Math 115
Worksheet Section 6.2

If $F' = f$, all antiderivatives of f have the form $F(x) + C$. Thus, we say

$$\int f dx = \underline{\hspace{3cm}}$$

Problem 1. Find an antiderivative of $f(x) = 5x - \sqrt{x}$.

Problem 2. Find the general antiderivatives of $g(z) = z + e^z$ and $h(t) = \dfrac{7}{\cos^2(t)}$.

Problem 3. Find an antiderivative of $p(x) = 2 + 4x + 5x^2$ with $F(1) = 1$. Are there any other solutions?

Problem 4. Find the following indefinite integrals.

(a) $\displaystyle\int \frac{x+1}{x}\, dx.$

(b) $\displaystyle\int \left(\frac{3}{t} - \frac{2}{t^2}\right) dt$

(c) $\displaystyle\int t^3(t^2+1)dt$

Problem 5. Find $\displaystyle\int_1^3 \frac{1}{y}\, dy$ and $\displaystyle\int_0^{\pi/4} \sin(t) + \cos(t)\ dt.$

Problem 6. Water is pumped into a cylindrical tank, standing vertically, at a decreasing rate given at time t minutes by

$$r(t) = 120 - 6t \text{ cubic feet/min, for } 0 \leq t \leq 10.$$

The tank has radius 5 ft and is empty when $t = 0$. Find the depth of water in the tank at $t = 4$.

Problem 7. In drilling an oil well, the total cost, C, consists of fixed costs (independent of the depth of the well) and marginal costs, which depend on depth; drilling becomes more expensive, per meter, deeper into the earth. Suppose the fixed costs are $1,000,000$ riyals and the marginal costs are

$$C'(x) = 4000 + 10x \text{ riyals/meter}$$

where x is the depth in meters. Find the total cost of drilling a well x meters deep.

Problem 8. The area under the graph of $1/\sqrt{x}$ from $1 \leq x \leq b$ is 6. Find b.

Problem 9. Sketch the parabola $y = x(x - \pi)$ and the curve $y = \sin x$, showing their points of intersection. Find the exact area between the two graphs.

Problem 10. (Fall 2015 Final Exam) Which of the following functions are antiderivatives of $f(x) = \frac{1}{x}$? Circle all such functions.

(a) $\ln(|x+1|)$

(b) $\ln(|x|)$

(c) $\ln(|x|) + 2$

(d) $\ln(4|x|)$

(e) $4\ln(|x|)$

(f) None of these

Problem 11. Which of the following functions are antiderivatives of $f(x) = e^{x/2}$?

(a) $e^{x/2}$

(b) $2e^{x/2}$

(c) $2e^{(1+x)/2}$

(d) $2e^{x/2} + 1$

(e) $e^{x^2/4}$

(f) None of these

Problem 12. (Fall 2013 Final Exam) A basketball player is running sprints in Crisler Center. She begins in the middle of the "M" at the center of the court and runs north and south. Her velocity, in meters per second, for the first 9 seconds is $v(t) = t\sin(\frac{\pi}{3}t)$, where t is the number of seconds since she started running. She is running north when $v(t)$ is positive and south when $v(t)$ is negative.

(a) Show that the function

$$f(t) = \frac{9}{\pi^2}\sin\left(\frac{\pi}{3}t\right) - \frac{3}{\pi}t\cos\left(\frac{\pi}{3}t\right)$$

is an antiderivative of $v(t)$.

(b) Where on the court is the player after the 9 seconds? Show all your work and give your answer in exact form (no decimal approximations).

(c) What is the total distance traveled by the player in the 9 seconds? Show all your work and give your answer in exact form (no decimal approximations).

Problem 13. (a) What is the average value of $f(t) = \sin t$ over $0 \le t \le 2\pi$? Why is this a reasonable answer?

(b) Find the average value of $f(t) = \sin t$ over $0 \le t \le \pi$.

Problem 14. Decide if each of the following equalities is true or false.

(a) $\int e^{2x} dx = 2e^{2x} + C$

(b) $\int e^{x^2} dx = 2x \cdot e^{x^2} + C$

(c) $\int 3\cos x dx = 3\sin x + C$

(d) $\int 5\sin\left(\frac{x}{5}\right) dx = 5\cos\left(\frac{x}{5}\right) + C$