.
 - Zeros of $q(x)$ give rise to _____.
- (c) When does $r(x)$ have a horizontal asymptote?

Problem 2. Which function dominates as $x \rightarrow \infty$?

- (a) e^x or $10^{1000} \ln(x)$ (b) $1000x^4$ or $0.2x^5$ (c) $10e^{0.1x}$ or $5000x^2$
- (d) $100x^5$ or 1.05^x (e) x^4 or $\ln(x)$ (f) e^x or 2.71^x

Problem 3. Which of the following functions have the given properties?

- (a) $y = \frac{x^2-2}{x^2+2}$ (b) $y = \frac{x^2+2}{x^2-2}$ (c) $y = (x-1)(1-x)(x+1)^2$
- (d) $y = x^3 - x$ (e) $y = x - \frac{1}{x}$ (f) $y = (x^2 - 2)(x^2 + 2)$

- (i) A polynomial of degree 3.
 (ii) A polynomial of degree 4.
 (iii) A rational function that is not a polynomial.
 (iv) Exactly two distinct zeros.
 (v) Exactly one vertical asymptote.
 (vi) More than two distinct zeros.
 (vii) Exactly two vertical asymptotes.
 (viii) A horizontal asymptote.

Problem 4. (1.6 #41–44) Assuming the window is large enough to show end behavior, identify the graph as that of a rational function, exponential function, or logarithmic function.

41.

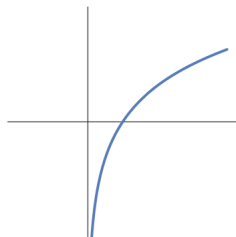


Figure 1.90

42.

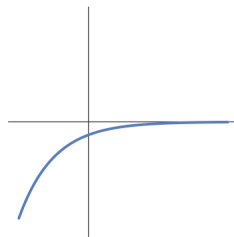


Figure 1.91

43.

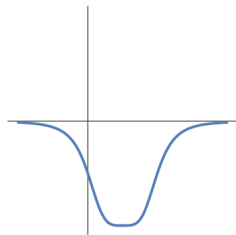


Figure 1.92

44.

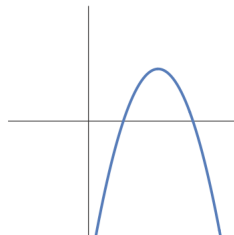


Figure 1.93

Problem 5. (Fall 2017 Exam 1) Consider the rational function r defined by

$$r(x) = \frac{3(x - \sqrt{2})(\pi x + 7)^2(x + 1)}{(x + 1)(x - \sqrt{3})}.$$

For all of the following parts of this problem, leave your answers in exact form.

- What is the domain of $r(x)$?
- Find the equations of all vertical asymptotes of $r(x)$. If there are none, write none.
- Let $p(x) = x^2 + 1.2x - 5$. Find the equations of all horizontal asymptotes of $\frac{r(x)}{p(x)}$. If there are none, write NONE. Show your work or reasoning to justify your answer.

Problem 6. (Winter 2017 Exam 1) A group of students planted a pine tree and an oak tree alongside the Diag. Let $P(t)$ and $O(t)$ be the height (in feet) of the pine and the oak t years after they were planted, where

$$P(t) = 170 - 165A^{-0.02t} \text{ and } O(t) = \frac{140}{2 + 100e^{-0.3t}}$$

where $A > 1$ is a constant.

- How tall (in feet) were each of the trees when they were planted?
- Ten years after the trees were planted, the height of the pine was 38 ft. Find the value of A . Find your answer algebraically and show all your work.
- How many years after being planted does it take the oak to be 38 ft? Find your answer algebraically and show all your work.

Problem 7. Use a graphing calculator or Desmos to graph $y = x^4$ and $y = 3^x$. Determine approximate domains and ranges that give each of the graphs in the figure below.

