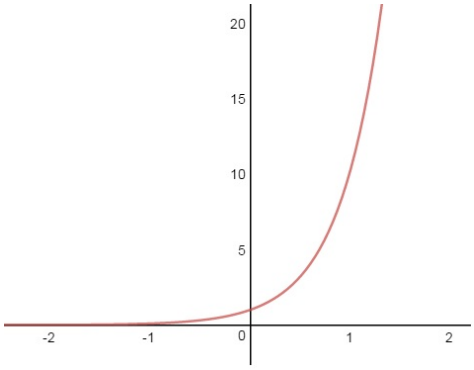
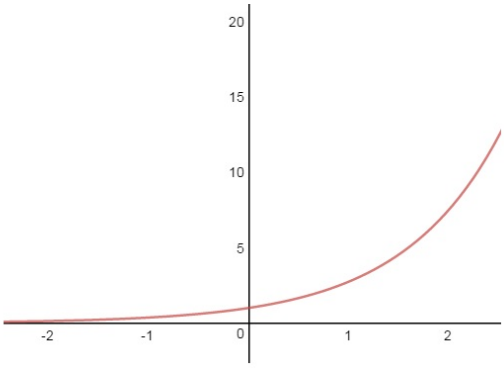
Math 115
Worksheet Section 1.4

Logarithms have a reputation for being confusing. One of the best ways to really understand them is to remember that they're the inverses of exponential functions.

Problem 1. (a) Use these graphs of $f(x) = 10^x$ and $g(x) = e^x$ respectively to estimate $f^{-1}(15)$ and $g^{-1}(5)$.





(b) The base-10 logarithm of x, written $\log_{10}(x)$ or just $\log(x)$, is the exponent we need to put on the 10 so that we get x. That is, $\log_{10} x = c$ means...

(c) The natural logarithm of x, written $\ln(x)$ is similar, except the base is e instead. So, $\ln(x) = c$ means..

(d) Estimate $\log_{10}(15)$ and $\ln(5)$.

Problem 2. (a) What is $\log(1000)$? $\log(1,000,000)$? $\ln(e^{42})$? $\ln(1)$?

(b) (1.4 #45)

At time t hours after taking the cough suppressant hydrocodone bitartrate, the amount, A, in mg, remaining in the body is given by $A = 10(0.82)^t$.

(a) What was the initial amount taken?
(b) What percent of the drug leaves the body each hour?
(c) How much of the drug is left in the body 6 hours after the dose is administered?
(d) How long is it until only 1 mg of the drug remains in the body?

Note: You should be comfortable using **all** the logarithm properties in the box on page 30.

(c) Lily is studying a particular radioactive isotope of radon. Ten minutes into her experiment, she measures that only 15% of the radon remains. Find the **half-life** of this isotope. (You don't need any special formulas to do this!)

(d) (1.4 #55)

In 2010, there were about 246 million vehicles (cars and trucks) and about 308.7 million people in the US.[33] The number of vehicles grew 15.5% over the previous decade, while the population has been growing at 9.7% per decade. If the growth rates remain constant, when will there be, on average, one vehicle per person?

Problem 3. Find the inverse of the function $f(x) = 2^{3^x}$ and state its domain and range.

Problem 4. For each of the following functions, sketch its graph, and find the domain, range, and asymptotes (on a separate sheet of paper).
(a) $f(x) = 2^{-x+1}$, (b) $f(x) = 3 + 2^x$, (c) $f(x) = \log_3(x-1)$, (d) $2 - \log_2(x)$.

Problem 5. Solve each one of the following equations, showing every step of your work (on a separate sheet of paper).
(a) $3^{2x-7} = 27$, (b) $5^{4-x} = \frac{1}{125}$, (c) $3^{2x} - 3^x - 6 = 0$, (d) $\log_2(5-x) + \log_2(5+x) = 4$.