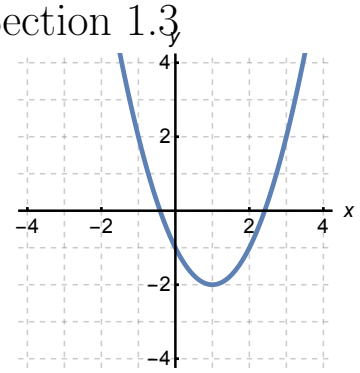
Math 115
Worksheet Section 1.3

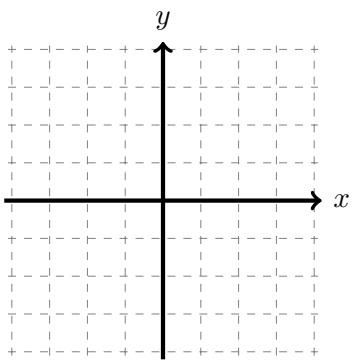
Problem 1.

x	-1	0	1	3
$f(x)$	2	-1	-2	2

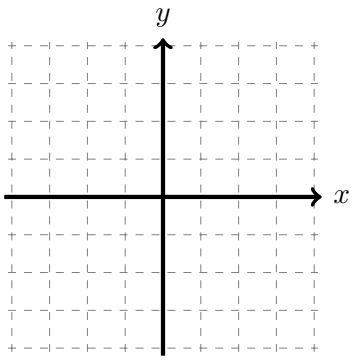


Use the graph provided for $f(x)$ to sketch the graph and fill in some values of the table for some of the related functions below.

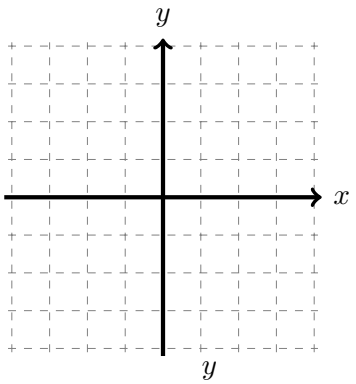
x				
$f(x)-2$				



x				
$f(x-2)$				

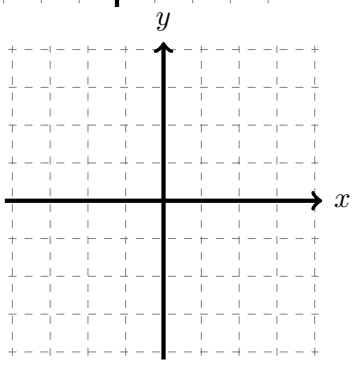


x				
$-2f(x)$				

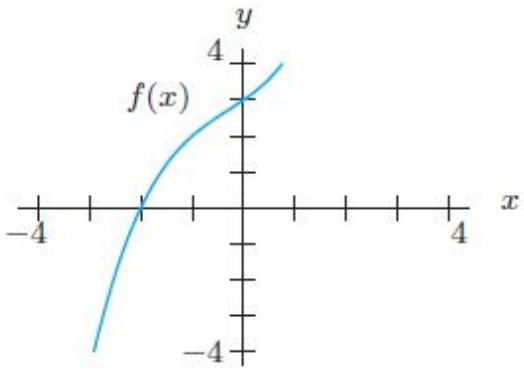


Consider $f(x)$ above. First, shift left by 3, then make the graph wider by a factor of 2. Sketch this function, and write down a formula for it in terms of f.

Now, start again, but this time, first make the graph wider by a factor of 2, then shift left by 3. Again, sketch it and find a formula. Is it the same function that you got previously?



Problem 2. (a) For the following graph, estimate $f^{-1}(0)$ and $f^{-1}(2)$, and sketch f^{-1}.



(b) For the following table, find $f(2)$, $f^{-1}(2)$ and $f^{-1}(4)$.

x	1	2	3	4	5	6	7
$f(x)$	3	−7	19	4	178	2	1

(c) Sketch graphs for the heights of a skydiver and a bungee jumper respectively. Are they invertible?

(d) A function has an inverse if and only if:

(e) Is $y = f(x) = 3x - 5$ invertible? If so, find the inverse.

Let $C = f(A)$ be the cost, in dollars, of building a store of area A square feet. In terms of cost and square feet, what do the following quantities represent?

(a) $f(10{,}000)$

(b) $f^{-1}(20{,}000)$

(f) (1.3 #19)

Function Composition

(g) (1.3 #9) Let $f(x) = \sqrt{x+4}$ and $g(x) = x^2$. Find $f(g(x))$ and $g(f(x))$.

(h) (1.3 #48–51)

In Problems 48–51 the functions $r = f(t)$ and $V = g(r)$ give the radius and the volume of a commercial hot air balloon being inflated for testing. The variable t is in minutes, r is in feet, and V is in cubic feet. The inflation begins at $t = 0$. In each case, give a mathematical expression that represents the given statement.

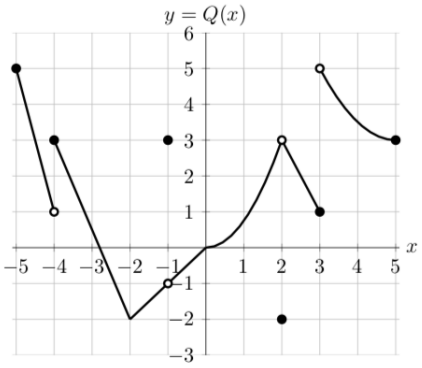
48. The volume of the balloon t minutes after inflation began.

49. The volume of the balloon if its radius were twice as big.

50. The time that has elapsed when the radius of the balloon is 30 feet.

51. The time that has elapsed when the volume of the balloon is 10,000 cubic feet.

Problem 3. (Fall 2017 Exam 1 Problem 4) The graph of a function $Q(x)$ with domain $[-5, 5]$ is shown below.



(a) On which of these intervals is Q invertible? Circle <u>all</u> that are true.

$[-4, -1]$ $[-2, 3]$ $[2, 5]$ $[-2, 2]$ None of these.

(b) For which values of $-5 < x < 5$ is the function Q not continuous?

Problem 4. (Winter 2018 Exam 1 Problem 2) A group of divers recently discovered one of the most submerged caverns in the world. As part of the exploration team, Elena descended into the caverns to take measurements of the temperature and pressure at different depths in the water. Elena started her descent at 8 am and reached the bottom of the caverns at 8:30 am. Let

- $A(t)$ be Elena's depth (in meters) during her descent t minutes after 8 am,
- $B(p)$ be the depth (in meters) at which Elena measures a water pressure of p kPa (kilo-Pascals),
- C(m) be the water temperature (in degrees Celsius) at a depth of m meters.

Assume all these functions are invertible.

(a) Find mathematical expressions that represent each of the sentences below.

i. The temperature of the water in degrees Celsius when its pressure is 118 kPa.

ii. The water pressure, in Pascals, 2 meters under the water surface (1 kPa =1000 Pascals).

(b) At 8:02am, Elena started recording all the data that she was measuring. Let $F(x)$ be Elena's depth (in meters) x seconds after she started recording data. Find a formula for $F(x)$ in terms of any of the functions A, B or C.

Problem 5. Consider the graph of the function m below.

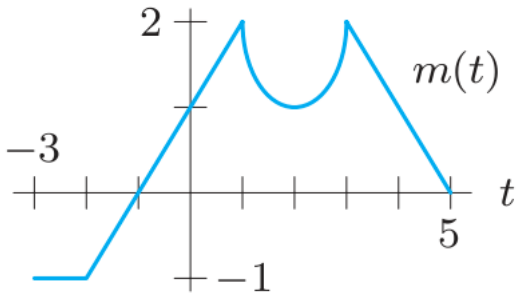


Figure 1: Graph of m

(a) What is the domain of m?

(b) What is the range of m?

(c) On which interval(s) is the function constant?

(d) On which interval(s) is the function linear?

(e) On which interval(s) is the function increasing?

(f) On which interval(s) is the function decreasing?

(g) Graph the following functions:

$$\text{(i) } n(t) = m(t) + 2, \quad \text{(ii) } k(t) = m(t+1), \quad \text{(iii) } z(t) = 2m(t) - 2$$