The drawbacks of Delta fleet is that its fleet is highly non-uniform. It needs multiple and different maintenance facilities, holds expensive spare parts inventory for many part types, employ technicians with different trainings for different aircraft types.

1.3.2 Southwest

The benefits of Southwest strategy is from the uniformity of aircraft in Southwest can have similar maintenance facilities with similar tools, smaller set of spare parts inventory, less types of technician, exchangeability of equipment, etc.

The drawbacks is in the types of customer and market it can server.

1.4 Uniformity among certain handling characteristics

In logistics, goods can be handled in the same way. In that way of handling, the goods are considered uniform. For example, UPS has letters, small packages, cases and pallets. Each type is handled the same. All small packages, although different sizes, shapes and firmness, they are handled the same by the operators since the operators are flexible. The concept of unit load is to take advantage of handling uniformity in transportation, stacking, equipment design, etc.

1.5 Summary of economies of deterministic uniformity

Products, services, operations and facilities can be nearly deterministic but variable. Such variations leads to higher cost. Therefore, when you add variety or introduce differences, there is costs associated with your decision, sometimes severe. If you have to add the variations, you should look for synergy among products, services, operations, personnel or equipment to maximize economies of scope, or adds minimum cost with the increased variety.

2. ECONOMIES OF SCOPE

The economies of scope is the cost savings of producing *n* products or services as a whole minus the total cost of producing them individually. The economy of scope can be expressed as

$$\text{Econoimes of Scope} = TC(q_1, \ldots q_i \ldots q_n) - \sum_n TC(q_i)$$

If this negative, there exist economies of scope.

Let's consider the economies of scope in different types of the deterministic economies of uniformity.

In supply chain, there is some initial investment cost I, such as building, equipment, fleet, etc. To build a product or provide a service type i, a minor setup K_i is needed such as adding the storage system or setup the tools on the machine, or modifying the trucks for certain services such as temperature control, lift gate, etc. During operations, the change of products or services will incur a changeover cost *K*. Typically, $I >> K_i$. In production or service, there is a variable cost c_i per unit produced or serviced. The average cost of building q_i products *i* for $i = 1, 2, \ldots n$ products is

$$AC = \frac{I}{\sum_{i=1,\ldots n} q_i} + \frac{\sum_i \left(K_i + c_i q_i\right)}{\sum_{i=1,\ldots n} q_i} = \frac{1}{\sum_{i=1,\ldots n} q_i}\left\{I + \sum_{i=1,\ldots n}\left[K_i + c_i q_i\right]\right\}$$

If you can reduce the cost of change over between products, you can enjoy the economies of scope.

2.1 Economies of scope among different products

If different products can be produced with low changeover cost, you can enjoy economies of scope. This is what flexible systems, computer controlled systems can achieve. They can basically treat a class of products the same. For example, the modern car assembly lines can assemble different models on the same line, even sedans, coups or even SUVs, on the same line.

The flexibility also lower than invest in separate lines. It can also enjoy economies of scope because different products are processed on the same line.

2.2 Economies of scope of complementary seasonality

Some products have different seasonality. Summer versus winter, earlier in the month vs. later in the month (office supplies). This can be within a week or a day.

2.3 Economies of scope among different facility

Note that Southwest has many different types of Boeing 737s with different seating capacity, seating arrangement and range. However, their engine, seats, windows, doors, controls, etc. are mostly identical.

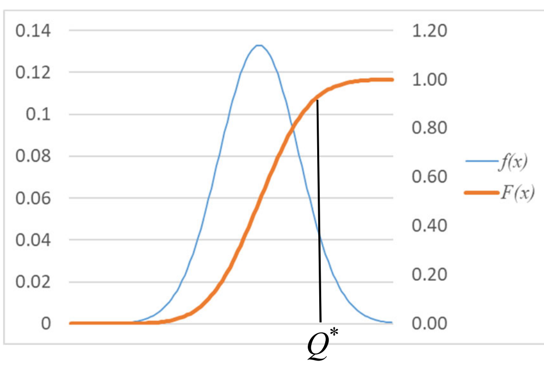
3. THE COST OF UNCERTAINTY

The demand, weather, an infection, probability of getting sick, the exams, the possibilities of interruptions are usually uncertain. As a supply chain engineer, you must make decisions against the uncertainty. However, one thing is certain: if your decision is not spot on, you will be either too much or too little. There is cost either way. If the uncertainty can be captured by a distribution, either continuous or discrete, and the expected cost of being too much or too little can be quantified and find the optimum solution.

3.1 Newsvendor model

If your decision is trade off too much or too little against a single random variable, it can often be modeled as a News Vendor model. Newsvendor model has very loose assumptions with the guaranteed optimality and therefore, extremely useful! The story goes that a newsvendor has to decide how many papers to acquire in the early morning based on the type of news of the day and his/her estimation of the random demand. If he/she runs short, there is an underage cost. Otherwise, he/she will find useless old "news" at the end of day, or an overage cost. Although this model rooted in such background, it applies whenever a decision is against a single known random variable with known overage cost and underage cost per decision.

If the random variable can be modeled with a distribution $f(x)$ and $F(x)$, or in discrete case $p(x)$ and $P(x)$, the decision is to find the optimum $x^* = Q$ that minimize the expected total expected cost considering both the overage cost and underage cost.



If the outcome x is less than the Q, there is an overage cost. If x is more than Q, there is underage cost.

c_o overage cost per unit per decision cycle

c_u underage cost per unit per decision cycle.

The total expected cost with respect to Q is

$$C(Q) = c_o \int_{-\infty}^{Q} (Q - x) f(x) dx + c_u \int_{Q}^{\infty} (x - Q) f(x) dx$$

For discrete distribution, we have

$$C(Q) = c_o \sum_{Q_i<Q} (Q - Q_i)p(Q_i) + c_u \sum_{Q_i>Q} (Q_i - Q)p(Q_i)$$

In verbal form, this is

$$C(Q) = c_o E[\text{number of units over}] + c_u E[\text{number of units under}]$$

You can apply first order condition to find the optimum solution. Since the decision variable Q is also in the limits, you need to apply Leibniz rule.

$$\frac{d}{dQ}\int_{a(Q)}^{b(Q)} f(Q,x)dx = \int_{a(Q)}^{b(Q)} \frac{\partial}{\partial Q} f(Q,x)dx + f(b(Q),Q)\frac{\partial b(Q)}{\partial Q} - f(a(Q),Q)\frac{\partial a(Q)}{\partial Q}$$

Apply Leibniz rule, and set the first order condition, we have

$$\begin{aligned}\frac{dC(Q)}{d(Q)} &= c_o\int_{-\infty}^{Q}\frac{\partial(Q-x)f(x)}{\partial Q}dx + (Q-Q)f(Q)\frac{dQ}{dQ} - (-\infty-Q)f(-\infty)\frac{d(-\infty)}{dQ} + \int_{Q}^{\infty}\frac{\partial(x-Q)f(x)}{\partial Q}dx + 0 + 0 \\ &= c_0\int_{-\infty}^{Q} 1 * f(x)dx + c_u\int_{Q}^{\infty}(-1)f(x)dx \\ &= c_0F(Q) - c_u(1-F(Q)) \to 0\end{aligned}$$

$$F(Q^*) = \frac{c_u}{c_u + c_o}$$

You can find the expected cost and number of overage and underage by plugging the result back into the model.

$$C(Q) = c_o[(Q-\mu)F(Q) + \sigma\phi(z)] + c_u[(\mu - Q)(1 - F(Q)) + \sigma\phi(z)]$$

Where

$$E[\text{Over}] = (Q-\mu)F(Q) + \sigma\varphi(z), \quad E[\text{Under}] = (\mu - Q)(1 - F(Q)) + \sigma\varphi(z)$$

Here, F can be any continuous distribution. The right hand side is called **critical ratio**. You can find proof that if the distribution is discrete, the equal sign becomes greater or equal, or

$$P(Q^*) \geq \frac{c_u}{c_u + c_o}$$

The optimum quantity must be the discrete quantity at or above the critical ratio. In supply chain, the quantity is normally discrete, or integer.

In supply chain, Newsvendor is useful in many practical situations.

1. The scheduling problem in services. The arrivals or service times are normally uncertain. If you are consider one random variable, and the capacity or the schedule will lead to idle times, possibly underage, or wait times, possibly overage. Such can possibly be modeled as Newsvendor problems.
2. Capacity design in services. The demand is normally uncertain. If you can find the demand distribution, and cost for too much or too little, this can be modeled as Newsvendor problem.
3. Probabilistic inventory problems
    a. Perishable goods, such as newspaper, fruit and vegetables, fashion,
    b. One time order such as in fashion, seasonal products, etc. This is similar to perishable goods.
    c. Problem with fixed order or delivery cycles. Such problems normally have 2 or more decision variables. Since the order cycle times are fixed, the setup cost per unit time is fixed when determining the cycle length. The problem is reduced to a single variable problem. This is a common practice in supply chain.
4. Determine best leadtime if leadtime is uncertain
5. Determine fleet size or drivers
6. Determine the area of vaccine distribution
7. …

It also applies to many other problems that can be reduced to a single random variable decision problem with known distribution, overage and underage costs not related to supply chain.

3.2 The notation in inventory problems

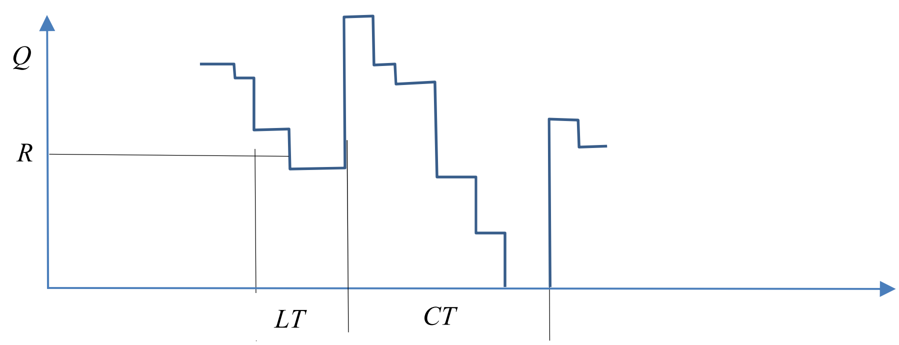
We may encounter some inventory terms in supply chain context. You may find the same term called differently by different group or text. The following defines what the intended meanings in this class.

The inventory level over time may look like the figure below. Each entry can be given or a decision. Each entry can be deterministic or probabilistic.

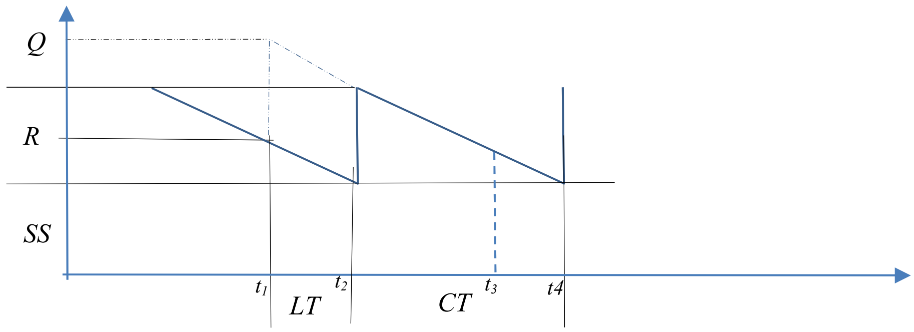
- Inventory Review time: It is time interval inventory level is reviewed to determine it is time to add, and if yes, how much. In modern systems with warehousing management system (WMS), the inventory level is updated whenever a bar code (or RFID) is scanned. This make it almost real time. In traditional systems however, inventory are updated only at certain point when the inventory are tallied, rather infrequently, say, every week.
- Inventory Reorder time: The time an order is placed. Inventory control policies are often divided into continuous review and periodic review policies.

- Replenishment: It is the activity to restock the inventory. There can be delay between the ordering and restock.
- Order leadtime (*LT*). It is the time from when the order is placed to when the goods arrived and replenished. In developed economies, this quantity is rather certain.
- Inventory cycle time (*CT*): It is the average time between replenishment.
- Production or order quantity Q: This can be a fixed amount such as a unit load: a truckload, a container, a pallet. This can also be a variable depending on the current inventory level and the target level to stock up to.
- Reorder point R: It is an inventory level when the inventory is reduced below this point, an order is placed.
- Window of uncertainty (*W*): The duration of uncertainty is important to find the safety stocks, $W = LT + CT$, more later.
- Order-up-to level: It is the expected level of inventory to be reached when an order is placed.
- Safety stock (*SS*): It is the expected inventory level right before the restock.
- Cycle stock (*CS*): It is the average inventory level that cycles between the minimum and maximum inventory.
- Stock out: It is when the inventory is zero. During stock out, there may or may not be orders. If there is order, and the restock comes soon enough to still satisfy the order by some exception handling, such as express service.
- Back order: an order if filled up restock, it can be late reaching the customer and incurs a penalty. There can also be a cost of loss of good will.
- Lost sales: the customer may not want the product or worse take the business to others, such as a competitor.

In supply chain, orders are often in various lumpy discrete quantities. Graphically, the actual inventory can take the form of



In modeling, the inventory depletion is simplified to a descending continuous straight line. The model with parameters are shown in the figure below.



What is the window of uncertainty? Refer to the figure above. At time t_1, the inventory level drops to reorder point. An order will be placed. The order will arrive at t_2 after leadtime. Another order will be placed at t_3. However, it will arrive at t_4 that cannot cover the uncertainty exposed from t_1 to t_4. Therefore, the window of uncertainty is

$$W = LT + CT$$

In inventory literature of or books, there is a set of policies called "periodic review". The inventory review time referred to the interval that the inventory is reviewed. In modern system, the inventory level is updated almost in realtime: whenever an item is touched or moved. However, the decision to restock or replenishment are must less frequent or periodic. Therefore, you should consider "periodic review" policy as periodic restock or replenishment policy, which are very prevalent in modern supply chain.

Example: A client orders goods from a supplier every Friday after the business closes. Therefore, $CT = 1$ week. The supplier has the goods in stock and the goods will be delivered to the client first thing Monday morning. Since there is no business from the Friday night to Monday morning, the order leadtime L can be considered zero, or $L = 0$. The total window of uncertainty is $CT + L = 1 + 0 = 1$ week, or 1 week that follows the delivery.

On the other hand, if it takes the supplier a week to deliver, or $L = 1$, then the total window of uncertainty is $CT + L = 1 + 1 = 2$ weeks, the week the order is placed plus the week after the order arrives or before next order arrives.

3.3 Newsvendor problem with repeated and constant order cycles

In the above illustration, if the replenishment interval is a constant, it will be the order cycle time. The fixed order cycle is quite common in supply chain, such as weekly order, or monthly delivery. A car assembly plant takes delivery of certain parts every week, some every day, still others every 2 hours. If you order in a fixed cycle, say every 4 weeks, the safety stock can be determined by newsvendor problem. The average order quantity in a stable fixed order cycle should be near EOQ. Once the cycle time is determined, the average cycle stock is determined. For example, if the cycle length is T weeks, and the demand rate is D per week. If the $T = 2$, $D = 100$, the cycle stock should be 2 * 100 = 200.

The main difference between durable goods and perishable goods is in the modeling of the overage cost. When the durable goods are sold in future cycles at the same price, the cost of overage will be the cost of holding safety stock. We do not account the holding cost of the cycle stock because it is a constant after the cycle length is fixed. Therefore, c_o in this case will be holding cost. Or,

$$c_o = h$$

Note that the dimension of overage cost is cost per unit per decision cycle while holding cost is cost per unit per unit time. For example, the holding cost is per week while the order cycle is 3 weeks. You need to find the holding cost per decision cycle, which is 3 weeks. The underage cost can come from the lost of profit, or the difference between retail price p_r and wholesale price p_w. The underage cost can also include the cost of lost of good will cost c_g because the customer may not come back next time, and lead to long term losses. Therefore, the critical ratio for the durable goods in constant ordering cycles is

$$F(Q^*) = \frac{c_u}{c_u + c_o} = \frac{c_u}{c_u + h} = \frac{p_r - p_w + c_g}{p_r - p_w + c_g + h}$$

$$Q = \mu + \sigma z$$

Typically, the holding cost per unit per unit time is lower compared to other cost items. As a result, the critical ratio tends to be higher than 0.5. If the demand distribution is symmetric, the order quantity would be much higher than the expected demand. Note, the resultant Q include both cycle stock and cycle stock. In each ordering cycle, the expected order quantity should be μ, not Q. Or, you will violate material conservation. The expected safety stock is σz. In practice, this can be implemented as a reorder point system if it is convenient for fixed order quantity, such as a pallet. Whenever the stock level drops to σz, place an fixed order quantity. Since there is delay between the time of ordering and time of arrival, additional consideration must be observed. Another way to implement is to consider Q as order up to level. This is applicable if the delivery is in a fixed sequence.

3.4 The cost of variability in newsvendor model

We can find the expected cost at the order quantity Q for each cycle

$$C(Q) = c_o Q F(Q) - c_o \int_{-\infty}^{Q} x f(x)dx + c_u \int_{Q}^{\infty} x f(x)dx - c_u Q\left(1 - F(Q)\right)$$

If we can solve $\int_{-\infty}^{Q} x f(x)dx$, we can find the cost in closed form. If demand is normal, we can express it in the standard normal form

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}=\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$$

Let $x=\mu+\sigma z$, $dx=\sigma dz$, when $x=-\infty$, $z=-\infty$, when $x=Q$, $z=\frac{Q-\mu}{\sigma}$. Then,

$$\begin{aligned}\int_{-\infty}^{Q} xf(x)dx &= \int_{-\infty}^{\frac{Q-\mu}{\sigma}} (\mu+\sigma z)\frac{1}{\sigma}\phi(z)\sigma dz \\ &= \mu\Phi\left(\frac{Q-\mu}{\sigma}\right)+\sigma\int_{-\infty}^{\frac{Q-\mu}{\sigma}} z\phi(z)dz \\ &= \mu F(Q)-\sigma\phi\left(\frac{Q-\mu}{\sigma}\right) \\ &= \mu F(Q)-\sigma\phi(z)\end{aligned}$$

Similarly,

$$\int_{Q}^{\infty} xf(x)dx=\int_{-\infty}^{\infty} xf(x)dx-\int_{-\infty}^{Q} xf(x)dx=\mu-\left(\mu F(Q)-\sigma\phi(z)\right)$$

Then, the total cost per order cycle will be

$$\begin{aligned}C(Q)&=c_oQF(Q)-c_o\mu F(Q)+c_o\sigma\phi(z)+c_u\mu-c_u\mu F(Q)+c_u\sigma\phi(z)-c_uQ(1-F(Q)) \\ &=c_o[(Q-\mu)F(Q)+\sigma\phi(z)]+c_u[(\mu-Q)(1-F(Q))+\sigma\phi(z)]\end{aligned}$$

The quantity in bracket after c_o is the expected number of parts leftover, and the after c_u is the expected number of parts short.

Note, Q in fixed order cycles should be considered the order-up-to level. For example, if Q is 100, the average demand in an ordering cycle is 70, the safety stock will be 30. Initial order can be 100. In the 2nd cycle, the actual order quantity should be 100 minus the sum of on-hand inventory and pipeline inventory.

Example: A dealer orders and sells windows. The retail price is \$300 per window. The wholesale price the dealer pays the manufacturer is \$200 per window. The unsold units by the late fall will be sold to the secondary market at \$60 per window. The dealer estimates the demand in the year can be approximated by a normal distribution $N(300, 100^2)$ per year. The production cost per window is \$120. The setup cost for each order is \$1000.

1. What is the quantity that the dealer should order from the manufacturer?
This is model for perishable goods.

$$c_o = cost\ of\ left\ over\ per\ unit = 200 - 60 = 140$$

$c_u = cost\ of\ lost\ profit\ per\ unit = 300 - 200 = 100$
$F(Q^*) = 100/(100+140) = 0.4167$
$z = \Phi^{-1}(0.417) = -\Phi^{-1}(1-0.417) = -0.21$
$\phi(-0.21) = \phi(0.21) = 0.39$
$Q^* = 300 - 100 * 0.21 = 279$

2. What is the cost per order cycle due to variability?

$E(unsold) = (279-300) * 0.417 + 100 * 0.39 = 30.24$
$E(number\ short) = (300-279)(1-0.417) + 100 * 0.39 = 51.24$
$C(279) = 140 * 30.24 + 100 * 51.24 = 4{,}237 + 5{,}124 = \$9{,}361$

3. What does this cost mean? If there is no variability, what is the profit for the dealer?

If there is no variability, or $\sigma = 0$, the profit $(300 - 200)\ D = 100 * 300 = 30{,}000$. With the variability, we can verify the profit will be ~ (30,000 – 9,361). This is an approximate due to rounding.

This is an important concept. The optimum value only minimize the cost for given uncertainty. You can also work to reduce the uncertainty itself, which can lead to higher impact!

4. What will be the cost of uncertainty for the manufacturer in this case?

Zero because the dealer orders from the manufacturer. Manufacturer will only make what is ordered, and therefore, takes no risk. Later in the Contract section, we will study how to share the risk to benefit both partners in the supply chain more.

Example: If the demand for a commodity part is $N(1000, 300^2)$ per week. The firm orders every 4 weeks (or one month) in the night before delivery. The retail price is \$100, the wholesale price is \$65. The stock out is lost sales. The holding cost is 12% of the value of the part per year. The loss of good will cost is \$1 per unit per occurrence.

Solution:
This is a repeated cycle problem.
$c_o = h = 65 * (0.12/52)*4 = 0.6$, $c_u = 100 - 65+5 = 40$.
$F(Q^*) = \frac{40}{40+0.6} = 0.985$. From the normal table, $z = 2.17$. $\phi(2.17) = 0.038$.
Window of uncertainty: 0+4 = 4. The average demand in 4 weeks is 4,000. The variance in 4 weeks will be $4*300^2$, or 600^2.

$Q = 4000 + 600 * 2.17 = 5{,}302$. This should be considered order-up-to level. The safety stock is 1,302. The cost of uncertainty per order cycle is

$C(5302) = 0.6[(5302 - 4000)0.985 + 600 * 0.038] + 40[(4000 - 5302)(1 - 0.985) + 600 * 0.038]$

$C(5302) = 0.6[(1302)0.985 + 22.8] + 40[(-1302)(0.015) + 22.8] = 0.6[1305.3] + 40[3.27] = 783.18 + 130.80 = 913.98$

The expected number of parts not sold before next order arrives is 1305.3. The expected number of parts short is 3.27. $913.98 or about $914 is the cost of uncertainty. If there is no demand variation, you would order 4,000. There is not overage or underage cost. The total profit 4000*(100-65) = $140,000. With variability, the expected overage cost is 783.18, underage cost is 130.80. The total cost due to variability is 913.98. This cost of variability is not very high since the goods can still be sold at the same price later. If the variability is higher, say VAR = 1500^2, the cost will be much higher. If the variance is significantly higher than 1,500, the distribution cannot be normal.

Here, Q should be considered order up to level. If you order Q in the first month, you may have left over, or stock out at the end of month. Since you have safety stock, on average, you should have 1,182 left. Let's assume that you have I_1 left in inventory, where I_1 can be 0 (or negative based on the model). In the second cycle, you should enough to bring the stock up to Q, In another word, you should order Q - I_1. That is way Q should be considered "order up to" level. In long run, your expected order quantity will be μ, which is your cycle stock. The expected end of cycle inventory should be the safety stock, or Q - μ.

It can cost a lot if the leftover has to go to salvage. We will discuss this later in the contract.

3.5 Newsvendor model for perishable or one-time order products

For perishable goods or one-time ordering, if the purchasing price is p_w, the salvage value is p_s and the retail price is p_r, we have

$$c_u = p_r - p_w$$
$$c_o = p_w - p_s$$

In reality, a potential customer may go elsewhere in the future if you stock out. There is also a loss of good cost c_g, then,

Example: A tool dealer orders tools from a manufacturer and then sell. The retail price is $1000. The whole sale price is $700. The unsold units are salvaged to a secondary market at $100. The demand is estimated to follow a normal distribution $N(150, 29^2)$. We will see this example again in contract when we calculate profit or try to maximize profit via contract.

$$c_o = \textit{cost of left over per unit} = 700 - 100 = 600$$
$$c_u = \textit{cost of lost profit per unit} = 1000 - 700 = 300$$
$$F(Q^*) = 300/(600 + 300) = 0.333$$
$$z = \Phi^{-1}(0.333) = -\Phi^{-1}(1 - 0.333) = -0.43$$

$$\phi(-0.43) = \phi(0.43) = 0.364$$
$$Q^* = 150 - 29 * 0.43 = \lceil 137.53 \rceil = 138$$

$$C(138) = 700(138-150) - 100[(-12)*0.333 + 29*0.364] + 1000[12*0.667 + 29*0.364]$$
$$= -8{,}400 - 656 + 18{,}560$$
$$= \$9{,}604$$

What does this cost mean?

Let's compute the profit if there is no variability. The demand is a constant 150. The vendor should order 150. Every tool will be sold at $1,000. The total profit is 150 * (1000 – 700) = $45,000. The actual profit, you can verify will be the difference between these two. This means that the cost of uncertainty, even when the vendor applied the "optimum" order quantity, is $9,604. If he orders a quantity different from 138, the cost will be even higher!

In this example, the fact that the vendor ordered less than the expected demand, he "saved" 8,400. He also expect to get 656 from salvage, also sort of savings. However, he lost a lot of sales because of the low order quantity. The cost of lost sales is $18,560.

3.6 The cost of demand lead time uncertainty

The duration of window of uncertainty itself can be uncertain. Below are some definitions and their relationship when the demand or time are subject to uncertainty. The uncertainty may or may not be normal. Here, we use σ to represent the square root of the variance which applies to any distribution.

- The demand in unit time and its variation $D \sim N(D, \sigma^2)$
- Leadtime variability or uncertainty $W \sim N(\mu_W, \sigma_W^2)$
- The demand in leadtime $D_W \sim N(\mu_W, \sigma_W^2)$,

Where the value of σ_L depends on the following:

$\sigma_W = \sqrt{\mu_W \sigma^2 + D^2 \sigma_W^2}$, if both demand and leadtime are uncertain

Example Please compare the demand variability during order leadtime or order cycle time in a continuous review (replenishment) inventory system in the following three cases. I picked these data so that one of them is constant, and the other have the same coefficient of variability, $\frac{\sigma}{\mu}$.

1. Leadtime is 2 weeks, constant. The demand $D \sim N(1000, 300^2)$.

$$\sigma_L = \sqrt{2} * 300 = 424$$

2. Leadtime $T \sim N(2, 0.6^2)$. The demand is constant at 1000.

   $\sigma_L = 1000 * 0.6 = 600 > 424$

3. Both demand and leadtime are uncertain.

   $\sigma_L = \sqrt{L\sigma^2 + \mu^2\sigma_T^2} = \sqrt{2 * 300^2 + 1000^2 * 0.6^2} = \sqrt{180{,}000 + 360{,}000} = 734.85$

Summary: The leadtime variability lead to higher demand variability during the leadtime than the demand variability.

Conclusion: the variation in lead time lead to higher cost than the demand variability.

LT \ Demand	$CV_D = 0$	$CV_D > 0$
0	0	0
Constant	0	H
$CV_{LT} > 0$	Higher	Highest

4. UNCERTAINTY IDENTIFICATION AND REDUCTION

Some industrial engineers take the uncertainty as given, and apply optimization, probability or statistics, stochastic process, simulation, etc. to find the solutions. An outstanding industrial engineer should not simply take the uncertainty as granted. Instead, should be actively identify the cause and find ways to reduce the variability. Here are some categories of cases one can consider.

Customer demand variation in product or service is a way of life. However, the variation can be both systematic variations and uncertain. For example, if the demand is seasonal, or influenced by scheduled events such as games, events, weather, you can use knowledge or information, or regression to find the deterministic components of the demand. The remaining random variation will have smaller variance of standard deviation (normal). The variation can also be due to events. When ISyE is organizing a large information session, they will order a lot of Pizzas. The demand is known well in advance. However, the Pizza joint does not know. In construction, the requirements for building materials, plumbing, windows, heating and air conditioning are know much earlier than when the orders are placed. As IEs, your job include this very critical part. Do not jump into the conclusion that the demand is probabilistic or an outlier of data is not important. Investigate. For example, when researchers plot skin cancer, they found the occurrences in Australia is much higher than other countries, they initially treat it as an outlier and took out the point. They later found it is real because of ozone hole there.