

# Math 115

## Worksheet Section 3.6

**Problem 1.** (a) Use the chain rule to differentiate (with respect to  $x$ ) both sides of the identity  $f(f^{-1}(x)) = x$ .

(b) Having done that, solve for  $\frac{d}{dx}f^{-1}(x)$ .

(c) Now with this trick, find the derivatives (as functions of  $x$ ) of the following functions.

(a)  $f(x) = \ln(x)$

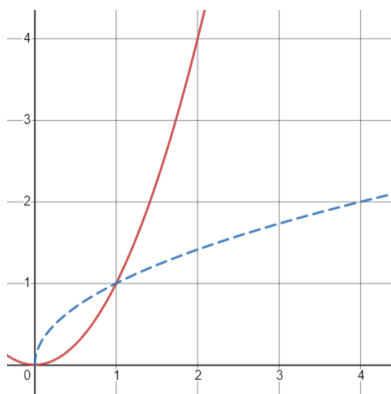
(d)  $f(x) = \arctan(x)$

(b)  $f(x) = \log_{10}(x)$

(e)  $f(x) = \arccos(x)$

(c)  $f(x) = \arcsin(x)$

**Problem 2.** Consider the following graph of a function  $f$  (solid line) and its inverse  $f^{-1}$  (dotted line):



Use the fact that  $f'(2) = 4$  and the general formula to find  $(f^{-1})'(4)$ . Then, use the graph to explain your answer.

**Problem 3.** The number of years,  $T$ , it takes an investment of \$1000 to grow to  $\$F$  in an account which pays 5% interest compounded continuously is given by

$$T = g(F) = 20 \ln(0.001F).$$

Find  $g(5000)$  and  $g'(5000)$ . Give units with your answers and interpret them in terms of money in the account.

**Problem 4.** At a particular location,  $f(p)$  is the number of gallons of gas sold when the price is  $p$  dollars per gallon.

(a) What does the statement  $f(2) = 4023$  tell you about gas sales?

(b) Find and interpret  $f^{-1}(4023)$ .

(c) What does the statement  $f'(2) = -1250$  tell you about gas sales?

(d) Find and interpret  $(f^{-1})'(4023)$ .

**Problem 5.** Solve the following textbook problems

In Problems 57–59, use Figure 3.31 to estimate the derivative. In Problems 60–62, use Figure 3.32 to calculate the derivative.

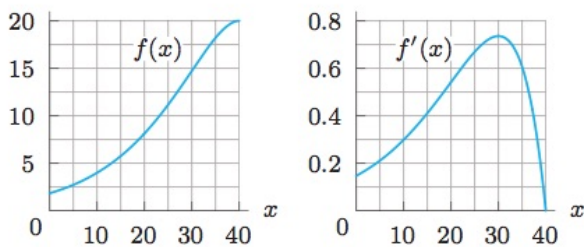


Figure 3.31

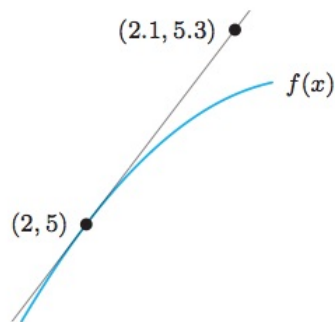


Figure 3.32

57.  $(f^{-1})'(5)$     58.  $(f^{-1})'(10)$     59.  $(f^{-1})'(15)$     60.  $h'(2)$  if  $h(x) = (f(x))^3$   
 61.  $k'(2)$  if  $k(x) = (f(x))^{-1}$   
 62.  $g'(5)$  if  $g(x) = f^{-1}(x)$

**Problem 6.** On what intervals is  $\ln(x^2 + 1)$  concave up?

**Problem 7.** (Fall 2017 Exam 2) Let  $A$  and  $B$  be two constants and

$$h(x) = \begin{cases} 2Bx + A \ln(x) & \text{if } 0 < x \leq 1 \\ \frac{4A}{x} + Bx - 1 & \text{if } 1 < x \leq 2 \end{cases}$$

Find all the values of  $A$  and  $B$  that make the function  $h(x)$  differentiable on the interval  $0 < x < 2$ . If no such values exist, write none. Justify your answer.

**Problem 8.** (Winter 2018 Exam 2) Some values of the twice differentiable function  $f(x)$  and of its first and second derivative are given by the following table:

$x$	0	1	2	4	5	6	7
$f(x)$	1			4	4.3	5	
$f'(x)$			8		0.25	0.6	2
$f''(x)$	4				0.1	0.2	

Suppose the function  $f(x)$  is defined and invertible for  $-\infty < x < \infty$ . In the following questions, you will find some of the missing values using the information given. If there is not enough information given to answer the question, write “NEI”. Show your work.

- (a) The function  $a(x) = \ln(1 + f(x))$  satisfies  $a'(2) = 2$ . Find  $f(2)$ .  
 (b) Let  $b(x) = f(x)f'(x)$  and  $b'(0) = 4$ . Find  $f'(0)$ .  
 (c) Let  $h(x) = f^{-1}(x)$ . Find the value of  $h'(5)$ .

**Problem 9.** (Fall 2016 Exam 2) Pepukai is studying the effect of the availability of water on the fruit productivity of Michigan apple trees. She observes that Michigan apple trees produce very few apples if they have too little water. She determines a function  $p(w)$  that models the total weight, in pounds, of all the apples that an average Michigan apple tree produces in a season when it is watered with  $w$  gallons of water every week. The domain of  $p$  is  $[5, 40]$ . Some values of the function  $p$  and its derivative  $p'$  are shown in the table below.

$w$	10	15	20	25	30
$p(w)$	25	96	118	129	135
$p'(w)$	96	13	4	2	1

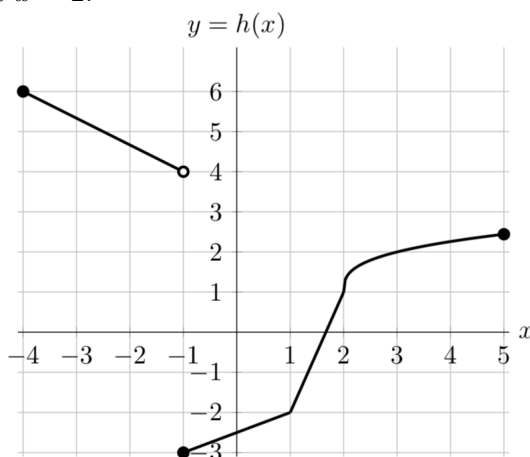
The function  $p$  is invertible and the functions  $p$ ,  $p'$ , and  $p^{-1}$  are all differentiable. Furthermore, the function  $p'$  is always decreasing.

- (a) Find  $(p^{-1})'(96)$ .  
 (b) Circle the one statement that is best supported by the equation

$$(p^{-1})'(10) = 0.01$$

- (i) To increase the total weight of apples produced in a season by an average Michigan apple tree from 10 pounds to 11 pounds, the tree should be watered with about 0.01 additional gallons of water every week.  
 (ii) If an average Michigan apple tree produces 10 pounds of apples in a season, watering the tree with 1 extra gallon every week increases the total weight of apples produced by the tree in a season by about 0.01 pounds.  
 (iii) If the amount of water that an average Michigan apple tree is watered with increases from 10 gallons every week to 10.1 gallons every week, the total weight of apples produced by the tree in a season increases by about 10 pounds.  
 (iv) If the amount of water that an average Michigan apple tree is watered with increases from 10 gallons every week to 10.1 gallons every week, the total weight of apples produced by the tree in a season increases by about 0.001 pounds.

**Problem 10.** (Fall 2017 Exam 2 (adapted)) Consider the graph of  $h(x)$  below. Note that  $h$  is linear on the intervals  $[-4, -1]$ ,  $[-1, 1]$ , and  $[1, 2]$ , differentiable on  $(2, 5)$ , and has a sharp corner at  $x = 2$ .



Find the exact value of the following expressions. If there is not enough information provided to find the value, write NI. If the value does not exist, write DNE. Show all your work.

- (a) Let  $g(x) = xh(x)$ . Find  $g'(-2)$ .  
 (b) Let  $p(x) = h^{-1}(x)$ . Find  $p'(0)$ .  
 (c) Find  $h'(-1)$ .