Math 115
Worksheet Section 3.7

Problem 1. (a) Find $\dfrac{dy}{dx}$ given that $x^2 + y^2 - 4x + 7y = 15$.

(b) Under what conditions on x and/or y is the tangent line to this curve horizontal?

(c) Find $\dfrac{dx}{dy}$ (pay attention to the order!)

(d) Under what conditions on x and/or y is the tangent line to this curve vertical?

Problem 2. Find dy/dx for $e^{\cos(y)} = x^3 \arctan(y)$.

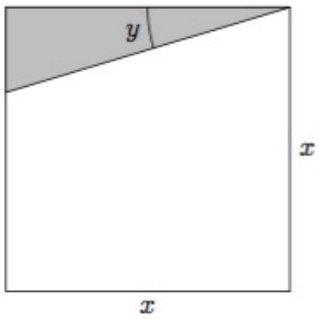
Problem 3. Consider the curve given by the equation $e^{y^2} = x^3 + 9$.

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of the two points on the curve where the tangent line is horizontal.

(c) Find any points on the curve where there is a vertical tangent line.

Problem 4. Valerie is building a square chicken coop with side length x. Because she needs a separate place for the rooster, she needs to put fence around the square and also along the diagonal line shown. The fence costs \$20 per linear meter, and she has a budget of \$900.



$$80x + \frac{20x}{\cos(y)} = 900$$

(a) Verify that the equation above gives the relationship between x and the angle y (in radians).

(b) If Valerie builds the coop with $y = \pi/6$ (and wants to use her whole budget), find the side length x.

(c) Find slope of the curve at this point, and interpret what it tells Valerie.

Problem 5. (Winter 2018 Exam 2) Find $\dfrac{dy}{dx}$ for the implicit function given by

$$2^{x+y} + \sin(x)\cos(y) = 5 - x.$$

Problem 6. (Fall 2016 Exam 2) Let a and b be constants. Consider the curve $\mathcal{C}$ defined by the equation

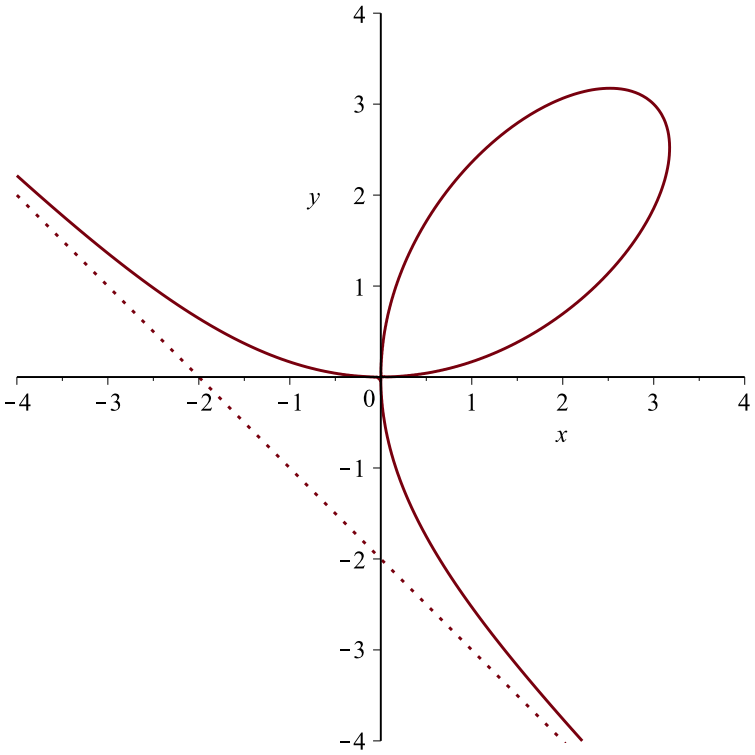
$$\cos(ax) + by\ln(x) = 3 + y^3.$$

Find the formula for $\frac{dy}{dx}$ in terms of x and y. The constants a and b may appear in your answer.

Problem 7. The folium of Descartes (a.k.a. Descartes' leaf) was first discovered in 1638 and is defined as the curve

$$x^3 + y^3 = 6xy.$$

The curve forms a loop in the first quadrant, and it is symmetric about $y = x$:

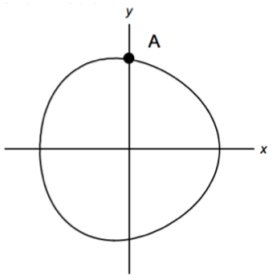


(a) Show that the point $(x, y) = (3, 3)$ lies on the curve.

(b) Find the equation of the tangent line to the curve at the point $(x, y) = (3, 3)$.

(c) For what value(s) of x (if any) will the tangent line to this curve be horizontal?

Problem 8. (Fall 2017 Exam 2) Let $\mathcal{C}$ be the curve given by the equation $81 - (x^2 + y^2)^2 = 2xy^2$. The graph of $\mathcal{C}$ is shown below.



(a) Find the coordinates (x, y) of the point A.

(b) Find $\dfrac{dy}{dx}$.

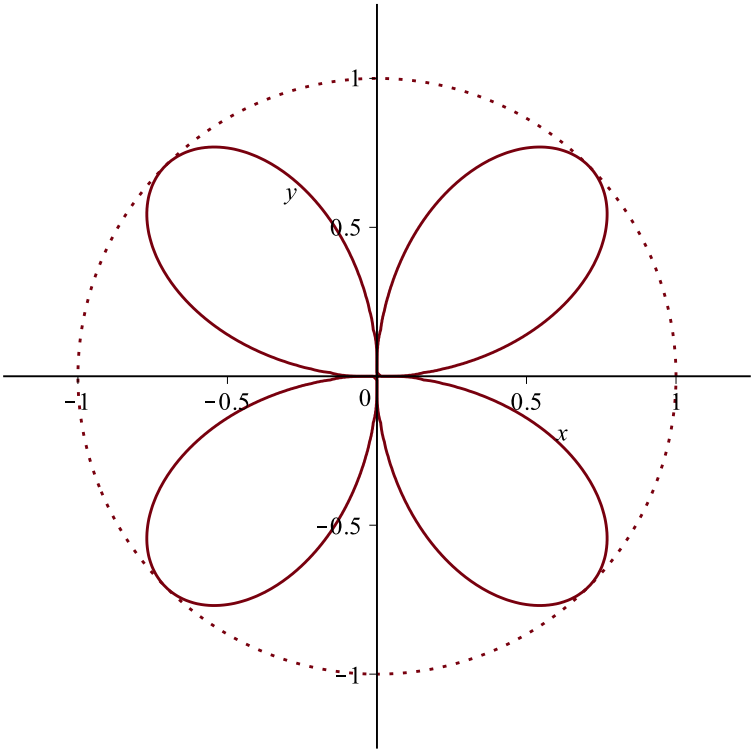
(c) Find the equation of tangent line $L(x)$ to the graph of $\mathcal{C}$ at A.

Show your computations step by step.

Problem 9. A *quadrifolium* is a 4-petaled rose curve given by

$$\left(x^2 + y^2\right)^3 = 4x^2y^2$$

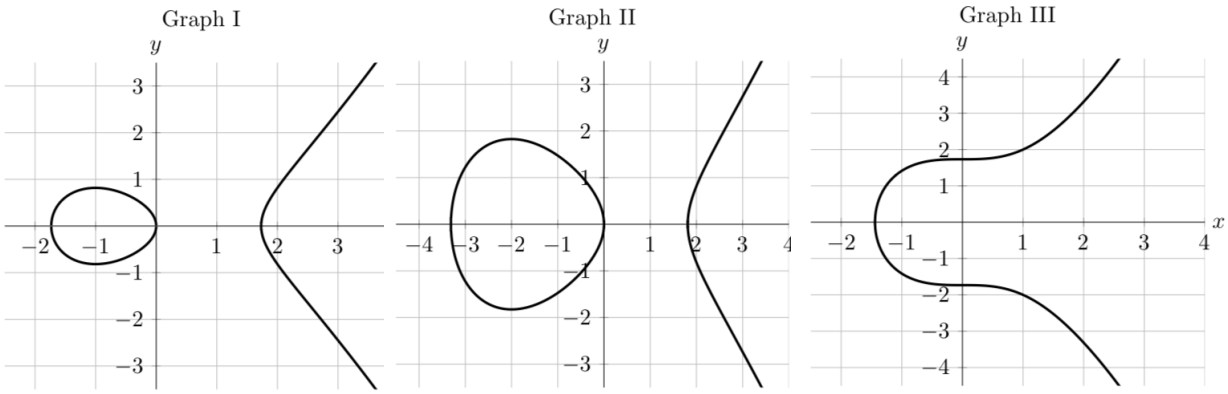
and which is shown below as the solid curve. It lies inside the unit circle $x^2 + y^2 = 1$.



(a) Show that the point $(x, y) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ lies on the quadrifolium.

(b) Find the equation of the tangent line to the quadrifolium at $(x, y) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Problem 10. (Winter 2018 Exam 2) Each the following is the graph of an implicit function.



Match each of the graphs above to the formula below that gives the slope at each point on the graph.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2}{2y}$

(b) $\dfrac{dy}{dx} = \dfrac{(x-1)(x+2)}{2y}$

(c) $\dfrac{dy}{dx} = \dfrac{x^2-1}{2y}$

(d) $\dfrac{dy}{dx} = \dfrac{(y-1)(y+2)}{2x}$