Math 115
Worksheet Section 3.3

Warm-up question

$(f(x)g(x))' =$ $\left(\frac{f(x)}{g(x)}\right)' =$

Problem 1. Differentiate $f(x) = e^{2x}$ by writing it as $e^x \cdot e^x$.

Problem 2. Using only the quotient rule and the derivatives of sine and cosine, find the derivative of the function $f(x) = \tan(x)$.

Problem 3. For what intervals is $f(x) = xe^x$ concave up?

Problem 4. The quantity, q, of a certain skateboard sold depends on the selling price, p, in dollars, so we write $q = f(p)$. You are given that $f(140) = 15,000$ and $f'(140) = -100$.

(a) What do $f(140) = 15,000$ and $f'(140) = -100$ tell you about the sales of skateboards?

(b) The total revenue, R, earned by the sale of skateboards is given by $R = pq$. Find $\frac{dR}{dp}|_{p=140}$. (This is different notation for $R'(140)$.)

(c) What is the sign of $\frac{dR}{dp}|_{p=140}$? If the skateboards are currently selling for \$140, what happens to revenue if the price is increased to \$141?

Problem 5. Find the equation of the tangent line to the graph of $f(x) = \frac{2x-5}{x+1}$ at $x = 0$.

Problem 6. (a) Differentiate $y = \frac{e^x}{x}, y = \frac{e^x}{x^2}$, and $y = \frac{e^x}{x^3}$.

(b) What do you anticipate the derivative of $y = \frac{e^x}{x^n}$ will be? Confirm your guess.

Problem 7. Let $f(v)$ be the gas consumption (in liters/km) of a car going at velocity v (in km/hr). In other words, $f(v)$ tells you how many liters of gas the car uses to go one kilometer if it is going at velocity v. You are told that

$$f(80) = 0.05 \text{ and } f'(80) = 0.0005\,.$$

(a) Let $g(v)$ be the distance the same car goes on one liter of gas at velocity v. What is the relationship between $f(v)$ and $g(v)$? Find $g(80)$ and $g'(80)$ and give practical interpretations of these values.

(b) Let $h(v)$ be the gas consumption in liters per hour. In other words, $h(v)$ tells you how many liters of gas the car uses in one hour if it is going at velocity v. What is the relationship between $h(v)$ and $f(v)$? Find $h(80)$ and $h'(80)$ and give practical interpretations of these values.

Problem 8. Find the derivative of the following functions:

i. $f(x) = x \cdot 2^x$

ii. $f(t) = \sin(5)(t^2 + 3)e^t$

iii. $f(w) = \dfrac{w^{3.2}}{5^w}$

iv. $f(t) = \dfrac{t-3}{t+3}$

v. $f(x) = 2t \cdot x \cdot e^t - \dfrac{1}{\sqrt{t}}$

vi. $f(y) = \dfrac{4}{3 + \sqrt{y}}$

vii. $f(z) = \dfrac{az+b}{cz+k}$

viii. $f(x) = (2 - 3x^2)(6x^e - 3\pi)$

ix. $f(x) = (3x^2 + \pi)(e^x - 4)$

Problem 9. (Fall 2017 Exam 2) Let g be a twice differentiable function defined on $-1 < x < 11$. Some values of $g(x)$, $g'(x)$ and $g''(x)$ are shown in the table below.

x	0	2	4	6	8	10
$g(x)$	-2	-1	3	4	5	6
$g'(x)$	0.5	2	?	5	1	2
$g''(x)$	2	1	5	-3	-1	0.5

(a) Let $k(x) = g(x)g'(x)$. Find the value of $g'(4)$ if $k'(4) = 15$.

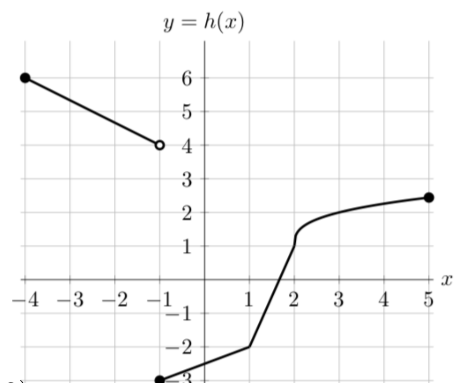
(b) Let $r(x) = \frac{\sin(x)}{g(x)}$. Find $r'(0)$.

Problem 10. Consider the family of functions

$$f(x) = \frac{ax^b}{e^x}, \quad x \geqslant 0$$

where $a > 0$ and $b > 1$ are positive constants. Find the values of x for which the tangent lines of $y = f(x)$ are horizontal. Your answer will contain a and b.

Problem 11. (Fall 2017 Exam 2) Consider the graph of $h(x)$ below. Note that h is linear on the intervals $[-4, -1)$, $[-1, 1]$ and $[1, 2]$, differentiable on $(2, 5)$, and has a sharp corner at $x = 2$.



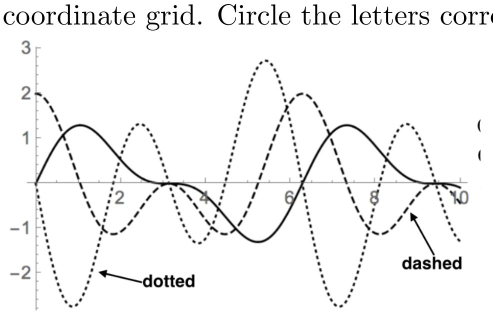
Let $g(x) = xh(x)$. Find $g'(-2)$.

Problem 12. The equation of motion for a particle is given by $s(t) = \cos(t)\sin(t) + \sin(t)$.

(a) Find the velocity and acceleration at time t.

(b) Use $\sin^2(x)+\cos^2(x) = 1$ to write the velocity function $s'(t)$ in terms of the cosine function. Then factor the result in order to find the exact times when the velocity is 0.

(c) Here's a graph of the position, velocity, and acceleration functions together on the same coordinate grid. Circle the letters corresponding to correct statements:



(a) The graph of the position function is dotted.

(b) The graph of the position function is solid.

(c) The graph of the velocity function is dashed.

(d) The graph of the velocity function is dotted.