## 1.1 Describing a set

**Exercise.** 1.1 Which of the following are sets?

$(d)\,\{1,\{2\},3\}$ [correct ✓]

**(e)** $\{1,2,a,b\}$ $[correct✓]$

**Exercise.** 1.3 Determine the cardinality of each of the following sets:

**(a)** A=$\{1,2,3,4,5\}$

$|A|=5$ [correct ✓]

**(b)** B=$\{0,2,4,...,20\}$

$|B|$ = 11 [correct ✓]

**(c)** C = $\{25,26,27...,75\}$

$|C|=$ 51 [correct ✓]

**(d)** D = $\{\{1,2\},\{1,2,3,4\}\}$

$|D|=$ 2 [correct ✓]

**(e)** E = $\{\emptyset\}$

$|E|=1$ [correct ✓]

**(f)** $F=\{2,\{2,3,4\}\}$

$|F|=2$ [correct ✓]

**Exercise.** 1.5

**(a)** A=$\{-1,-2,-3,\ldots\}$

$A=\{x=-y,y\in\mathbb{N}\}$ [correct ✓]

**(b)** B=$\{-3,-2,\ldots,3\}$

$B=\{x\in\mathbb{Z}:-3\leq x\leq 3\}$ [correct ✓]

**(c)** $C=\{-2,-1,1,2\}$

$C=\{x\in Z:-2\leq x\leq 2,x\neq 0\}$ [correct ✓]

**Exercise.** 1.7

**(a)** $A=\{\ldots,-4,-1,2,5,8,\ldots\}$

$A=\{3x-1:x\in\mathbb{Z}\}$ [correct ✓]

**(b)** $B=\{\ldots,-10,-5,0,5,10,\ldots\}$

$B = \{5x : x \in \mathbb{Z}\}$ [correct ✓]

**(c)** $C = \{1, 8, 27, 64, 125, \ldots\}$

$C = \left\{x^3 : x \in \mathbb{N}\right\}$ [correct ✓]

**Exercise.** 1.9

**For** $A = \{2, 3, 5, 7, 8, 10, 13\}$, let $B = \{x \in A : x = y + z, y \in A, z \in A\}$ and $C = \{r \in B : (r + s) \in B$ for some s∈B$\}$ . Determine C

$B = \{5, 7, 8, 10, 13\}$ [correct ✓]

$C = \{10, 13\}$ *[incorrect]* $C = \{5, 8\}$ *(I was looking for $r + s \in B$, should have been looking for r)*

## 1.2 Subsets

**Exercise.** 1.11

**Let** $(a, b)$ be an open interval of real numbers, and let $c \in (a, b)$. Describe an open interval I centered at c such that $I \subseteq (a, b)$ .

**Let** $d, e \in \mathbb{R} : a < d \leq c \leq e < b$ and $c - d = e - c$. Then $(c, e)$ describes an open interval I centered at c

**[correct** ✓]

**Exercise.** 1.13

**For** a universal set $U = \{1, 2, \ldots, 8\}$ and two sets $A = \{1, 3, 4, 7\}$and $B = \{4, 5, 8\}$draw a venn diagram that represents these sets (done on paper)

**Exercise.** 1.15

**Find** $\mathcal{P}(A)$ for $A = \{0, \{0\}\}$

$\mathcal{P}(A) = \{\emptyset, \{0\}, \{\{0\}\}, \{0, \{0\}\}\}$ [correct ✓]

**Exercise.** 1.17

**Find** $\mathcal{P}(A)\, and\, |\mathcal{P}(A)|$ for $A = \{0, \emptyset, \{\emptyset\}\}$

$\mathcal{P}$**(A)**=$\{\emptyset, \{0\}, \{\emptyset\}, \{\{\emptyset\}\}, \{0, \emptyset\}, \{0, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}, \{0, \emptyset, \{\emptyset\}\}\}$

$|\mathcal{P}(A)| = 8$ [correct ✓]

**Exercise.** 1.19

**Give** an example of a set S such that

**(a)** $S \subseteq \mathcal{P}(\mathbb{N})$

$S = \{\emptyset\}$ [correct ✓]

**(b)** $S \in \mathcal{P}(\mathbb{N})$

$S = \emptyset$ [correct ✓]

**(c)** $S \subseteq \mathcal{P}(\mathbb{N})$ and $|S| = 5$

$S = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$ [correct ✓]

**(d)** $S \in \mathcal{P}(\mathbb{N})$ and $|S| = 5$

$S = \{1, 2, 3, 4, 5\}$ [correct ✓]

**Exercise.** 1.21

**Three** subsets A, B, C of $\{1, 2, 3, 4, 5\}$ have the same cardinality. Furthermore,

**(a)** 1 belongs to A and B, but not to C

**(b)** 2 belongs to A and C, but not to B

**(c)** 3 belongs to A and exactly one of B and C

**(d)** 4 belongs to an even number of A, B and C

**(e)** 5 belongs to an odd number of A, B and C

**(f)** The sums of the elements in two of the sets A, B and C differ by 1

**What** is B?

$A = \{1, 2, 3\}$

$B = \{1, (3), 4\}$

$C = \{2, (3), 4\}$

**(d)** 4 belongs to an even number of A, B and C i.e. to none or two of A,B,C

**If** $4 \notin A, B, C$ , then $|B|$ or $|C| = 2$ , therefore 4 belongs to two of A,B,C

**If** $4 \in A$, then $|A| = 4$ and $|B|$ or $|C|$ =3

**Therefore** $4 \notin A$ , so $4 \in B, C$

**(e)** 5 belongs to an odd number of A, B and C

**If** $5 \in A, B, C$ then $|A| = 4$, while either $|B|$ or $|C|$ =3

**Therefore** 5 belongs to **one** of A,B,C and $5 \notin A$

**If** 3 and 5 are both in the same set B or C, then $|B|$ or $|C| = 4$ and the other will be 3

**Therefore** 3 and 5 must belong to different sets B and C

**If** $B = \{1, 3, 4\}$then $C = \{2, 4, 5\}$

**the** sum of the elements of $B = 8$

**the** sum of the elements of $A = 6$

**the** sum of the elements of $C = 11$ , which contradicts (f)

**Therefore**

$B = \{1, 4, 5\}$**,** sum of elements 10 [correct ✓]

$C = \{2, 4, 5\}$**,** sum of elements 11

$A = \{1, 2, 3\}$**,** sum of elements 6

## 1.3 Set Operations

**Exercise.** 1.23

**Give** examples of two sets A and B such that $|A - B| = |A \cap B| = |B - A| = 3$

$A = \{1, 2, 3, 4, 5, 6\}, B = \{4, 5, 6, 7, 8, 9\}$

$A - B = \{1, 2, 3\}, A \cap B = \{4, 5, 6\}, B - A = \{7, 8, 9\}$ [correct ✓]

**Exercise.** 1.24

**Give** examples of three sets A,B and C such that $B \neq C$ but $B - A = C - A$

$B = \{1, 2, 3\}, C = \{1, 2, 4\}, A = \{3, 4\}$

**Exercise.** 1.25

**Give** examples of three sets A, B and C such that

**(a)** $A \in B, A \subseteq C\, and\, B \nsubseteq C$

$A = \{1\}, B = \{\{1\}\}, C = \{1\}$

**(b)** $B \in A, B \subset C\, and\, A \cap C \neq \emptyset$

$A = \{\{1\}\}, B = \{1\}, C = \{1, \{1\}\}$

**(c)** $A \in B, B \subseteq C\, and\, A \nsubseteq C$

$A = \{1\}, B = \{\{1\}\}, C = \{\{1\}\}$ [correct ✓ (I think... I got different answers, probably because I didn't apply any condition that $A \neq B \neq C$ , the question does not specify such a condition)]

**Exercise.** 1.27

**Give** an example of a universal set U, two sets A and B and accompanying Venn diagram such that |A∩B|=|A-B|=|B-A|=|$\overline{A \cup B}$|=2

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$ [correct ✓]

**Exercise.** 1.29

**Let** $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$

**(a)** Determine which of the following are elements of A: $\emptyset, \{\emptyset\}$ [correct ✓]

**(b)** Determine $|A|$ . $|A| = 3$ [correct ✓]

**(c)** Determine which of the following are subsets of A: $\{\emptyset\}$,$\{\emptyset, \{\emptyset\}\}$ ["all of $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$are subsets of A"\

**(d)** $\emptyset \cap A = \emptyset$ [correct ✓]

**(e)** $\{\emptyset\} \cap A = \emptyset$ ["$\{\emptyset\}$"]

**(f)** $\{\emptyset, \{\emptyset\}\} \cap A = \{\emptyset, \{\emptyset\}\}$ [correct ✓]

**(g)** $\emptyset \cup A = A$ [correct ✓]

**(h)** $\{\emptyset\} \cup A = A$ [correct ✓]

**(i)** $\{\emptyset, \{\emptyset\}\} \cup A = A$ [correct ✓]

**Exercise.** 1.31

**Give** an example of four different sets A, B, C and D such that

**(1)** $A \cup B = \{1, 2\}$ and $C \cap D = \{2, 3\}$ and

**(2)** if B and C are interchanged, and ∪ and ∩ are interchanged, then we get the same result

**in** other words, $A \cap C = \{1, 2\}$ *and* $B \cup D = \{2, 3\}$

$A = \{1, 2\}, B = \{\}, C = \{1, 2, 3\}, D = \{2, 3\}$ [correct ✓ although their answer has $B = \{2\}$]

**Exercise.** 1.33

**Give** an example of two nonempty sets A and B such that $\{A \cup B, A \cap B, A - B, B - A\}$ is the power set of some set

$A = \{1\}, B = \{2\}$ [correct ✓]

**Exercise.** 1.35

**Give** examples of a universal set U and sets A, B and C such that each of the following sets contains exactly one element:

$A \cap B \cap C, (A \cap B) - C, (A \cap C) - B, (B \cap C) - A, A - (B \cup C), B - (A \cup C), C - (A \cup B), \overline{A \cup B \cup C}$

$A = \{1, 2, 3, 5\}, B = \{1, 2, 4, 6\}, C = \{1, 3, 4, 7\}, U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ [correct ✓]

## 1.4 Indexed Collections of Sets

**Exercise.** 1.37

**Let** $A = \{1, 2, 5\}, B = \{0, 2, 4\}, C = \{2, 3, 4\}\ and\, S = \{A, B, C\}$

**Determine** $\bigcup_{x \in S} X$ and $\bigcup_{x \in S} X$

$\bigcup_{x \in S} X = \{0, 1, 2, 3, 4, 5\}, \bigcap_{x \in S} X = \{2\}$

**Exercise.** 1.39

**Let** $A = \{a, b, \ldots, z\}$ be the set consisting of the letters of the alphabet. For $\alpha \in A$, let $A_\alpha$ consist of $\alpha$ and the two letters that follow it, where $A_y = \{y, z, a\}\ and\, A_z = \{z, a, b\}$. Find a set $S \subseteq A$ of smallest cardinality such that $\bigcup_{\alpha \in S} A_\alpha = A$

$S = \{a, d, g, j, m, p, s, v, y\}$ $|S| = 9$

**Explain** why your set S has the required properties

**There** are 26 letters in the alphabet, each index value e.g. a,d,g corresponds to a collection of three successive letters, therefore if each index letter is chosen to every 3rd letter, then 8 index letters will correspond to $|\bigcup_{\alpha \in S} A_\alpha| = 24$ , and 9 index letters will be needed so that all 26 letters of the english alphabet (all 26 members of the set A) are contained in $\bigcup_{\alpha \in S} A_\alpha$

**Exercise.** 1.41

**For** each of the following, find an indexed collection $\{A_n\}_{n \in \mathbb{N}}$ of distinct sets (that is, no two sets are equal) satisfying the given conditions.

**(a)** $\bigcap_{n=1}^{\infty} A_n = \{0\}\ and\, \bigcup_{n=1}^{\infty} A_n = [0, 1]$

$A_n = \left\{\frac{0}{n}, \frac{1}{n}, \ldots, \frac{n}{n}\right\}$ [INCORRECT]

$\{A_n\}_{n \in \mathbb{N}}$ ,**where** $A_n = \left\{x \in \mathbb{R} : 0 \leq x \leq \frac{1}{n}\right\} = \left[0, \frac{1}{n}\right]$

**[my** answer did not include the irrational numbers. I was trying to use the natural numbers for both the boundaries and for each value (therefore leaving out the irrational numbers), the given solution uses the Real numbers as the 'source' of the contents of A, using the Natural numbers only as the boundaries]

**(b)** $\bigcap_{n=1}^{\infty} A_n = \{-1, 0, 1\}$ *and* $\bigcup_{n=1}^{\infty} A_n = \mathbb{Z}$

$\{A_n\}_{n\in\mathbb{N}}$ *where* $A = \{a \in \mathbb{Z} : |a| \leq n\} = \{-n, -(n-1), \ldots, 0, \ldots, (n-1), n\}$

**[correct** ✓]

**Exercise.** 1.43

**For** $r \in \mathbb{R}^+$, let $A_r = \{x \in \mathbb{R} : |x| < r\}$ . Determine $\bigcup_{r\in\mathbb{R}^+} A_r$ and $\bigcap_{r\in\mathbb{R}^+} A_r$

$\bigcup_{r\in\mathbb{R}^+} A_r = \mathbb{R}$ [correct ✓]

$\bigcap_{r\in\mathbb{R}^+} A_r = \{0\}$ [correct ✓]

**Exercise.** 1.45

**For** $n \in \mathbb{N}$, let $A_n = \left(-\frac{1}{n}, 2 - \frac{1}{n}\right)$ . Determine $\bigcup_{n\in\mathbb{N}} A_n$ and $\bigcap_{n\in\mathbb{N}} A_n$

$\bigcup_{n\in\mathbb{N}} A_n = (-1, 2)$ [correct ✓]

$\bigcap_{n\in\mathbb{N}} A_n = [0, 1)$ ["[0,1]" - I don't understand how the righthand side of the interval can include 1 i.e. 1] when the 'lowest' value of $2 - \frac{1}{n}$ is 1, so the righthand side of the interval is 1) ?

## 1.5 Partitions of Sets

**Exercise.** 1.47

**Which** of the following are partitions of $A = \{1, 2, 3, 4, 5\}$?

**(b)** $S_2 = \{\{1, 2\}, \{3, 4, 5\}\}$ [correct ✓]

**Exercise.** 1.49

**Give** an example of a set A with $|A| = 4$ and two disjoint partitions $S_1$ and $\mathrm{S}_2$ of A with $|S_1| = |S_2| = 3$

$A = \{1, 2, 3, 4\}$ $S_1 = \{\{1\}, \{2\}, \{3, 4\}\}$ $S_2 = \{\{1, 2\}, \{3\}, \{4\}\}$ [correct ✓]

**Exercise.** 1.51

**Give** an example of a partition of $\mathbb{Q}$ into three subsets.

$\{\{x \in \mathbb{Q} : x < 0\}, \{0\}, \{x \in \mathbb{Q} : x > 0\}\}$ [correct ✓]

**Exercise.** 1.53

**Give** an example of a partition of $\mathbb{Z}$ into four subsets.

$\{\{4x : x \in \mathbb{Z}\}, \{4x + 1 : x \in \mathbb{Z}\}, \{4x + 2 : x \in \mathbb{Z}\}, \{4x + 3 : x \in \mathbb{Z}\}\}$ [correct ✓] (but their answer is different)

**Exercise.** 1.55

**A** set S is partitioned into two subsets S1 and S2. This produces a partition P1 of S where P1 = {S1, S2} and so |P1| = 2. One of the sets in P1 is then partitioned into two subsets, producing a partition P2 of S with |P2| = 3. A total of |P1| sets in P2 are partitioned into two subsets each, producing a partition P3 of S. Next, a total of |P2| sets in P3 are partitioned into two subsets each, producing a partition P4 of S. This is continued until a partition P6 of S is produced. What is |P6|?

**21** [correct ✓]

## 1.6 Cartesian Products of Sets

**Exercise.** 1.57

**Let** $A = \{x, y, z\}$and $B = \{x, y\}$ . Determine $A \times B$

$A \times B = \{\{x, x\}, \{x, y\}, \{y, x\} \{y, y\}, \{z, x\}, \{z, y\}\}$ [correct ✓]

**Exercise.** 1.59

**For** $A = \{a, b\}$, determine $A \times \mathcal{P}(A)$ .

$\mathcal{P}$**(A)**=$\{\emptyset, a, b, \{a, b\}\}$ [incorrect: should be $\mathcal{P}$(A)=$\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ , as each element is a SUBSET of A]

$A \times \mathcal{P}(A) = \{\{a, \emptyset\}, \{a, \{a\}\}, \{a, \{b\}\}, \{a, \{a, b\}\}, \{b, \emptyset\}, \{b, \{a\}\}, \{b, \{b\}\}, \{b, \{a, b\}\}\}$ [correct ✓] - corrected after feedback from first section

**Exercise.** 1.61

**For** $A = \{1, 2\}$ and $B = \{\emptyset\}$, determine $A \times B$ and $\mathcal{P}(A) \times \mathcal{P}(B)$ .

$A \times B = \{\{1, \emptyset\}, \{2, \emptyset\}\}$ $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ $\mathcal{P}(B) = \{\emptyset, \{\emptyset\}\}$

$\mathcal{P}(A) \times \mathcal{P}(B) = \{\{\emptyset, \emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\{1\}, \emptyset\}, \{\{1\}, \{\emptyset\}\}, \{\{2\}, \emptyset\}, \{\{2\}, \{\emptyset\}\}, \{\{1, 2\}, \emptyset\}, \{\{1, 2\}, \{\emptyset\}\}\}$ [correct ✓]

**Exercise.** 1.63

**List** the elements of the set $S = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : |x| + |y| = 3\}$

$S = \{\{\pm 3, 0\}, \{\pm 2, \pm 1\}, \{\pm 1, \pm 2\}, \{0, \pm 3\}\}$ [correct ✓]

**Exercise.** 1.65

**For** $A = \{x \in \mathbb{R} : |x - 1| \leq 2\}$ and $B = \{y \in \mathbb{R} : |y - 4| \leq 2\}$, give a geometric description of the points in the xy-plane belonging to $A \times B$.

**The** points belonging to $A \times B$ correspond to a square, with vertices at $(-1, 2), (-1, 6), (3, 2)\,(3, 6)$ .[correct ✓]