Math 115
Worksheet Section 6.1

The function $F(x)$ is an antiderivative of $f(x)$ if ____________________.

If $F(x)$ is an antiderivative of the differentiable function $f(x)$,

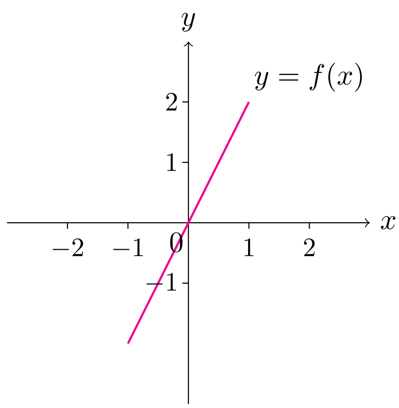
(a) if f is positive on an interval, then F is ____________________ on that interval.

(b) if f is increasing on an interval, then F is ____________________ on that interval.

Problem 1. For each of the given functions f, sketch the graphs of its antiderivatives F, G, H and K such that $F(0) = 0$, $G(0) = 1$, $H(0) = 2$ and $K(0) = -2$.

(a) f is the constant function $f(x) = 0$.

(b) f is the constant function $f(x) = 1$.



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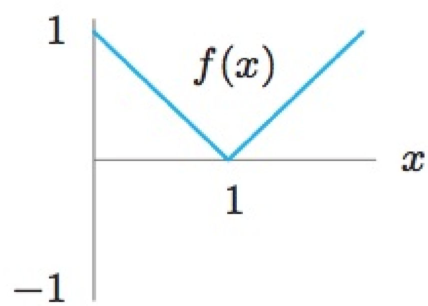


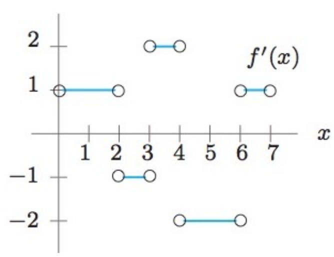
(d)

Problem 2. Let $F(x)$ be the antiderivative of $f(x)$.

(a) If $\int_2^5 f(x)dx = 4$ and $F(5) = 10$, find $F(2)$.

(b) If $\int_0^{100} f(x)dx = 0$, what is the relationship between $F(100)$ and $F(0)$?

Problem 3. Assume f' is given by the graph below. Suppose f is continuous and that $f(0) = 0$.



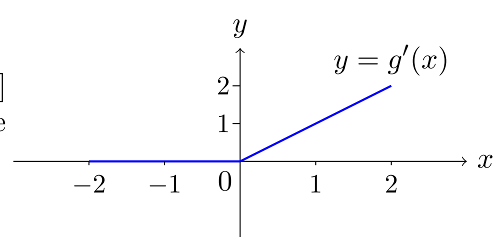
(a) Find $f(3)$ and $f(7)$.

(b) Find all x with $f(x) = 0$.

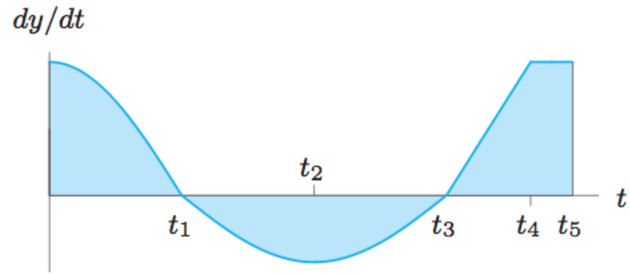
(c) Sketch a graph of f over $0 \le x \le 7$.

Problem 4.

The function g is defined on the interval $[-2, 2]$ and has $g(-1) = 1$. The graph of its derivative $g'(x)$ is given. Sketch the graph of g.



Problem 5. The graph of $\dfrac{dy}{dt}$ against t is below. The three shaded regions each have area 2. If $y = 0$ when $t = 0$, draw the graph of y as a function of t, labeling the known y-values, maxima and minima, and inflection points. Mark $t_1, t_2, \ldots, t_5$ on the t axis.



Problem 6. The Quabbin Reservoir in the western part of Massachusetts provides most of Boston's water. The graph below represents the flow of water in and out of the Quabbin Reservoir throughout 2007.

(a) Sketch a graph of the quantity of water in the reservoir as a function of time.

(b) When, in the course of 2007, was the quantity of water in the reservoir largest? Smallest? Mark and label these points on the graph you drew in part (a).

(c) When was the quantity of water increasing most rapidly? Decreasing most rapidly? Mark and label these times on both graphs.

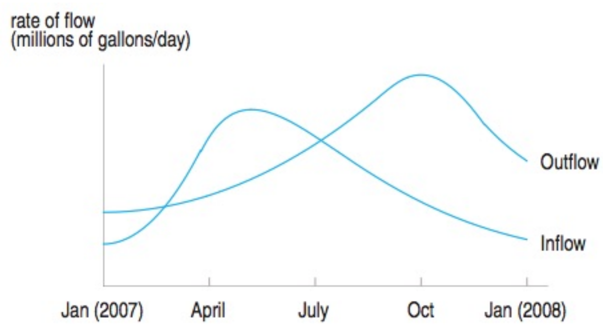
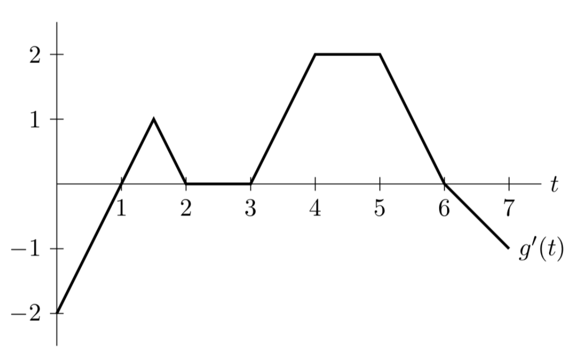


Figure 6.22

Problem 7. (Fall 2013 Final Exam) The function $g(t)$ is the volume of water in the town water tank, in thousands of gallons, t hours after 8 A.M. A graph of $g'(t)$ is shown below. Note that $g'(t)$ is a piecewise-linear function. Suppose that $g(3) = 1$.

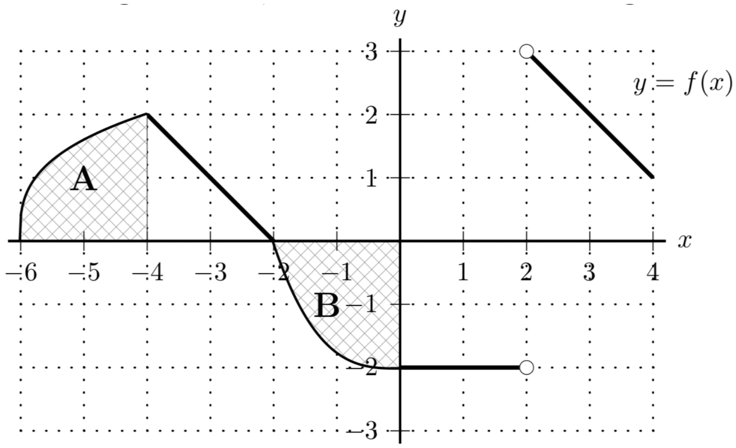


(a) Write an integral which represents the average rate of change, in thousands of gallons per hour, of the volume of water in the tank between 9 A.M. and 1 P.M. Compute its exact value.

(b) At what time does the tank have the most/least water in it?

(c) Sketch a graph of $g(t)$ and give both coordinates of the point on the graph at $t = 7$.

Problem 8. (Fall 2014 Final Exam) A portion of the graph of $y = f(x)$ is shown below. The area of the region A is 3, and the area of the region B is 3. Let $F(x)$ be the continuous antiderivative of $f(x)$ with $F(0) = 1$ whose domain includes the interval $-6 \leqslant x \leqslant 4$.

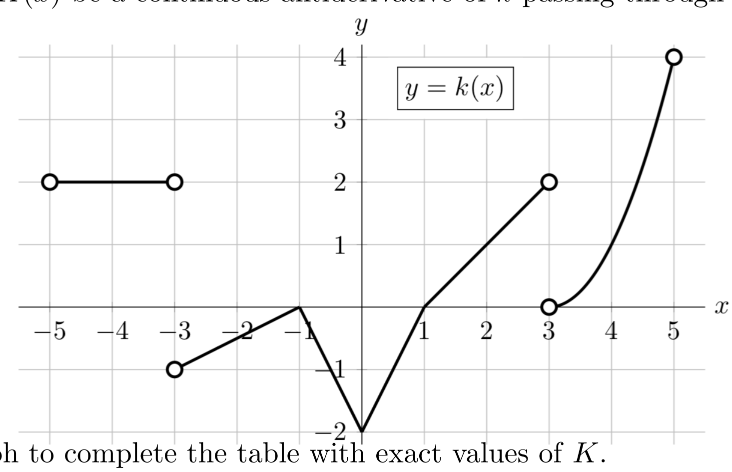


(a) For what value(s) of x with $-6 < x < 4$ does $F(x)$ have local extrema?

(b) Sketch the graph of $y = F(x)$ on the interval $-6 \leqslant x \leqslant 4$. Pay close attention to the following:

- the value of $F(x)$ at each of $x = -6, -4, -2, 0, 2, 4$;
- where F is/is not differentiable;
- where F is increasing/decreasing/constant;
- the concavity of the graph of $y = F(x)$.

Problem 9. (Fall 2016 Final Exam) A portion of the graph of k is shown below. Note that for $3 < x < 5$, the graph of $k(x)$ is a portion of the graph obtained by shifting $y = x^2$ three units to the right. Let $K(x)$ be a continuous antiderivative of k passing through the point $(-1, 1)$.



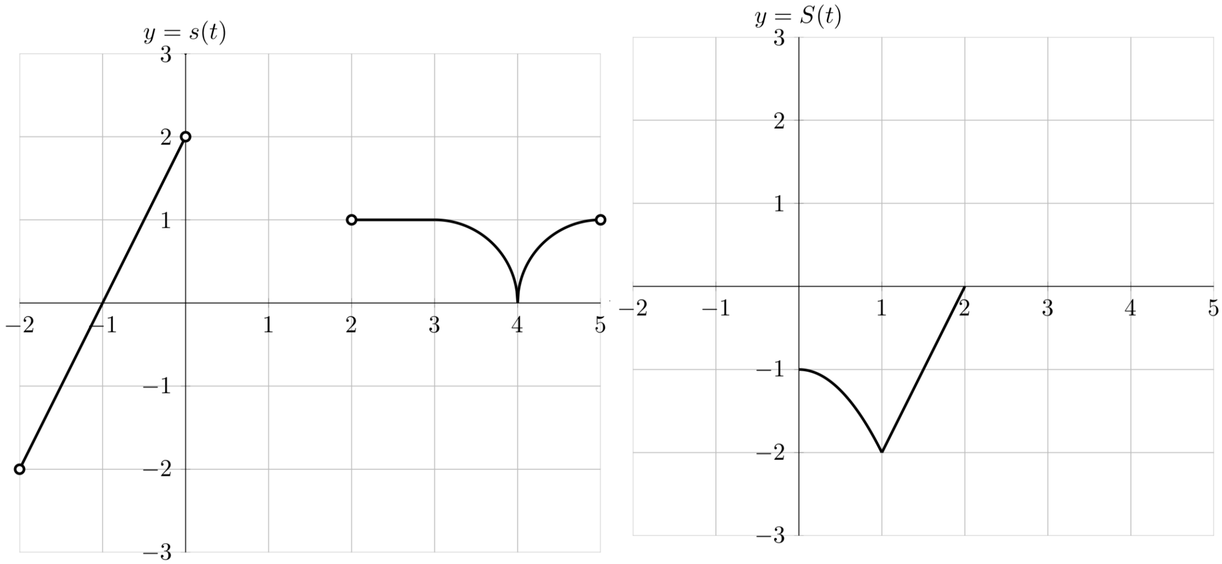
(a) Use the graph to complete the table with exact values of K.

x	-5	-3	-1	1	3	5
$K(x)$						

(b) Sketch a detailed graph of $y = K(x)$ for $-5 < x < 5$. Pay close attention to the following:

- where $K(x)$ is and is not differentiable,
- the values of $K(x)$ you found in the table above,
- where $K(x)$ is increasing/decreasing/constant, and the concavity of $K(x)$.

Problem 10. (Fall 2017 Final Exam) A portion of the graphs of two functions $y = s(t)$ and $y = S(t)$ are shown below. Suppose that $S(t)$ is the continuous antiderivative of $s(t)$ passing through the point $(0, -1)$. Note that the graphs are linear anywhere they appear to be linear, and that on the intervals $(3, 4)$ and $(4, 5)$, the graph of $s(t)$ is a quarter circle.



(a) Use the portions of the graphs to fill in the exact values of $S(t)$ in the table below.

t	-2	-1	0	2	3	5
$S(t)$						

(b) Sketch the missing portions of both s and S over the interval $-2 < t < 5$. Pay attention to:

- the values of $S(t)$ from the table above;
- where S is and is not differentiable;
- the concavity of the graph $y = S(t)$.
- where S and s are increasing/decreasing/constant;

Problem 11. (Winter 2016 Final Exam) Which of the following is an antiderivative of the function $f(x) = \cos(x)$? Circle all the correct options.

(a) $\dfrac{\cos(x)}{2}$

(b) $\sin(x) + 5$

(c) $\cos\left(x - \dfrac{\pi}{2}\right)$

(d) $\ln\left(3e^{\sin(x)}\right)$

(e) $19 - \sin(x)$

(f) None of these

Problem 12. (Fall 2017 Final Exam) Which of the following is an antiderivative of the function $f(x) = \dfrac{1}{x} + \cos(x)$? Circle all the correct options.

(a) $-\dfrac{1}{x^2} - \sin(x)$

(b) $\ln(5x) + \sin(x)$

(c) $\ln(x) + \sin(x) - 20$

(d) $\ln\left(\dfrac{1}{x}\cos(x)\right)$

(e) $\dfrac{1}{x^2} + \sin(x)$

(f) None of these