Head to head collision

□→←□

$$\frac{1}{speed_a}(t - start_a) = d_a$$

$$\frac{+1}{speed_b}(t - start_b) = d_b \quad = +1 + -d_a$$

$$d_a + d_b = 1$$

System

$$d_a = 1 - d_b$$

$$d_b = 1 - d_a$$

$$\frac{-1}{b_{speed}}(t - b_{start}) + 1 = d_a$$

$$\frac{-1}{b_{speed}}(t - b_{start}) + 1 = \frac{1}{a_{speed}}(t - a_{start})$$

$$\frac{-t}{b_{speed}} + \frac{b_{start}}{b_{speed}} + 1 = \frac{t}{a_{speed}} - \frac{a_{start}}{a_{speed}}$$

$$\frac{a_{start}}{a_{speed}} + \frac{b_{start}}{b_{speed}} + 1 = \frac{t}{a_{speed}} + \frac{t}{b_{speed}}$$

$$\frac{a_t}{a_s} + \frac{b_t}{b_s} + 1 = \frac{t \cdot b_s}{a_s \cdot b_s} + \frac{t \cdot a_s}{b_s \cdot a_s}$$

$$= \frac{b_s t + a_s t}{a_s b_s}$$

$$a_s b_s\left(\frac{a_t}{a_s} + \frac{b_t}{b_s} + 1\right) = b_s t + a_s t$$

$$a_t b_s + b_t a_s + a_s b_s = (b_s + a_s)t$$

$$\frac{a_t b_s + b_t a_s + a_s b_s}{b_s + a_s} = t$$

solved for t

head to head
Checks

a [0, 10] $t=5$ ✓

b [0, 10]

$$\frac{0+0+10\cdot 10}{10+10}=\frac{100}{20}=5$$

a [2, 12] $t=$ ~~16~~ 6 ✓

b [0, 10]

$$\frac{2\cdot 10+0+10\cdot 10}{10+10}=\frac{20+100}{20}=\frac{12}{2}=6$$

a [3, 18] $t=8\frac{1}{2}$ ✓

b [4, 14]

$$\frac{3\cdot 10+4\cdot 10+10\cdot 10}{10+10}=\frac{30+40+100}{20}=\frac{17}{2}=8\frac{1}{2}$$

a [3, 17] $t=?=8\frac{3}{19}$ Looks Pretty right

b [5, 11]

$b_s=11-5=5$

$a_s=17-3=14$

$$\frac{3\cdot 5+5\cdot 14+5\cdot 14}{5+14}$$

$$\frac{15+70+70}{19}=\frac{155}{19}\quad \frac{95}{19}+\frac{60}{19}=5+3+\frac{3}{19}=8\frac{3}{19}$$