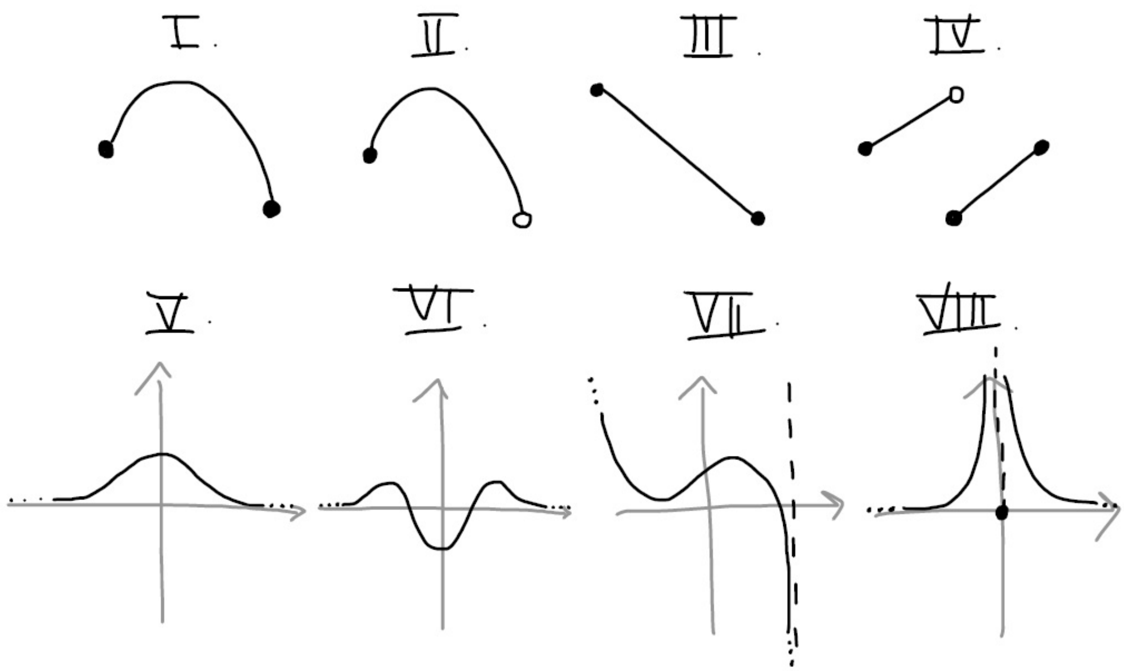
Math 115
Worksheet Section 4.2

A point p in the domain of f is a **global maximum** of f on a given interval if $f(p) \geq f(x)$ for all x in the interval.

A point p in the domain of f is a **global minimum** of f on a given interval if $f(p) \leq f(x)$ for all x in the interval.

Problem 1. For each of the following graphs, locate any global maxima or minima on the interval drawn, if they exist.



Problem 2. Let $g(x) = x^3 e^{-x}$.

(a) Find any global minima and maxima of g on $(1, \infty)$.

(b) Now find any global minima and maxima of g on $[0, \infty)$.

(c) Now find any global minima and maxima of g on $(-\infty, \infty)$.

Problem 3. Find any global extrema of the following function on its domain.

$$h(x) = \begin{cases} 2e^{x-1} & 0 \leq x \leq 1 \\ (x-3)^2 - 2 & 1 < x \leq 4 \end{cases}$$

Problem 4. (Winter 2018 Exam 2) The amount of chlorine in a chemical reaction $C(t)$ (in gallons) t seconds after it has been added into a solution is given by the function

$$C(t) = 2 - 3(t-5)^{\frac{4}{5}}(t-1)e^{-t} \qquad \text{for } t \geqslant 0.$$

Notice that

$$C'(t) = \frac{3(t-6)(5t-9)e^{-t}}{5(t-5)^{\frac{1}{5}}} \qquad \text{for } t \geqslant 0.$$

(a) Use calculus to find the time(s) (if any) at which the amount of chlorine in the solution is the greatest and the smallest.

Problem 5. (Fall 2017 Exam 2) Blizzard the snowman and his mouse friend Gabe arrived in Montana, where it has recently snowed. Since Blizzard is still melting, they decide to use this time to pack extra snow onto Blizzard, to help him make it to the North Pole. Let $H(t)$ be Blizzard's height, in inches, if Blizzard and Gabe stay in Montana for t hours. On the interval $1 \leqslant t < \infty$, the function $H(t)$ can be modeled by

$$H(t) = 35 + 10e^{-\frac{t}{6}}(t-2)^{\frac{1}{3}}.$$

Notice that

$$H'(t) = \frac{-5e^{-\frac{t}{6}}(t-4)}{(t-2)^{\frac{1}{3}}}.$$

(a) Find all values of t that give global extrema of the function $H(t)$ on the interval $1 \leqslant t < \infty$. Use calculus to find your answers, and be sure to show enough evidence that the point(s) you find are indeed global extrema. For each answer, write none if appropriate.

(b) Assuming Blizzard stays in Montana for at least 1 hour, what is the tallest height Blizzard can reach? Remember to include units.

Problem 6. (Winter 2017 Exam 2) At Happy Hives Bee Farm, the population of bees, in thousands, t months after the farm opens, can be modeled by $g(t)$, where

$$g(t) = \begin{cases} 20 + \frac{1}{3}e^{4-t} & \text{for } 0 \leqslant t \leqslant 4 \\ -\frac{1}{6}t^3 + \frac{9}{4}t^2 - 7t + 23 & \text{for } 4 < t \leqslant 8 \end{cases}$$

and

$$g'(t) = \begin{cases} -\frac{1}{3}e^{4-t} & \text{for } 0 \leqslant t \leqslant 4 \\ -0.5(t-2)(t-7) & \text{for } 4 < t \leqslant 8 \end{cases}$$

(a) Find the values of t that minimize and maximize $g(t)$ on the interval $[0, 8]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema. For each answer, write none if appropriate.

(b) What is the largest population of bees that occurs in the first 8 months the farm is open?

Problem 7. (Fall 2017 Exam 2) Sketch graphs of functions $f(x)$ and $g(x)$ satisfying the conditions below, or say that no such function exists.

(a) A function $f(x)$ defined on the interval $(0, 4)$ that satisfies

- $f'(x) > 0$ for all $x \neq 2$, and
- $x = 2$ is a global minimum.

(b) A function $g(x)$ defined on the interval $(0, 4)$ that satisfies

- $\lim_{x \to 2^-} g'(x) = \infty$, and
- $\lim_{x \to 2^+} g'(x) = 0$.

Problem 8. (Winter 2017 Exam 2) Sketch the graph of a single function $y = h(x)$ satisfying all the following:

- The function $h(x)$ is defined for $-7 \leqslant x \leqslant 7$.
- $h(x)$ has global maximums at $x = -7$ and $x = 3$.
- $h(x)$ has an inflection point at $x = -5$.
- $h(x)$ is continuous at $x = -3$ but not differentiable at $x = -3$.
- $h(x)$ has a local minimum at $(-1, -4)$ but is not continuous at $x = -1$.
- $h(x)$ has a critical point at $(2, 5)$ that is neither a local maximum or a local minimum.
- $h(x)$ satisfies the conclusion of the Mean Value Theorem on $[4, 7]$ but not the hypothesis of this theorem.

Problem 9. (Winter 2016 Exam 2) Consider a continuous function T with the following properties.

- $T(v)$ is defined for all real numbers v.
- The critical points of $T(v)$ are the four points $v = 3, v = 5, v = 7$, and $v = 8$. ($T(v)$ has no other critical points.)

Some values of T are shown in the following table:

v	0	3	5	7	8	10
$T(v)$	21	9	13	19	11	21

For each of a.-f. below, use the answer blank provided to list all the values v at which $T(v)$ attains the specified global extremum. If there is not enough information provided to give an answer, write "not enough info". If $T(v)$ does not attain the specified global extremum on the specified interval, write "none".

For what value(s) v does $T(v)$ attain its...

(a) global minimum on the interval $0 \leq v \leq 10$?

(b) global maximum on the interval $0 \leq v \leq 10$?

(c) global minimum on the interval $0 < v \leq 10$?

(d) global maximum on the interval $0 < v \leq 10$?

(e) global minimum on the interval $(-\infty, \infty)$?

(f) global maximum on the interval $(-\infty, \infty)$?

Problem 10. (Winter 2016 Exam 2) Sketch the graph of a single function $y = g(x)$ satisfying all the following:

- $g(x)$ is defined for all x in the interval $-6 < x < 6$.
- $g(x)$ has at least 5 critical points in the interval $-6 < x < 6$.
- The global maximum value of $g(x)$ on the interval $-5 \leqslant x \leqslant -3$ is 4, and this occurs at $x = -4$.
- $g(x)$ is not continuous at $x = -2$.
- $g'(x)$ (the derivative of g) has a local maximum at $x = 0$.
- $g(x)$ is continuous but not differentiable at $x = 1$.
- $g''(x) \geqslant 0$ for all x in the interval $2 < x < 4$.
- $g(x)$ has at least one local minimum on the interval $4 < x < 6$ but does not have a global minimum on the interval $4 < x < 6$.
- $g(x)$ has an inflection point at $x = 5$.