

# Math 115

## Worksheet Section 3.9

### Warm-up question

The linear approximation or local linearization of  $f(x)$  at  $x = a$  is given by  $L(x) = \underline{\hspace{2cm}}$

**Problem 1.** (a) Find the linear approximation of  $\ln(x)$  at  $x = 1$ .

(b) Use your approximation to approximate  $\ln(1.1)$

(c) Is your answer an underestimate or overestimate of  $\ln(1.1)$ ? Why?

**Problem 2.** Find the linear approximation of  $\cos(x)$  at  $x = 0$ .

**Problem 3.** (Winter 2017 Exam 2) A group of biology students is studying the length  $L$  of a newborn corn snake (in cm) as a function of its weight  $w$  (in grams). That is,  $L = G(w)$ . A table of values of  $G(w)$  is shown below.

$w$	5	10	15	20	25
$G(w)$	24.5	31.6	38.7	44.7	50
$G'(w)$	2.23	1.58	1.30	1.12	1.05

Assume that  $G'(w)$  is a differentiable and decreasing function for  $0 < w < 25$ .

(a) Find a formula for  $H(w)$ , the tangent line approximation of  $G(w)$  near  $w = 20$ .

(b) Use the tangent line approximation of  $G(w)$  near  $w = 20$  to approximate the length of a corn snake that weighs 22 grams.

(c) Is your answer in part (b) an overestimate or an underestimate? Write a sentence to justify your answer.

(d) In their study of the growth of corn snakes, they found the results of a recent article that states that the average weight  $w$  of a corn snake (in grams)  $t$  weeks after being born is given by  $w = \frac{1}{5}t^2$ . Let  $S(t) = G(\frac{1}{5}t^2)$  be the length of a corn snake  $t$  weeks after being born. Find a formula for  $P(t)$ , the tangent line approximation of  $S(t)$  near  $t = 5$ .

**Problem 4.** (Fall 2016 Exam 2) Let  $h(x) = x^x$ . For this problem, it may be helpful to know the following formula:

$$h'(x) = x^x(\ln(x) + 1)$$

Write a formula for  $p(x)$ , the local linearization of  $h(x)$  near  $x = 1$ .

**Problem 5.** (Fall 2017 Exam 2) Let  $g$  be a twice differentiable function defined on  $-1 < x < 11$ . Some values of  $g(x)$ ,  $g'(x)$  and  $g''(x)$  are shown in the table below.

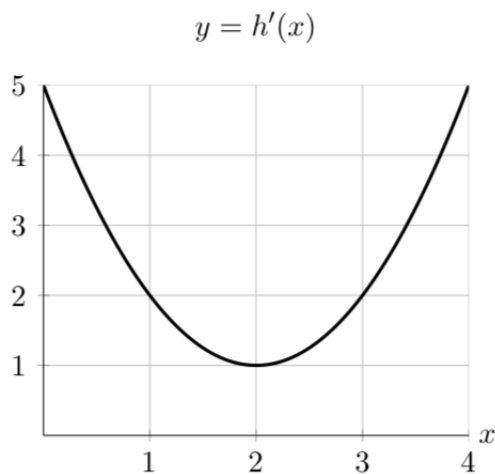
$x$	0	2	4	6	8	10
$g(x)$	-2	-1	3	4	5	6
$g'(x)$	0.5	2	?	5	1	2
$g''(x)$	2	1	5	-3	-1	0.5

Let  $j(x) = g(14 - 4x)$ .

- Use the values from the table to find a formula for  $L(x)$ , the linear approximation to  $j(x)$  at  $x = 2$ .
- Find an approximate value for  $j(2.25)$  using your formula for  $L(x)$ .
- Is your approximation in part (b) an overestimate or an underestimate? Circle your answer and give a justification of your answer.  

Overestimate
Underestimate
Not enough information

**Problem 6.** (Winter 2018 Exam 2) Below is the graph of  $h'(x)$ .



- Find a formula for the tangent line approximation  $L(x)$  to the function  $h(x)$  near  $x = 2$  if the point  $(2, -3)$  lies on the graph of  $y = h(x)$ . Your answer should not include the letter  $h$ .
- Use the tangent line approximation to  $h(x)$  near  $x = 2$  to approximate the value of  $h(1.6)$ .
- Is your approximation in part (b) an overestimate, an underestimate or is there not enough information to determine that?