

Math 115 Fall 2024
Worksheet 15 – MVT

Warm-up questions

What are the hypothesis of the mean value theorem?

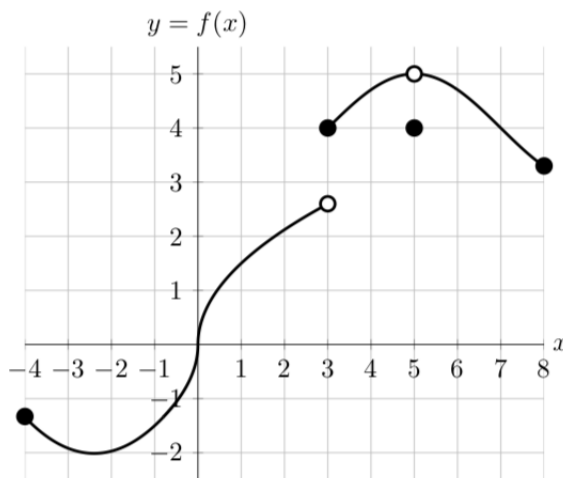
What is the conclusion of the mean value theorem?

Problem 1. Give an example of a function f that is...

- (a) continuous on the interval $[-1, 1]$ but that does not satisfy the hypothesis of the Mean Value Theorem on that interval, and for which the conclusion of the Mean Value Theorem is false.
- (b) continuous on the interval $[-1, 1]$ but that does not satisfy the hypothesis of the Mean Value Theorem on that interval, and for which the conclusion of the Mean Value Theorem is true.
- (c) differentiable on the interval $(0, 2)$ but that does not satisfy the hypothesis of the Mean Value Theorem on $[0, 2]$.

Problem 2. (Winter 2018 Exam 2) The graph of the function f with domain $-4 \leq x \leq 8$ is shown below. The function $f(x)$ satisfies:

- $f(x) = 1.5x^{\frac{1}{3}}$ for $-1 < x < 1$, and
- $f(x) = 4 + \sin\left(\frac{\pi}{4}(x - 3)\right)$ for $3 \leq x < 5$ and $5 < x \leq 8$.



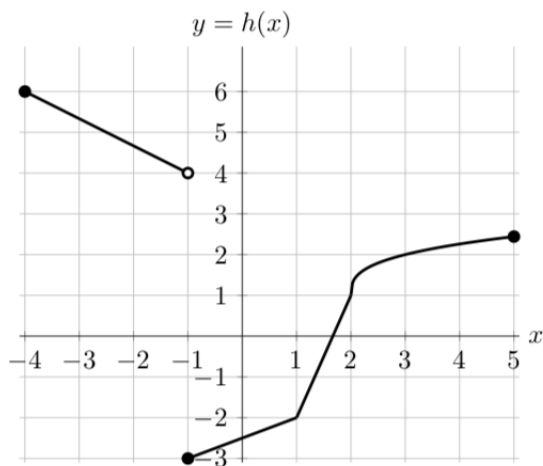
- (a) On which of the following intervals is the conclusion of the Mean Value Theorem true?

$[-4, 0]$ $[0, 5]$ $[1, 3]$ $[3, 7]$ none of these

- (b) On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, 0]$ $[0, 5]$ $[1, 3]$ $[3, 7]$ none of these

Problem 3. (Fall 2017 Exam 2) Consider the graph of $h(x)$ below. Note that h is linear on the intervals $[-4, -1)$, $[-1, 1]$ and $[1, 2]$, differentiable on $(2, 5)$, and has a sharp corner at $x = 2$.



(a) On which of the following intervals is the conclusion of the Mean Value Theorem true?

$[-4, -1]$ $[-2, -1]$ $[0, 4]$ $[2, 5]$ none of these

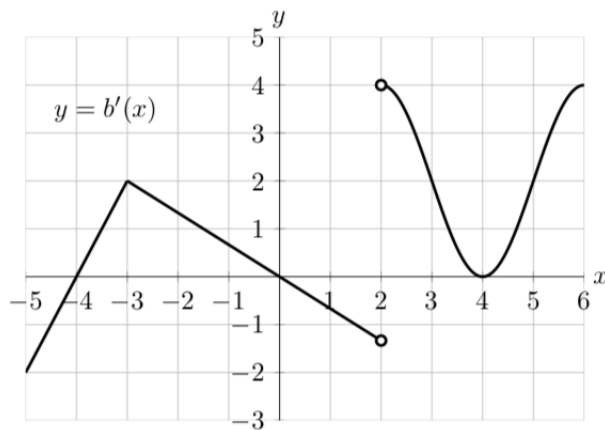
(b) On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, -1]$ $[-2, -1]$ $[0, 4]$ $[2, 5]$ none of these

(c) For which values given below is the function $m(x) = h(h(x))$ not differentiable? Circle all that apply.

$x = -3$ $x = -1$ $x = 2$ $x = 3$ $x = 4$ none of these

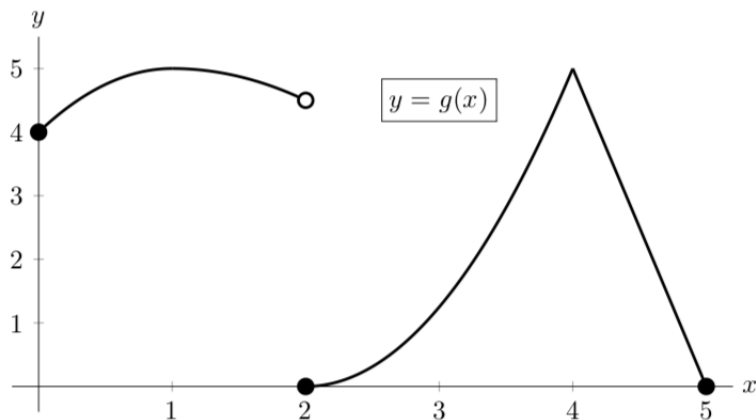
Problem 4. The graph of a portion of the derivative of $b(x)$ is shown below. Assume that $b(x)$ is defined and continuous on $[-5, 6]$.



On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, -2]$ $[-2, 2]$ $[1, 4]$ $[-5, 6]$ none of these

Problem 5. (Fall 2016 Exam 2) The entire graph of a function $g(x)$ is shown below. Note that the graph of $g(x)$ has a horizontal tangent line at $x = 1$ and a sharp corner at $x = 4$.



- (a) On which of the following intervals does the function $f(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle the correct answer(s):

$[0, 2]$ $[1, 3]$ $[2, 4]$ $[3, 5]$ None of these

- (b) On the interval $[8, 12]$ the hypotheses of the Mean Value Theorem are true for the function $f(x)$. What does the conclusion of this theorem say in this interval?

Answer:

Problem 6. (Winter 2016 Exam 2) Let $h(x)$ be a twice differentiable function defined for all real numbers x . (So h is differentiable and its derivative h' is also differentiable.) Some values of the derivative of h are given in the table below.

x	-8	-6	-4	-2	0	2	4	6	8
$h'(x)$	3	7	0	-3	-5	-4	0	-2	6

- (a) Circle all the intervals which must contain a number c such that $h''(c) = 2$.

$-8 < x < -6$ $-4 < x < -2$ $-2 < x < 0$ $6 < x < 8$

- (b) Suppose that $h''(x) < 0$ for $x < -8$ and $h(-8) = 7$. Circle all the numbers below which could equal the value of $h(-10)$.

-2 -1 0 1 2 None of these