Math 115
Worksheet Section 3.2

Warm-up question

$(e^x)' =$ _____ $(a^x)' =$ _____ $(\sin(x))' =$ _____ $(\cos(x))' =$_____

Problem 1. An animal population is given by $P(t) = 300(1.044)^t$ where t is the number of years since the study of the population began. Find $P'(5)$ and provide a practical interpretation of your result.

Problem 2. Find the quadratic polynomial $g(x) = ax^2 + bx + c$ which best fits the function $f(x) = e^x$ at $x = 0$, in the sense that

$$g(0) = f(0) \text{ and } g'(0) = f'(0) \text{ and } g''(0) = f''(0)$$

Using a computer or calculator, sketch the graphs of f and g on the same axes. What do you notice?

Problem 3. Using the equation of the tangent line to the graph of e^x at $x = 0$, show that

$$e^x \geq 1 + x$$

for all values of x. A sketch may be helpful.

Problem 4. Find the 50^{th} derivative of $y = \cos(x)$.

Problem 5. Find the tangent lines to $f(x) = \sin(x)$ at $x = \frac{4\pi}{3}$ and at $x = \frac{10\pi}{3}$. Graph them, along with $\sin(x)$. What do you notice? Express the second as a transformation of the first.

Problem 6. Are the following statements true or false? Give an explanation for your answer.

(a) If $f(x)$ is increasing, then $f'(x)$ is increasing.

(b) There is no function such that $f'(x) = f(x)$ for all x besides the constant function $f(x) = 0$.

(c) There is no function such that $f'(x) = -f(x)$ for all x besides the constant function $f(x) = 0$.

(d) There is no function such that $f''(x) = -f(x)$ for all x besides the constant function $f(x) = 0$.

(e) If $f(x)$ is defined for all x, then $f'(x)$ is defined for all x.

Problem 7. For what value(s) of a are $y = a^x$ and $y = x + 1$ tangent at $x = 0$?

Problem 8. (Winter 2017 Exam 3) A Math 115 coordinator is trying to create functions with certain properties in order to test students' understanding of various calculus concepts. She wants a function $f(x)$ of the form

$$f(x) = \begin{cases} ax^2 + ax + be^x & \text{for } x < 0 \\ a + 2\cos(x) & \text{for } x \geq 0 \end{cases}$$

where a and b are constants. Find all value(s) of a and b for which $f(x)$ be differentiable at $x = 0$. Show enough work to justify your answer.

Problem 9. Explain for which values of $a > 0$ the function a^x is increasing and for which values it is decreasing. Is this consistent with your formula for $(a^x)'$?

Problem 10. Give an example of:

(a) An exponential function for which the derivative is always negative.

(b) A function f such that $f'''(x) = f(x)$.

Problem 11. In 2009, the population of Mexico was 111 million and growing 1.13% annually, while the population of the US was 307 million and growing 0.975% annually. If we measure growth rates in people/year, which population was growing faster in 2009? Note $\ln(1.0113) \approx 0.01124$ and $\ln(1.00975) \approx 0.009702$.