2. ECONOMIES OF SCALE

The Economies of Scale refers to the effect that the average cost of production decreases as the production quantity increase. This is also called scale effect, which is an important factor in the analysis, design and strategy in supply chain. Scale effect is very strong in many industries. It is part of the reason certain industry becomes bigger and bigger, and part of the motivation in mergers and acquisitions.

There is also Diseconomies of scale in which the average cost increases with quantity when the quantity reaches to certain level. Larger quantity may need more storage, overtime pay, cost of outsourcing, extra overhead in management, organization, transportation, etc. The firms in many industrial sectors stay small because of this. Examples can be industry for repair or customized services. The consolidation will require more management structure without additional cost benefits. However, the information technology, artificial intelligence may offer opportunities to change that.

Quantitatively, we can define the economies of scale with some simple definitions. Let

q	Production quantity
c	Variable cost per unit of production or marginal cost for adding another unit.
$TC(q)$	Total cost
$AC(q)$	Average cost
$MC(q)$	Marginal cost, unit incremental cost

With these definitions, we have

$$AC(q) = \frac{TC(q)}{q}$$

For the marginal cost in discrete space, we can consider adding the current unit or another unit. Let's use the latter. We have

$$\begin{aligned} MC(q) &= \frac{d(TC(q))}{dq} \\ &= \frac{TC(q+1) - TC(q)}{q+1-q} \\ &= TC(q+1) - TC(q) \end{aligned}$$

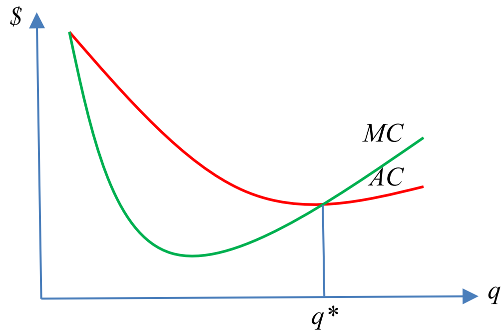


Figure 1 Average cost and marginal cost

At q^*, the $AC(q^*) = MC(q^*)$, and $\frac{dAC(q^*)}{dq} = 0$. When $q < q^*$, there is economies of scale. When $q > q^*$, there is diseconomies of scale.

2.1 Factors Contribute to the Scale Effect

In supply chain, the economies of scale can be attributed to certain factors. Many of these can be quantified.

1. Internal factors of scale effect
   a. Shared resources: higher production leads to lower shared resources such as investment cost per unit
   b. Technological: higher production leads to the use of mechanization, automation, IT, high throughput processes that leads to lower marginal cost or cost per unit.
   c. Batching: higher production leads to larger economic batches to reduce setup cost or setup time per unit.
   d. Structural: division of labor, such as lower cost in flow lines than in job shops,
   e. Shared overhead: marketing, management, facilities.
2. External Sources of scale effect
   a. Unit load: The cost of transportation and handling per unit decreases with the increase size of the unit load, such as cases vs. pallets vs. truckloads.
   b. Quantity discount: higher production requires more raw materials & services and enjoy quantity discount.

In supply chain, these factors can be quantified with the knowledge and tools you already have or will learn. We will discus 1.a, 1.b, 1.c. and 2.a. We will only provide some high level overview for 1.d, 1.e, and 2.b, although there is details on 2.b in this section. You should be able to perform the analysis in many of these without any help from formula sheets or similar.

2.2 Scale effect due to lower shared investment cost

A firm can invest in a building, facilities, a piece of equipment, a truck, such as a plant, a distribution center (DC), a truck fleet or a call center, a consulting firm. The initial investment cost is often significant. The firm can finance the initial cost in many ways. Regardless, let's make it simple and call its net present value of the investment cost *I*. The facility invested can be used to produce or provide value added services. The cost will be shared among all the products produced, or goods stored, handled and transported or service provided. If the unit cost of a product is *c*, the total cost will be

$$TC(q) = I + cq$$

The average cost is

$$AC(q) = \frac{I}{q} + c$$

The marginal cost is

$$MC(q) = \begin{cases} I + c & q = 0 \\ c & 0 < q < Capacity \end{cases}$$

You can also set this up on annual basis where *I* represents the annual cost of the investment, and *q* the annual production quantity.

In this very simple form, the average cost is a decreasing function of production quantity, or the scaling effect clearly as long as *q* is below capacity. The higher the supply, the more units will share the investment cost and less cost per unit. For example, if investment is made to acquire land, building and machinery to produce certain products or service, the more products and services provided, the less share of investment cost. In US, factories rarely run full capacity (24/7/365). In fact, many do not even run full in one shift. The added production, if there is demand, can help to accelerate the recovery of the capital. You can also see this from the perspective of marginal cost. The cost of an additional unit is the only incremental cost *c*, independent of the investment. Please also note, there is diminish cost reduction as *q* becomes large.

When the capacity is reached, there are many possible strategies such as over time, 2 shifts, 10-hour work days, open on Saturdays, outsource, etc. It the demand is ls only 10% above capacity, the firm can run overtime. Many firms pay 150%, or even 200% for overtime. If the demand is 25% above capacity, the firm can run 10-hour days with 4 working days. The worker can rotate to get 40 hour work days. If the demand is 80% above capacity and increasing, you can run 2 shifts and consider additional investment for long run. For all these cases, you will need to adjust the total cost function to find the average and marginal costs.

2.3 Economies of Scale with Respect to Technology

When the production quantity is low, one would invest with manual machinery with lower initial cost. However, the variable cost or marginal cost is often high. High production quantity can justify higher investment cost for mechanized systems or even computerized, automated processes with higher production rate and lower variable cost. You can consider the equipment used for printing, food, plastics, electronics, toy, furniture or other industries. Another factor is technology from flexibility perspective. When the production quantity is low but the each job is different, the job shop or process based system is most suitable. For example, a large restaurant. A system is divided into functional areas. The different types of food are processed and served. The system is flexible but not cost efficient. On the other hand, a factory that produces dinner meals are designed as flow lines with high degree of automation, high throughput and low marginal cost or incremental cost.

Let the investment cost be $I = \{I_0, I_1, I_2, ...\}$, and a unit production cost $c = \{c_0, c_1, c_2, ...\}$, corresponding to the investment of different systems. The total cost for each investment and incremental cost can be shown with a straight line in Figure 2 Total cost with different investment. At each quantity range, one of the investment options yields the minimum cost, and should be adopted, shown in the green dashed line, or

$$TC(q) = \min_i \{I_i + c_i(q)q\}$$

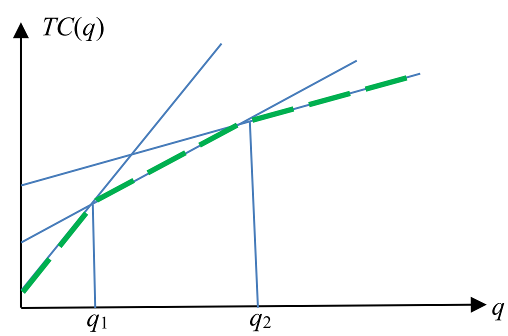


Figure 2 Total cost with different investment.

Example: A company can make a product with existing highly manual process at $100,000 per order. The capacity of the system is 10 orders per year. If the demand is 11 to 20 or more, the cost is $140,000 per order. If the demand is more than 20, the company cannot satisfy the demand. It can also invest $900,000 and cut down on the variable cost to $10,000 per order, up to 30 orders per year.

1. Please find the marginal cost and average cost as a function of the production quantity for the current system.
2. Repeat for the new system.
3. Please discuss what is the best strategy.

The company carries about same level of inventory regardless of the technology adopted.

F={0,900}, c={100,10}.

Assume that the current system is fully depreciated. Its investment cost is a sunk cost and therefore not relevant. The total cost is:

1. The average and marginal cost of the existing system are

$$TC_0(q) = \begin{cases} 100q, & if\ q \le 10 \\ 100 * 10 - 140(q - 10), & if 10 < q \le 20 \end{cases}$$

$$AC_0(q) = \begin{cases} 100, & if\ q \le 10 \\ 140 - \dfrac{400}{q}, & if\ q > 10 \end{cases}$$

$$MC_0(q) = \begin{cases} 100, & if\ q < 10 \\ 140, & if\ q = 10 \\ 140, & 10 < q < 20 \end{cases}$$

2. The total cost for the new system is

$$TC_1(q) = 900 + 10q, \quad if\ q \le 30$$

$$AC_1(q) = \frac{900}{q} + 10, \quad if\ q \le 30$$

$$MC_1(q) = \begin{cases} 900 + 10, & if\ q = 0 \\ 10, & if\ 0 < q < 30 \end{cases}$$

3. We can set the total cost of 2 systems equal to find the break even point.

$$100q = 900 + 10q$$

We have $q = 10$. When $q < 10$, the cost in current system is lower. If $q = 10$, both system yield same cost. If $q > 10$, invest new system will lead to lower cost. If the current production is near 10 with increasing tread, the investment may be a much better position because in addition to lower cost, it has higher potential to handle more orders.

2.3.1 KIVA robots example for Investment and Technology

Kiva Systems started in 2003 and developed robots to move shelves in the DCs. You can easily find videos on the internet how it works.

The ones shown in picture can carry up to 1000 lbs. They also can produce unit for pallet that can lift pallet at 3,000 lbs. Max velocity 1.3 meters/second. In comparison, walking speed of 5K/hour = 1.38 m/second.

Staples experimented the system in 2007, and planned to do more. However, Amazon bought Kiva Systems for $775 Million in 2012. Many considered Amazon used a "foreclosure" strategy to stem Staple's competition (and potentially others). It devastated Staples. Some viewed Amazon being anti-competitive. It took a while for Amazon to decide what to do. In 2015, Amazon changed the name of the division to Amazon Robotics, LLC.

Amazon fulfillment center design varies by generations. Started manual intensive, then mechanized, highly automated, then manual + IT, etc. An Amazon fulfillment center can carry millions of different SKUs. In June 2013, Amazon announced that it installed 1400 Kiva in 3 Fulfillment Centers. In the company filing to the local government, it claimed that it invest $45 mil in Ruskin, Florida, $26.1 in equipment. Assuming 500 robots in each center. It is on the order of $50,000 each robot. For such expenditure, it is important to get high utilization to justify its investment.

Amazon deployed the system in busy season in the 8th generation fulfillment Centers in 2014 with 15,000 KIVA robots in 10 US Fulfillment Centers. Amazon believed it can cut operation cost by 20%, and speed up order picking. Order picking of pieces is the most expensive operation in a DC. The manual picking is on the order of 10c per pick in efficient order picking systems. Let's use the following estimated values as an example.

We can compute the cost curves for manual system and KIVA systems. Let's assume that everything else is equal between the two systems. The company invested $45,000,000 extra for the KIVA systems. Then, for the manual system, *I* is 0, and *c* is 10¢ per pick. For the Kiva system, *I* is $45,000,000, and *c* is lower. Based on Amazon's claim, it leads to 20% savings. The incremental cost must be lower than 10¢ * 0.8 = 8¢ because of extra investment cost. Let's assume *c* = 5¢ in Kiva system per pick, or $50 per million picks. Then, for the manual system,

$$TC(q) = I + cq = 100q$$

The average cost is

$$AC(q) = 100$$

The marginal cost is

$$MC(q) = 100$$

For the robotic system,

$$TC(q) = 4500 + 50q$$

$$AC(q) = \frac{4500}{q} + 50$$

$$MC(q) = \begin{cases} 4500 + 50 & q = 0 \\ 50 & 0 < q \le Capacity \end{cases}$$

Which system is better? It depends on *q*. Let's find the breakeven number of picks by setting average cost equal between two technologies.

$$100 = \frac{45000}{q} + 50$$
$$q = 900 \text{ mil picks!}$$

In Amazon, this is a possibility over multiple years. However, operations such as Barnes and Noble cannot justify such investment. As a result, their operational cost will be higher from order picking perspective. Amazon has many other scale benefits over Barns and Nobel due to scale, such as bargaining power, transportation, etc. The comparison of average costs of two systems is shown in Figure

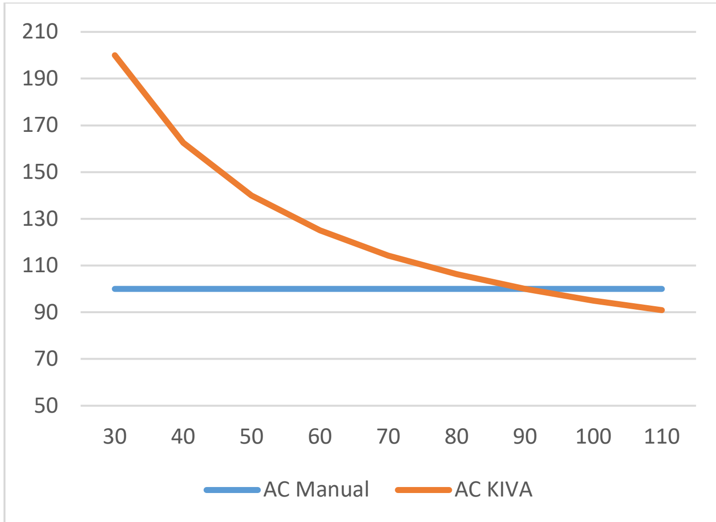


Figure 2. 3 Comparison of average cost between manual and KIVA order picking systems.

When does the diseconomies of scale emerge? It does not show in this simple relationship. For example, if orders picked from a DC leads to more transportation distance and more cost, the overall cost can increase. To quantify, more information is needed.

What if we measure this in Social and Environmental perspective? One of the objectives in macroeconomics is full employment. Robots replace people and lead to less employment. It is not necessary bad by itself because monotonous order picking does not provide fulfilling work experience. The robot carries a pot with many products from the storage area to the picker at the maximum speed of 1.3 meters/second. In comparison, walking speed of 5K/hour = 1.38 m/second, or basically similar to the speed a KIVA robot. Robot can consume significant amount of energy when carrying a pot with many products on. Each robot also has a lot of embedded energy: the energy used to excavate the materials, convert the materials, manufacturing, transporting, and assembly. On the other hand, walking is good for health, is not slower, and provide higher level of employment.

In practice, there are many other factors related to economies of scale. Let's look at a more complex example.

2.4 A Comprehensive Example for Car Assembly

A leading car maker is to build a new plant. Let's assume that the annual equivalent investment cost is \$400 mil. Let's consider only the initial investment and incremental labor cost c, and $2c$ in over time. Here, c is \$300 per car. For simplicity, the cost of parts, overhead, utilities, insurance, etc. are not considered here. If it runs one shift, it can produce up to 80,000 per year. If the production is higher than 80,000, it can add overtime with double pay. If the demand is approaching 160,000 per year, it can run 2 shifts. If the demand is between 80,000 and 160,000, say 100,000, it can ran 10 hour shifts with rotating schedule, or second shift on some days to avoid overtime pay. If the demand is approaching 240,000, it can run 3 shifts. It can add overtime if production is more than 240,000, until the maximum capacity of 320,000 if running 24/7/364. If the demand is higher than 320,000, it can retool the other plants, or outsource to other companies at an extra annual cost of $100 mil, plus $1,500 incremental cost per car. The total cost, the average cost and marginal cost are

$$TC(q) = \begin{cases} I + cq = 400{,}000 + 0.3q & q \le 80{,}000 \\ I + 2cq = 424{,}000 + 0.6(q - 80{,}000) & 80{,}000 < q < ? \\ \ldots & \\ I + cq = 400{,}000 + 0.3q & ? \le q \le 160{,}000 \\ \ldots & \\ I + 2cq = 472{,}000 + 0.6(q - 240{,}000) & 240{,}000 < q \le 320{,}000 \\ I + 100 + 1.5q = 620{,}000 + 1.5(q - 320{,}000) & 320{,}000 < q \le ? \end{cases}$$

Note I put a "?" between 80,000 and 160,000. This is because we do not have the cost of utilities, overhead, etc. associated with running the second shift. You should realize that that over time pay would work only if the demand is slightly higher than what can be done in the regular shift. Once the demand is significantly higher, it is better to adjust in other ways, such as 10 hour work days with rotating schedule such as each person work 4 days per week. If the demand is higher, a second shift can be added in some days, etc.

$$AC(q) = \begin{cases} \dfrac{400{,}000}{q} + 0.3 & q \le 80{,}000 \\ \dfrac{376{,}000}{q} + 0.6 & 80{,}000 < q < ? \\ \ldots & \\ \dfrac{400{,}000}{q} + 0.3 & ? < q < 160{,}000 \\ \ldots & \\ \dfrac{280{,}000}{q} + 0.6 & 240{,}000 < q \le 320{,}000 \\ \dfrac{140{,}000}{q} + 1.5 & 320{,}000 < q \le ? \end{cases}$$

The average cost is a decreasing function of q, until the excess production cost more in overtime pay or outsource pay. The marginal cost (not to include the discontinuity at the boundaries are

$$AC(q)=\begin{cases}0.3 & q\le 79{,}999\\ 0.6 & 80{,}000<q<?\\ \ldots & \\ 0.3 & ?<q<160{,}000\\ \ldots & \\ 0.6 & 240{,}000<q\le 320{,}000\\ 1.5 & 320{,}000<q\le ?\end{cases}$$

The marginal cost is a step function. The singularity point may not be important because it is unlikely the quantity is at that point.

Figure 2.2 depict the total cost, average cost and marginal cost of investment and labor.

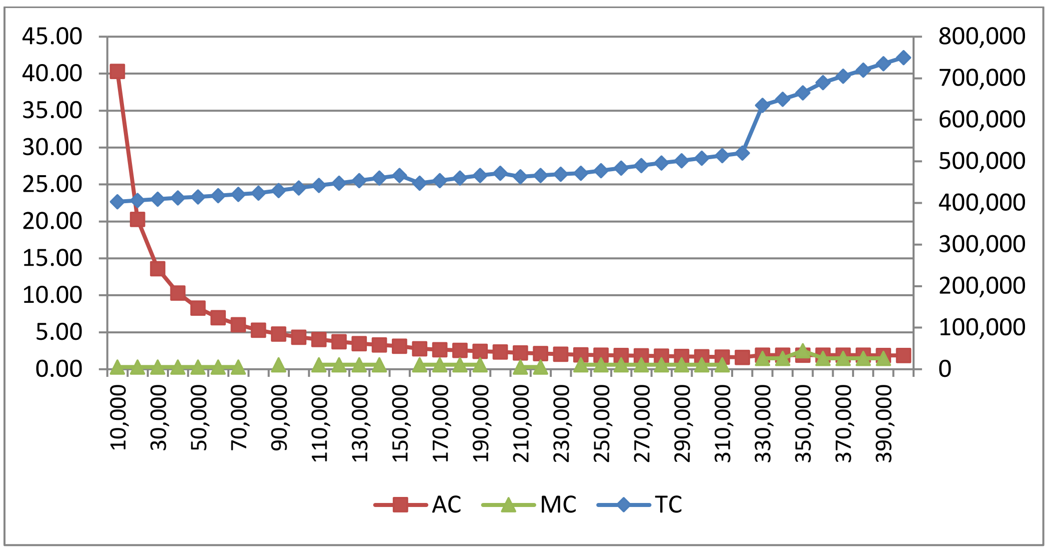


Figure 2. 4 Total cost, average cost and marginal cost of auto plant.

2.5 The Economies of Scale Related to Batching or Lot Sizing

Most productive systems produces multiple products. It is necessary to changeover between batches of different products. A change incur changeover cost. The larger the batch, the more product can share the changeover cost. The larger the overall production quantity normally lead to larger economic batch sizes. Therefore, the batching is directly related to the economies of scale, but not identical. The economies of scale is the cost implication of the total demand or annual demand. The batching is how to batch the total or annual productions into economic sizes.

People batch to save on something. When you need a bottle of water, you pick up more than one bottle to save another trip when you need water again. The trip incur setup time, effort and money if you drive with fuel, wear and tear of the automobile, and possibly parking. However, you probably do not buy a whole pallet of water, or the supply for the whole year. Buying too much set you back on your money reserve, takes up limited space, and probably difficult to transport. Further, you may run a risk that you may not be able to finish before you move. Business must make batching decisions.

On the same trip, you may also pick up tooth paste, a cup, peanut butter, or other things you may need. This is batching of different things to share the cost of the trip. Batching can take many forms and dimensions:

1. The batching of same item to reduce total cost of setups and inventories. Large batches lead to less change over costs but higher inventory cost. The economic batch size, or commonly known as economic order quantity (EOQ) minimizee the total cost due to setup and holding inventories. The first solution was published by Harris Ford in journal Factory in 1913 and reprinted in Operations Research in 1990. The details of this model is presented in next section on Economic Order Quantity (EOQ).
2. The batching of same items to balance between throughput and cycle time. In many operations, the holding cost is unimportant. For example, documents to be processed, the batch time is much shorter than a day. Large batches reduces times needed in setup so to increase the throughput. However, large batches increases batch processing time. If the jobs must visit many stations, such as patients, documents, the total cycle time can add up to very long time. Batching delay can also cause delays of bottleneck stations or reduce throughput of a bottleneck station. These are discussed in the Economies of Speed section.
3. The batching of different items to share the cost of handling or transportation
    a. Batching different inbound or outbound channels. Batching inbound channels, such as through a consolidation point, can enjoy economies of scale in transportation, receiving, information handling. Batching outbound channels can also enjoy economies of scale, unit load, order picking efficiency, etc.
    b. Batching over time to enjoy economies of scale in internal operations. You can relate to Amazon fulfillment center. Amazon does not batch express orders. Each order incurs high cost. It batch orders from Prime members to a few hours to reduce the cost. It batch orders from non-Prime members to days to further reduce the cost. The cost savings are from scale effect, work sharing, variation pooling (later), similar to grocery shopping of many items cost less per item.

4. Batching to unit loads. Totes, cartons, pallets and trucks are used in distribution centers and transportation. Each can be considered a "unit load". There is infrastructure for handling and transportation developed to fit human, forklift, road truck capacities. A load will need one transporter, one movement, regardless how much is held in the unit load. As a result, it is always desirable to fill up the unit load to reduce cost.

2.3.5 Economic Order Quantity (EOQ) for single product

There is batching problems in inventory systems that is unique and common in supply chain of goods. Let's first consider the single SKU. Here are some assumptions.

Assumptions

1. The products can be analyzed singly
2. The demand is continuous with constant rate
3. Replenishment is instantaneous
4. No shortage is allowed (or the shortage cost is dominate holding cost)

We can define the following:

K Setup cost or change over cost per setup or per changeover. This can be the transportation cost each trip, adjustment needed for a particular product, paper work needed to issue a purchasing order, etc. We assume that it is not a function of purchasing quantity.

h Holding cost per unit per unit time.

c The cost of each unit.

D Constant and continuous demand rate per unit time. The time unit can be per week, per month, or per year. This is linearly related to production or demand quantity q used in economies of scale analysis.

Q Order quantity each time, a decision variable. Here, we used a capital Q to differentiate from a lower case q earlier for the total production quantity per unit time or for the life of a facility. In inventory problems, increasing Q will lead to less frequent production, less change over but higher inventories. The total production is the same.

We further assume that K and h and c cannot be function of Q. Many of these assumptions can be relaxed to accommodate more complex problems.

Under all the assumptions, the inventory level over time can be shown in Figure 6

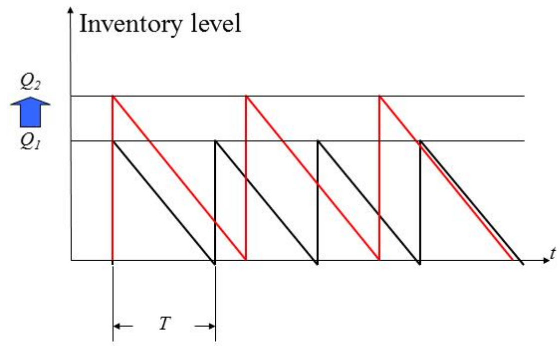


Figure 5 Inventory level changes over time at different order quantities.

The slope is the demand rate. The lowest inventory level is zero because no shortage is allowed, and above zero will lead to higher inventory cost. The highest iventory level is reached just after replenishment. The objective is to minimize the total cost per unit time, which can be written as the sum of setup cost, holding cost and purchasing price per unit time as

$$\min C(Q) \; = K\frac{D}{Q} + h\frac{Q}{2} + Dc \quad \$ / \text{time}$$

The first term is the setup cost times order frequency per unit time. For example, if you buy 2 times in a year, you would incur 2 setups in a year. The second term is the holding cost. It is the holding cost times half of order quantity Q, or the average inventory level. The more in each order, the more inventory is to be carried. The last term is the prices you pay for the goods. The last term is normally the highest in the total cost. However, it is independent of order quantity as long as the cost is not a function of order quantity (excluding quantity discount).

The decision variable is the order quantity. Only the first two terms are influenced by the order quantity, and can be considered inventory related cost.

There is economies of scale due to the fixed ordering cost. Each time you order, there is a quantity independent transaction cost, such as a trip, station adjustment, paper work, etc. The more you order, more pieces will share this ordering cost and enjoy the economies of scale and therefore lower the cost per unit. However, the more you order, the more you have to hold on. The units you hold on requires capital, occupy space and subjects to risk (loss, damage, price drop, obsolescence, etc.). These are shown in Figure 2. 6 The trends in inventory related cos as a function *of* Q..

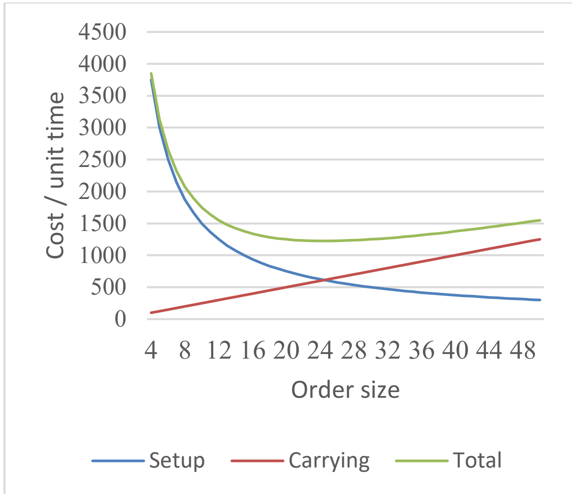


Figure 2. 6 The trends in inventory related cos as a function of Q..

If you apply first order condition to the total cost per unit time model above, you can find the optimum order quantity, commonly refer to as Economic Order quantity (EOQ),

$$Q^* = EOQ = \sqrt{\frac{2KD}{h}}$$

In industry, a more useful measure of the quantity is how long does the inventory last from each order, or inventory cycle time T, or in industry, people call days of supply or DOS.

$$T^* = \sqrt{\frac{2K}{Dh}}$$

Example. The demand in an auto repair shop for a popular filter is 200 per month. The ordering cost is $100 per order, regardless of order size. The cost of a filter is $4. The shop set holding cost at 12% per year. Here, h = 4 * 0.01 = 0.04/month. What are *EOQ*, the order cycle time and cost at EOQ?

$$EOQ = \sqrt{\frac{2*100*200}{0.04}} = 1{,}000$$

$$T^* = \frac{EOQ}{D} = \frac{1000}{200} = 5 \text{ months/order}$$

$$C(1000) = 100\frac{200}{1000} + 0.04\frac{1000}{2} + 200*4 = 20 + 20 + 800 = 840$$

The cost due to setup and carrying is $20 per month, respectively. The cost of buying for the items are $800 per month, much higher than the cost due to setup and carrying. The breakdwons at the different quantities are in the table below.

	Setup	Holding	purchasing	Total
200	100.0	4	800	104.0
400	50.0	8	800	58.0
600	33.3	12	800	45.3
800	25.0	16	800	41.0
1000	20.0	20	800	40.0
1200	16.7	24	800	40.7
1400	14.3	28	800	42.3
1600	12.5	32	800	44.5
1800	11.1	36	800	47.1

The Characteristics of *EOQ*:

1. Proportional to the square root of the demand. Therefore, batching is related to the square root of economies of scale.
2. Proportional to the square root of the setup cost. If you can reduce the setup cost, you can reduce the inventory level, in a lesser way.
3. Proportional to the inverse of the square root of holding cost. Does this mean a higher holding cost is better? You can jack up holding cost by using fancier facilities. How can that make sense?
4. The total cost function is not sensitive to the order quantity near the *EOQ*. See Figure 2. 6 The trends in inventory related cos as a function of Q..
5. At the optimum, the total setup cost is the same as total holding cost
6. The annual purchasing cost *Dc* is constant. It is typically much higher than the setup cost plus the holding cost. This is not true if cost is a function of *Q*, such as in quantity discount.
7. Buyers of often face challenges that *EOQ* < minimum order quantity (*MOQ*). *MOQ* is often vender's *EOQ*, or some convenient quantity based on historical reasons. You should working with venders, or expand your market to increase your own *EOQ*.
8. In reality, the strict assumptions of *EOQ* rarely holds. However, *EOQ* is quite robust, meaning it provides excellent guidelines on order quantities even when some assumptions are violated.

Lean has many dimensions. A very important one is directly linked to *EOQ*. If *EOQ* is high, the system will carry a lot of inventories, work in process or finished goods, and by definition, not lean. What can you do to reduce *EOQ*? You can reduce *K*, *D*, or increase *h*. No one with the right mind would try to actually decrease demand or increase holding cost, although I have encountered students stated such solutions in test! However, there is something to consider about increasing the accounting of the holding cost. It is not about increasing the actual holding cost but about how to account holding cost accurately. Many companies use the discount rate, internal rate or return or other financial measures to account for holding cost. This can be much lower than what the holding cost should be. HP published a case study on inventory driven cost on electronics. It found that the traditional way of accounting holding cost only capture a small portion of the total inventory driven costs (Gallioni, Montgros, Slagmulder, Wassenhove and Wright, 2005). The other cost can be obsolescence, rebate, returns, price drop, handling, storage, shrinkage, and many others.

The most effective way to reduce *EOQ* drastically is through the reduction of setup cost. Toyota started Just-in-time (JIT) production. JIT demand small batch sizes. The first thing Toyota did to promote JIT was the setup reduction through an effort called Single Minute Exchange of Dies (SMED). It was related to the change of dies for metal stamping of car body panels. The panels are shaped with very large and heavy die sets. It would take several hours to change and adjust a new die set. The stamping process on the other hand is very fast, at a few second a piece. After the die change, one would naturally stamp many panels, and leads to a lot of inventory and WIP. The SMED effort lead to tremendous reduction in setup time, so was the batch sizes.

2.6 The Economies of Scale from the Demand Side

So far, we found that the scale effect in batch has a saddle point. We also concluded that it is absurd to reduce the demand to become more lean. We know that the inventory cost per unit time increases with the demand. I have heard consultant apologize to the client that the increase in demand will increase inventory cost, which can be shown in the cost function. However, is it a good measure? Would inventory cost per unit per unit time be more meaningful?

You can verify that the total cost at *EOQ* by substituting it in the cost equation, or

$$C\left(EOQ\right)=\sqrt{2KDh}+Dc$$

To convert this quantity to per unit per unit time, we can divide this by the demand per unit time *D*, the average inventory cost per unit per unit time is

$$AC\left(EOQ\right)=\frac{\sqrt{2KDh}+Dc}{D}=\sqrt{\frac{2Kh}{D}}+c$$

This is a decreasing function of the demand! Therefore, increasing demand not only increase the revenue, it reduces the average inventory cost at the same time.

In traditional industrial engineering or supply chain, the focus is only on cost reduction. From broader economic perspective, you should also consider expanding the demand.

2.3.2 Batching of different materials from in-bound, outbound or material handling

In material handling, once setup the positions, tools, work space, it is quicker to do repetitive work. The cost of setting up are shared by all pieces to be handled, to reduce cost per piece. This can be true in document handling, handling and paper work in receiving or shipping. For example, to receive pallets delivered by a truck, a person would communicate with the delivery personal or find the delivery information, such as Advanced Shipping Notification (ASN), get the loading dock and fork lift ready, open the trailer, etc. The unloading of pallet will ensure. Upon completion, the setup must be shut down in preparation for future delivery or shipping. If the cost of all the setup and shut down is K to unload n pallets and the cost of actual unloading of each pallet is c, the cost for receiving each pallet is $c_{each} = c + \frac{K}{N}$.

In transportation, a person, a fork lift, a truck can handle or transport certain amount of materials. Often, there is not enough material to make up the full load. In such cases, the more one can load into the transporter, the more economies of scale is to enjoy.

Many companies outsource transportation services to logistics providers. The inbound goods can come from different sources. If the company work with the logistics provider to consolidate the delivery from different sources, more shipment will share the fixed cost. An example you can relate to. When you order multiple items from Amazon, different items may be stored in different fulfillment centers. The Amazon will first consolidate the items. After consolidation, the items will be transported (and handled) together. All the items will share the cost, especially in the most expensive last-mail.

The above apply to outbound shipment also.

2.3.3 Batching customer orders over time in delivery and order picking

Consider a UPS driver who delivers packages in a zone. Assume there are 240 deliveries in a day, or 9 hours, in this zone. Assuming the route in the zone is 90 miles. If the delivery window is one hour, there are 30 deliveries in 90 miles on average. The average distance travelled, which is not value added function, is 3 miles between two deliveries. This may be necessary for express orders. On the other hand, if the delivery window is one day, there will be 240 deliveries. The average travel between deliveries is only 0.375 miles. It is similar inside distribution centers, one of the effective ways to reduce cost is to batch orders or batch materials over time. We already mentioned example in Amazon. The longer the window of the batching, the lower the cost per pick. An Amazon fulfillment DC with 1000 people may pick 15,000 orders in an hour with 1.5 lines and 2.5 items in each order. The biggest waste in order picking is the travel by the pickers. The travel is not a value added activity. If the batch is large, the pick density can be increased. The picked items can also be batched in material handling: the cost of walking with 1 item vs. walking with 100 items in a push card is about the same, but with much lower cost per item.

Netflix figured out the extremely low cost of delivery and return of DVDs via US Postal Services (USPS). USPS makes stops at every address nearly every day, with special designed truck that the driver does not have to come off. The marginal cost to add a DVD at each address is very low. Netflix knows that, and get great deal. This is one of the major factors that make Netflix so competitive. However, they now have to compete with streaming services with even lower marginal cost. You can link to your own experience also. Assume you go to a grocery store to get water or another item each day. Let's assume the trip takes 15 minutes. You can batch the orders into per 5 days. The trip may be 20 minutes for 5 items. The unit time is now only 4 minute each. However, for Amazon, the revenue generated from general order is less than from Prime order, even less than express order, which leads to lower revenue and possibly profit per unit.

Wal-Mart batch their orders from the DC to their stores into one to a couple of days per delivery. Their high volume sales allows extremely high order picking efficiency or low cost. I

2.3.2 Batching into unit load

Many operations are based on unit load. A unit size is designed for the capabilities of an operator, a machine, or a transporter. For example, a shipping carton or a tote are designed so that an average person can pickup or move without difficulty. A pallet is designed for fast pickup, move and position by various types of forklifts. A sea container is designed to be loaded and secured rapidly in container ships, the trucks and train carts. Large industries are developed to support different types of unit loads in terms of packing, storage, moving, transporting, protecting, etc. Therefore, it is often efficient to work with standard unit loads. There are international (ISO containers, IOS pallets), national (tractor and trailers) and industrial (cages and totes for car parts, totes for grocery) standards defined and followed. The use of unit load can avoid exception handling and reduce cost.

In batch process in manufacturing, such as baking food, reacting chemical agents in a tank, a batch of jobs would start and end at the same time regardless of amount in the batch. Therefore, it is most efficient to process full batches. The condition is that the there is sufficient demand to justify the full batch. If the demand is low, large batches lead to high inventory carrying cost, and possible obsolescence.

You can find examples in office work, in health systems, or many other systems. Therefore, a system with production less than unit load will lead to higher cost due to wasted capacity. In transportation, some call this shipping air.

A case, or a shipping carton or a tote is the smallest unit load in supply chain logistics. It should have the size and weight that an average person can handled manually. They provide volume, convenience, and protection in moving and storage for a set of same or different items. Without them, a person may only handle limited amount of loose materials, such as vegetables, cups, parts, tools, water, etc.

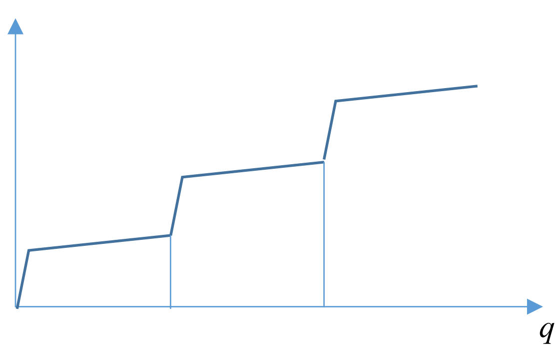
A pallet or a skid, a crate, a wire basket is the next size up in supply chain. Some types of forklift can be used to pick up, deposit and move one or more pallets quickly. There is ISO, US or industrial standard that defines their dimensions and weights. The pallet does not define the height of stacking of goods on the pallet. The height can be an important design parameter for a particular supply chain.

Container is the next size up (or a truck). There are ISO standard containers (20', or TEU and 40' of 8' or 9.5' high). There is entire infrastructure on ocean liners, train carts and truck trailers and chassis designed to handle these containers. Regardless of amount of materials in a container, it is handled and charged as a container. Similarly, there is trucks. A truck will require a tractor and a driver no matter how much goods are in the truck.

Related to containers is the intermodal containers or trailers in US. These containers or trailers have the same handling interface as standard sea containers so that the container cranes, forklift, etc. They are often longer at 53' or 48', some with the wheels others without.

A truckload is also a unit load. In US, the popular lengths are 53', 48'. There are also 28' pubs a tractor can haul 2 at a time.

In unit handling, regardless of if a case, a pallet, a container and a truck is full, it will be handled and charged as a unit load. A forklift can move a pallet weather it is full or 30% full. The cost difference is small. This is the same for case and truck. A trailer with half load would need the same driver, same engine, pay the same toll, burn similar amount of fuel. Therefore, the total cost curve with unit load will be



The lowest cost quantity per unit are in full unit load. For example, a truckload costs $1,500 from origin to destination. It can take up to 20 tons for up to 30 pallets. A truck can be cubed out for bulky items or weight out for heavy loads. If you carry paper towels, 30% of the weight will need 100% volume. This is called cubed out. On the other hand, if you carry sheet metal, 8% of the volume will reach 100% weight capacity, or weight out. If you have less goods to transfer, you can consider a less than truck load (LTL) carrier. If you only have a few lbs to carry, you can use a package delivery company. Assume that LTL charges (after negotiation) $100 per pallet up to one ton each. If each pallet weighs 0.8 ton. The cost as a function of number of pallets q will be

$$TC(q) = \min \begin{cases} 100q \\ 1500\left\lceil \frac{q}{30} \right\rceil \end{cases}$$

You can find that the breakeven point is q = 15.

$$TC(q) = \begin{cases} 100q & 1 \le q \le 15 \\ 1500 & 15 \le q \le 30 \end{cases}$$

$$AC(q) = \begin{cases} 100 & 1 \le q \le 15 \\ \frac{1500}{q} & 15 \le q \le 30 \end{cases}$$

$$MC(q) = \begin{cases} 100 & 1 \le q \le 15 \\ 0 & 15 \le q < 30 \end{cases}$$

Beyond one truckload, you will figure out a way to maintain truckload by adjusting time window of batching, batching or consolidating from different channels.

The quantity discount is often associated to unit load. For example, Relay copy paper \$37.99 per case, \$1150/pallet of 40 cases, or \$28.75 per pallet. Amazon, Staples may buy by truckload. The cost will be even lower. Part of the reason is less handling and storage cost. For example, moving 40 cases of paper from A to B needs 40 trips. Moving a pallet of 40 cases need one trip, with the help of a forklift or a pallet truck (manual). Loading 1500 cases in a truck takes several hours while loading a truck with 30 pallet may take 20 minutes.

2.3.4 Diseconomies of batching

We already mentioned that large batch may tie capital in inventory with uncertainty and occupy space. However, when the time is within a day or a few hours, money may not be relevant. For example, order picking in an Amazon fulfillment center. When an order is batched by an hour, or 2 hours, or 4 hours or a day, the cost difference due to inventory is not important. The longer the window of batching, the slower the order fulfillment process. Slow picking can sway some customers away and shrink the market. More detailed discussion will be in the economies of speed section.

2.3.6 Batching delays

2.4 Quantity discount

It is common, especially in US, to enjoy quantity discount. There are many reasons that seller would provide quantity discount. It could be that it passes its cost savings to you. For example, most sellers would charge you much less if you buy by unit load such as a case, a pallet or a truck load. The quantities less than those unit load needs more handling in order picking, packaging or transportation per unit. A fork lift can handle thousands of items on a pallet in one trip. 20% of a truckload still need the same driver, the truck and all the overhead associated with driving the truck. Another reason can be to compete for the market share. Customers buy more of their products would buy less from others. It can also be used to smooth the demand to manage its inventories and production.

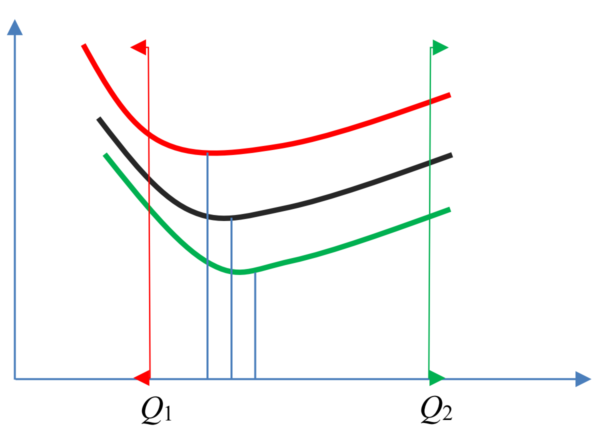
The quantity discount can take many forms. One of them, and probably the most common, is all unit quantity discount. It means that if you reach certain level, you will be a discounted price for all units you buy. For example, a 3-level pricing can be expressed as

$$c(Q)\ \$/\text{unit} = \begin{cases} c_0 & 1 \le Q < Q_1 \\ c_1 & Q_1 \le Q < Q_2 \\ c_2 & Q_2 \le Q \end{cases}$$

Here, cD is no longer a constant. You must include it in the inventory cost per unit time, or

$$C(Q)\ \$/\text{unit time} = \begin{cases} K_0 \dfrac{D}{Q} + h\dfrac{Q}{2} + c_0 D & 1 \le Q < Q_1 \\ K_1 \dfrac{D}{Q} + h\dfrac{Q}{2} + c_1 D & Q_1 \le Q < Q_2 \\ K_2 \dfrac{D}{Q} + h\dfrac{Q}{2} + c_2 D & Q_2 \le Q \end{cases}$$

Let's consider how does this work



The optimum solution for this piecewise convex functions will be among a finite set of possible solutions that include *EOQ*s and boundary point with lower prices. You can reduce the set by removing all *EOQ*s and boundaries below the first realizable *EOQ*. You then compare the cost at each of this finite of potential solutions. The one yields the lowest cost is the optimum solution. In the figure shown above, the red line is for the highest price for quantities less than Q_1. The black line is for the second price range. The green one is for the lowest price. Each has its ow *EOQ*. The red arrow line show the fissile quantity for the first price and green for the 3rd. In this particular case, the lowest price is at intersection of the green line with green vertical arrow, and that is the optimum point. If Q_2 is higher, then the *EOQ* at the 2nd price range yield the minimum cost.

The decreasing price per unit leads to the scale effect. However, there is a downside of purchasing a lot. You may have carry inventory. In addition that the fund in the inventory has poor liquidity, the holding cost money in storage, handling. The goods can be damaged, obsolete, devalued or lost. These are the diseconomies of scale not all considered in the model. Therefore, we should first look at the economies and diseconomies of scale in inventories.

Another impact of quantity discount is waste. People often buy more without actual need and lead to waste. For food, this can also lead to obesity, spoilage, nutrition reduction, etc.

Example: A distributor wanted to expand its portfolio to include an electronic device. He can buy by case (case), pallets, or truckload. A pallet can hold 20 cases, and a truck can hold 26 pallets. If the order quantity is less than a pallet, he would use a package delivery service. The setup cost is \$50 per order, the cost is \$100 per case, and the cost of delivery is \$10 per case. If the quantity is in pallets, the cost is \$1,800 per pallet, or \$90 per case. He would use a less than truck load service. The setup cost is \$100 plus delivery cost of \$60 per pallet delivered. If the quantity is over 10 pallets, he would use a special delivery. The price is \$82 per unit if over 200 units. The setup cost is \$800 per order, as long as it is less than a truckload of 26 pallets. The carrying cost for this product with short life cycle is 50% per year. The estimated demand is 300 per year. What should be best order quantity?

Solution: The potential solutions are at either the *EOQ*s or the boundary locations. There are some irregularities in this problem

$$C(Q)\ \$/\text{unit time} = \begin{cases} 50\dfrac{300}{Q} + 100 * 0.5\dfrac{Q}{2} + (100+10) * 300 & 1 \le Q < 20 \\ 100\dfrac{300}{Q} + 90 * 0.5\dfrac{Q}{2} + (90+3) * 300 & Q = 20,40,...200 \\ 800\dfrac{300}{Q} + 82 * 0.5\dfrac{Q}{2} + (82+0) * 300 & Q = 200,400,...5200 \end{cases}$$

The delivery cost can be either dependent or independent of the delivery quantity (truckloads). The equation only provided the purchasing cost, holding cost and setup cost. Where should the quantity dependent delivery cost go? The delivery cost is not related to actual holding of inventory, but it is related to the cost of capital. The cost of capital should be much lower than 50% per year, and therefore, we do not have to include it in holding cost. We set $h = c * 0.5$.

$$EOQ_{100} = \sqrt{\frac{2*50*300}{100*0.5}} = 24.5, \text{ a pallet is 20, you can order 24 using package delivery, may be costly}$$

$$EOQ_{90} = \sqrt{\frac{2*100*300}{90*0.5}} = 36.5, \text{ order 40 or 2 pallets}$$

$$\mathrm{EOQ}_{82} = \sqrt{\frac{2*800*300}{82*0.5}} = 108.2, \text{ minimum order is } 20*10 = 200.$$

The EOQ_{100} for cases is 24.5. We can round it to 24 or 25. However, at any quantity above 20, you could enjoy lower price. Therefore, this *EOQ* cannot yield low cost and should not be considered.

EOQ_{90} is 36.5, almost 2 pallets. Therefore, you should round it to the closest pallets, or $Q = 40$. The total cost is much lower than ordering 24 (or 25). You could try to round it down to $Q = 20$. Notice that total cost is only about 1% higher! Inventory cost is not sensitive as long as you are near the optimum! You cannot stop here because ordering by truckloads might yield even lower cost. *EOQ* at truckload prices is 108.2. This is not realizable because the seller would not sell you at the truckload price. You would have to buy at least 200 cases. The total cost is $29,900, about 1% higher than ordering 2 pallets at a time. Therefore, the best solution is Q = 40 or 2 pallets. On the average, you would need to order every 40/300 = 0.1333 year, or 1.6 months.

Table 2. 1 Inventory costs calculation for quantity discount.

	K	*c*	*h*	*EOQ*	*Q*	Setup	Carrying	Purchase	Variable delivery	Total
< 20	50	100	50	24.5	24	625	600	30000	3000	34225
20 - 200	100	90	45	36.5	40	750	900	27000	900	29550
20	100	90	45		20	1500	450	27000	900	29850
>= 200	800	82	41	108.2	200	1200	4100	24600	0	29900

The columns of Setup, Carrying and Purchase corresponds to the first, second and third term in the equation above. The plot of the cost as a function of purchasing quantity is shown in below.

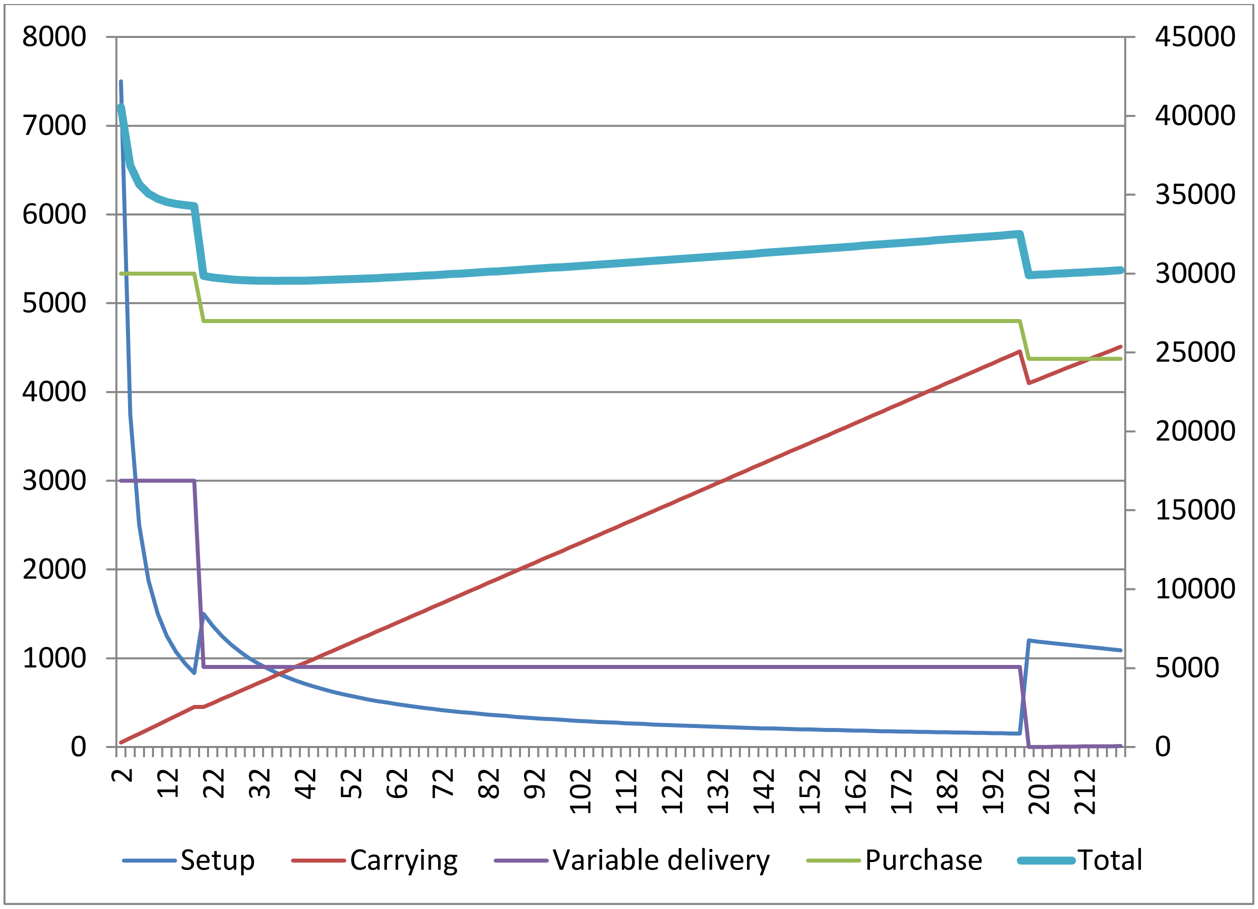


Figure 2. 7 The inventory related cost with quantity discount.

2.7 Summary of Economies of Scale

Economies of scale is a very strong economic force. A larger production quantity almost always incur lower cost per unit when the production capacity is sufficient. Even when the capacity is insufficient, and the marginal cost starts to trend up, the average cost may still be lower than its smaller scale counterpart may.

Batching is also kind of scale effect. In many situations, batching is an important mean for cost reduction. However, larger batches can also come with slower speed, longer cycle times, delays, and excess inventories. etc.

In economics, the economies of scale are often explained as a concept with illustrations. In supply chain, many important factors of the scale effect can be quantified explicitly with sufficient accuracy for decision-making. This section included some of these factors.

Exercises

1. What are the main forces of economies of scale?
2. Many industries consolidate from cottage to fragmented to loose oligopoly and to tight oligopoly. What are the driving forces behind the consolidation?
3. In a simple production system involving a large investment and fixed incremental cost with production capacity of *C*, how do you quantify the economies of scale?
4. What are the benefits of large batch sizes and small batch sizes?
5. What is the biggest hurdle in batch size reduction?
6. If everything else equal and you managed to increase demand, what will happen to the monthly inventory cost? How about average inventory cost per item?
7. What are the reasons for quantity discount?
8. In anticipation of wide adoption of 3-D printers, a firm is considering major investment to make 3-D printers. It narrows down the options to two types of production systems: A and B.

System	Annual Equivalent Investment $/year	Production cost per printer $/printer	Change over cost $/changeover	Product types	Demand per type Units/week
A	3,000,000	1,000	400	2	600
B	5,000,000	1,000	10	4	500

1.1. Please quantify the economies of scale considering only the investment, production cost and production quantity perspective.

$$\sum D_A = 2 * 600 = 1200/week \ = 62400/year$$

$$\sum D_B = 4 * 500 = 2000/week \ = 104000/year$$

$$AC_A = \frac{3,000,000}{62400} + 1000 = 1048.1/year$$

$$AC_B = \frac{5,000,000}{104000} + 1000 = 1048.1/year$$

1.2. (relevant to later sections) Assuming adjusted holding cost h' = \$0.1 per unit per week for all product types, what should be the common manufacturing cycle times and average batch sizes for each types of system (assuming sufficient production capacity for both systems)?

$$T^* = \sqrt{\frac{2\sum K}{\sum h'D}} = \sqrt{\frac{2*2*400}{2*0.1*600}} = \sqrt{\frac{2*400}{0.1*600}} = 3.65\,weeks,\ Q_1 = Q_2 = 2190$$

$$T^* = \sqrt{\frac{2\sum K}{\sum h'D}} = \sqrt{\frac{4*2*400}{4*0.1*600}} = \sqrt{\frac{2*10}{0.1*500}} = 0.63\,weeks,\ Q_1 = \cdots Q_4 = 315$$

1.3. (relevant to later sections) Which system would you select and why? You must consider all the relevant factors and parameters in the formulas in 4301.

B is much better than A even the average and marginal costs are similar. If we count the share of the setup cost, B will have slightly lower average cost also. Other advantages

- Economies of scale: smaller batches and smaller cycle stock or inventories. More total products, more revenue (not from cost view), better system utilization.
- Economies of scope: it can handle 2 more product types. In such market, the demand can change. The flexibility will allow it to adapt to other designs.
- Economies of speed: the lower setup cost leads to shorter LT and less SS (lower variability).

9. Please complete the car plant example in the notes. We will add the cost of parts and operating cost. The cost of parts and supplies are $12,000 per car up to 100,000 cars per year, and $11,000 per car above 100,000 cars per year. The operating cost is $80 mil per shift per year. You can use fractions of operating cost if applicable. Please use $1,000 as monetary units.

The answer below has been developed assuming that the operating cost can be pro-rated in direct proportion of q/80,000, and assuming that the range where it is sensible to pay overtime instead of changing the shift configuration is smaller than 20,000 cars. If you assumed that the operating cost is proportional to the length of the shift but not directly proportional to the quantity produced, then you should have had an operating cost of 100,000 per year for the times when you ran 10hr shifts; 160,000 for the intervals with two shifts; and so on. Also, you would have had to add additional ?<q<? interval definitions to the above piecewise linear function, in the 10hr shift domain…

$$TC(q) = \begin{cases} 400{,}000 + 0.3q + 12q + 80{,}000 \cdot \dfrac{q}{80{,}000} & q \le 80{,}000 \\ 424{,}000 + 0.6(q - 80{,}000) + 12q + 80{,}000 \cdot \dfrac{q}{80{,}000} & 80{,}000 < q \le ? \\ 400{,}000 + 0.3q + 12q + 80{,}000 \cdot \dfrac{q}{80{,}000} & ? < q \le 100{,}000 \\ 400{,}000 + 0.3q + 11q + 80{,}000 \cdot \dfrac{q}{80{,}000} & 100{,}000 < q \le 160{,}000 \\ 448000 + 0.6(q - 160{,}000) + 11q + 80{,}000 \cdot \dfrac{q}{80{,}000} & 160{,}000 < q \le ? \\ 400{,}000 + 0.3q + 11q + 80{,}000 \cdot \dfrac{q}{80{,}000} & ? < q \le 240{,}000 \\ 472{,}000 + 0.6(q - 240{,}000) + 11q + 80{,}000 \cdot \dfrac{q}{80{,}000} & 240{,}000 < q \le 320{,}000 \\ 520{,}000 + 100{,}000 + 1.5(q - 320{,}000) + 11q + 80{,}000 \cdot \dfrac{q}{80{,}000} & 320{,}000 < q \le ? \end{cases}$$

$$AC(q) = \begin{cases} \dfrac{400{,}000}{q} + 13.3 & q \le 80{,}000 \\ \dfrac{376{,}000}{q} + 13.6 & 80{,}000 < q < ? \\ \dfrac{400{,}000}{q} + 13.3 & ? < q < 100{,}000 \\ \dfrac{400{,}000}{q} + 12.3 & 100{,}000 < q < 160{,}000 \\ \dfrac{352{,}000}{q} + 12.6 & 160{,}000 < q < ? \\ \dfrac{400{,}000}{q} + 12.3 & ? < q < 240{,}000 \\ \dfrac{328{,}000}{q} + 12.6 & 240{,}000 < q \le 320{,}000 \\ \dfrac{140{,}000}{q} + 13.5 & 320{,}000 < q \le ? \end{cases}$$

- $$MC(q) = \begin{cases} 13.3 & q \le 80{,}000 \\ 13.6 & 80{,}000 < q < ? \\ 13.3 & ? < q < 100{,}000 \\ 12.3 & 100{,}000 < q < 160{,}000 \\ 12.6 & 160{,}000 < q < ? \\ 12.3 & ? < q < 240{,}000 \\ 12.6 & 240{,}000 < q \le 320{,}000 \\ 13.5 & 320{,}000 < q \le ? \end{cases}$$

10. A firm will start to make gears for a heavy equipment OEM. It can invest in a traditional gear machine at a cost of $20,000, or a computerized machine at $45,000. The operating cost per piece using the traditional machine is $50 per piece, the computerized machine at $35 per piece. The maximum capacity of the machines are 5,000. Beyond that, the company must buy another machine. The maximum market size is estimated to be 6,000. Please find total cost, average cost and marginal cost as a function of production quantity from 1 to 6,000.

    Total cost as a function of the production quantity, q, will be defined in multiple intervals. In the first one, for small values of q, production is less costly using traditional machinery. Then, if q is planned to be larger than a certain value, q_1, computerized machinery should be used, up until q=5000, when any machine reaches full capacity. After q=5000, the remaining 1,000 units should be produced in the most economical way, which is achieved, as we will show next, by using a traditional machine:

    To find the value of the cutoff point q* at which it becomes cheaper to use a computerized system, we solve the equation

    $$20{,}000 + 50 \cdot q = 45{,}000 + 35 \cdot q \;\Rightarrow\; q^* = 1666.66 \cong 1667$$

    Hence, if production is planned to be less than 1667 units, buying one traditional machine is better; if q is planned to be between 1667 and 5000, the computerized machine will be most economical; and if q is planned to be between 5000 and 6000, the computerized system should be used at full capacity and the remaining production should be carried out using traditional equipment.

    $$TC = \begin{cases} 20{,}000 + 50q & for\ 1 \le q < 1667 \\ 45{,}000 + 35q & for\ 1667 \le q \le 5000 \\ 65{,}000 + 35(5{,}000) + 50(q - 5{,}000) & for\ 5000 < q \le 6000 \end{cases}$$

$$AC = \begin{cases} \frac{20{,}000}{q} + 50 \ \ for\ 1 \le q < 1667 \\ \frac{45{,}000}{q} + 35\ for\ 1667 \le q \le 5000 \\ \frac{-10{,}000}{q} + 50\ for\ 5000 < q \le 6000 \end{cases}$$

$$MC = \begin{cases} 50\ for \quad\ \ 1 \le q < 1667 \\ 35\ \ for\ 1667 \le q \le 5000 \\ 50\ \ for\ 5000 < q \le 6000 \end{cases}$$

11. If the demand is 30 per week, the holding cost is \$2/unit/week, and the fixed (setup) cost is \$50 per setup, what is the economic order quantity?

$EOQ = \sqrt{\frac{2*50*30}{2}} = 38.7$, you can order 39 or 40 for an even number, or 4 packages of 10.

12. (3) In order to reduce batch sizes by 30%, by how much must the setup cost or setup time be cut?

$$EOQ_{New} = 0.7EOQ_{Old} = 0.7\sqrt{\frac{2K_{Old}D}{h}} = \sqrt{\frac{2K_{New}D}{h}}$$

$$K_{New} = 0.49K_{Old}\text{, or 51\% Cut}$$

13. A copy center orders laser printer toners from a specialty supplier. The price is \$100 per cartridge, \$950 per carton of 10 cartridges and \$9,000 for 10 cartons. The demand is 50 cartridges per year. The ordering cost is \$15. The carrying cost is 30% of the value per year.

    a. Please find the <u>sufficient and necessary</u> set of order quantities to be considered for finding the optimum solution. (Deduction will apply if you include quantities that should not be considered).

       EOQ_{100} = 7.07, less than a carton, round to 7.

       EOQ_{95} = 7.25, less than a carton, not realizable

       EOQ_{90} = 7.45, far less than 10 cartons, not realizable

       Necessary and sufficient set = (7, 10, 100)

    b. Find the optimum order quantity.

       G(7) = 5,212.14

G(10) = 4,967.50

G(100) = 5,857.50

$Q^* = 10$, or one carton each order

c. If the cartridge storage can only hold 85% of the optimum order quantity, what would you do?

One carton will not fit. The next best straight solution is 7, or EOQ at highest cost. However, if extra space can be made available at low cost (do you know how to calculate?), it can be an option also.

15. An electronics assembly plant uses a special type of connector. The prices at different quantities are given below. For less than 100, $3 each, for multiples of 100 and up to 2900, $2 each. For quantities of multiples of 3000 and above, the cost is $1.50.

15.1. What are the average costs per unit at different quantities based on price alone.

15.2. If the demand is 1,000 per month, the holding cost is 2% of the value of the connector per month, the setup cost in purchasing up to 100 is $100, the setup cost for purchasing from 100 to 2,900 is $300, and anything above is $1,000, please find the best order quantity (consider both prices and inventory cost).

15.3. If the demand is 10,000 per month, what is the best order quantity?

15.1 $$AC(q)=\begin{cases} 3 & q<100 \\ 2 & 100\le q\le 2900, \text{ or } q=100,200,...2900 \\ 1.5 & 3000\le q, \text{ or } q=3000,6000,9000... \end{cases}$$

15.2 $$Q=\begin{cases} \sqrt{\dfrac{2*100*1000}{3*0.02}}=1826>99, \text{ we can only use the boundary quantity } 99 \\ \sqrt{\dfrac{2*300*1000}{2*0.02}}=3873>2900, \text{ we can only use boundary quantity } 2900 \\ \sqrt{\dfrac{2*1000*1000}{1.5*0.02}}=8165, \text{ use } 9000, \text{ because it is closer to } 8165 \text{ than } 6000. \end{cases}$$

C(q)	K	EOQ	Q	Setup	Carring	Purchasing	Total
3	100	1826	99	1010	2.97	3000	4013
2	300	3873	2900	103	58	2000	2161
1.5	1000	8165	9000	111	135	1500	1746

15.3 EOQ = 25,820. You can use either 24,000 or 27,000

Total cost (Q*) = 370 + 405 + 15000 = 15,775!

The total cost is higher than 1,746 with demand of 1000 per month. Does this mean diseconomies of scale?

This higher cost is shared by 10 times of the demand per month. The average cost per item at 1000/month is 1.75/piece, with 1.50 for the purchasing and 0.25 for inventory related cost. The average cost per item at 10000/month is 1.58/piece, with 1.50 for purchasing and 0.08/piece for inventory related cost, or a savings of 0.17/piece. This is pretty significant savings, or economy of scale!

15. A central library uses 50 rolls of 1/2" * 18 yards double-sided foam tapes per year. You found Uline offers the rolls by 3M (model 4004), http://www.uline.com/ProductDetail.asp?model=S-10057. It is estimated that the ordering cost is $25 per order and the annual holding cost is 15% of the value. The pricing is summarized as follows

$$C(q) = \begin{cases} 35.25, & 2 \le q < 12 \\ 34.15, & 12 \le q < 24 \\ 31.95, & 24 \le q \end{cases}$$

15.1. What are the potential optimal order quantities?

TC(12) = 1898.39 (you do not need to calculate this, I hope you know why)

TC(22) = 1820.67

TC(24)=1707.09, order 24 each time approximately once in 6 months.

15.2. What is the optimum order quantity and its associated annual cost.

The best order quantity is 24, with cost of $1,707.09.

16. A company plans to build a solar powered charger it invented. The production requires an initial investment of either $100,000 or $150,000. The corresponding production costs would be $12 and $10 per unit, respectively.

16.1. Please find the total cost function with respect to the production quantity.

First, find the transition point: 100,000 + 12q = 150,000 + 10q, q = 25,000 (2 pts)

$$TC(q)=\begin{cases}0 & q=0 \quad \text{(no deductions)}\\ 100{,}000+12q & 1\le q<25{,}000 \ \ \text{(4 points)}\\ 150{,}000+10q & 25{,}000\le q \quad \text{(4 points)}\end{cases}$$

16.2. (6) Please find the marginal cost with respect to the production quantity.

$$MC(q)=\begin{cases}100{,}012 & q=0 \quad \text{(no points)}\\ 12 & 1\le q<24{,}999 \quad \text{(this and next line 4 points)}\\ 12 & q=24{,}999\,*\\ 10 & 25{,}000\le q\end{cases}$$

* requires different calculation.

16.3. Please find the average cost with respect to the production quantity.

$$AC(q)=\begin{cases}12+\dfrac{100{,}000}{q} & 1\le q<25{,}000\\ 10+\dfrac{150{,}000}{q} & 25{,}000\le q\end{cases}$$

16.4. * Please quantify the economies of scale of this problem mathematically.

Economies of scale: $AC(q)$ is a decreasing function, or $\dfrac{dAC(q)}{dq}<0$, in this case,

$\dfrac{dAC(q)}{dq}<-\dfrac{\text{Constan}}{q^2}$ is a decreasing function (you can show more …)

References

Harris Ford, "How Many Parts to Make at Once," Factory, in 1913, Vol 10, No 2. pp 135 – 136. It was reprinted in Operations Research, Vol 38, No 6, p 947 -.

Gallioni, G., Montgros, X, Slagmulder, R., Wassenhove, L., and Wright, L., "Inventory-Driven Cost," Harvard Business Review Case, March 2005.