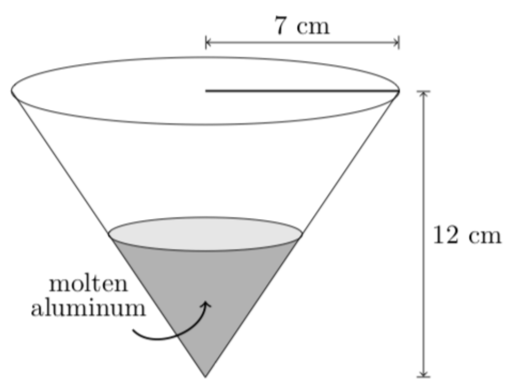
Math 115
Worksheet Section 4.6 (continued)

Problem 1. A rectangle has one side of length 8cm. How fast is the diagonal changing at the instant when the other side is 6cm and increasing at 3cm per minute?

Problem 2. Grit, which is spread on roads in winter, is stored in mounds which are in the shape of a cone. As grit is added to the top of the mound at 2 cubic meters per minute, the angle between the slant side of the cone and the vertical remains 45°. How fast is the height of the mound increasing when it is half a meter high?

Problem 3. The London Eye is a large Ferris wheel that has diameter 135 meters and revolves continuously. Passengers enter the cabins at the bottom of the wheel and complete one revolution in about 27 minutes. One minute into the ride, a passenger is rising at 0.06 meters per second. How fast is the horizontal motion of the passenger at that moment?

Problem 4. (Fall 2016 Final Exam) Uri is filling a cone with molten aluminum. The cone is upside-down, so the base is at the top of the cone and the vertex at the bottom, as shown in the diagram. The base is a circular disk with radius 7 cm and the height of the cone is 12 cm. Recall that the volume of a cone is $\frac{1}{3}Ah$, where A is the area of the base and h is the height of the cone (i.e., the vertical distance from the vertex to the base). (Note that the diagram may not be to scale.)

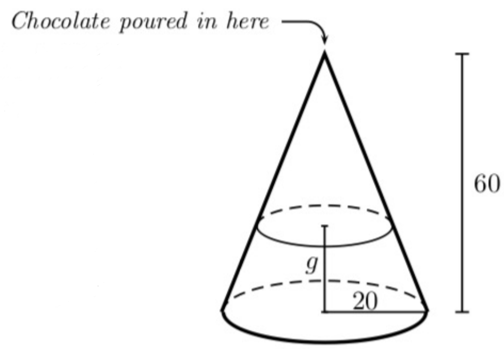


(a) Write a formula in terms of h for the volume V of molten aluminum, in cm^3, in the cone if the molten aluminum in the cone reaches a height of h cm.

(b) The height of molten aluminum is rising at 3 cm/sec at the moment when the molten aluminum in the cone has reached a height of 11 cm. What is the rate, in cm^3/sec, at which Uri is pouring molten aluminum into the cone at that moment?

(c) The height of molten aluminum is rising at 3 cm/sec at the moment when the molten aluminum in the cone has reached a height of 11 cm. What is the rate, in cm^2/sec, at which the area of the top surface of the molten aluminum is increasing at that moment?

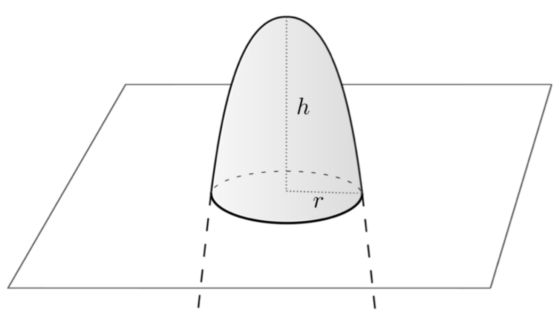
Problem 5. (Winter 2015 Final Exam) Having taken care of Sebastian and sent Erin into the hands of the Illumisqati, King Roderick is pleased that his plan is proceeding well. Our wicked villain decides to relax with a handmade chocolate before he heads to his farmhouse. The process of making the chocolate involves pouring molten chocolate into a mould. The mould is a cone with height 60 mm and base radius 20 mm. Roderick places the mould on the ground and begins pouring the chocolate through the apex of the cone.



(a) Let g be the depth of the chocolate, in mm, as shown in the diagram above. What is the value of g when Roderick has poured a total of 20,000 mm^3 of chocolate into the mould? Show your work carefully, and make sure your answer is accurate to at least two decimal places.

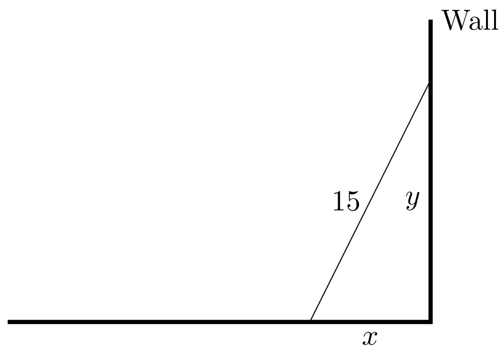
(b) How fast is the depth of the chocolate in the mould (g in the diagram above) changing when Roderick has already poured 20,000 mm3 of chocolate into the mould if he is pouring at a rate of 5,000 mm^3 per second at this time? Show your work carefully and make sure your answer is accurate to at least two decimal places. Be sure to include units.

Problem 6. (Winter 2018 Final Exam) A group of meteorologists observe that the sea level is rising by observing a piece of a rock in the sea. Only the tip of the rock is visible, and as the sea water rises, less and less of the rock is above water. Let h and r be the height and radius (in inches), respectively, of the part of the rock that is above the sea. The volume of the rock (in cubic inches) is then given by the formula $V = \frac{\pi}{2}(1 + r^2)h$.

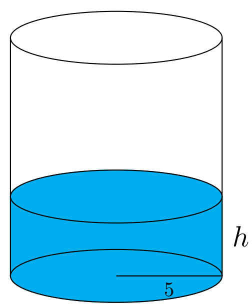


The meteorologists notice that, as the level of the sea is rising, the radius and volume of the rock are changing. A year after they started taking the measurements, the radius and height of the rock are 5 and 46 inches, respectively. They notice that at that time, the radius is decreasing at a rate of 0.05 inches per year, which makes the volume change at a rate of 80 cubic inches per year. At what rate is the height of the rock changing at that time? Be sure to include units. Is the height of the part of the rock that is above the sea increasing or decreasing?

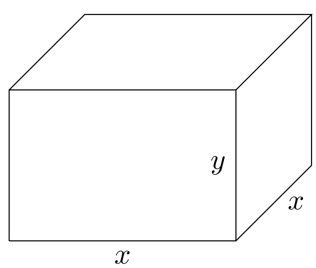
Problem 7. A 15-ft ladder leaning against a wall begins to slide. How fast is the top of the ladder sliding down the wall at the instant of time when the bottom of the ladder is 9 ft from the wall and sliding away from the wall at the rate of 6 ft/sec?



Problem 8. A coffee pot in the form of a circular cylinder of radius 5 in. is being filled with water flowing at a constant rate. If the water level is rising at the rate of 0.7 in./sec, what is the rate at which water is flowing into the coffee pot?



Problem 9. Suppose we have a constantly changing rectangular box with fixed volume of 100 in^3 and a square base where the sides of the base are increasing by 5 inches every second. If this necessarily flexible material costs $\$10, \$20$, and $\$30$ per square inch, for the top, bottom, and sides, respectively, then find the rate of change of the cost when the sides of the base are 10 inches.



Problem 10. Challenge: Suppose that water is flowing out of a hole at the bottom of a cone into a cylinder located below the bottom of the cone. Suppose that the height of the water in the cone is decreasing at a constant rate of 2 cm/sec. The cone has radius 10 cm and height 20 cm. The cylinder has radius 30 cm and height 70 cm. For simplicity, assume that as soon as water leaves the hole of the cone it instantaneously goes into the cylinder. In other words, it takes 0 time for the water to fall from the hole into the cylinder. What is the rate of change of the height of the water in the cylinder when the height of the water in the cone is 4 cm? **(Hint: Use Similar Triangles)**