Math 115
Worksheet Section 4.5

Warm-up questions

The **cost function** $C(q)$ gives the cost of producing a quantity q of a certain good.

The **revenue function** $R(q)$ gives the revenue received from selling a quantity q of some good.

The **profit** $\pi(q) =$_____ gives the total profit from producing and selling q of that good.

To decide whether a company's profit would increase or decrease if the company increased or decreased production of a certain good, we might look at the marginal cost and marginal revenue:

The **marginal cost** is given by $\mathrm{MC}(q) =$ _____ $\approx$ _____

The **marginal revenue** is given by $\mathrm{MR}(q) =$ _____ $\approx$ _____

When can maximum profit occur?

How do we identify the fixed cost of producing a certain good?

Problem 1. The total cost of producing q items is approximated by $C(q) = 5000 + 2.4q$ and the total revenue of selling q items is approximated by $R(q) = 4q$, both in dollars. Find the fixed cost, marginal cost per item, and the price at which this commodity is sold.

Problem 2. A car wash operator pays \$35,000 for a franchise, then spends \$10 per car wash, which costs the consumer \$15. Find the cost, revenue, and profit functions.

Problem 3. Revenue of selling q items is given by $R(q) = 450q$, and cost is given by $C(q) = 10,000 + 3q^2$. At what quantity is profit maximized? What is the total profit at this production level?

Problem 4. When production is $q = 2000$ units, marginal revenue is \$4 per unit and marginal cost is \$3.25 per unit. Do you expect maximum profit to occur at a production level above or below 2000 units? Explain.

3. [12 points] Oren plans to grow kale on his community garden plot, and he has determined that he can grow up to 160 bunches of kale on his plot. Oren can sell the first 100 bunches at the market and any remaining bunches to wholesalers. The revenue in dollars that Oren will take in from selling b bunches of kale is given by

$$R(b) = \begin{cases} 6b & \text{for } 0 \le b \le 100 \\ 4b + 200 & \text{for } 100 < b \le 160. \end{cases}$$

a. [2 points] Use the formula above to answer each of the following questions.

   i. What is the price (in dollars) that Oren will charge for each bunch of kale he sells at the market?

   Answer: _____

   ii. What is the price (in dollars) that Oren will charge for each bunch of kale he sells to wholesalers?

   Answer: _____

For $0 \le b \le 160$, it will cost Oren $C(b) = 20 + 3b + 24\sqrt{b}$ dollars to grow b bunches of kale.

b. [1 point] What is the fixed cost (in dollars) of Oren's kale growing operation?
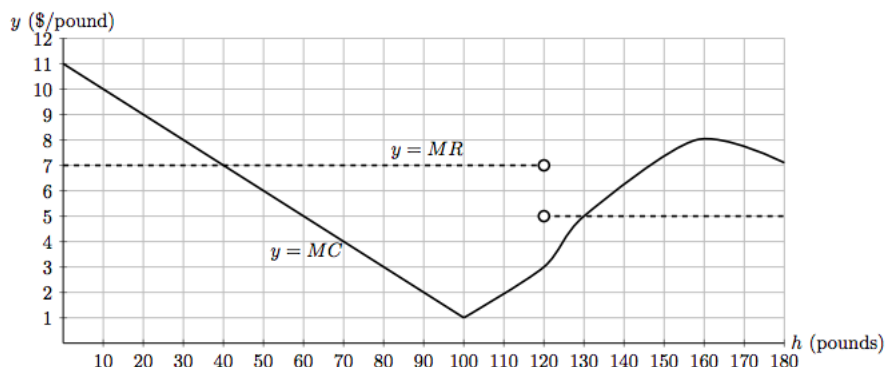
   Answer: _____

c. [4 points] At what production level(s) does Oren's marginal revenue equal his marginal cost?

   Answer: _____

d. [5 points] Assuming Oren can grow up to 160 bunches of kale, how many bunches of kale should he grow in order to maximize his profit, and what is the maximum possible profit? *You must use calculus to find and justify your answer. Be sure to provide enough evidence to justify your answer fully.*

   Answer: bunches of kale: _____ and max profit: _____

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Problem 5. (Winter 2017 Final Exam) The Happy Hives Bee Farm sells honey. The graph below shows marginal revenue MR (dashed) and marginal cost MC (solid), in dollars per pound, where h is the number of pounds of honey.

Use the graph to estimate the answers to the following questions. If an answer can't be found with the information given, write *NEI*.

(i) For what value(s) of h in the interval $[0, 180]$ is the cost function C minimized?

(ii) For what value(s) of h in the interval $[0, 180]$ is MC minimized?

(iii) For what value(s) of h in the interval $[0, 180]$ is profit maximized?

(iv) What are the fixed costs of the farm?

(v) For what values of h in the interval $[0, 180]$ is the profit function concave up?

Problem 6. (Fall 2016 Final Exam) Yukiko has a small orchard where she grows Michigan apples. After careful study last season, Yukiko found that the total cost, in dollars, of producing a bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a} \qquad \text{for } 0 \leqslant a \leqslant 320.$$

Qabil has promised to buy up to 100 bushels of apples for his famous apple ice cream. If Yukiko has any remaining apples, she has an agreement to sell them to Xanthippe's cider mill at a reduced price. Let $R(a)$ be the revenue generated from selling a bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{for } 0 \leqslant a < 100 \\ 200 + 50a & \text{for } 100 < a \leqslant 320 \end{cases}$$

(a) How much will Xanthippe's cider mill pay per bushel?

(b) What is Yukiko's fixed cost?

(c) For what quantities of bushels of apples sold would Yukiko's marginal revenue equal her marginal cost? Write none if appropriate.

(d) Assuming Yukiko can produce up to 320 bushels of apples, how many bushels should she produce in order to maximize her profit, and what would that maximum profit be? You must use calculus to find and justify your answer. Make sure to provide enough evidence to justify your answer fully.

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Problem 7. (Fall 2017 Final Exam) Jane has a company that produces a protein powder for an energy shake. The cost, in dollars, of producing m pounds of protein powder is given by the function

$$C(m) = \begin{cases} \frac{1}{4}(m+2)^2 + 8 & \text{for } 0 \leqslant m < 16 \\ 2m + 57 & \text{for } 16 \leqslant m \leqslant 30 \end{cases}$$

The revenue, in dollars, of selling m pounds of protein powder is given by

$$R(m) = 5m.$$

(a) What is the price, in dollars, at which Jane sells each pound of the protein powder?

(b) What is the fixed cost, in dollars, of producing Jane's protein powder?

(c) Find all values of $16 \leqslant m \leqslant 30$ for which Jane's profit is positive.

(d) Find all the values of $0 \leqslant m \leqslant 30$ where the marginal cost is equal to the marginal revenue for the protein powder. Show all your work to justify your answer.

(e) What is the maximum profit that Jane can make if she sells at most 30 pounds of protein powder? Use calculus to find and justify your answer, and make sure to provide enough evidence to fully justify your answer.

Problem 8. (Fall 2013 Final Exam) Link has started a business selling winter clothes for cats. Among his most successful products are his new kitten mittens. He is currently selling his mittens for $7 per set. Below is a graph of Link's marginal cost MC(q) and marginal revenue MR(q), in dollars per set of mittens, if he makes q sets of mittens this winter. Due to a shortage of yarn, Link can make a maximum of 200 sets of mittens this winter. In order to start making mittens, Link must spend $40 on knitting supplies (in other words, it costs $40 to make 0 sets of mittens).
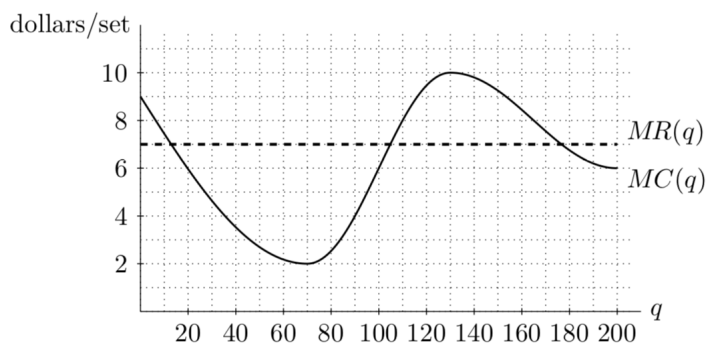
(a) Approximately how many sets of mittens should Link make this winter in order to maximize his profit?

Warning: we do not know how to answer the following question yet, but we will soon!

(b) If the price per set is raised to $9, approximately how many sets of mittens should Link make in order to maximize his profit?

(c) Link makes a deal with a store that would like to buy his cat hats. If the store buys up to 50 hats, then each one will cost $10. If the store buys more than 50 hats, then Link will reduce the price of the entire order by $0.05 per hat for every additional hat over 50. (For example, if the store buys 52 hats, they will pay $9.90 per hat.) Write a formula for a function L(q) which gives Link's revenue if he sells q hats to the store.