

Math 115

Worksheet Section 5.1

Problem 1. The record time for the 100 meter dash is 9.58 seconds, set by Usain Bolt at a race in 2009. Let $v(t)$ be Bolt's velocity, in meters per second, t seconds after Bolt starts the race. Several values of $v(t)$ are shown below.

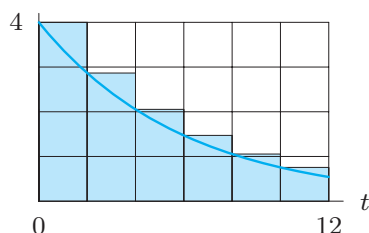
t	0	0.5	1	1.5	2	2.5	3
$v(t)$	0.6	3.5	5.8	7.7	9.1	10.1	10.6

- Assume that Usain keeps his velocity constant in each of the intervals $[0, 0.5)$, $[0.5, 1)$, $[1, 1.5)$, $[1.5, 2)$, $[2, 2.5)$ and $[2.5, 3)$. Compute the total distance Usain ran in the first 3 seconds.
- Now assume instead that Usain keeps his velocity constant in the intervals $(0, 0.5]$, $(0.5, 1]$, $(1, 1.5]$, $(1.5, 2]$, $(2, 2.5]$ and $(2.5, 3]$ and compute the total distance he ran in this situation.
- For each of the situations described in (a) and (b), make a sketch of the graph of the function $v(t)$. Can you represent each of the sums computed in (a) and (b) on the respective graph?

Assume now that $v(t)$ is a continuous increasing function for the first three seconds of the race.

- Based on the values provided on the table make a new sketch of $v(t)$.
- What information about the graph in (d) do the sums in (a) and (b) provide?
- Estimate the error when you compute the area below the graph of $v(t)$ using (a) and (b).

Problem 2. The figure below shows the velocity of a car for $0 \leq t \leq 12$ and the rectangles used to estimate of the distance traveled.



- Do the rectangles represent a left or a right sum?
- Do the rectangles lead to an upper or a lower estimate?
- What is the value of n ?
- What is the value of Δt ?
- Give an approximate value for the estimate.

Problem 3. The velocity $v(t)$ in the table below is decreasing for $2 \leq t \leq 12$.

t	2	4	6	8	10	12
$v(t)$	44	42	41	40	37	35

Using $n = 5$ subdivisions to approximate the total distance traveled, find

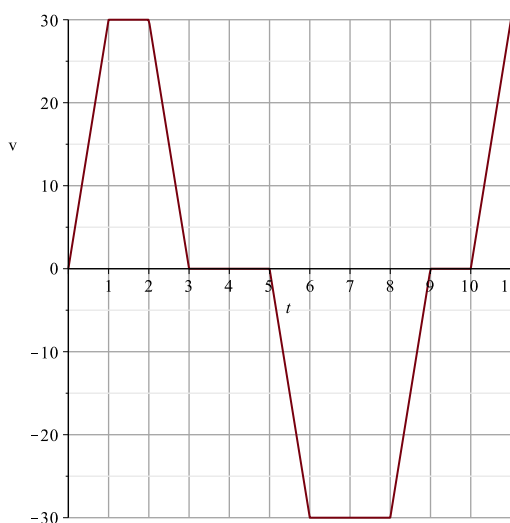
- An upper estimate.
- A lower estimate.

Problem 4. A particle's velocity is given by $v(t)$, where

$$(a) \quad v(t) = \begin{cases} 2, & 0 \leq t \leq 3 \\ -8 + 2t, & 3 < t \leq 6 \end{cases} \quad (b) \quad v(t) = 3t^2, \quad 0 \leq t \leq 6.$$

- (i) For (a), compute the particle's displacement (signed area under the curve) between $t = 0$ and $t = 6$ using geometry.
- (ii) For (b), estimate the displacement of the particle, using left and right sums with $n = 6$.

Problem 5. A bicyclist starts from home and rides back and forth along a straight east/west highway. Her velocity $y = v(t)$ at time t (in minutes) measured in feet per second is given below (positive velocities indicate travel toward the east, negative toward the west).



- (a) On what time intervals is she stopped?
- (b) How far from home is she the first time she stops, and in what direction?
- (c) At what time does she bike past her house?
- (d) If she maintains her velocity at $t = 11$, how long will it take her to get back home?

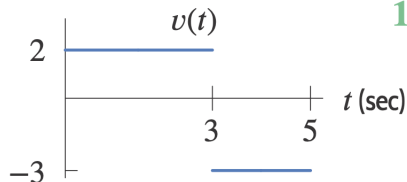
Problem 6. The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life. Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

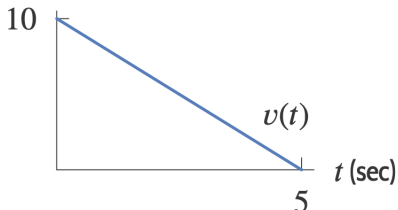
- (a) Using six subdivisions, find an overestimate and an underestimate for the total weight gained by a baby over its first 54 weeks of life.
- (b) How frequently over the 54 week period would you need the data for $g(t)$ to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 8 oz?

Problem 7. The following graphs show the velocity, in cm/sec, of a particle moving along a number line. Find the change in position and total distance traveled between times $t = 0$ and $t = 5$ seconds.

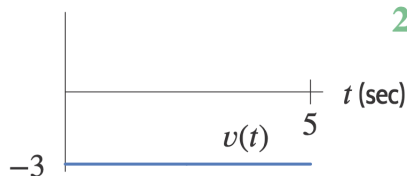
17.



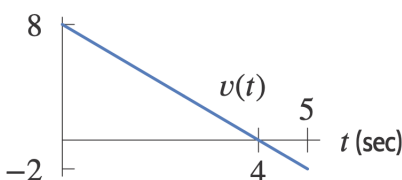
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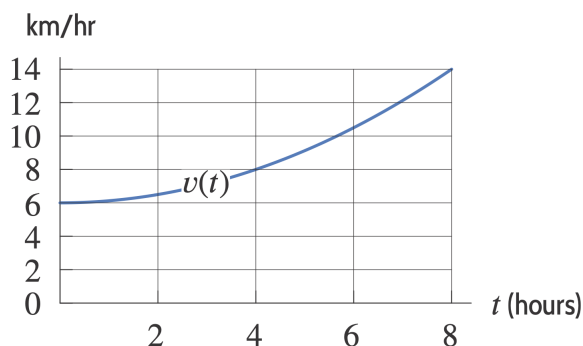


20.



Problem 8. The velocity of an object for $0 \leq t \leq 8$ is shown below. Calculate the following estimates of the distance the object travels between $t = 0$ and $t = 8$ and indicate whether each is an upper or lower estimate of the distance traveled.

- (a) A left sum with $n = 2$ subdivisions
- (b) A right sum with $n = 2$ subdivisions



Problem 9. Find the difference between upper and lower estimates of the distance traveled at velocity $f(t)$ on the interval $a \leq t \leq b$ for n subdivisions: $f(t) = e^{-t^2/2}$, $a = 0$, $b = 2$, $n = 20$.

Problem 10. At time t , in second, your velocity, v , in meters/second, is given by

$$v(t) = 1 + t^2 \text{ for } 0 \leq t \leq 6$$

Use $\Delta t = 2$ to estimate the distance traveled during this time. Find the upper and lower estimates, and then average the two.