

# Math 115

## Worksheet Section 5.1

**Problem 1.** The record time for the 100 meter dash is 9.58 seconds, set by Usain Bolt at a race in 2009. Let  $v(t)$  be Bolt's velocity, in meters per second,  $t$  seconds after Bolt starts the race. Several values of  $v(t)$  are shown below.

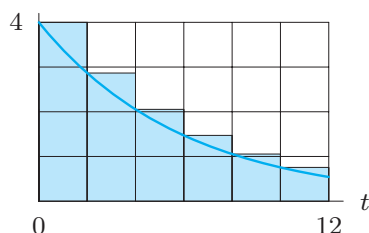
$t$	0	0.5	1	1.5	2	2.5	3
$v(t)$	0.6	3.5	5.8	7.7	9.1	10.1	10.6

- Assume that Usain keeps his velocity constant in each of the intervals  $[0, 0.5)$ ,  $[0.5, 1)$ ,  $[1, 1.5)$ ,  $[1.5, 2)$ ,  $[2, 2.5)$  and  $[2.5, 3)$ . Compute the total distance Usain ran in the first 3 seconds.
- Now assume instead that Usain keeps his velocity constant in the intervals  $(0, 0.5]$ ,  $(0.5, 1]$ ,  $(1, 1.5]$ ,  $(1.5, 2]$ ,  $(2, 2.5]$  and  $(2.5, 3]$  and compute the total distance he ran in this situation.
- For each of the situations described in (a) and (b), make a sketch of the graph of the function  $v(t)$ . Can you represent each of the sums computed in (a) and (b) on the respective graph?

Assume now that  $v(t)$  is a continuous increasing function for the first three seconds of the race.

- Based on the values provided on the table make a new sketch of  $v(t)$ .
- What information about the graph in (d) do the sums in (a) and (b) provide?
- Estimate the error when you compute the area below the graph of  $v(t)$  using (a) and (b).

**Problem 2.** The figure below shows the velocity of a car for  $0 \leq t \leq 12$  and the rectangles used to estimate of the distance traveled.



- Do the rectangles represent a left or a right sum?
- Do the rectangles lead to an upper or a lower estimate?
- What is the value of  $n$ ?
- What is the value of  $\Delta t$ ?
- Give an approximate value for the estimate.

**Problem 3.** The velocity  $v(t)$  in the table below is decreasing for  $2 \leq t \leq 12$ .

$t$	2	4	6	8	10	12
$v(t)$	44	42	41	40	37	35

Using  $n = 5$  subdivisions to approximate the total distance traveled, find

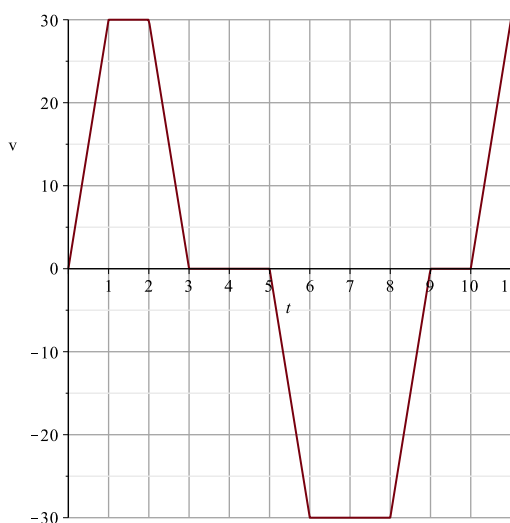
- An upper estimate.
- A lower estimate.

**Problem 4.** A particle's velocity is given by  $v(t)$ , where

$$(a) \quad v(t) = \begin{cases} 2, & 0 \leq t \leq 3 \\ -8 + 2t, & 3 < t \leq 6 \end{cases} \quad (b) \quad v(t) = 3t^2, \quad 0 \leq t \leq 6.$$

- (i) For (a), compute the particle's displacement (signed area under the curve) between  $t = 0$  and  $t = 6$  using geometry.
- (ii) For (b), estimate the displacement of the particle, using left and right sums with  $n = 6$ .

**Problem 5.** A bicyclist starts from home and rides back and forth along a straight east/west highway. Her velocity  $y = v(t)$  at time  $t$  (in minutes) measured in feet per second is given below (positive velocities indicate travel toward the east, negative toward the west).



- (a) On what time intervals is she stopped?
- (b) How far from home is she the first time she stops, and in what direction?
- (c) At what time does she bike past her house?
- (d) If she maintains her velocity at  $t = 11$ , how long will it take her to get back home?

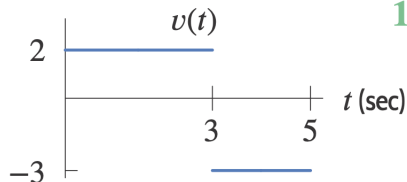
**Problem 6.** The table below gives the expected growth rate,  $g(t)$ , in ounces per week, of the weight of a baby in its first 54 weeks of life. Assume for this problem that  $g(t)$  is a decreasing function.

week $t$	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

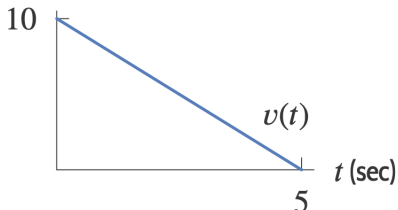
- (a) Using six subdivisions, find an overestimate and an underestimate for the total weight gained by a baby over its first 54 weeks of life.
- (b) How frequently over the 54 week period would you need the data for  $g(t)$  to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 8 oz?

**Problem 7.** The following graphs show the velocity, in cm/sec, of a particle moving along a number line. Find the change in position and total distance traveled between times  $t = 0$  and  $t = 5$  seconds.

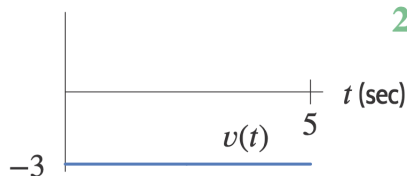
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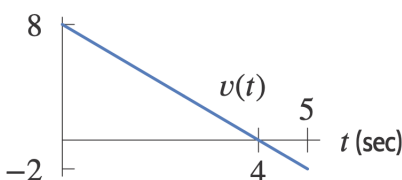
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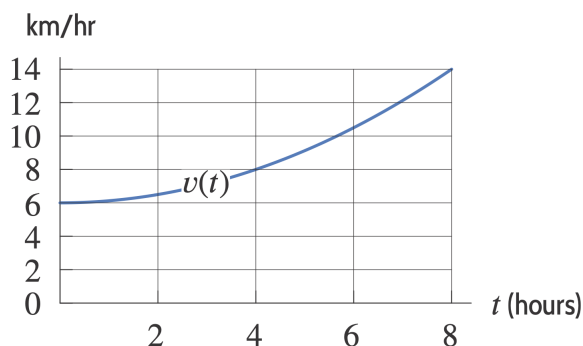


20.



**Problem 8.** The velocity of an object for  $0 \leq t \leq 8$  is shown below. Calculate the following estimates of the distance the object travels between  $t = 0$  and  $t = 8$  and indicate whether each is an upper or lower estimate of the distance traveled.

- (a) A left sum with  $n = 2$  subdivisions
- (b) A right sum with  $n = 2$  subdivisions



**Problem 9.** Find the difference between upper and lower estimates of the distance traveled at velocity  $f(t)$  on the interval  $a \leq t \leq b$  for  $n$  subdivisions:  $f(t) = e^{-t^2/2}$ ,  $a = 0$ ,  $b = 2$ ,  $n = 20$ .

**Problem 10.** At time  $t$ , in second, your velocity,  $v$ , in meters/second, is given by

$$v(t) = 1 + t^2 \text{ for } 0 \leq t \leq 6$$

Use  $\Delta t = 2$  to estimate the distance traveled during this time. Find the upper and lower estimates, and then average the two.