Math 115

Worksheet Section 2.2

Problem 1. (a) What is the definition of the derivative of the function f(x) at x = c?

(b) Compute the derivative of $g(x) = 3x^2$ at x = 10 algebraically. In other words, use algebra to find the limit from the definition exactly using limit computations we learned from the previous sections.

Problem 2. (Winter 2018 Exam 1) Let $m(x) = (1+x^2)^{3x-4}$. Which of the limits below represents m'(2)? There is only one correct answer. Be sure to explain your reasoning on the board.

(a)
$$\lim_{h\to 0} \frac{(1+x^2)^{3x-4} + h - 25}{h}$$

(d)
$$\lim_{h\to 0} \frac{(1+(2+h)^2)^{3h+2}-25}{h}$$

(b)
$$\lim_{h\to 0} \frac{(1+h^2)^{3h-4}-25}{h}$$

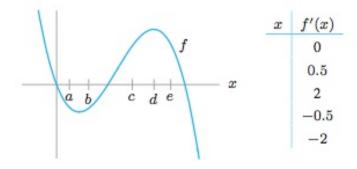
(e)
$$\lim_{h \to 0} \frac{(5+h^2)^{3h+2} - 25}{h}$$

(c)
$$\lim_{h\to 0} \frac{(1+(2+h)^2)^{3h-4}-25}{h}$$

(f)
$$\lim_{h\to 0} \frac{(1+h^2)^{3h+2}-25}{h}$$

Problem 3. (2.2 #27) Create a table using difference quotients to approximate the derivative of x^x at x = 2 to one decimal place. You may use a calculator for this problem.

Problem 4. (2.2 #11) Match the derivatives in the table with the points a, b, c, d, e.



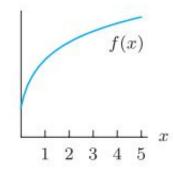
Problem 5. (2.2 # 15) For each of the following pairs, use the graph to decide which is larger. Explain.

(a)
$$f(3)$$
 or $f(4)$?

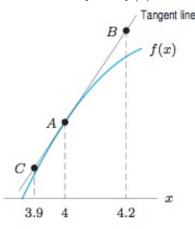
(b)
$$f(3) - f(2)$$
 or $f(2) - f(1)$?

(c)
$$\frac{f(2) - f(1)}{2 - 1}$$
 or $\frac{f(3) - f(1)}{3 - 1}$?

(d)
$$f'(1)$$
 or $f'(4)$?



Problem 6. (2.2 #17) The given function f has f(4) = 25 and f'(4) = 1.5. Find the coordinates



of the points A, B, C.

Problem 7. (Winter 2016 Exam 1) Consider the function g defined by

$$g(x) = \begin{cases} \frac{1}{e^x - 1} & \text{if } x < \frac{1}{2} \\ \cos(x^x) & \text{if } \frac{1}{2} \le x < 5 \\ \frac{x^2}{(x - 1)(6 - x)} & \text{if } x \geqslant 5 \end{cases}$$

- (a) Use the limit definition of the derivative to write an explicit expression for g'(3). Your answer should not involve the letter g. Do not attempt to evaluate or simplify the limit.
- (b) Find all vertical asymptotes of g, if there are any.

Problem 8. (Winter 2018 Exam 1) Sketch the graph of a single function y = f(x) satisfying all of the following conditions:

- The domain of f(x) is the interval $-8 < x \le 6$.
- f(x) is continuous on the interval -8 < x < -2.
- f'(-7) = 0.
- f(x) is decreasing and concave up for all x in the interval -6 < x < -4.
- The average rate of change of f(x) is equal to 0.5 between x = -5 and x = -2.
- f(0) = 2 and f'(0) = -1.
- $\lim_{x\to 2^-} f(x) = f(2)$ and $\lim_{x\to 2^+} f(x) < \lim_{x\to 2^-} f(x)$. f(x) has constant rate of change on the interval $3 \leqslant x \leqslant 6$.

Make sure that your graph is large and unambiguous.