Example Historically, the average demand for a type of vegetables at a store is 30 cases per day, with variance of 30^2. The store runs 6 days a week from Tuesday through Sunday. You noticed that the demand on Tuesdays through Thursday are very different from Fridays through Sundays. The further study show that the average weekday sales 15 cases per day while the weekend is 45 cases per day.

1. If the variance in the weekdays and weekends are the same, what should be their values?
2. If the coefficient of variation are the same between weekdays and weekends, what should be the values?
3. Assuming CVs are the same and the variance in the weekdays and weekends are somewhat negatively correlated with correlation factor of 0.3, what should be their variances?

If you treat the demand as week without seasons, what will be the coefficient of variation? If you add the seasonality analysis, what will be the coefficient of variation.

3. You can also use pricing, promotion and marketing to reduce the demand variations. The success of discount store's before e-commerce's emergence show that uniform and low pricing strategy worked much better than the traditional retailers using "sales" strategies. One of the element in the Toyota production system is to use promotion as a tool to smooth out the demand because JIT system does not work well with high fluctuations.

4. Reducing order leadtimes and the use of incentives can also convert the business model from make-to-stock. A make-to-stock supplier takes much lower risks. Make-to-order applies both to product or service. For example, a service provider can require customers to schedule for services instead of first-come-first served.

5. Breakdowns in the process also appears to be inevitable. However, training, preventative maintenance, the investment in more reliable tools, equipment, hire more reliable servers, etc. can significantly reduce the variability. This is part of the focus in six-sigma movement.

6. The use of artificial scarcity can also help to reduce variability. More in the Economies of Speed.

5. REDUCING RELATIVE VARIABILITY VIA POOLING

One important way to reduce variation to pool the sources of variability. The theory behind pooling is a statistical property. When two independent variables are combined or pooled, the relative variability reduces. The common ways to measure variability include: variance, standard deviation, confidence interval, range, etc. The single parameters of variance and confidence are for absolute measure. The others express both require two parameters. Some concepts require most statistical background. A single parameter and easily understandable concept of relative variability is **coefficient of variation**. The relative variability is often more useful than the absolute one. If your job is to plan to supply for the following, which is less certain:

1. Daily restock of markers in an Amazon fulfillment center, or
2. Daily restock of markers in GT bookstore.

The first have much higher variance or standard deviation than the 2nd because the total demand is in the thousands, while the second is probably less than a dozen. However, the first is near constant while the second varies a lot from zero to many.

The coefficient of variation is defined as

$$CV = \frac{\sqrt{VAR}}{\mu} = \frac{\sigma}{\mu}$$

Or squared coefficient of variation

$$SCV = \frac{VAR}{\mu^2}$$

If there are *n* independent random variables, the mean and variance of the pooled variable are

$$\mu_{Pool} = \sum_i \mu_i \,,$$

$$VAR_{Pool} = \sum_i VAR_i + \sum_{i \neq j} COV_{i,j}$$

5.1 The mathematical foundation of pooling effect

If the variables are not correlated, or the $COV_{i,j} = 0$. The coefficient of variation of the random variable pooled from *n* sources is

$$CV_{Pool} = \frac{\sqrt{\sum_i VAR_i}}{\sum_i \mu_i} = \frac{\sigma^2}{\sum_i \mu_i}$$

Without independent assumption, the general expression is

$$CV_{Pool} = \frac{\sqrt{\sum_{\forall i,j} COV_{i,j}}}{\sum_i \mu_i} = \frac{\sqrt{\sum_i VAR_i + \sum_{i \neq j} COV_{i,j}}}{\sum_i \mu_i}$$

Where the covariance term can be positive, negative or 0, and

$$0 < CV_{pool} < \max_i \{CV_i\}$$

The $CV_{pool} \rightarrow \max_{i}\{CV_i\}$ can occur only when the random variables are perfectly and positively correlated, which is no longer random. If all random independent variables have the same mean and variance, we can compute

$$CV_{Comb} = \frac{\sqrt{nVAR}}{n\mu} = \frac{1}{\sqrt{n}}\frac{\sqrt{VAR}}{\mu} = \frac{1}{\sqrt{n}}CV$$

This means that when *n* random variables with same mean and variance are pooled or combined, the coefficient of variation is reduced by a factor of $\frac{1}{\sqrt{n}}$, this is called pooling effect.

5.2 The Impact of Correlations

Note that this simple result has several assumptions. In reality, the means of the random variables may be different and the random variables are correlated.

Consider 2 stores carrying the same product. If they are the same company, both sales will go up when the company runs a promotion. In this case, $COV > 0$. The pooling effect will be reduced. You can see in the example, even that is the case, the pooling effect can still quite strong.

If the two stores are independent, the one runs the promotion and the other not. One will experience increased demand while the other may experience reduced demand. In this case, the $COV < 0$, and the pooling effect is even stronger. If two random variables with the same mean are perfectly negatively correlated, $CV = 0$. However, the process is no-longer random.

Therefore, the quantitative modeling must be associated with proper assumptions. The simplistic generalization can be dangerous.

Example A distribution center (DC) supplies to 100 retail stores. The demand for bottled water at all stores have the same average of 100 cases with variance of 60^2 per week. The demand from different stores are independent.

1. What is the coefficient of variation for each store?
2. What is the coefficient of variation for the DC if the demands from stores are independent, or their $COV = 0$.
3. If the demands among the stores are positively correlated, what should be the range of the coefficient of variation for the DC?
4. If the stores are in pairs, and the demands between pairs are independent but the demands between the two stores in the pair are negatively correlated, what should be the range of coefficient of variation?

Solutions

1. $CV_{sStore} = \frac{60}{100} = 0.6$
2. $CV_{DC} = \frac{\sqrt{100*60^2}}{100*100} = \frac{\sqrt{100}*60}{100*100} = \frac{1}{\sqrt{100}}\frac{60}{100} = \frac{1}{10}0.6 = \frac{1}{6}CV_{Store}$
3. If the demands are positively correlated, the CV for the DC should be higher than calculated above. The maximum will be CV for the store. It will require all stores vary in unison, almost impossible.
4. If demands are negatively correlated, the CV for the DC will be lower than calculated above. If each pair are perfectly correlated with coefficient of -1, the CV for the DC will be zero.

Wait a minute. Do you see something wrong here? Have you heard of bull whip effect? The bull whip effect refers to that as you much upstream of the supply chain, the variability becomes higher and higher. The pooling effect suggests the opposite is true. Where is the inconsistency?

In practice, the demand are often correlated. If the deamd are correlated, you should first find the covariance.

Example Let's use a VERY small data set to show the correlated demand. A distributor supplies to 3 stores. The demand in the 6 days in last week were

	D_1	D_2	D_3
1	13	10	4
2	1	1	18
3	4	1	11
4	10	11	9
5	7	6	6
6	11	12	2

Please find

1. The Coefficient of variation for each store,
2. The coefficient of variation if we assume the demand among 3 stores are independent.
3. The coefficient of variation considering the correlation.

Solutions

1. $VAR_i = E[X^2] - (\mu_i)^2$. You will find the result is smaller than what hat Excel calculates. This is because of biased estimate vs. unbiased sample.

mu	7.67	6.83	8.33
Var	20.67	24.57	33.07
CV	0.59	0.73	0.69

2. If we assume that the 3 demands are not correlated, the Coefficient of variation will be the square root of the sum of the variances divided by the sum of the means, or

$$CV = \frac{\sqrt{20.67 + 24.57 + 33.07}}{7.67 + 6.83 + 8.33} = 0.39$$

This is smaller than any of the three CVs individually. This is the pooling effect.

3. $COV(X,Y) = E(XY) - \mu_x\mu_y$. For D_1 and D_2, you multiply the demand for each day, then find the average, subtract the product of means, will give you 69.83 – 7.67 * 6.83 = 17.44. You can find the pair between 1 & 2, 1 & 3, 2 & 3, 2 & 1, 3 & 1 and 3 & 2, or you can put everything in a matrix.

	D_1	D_2	D_3
D_1	20.67	17.44	-19.22
D_2	17.44	24.57	-18.94
D_3	-19.22	-18.94	33.07

The cell with D_1 & D_1 is VAR_1. This is a symmetric matrix. It is clear that D_1 and D_2 are pretty highly positively correlated, while D_1 & D_3 are strongly negatively correlated. D_2 & D_3 are also strongly correlated, which should make sense. The numerator for CV will be the sum of the 9 elements, while the denominator is the sum of the means, or $CV_{Pooled} = 0.27$. This means overall, the demands are quite negatively correlated. The CV_{pooled} will be

$$0 \le CV_{pooled} \le \max_i\{CV_i\}.$$

It is very difficult to reach 0 because it would require perfect negative correlation. It is also difficult to reach the maximum. See some old homeworks.

5.3 Pooling and Bullwhip Effect

In supply chain, people often talk about bullwhip effect. Bullwhip effect refers to the fact as you march up the supply chain, the demand variability increases. However, it appears that the pooling effect contradicts the bullwhip effect. Which is correct?

Note that bullwhip effect was demonstrated in a pure chain: one supplier – one distributor – one store. In such setting, there is no pooling but bullwhip effect caused by delays, people's risk averse behavior, limited chain visibility etc. However, most distribution systems involve a supply tree. Each supplier supplies many clients. In such situation, pooling effect also presents. In the tree structure, the bullwhip effect still exists in each branch. Therefore, in reality, both bullwhip effect and pooling effect exist. The level of the effect depends on many other factors. In distribution channels, since the supplier supplies to many clients, or n is very large, the pooling effect can be much stronger than bullwhip effect.

5.4 Dimensions to pool

The pooling can be used to reduce uncertainty in many different ways.

1. **Channels**: Pooling can be applied to one SKU when channels are combined, as in the above example.
2. **Time**: The longer the time duration, the lower is the variability or uncertainty. Please consider the demand of a coffee shop on a daily, weekly, monthly or annual bases. Do you know how to quantify?
3. **Space**: The pooling can also occur in space. The variability of space requirement in randomized storage is much lower than in dedicated storage due to pooling effect.
4. **Material handling or transportation**: SKUs can be pooled to reduce the variability in handling of pallets, trucks, etc. You can have mixed truckload, mixed pallets, mixed cases that enjoy the pooling.
5. **Machines or work stations**: A pool of identical machines with variable randomness exhibits less variability as a whole from the outside.
6. **Personnel:** You can have cross trained workforce to pool and reduce the problem with demand variability.
7. **Among countries with different currency:** More later in off-shoring
8. **Among suppliers:** To source from different suppliers can enjoy pooling effect of the risks.

Example on pooling in time A car manufacturer can assemble a car in 1 day. It can choose the number of days to promise its customers when to delivery. The daily demand can be approximated by $N(500, 100^2)$.

1. If it promises the assembly leadtimes of 5 days, what will be the distribution of the "pooled" demand? Be careful how many days you pool.
2. What if it promise 65 days of delivery leadtime?

Solution:

Assumptions: the daily demands are iid. This is pooling over time.

$$\sigma_L^2 = L\sigma^2$$

Note, there is no square over L. This is because of "pooling effect" among independent demand in different days.

1. For 5 day, $\mu = 4*500 = 2{,}000, VAR = 4*100^2$. $CV = \frac{200}{2000} = 0.1$
2. For 65 days, $\mu = 64*500 = 32{,}000, VAR = 64*100^2$. $CV = \frac{800}{32000} = 0.025$ This is nearly a constant!

In reality,

- The daily demand may not be iid. Someone buys today would not buy tomorrow. The demand in successive days can be negatively correlated due to special situations such as a holiday, bad weather.
- The demand within a week tends to be seasonal.

Example on pooling in space Georgia Tech used to let designate classrooms to the departments. For example, ISyE has many classrooms at our disposal in Instruction Center. To better utilize the space, around 2009, Georgia Tech started to coordinate space by GT Space Planning, or all classrooms are "pooled" throughout the campus. In warehousing, you can adopt dedicated storage strategy. It provides convenience in searching. However, in dedicated storage, an empty space dedicated for one SKU cannot be used for another and lead to wasted space. You can employ randomized storage to enjoy pooling effect. An empty space can be used for any SKU. So, the space requirements of different SKUs are "pooled". To enjoy pooling effect, one must have a way to track the location of the SKUs. Warehouses with Warehouse Management Systems (WMS) often employ randomized storage for the reserve, or low velocity products, and hybrid strategy for high velocity SKUs. Retailers normally use dedicated method to display merchandize using dedicated storage strategy.

Example A distribution center handles pallets. It has 10,000 SKUs. If the demand is at constant rate, and there is no safety stock, the pallets on hand can be approximated by a $U(0, 20)$. If the company would allocate 95% space for all the SKUs, what are the space requirements for dedicated and randomized storages.

For dedicated storage: 20 * 0.95 * 10,000 = 190,000.

Let's assume the correlation among SKUs are 0. The parameters with each SKU is

$$\mu = 10, VAR = \frac{1}{12}\left(b^2 - a^2\right) = \frac{1}{12}\left(20^2\right) = 33.333, \sqrt{VAR} = 5.77,\ CV = 0.577$$

The parameters for the randomized storage

$$\mu = 100{,}000, VAR = 33.333 * 10{,}000 = 333{,}333, \sqrt{VAR} = 577,\ CV = 0.00577$$

The space requirement for randomized storage is

$\mu + \sigma z = 100{,}000 + 577 * \Phi^{-1}(.95) = 100{,}000 + 577 * 1.64 = 100{,}946$, or 88% savings!

This is very significant reduction! How much would you expect in reality?

Example Same as above but allocate 100% space for all SKUs.

For dedicated storage: 20 * 1 * 10,000 = 200,000

For randomized storage, to allocate 100% space, and assume normal distribution $N(20*10{,}000, 577^2)$. $\Phi^{-1}(1) = \infty$! Let's try 6 sigma, or z = 6, $1 - \Phi(6) = 9.87E^{-10}$, very small. Even not zero we can ignore. The space requirement will be 100,000 + 577 * 6 = 103,462, or 93% savings!

In reality, the savings will be lower because many SKUs are replenished at the same time. However, the savings is still very significant. If the inventory levels does follow uniform distribution, the space utilization in dedicated storage is only 50%. In the shared space utilization in practice, a utilization of 85% or higher is common.

5.5 Reducing the Negative Impact of Variability with Flexibility

Different items or services often cannot batch because they are different. However, if the processes, machines or operators are flexible or cross trained to the level that the changeover time or cost is minimum or nothing, a family of product or services can be considered the same and be batched to reduce cost. More details are in the part on Economies of Scope.

5.6 Reducing the Negative Impact of Variability with Speed

The negative impact of variability increases with leadtime. You can reduce the impact by reducing leadtimes. More discussion and modeling are in the Economies of Speed.

6. COST OF SUPPLY CHAIN UNCERTAINTY AT MACRO LEVEL

From the efficient allocation of resources to supply toward human needs perspective, what will be the pros and cons of the uncertainty?

6.1 The benefits of uncertainty at macro-level

One may say it provides opportunities for insurance and financial industry. It provides challenges and jobs for engineers, software development. One benefits may be increase employment or livelihood for humans. For those who can deal with uncertainty better, these can also add esteem of opportunities for self fulfillment.

6.2 The cost of uncertainty at macro-level

Through the simple newsvendor model, the uncertainty always add cost to the basic physiological and safety needs in the society, even though some people may benefit. It add anxiety and stress. Even for those who can benefit from it financially, the excessive added stress can lead to various problems.

7 SUMMARY OF ECONOMIES OF UNIFORMITY

The easiest cases to handle in supply chain is everything is uniform, no deterministic variations or uncertainty. Any variations leads to higher cost or inefficiency. However, variety is the spice of life, and the uncertainty offers challenges. Together, they offer opportunities for engineers or managers opportunities to do better in competition. The tools used for deterministic variations is often deterministic optimizations in MRP, scheduling, vehicle routing systems. The methods for uncertainty are stochastic models. The simple ones we references here are newsvendors. The queueing models are also useful in the supply chain modeling. Please do not take variation and uncertainty as given. As engineer, you should also actively working on reducing the variability and uncertainty, such as SKU reduction, equipment streamlining, identifying the systematic errors in data, such as seasonality or event based demand variations, and try to reduce the relative variation through pooling in channels, time, space, personnel, etc. When working on a project, you should not lose site on how the project on hand serve human needs.

EXERCISES

1. What are the types of variations and how do you go about dealing with them?
2. What are the negative impact of variations in supply chain?
3. What are the important factors safety stock depend on?
4. Why is variance alone not the best indicator for relative variability?
5. What is the mathematical foundation for pooling effect?
6. In the HBR case, Barilla run promotions with timed discount and quantity discount. What are the positive and negative effect of such promotions?
7. The demand distribution can be approximated by $N(200, 80^2)$ per week. The safety factor is set at 2. The procurement found two suppliers. Supplier A has a 2-week leadtime with no variation. Supplier B's leadtime can be approximated by $N(1, 0.4^2)$ in weeks.
   7.1. What is the average and standard deviation of the demand during leadtime for A?

$$\mu = 200 * 2 = 400 \text{ (2 points)}$$
$$\sigma = 80 * \sqrt{2} = 113.14 \text{ (2 points)}$$

   7.2. What is the average and standard deviation of the demand during leadtime for B?

$$\mu = 200 \text{ (2 points)}$$
$$\sigma = \sqrt{1 * 80^2 + 200^2 * 0.4^2} = \sqrt{80^2 + 80^2} = 113.14 \text{ (4 points)}$$

   Note, std are the same but CV is not. How about safety stock?

8. The current leadtime is LT and it follows some distribution $D_L(\mu_L, 0)$. The demand in the leadtime follows $D_D(\mu_D, VAR_D)$. If you can cut the leadtime in half, what should be the new safety stock level in terms of current?

   The current $S.S. = z\sqrt{\mu_L}\sqrt{VAR}$.

   If the leadtime is cut in half, the new safety stock will be $S.S.' = z\sqrt{\frac{\mu_L}{2}}\sqrt{VAR} = \frac{1}{\sqrt{2}}S.S.$

9. The weekly demand of a product can be approximated by a normal distribution $N(500,200^2)$. The firm orders this product from its provider every 2 weeks. The order leadtime is 1 week. The safety factor is set to be at 2.5.
   9.1. What is the approximate level of cycle stock?

   $Cycle\ stock = 500 * 2 = 1000$, this is because they order every 2 weeks. The average demand in 2 week period is above.

   9.2. What should be the safety stock?

$$Safety = 2.5 * 200 * \sqrt{2 + 1} = 866$$

9.3. A new supplier provides a service with 1 day leadtime (0.2 week). Its delivery route comes every week. Therefore, you can order every week. How much will be your cycle stock and safety stock?

$Cycle = 500 * 1 = 500, Safety = 2.5 * 200 * \sqrt{1 + 0.2} = 548$

With the new supplier, the average stock will be reduced from 1366 to 798! In addition to lower carrying cost, the setup cost can be lower because the client is in-route of the supplier's deliveries. The other benefits can be faster response time (1.2 weeks vs. 3 weeks), etc.

10. (40) The weekly demand of a special types of tires can be approximated by $N(100, 40^2)$. The firm orders the tires every 2 weeks. The setup cost is $120 per order. The tire sells at $180 per tire. The wholesale price is $120. The holding cost is 0.5% of the value per week. When there is a shortage, the firm set a cost of good will at $2 per tire per shortage.

First realize that T=2 weeks is the cycle time. Bi-weekly demand is $D \sim N(200, \sqrt{2 \cdot 40^2})$. Also, $h = 1.2, c_o = 1.2 \text{ and } c_u = 62$.

10.1. Please find the cycle stock and cost associated with cycle stock per order cycle.

Cycle Stock quantity: $Q_{cs} = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 \cdot 120 \cdot 200}{1.2}} = 200$

Cycle stock cost: $K\frac{D}{Q} + h\frac{Q}{2} = 120 \cdot \frac{200}{200} + 1.2 \cdot \frac{200}{2} = 240$

10.2. Please find the safety stock and the cost associated with uncertainty per year.

Safety stock:

$Q^* = Q_{cs} + Q_{ss}\,, \qquad F(Q^*) = \frac{c_u}{c_o + c_u} = 0.981$

$Q_{ss} = z\sigma, \qquad z = 2.07, \qquad \varphi(z) = \varphi(2.07) = 0.0468$

$\Rightarrow Q_{ss} = 2.07 \cdot \sqrt{2 \cdot 40^2} = 117.09 \approx 118$

$\Rightarrow Q^* = 200 + 118 = 318$

Cost of uncertainty:

$c_o[(Q - \mu)F(Q) + \sigma\phi(z)] + c_u[(\mu - Q)(1 - F(Q)) + \sigma\phi(z)]$

$= 1.2[118 \cdot 0.981 + \sqrt{2 \cdot 40^2} \cdot 0.0468] + 62[(-118)(0.02) + \sqrt{2 \cdot 40^2} \cdot 0.0468]$

$= 1.2[118.4] + 62[0.287]$

$= \$159.87$

10.3. If the demand is constant at 100 per week, what should be the profit?

$\text{Profit} = \text{revenue} - \text{cost} = 200(180) - [120 + 200(120)] - 1.2 * 200/2 = \$11{,}760$ bi-weekly. The last term is the holding cost, or unit holding cost per 2 week period times half of the order quantity.

$$Anual\ profit = 26 * 11{,}760 = 305{,}760$$

11. A company procures a popular model of tires from company Even. The monthly demand can be approximated by a normal distribution $N(10000, 3000^2)$. The order leadtime is 2 weeks, constant (approximately 0.5 month). Company Odd is trying to get your business with lower price. Their average leadtime is also 2 weeks. However, history reflects that their order leadtime varies and can be approximated with normal distribution with coefficient of variation of 0.3. What is the standard deviation you should use to calculate the safety stock for company Even?

$$\sigma_L = \sqrt{L}\sigma = \sqrt{0.5} * 3000 = 2121.32$$

11.1. What is the standard deviation you should use to calculate the safety stock for company Odd?

$$\sigma_L = \sqrt{L\sigma^2 + D^2\sigma_T^2} = \sqrt{0.5(3000)^2 + 10{,}000^2(0.3 * 0.5)^2} = 2598.08$$

11.2. If the safety factor is 2.5, and holding cost is \$1 per tire per month, how much cheaper does company Odd must offer you to justify the extra safety stock?

To solve this problem, in general terms, one would have to calculate the difference in total cost between the alternatives. However, the drop on price needed to compensate for the higher stock levels cannot be calculated precisely from the information given without making some strong assumptions (like ignoring that a price change would affect all the units bought, not just the cycle stock… and this would in turn change the optimal order quantity).

Nonetheless, the question asks just on the safety cost part. The extra safety cost has the main influence and therefore, provides a good approximation. It can be found by calculating the cost of holding safety stock on each scenario, and dividing the difference by an estimate of the order quantity.

$Q_{ss.Even} = 2.5 * 2121.32 = 5303.3 \approx 5304$

$Q_{ss.Odd} = 2.5 * 2598.08 = 6495.2 \approx 6496$

$$C(S.S._{Odd}) - C(S.S._{Even}) \approx 1192$$

The extra holding cost will be h*1192 per 2 week period, or = ½ * 1192 = 596. This extra cost will be share by the 5,000 average order quantity, or \$0.119, or \$0.12.

Another possible approach was to calculate the cost of uncertainty from

$$c_o[(Q-\mu)F(Q) + \sigma\phi(z)] + c_u[(\mu - Q)(1 - F(Q)) + \sigma\phi(z)]$$

By assuming that F(Q) would remain invariant under both scenarios (technically not true).

12. A DC supplies 100 retail stores. For one popular item, the average weekly demand at each store is 1000. The variance is 1000^2.

12.1. What is the coefficient of variation in each store?

$$CV_{store} = \frac{\sqrt{1000^2}}{1000} = 1$$

12.2. What is the coefficient of variation at the DC if the demands at stores are not correlated?

$$CV_{DC} = \frac{\sqrt{100 * 1000^2}}{100 * 1000} = \frac{1}{10}$$

12.3. Will your result in 3.2 become higher or lower if the demand at the stores are positively correlated?

In this case the coefficient of variation at the DC will increase

12.4. In what situation the demands to be positively correlated.

For example, if they are located in a close geographical region, if they all target the same demographics, etc.

13. (10) You are pooling among two sources. The means and variances of the two sources are μ_i and VAR_i, for $i = 1,2$.

$$CV_{pool} = \frac{\sqrt{VAR_1 + VAR_2 + 2COV_{1,2}}}{\mu_1 + \mu_2}$$

Now, from the correlation coefficient definition, we can find bounds on the value of the covariance of a sum of two random variables $-\sigma_1\sigma_2 \leq COV_{1,2} \leq \sigma_1\sigma_2$

13.1. What is the minimum value of the CV_{pool}? Please show the condition for this minimum quantitatively.

The value of the covariance is minimized when the two variables have a perfect negative correlation: $-\sigma_1\sigma_2 = COV_{1,2}$

In this case,

$$CV_{pool} = \frac{\sqrt{VAR_1 + VAR_2 + 2COV_{1,2}}}{\mu_1 + \mu_2} = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2}}{\mu_1 + \mu_2} = \frac{|\sigma_1 - \sigma_2|}{\mu_1 + \mu_2}$$

The minimum occurs when $\sigma_1 = \sigma_2$, and $CV_{pool} = 0$

13.2. What is the maximum value of the CV_{pool}? Please show the conditions for this maximum quantitatively.

The coefficient of variation is maximized when there is a perfect positive correlation between the two sources: $+\sigma_1\sigma_2 = COV_{1,2}$

In this case,

$$CV_{pool} = \frac{\sqrt{VAR_1 + VAR_2 + 2COV_{1,2}}}{\mu_1 + \mu_2} = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2}}{\mu_1 + \mu_2} = \frac{\sigma_1 + \sigma_2}{\mu_1 + \mu_2}$$

The maximum occurs when $\sigma_1 = \sigma_2$ and $\mu_1 = \mu_2$, and $CV_{pool} = CV_1 = CV_2$. Since perfect correlation would mean certainty for one of the variables, pooling effect almost always exist, to some extent, even when two random variables are positively correlated.

14. The daily demands of a fruit juice at 4 stores are given in the table below.

	S1	S2	S3	S4
1	178	98	244	188
2	135	64	246	232
3	129	196	157	185
4	187	136	241	169
5	181	163	304	72
6	52	242	135	317
7	285	79	347	27
8	193	114	270	200
9	193	52	269	116
10	264	142	357	130
11	121	52	213	218
12	74	244	120	321
13	201	202	286	108
14	265	123	359	28
15	289	230	360	93
16	280	146	366	0
17	200	160	225	9
18	171	126	208	157
19	63	115	158	271
20	265	191	304	41
21	80	118	148	236
22	187	139	281	192
23	185	211	249	141
24	201	120	228	168
25	194	123	224	139
26	200	146	245	155
27	85	84	203	117
28	213	146	245	105
29	224	112	315	20
30	217	61	264	31
31	121	110	246	156
32	280	78	343	0
33	70	119	100	178
34	177	84	207	165
35	98	109	179	265
36	210	110	333	48
37	187	133	282	140
38	191	197	302	104
39	138	244	179	126
40	227	95	343	105
41	215	90	273	77
42	96	235	179	175
43	201	113	324	62
44	60	95	101	202
45	98	160	195	159
46	59	230	111	285
47	72	112	110	267
48	110	205	180	145
49	55	60	129	244
50	261	142	293	32

51	162	137	265	147
52	236	163	300	140
53	172	170	203	65
54	182	231	234	41

14.1. (10) Please find the coefficient of variation of daily demand for each store.

	S1	S2	S3	S4
MEAN	170.19	139.94	240.41	139.15
VAR	4603.56	2872.24	5475.65	6769.27
CV	0.40	0.38	0.31	0.59

14.2. (20) Please find the coefficient of variation of store pairs S1 and S2, S1 and S3, and S1 and S4. What can you conclude based on your calculations?

	S1 and S2	S1 and S3	S1 and S4
∑ MEAN	310.13	410.59	309.33
∑ VAR	7475.80	10079.21	11372.83
∑ COV	-677.39	9180.44	-8908.31
CV	0.27	0.34	0.16

14.3. (10) Repeat 1.2 if you assume independence between S1 and other stores and compare your solutions to those in 1.2. What can you conclude?

Assume Indep.	S1 and S2	S1 and S3	S1 and S4
∑ MEAN	310.13	410.59	309.33
∑ VAR	7475.80	10079.21	11372.83
CV	0.28	0.24	0.34

Note, the independent assumption cause significant errors between S1 and S3, and S1 and S4.

14.4. (10) Please find the coefficient of variation for a distribution center that delivers to 4 stores every day.

	DC
∑ MEAN	689.69
∑ VAR	19720.72
∑ COV	-9855.39
CV	0.14

If you assume independence, the CV will be much higher.

15. (20) Other than pooling, what can you do to reduce variability? When possible please be specific which parameter can be changed with each strategy you employ.

Reduce variance through

- Manage your demand using promotions, etc.
- Identify the systemic component from the random component in the variability, such as seasonality, events, nature, political and economic (exchange rate, fed cutting of discount rate, oil price change), etc. The removal of the deterministic component will reduce the variance of the demand.
- Reduce the unexpected interruptions through training, process improvement (six sigma). Adopt good preventative maintenance.

Reduce leadtimes or cycle times. Using normal as example. The effective variation $\sigma_L = \sqrt{L}\sigma$. In fixed interval ordering, it will be order cycle time. You can work to reduce leadtime with your suppliers or work to reduce your setup so that to reduce your cycle time.