Math 115
Worksheet Section 3.1

Problem 1. Find the derivatives of the following functions

(a) $a(x) = x^{12}$

(b) $b(x) = x^{3/4}$

(c) $c(x) = x^{-3/4}$

(d) $d(x) = \ln e^{ax}$ for a a constant

(e) $e(x) = \sqrt{x}(x+1)$

(f) $f(x) = \frac{x^2+1}{x}$

(g) $g(x) = 3x^2 + \frac{12}{\sqrt{x}} - \frac{1}{x^2}$

Problem 2. What is the derivative of $f(x) = x^{\frac{1}{5}}$? Is f differentiable at $x = 0$?

Problem 3. On what intervals is the graph of $g(x) = x^4 - 4x^3$ both decreasing and concave up?

Problem 4. For what values of x is the function $f(x) = x^5 - 5x$ both increasing and concave up?

Problem 5. The n^{th} derivative of f, $f^{(n)}(x)$, is the result of differentiating $f(x)$ n times. Consider the function $f(x) = x^7 + 5x^5 - 4x^3 + 6x - 7$.

(a) Find the *8th* derivative of $f(x)$. Think ahead!

(b) Find the 7th derivative of $f(x)$.

Problem 6. (a) Find values for a and b so that the function k is both continuous and differentiable everywhere.

$$k(x) = \begin{cases} ax + 2 & x < 0 \\ b(x-1)^2 & x \geq 0 \end{cases}$$

(b) What is $k'(x)$?

Problem 7. Let $p(x)$ be a seventh-degree polynomial with 7 distinct zeros. How many zeros does $p'(x)$ have? Hint: use MVT to solve this.

Problem 8. At a time t seconds after it is thrown up in the air, a tomato is at a height of $f(t) = -4.9t^2 + 25t + 3$ meters.

(a) What is the average velocity of the tomato during the first 2 seconds? Give units.

(b) Find (exactly) the instantaneous velocity of the tomato at $t = 2$. Give units.

(c) What is the acceleration at $t = 2$?

(d) How high does the tomato go?

(e) How long is the tomato in the air?

Problem 9. (a) Find an equation of the line tangent to the graph of $f(x) = \sqrt{x}$ at the point $(4, 2)$ on the graph.

(b) For $f(x) = \sqrt{x}$, what is $\lim_{x\to\infty} f'(x)$? How is this consistent with the graph of $f(x)$?

Problem 10. (Fall 2018 Exam 2) Let A and B be constants and

$$k(x) = \begin{cases} 3x + \frac{B}{x} & \text{for } 0 < x < 1 \\ Bx^2 + Ax^3 & \text{for } 1 \leq x \end{cases}$$

Find the values of A and B that make the function $k(x)$ differentiable on $(0, \infty)$. Show all your work to justify your answers. If there are no such values of A and B, write none.

Problem 11. Suppose p is a cubic polynomial function, meaning that $p(x) = a_3x^3+a_2x^2+a_1x+a_0$ for some constants a_0, a_1, a_2, a_3, with $a_0 \neq 0$.

(a) Write expressions for $p(0)$, $p'(0)$, $p''(0)$ and $p'''(0)$ depending on a_0, a_1, a_2, and a_3.

(b) Find the formula for a cubic polynomial function q that satisfies

$$q(0) = 2, \quad q'(0) = -1, \quad q''(0) = 5, \quad q'''(0) = 4.$$

Problem 12. Assume that f'' and g'' exist and that f and g are concave up for all x. Are the following statements true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample.

(a) $f(x) + g(x)$ is concave up for all x.

(b) $f(x) - g(x)$ cannot be concave up for all x.

Problem 13. Let $f(x) = x^4 - 3x^2 + 1$.

(a) Show that $f(x)$ is an even function.

(b) Show that $f'(x)$ is an odd function.

(c) Are all polynomials of even degree even functions?

Problem 14. (Winter 2016 Exam 3) For constants A and B, consider the function h defined by

$$h(t) = \begin{cases} (At)^2 - 48 & \text{if } t < 2 \\ Bt^3 & \text{if } t \geqslant 2. \end{cases}$$

Circle <u>all</u> pairs of values of A and B such that $h(t)$ is differentiable.

i. $A = 3, B = 3$	iii. $A = -6, B = 12$	v. $A = 18, B = 12$
ii. $A = 6, B = 12$	iv. $A = 0, B = 0$	vi. NONE OF THESE