Math 115
Worksheet Section 4.3

Optimization questions: what to do?

Read the problem carefully:

- What is the quantity to be optimized?
- What information is given?
- Draw a picture that helps you organize the information given.
- Introduce variables to label the information given to you.
- What formulas are available that could be useful?
- Write a function for the quantity you will optimize using your formulas.
- Determine the domain of your function.
- Use what you learned in sections 4.1 and 4.2 to find your answer(s).

Problem 1. City council is planning to construct a new sports ground in the shape of a rectangle with semicircular ends. A running track 400 meters long is to go around the perimeter.

(a) What choice of dimensions will make the rectangular area in the center as large as possible?

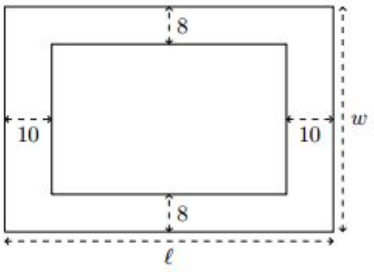
(b) What dimensions would maximize the total area enclosed by the running track?

Problem 2. The sum of two positive numbers is 48. What is the smallest possible value of the sum of their squares?

Problem 3. A factory makes cylindrical cans of volume 500 cubic centimeters. Suppose the metal for the sides of the can costs 1 cent per cm^2, and the metal for the top and the bottom costs twice as much. Find the dimensions of the can that minimizes its cost.

Problem 4. A wire six meters long is cut into 12 pieces. These pieces are welded together at right angles to form the frame of a box with square base. Where should the cuts be made to maximize the total surface area of the box?

Problem 5. (Fall 2013 Exam 2) Liam wants to build a 1600 square foot, rectangular swimming pool behind his new house. He also wants 8-foot wide decks on two sides of the pool and 10-foot wide decks on the other two sides of the pool (see the diagram below).



(a) Let ℓ and w be the length and width (in feet) of the pool area including the decks as shown in the diagram. Write a formula for ℓ in terms of w.

(b) Write a formula for the function $A(w)$ which gives the total area (in square feet) of the pool and the decks in terms of only the width w. Your formula should not include the variable ℓ. (This is the function Liam would minimize in order to find the minimum area that his pool and deck will take up in his yard. You do not need to do the optimization in this case.)

(c) What is the domain of w?

Problem 6. (Fall 2016 Exam 2) Suma is making cylindrical paper cups that will be used to serve milkshakes at Qabil's Creamery. She rolls paper into a cylinder and then attaches it to the base. The thicker material that she uses for the base costs \$4.30 per square meter, and the lighter material that she uses for the vertical part of the cup costs \$2.20 per square meter. The radius of the circular base is r meters, and the height of the cup is h meters. It may be helpful to know that the surface area of the vertical portion of the cup is $2\pi rh$. Note: the top of the cup is left open.

Throughout this problem, assume that the material that Suma uses to make one paper cup costs \$0.12.

(a) Find a formula for h in terms of r.

(b) Let $V(r)$ be the volume (in cubic meters) of the cup that Suma makes given that the material for the cup costs \$0.12 and the radius of the cup is r meters. Find a formula for $V(r)$. The variable h should not appear in your answer. (Note: This is the function that Suma would use to find the value of r maximizing the volume of the cup, but you should not do the optimization in this case.)

(c) In the context of this problem, what is the domain of $V(r)$?

Problem 7. (Fall 2015 Exam 2) The engineer Elur Niahc has been commissioned to build a park for the citizens of Srebmun Foyoj. The park will consist of a square attached to a rectangular dog park. The fencing for the dog park costs \$4 per linear meter, and the fencing for the three remaining sides of the square portion of the park costs \$6 per linear meter.

(a) Assume that Elur spends \$2400 on fencing. The resulting park will have width w meters, and the length of the dog park will be l meters. Find a formula for l in terms of w.

(b) Let $A(w)$ be the total area (in square meters) of the resulting park (including the dog park) if the width is w meters and Elur spends \$2400 on fencing. Find a formula for the function $A(w)$. The variable l should not appear in your answer. (Note: This is the function that Elur would use to find the value of w maximizing the area of the park, but you should not do the optimization in this case.)

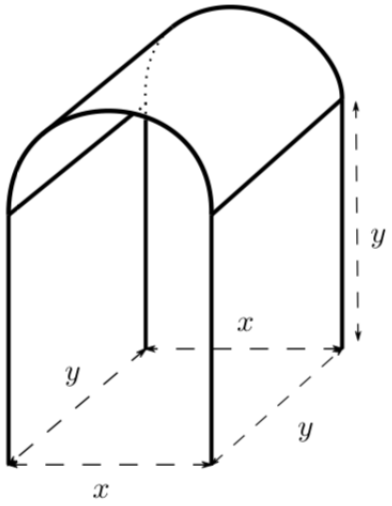
(c) In the context of this problem, what is the domain of $A(w)$?

Problem 8. (Winter 2018 Exam 2) The following formulas may be useful in this problem:

- the surface area of a cylinder of radius r and length ℓ is $2\pi r\ell$,
  and
- the volume of a cylinder of radius r and length l is $\pi r^2\ell$.

The Public Transit Authorities (PTA) are designing rain shelters for their bus stops. They decide to place a roof in the shape of half a cylinder on four vertical legs of height y feet. The four legs are placed in a rectangle on the ground with width x feet and length y feet.
The costs of production are:

- \$25 for each foot of the total length of the legs,
- \$40 for each square foot of the area of the roof.



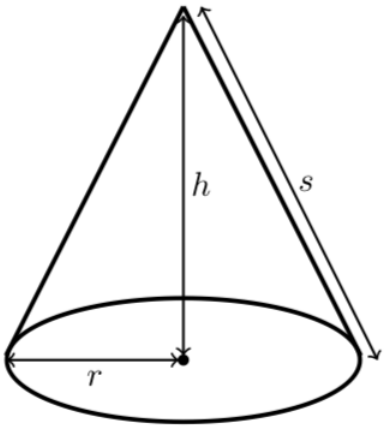
The PTA would like to spend exactly \$5000 on one rain shelter.

(a) Write a formula for y in terms of x.

(b) Find a formula for the total volume in cubic feet covered by the shelter, $V(x)$, if the width of the dashed rectangle has length x feet.

(c) The PTA wants to make sure that each of the sides of the rectangle has length at least 5 feet, and the height (that is, y) of the shelter is at least 8 feet. In the context of the problem, what is the domain of the function $V(x)$?

Problem 9. (Fall 2017 Exam 2) Jane is designing a water tank using a cone of height h meters and a circular base of radius r meters as shown below.

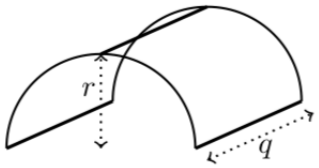


r = radius
h = height
s = length of slant side

(a) The cost of the material for the tank is 3 dollars per square meter for the circular base and 5 dollars per square meter for the cone (without the base). The area, A, of the material used for the cone (without the base) is given by the formula $A = \pi rs$ where s is the length of the slant side of the cone, in meters. Find a formula for s in terms of the radius r if Jane plans to spend 200 dollars on the water tank. Your answer should not include the variable h.

(b) Find a formula for $V(r)$, the volume of the tank (in cubic meters) in terms of the radius r. Recall that the volume of a cone of radius R and height $H$is $\frac{1}{3}\pi R^2 H$. Your answer should not include the variables h and/or s.

Problem 10. (Winter 2017 Exam 2) Duncan's person is making him a new tent in the shape of half a cylinder. She plans to use wire to make the tent frame. This will consist of two semicircles of radius r (measured in inches) attached to three pieces of wire of length q (also measured in inches), as shown in the diagram below. She has 72 inches of wire to use for this.



(a) Find a formula for r in terms of q.

(b) Let $V(q)$ be the volume (in cubic inches) of the space inside the tent after the fabric is added, given that the total length of wire is 72 inches and the length of the tent is q inches. (Recall that the tent shape is half of a cylinder.) Find a formula for $V(q)$. The variable r should not appear in your answer.

(c) In the context of this problem, what is the domain of $V(q)$?