Math 115
Worksheet Section 3.4

Problem 1. (a) Suppose the squirrel population in my neighborhood grows based on acorn availability, at a rate of 2 squirrels per bushel. Acorn availability is currently growing at a rate of 5 bushels per week. How fast are the squirrels taking over my neighborhood?

(b) Now suppose that a better model is

$$s(a(t)) = 2(a(t))^2 = 2\left(\sin\left(\frac{\pi}{6}t\right) + 3\right)^2.$$

How fast are the squirrels taking over my neighborhood with this model?

Problem 2. Find the derivatives of the following.

(a) $y = \tan\left(\frac{x}{2}\right)$

(b) $y = \sqrt{e^{-3t^2} + 5}$

(c) $f(x) = (2x+1)^{10}(3x-1)^7$

Problem 3. A yam is put into a 200°C oven. Newton's law of cooling (and warming) tells us that its temperature T after t minutes is

$$T(t) = 200 - ae^{-bt}$$

for some constants a and b. After 30 minutes, the temperature of the yam is 120°C and increasing at an instantaneous rate of 2°C per minute. Find a and b.

Problem 4. Prove the quotient rule by writing $h(x) = f(x)(g(x))^{-1}$ and taking the derivative using the product rule and the chain rule. Show your work carefully!

Problem 5. (Fall 2016 Exam 2) The table below gives several values for the function f and its derivative f'. You may assume that f is invertible and differentiable.

w	-2	-1	0	1	2
$f(w)$	1	0	-2	-3	-5
$f'(w)$	-3	-1.5	-0.5	0	-4

For each of the parts below, find the exact value of the given quantity. If there is not enough information provided to find the value, write *not enough info*. If the value does not exist, write *does not exist*. You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

(a) Let $h(w) = \frac{f(w)}{6+w}$. Find $h'(-2)$.

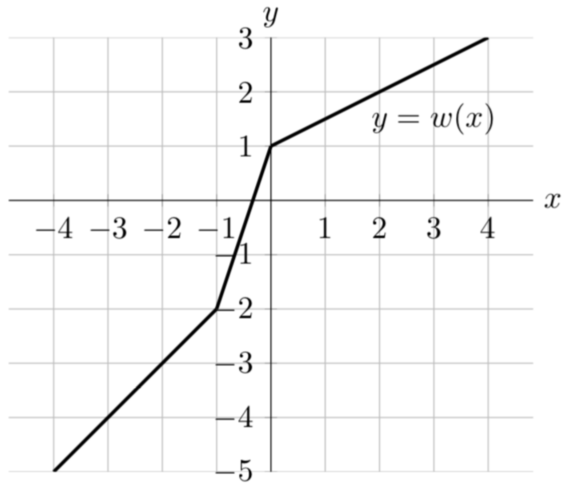
(b) Let $k(w) = 3^{f(2w)}$. Find $k'(-1)$.

(c) Let $p(w) = f(f(-w+1))$. Find $p'(1)$.

(d) Let $r(w) = w(f(w))^2$. Find $r'(2)$.

Problem 6. Use the chain rule to prove $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ by writing $e^{\ln(x)} = x$ and taking the derivative of both sides.

Problem 7. (Winter 2017 Exam 2) A portion of the graph of the function $w(x)$ is shown below.



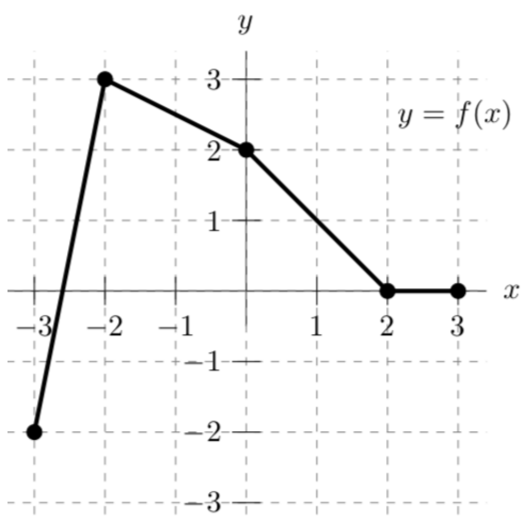
For each of the parts below, find the value of the given quantity. If there is not enough information provided to find the value, write *not enough info*. If the value does not exist, write *does not exist*. You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown. All your answers must be in exact form.

(a) Let $h(u) = \ln(3w(u))$. Find the value of $h'(1)$.

(b) Let $n(x) = \frac{w(x)}{1-x^2}$. Find $n'(-2)$.

(c) Let $s(x)$ be the exponential function $s(x) = 4^{w(x)}$. Find $s'(2)$.

Problem 8. (Winter 2016 Exam 2) Let f be the piecewise linear function with graph shown below.



The table below gives several values of a differentiable function g and its derivative g'. Assume that both $g(x)$ and $g'(x)$ are invertible.

x	-2	-1	0	2	5
$g(x)$	21	11	5	-1	-3
$g'(x)$	-12	-8	-4	-2	-0.4

For each of parts below, find the value of the given quantity. If there is not enough information provided to find the value, write *not enough info*. If the value does not exist, write *does not exist*.

(a) Let $j(x) = e^{g(x)}$. Find $j'(2)$.

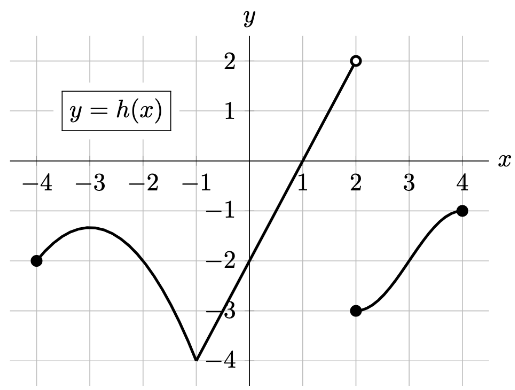
(b) Let $k(x) = f(x)f(x+2)$. Find $k'(-1)$.

(c) Let $h(x) = 3f(x) + g(x)$. Find $h'(-2)$.

(d) Let $m(x) = g(f(g(x)))$. Find $m'(2)$.

(e) Let $l(x) = \frac{f(x)}{2g(x)}$. Find $l'(-1)$.

Problem 9. (Fall 2019 Exam 2) Shown below is the graph of $h(x)$

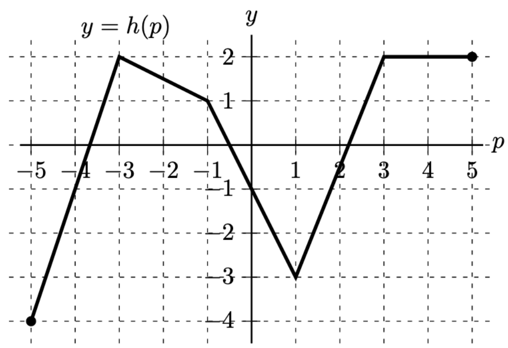


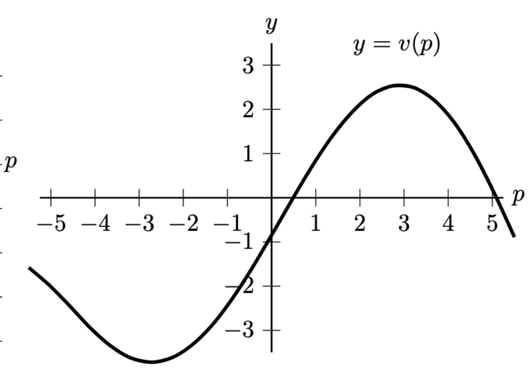
Define the function $k(x)$ such that

$$k(x) = \begin{cases} h(x) & -4 \le x < 1 \\ A^2 \sin(Ax + B) & 1 \le x \le 4 \end{cases}$$

where A and B are constants. Find one pair of values for A and B that make $k(x)$ differentiable at $x = 1$. Show your work

Problem 10. (Fall 2015 Exam 2) The graphs of two functions, $h(p)$ and $v(p)$, are shown below.





The following questions concern the functions B, W, and Q defined as follows:

$$B(p) = \frac{h(2p)}{h(4p)}, \qquad W(p) = h(h(p)), \qquad \text{and} \qquad Q(p) = e^{-v(p)}$$

Assume that the first and second derivatives of $v(p)$ are defined everywhere, i.e. that both v and v' are differentiable on $(-\infty, \infty)$. Note that the graph of $h(p)$ consists of line segments whose endpoints have integer (whole number) coordinates. Find the exact value of each of the quantities in (a) and (b). below. If the value does not exist, write does not exist. Remember to show your work carefully.

(a) $B'(-1)$

(b) $W'(2)$

(c) On the interval $-2 < p < 2$, is $Q(p)$ always increasing, always decreasing, or neither? Show your work and explain your reasoning.