Math 115
Worksheet Section 5.2

Problem 1. Find the integral

$$\int_0^{10} x - 5 \; dx$$

by finding the area of the region between the curve and the horizontal axis.

Problem 2. Consider the function

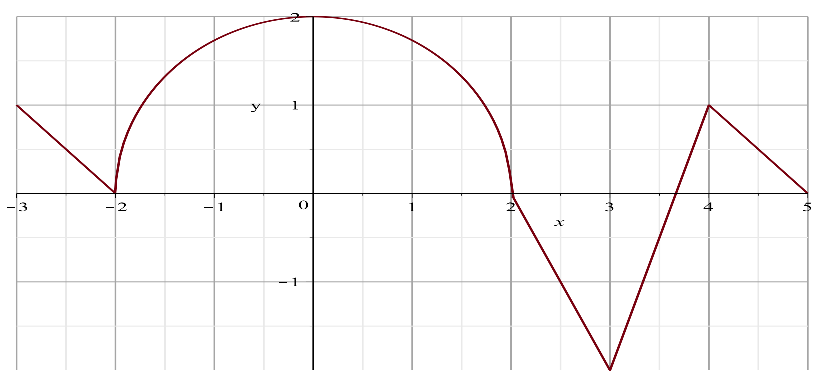
$$f(x) = \begin{cases} 1 - x, & 0 \leqslant x \leqslant 1, \\ x - 1, & 1 < x \leqslant 2. \end{cases}$$

(a) Sketch the graph of f.

(b) Find $\displaystyle\int_0^2 f(x)\, dx$.

(c) Find the 4-term left Riemann sum approximation of the definite integral you just computed. How does your approximation compare to the exact value?

Problem 3. The plot below shows $y = g(x)$.



Find the exact value of

(a) The definite integral $\displaystyle\int_{-3}^{5} g(x)\, dx$.

(b) The definite integral $\displaystyle\int_{-3}^{5} |g(x)|\, dx$.

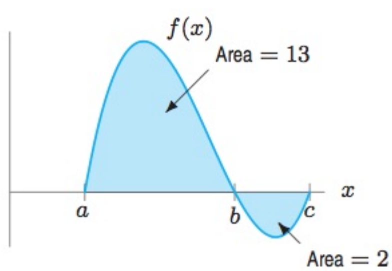
Problem 4. Use the figure below to compute the values of

(a) $\displaystyle\int_a^b f(x)dx$

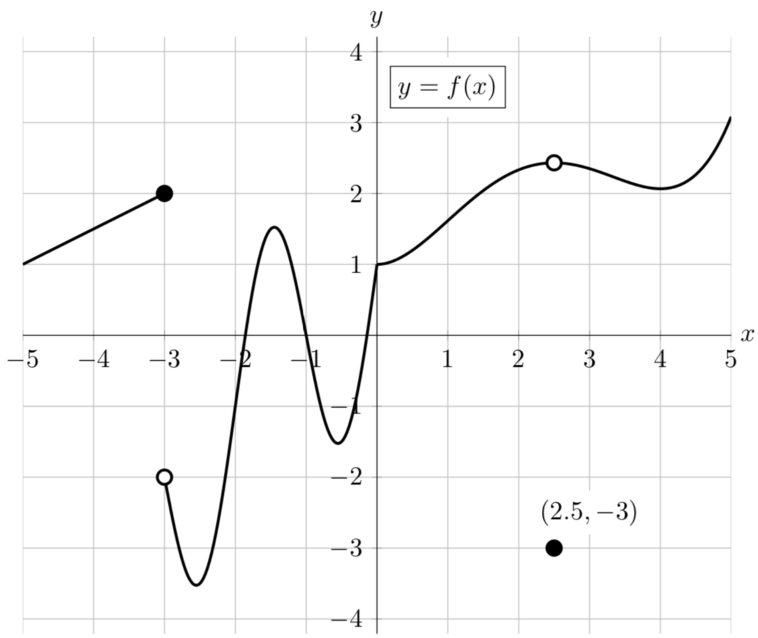
(b) $\displaystyle\int_b^c f(x)dx$

(c) $\displaystyle\int_a^c f(x)dx$

(d) $\displaystyle\int_a^c |f(x)|dx$



Problem 5. (Fall 2016 Final Exam) A portion of the graph of a function f is shown below.



(a) For which of the values of c is $\lim_{x \to c^-} f(x) = f(c)$?

$c = -3$ $c = -1$ $c = 0$ $c = 1$ $c = 2.5$ none

(b) For which of the following values of c is $f(x)$ continuous at $x = c$?

$c = -3$ $c = -1$ $c = 0$ $c = 1$ $c = 2.5$ none

(c) For which of the following values of c does f appear to be differentiable at $x = c$?

$c = -3$ $c = -1$ $c = 0$ $c = 1$ $c = 2.5$ none

(d) Rank the following quantities in order from least to greatest:

I. The number 0.

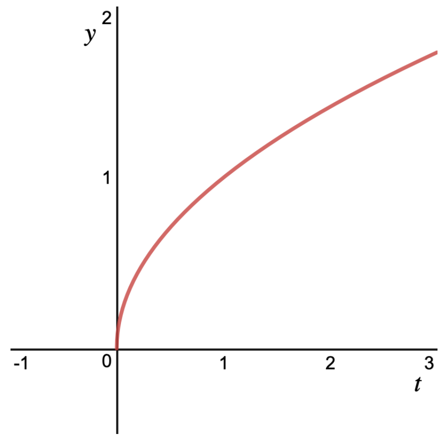
II. $f(1)$.

III. $\int_{-1}^{1} f(x)dx$.

IV. The left-hand Riemann sum with 2 subintervals for $\int_{-1}^{1} f(x)dx$.

V. The right-hand Riemann sum with 2 subintervals for $\int_{-1}^{1} f(x)dx$.

Problem 6. We want to compute $\int_0^2 \sqrt{t}\, dt$. Below is a portion of the graph of $f(t) = \sqrt{t}$.



(a) For this integral, are left sums always overestimates, always underestimates, or could they be either? What about right sums?

(b) Use a Riemann sum with 5 equal subdivisions to find a lower estimate for the integral. Show your answer to three decimal places.

(c) Use a Riemann sum with 5 equal subdivisions to find an upper estimate for the integral. Show your answer to three decimal places.

(d) Repeat (b) and (c) with 10 equal subdivisions. Show your answers to three decimal places.

Problem 7. For each of the following statements, must the statement be true for all continuous functions $f(x)$ and $g(x)$? Explain your answer.

(a) $\int_0^2 f(x)\, dx \leqslant \int_0^3 f(x)\, dx$.

(b) $\int_0^2 f(x)\, dx = \int_0^2 f(t)\, dt$.

(c) If $\int_2^6 f(x)\, dx \leqslant \int_2^6 g(x)\, dx$, then $f(x) \leqslant g(x)$ for all $2 \leqslant x \leqslant 6$.

Problem 8. Sketch the graph of a function f (you do not need to give a formula for f) on an interval $[a, b]$ with the property that with $n = 2$ subdivisions,

$$\int_a^b f(x)dx < \text{ Left-hand sum } < \text{ Right-hand sum.}$$

Problem 9. Without computing the sums, find the difference between the right- and left-hand Riemann sums if we use $n = 500$ subintervals to approximate $\int_{-1}^{1} (2x^3 + 4)dx$.

Problem 10. Compute the following integrals by interpreting them in terms of area.

(a) $\int_{-1}^{2} |x-1|dx$ (b) $\int_{0}^{1} \sqrt{1-x^2}dx$ (c) $\int_{-\pi}^{\pi} \sin(x)dx$

Problem 11. Give an example of a function f such that $\int_{1}^{3} f(x)dx < \int_{1}^{2} f(x)dx$.

Problem 12. Decide whether the following statement is true or false and justify your answer.

(a) On the interval $a \leq t \leq b$, the integral of the velocity is the total distance traveled from $t = a$ to $t = b$.

(b) A 4-term left-hand Riemann sum approximation cannot give the exact value of a definite integral.

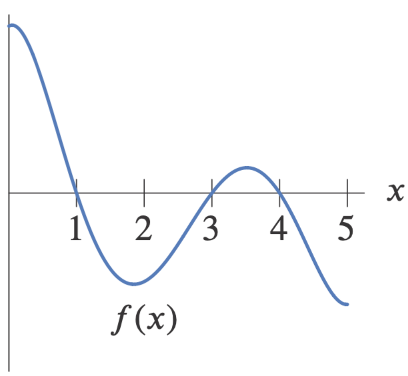
(c) If $f(x)$ is decreasing and $g(x)$ is increasing, then $\int_{a}^{b} f(x)dx \neq \int_{a}^{b} g(x)dx$.

Problem 13. Graph a continuous function $f(x) \geq 0$ on $[0, 10]$ with the given properties.

(a) The maximum value taken on by $f(x)$ for $0 \leq x \leq 10$ is 1. In addition, $\int_{0}^{10} f(x)dx = 5$.

(b) The maximum value taken on by $f(x)$ for $0 \leq x \leq 10$ is 5. In addition, $\int_{0}^{10} f(x)dx = 1$.

Problem 14. Use the figure to find limit a and b in the interval $[0, 5]$ with $a < b$ satisfying the given condition.



(a) $\int_{0}^{b} f(x)dx$ is as large as possible.

(b) $\int_{a}^{4} f(x)dx$ is as small as possible.

(c) $\int_{a}^{b} f(x)dx$ is as large as possible.

(d) $\int_{a}^{b} f(x)dx$ is as small as possible.