Exercise. 1.1 Which of the following are sets?

$(d)\,\{1,\{2\},3\}$ [correct ✓]

(e) $\{1,2,a,b\}$ [*correct*✓]

Exercise. 1.3 Determine the cardinality of each of the following sets:

(a) A=$\{1,2,3,4,5\}$

$|A|=5$ [correct ✓]

(b) B=$\{0,2,4,...,20\}$

$|B|$ = 11 [correct ✓]

(c) C = $\{25,26,27...,75\}$

$|C|$= 51 [correct ✓]

(d) D = $\{\{1,2\},\{1,2,3,4\}\}$

$|D|$= 2 [correct ✓]

(e) E = $\{\emptyset\}$

$|E|=1$ [correct ✓]

(f) $F=\{2,\{2,3,4\}\}$

$|F|=2$ [correct ✓]

Exercise. 1.5

(a) A=$\{-1,-2,-3,\ldots\}$

$A=\{x=-y,y\in\mathbb{N}\}$ [correct ✓]

(b) B=$\{-3,-2,\ldots,3\}$

$B=\{x\in\mathbb{Z}:-3\leq x\leq 3\}$ [correct ✓]

(c) $C=\{-2,-1,1,2\}$

$C=\{x\in Z:-2\leq x\leq 2,x\neq 0\}$ [correct ✓]

Exercise. 1.7

(a) $A=\{\ldots,-4,-1,2,5,8,\ldots\}$

$A=\{3x-1:x\in\mathbb{Z}\}$ [correct ✓]

(b) $B=\{\ldots,-10,-5,0,5,10,\ldots\}$

$B = \{5x : x \in \mathbb{Z}\}$ [correct ✓]

(c) $C = \{1, 8, 27, 64, 125, \ldots\}$

$C = \left\{x^3 : x \in \mathbb{N}\right\}$ [correct ✓]

Exercise. 1.9

For $A = \{2, 3, 5, 7, 8, 10, 13\}$, let $B = \{x \in A : x = y + z, y \in A, z \in A\}$ and $C = \{r \in B : (r + s) \in B$ for some s∈B$\}$. Determine C

$B = \{5, 7, 8, 10, 13\}$ [correct ✓]

$C = \{10, 13\}$ *[incorrect]* $C = \{5, 8\}$ *(I was looking for $r + s \in B$, should have been looking for r)*

Exercise. 1.11

Let (a, b) be an open interval of real numbers, and let $c \in (a, b)$. Describe an open interval I centered at c such that $I \subseteq (a, b)$.

Let $d, e \in \mathbb{R} : a < d \le c \le e < b$ and $c - d = e - c$. Then (c, e) describes an open interval I centered at c

[correct ✓]

Exercise. 1.13

For a universal set $U = \{1, 2, \ldots, 8\}$ and two sets $A = \{1, 3, 4, 7\}$and $B = \{4, 5, 8\}$draw a venn diagram that represents these sets (done on paper)

Exercise. 1.15

Find $\mathcal{P}(A)$ for $A = \{0, \{0\}\}$

$\mathcal{P}(A) = \{\emptyset, \{0\}, \{\{0\}\}, \{0, \{0\}\}\}$ [correct ✓]

Exercise. 1.17

Find $\mathcal{P}(A)\, and\, |\mathcal{P}(A)|$ for $A = \{0, \emptyset, \{\emptyset\}\}$

$\mathcal{P}$**(A)**$=\{\emptyset, \{0\}, \{\emptyset\}, \{\{\emptyset\}\}, \{0, \emptyset\}, \{0, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}, \{0, \emptyset, \{\emptyset\}\}\}$

$|\mathcal{P}(A)| = 8$ [correct ✓]

Exercise. 1.19

Give an example of a set S such that

(a) $S \subseteq \mathcal{P}(\mathbb{N})$

$S = \{\emptyset\}$ [correct ✓]

(b) $S \in \mathcal{P}(\mathbb{N})$

$S = \emptyset$ [correct ✓]

(c) $S \subseteq \mathcal{P}(\mathbb{N})$ and $|S| = 5$

$S = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$ [correct ✓]

(d) $S \in \mathcal{P}(\mathbb{N})$ and $|S| = 5$

$S = \{1, 2, 3, 4, 5\}$ [correct ✓]

Exercise. 1.21

Three subsets A, B, C of $\{1, 2, 3, 4, 5\}$ have the same cardinality. Furthermore,

(a) 1 belongs to A and B, but not to C

(b) 2 belongs to A and C, but not to B

(c) 3 belongs to A and exactly one of B and C

(d) 4 belongs to an even number of A, B and C

(e) 5 belongs to an odd number of A, B and C

(f) The sums of the elements in two of the sets A, B and C differ by 1

What is B?

$A = \{1, 2, 3\}$

$B = \{1, (3), 4\}$

$C = \{2, (3), 4\}$

(d) 4 belongs to an even number of A, B and C i.e. to none or two of A,B,C

If $4 \notin A, B, C$, then $|B|$ or $|C| = 2$, therefore 4 belongs to two of A,B,C

If $4 \in A$, then $|A| = 4$ and $|B|$ or $|C|$ =3

Therefore $4 \notin A$, so $4 \in B, C$

(e) 5 belongs to an odd number of A, B and C

If $5 \in A, B, C$ then $|A| = 4$, while either $|B|$ or $|C|$ =3

Therefore 5 belongs to **one** of A,B,C and $5 \notin A$

If 3 and 5 are both in the same set B or C, then $|B|$ or $|C| = 4$ and the other will be 3

Therefore 3 and 5 must belong to different sets B and C

If $B = \{1, 3, 4\}$then $C = \{2, 4, 5\}$

the sum of the elements of $B = 8$

the sum of the elements of $A = 6$

the sum of the elements of $C = 11$, which contradicts (f)

Therefore

$B = \{1, 4, 5\}$**,** sum of elements 10 [correct ✓]

$C = \{2, 4, 5\}$**,** sum of elements 11

$A = \{1, 2, 3\}$**,** sum of elements 6

Exercise. 1.23

Give examples of two sets A and B such that $|A - B| = |A \cap B| = |B - A| = 3$

$A = \{1, 2, 3, 4, 5, 6\}, B = \{4, 5, 6, 7, 8, 9\}$

$A - B = \{1, 2, 3\}, A \cap B = \{4, 5, 6\}, B - A = \{7, 8, 9\}$ [correct ✓]

Exercise. 1.24

Give examples of three sets A,B and C such that $B \neq C$ but $B - A = C - A$

$B = \{1, 2, 3\}, C = \{1, 2, 4\}, A = \{3, 4\}$

Exercise. 1.25

Give examples of three sets A, B and C such that

(a) $A \in B, A \subseteq C \, and \, B \nsubseteq C$

$A = \{1\}, B = \{\{1\}\}, C = \{1\}$

(b) $B \in A, B \subset C \, and \, A \cap C \neq \emptyset$

$A = \{\{1\}\}, B = \{1\}, C = \{1, \{1\}\}$

(c) $A \in B, B \subseteq C \, and \, A \nsubseteq C$

$A = \{1\}, B = \{\{1\}\}, C = \{\{1\}\}$ [correct ✓ (I think... I got different answers, probably because I didn't apply any condition that $A \neq B \neq C$, the question does not specify such a condition)]

Exercise. 1.27

Give an example of a universal set U, two sets A and B and accompanying Venn diagram such that |A∩B|=|A-B|=|B-A|=|$\overline{A \cup B}$|=2

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$ [correct ✓]

Exercise. 1.29

Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$

(a) Determine which of the following are elements of A: $\emptyset, \{\emptyset\}$ [correct ✓]

(b) Determine $|A|$. $|A| = 3$ [correct ✓]

(c) Determine which of the following are subsets of A: $\{\emptyset\}$,$\{\emptyset, \{\emptyset\}\}$ ["all of $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$are subsets of A"\

(d) $\emptyset \cap A = \emptyset$ [correct ✓]

(e) $\{\emptyset\} \cap A = \emptyset$ ["$\{\emptyset\}$"]

(f) $\{\emptyset, \{\emptyset\}\} \cap A = \{\emptyset, \{\emptyset\}\}$ [correct ✓]

(g) $\emptyset \cup A = A$ [correct ✓]

(h) $\{\emptyset\} \cup A = A$ [correct ✓]

(i) $\{\emptyset, \{\emptyset\}\} \cup A = A$ [correct ✓]

Exercise. 1.31

Give an example of four different sets A, B, C and D such that

(1) $A \cup B = \{1, 2\}$ and $C \cap D = \{2, 3\}$ and

(2) if B and C are interchanged, and $\cup$ and $\cap$ are interchanged, then we get the same result

in other words, $A \cap C = \{1, 2\}\ and\, B \cup D = \{2, 3\}$

$A = \{1, 2\}, B = \{\}, C = \{1, 2, 3\}, D = \{2, 3\}$ [correct ✓ although their answer has $B = \{2\}$]

Exercise. 1.33

Give an example of two nonempty sets A and B such that $\{A \cup B, A \cap B, A - B, B - A\}$ is the power set of some set

$A = \{1\}, B = \{2\}$ [correct ✓]

Exercise. 1.35

Give examples of a universal set U and sets A, B and C such that each of the following sets contains exactly one element:

$A \cap B \cap C, (A \cap B) - C, (A \cap C) - B, (B \cap C) - A, A - (B \cup C), B - (A \cup C), C - (A \cup B), \overline{A \cup B \cup C}$

$A = \{1, 2, 3, 5\}, B = \{1, 2, 4, 6\}, C = \{1, 3, 4, 7\}, U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ [correct ✓]