Math 115
Worksheet Section 5.4

Problem 1. Find each integral, given that $\int_a^b f(x)dx = 8$, $\int_a^b (f(x))^2 dx = 12$, and $\int_a^b g(t)dt = 2$.

(a) $\int_b^a (f(x))^2 dx - \left(\int_a^b f(x)dx \right)^2$

(b) $\int_a^b (c_1 g(x) + (c_2 f(x))^2) dx$

(c) $\int_{a+5}^{b+5} f(x-5)dx$

Problem 2. If $f(x)$ is an odd function and $\int_{-2}^{3} f(x)dx = 30$, find $\int_2^3 f(x)dx$.

Problem 3. Without computation, show that $2 \leq \int_0^2 \sqrt{1+x^3}dx \leq 6$. *Hint: carefully draw a graph.*

Problem 4. Without computation, explain why both of the following statements must be false.

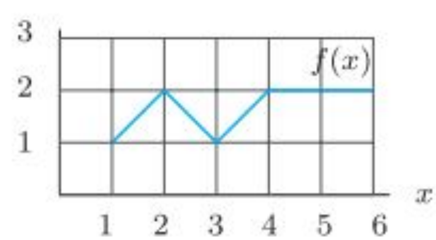
$$(a) \int_{-2}^{-1} e^{x^2} dx = -3 \qquad (b) \int_{-1}^{1} \left| \frac{\cos(x+2)}{1+\tan^2 x} \right| dx = 0$$

Problem 5. (a) Let $s(t)$ be the position function of a particle and let $v(t)$ be its velocity. Give a formula for the average velocity of the particle from $t = a$ to $t = b$ in terms of $s(t)$ and give another formula in terms of $v(t)$.

(b) For any function $f(x)$, what is the average value of the function from $x = a$ to $x = b$?

Problem 6. A function f is graphed below

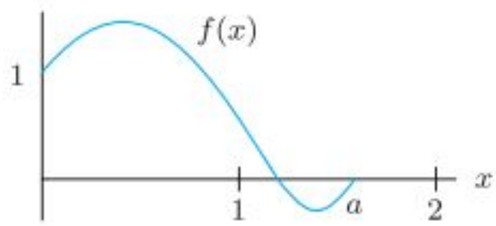
(a) Use the figure to find $\int_1^6 f(x)dx$



(b) What is the average value of f on $[1, 6]$?

Problem 7. If the average value of f on the interval $2 \leq x \leq 5$ is 4, find $\int_2^5 (3f(x) + 2)dx$.

Problem 8. The graph of f is given below. List the following quantities from least to greatest.



(I) $f'(1)$

(II) The average value of $f(x)$ on $0 \leq x \leq a$.

(III) The average value of the rate of change of $f(x)$, for $0 \leq x \leq a$.

(IV) $\int_0^a f(x)dx$

Problem 9. A bar of metal is cooling from 1000°C to room temperature, 20°C. The temperature, H, of the bar t minutes after it starts cooling is given, in °C, by

$$H = 20 + 980e^{-0.1t}.$$

(a) Find the temperature of the bar at the end of one hour.

(b) Write an integral that gives the average value of the temperature over the first hour.

(c) Is your integral in (b) greater or smaller than the average of the temperatures at the beginning and the end of the hour? Explain this in terms of the concavity of the graph of H.

Problem 10. (Fall 2016 Final Exam) The table below gives several values of a function $q(u)$ and its first and second derivatives. Assume that all of $q(u)$, $q'(u)$, and $q(u)$ are defined and continuous for all real numbers u.

u	0	1	2	3	4	5	6
$q(u)$	30	23	19	20	24	25	24
$q'(u)$	0	-6	-2	1	3	1	-2
$q''(u)$	-9	5	4	3	2	-5	0

Compute each of the following. Do not give approximations. If it is not possible to find the value exactly, write not possible.

(a) Compute $\int_5^2 q''(t)\, dt$.

(b) Suppose $q(u)$ is an even function. Compute $\int_{-5}^5 (q'(u) + 7)\, du$.

(c) Suppose $q(u)$ is an even function. Compute $\int_{-5}^5 q(u)\, du$.

Problem 11. Suppose n is a positive integer, f is a decreasing, continuous function on $[2,6]$, the value of the left Riemann sum with n equal subdivisions for $\int_2^6 f(x)\,dx$ is A, and $f(2) = f(6)+8$. Circle all the statements that must be true.

(a) A is an overestimate for $\int_2^6 f(x)\,dx$.

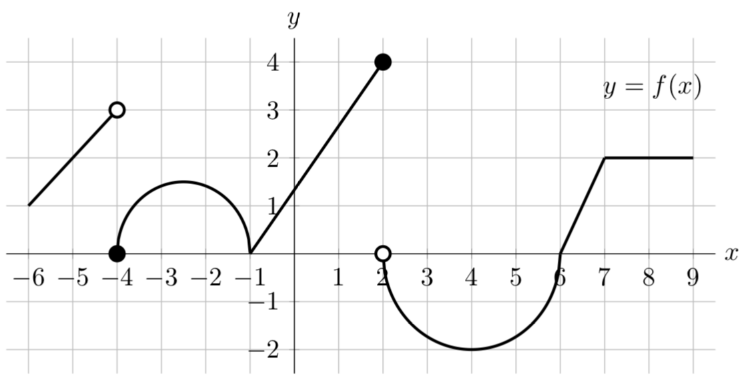
(b) $\int_2^6 f(x)\,dx = 8$.

(c) $\int_1^5 f(x+1)\,dx = \int_2^6 f(x+1)\,dx$.

(d) The left Riemann sum for $\int_2^6 (f(x))^2\,dx$ with n equal subdivisions is A^2.

(e) none of these

Problem 12. (Winter 2017 Final Exam) The graph of f shown below consists of lines and semicircles.



Use the graph above to calculate the answers to the following questions. Give your answers as exact values. You do not need to show work. If any of the answers cannot be found with the information given, write *nei*.

(a) Find the average value of $f(x)$ on $[-4, 2]$.

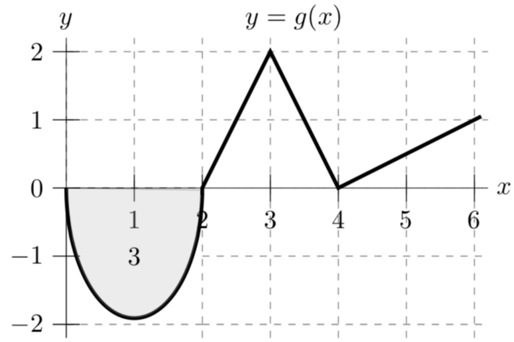
(b) Find the value of $\int_4^9 |f(z)|\,dz$.

(c) Find the value of $4 \leqslant T \leqslant 9$ such that $\int_4^T f(z)\,dz = 0$.

(d) Find the value of $\int_{-8}^{-7} f(x+2)+1\,dx$.

Problem 13. (Winter 2013 Final Exam) If $\int_{-1}^{4} 2f(x) - 3\,dx = -31$, find $\int_{-1}^{4} f(x)\,dx$.

Problem 14. (Winter 2016 Final Exam) A portion of the graph of a continuous function $g(x)$ is shown below. Assume that the area of the shaded region is 3 (as indicated on the graph), and note that $g(x)$ is piecewise linear for $2 < x < 6$.



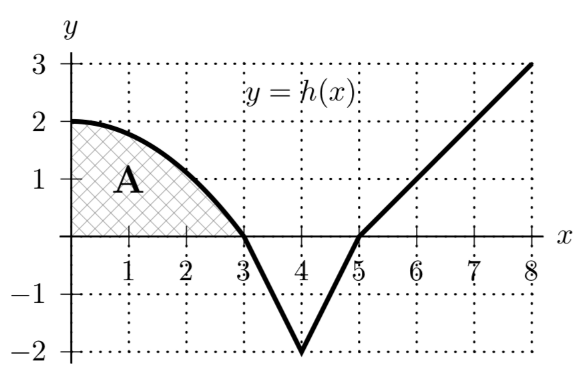
(a) Find $\displaystyle\int_0^6 g(x)\,dx$.

(b) Find $\displaystyle\int_0^2 5 - 4g(x)\,dx$.

(c) Suppose $C(x) = \ln(g(x))$. Find $C'(2.5)$.

(d) Find $\displaystyle\int_2^4 g(x+2) - g(x-2)\,dx$.

Problem 15. (Winter 2014 Final Exam) The graph of a function $h(x)$ is shown below. The area of the shaded region A is 4, and $h(x)$ is piecewise linear for $3 \leqslant x \leqslant 6$.



Compute each of the following. If there is not enough information to compute a value exactly, write *not enough info*.

(a) Find $\displaystyle\int_0^3 h(t) + 2\,dt$.

(b) Find the average value of $h(x)$ on the interval $[0, 4]$.

(c) Let $J(x) = \sin(\pi h(x))$. Find $J'(3.5)$.

(d) Let $g(x) = e^x$. Find $\displaystyle\int_6^7 g'(x)h(x) + g(x)h'(x)\,dt$.