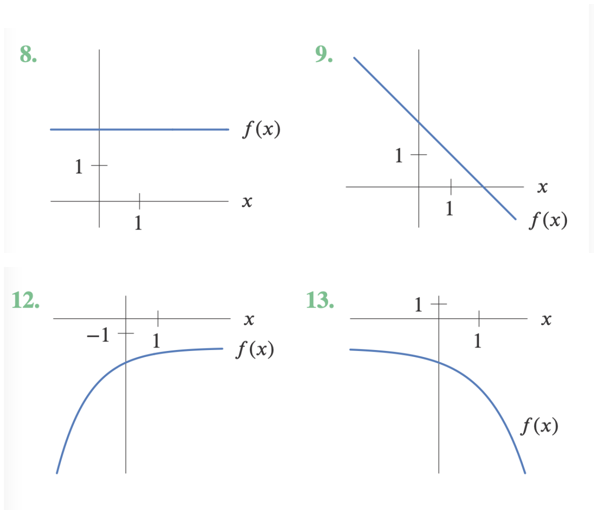
Math 115

Worksheet Section 2.5

Warm-up questions

If $f''(x) < 0$ on an interval, then f is ______________ on that same interval.
If $f''(x) > 0$ on an interval, then f is ______________ on that same interval.
If the graph of f is ______________ on an interval where f'' exists, then $f''(x) \geqslant 0$ on that interval.
If the graph of f is ______________ on an interval where f'' exists, then $f''(x) \leqslant 0$ on that interval.
The instantaneous acceleration at a point is ____.

Problem 1. For each, is $f'(x)$ is positive, negative, or zero? What about $f''(x)$?



Problem 2. Define a position function $s(t)$ and suppose $s'(t) = v(t)$ is given in the following table. What can you say about $v(t)$? What about the acceleration function $v'(t) = a(t)$?

Time, t (sec)	0	1	2	3	4	5
Velocity, $v(t)$ (ft/sec)	0	30	52	68	80	88

Problem 3. At which of the marked x-values in the graph below can the following statements be true?

(a) $f(x) < 0$

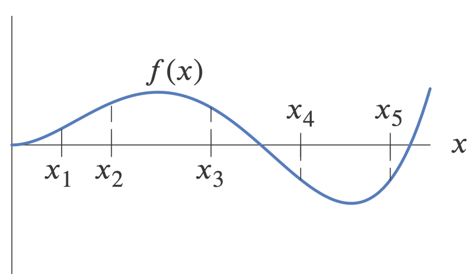
(b) $f'(x) < 0$

(c) $f(x)$ is decreasing

(d) $f'(x)$ is decreasing

(e) The slope of the tangent lines of $f(x)$ is positive

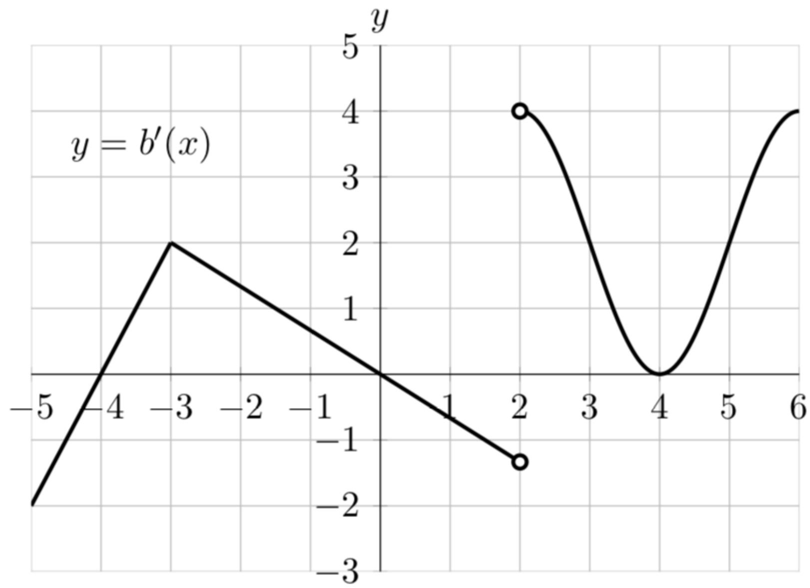
(f) The slope of the tangent lines of $f(x)$ is increasing



Problem 4. A function f has $f(5) = 20, f'(5) = 2$ and $f''(x) < 0$ for $x \geq 5$. Which of the following are possible values for $f(7)$ and which are impossible?

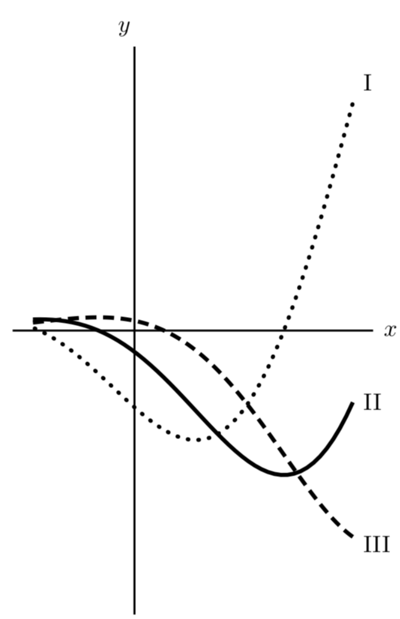
(a) 26 (b) 24 (c) 22

Problem 5. (Winter 2017 Exam 2) The graph of a portion of the derivative of $b(x)$ is shown below. Assume that $b(x)$ is defined and continuous on $[-5, 6]$.

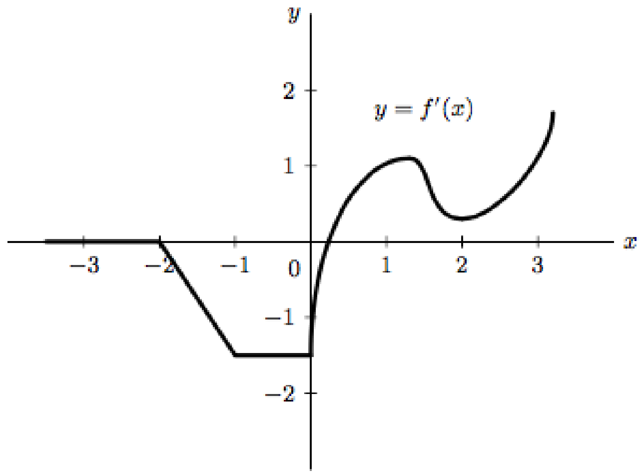


For what values of x is $b(x)$ concave up? Write your answer using inequalities or interval notation.

Problem 6. (Winter 2015 Exam 3) Shown on the axes below are the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which.



Problem 7. (Fall 2015 Exam 1) Below is the graph of $f'(x)$, the **derivative** of the function $f(x)$. Note that $f'(x)$ is zero for $x \leqslant -2$, linear for $-2 < x < -1$, and constant for $-1 < x < 0$.



For each of the following, circle all of the listed intervals for which the given statement is true over the entire interval. If there are no such intervals, circle NONE. You do not need to explain your reasoning.

(a) $f'(x)$ is increasing.

$-2 < x < -1$ $\quad$ $0 < x < 1$ $\quad$ $1 < x < 2$ $\quad$ $2 < x < 3$ $\quad$ NONE

(b) $f'(x)$ is concave up.

$0 < x < 1$ $\quad$ $1 < x < 2$ $\quad$ $2 < x < 3$ $\quad$ NONE

(c) $f(x)$ is increasing.

$-2 < x < -1$ $\quad$ $-1 < x < 0$ $\quad$ $0 < x < 1$ $\quad$ $1 < x < 2$ $\quad$ $2 < x < 3$ $\quad$ NONE

(d) $f(x)$ is linear but not constant.

$-3 < x < -2$ $\quad$ $-2 < x < -1$ $\quad$ $-1 < x < 0$ $\quad$ $0 < x < 1$ $\quad$ $1 < x < 2$ $\quad$ $2 < x < 3$ $\quad$ NONE

(e) $f(x)$ is constant.

$-3 < x < -2$ $\quad$ $-2 < x < -1$ $\quad$ $-1 < x < 0$ $\quad$ $0 < x < 1$ $\quad$ $1 < x < 2$ $\quad$ $2 < x < 3$ $\quad$ NONE

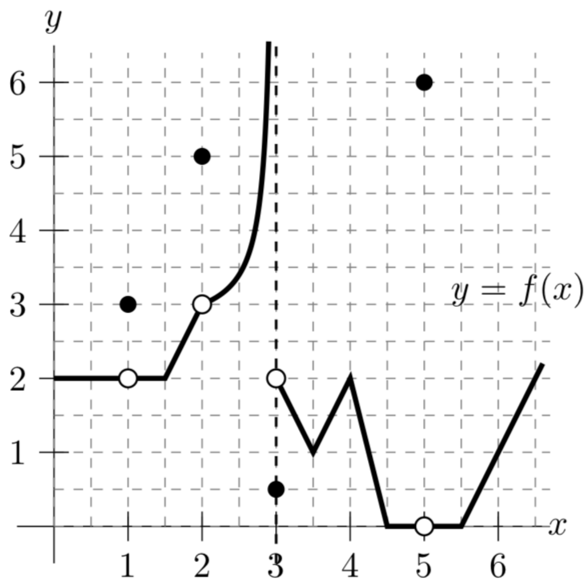
Problem 8. (Winter 2017 Exam 2) Some information about a function $f(x)$ is given in the table below.

x	-2	-1	0	1	2	3	4
$f'(x)$	-2	0	-2	0	1	0	-1
$f''(x)$	1	0	0	2	0	0	-2

Assume that $f''(x)$ is continuous on $[-2, 4]$ and that the values of $f'(x)$ and $f''(x)$ are strictly positive or strictly negative between consecutive table entries. Circle all of the intervals on which $f''(x)$ must be negative, if any exist.

$-2 < x < -1$ $\quad -1 < x < 0$ $\quad 0 < x < 1$ $\quad 1 < x < 2$ $\quad 2 < x < 3$ $\quad 3 < x < 4$ $\quad$ None of these

Problem 9. (Winter 2016 Exam 1) The graph of a function f is shown below.



Note: You may assume that pieces of the function that appear linear are indeed linear. Use the graph above to evaluate each of the expressions below.

(a) $f(1)$

(b) $\lim_{x \to 5} f(x)$

(c) $\lim_{q \to 3} f(q)$

(d) $\lim_{z \to 2} f(2)$

(e) $\lim_{r \to 6^-} f(r)$

(f) $\lim_{h \to 0} \dfrac{f(4.25 + h) - f(4.25)}{h}$

(g) $\lim_{p \to 0.5} \dfrac{f(p)}{p}$

(h) $\lim_{t \to 3} f(t) f(t + 2)$

(i) $\lim_{x \to 3^+} f(f(x))$

(j) $\lim_{s \to 1} f(f(s))$