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Introduction to Pair Trading





Learning Objectives

- Understand the advantages of trading a pair of stocks versus a single stock
- Identify the patterns of price movements that cause a pair trade to be profitable
- Construct a beta-hedged pair strategy



Agenda

Advantages of trading a pair of stocks versus a single stock

Price movements that cause a pair trade to be profitable

Beta Hedging a Stock Pair





Single Stock Trading

- Trading a single stock is betting on returns moving in a single direction
- If you are bullish you go long and hope for the price to rise
- If you are bearish you sell short and hope for the price to decrease



Short Selling Rules

- Selling a stock you don't currently own
- Legal requirement to borrow stock (usually from a stock lender) and then sell it
- Close trade by buying the stock back and returning it to the lender



Single Stock vs Pair Trading

- Trading a single stock is betting on returns moving in a single direction
- Pairs trading requires you to focus on relative performance rather than absolute performance



Agenda

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Price movements that cause a pair trade to be profitable

Beta Hedging a Stock Pair

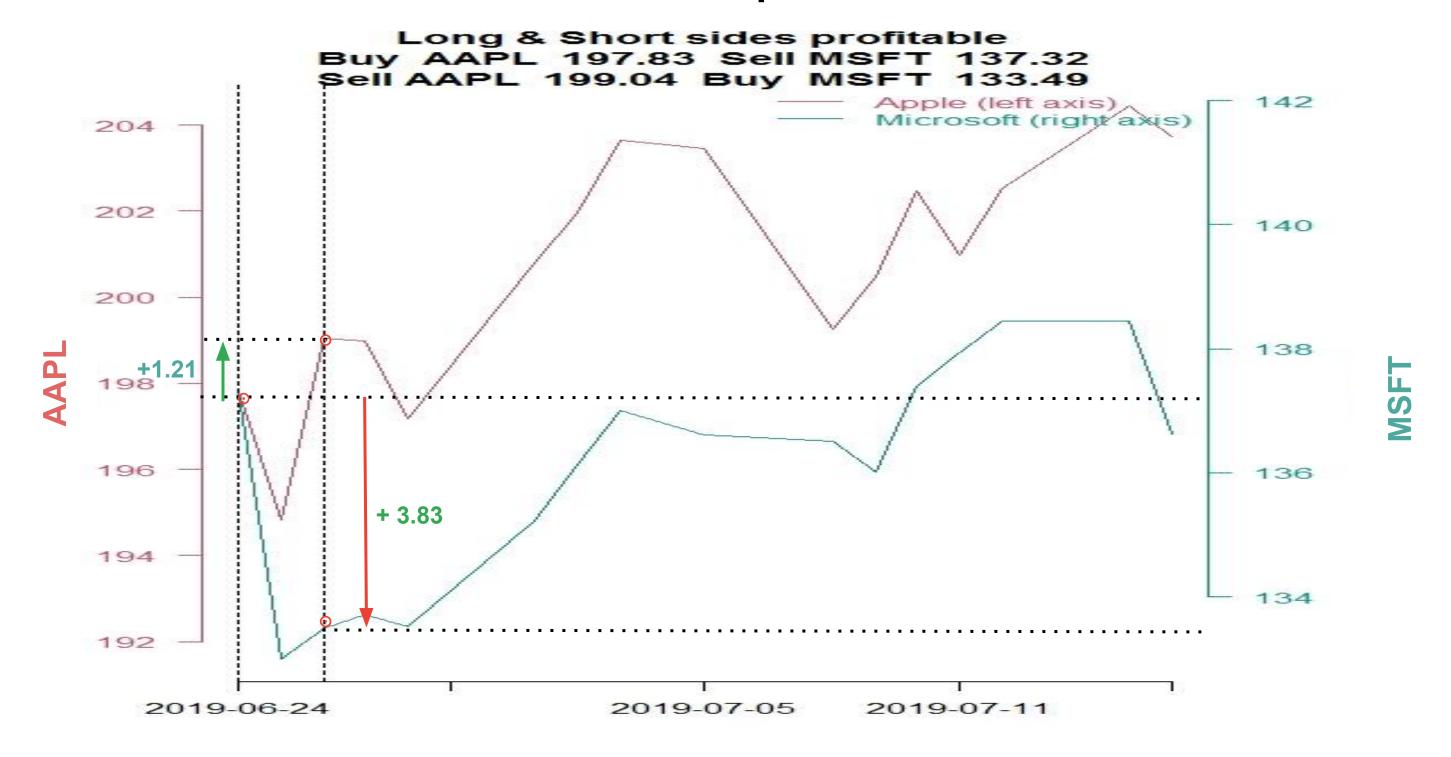




Profitable Scenarios for Pair Trading

- 1. AAPL | 5% MSFT | 5%
- 2. AAPL 15% MSFT 5%
- 3. AAPL | 5% MSFT | 15%
- → Long side outperforms
- → Profit is +10% for all 3 scenarios

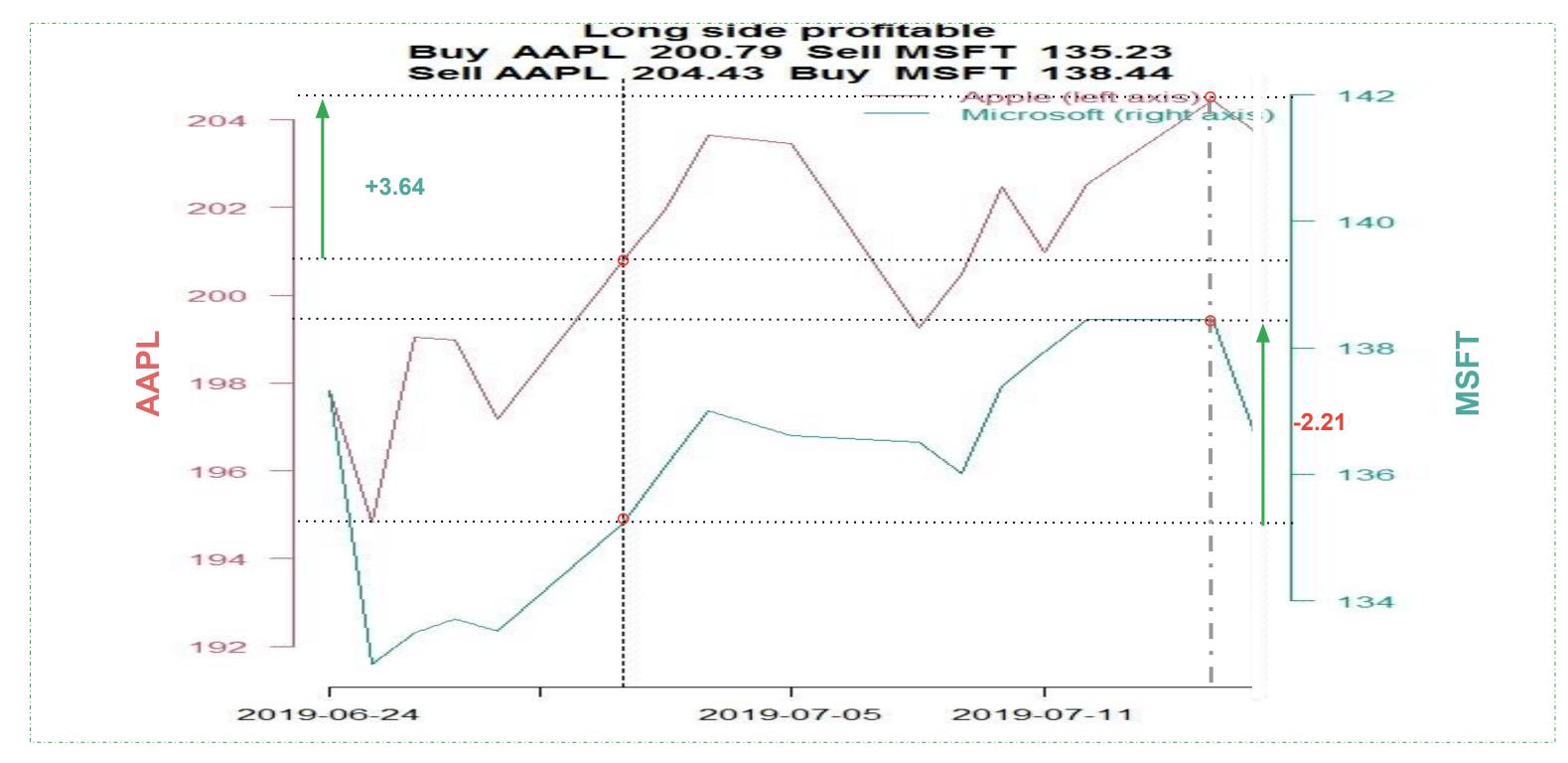
Profitable Pair Trade: AAPL up 1.21 & MSFT down 3.83







Profitable Pair Trade: AAPL up 3.64 & MSFT up 3.21







Agenda

Advantages of trading a pair of stocks versus a single stock

Price movements that cause a pair trade to be profitable

Beta Hedging a Stock Pair





Beta Hedging a Stock Pair

Beta is a stock's covariance with the overall market per unit of market risk

$$\beta_{AAPL} = \frac{COV(AAPL,MKT)}{VAR(MKT)}$$

Beta is a measure of a stock's systematic risk

$$\beta_{AAPL} = 1.10$$

⇒ AAPL has 10% more systematic volatility than the market

Beta Hedging a Stock Pair

For Large Cap Stocks:

⇒ MKT ≈ S&P 500

If S&P 500 is up 20%

⇒ AAPL expected to be up 22%

If S&P 500 is down 20%

⇒ AAPL expected to be down 22%





Capital Asset Pricing Model (CAPM):

$$E(R_{stock}) - R_{rf} = \beta_{stock} * [E(R_{mkt}) - R_{rf}]$$

 \Rightarrow a stock's expected excess return is equal to its β times the expected market risk premium [E(R_{mkt}) - R_{rf}]

3-month
$$\beta_{AAPL}$$
 = 1.10 β_{MSFT} = 0.96

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- Pair will not contain equal amounts of AAPL and MSFT
- Weighting must be Beta Neutral and also factor in relative stock prices.

$$\beta_{AAPL}^* P_{AAPL}^* Q_{AAPL} = \beta_{MSFT}^* P_{MSFT}^*$$
 Q_{MSFT}



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Suppose MSFT is \$135 & AAPL is \$200



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Mkt Val. AAPL is \$200*589 = \$117,800

Beta = 1.10 ⇒ adjusted market value is:

\$117,800*1.10 = \$129,580

Mkt Val. MSFT is \$135*1000 = \$135,000

Beta = 0.96 ⇒ adjusted market value is:

\$135,000*0.96 = \$129,600





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\$135,000*0.96 = \$129,600

With rounding you are beta neutral!





Beta Hedging Choosing Good Pair Candidates ?

- Constructing a Beta-hedged strategy is important for managing trading risk
- We still need a strategy to identify stocks that would make good pair candidates



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Picking Pairs





- Understand the role of correlation in selecting pairs within an industry sector
- Create a hierarchical clustering using a daily return matrix
- Construct a scree plot using principal components



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Agenda

Use correlation to select pairs

Identify hierarchical clusters of stocks and create a cluster plot

Construct a scree plot for stocks within an industry sector



Correlation between Stocks in Pair Trade

- Predicted correlation is negative
- Profit still depends on going long the the stock that will outperform...
- And going short the stock that will underperform

Short MSFT





Measuring Correlation

- Long-term negative correlation is unusual as almost all stocks have positive betas
- Correlations between pairs can be unstable and should be re-estimated regularly



S&P 500 Sectors

- 1. Financials
- 2. Utilities
- 3. Consumer Discretionary
- 4. Consumer Staples
- 5. Energy
- 6. Health Care
- 7. Industrials
- 8. Technology
- 9. Telecom
- 10. Materials
- 11. Real Estate



XLK Technology Sector Exchange-Traded Fund (ETF)

- Market-cap weighted exposure to entire sector
- XLK is the Technology sector fund
- AAPL and MSFT account for over 38% of XLK



Technology Sector: Top Sub-sectors

Sub-sectors form logical groups which can be useful for selecting pair candidates:

Software & IT Services 52.0%

Computers & Phones 18.6%

Semiconductors 17.4%

Communications & Network 5.4%





DataFrame for XLK

- Daily returns matrix (DataFrame)
 for each of the 68 stocks in XLK
- Each row is a trading day (Index)
- Each Column is one of the stocks
- Each cell (the Data) consists of a single stock's return on a particular day



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What is Clustering?

Cluster Analysis is a method of grouping similar objects:

- 1. Decide metric to to measure differences between objects
- 2. Decide number of clusters
- 3. Assign objects to clusters so as to minimize differences



Hierarchical Clustering Matrix

Use the matrix containing daily returns for each stock in XLK

- Columns contain data for a particular stock
- 2. **Rows** contain data for a particular **trading day**



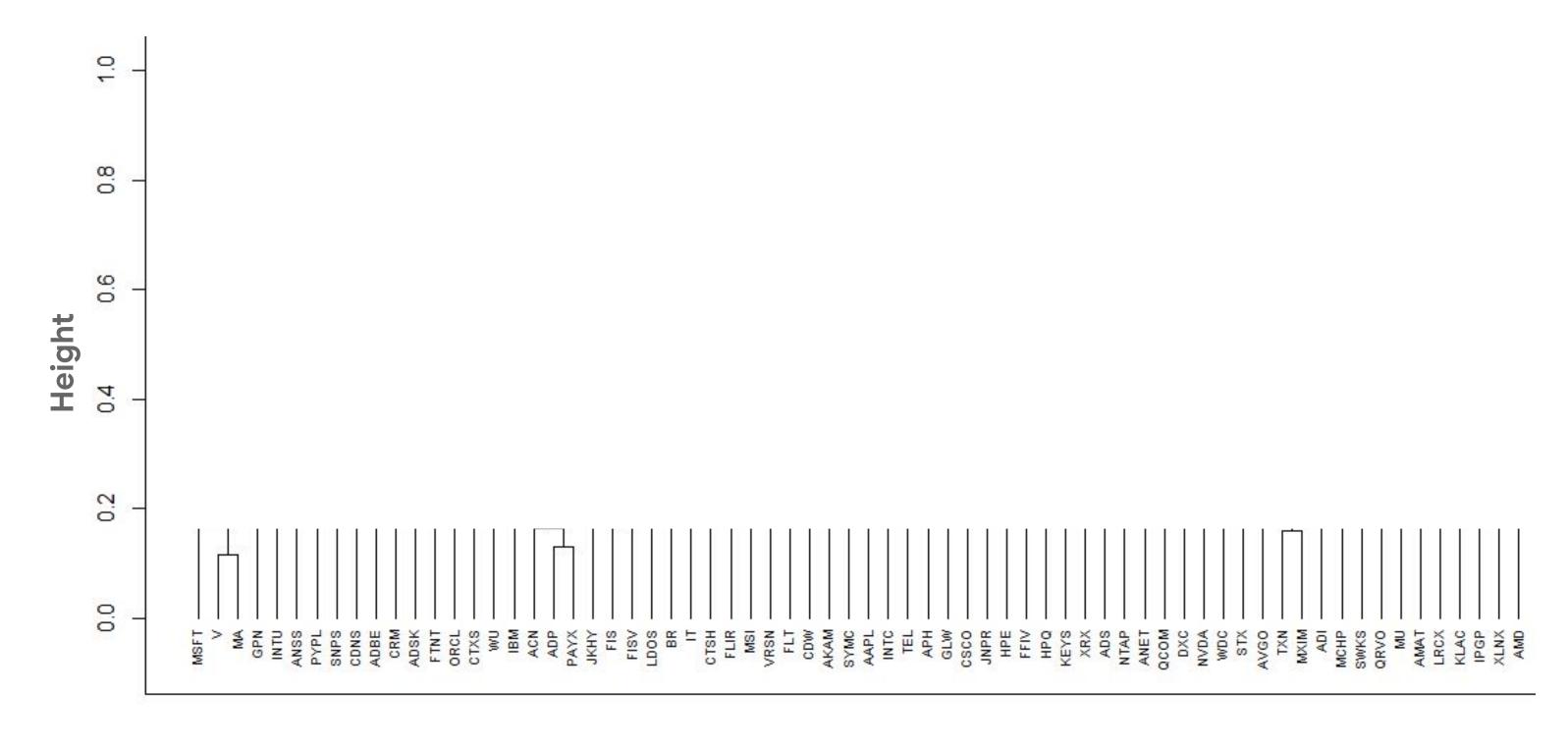
Computing Distance between Returns

Objective is to group stocks that have similar returns (minimize distance between returns).

1. Calculate distance using the Euclidean method

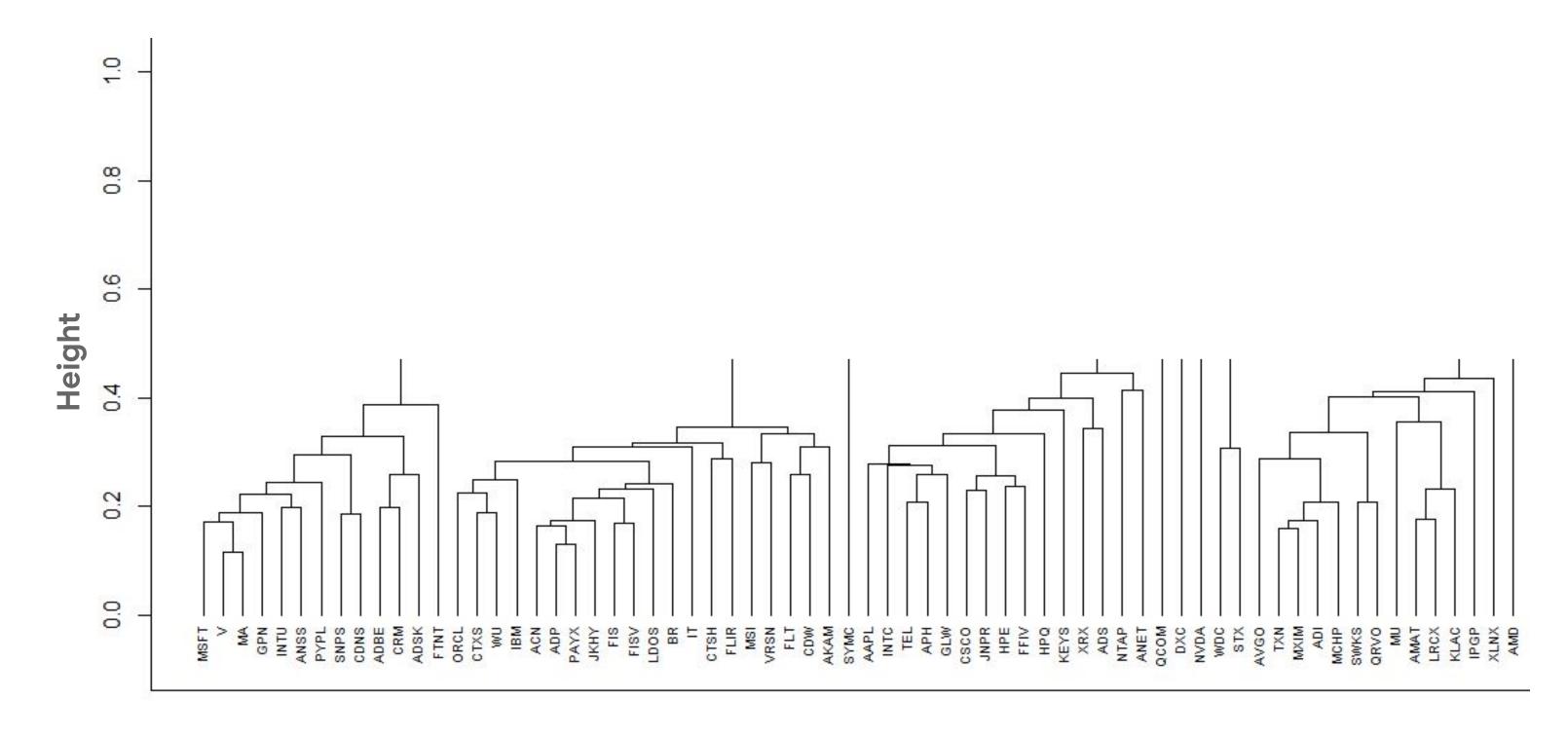
$$\mathbf{d} = \sqrt{\sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{y}_i)^2}$$

2. Iteratively cluster stocks based on minimizing distance



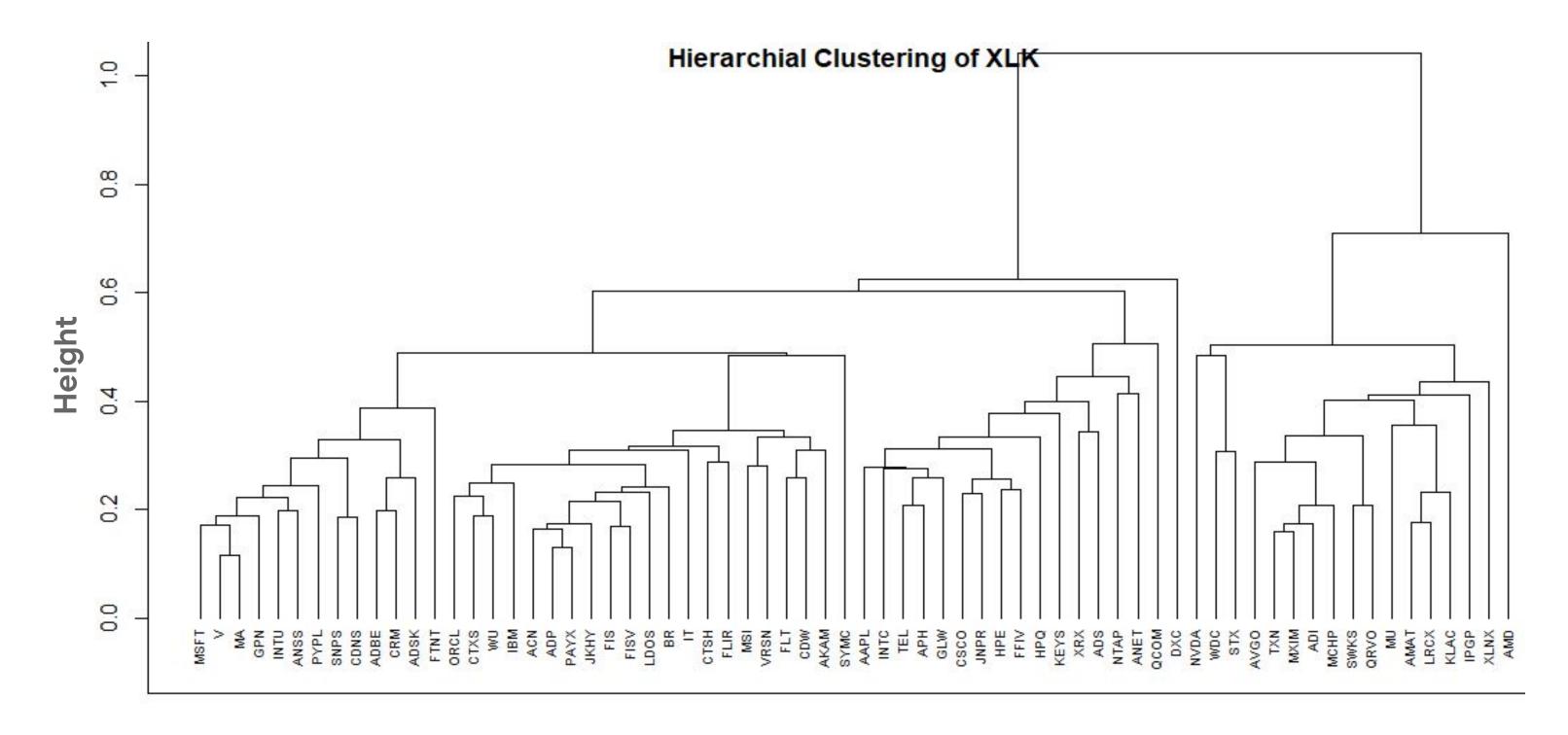






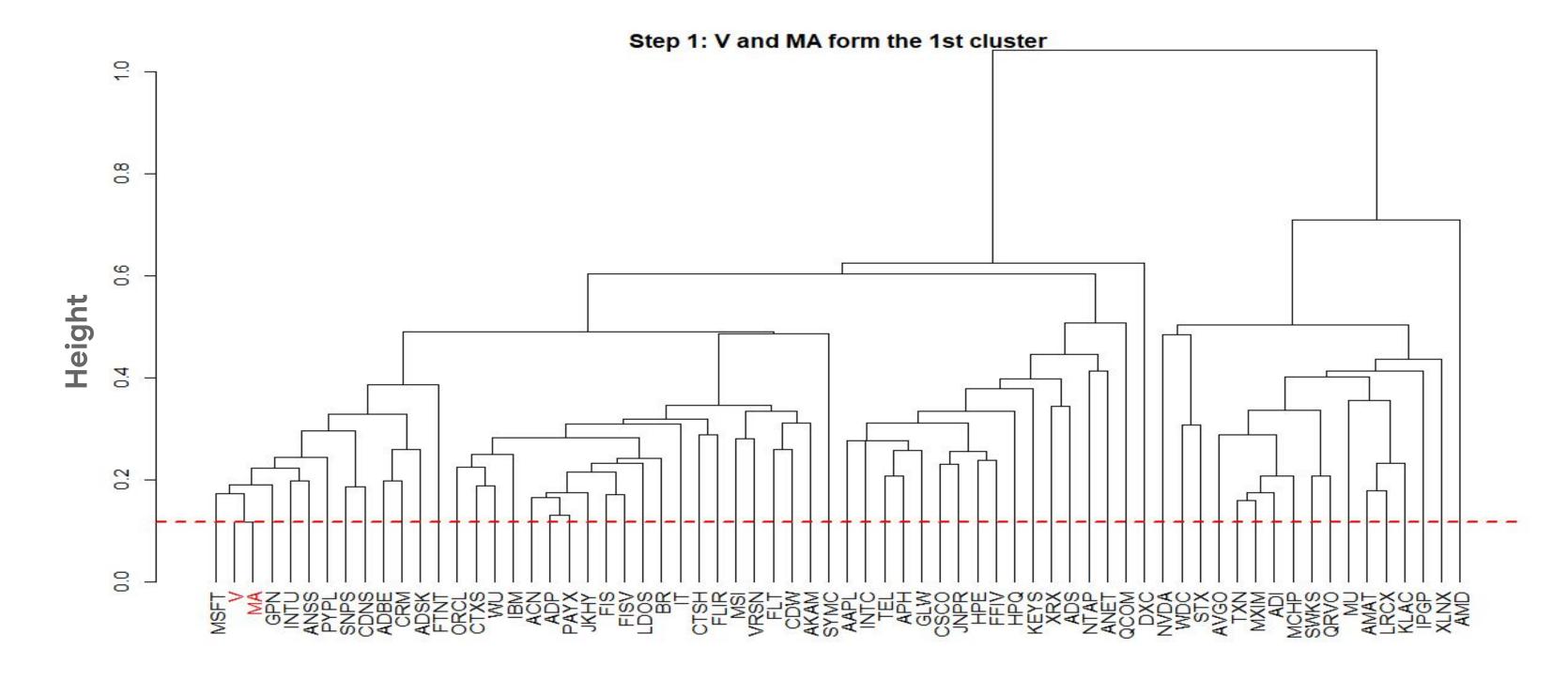






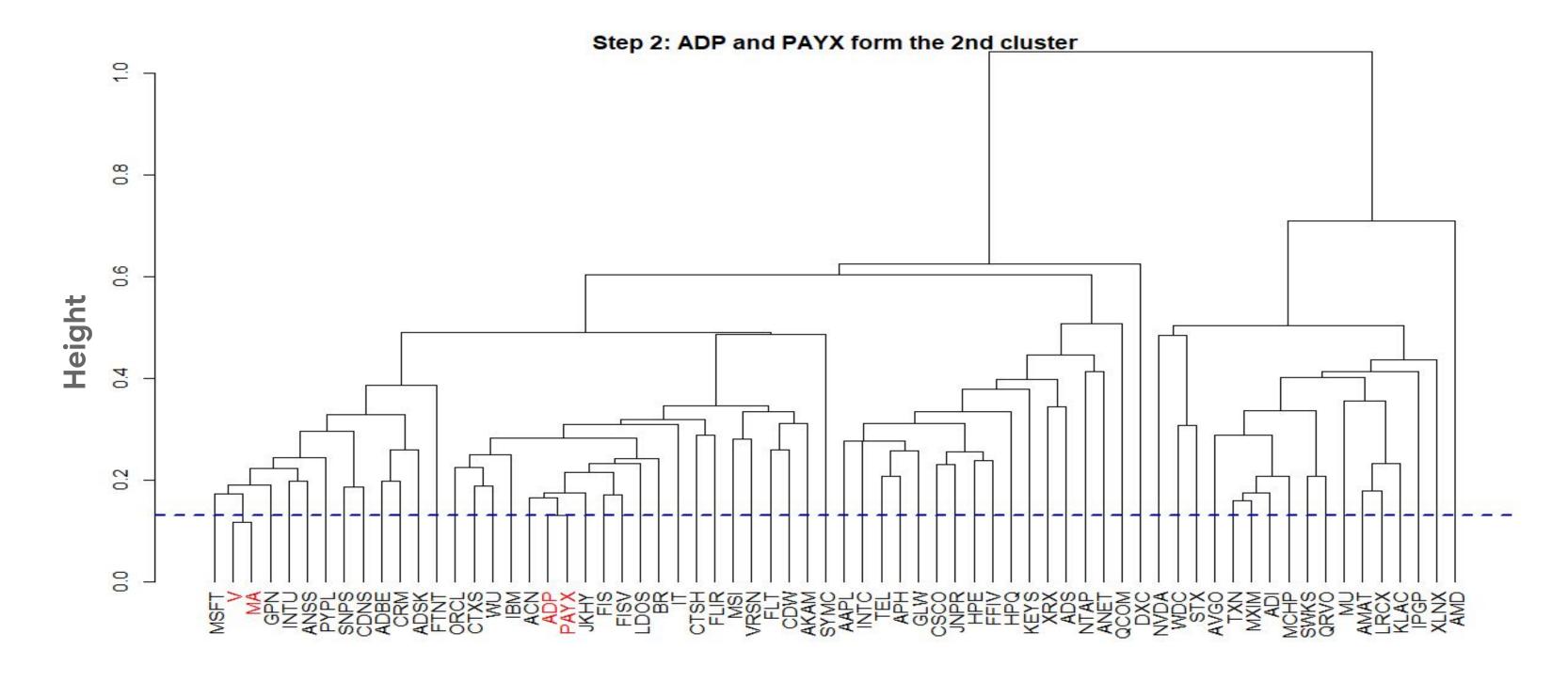






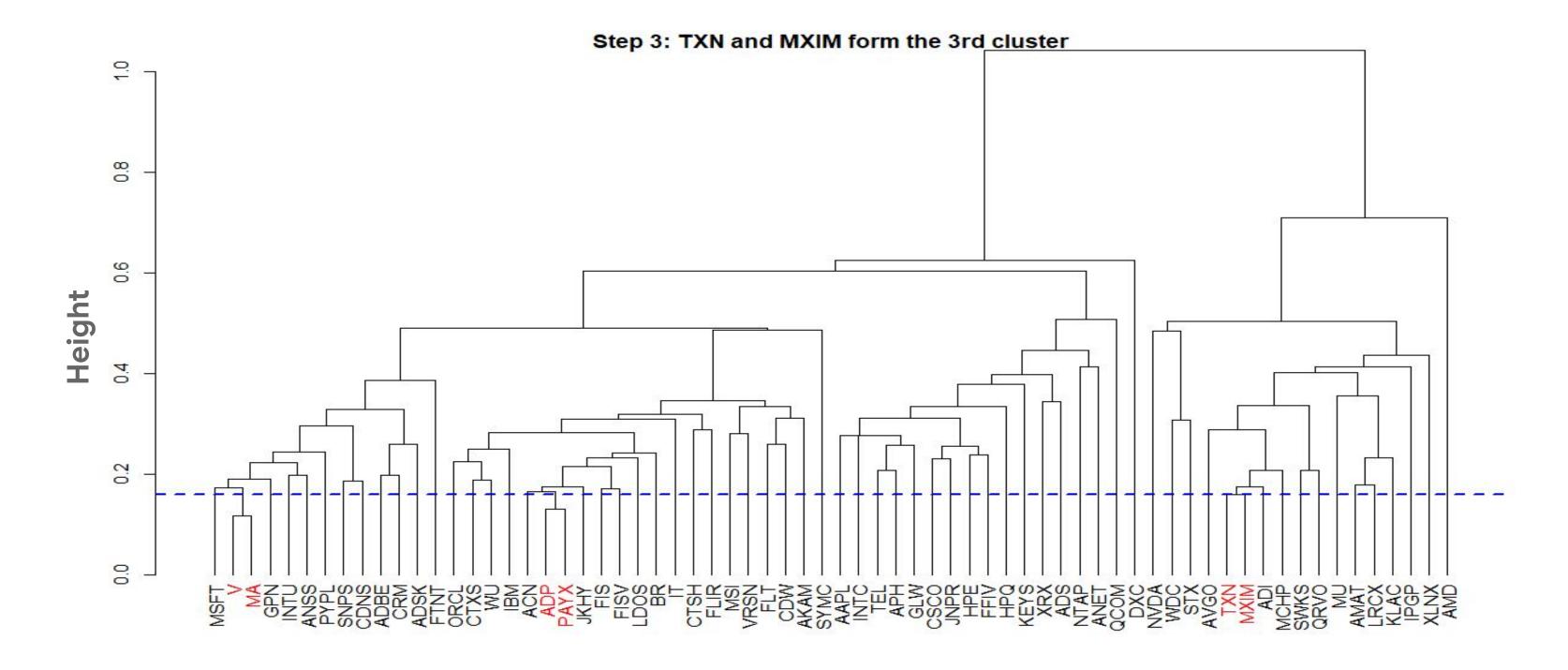






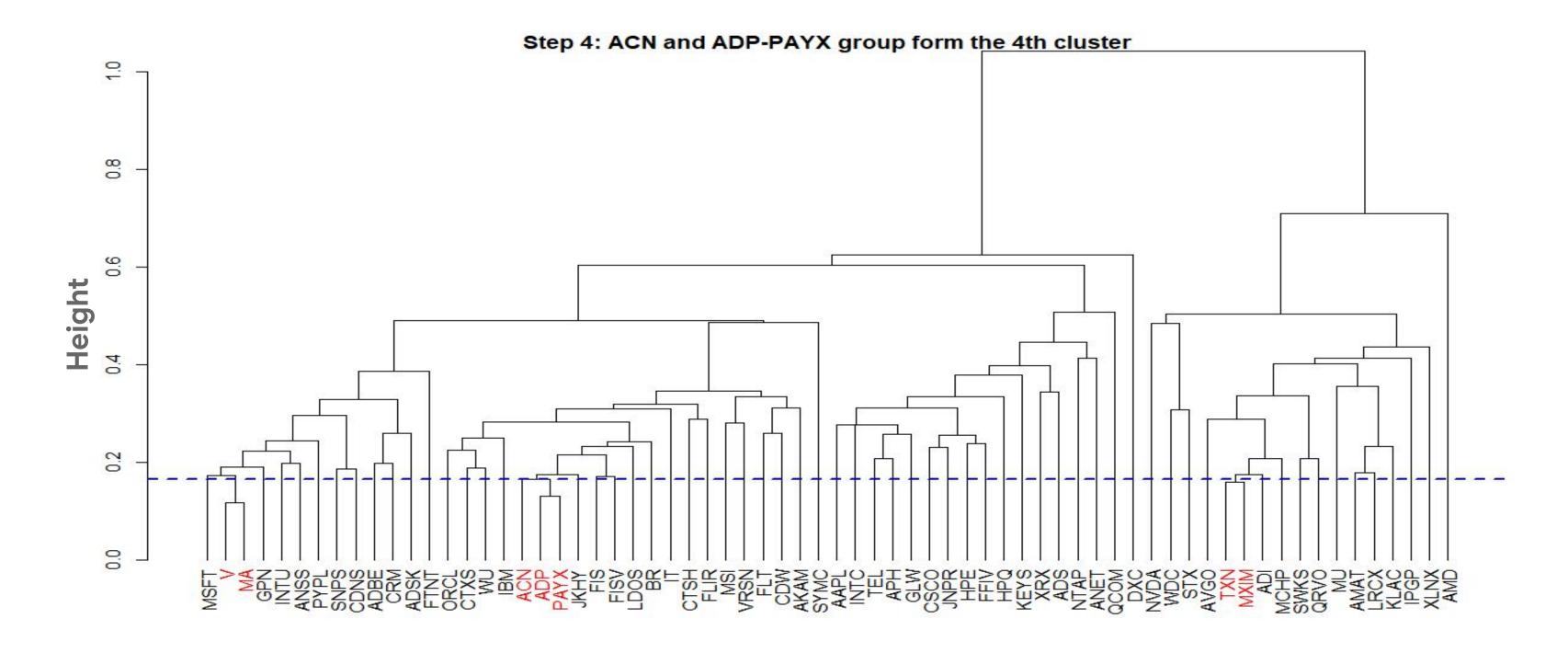






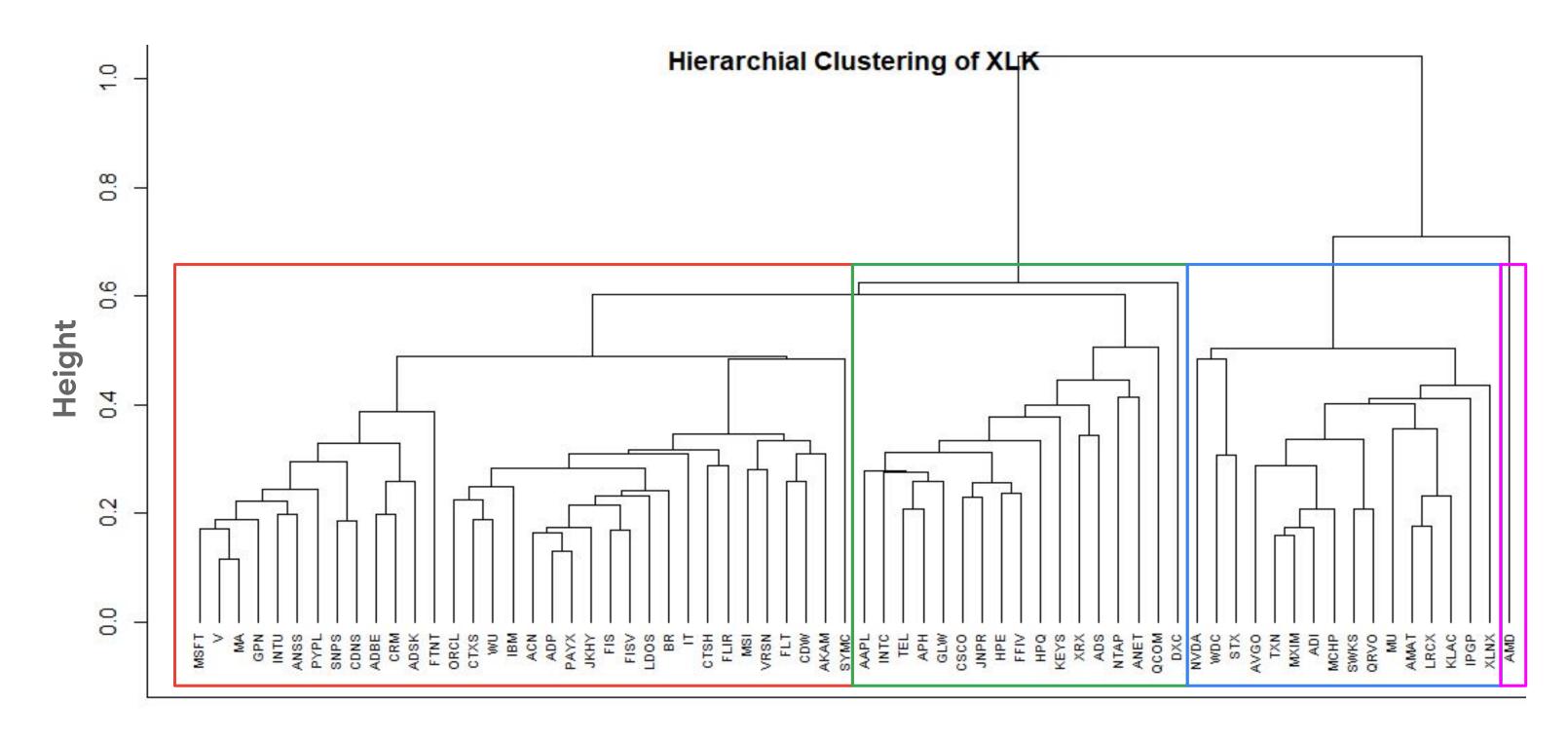








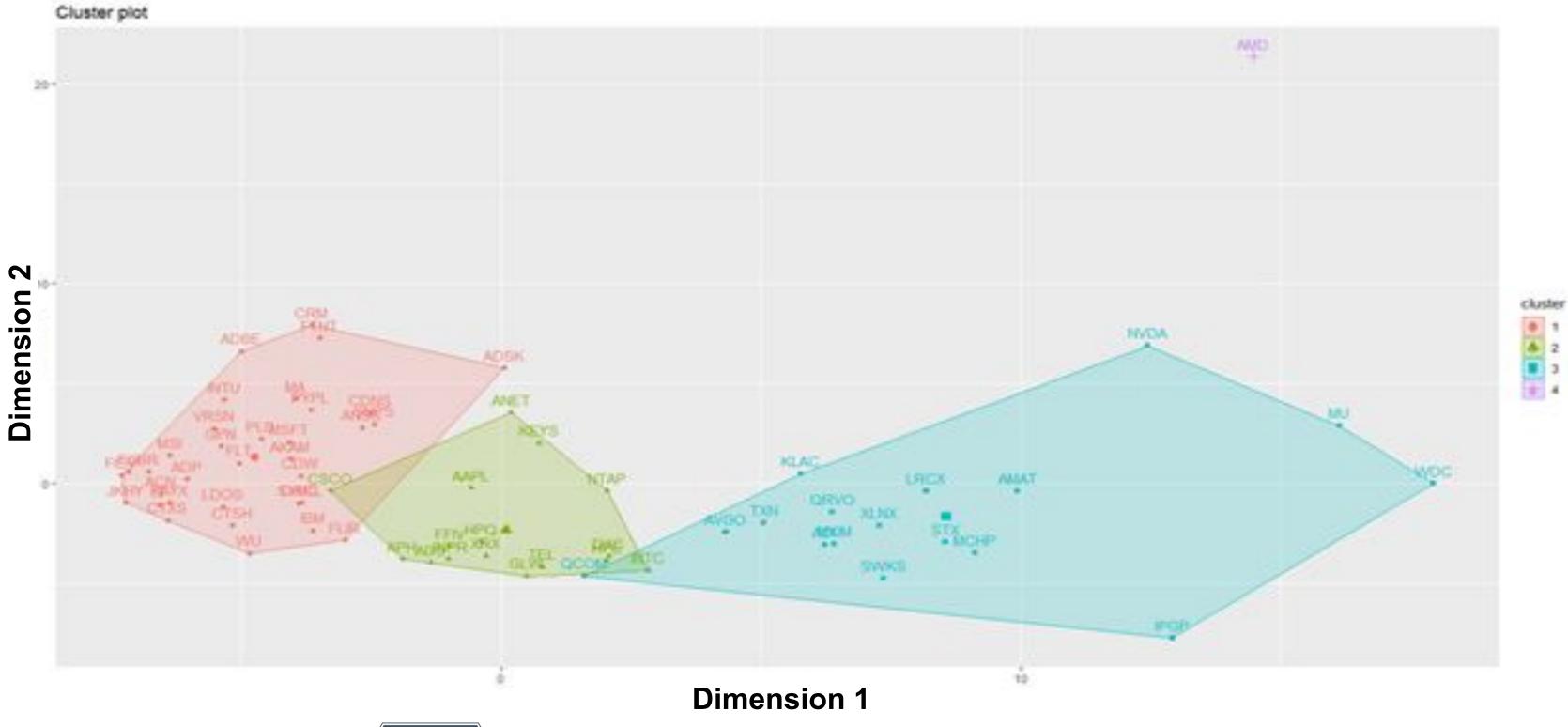






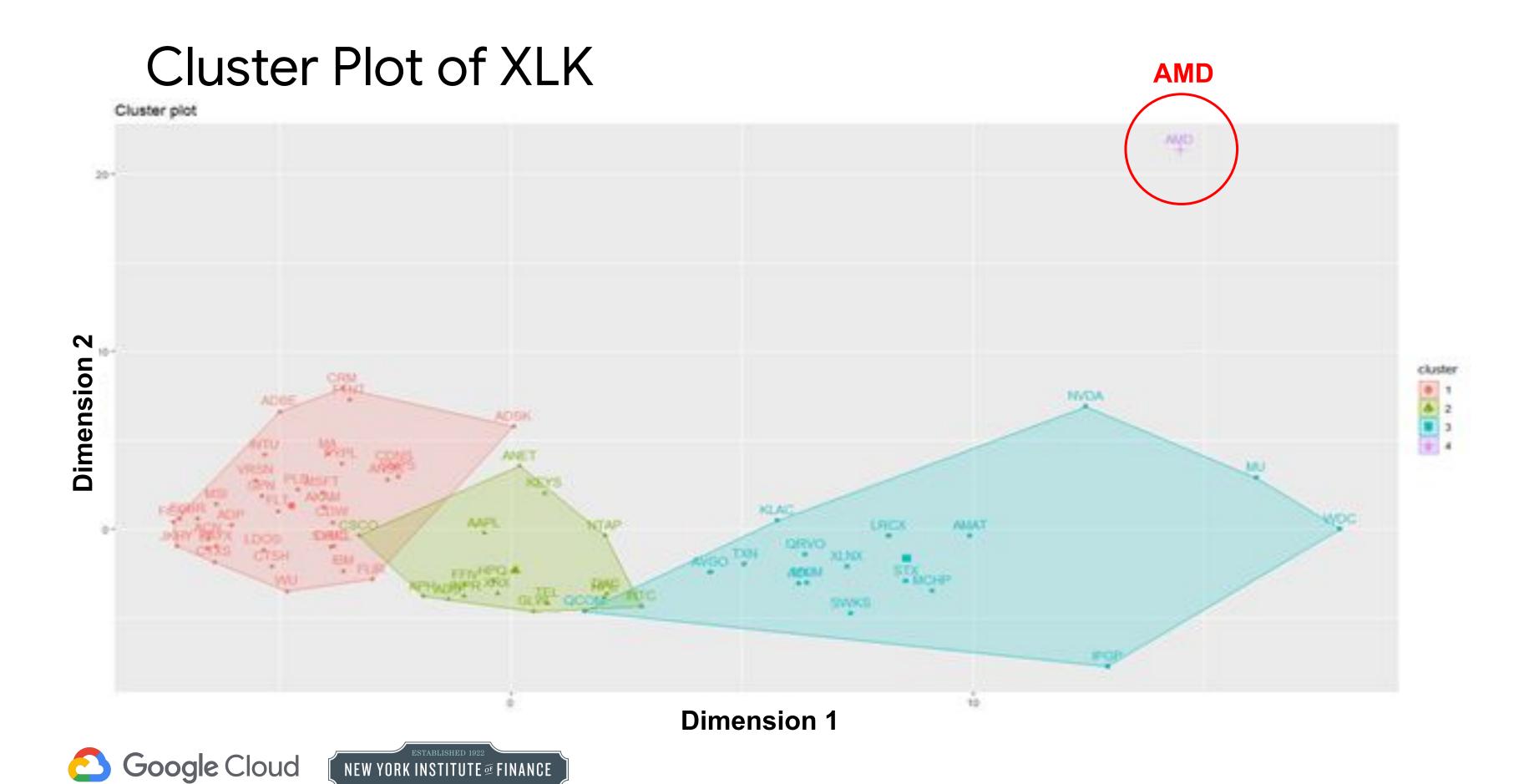


Cluster Plot of XLK









Agenda

Use correlation to select pairs

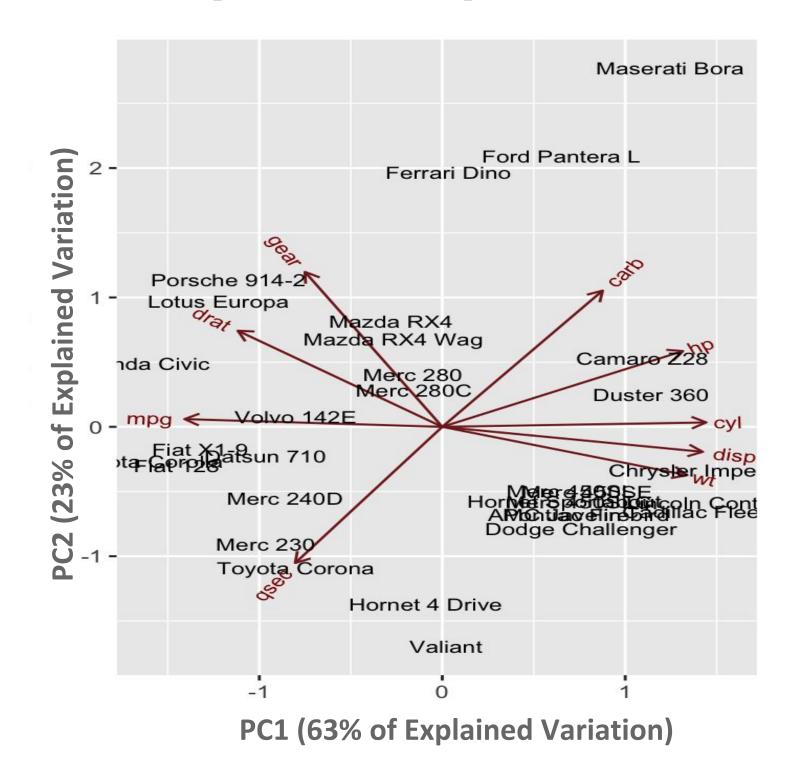
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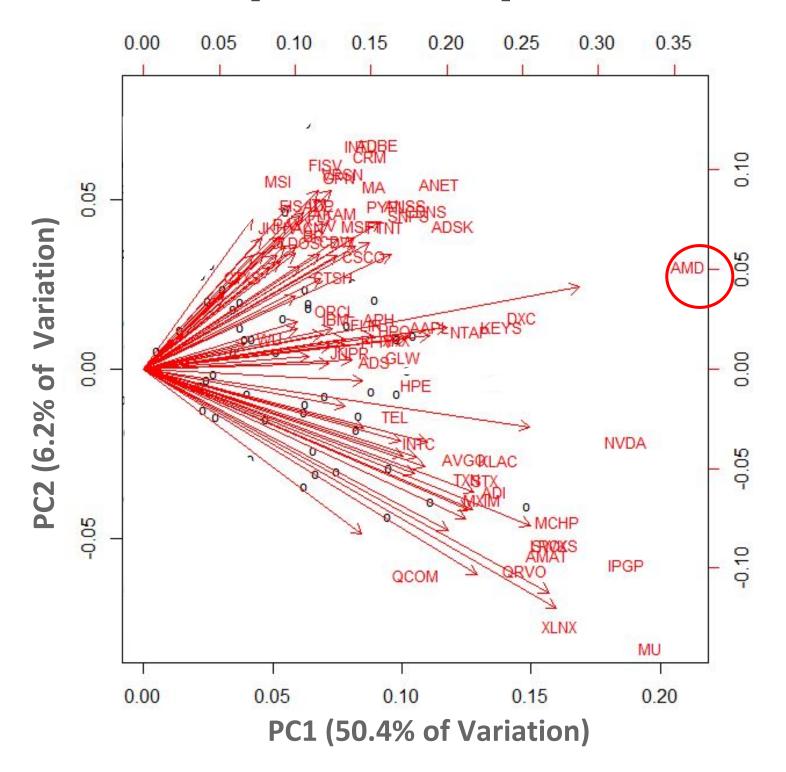
Principal Components







Principal Components







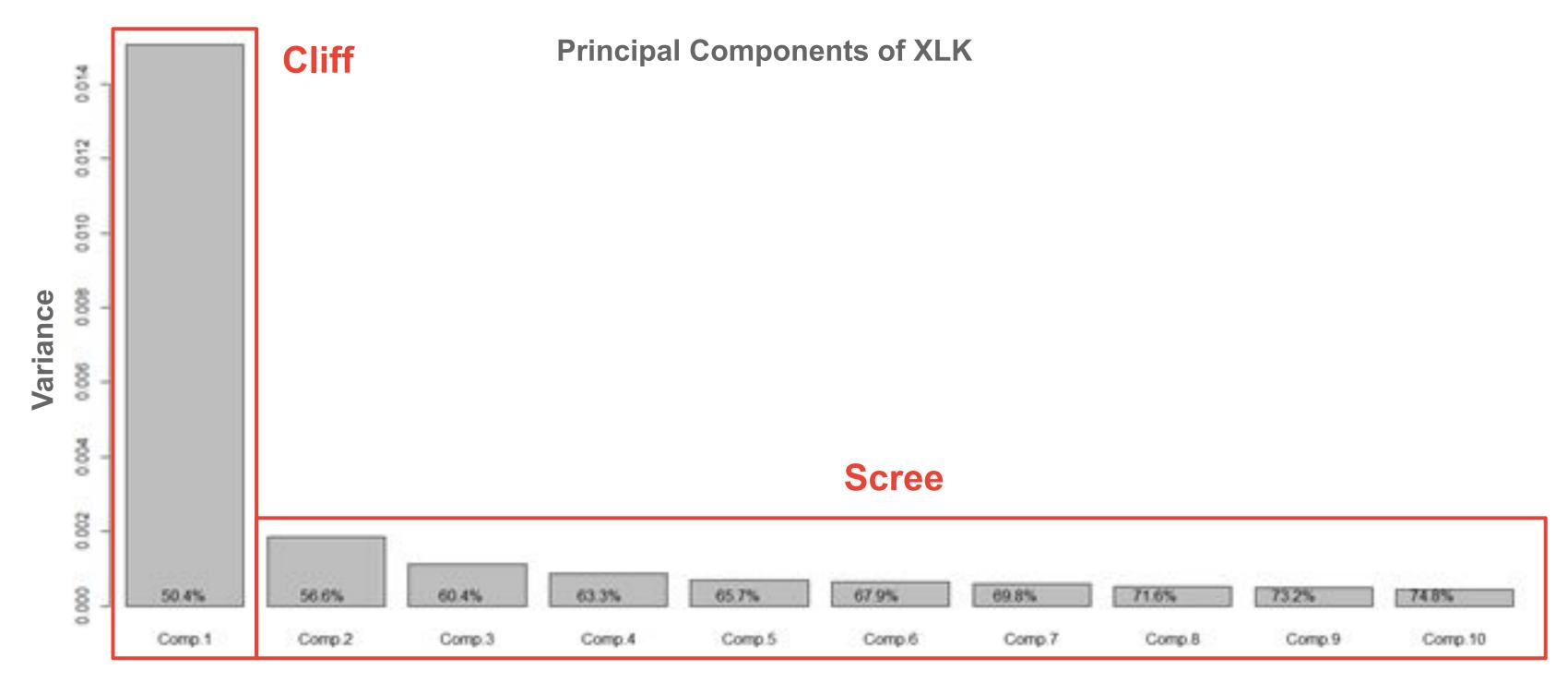


Scree Plots





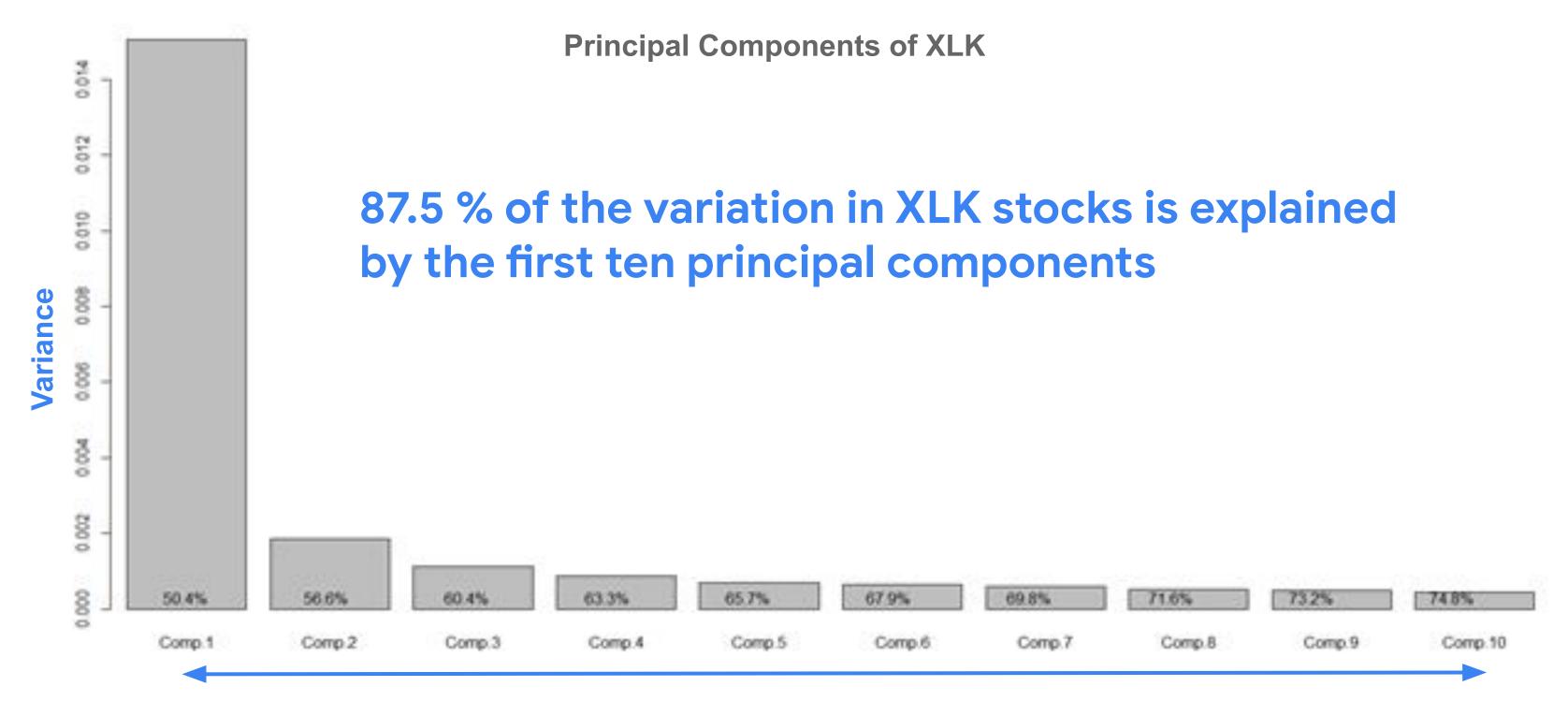
Scree Plot for Principal Components of XLK







Scree Plot for Principal Components of XLK







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How to Implement a Pair Strategy





- Understand what the loadings in a PCA indicate
- Use PCA loadings for the first two components to select a pair
- Design a beta weighted pair strategy for two stock in SLK



Agenda

Understand the significance of the first two principal components

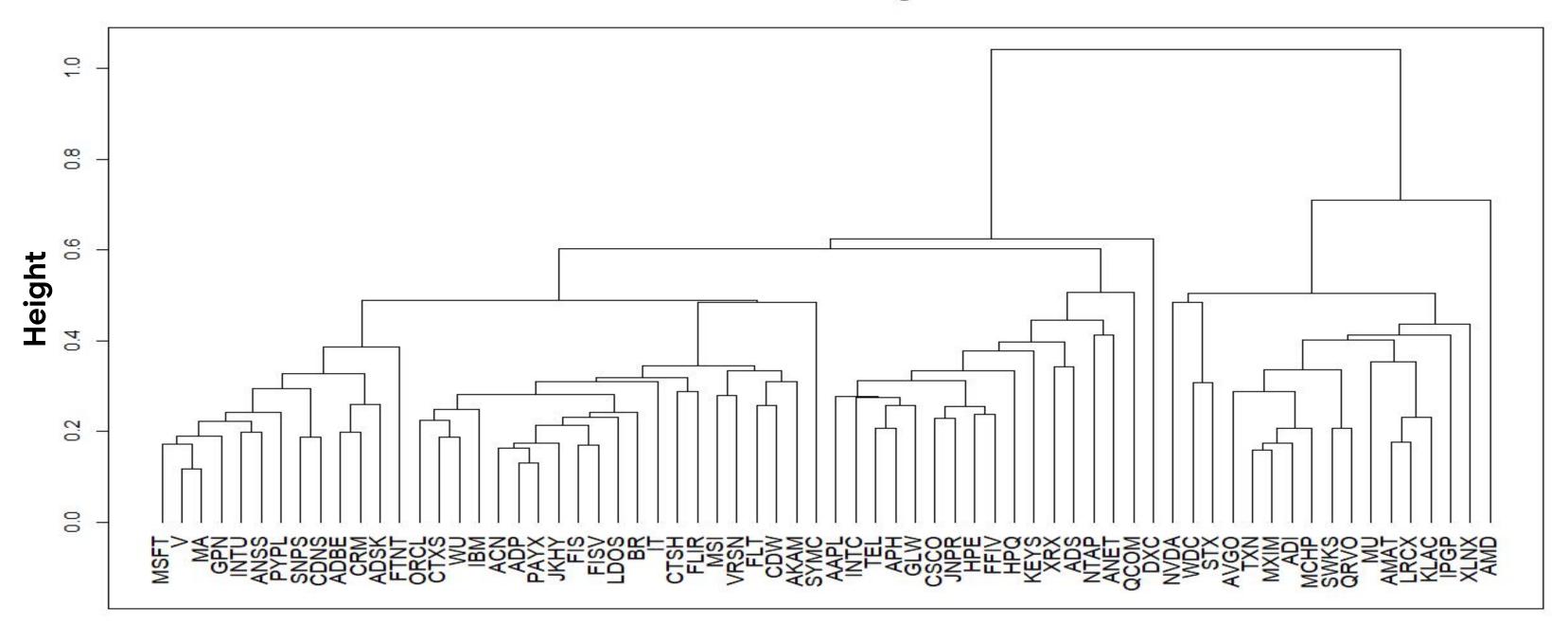
Use PCA component loadings to select pairs

Construct a beta-weighted pair





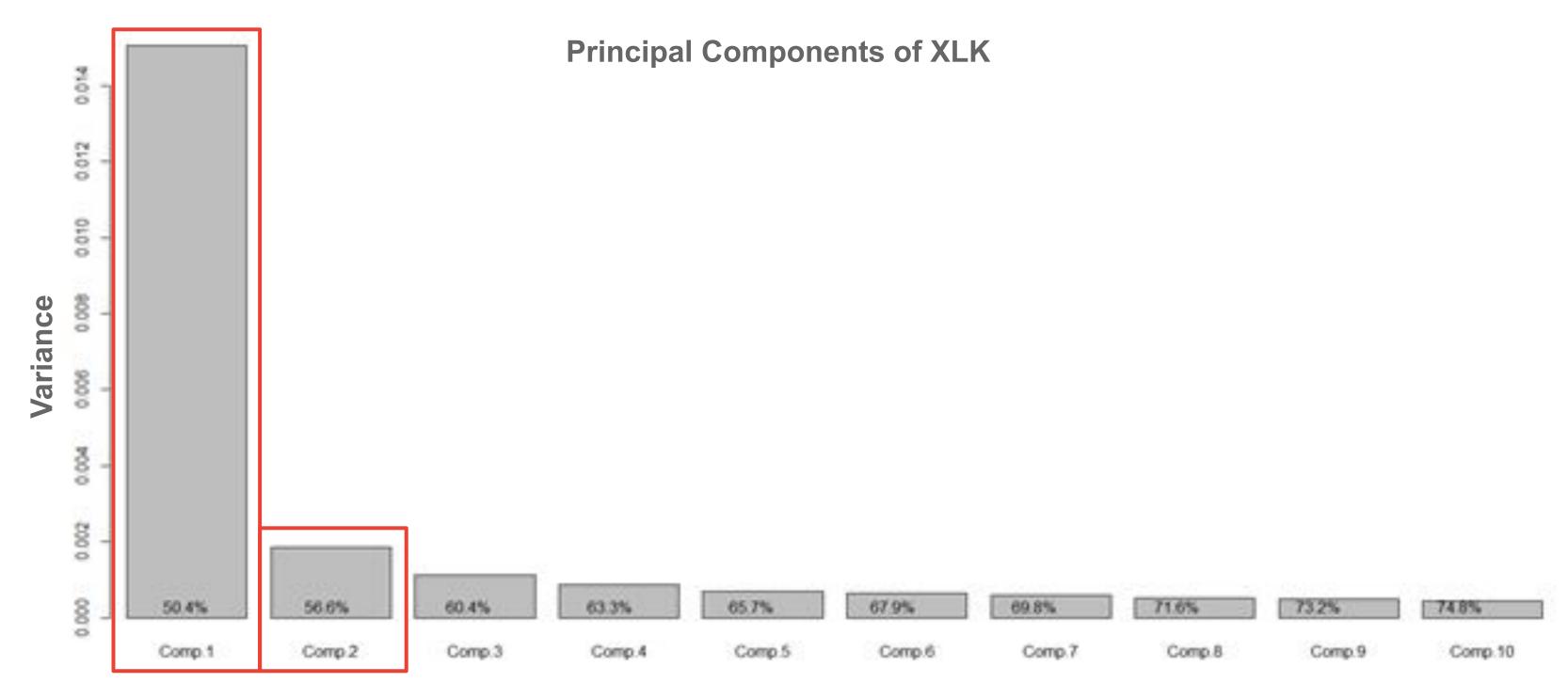
Hierarchial Clustering of XLK





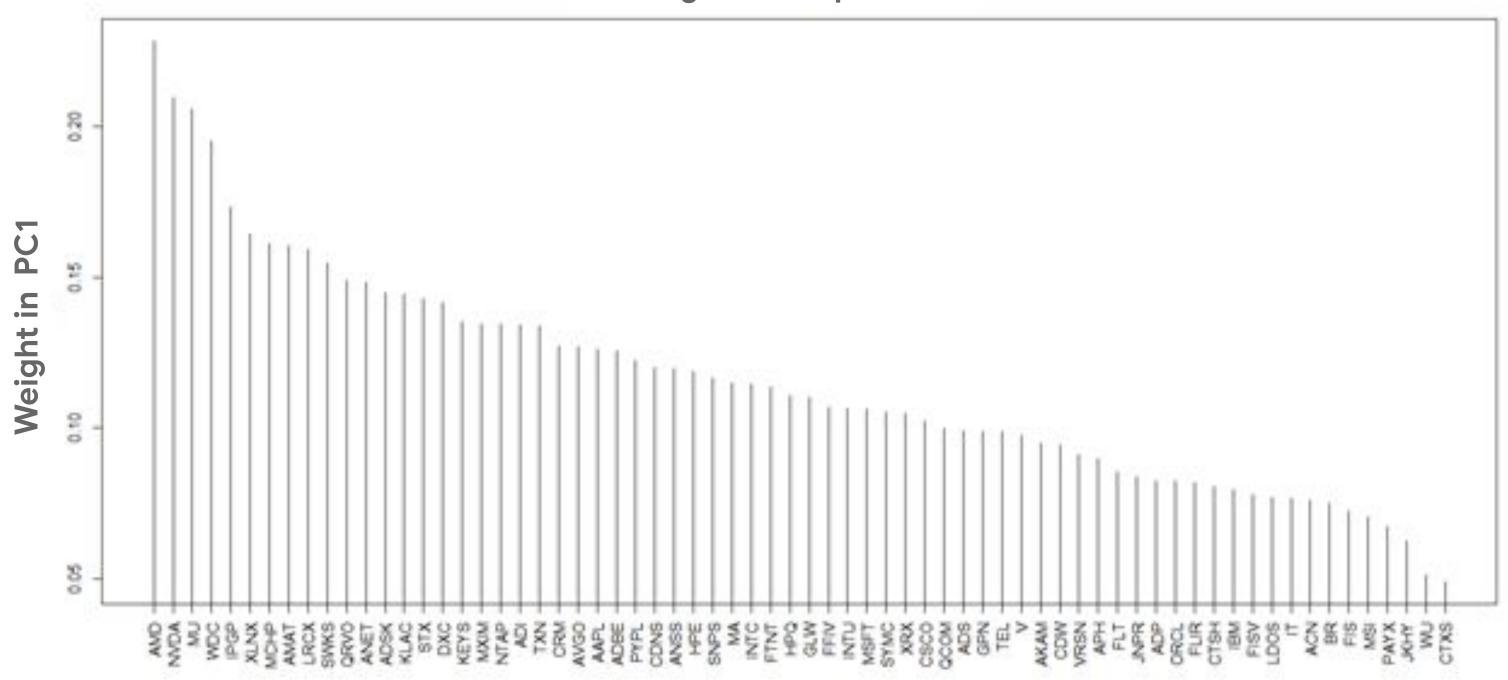


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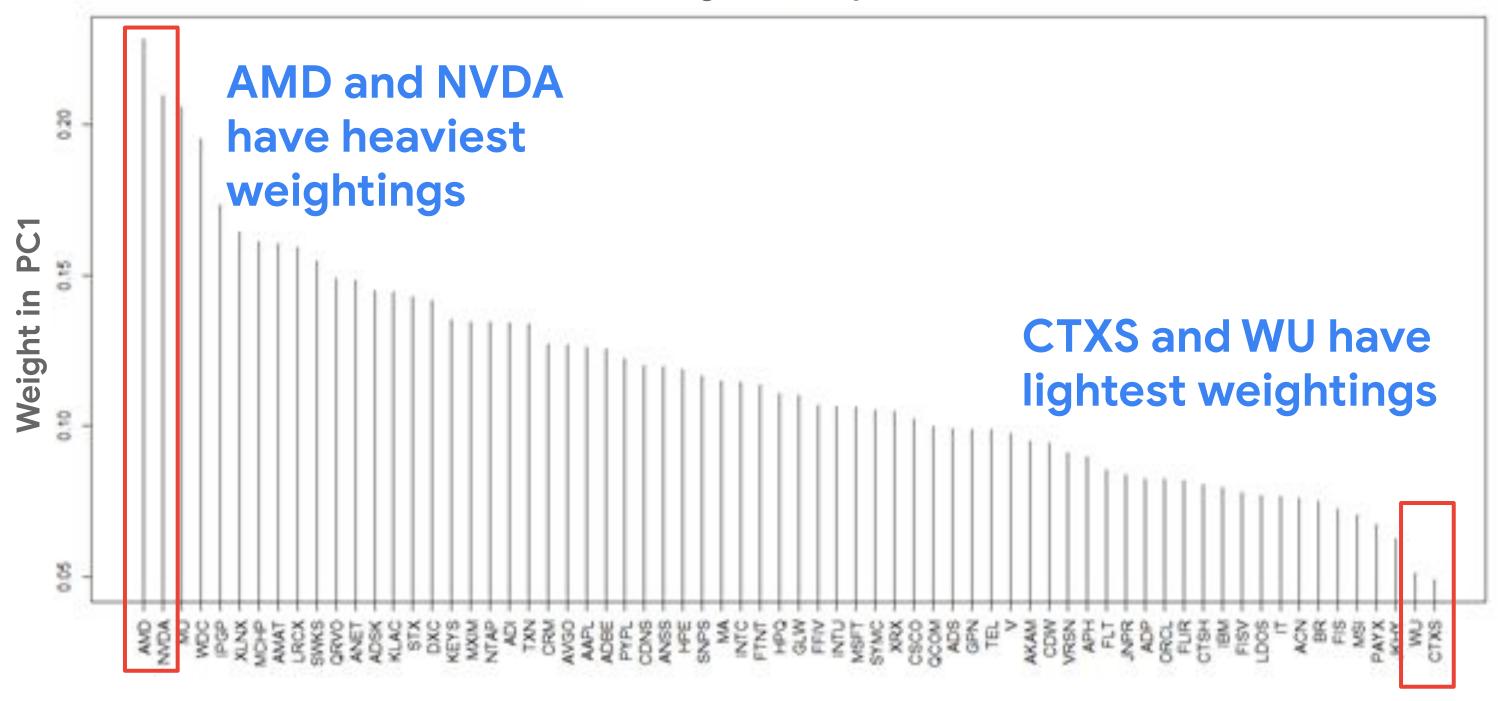






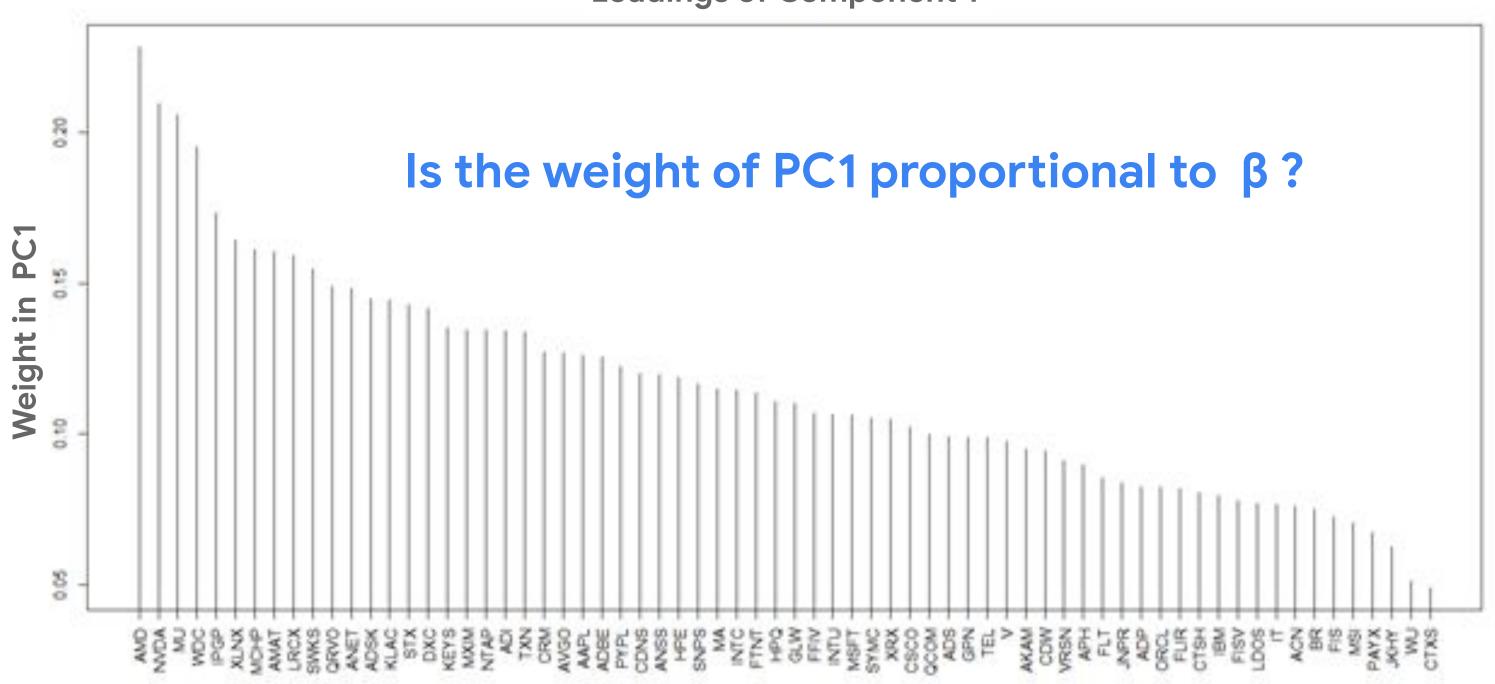
















Agenda

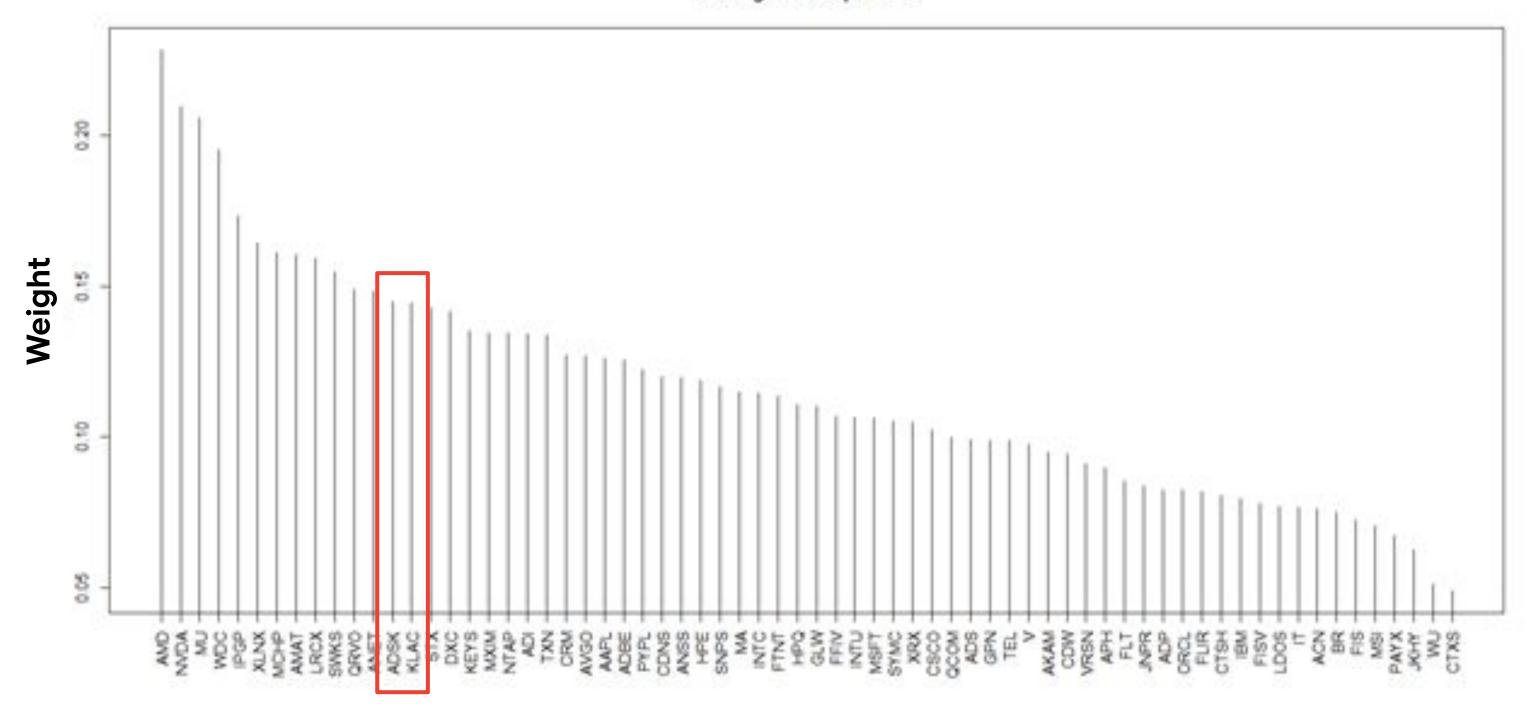
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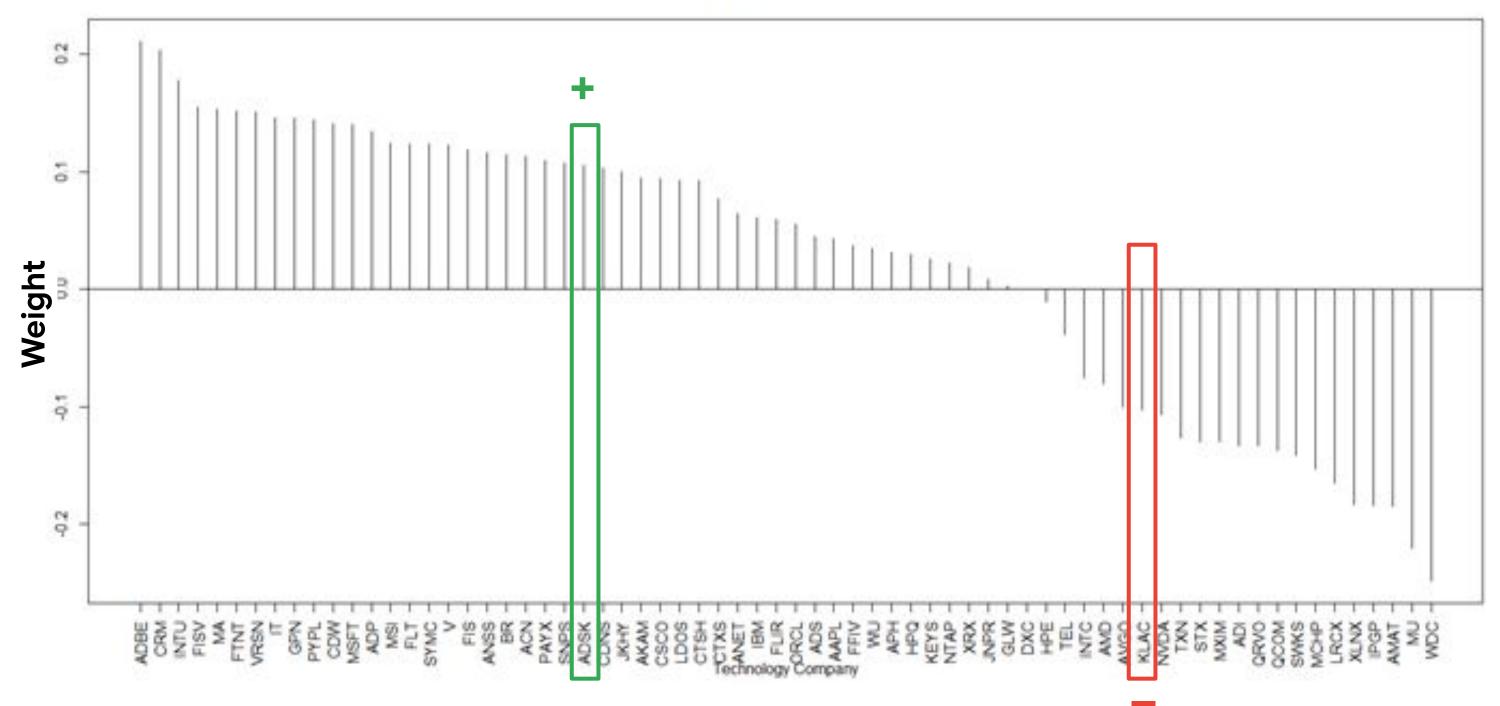






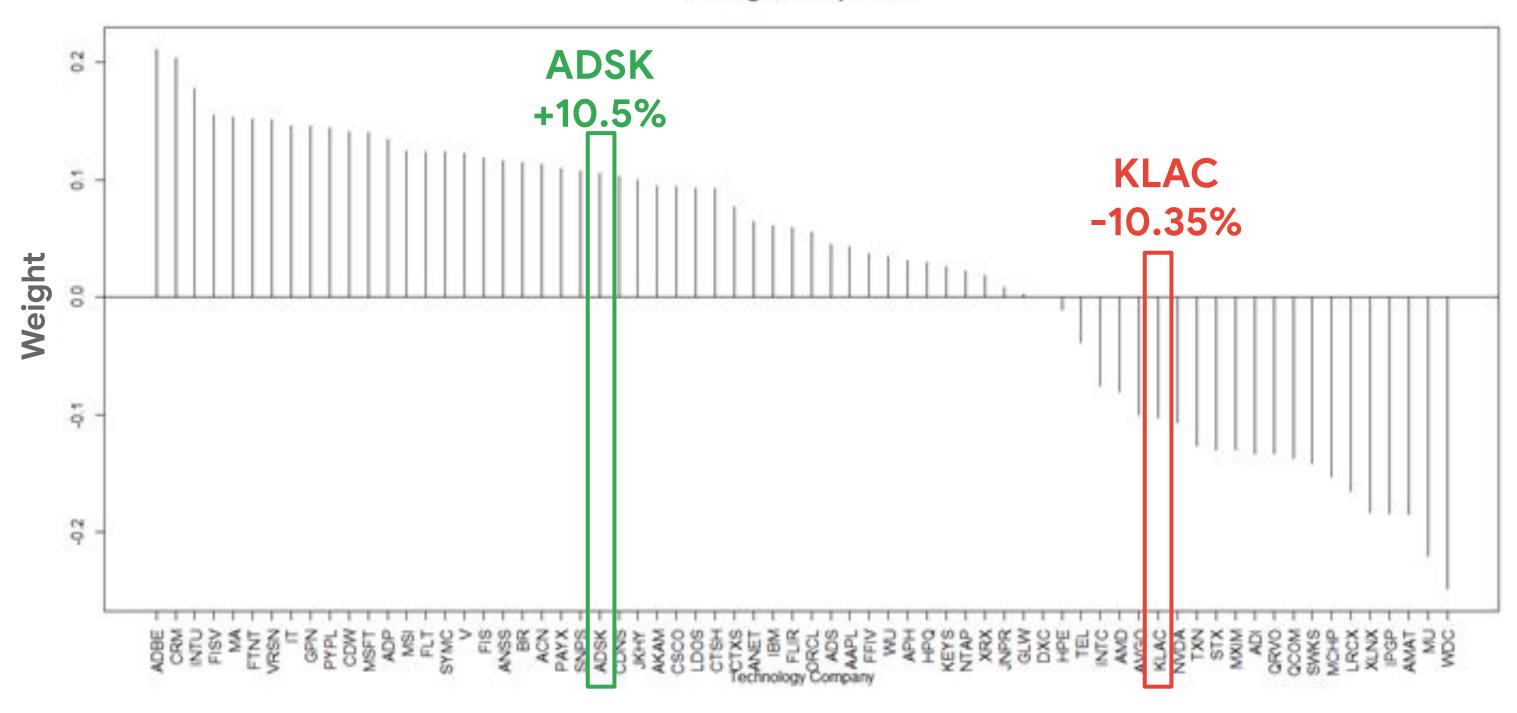








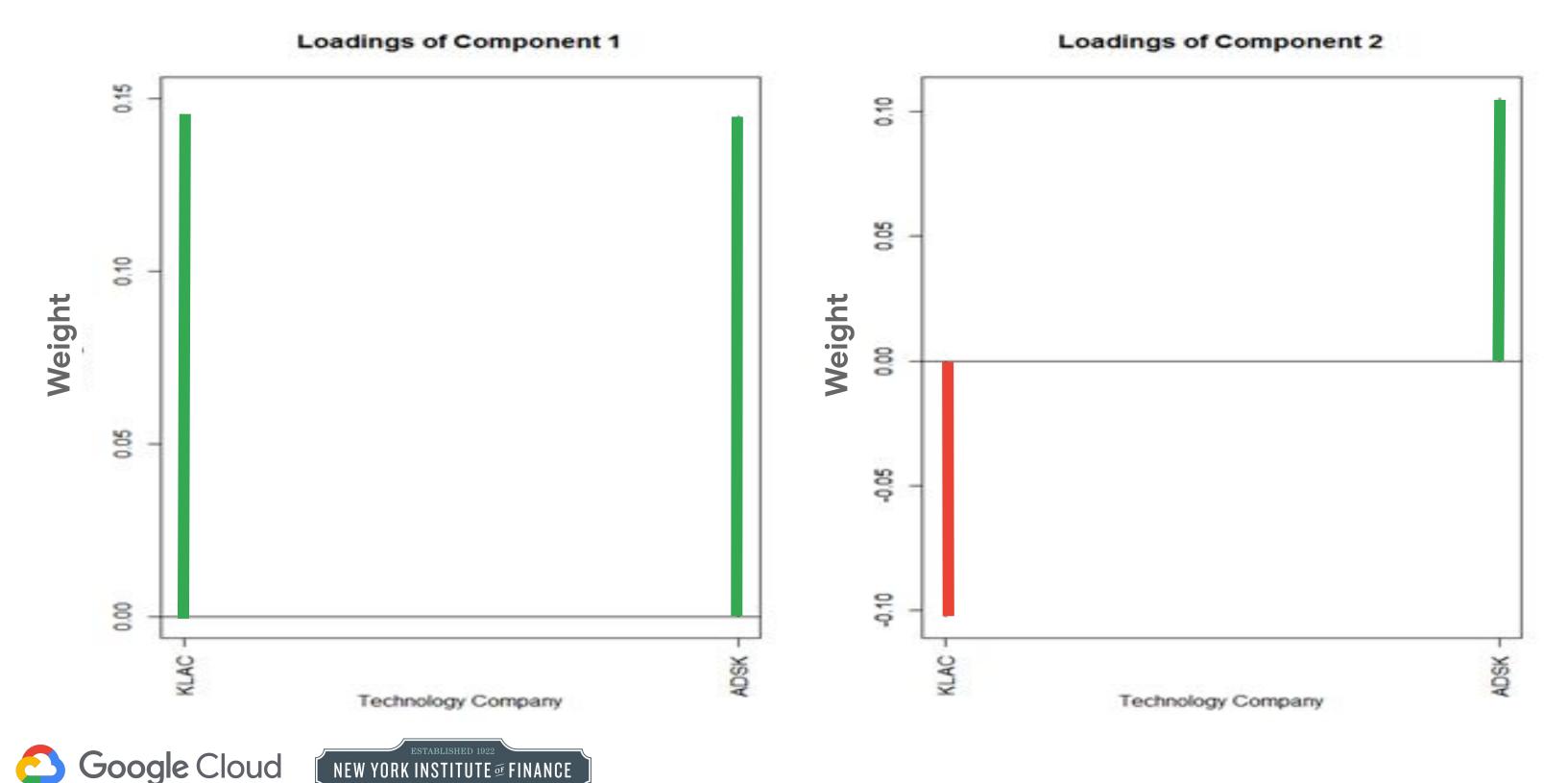








Loadings of KLAC and ADSK for Components 1 and 2



Agenda

Understand the significance of the first two principal components

Use PCA component loadings to select pairs

Construct a beta-weighted pair





Weighting Long and Short Components of Pair Trade

- KLAC trades at \$170 and has a beta of 1.56
- ADSK trades at \$145 and has a beta of 1.41
- Weighting must be Beta Neutral and also factor in relative stock prices.

$$\beta_{KLAC} * P_{KLAC} * 1,000 = \beta_{ADSK} * P_{ADSK} * Q_{ADSK}$$

1,000 shares of KLAC = \$170,000



Weighting Long and Short Components of Pair Trade

 β_{KLAC} * \$170,000 / $\beta_{ADSK} = P_{ADSK}$ * Q_{ADSK}

We need 1.56 * \$170,000 / 1.41 = \$188,085 worth of ADSK

At a share price of \$145, you would need about \$188,085/145 = 1297 shares of ADSK.

Rounding, we'll trade 1,300 shares of ADSK





Weighting Long and Short Components of Pair Trade

1.56 * 1000 sh * \$170/sh = 265,200

1.41 * 1300 sh * \$145/sh = \$265,785

Long 1000 KLAC



Short 1300 ADSK



Pair Strategy

When do you get in?

How do you define your entry signal?







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Evaluate Results of a Pair Trade





Learning Objectives

- Construct a trade blotter to record entry and exit data
- Quantify and implement a trading rule
- Identify six key performance metrics for pairs trading



Agenda

Construct a Trade Blotter

Quantify and Implement a Trading Rule

Evaluate Performance using Six Trading Metrics





The pair you trade

The direction of the trade

The entry date

The entry level

The exit level





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Agenda

Construct a Trade Blotter

Quantify and Implement a Trading Rule

Evaluate Performance using Six Trading Metrics





Specifying Pairs using PCA

- Use second component loadings to choose viable pairs for trading
- 2. Update PCA loading based on new data
- 3. Maintain that existing pair is still supported
- 4. Otherwise choose new pair based on current loadings



Mean-Reverting Strategy

- Assume KLAC and ADSK will have similar long-term performance
- Sell pair member that has outperformed
- Buy pair member that has underperformed
- Wait for spread to converge



- Typically, you will have an entry signal,
 a profit target, and a stop-loss
- If KLAC ADSK > \$30, enter the trade
- Your profit target is when KLAC -ADSK < \$25
- Your stop loss is when KLAC ADSK > \$35
- Your time out will be one month



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Construct a Trade Blotter

Quantify and Implement a Trading Rule

Evaluate Performance using Six Trading Metrics





Backtest and Evaluate Trading Rule

- Apply trading rules to historical data set
- Create hypothetical trade blotter to record results of trading rule
- Measure hypothetical performance of trading rule



Key Performance Metrics

- 1. Win Percentage
- 2. Total Return
- 3. Average Win/Loss Amounts
- 4. Extreme Win/Loss Amounts
- 5. Variance of Returns
- 6. Total Portfolio Value



Performance Metrics: Win Percentage

- Hit Target = Win
- Hit Stop Loss Loss

Win % = [Wins/Losses]*100

 75-80% puts odds in your favor and covers trading costs



Performance Metrics: Total Return

- Need to calculate total invested capital and how much was gained or lost when the trade was closed
- Remember to subtract trading costs such as fees and commissions
- Market impact costs will be reflected in the prices at which you transact



Performance Metrics: Average Win / Loss Amounts

- Average is for performance of both sides of pair combined
- PnL distribution can be asymmetric
- Average win is \$1 but
 average loss is \$10
- You exit trade profitably 90% of the time but still make a loss overall





Performance Metrics: Extreme Win and Loss Amounts

- PnL distribution can have have a "long fat tails"
- Significant probability of an extreme gain or loss that accounts for most of the expected gain/loss
- Implies low confidence in averages



Performance Metrics: Variance of Returns

- High variance/volatility of returns also implies low confidence in expected returns
- Your alpha may have a low Information Ratio (IR)
- Skilled vs Lucky? IR penalizes luck

IR = alpha / volatility return





Performance Metrics: Track Total Portfolio Value

- Track linear trend of portfolio value against market value
- Calculate correlation
- Steady, stable trend preferred to wild oscillations even if less profitable



Performance Metrics: Track Total Portfolio Value

- Prefer a return that is not "bumpy"
- Can measure bumpiness by calculating the second derivative (in discrete time) of portfolio value and then integrating it numerically

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Backtesting and Avoiding Overfitting





Learning Objectives

- Identify the steps required to create a pairs trading model
- Describe a method to modify pair choices dynamically
- Identify the steps required to backtest a static pairs model



Agenda

Backtesting a Pairs Trading Model

Modify model to Dynamically Update Pair Candidates

Detail Steps in a Static Model Backtest





Backtesting Static Pairs

- 1. Backtest KLAC-ADSK pair using several months of data
- Backtest other potential pairs that have been identified using PCA analysis
- 3. Rank pairs and perform an implementation backtest on top candidates



Selecting Optimal Pairs

Selection based on a weighted average of the following criteria.

- 1. Minimize the distance between stock loadings in Component 1.
- 2. Minimize the distance between the absolute value of stock loadings in Component 2.
- 3. Choose two stocks with opposite sign loadings in Component 2.



Agenda

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Modify model to Dynamically Update Pair Candidates

Detail Steps in a Static Model Backtest





Dynamically Update Principal Components

- Update principal components periodically by incorporating more recent price data
- 2. Select optimal pairs using updated PCA analysis



Dynamic Pair Updating: Practical Considerations

Decision to dynamically update pairs requires backtesting or qualitative analysis to determine optimal:

- 1. Update frequency
- 2. Selection criteria for new pairs





Agenda

Backtesting a Pairs Trading Model

Modify model to Dynamically Update Pair Candidates

Detail Steps in a Static Model Backtest





Backtest of a Static Example

Backtest incorporates varying three parameters:

- 1. Entry Signal
- 2. Profit-taking level
- 3. Stop Loss

Assume no time outs.



Combinations of Backtest Parameters

- 50 Different Entry Signals
- 50 Different Profit-taking levels
- 50 Different Stop Losses

50 x 50 x 50 ⇒ 125,000 potential combinations requiring efficient software to:

- Produce a trade blotter
- Summarize each blotter with our key metrics





Divide DataFrame into Training and Testing

Divide available historic price series into two unequal parts:

- Allocate two thirds of data for training our models
- Allocate one third of data for testing our models



Backtesting Potential Parameter Combinations

- Back test each parameter combination/create a trade blotter
- Summarize key metrics:
 - 1. Total, average and percent win
 - 2. Expected loss
 - 3. Volatility
 - 4. Steadiness of the return
- Define optimal criterion as Sharpe ratio: Total Return / Volatility





Measuring Performance of Selected Models

- Run selected models on test data
- Testing out-of-sample helps to identify overfitting model to training data
- Poor performance on testing data indicates overfitting



Measuring Performance of Selected Models

- Run top 50% of models on test data
- Produce new trade blotter
- Choose best models based on Sharpe Ratio or other risk-adjusted performance criterion



Compare Best Models from Testing Phase

- We have over 60,000 models to compare
- Select models with best overall performance based on your weighting of performance in the training and testing phase:

Metric = 33% Training + 67% Testing





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Next Steps: Improvements to Your Pairs Strategy





Learning Objectives

- Identify external factors that can affect asset prices
- Understand how external factors can impact trade profitability
- Learn how to incorporate external factors into trading models



Agenda

Time and Day-of-the Week Effect

Related Markets

Economics and Company Specific Events

Updates to Market Indices





- Market Open
- Market Close
- Intraday drop in trading Volume



- Segregate trades by time of day
- Compare profit metrics for each time slice
- Filter your data and backtest most profitable parts of day
- Run a seperate backtest on excluded data
- Understand when and how your model works





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Day-of-Week Effects

- Higher volume on some weekdays
- Higher volatility
- Avoid days with low volatility



Agenda

Time and Day-of-the Week Effect

Related Markets

Economics and Company Specific Events

Updates to Market Indices





Related Markets

- Options markets
- Expire on third Friday of each month
- Is there more volatility on Expiration days?
- Do market participants adjust their trading on Expiration days?



Agenda

Time and Day-of-the Week Effect

Related Markets

Economics and Company Specific Events

Updates to Market Indices





Economic Events

- Are economic events part of the rules you use to initiate or close trades?
- Are your results influenced by key economic reports
- Does your risk management model for the volatility that can occur after a "surprise" change in an economic indicator (adjust stop-loss orders prior to announcement)



Stock Specific Events

- Quarterly earnings
 announcements have a large impact on trading volume and volatility
- Announcements for either stock in a pair can trigger outsized gains and losses
- Adjusting risk management model is key



Agenda

Time and Day-of-the Week Effect

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Updates to Market Indices

- Additions and subtractions to indices have enormous impact on affected assets
- Changes are partially anticipated by market
- Can still have volatility of announcement of change



Improving Your Pairs Strategy

- Periodic events can be anticipated and modeled
- Infrequent events such as index additions and deletions are more difficult
- Can still have volatility of announcement of change

