

# Math 115

## Worksheet Section 2.2

**Problem 1.** (a) What is the definition of the derivative of the function  $f(x)$  at  $x = c$ ?

(b) Compute the derivative of  $g(x) = 3x^2$  at  $x = 10$  **algebraically**. In other words, use algebra to find the limit from the definition exactly using limit computations we learned from the previous sections.

**Problem 2.** (Winter 2018 Exam 1) Let  $m(x) = (1+x^2)^{3x-4}$ . Which of the limits below represents  $m'(2)$ ? There is only one correct answer. Be sure to explain your reasoning on the board.

(a)  $\lim_{h \rightarrow 0} \frac{(1+x^2)^{3x-4} + h - 25}{h}$

(d)  $\lim_{h \rightarrow 0} \frac{(1+(2+h)^2)^{3h+2} - 25}{h}$

(b)  $\lim_{h \rightarrow 0} \frac{(1+h^2)^{3h-4} - 25}{h}$

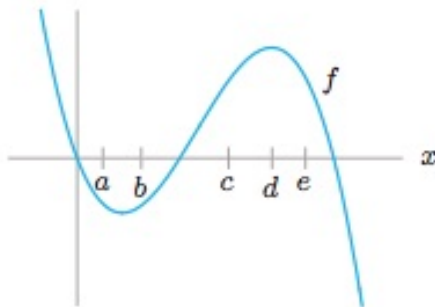
(e)  $\lim_{h \rightarrow 0} \frac{(5+h^2)^{3h+2} - 25}{h}$

(c)  $\lim_{h \rightarrow 0} \frac{(1+(2+h)^2)^{3h-4} - 25}{h}$

(f)  $\lim_{h \rightarrow 0} \frac{(1+h^2)^{3h+2} - 25}{h}$

**Problem 3.** (2.2 #27) Create a table using difference quotients to approximate the derivative of  $x^x$  at  $x = 2$  to one decimal place. You may use a calculator for this problem.

**Problem 4.** (2.2 #11) Match the derivatives in the table with the points  $a, b, c, d, e$ .



$x$	$f'(x)$
0	
0.5	
2	
-0.5	
-2	

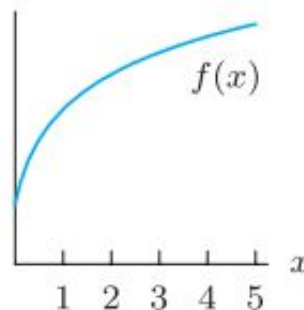
**Problem 5.** (2.2 # 15) For each of the following pairs, use the graph to decide which is larger. Explain.

(a)  $f(3)$  or  $f(4)$ ?

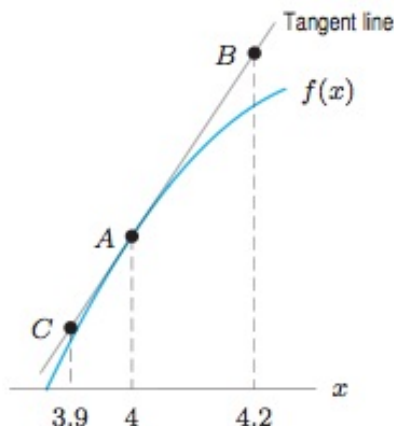
(b)  $f(3) - f(2)$  or  $f(2) - f(1)$ ?

(c)  $\frac{f(2) - f(1)}{2 - 1}$  or  $\frac{f(3) - f(1)}{3 - 1}$ ?

(d)  $f'(1)$  or  $f'(4)$ ?



**Problem 6.** (2.2 #17) The given function  $f$  has  $f(4) = 25$  and  $f'(4) = 1.5$ . Find the coordinates



of the points  $A, B, C$ .

**Problem 7.** (Winter 2016 Exam 1) Consider the function  $g$  defined by

$$g(x) = \begin{cases} \frac{1}{e^x - 1} & \text{if } x < \frac{1}{2} \\ \cos(x^x) & \text{if } \frac{1}{2} \leq x < 5 \\ \frac{x^2}{(x-1)(6-x)} & \text{if } x \geq 5 \end{cases}$$

- Use the limit definition of the derivative to write an explicit expression for  $g'(3)$ . Your answer should not involve the letter  $g$ . Do not attempt to evaluate or simplify the limit.
- Find all vertical asymptotes of  $g$ , if there are any.

**Problem 8.** (Winter 2018 Exam 1) Sketch the graph of a single function  $y = f(x)$  satisfying all of the following conditions:

- The domain of  $f(x)$  is the interval  $-8 < x \leq 6$ .
- $f(x)$  is continuous on the interval  $-8 < x < -2$ .
- $f'(-7) = 0$ .
- $f(x)$  is decreasing and concave up for all  $x$  in the interval  $-6 < x < -4$ .
- The average rate of change of  $f(x)$  is equal to 0.5 between  $x = -5$  and  $x = -2$ .
- $f(0) = 2$  and  $f'(0) = -1$ .
- $\lim_{x \rightarrow 2^-} f(x) = f(2)$  and  $\lim_{x \rightarrow 2^+} f(x) < \lim_{x \rightarrow 2^-} f(x)$ .
- $f(x)$  has constant rate of change on the interval  $3 \leq x \leq 6$ .

Make sure that your graph is large and unambiguous.