Math 115
Worksheet Extrema Review

Local Extrema and Inflection Points

Problem 1. (Winter 2020 Final) Consider the continuous function

$$g(x) = \begin{cases} -(x^2 + 12x + 37)e^{-x} + 17 & x \leq 0 \\ 7x^3 - 21x^2 - 168x - 20 & x > 0 \end{cases}$$

Note that

$$g'(x) = \begin{cases} (x+5)^2 e^{-x} & x < 0 \\ 21(x-4)(x+2) & x > 0 \end{cases}$$

(a) Find the critical points of $g(x)$

(b) Find the x-coordinate of all local extrema of $g(x)$, and classify each as a local maximum or a local minimum. Use calculus to find and justify your answers, and be sure to show enough evidence that you have found all local extrema.

Solution: See https://dhsp.math.lsa.umich.edu/exams/115exam3/w20/s5.pdf

Problem 2. (Fall 2019 Exam 2) Suppose $q(x)$ is a differentiable function defined for all real numbers x. The derivative and second derivative of $q(x)$ are given by

$$q'(x) = x^{2/3}(x-3)^{5/3}(x+5) \text{ and } q''(x) = \frac{10(x-3)^{2/3}(x-1)(x+3)}{3x^{1/3}}$$

(a) Find the x-coordinates of all critical points of $q(x)$. If there are none, write none.

(b) Find the x-coordinates of all critical points of $q'(x)$. If there are none, write none.

(c) Find the x-coordinates of all local maxima and local minima of $q(x)$. If there are non of a particular type, write none. Use calculus to find and justify you answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

(d) Find the x-coordinates of all inflection points of $q(x)$. If there are none, write none. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: See https://dhsp.math.lsa.umich.edu/exams/115exam2/f19/s3.pdf

Global Extrema

Problem 3. (Winter 2019 Exam 3) Consider the continuous function

$$f(x) = \begin{cases} -2 - \ln(x+2) & -2 < x \leq -1 \\ x2^{-x} & x > -1 \end{cases}$$

and its derivative

$$f'(x) = \begin{cases} -\frac{1}{x+2} & -2 < x < -1 \\ 2^{-x}(1 - x\ln(2)) & x > -1 \end{cases}$$

(a) Find all critical point(s) of $f(x)$. Write none if there are none.

(b) Find the x-coordinate of all global maxima and global minima of $f(x)$ on its domain $(-2, \infty)$. For each, write none if there are none. You must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers

Solution: See https://dhsp.math.lsa.umich.edu/exams/115exam3/f19/s10.pdf

Problem 4. (Winter 2016 Exam 2) Consider a continuous function T with the following properties.

- $T(v)$ is defined for all real numbers v.
- The critical points of $T(v)$ are the four points $v = 3, v = 5, v = 7$, and $v = 8$. ($T(v)$ has no other critical points.)

Some values of T are shown in the following table:

v	0	3	5	7	8	10
$T(v)$	21	9	13	19	11	21

For each of a.-f. below, use the answer blank provided to list all the values v at which $T(v)$ attains the specified global extremum. If there is not enough information provided to give an answer, write "not enough info". If $T(v)$ does not attain the specified global extremum on the specified interval, write "none".

For what value(s) v does $T(v)$ attain its...

(a) global minimum on the interval $0 \leq v \leq 10$?

(b) global maximum on the interval $0 \leq v \leq 10$?

(c) global minimum on the interval $0 < v < 10$?

(d) global maximum on the interval $0 < v < 10$?

(e) global minimum on the interval $(-\infty, \infty)$?

(f) global maximum on the interval $(-\infty, \infty)$?

Solution: See https://dhsp.math.lsa.umich.edu/exams/115exam2/w16/s9.pdf

Problem 5. (Fall 2018 Exam 2) Consider the function

$$f(x) = \begin{cases} 4 - x - x^{\frac{2}{3}} & -8 \leq x \leq 0 \\ 5xe^{-0.5x} + 4 & x > 0 \end{cases} \text{ and its derivative } f'(x) = \begin{cases} \frac{2+3x^{\frac{1}{3}}}{-3x^{\frac{1}{3}}} & -8 < x < 0 \\ 5(1 - 0.5x)e^{-0.5x} & x > 0 \end{cases}$$

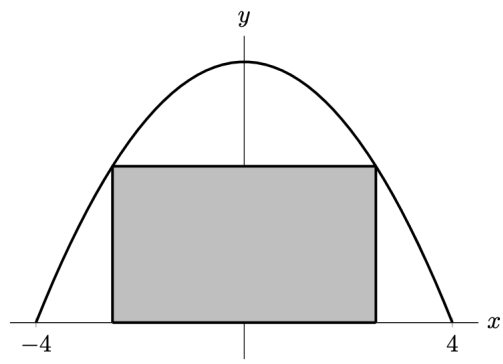
Find the x-coordinates of the global maximum and the global minimum of the function f(x) for $x \geq -8$. If one of them does not exist, write none in the answer line below. Use calculus to find your answers, and be sure to show enough evidence that the point(s) you find are indeed global extrema.

Optimization

Problem 6. A garden store plans to build a large rectangular sign on the interior wall at one end of their green- house. For x and y in meters, the curved roof of the greenhouse is described by the function

$$y = 5 - \frac{5}{16}x^2 \text{ for } -4 \leq x \leq 4$$

The curve is graphed below. The shaded rectangle is one possible sign that could be built.



Find the width and height of the sign with the maximum area. Use calculus to find your answers, and be sure to show enough evidence that the values you find do in fact maximize the area.

Solution: See https://dhsp.math.lsa.umich.edu/exams/115exam3/f19/s6.pdf

Putting it all together

Problem 7. (Winter 2016 Exam 2) Let $j(t)$ be a differentiable function with domain $(0, \infty)$ that satisfies all of the following:

- $j(5) = 0$
- $j(t)$ has exactly two critical points
- $j(t)$ has a local maximum at $t = 5$
- $j(t)$ has a local minimum at $t = 9$
- $\lim_{t \to 0^+} j(t) = -\infty$
- $\lim_{t \to \infty} j(t) = 0$

(a) Circle all of the following intervals on which $j'(t)$ must always be negative.

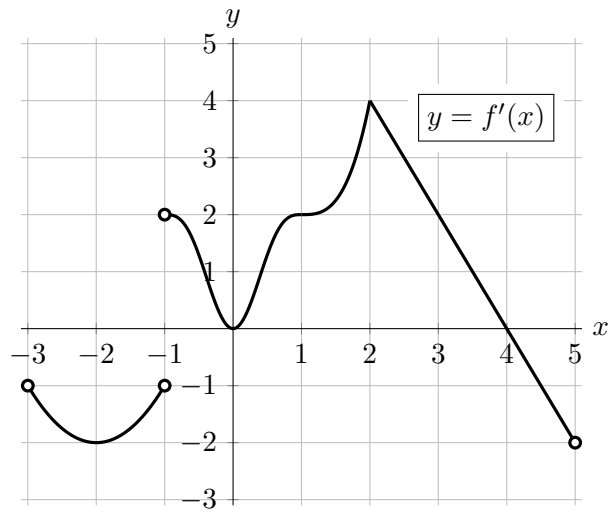
$$(0, 2) \qquad (2, 5) \qquad (5, 9) \qquad (9, \infty)$$

(b) Find all the values of t at which $j(t)$ attains global extrema on the interval $[1, 9]$. If not enough information is provided, write not enough info. If there are no such values of t, write none.

(c) Find all the values of t at which $j(t)$ attains global extrema on its domain. If not enough information is provided, write not enough info. If there are no such values of t, write none.

Solution: See https://dhsp.math.lsa.umich.edu/exams/115exam2/w19/s10.pdf

10. [12 points] Let $f(x)$ be a continuous function defined on $-3 < x < 5$. The graph of $f'(x)$ (the derivative of $f(x)$) is shown below. Note that $f'(x)$ has a sharp corner at $x = 2$.



For each of the following parts, circle all of the available correct answers.

a. [2 points] At which of the following values of x does $f(x)$ appear to have a critical point?

Solution:

$x = -2$ $\boxed{\text{x=-1}}$ $\boxed{\text{x=0}}$ $x = 1$ $x = 2$ $\boxed{\text{x=4}}$ NONE OF THESE

b. [2 points] At which of the following values of x does $f(x)$ attain a global maximum on the interval $[0, 3]$?

Solution:

$x = 0$ $x = 1$ $x = 2$ $\boxed{\text{x=3}}$ NONE OF THESE

c. [2 points] At which of the following values of x does $f(x)$ attain a local minimum?

Solution:

$x = -2$ $\boxed{\text{x=-1}}$ $x = 0$ $x = 1$ $x = 4$ NONE OF THESE

d. [2 points] Which of the following values of x are not in the domain of $f''(x)$?

Solution:

$\boxed{\text{x=-1}}$ $x = 0$ $x = 1$ $\boxed{\text{x=2}}$ NONE OF THESE

e. [2 points] At which of the following values of x does $f(x)$ appear to have an inflection point?

Solution:

$\boxed{\text{x=-2}}$ $\boxed{\text{x=-1}}$ $\boxed{\text{x=0}}$ $x = 1$ $x = 4$ NONE OF THESE

f. [2 points] On which of the following intervals is $f''(x)$ increasing over the entire interval?

Solution:

$\boxed{\text{(-3,-1)}}$ $(-1, 0)$ $(-1, 1)$ $(0, 2)$ NONE OF THESE