

## Worksheet Section 1.6

- Problem 1.** (a) Every exponential \_\_\_\_\_ function eventually dominates every \_\_\_\_\_.
- (b) Consider the rational function  $r(x) = \frac{p(x)}{q(x)}$ . If the polynomials  $p(x)$  and  $q(x)$  have no common zeroes, then:
- Zeros of  $p(x)$  give rise to \_\_\_\_\_.
  - Zeros of  $q(x)$  give rise to \_\_\_\_\_.
- (c) When does  $r(x)$  have a horizontal asymptote?

**Problem 2.** Which function dominates as  $x \rightarrow \infty$ ?

- (a)  $e^x$  or  $10^{1000} \ln(x)$     (b)  $1000x^4$  or  $0.2x^5$     (c)  $10e^{0.1x}$  or  $5000x^2$
- (d)  $100x^5$  or  $1.05^x$     (e)  $x^4$  or  $\ln(x)$     (f)  $e^x$  or  $2.71^x$

**Problem 3.** Which of the following functions have the given properties?

- (a)  $y = \frac{x^2-2}{x^2+2}$     (b)  $y = \frac{x^2+2}{x^2-2}$     (c)  $y = (x-1)(1-x)(x+1)^2$
- (d)  $y = x^3 - x$     (e)  $y = x - \frac{1}{x}$     (f)  $y = (x^2 - 2)(x^2 + 2)$

- (i) A polynomial of degree 3.  
 (ii) A polynomial of degree 4.  
 (iii) A rational function that is not a polynomial.  
 (iv) Exactly two distinct zeros.  
 (v) Exactly one vertical asymptote.  
 (vi) More than two distinct zeros.  
 (vii) Exactly two vertical asymptotes.  
 (viii) A horizontal asymptote.

**Problem 4.** (1.6 #41–44) Assuming the window is large enough to show end behavior, identify the graph as that of a rational function, exponential function, or logarithmic function.

41.

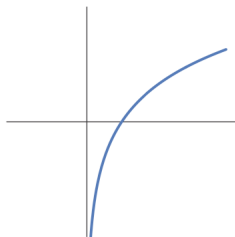


Figure 1.90

42.

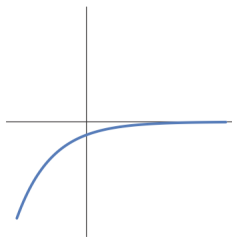


Figure 1.91

43.

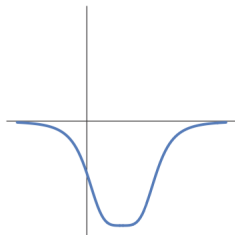


Figure 1.92

44.

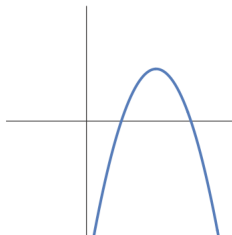


Figure 1.93

**Problem 5.** (Fall 2017 Exam 1) Consider the rational function  $r$  defined by

$$r(x) = \frac{3(x - \sqrt{2})(\pi x + 7)^2(x + 1)}{(x + 1)(x - \sqrt{3})}.$$

For all of the following parts of this problem, leave your answers in exact form.

- What is the domain of  $r(x)$ ?
- Find the equations of all vertical asymptotes of  $r(x)$ . If there are none, write none.
- Let  $p(x) = x^2 + 1.2x - 5$ . Find the equations of all horizontal asymptotes of  $\frac{r(x)}{p(x)}$ . If there are none, write NONE. Show your work or reasoning to justify your answer.

**Problem 6.** (Winter 2017 Exam 1) A group of students planted a pine tree and an oak tree alongside the Diag. Let  $P(t)$  and  $O(t)$  be the height (in feet) of the pine and the oak  $t$  years after they were planted, where

$$P(t) = 170 - 165A^{-0.02t} \text{ and } O(t) = \frac{140}{2 + 100e^{-0.3t}}$$

where  $A > 1$  is a constant.

- How tall (in feet) were each of the trees when they were planted?
- Ten years after the trees were planted, the height of the pine was 38 ft. Find the value of  $A$ . Find your answer algebraically and show all your work.
- How many years after being planted does it take the oak to be 38 ft? Find your answer algebraically and show all your work.

**Problem 7.** Use a graphing calculator or Desmos to graph  $y = x^4$  and  $y = 3^x$ . Determine approximate domains and ranges that give each of the graphs in the figure below.

