

# Math 115

## Worksheet Section 3.1

**Problem 1.** Find the derivatives of the following functions

- (a)  $a(x) = x^{12}$
- (b)  $b(x) = x^{3/4}$
- (c)  $c(x) = x^{-3/4}$
- (d)  $d(x) = \ln e^{ax}$  for  $a$  a constant
- (e)  $e(x) = \sqrt{x}(x + 1)$
- (f)  $f(x) = \frac{x^2+1}{x}$
- (g)  $g(x) = 3x^2 + \frac{12}{\sqrt{x}} - \frac{1}{x^2}$

**Problem 2.** What is the derivative of  $f(x) = x^{\frac{1}{5}}$ ? Is  $f$  differentiable at  $x = 0$ ?

**Problem 3.** On what intervals is the graph of  $g(x) = x^4 - 4x^3$  both decreasing and concave up?

**Problem 4.** For what values of  $x$  is the function  $f(x) = x^5 - 5x$  both increasing and concave up?

**Problem 5.** The  $n^{\text{th}}$  derivative of  $f$ ,  $f^{(n)}(x)$ , is the result of differentiating  $f(x)$   $n$  times. Consider the function  $f(x) = x^7 + 5x^5 - 4x^3 + 6x - 7$ .

- (a) Find the 8th derivative of  $f(x)$ . Think ahead!
- (b) Find the 7th derivative of  $f(x)$ .

**Problem 6.** (a) Find values for  $a$  and  $b$  so that the function  $k$  is both continuous and differentiable everywhere.

$$k(x) = \begin{cases} ax + 2 & x < 0 \\ b(x - 1)^2 & x \geq 0 \end{cases}$$

- (b) What is  $k'(x)$ ?

**Problem 7.** Let  $p(x)$  be a seventh-degree polynomial with 7 distinct zeros. How many zeros does  $p'(x)$  have? Hint: use MVT to solve this.

**Problem 8.** At a time  $t$  seconds after it is thrown up in the air, a tomato is at a height of  $f(t) = -4.9t^2 + 25t + 3$  meters.

- (a) What is the average velocity of the tomato during the first 2 seconds? Give units.
- (b) Find (exactly) the instantaneous velocity of the tomato at  $t = 2$ . Give units.
- (c) What is the acceleration at  $t = 2$ ?
- (d) How high does the tomato go?
- (e) How long is the tomato in the air?

**Problem 9.** (a) Find an equation of the line tangent to the graph of  $f(x) = \sqrt{x}$  at the point  $(4, 2)$  on the graph.

(b) For  $f(x) = \sqrt{x}$ , what is  $\lim_{x \rightarrow \infty} f'(x)$ ? How is this consistent with the graph of  $f(x)$ ?

**Problem 10.** (Fall 2018 Exam 2) Let  $A$  and  $B$  be constants and

$$k(x) = \begin{cases} 3x + \frac{B}{x} & \text{for } 0 < x < 1 \\ Bx^2 + Ax^3 & \text{for } 1 \leq x \end{cases}$$

Find the values of  $A$  and  $B$  that make the function  $k(x)$  differentiable on  $(0, \infty)$ . Show all your work to justify your answers. If there are no such values of  $A$  and  $B$ , write none.

**Problem 11.** Suppose  $p$  is a cubic polynomial function, meaning that  $p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$  for some constants  $a_0, a_1, a_2, a_3$ , with  $a_0 \neq 0$ .

(a) Write expressions for  $p(0)$ ,  $p'(0)$ ,  $p''(0)$  and  $p'''(0)$  depending on  $a_0, a_1, a_2$ , and  $a_3$ .

(b) Find the formula for a cubic polynomial function  $q$  that satisfies

$$q(0) = 2, \quad q'(0) = -1, \quad q''(0) = 5, \quad q'''(0) = 4.$$

**Problem 12.** Assume that  $f''$  and  $g''$  exist and that  $f$  and  $g$  are concave up for all  $x$ . Are the following statements true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample.

(a)  $f(x) + g(x)$  is concave up for all  $x$ .

(b)  $f(x) - g(x)$  cannot be concave up for all  $x$ .

**Problem 13.** Let  $f(x) = x^4 - 3x^2 + 1$ .

(a) Show that  $f(x)$  is an even function.

(b) Show that  $f'(x)$  is an odd function.

(c) Are all polynomials of even degree even functions?

**Problem 14.** (Winter 2016 Exam 3) For constants  $A$  and  $B$ , consider the function  $h$  defined by

$$h(t) = \begin{cases} (At)^2 - 48 & \text{if } t < 2 \\ Bt^3 & \text{if } t \geq 2. \end{cases}$$

Circle all pairs of values of  $A$  and  $B$  such that  $h(t)$  is differentiable.

i.  $A = 3, B = 3$

iii.  $A = -6, B = 12$

v.  $A = 18, B = 12$

ii.  $A = 6, B = 12$

iv.  $A = 0, B = 0$

vi. NONE OF THESE