Exercise. 1.1 Which of the following are sets?

(d) $\{1, \{2\}, 3\}$ [correct ✓]

(e) $\{1, 2, a, b\}$ [$correct$✓]

Exercise. 1.3 Determine the cardinality of each of the following sets:

(a) A=$\{1, 2, 3, 4, 5\}$

$|A| = 5$ [correct ✓]

(b) B=$\{0, 2, 4, ..., 20\}$

$|B|$ $= 11$ [correct ✓]

(c) C $= \{25, 26, 27..., 75\}$

$|C|=$ 51 [correct ✓]

(d) D $= \{\{1, 2\}, \{1, 2, 3, 4\}\}$

$|D|=$ 2 [correct ✓]

(e) E $= \{\emptyset\}$

$|E| = 1$ [correct ✓]

(f) $F = \{2, \{2, 3, 4\}\}$

$|F| = 2$ [correct ✓]

Exercise. 1.5

(a) A=$\{-1, -2, -3, \ldots\}$

$A = \{x = -y, y \in \mathbb{N}\}$ [correct ✓]

(b) B=$\{-3, -2, \ldots, 3\}$

$B = \{x \in \mathbb{Z} : -3 \leq x \leq 3\}$ [correct ✓]

(c) $C = \{-2, -1, 1, 2\}$

$C = \{x \in Z : -2 \leq x \leq 2, x \neq 0\}$ [correct ✓]

Exercise. 1.7

(a) $A = \{\ldots, -4, -1, 2, 5, 8, \ldots\}$

$A = \{3x - 1 : x \in \mathbb{Z}\}$ [correct ✓]

(b) $B = \{\ldots, -10, -5, 0, 5, 10, \ldots\}$

$B = \{5x : x \in \mathbb{Z}\}$ [correct ✓]

(c) $C = \{1, 8, 27, 64, 125, \ldots\}$

$C = \left\{x^3 : x \in \mathbb{N}\right\}$ [correct ✓]

Exercise. 1.9

For $A = \{2, 3, 5, 7, 8, 10, 13\}$, let $B = \{x \in A : x = y + z, y \in A, z \in A\}$ and $C = \{r \in B : (r + s) \in B$ for some s∈B$\}$. Determine C

$B = \{5, 7, 8, 10, 13\}$ [correct ✓]

$C = \{10, 13\}$ *[incorrect]* $C = \{5, 8\}$ *(I was looking for $r + s \in B$, should have been looking for r)*

Exercise. 1.11

Let (a, b) be an open interval of real numbers, and let $c \in (a, b)$. Describe an open interval I centered at c such that $I \subseteq (a, b)$.

Let $d, e \in \mathbb{R} : a < d \leq c \leq e < b$ and $c - d = e - c$. Then (c, e) describes an open interval I centered at c

[correct ✓]

Exercise. 1.13

For a universal set $U = \{1, 2, \ldots, 8\}$ and two sets $A = \{1, 3, 4, 7\}$and $B = \{4, 5, 8\}$draw a venn diagram that represents these sets (done on paper)

Exercise. 1.15

Find $\mathcal{P}(A)$ for $A = \{0, \{0\}\}$

$\mathcal{P}(A) = \{\emptyset, \{0\}, \{\{0\}\}, \{0, \{0\}\}\}$ [correct ✓]

Exercise. 1.17

Find $\mathcal{P}(A)\, and\, |\mathcal{P}(A)|$ for $A = \{0, \emptyset, \{\emptyset\}\}$

$\mathcal{P}$**(A)**$=\{\emptyset, \{0\}, \{\emptyset\}, \{\{\emptyset\}\}, \{0, \emptyset\}, \{0, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}, \{0, \emptyset, \{\emptyset\}\}\}$

$|\mathcal{P}(A)| = 8$ [correct ✓]

Exercise. 1.19

Give an example of a set S such that

(a) $S \subseteq \mathcal{P}(\mathbb{N})$

$S = \{\emptyset\}$ [correct ✓]

(b) $S \in \mathcal{P}(\mathbb{N})$

$S = \emptyset$ [correct ✓]

(c) $S \subseteq \mathcal{P}(\mathbb{N})$ and $|S| = 5$

$S = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$ [correct ✓]

(d) $S \in \mathcal{P}(\mathbb{N})$ and $|S| = 5$

$S = \{1, 2, 3, 4, 5\}$ [correct ✓]

Exercise. 1.21

Three subsets A, B, C of $\{1, 2, 3, 4, 5\}$ have the same cardinality. Furthermore,

(a) 1 belongs to A and B, but not to C

(b) 2 belongs to A and C, but not to B

(c) 3 belongs to A and exactly one of B and C

(d) 4 belongs to an even number of A, B and C

(e) 5 belongs to an odd number of A, B and C

(f) The sums of the elements in two of the sets A, B and C differ by 1

What is B?

$A = \{1, 2, 3\}$

$B = \{1, (3), 4\}$

$C = \{2, (3), 4\}$

(d) 4 belongs to an even number of A, B and C i.e. to none or two of A,B,C

If $4 \notin A, B, C$, then $|B|$ or $|C| = 2$, therefore 4 belongs to two of A,B,C

If $4 \in A$, then $|A| = 4$ and $|B|$ or $|C|$ =3

Therefore $4 \notin A$, so $4 \in B, C$

(e) 5 belongs to an odd number of A, B and C

If $5 \in A, B, C$ then $|A| = 4$, while either $|B|$ or $|C|$ =3

Therefore 5 belongs to **one** of A,B,C and $5 \notin A$

If 3 and 5 are both in the same set B or C, then $|B|$ or $|C| = 4$ and the other will be 3

Therefore 3 and 5 must belong to different sets B and C

If $B = \{1, 3, 4\}$then $C = \{2, 4, 5\}$

the sum of the elements of $B = 8$

the sum of the elements of $A = 6$

the sum of the elements of $C = 11$, which contradicts (f)

Therefore

$B = \{1, 4, 5\}$**,** sum of elements 10 [correct ✓]

$C = \{2, 4, 5\}$**,** sum of elements 11

$A = \{1, 2, 3\}$**,** sum of elements 6