Math 115
Worksheet Section 4.6

Steps for related rates problems

1. Draw a picture and define important variables.
2. Identify the quantity for which we want to compute the rate of change. Find a formula for it.
3. Treat each quantity in your equation as a function of time, and then take the derivative of both sides with respect to time.
4. Substitute any given quantities and rates of change into your derivative equation, paying particularly close attention to the sign of any rates of change.
5. If any quantity or rate of change (other than the one we want to compute) in your derivative equation is still unknown, see if you can use the original equation or any other equations relating the quantities to solve for those unknowns.
6. Use your derivative equation to solve for the desired rate of change.

Problem 1. A plane is climbing at 500 feet per minute, and the air temperature outside the plane is falling at 2°C per 1000 feet. What is the rate of change (as a function of time) of the air temperature just outside the plane?

Problem 2. The gravitational force, F, on a rocket at a distance, r, from the center of the earth is given by

$$F = \frac{k}{r^2}$$

where $k = 10^{13}$ newton $\cdot$ km^2. When the rocket is 10^4 km from the center of the earth, it is moving away at 0.2 km/sec. How fast is the gravitational force changing at that moment? Give units. (A newton is a unit of force.)

Problem 3. The average cost per item, C, in dollars, of manufacturing a quantity q of cell phones is given by

$$C = \frac{a}{q} + b, \quad \text{where } a, b \text{ are positive constants.}$$

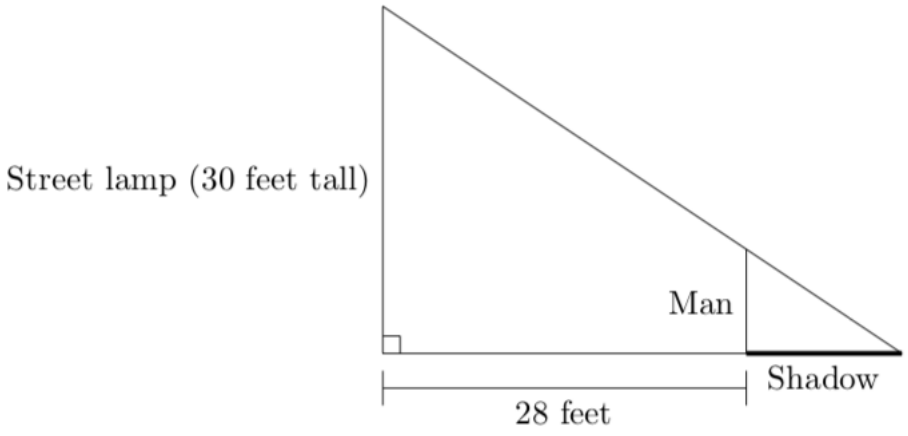
(a) Find the rate of change of C as q increases. What are its units?

(b) If production increases at a rate of 100 cell phones per week, how fast is the average cost changing? Is the average cost increasing or decreasing?

Problem 4. Car B is driving south, away from an intersection. Car A is approaching the intersection and is moving west. At what rate is the distance between the cars changing at the instant when car B is 40 miles from the intersection and traveling 50 mph and car A is 30 miles from the intersection and traveling at 45 mph? Are the cars getting closer together or farther apart at this time?

Problem 5. (Winter 2017 Final Exam) A cylindrical bar of radius R and length L (both in meters) is put into an oven. As the bar gains temperature, its radius decreases at a constant rate of 0.05 meters per hour and its length increases at a constant rate of 0.12 meters per hour. Fifteen minutes after the bar was put into the oven, its radius and length are 0.4 and 3 meters respectively. At what rate is the volume of the bar changing at that point? Be sure to include units.

Problem 6. (Winter 2016 Final Exam) A man, who is 28 feet away from a 30 foot tall street lamp, is sinking into quicksand. (See diagram below.) At the moment when 6 feet of him are above the ground, his height above the ground is shrinking at a rate of 2 feet/second.



(a) How long will the man's shadow (shown in bold in the diagram above) be at the moment when 6 feet of him are above the ground?

(b) At what rate is the length of the man's shadow changing at the moment 6 feet of him are above the ground? Is his shadow growing or shrinking at that moment?

Problem 7. (Fall 2014 Final Exam) Tommy and Gina were friends in high school but then went to college in different parts of the country. They thought they were going to see each other in Springfield over the December break, but their schedules didn't match up. In fact, it turns out that Tommy is leaving on the same day that Gina is arriving. Shortly before Gina's train arrives in Springfield, she sends a text to Tommy to see where he is, and Tommy sends a text response to say that, sadly, his train has already left. At the moment Tommy sends his text, he is 20 miles due east of the center of the train station and moving east at 30 mph while Gina is 10 miles due south of the train station and moving north at 50 mph.

(a) What is the distance between Gina and Tommy at the time Tommy sends his text? Remember to include units.

(b) When Tommy sends his text, are he and Gina moving closer together or farther apart? How quickly? You must show your work clearly to earn any credit. Remember to include units.

(c) Let $J(t)$ be the distance between Gina and Tommy t hours after Tommy sends his text. Use the local linearization of $J(t)$ at $t = 0$ to estimate the distance between Gina and Tommy 0.1 hours after Tommy sends his text. Remember to show your work carefully.